Assignment

Differentiability

Basic Level

1. If $f(x)=\begin{cases}1 & , \ x<0 \\ 1+\sin x & , \ 0\le x\le \pi/2\end{cases}$ then at $x=0$, the value of $f'(x)$ is equal to **[Rajasthan PET 1990]**

(a) 1 (b) 0 (c) ∞ (d) Derivative does not exist

2. If $f(x)=|x-3|$, then $f'(3)$ equals

(a) 0 (b) 1 (c) –1 (d) Does not exist

3. If $f(x)=\begin{cases}x\sin(1/x), & x\ne 0 \\ 0 \quad , & x=0\end{cases}$ then at x = 0, the function is

(a) Discontinuous (b) Continuous but not differentiable

(c) Both continuous and differentiable (d) None of these

4. If $f(x)=|x-3|$, then f is **[Rajasthan PET 1994]**

(a) Discontinuous at x = 2 (b) Not differentiable at x = 2

(c) Differentiable at x = 3 (d) Continuous but not differentiable at x = 3

5. If $f(x)=\begin{cases}x+1 & , \text{ when } x<2 \\ 2x-1 & , \text{ when } x\ge 2\end{cases}$, then $f'(x)$ at $x=2$ equals **[Rajasthan PET 1992; Karnataka CET 2002]**

(a) 0 (b) 1 (c) 2 (d) Does not exist

6. If $f(x)=\begin{cases}x^2\sin(1/x), & \text{when } x\ne 0 \\ 0 \quad , & \text{when } x=0\end{cases}$, then at $x=0$, value of $f'(x)$ equals **[Rajasthan PET 1991]**

(a) 1 (b) 0 (c) ∞ (d) Does not exist

7. If $\lim\limits_{x\to c}\dfrac{f(x)-f(c)}{x-c}$ exists finitely, then

(a) $\lim\limits_{x\to c}f(x)=f(c)$ (b) $\lim\limits_{x\to c}f'(x)=f'(c)$ (c) $\lim\limits_{x\to c}f(x)$ does not exist (d) $\lim\limits_{x\to c}f(x)$ may or may not exist

8. If $f(x)=\dfrac{|x-1|}{x-1}$, $x\ne 1$ and $f(1)=1$. Then which of the following statement is true

(a) Continuous for $x\le 1$ (b) Discontinuous at x = 1 (c) Differentiable at x = 1 (d) Discontinuous for x >1

9. Let $f(xy)=f(x)f(y)$ for all $x,y\in R$. If $f'(1)=2$ and $f(4)=4$, then $f'(4)$ equal to

(a) 4 (b) 1 (c) $\dfrac{1}{2}$ (d) 2

10. The derivative of $f(x)=|x|$ at $x=0$ is

(a) 1 (b) 0 (c) –1 (d) Does not exist

11. If $f(x)=\begin{cases} e^x+ax, & x<0 \\ b(x-1)^2, & x\ge 0\end{cases}$ is differentiable at $x=0$ then (a,b) is

(a) $(-3,-1)$ (b) $(-3,1)$ (c) $(3,1)$ (d) $(3,-1)$

12. At the point x = 1, the function $f(x)=\begin{cases} x^3-1; & 1<x<\infty \\ x-1; & -\infty<x\le 1\end{cases}$

(a) Continuous and differentiable (b) Continuous and not differentiable

(c) Discontinuous and differentiable (d) Discontinuous and not differentiable

13. The function $|x^3|$ is

(a) Differentiable everywhere (b) Continuous but not differentiable at $x=0$

(c) Not a continuous function (d) A function with range [0, ∞]

14. For the function $f(x)=|x^2-5x+6|$ the derivative from the right $f'(2+)$; and the derivative from left $f'(2-)$ are respectively

(a) 1, – 1 (b) –1, 1 (c) 0, 2 (d) None of these

15. Let $f(x)$ be an even function. Then $f'(x)$

(a) Is an even function (b) Is an odd function (c) May be even or odd (d) None of these

16. Let $f(x)$ be an odd function. Then $f'(x)$

(a) Is an even function (b) Is an odd function (c) May be even or odd (d) None of these

17. Let $g(x)$ be the inverse of the function $f(x)$ and $f'(x)=\frac{1}{1+x^3}$ .Then $g'(x)$ is equal to

(a) $\frac{1}{1+(g(x))^3}$ (b) $\frac{1}{1+(f(x))^3}$ (c) $1+(g(x))^3$ (d) $1+(f(x))^3$

18. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x=c$, then $g'(f(c))$ equals

(a) $f'(c)$ (b) $\frac{1}{f'(c)}$ (c) $f(c)$ (d) None of these

19. If $f(x)=\begin{cases} x+2, & -1<x<3 \\ 5, & x=3 \\ 8-x, & x>3\end{cases}$ then at $x=3, f'(x)=$ **[MP PET 2001]**

(a) 1 (b) –1 (c) 0 (d) Does not exist

20. If $f(x)=(x-x_0)\,g(x)$, where $g(x)$ is continous at x_0, then $f'(x_0)$ is equal to

(a) 0 (b) x_0 (c) $g(x_0)$ (d) None of these

21. Function $f(x)=|x|+|x-1|$ is not differentiable at **[Rajasthan PET 1996]**

(a) $x=1,-1$ (b) $x=0,-1$ (c) $x=0,1$ (d) $x=1,2$

22. If $f(x)=\begin{cases} e^x; & x\le 0 \\ |1-x|; & x>0\end{cases}$, then **[Roorkee 1995]**

(a) $f(x)$ is differentiable at $x=0$ (b) $f(x)$ is continuous at $x=0$

(c) $f(x)$ is differentiable at $x=1$ (d) $f(x)$ is continuous at $x=1$

23. The function which is continuous for all real values of x and differentiable at $x=0$, is

(a) $|x|$ (b) $\log x$ (c) $\sin x$ (d) $x^{1/2}$

Advance Level

24. The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval $(0, 2)$ is

[UP SEAT 1995]

(a) 1 (b) 2 (c) 3 (d) 4

25. If $f(x) = \begin{cases} ax^2 + b, & b \neq 0,\ x \leq 1 \\ bx^2 + ax + c, & x > 1 \end{cases}$.Then $f(x)$ is continuous and differentiable at $x = 1$ if

(a) $c = 0, a = 2b$ (b) $a = b, c \in R$ (c) $a = b, c = 0$ (d) $a = b, c \neq 0$

26. If $f(x) = \begin{cases} ax^2 - b, & |x| < 1 \\ \frac{1}{|x|}, & |x| \geq 1 \end{cases}$ is differentiable at $x = 1$, then

(a) $a = \frac{1}{2}, b = -\frac{1}{2}$ (b) $a = -\frac{1}{2}, b = -\frac{3}{2}$ (c) $a = b = \frac{1}{2}$ (d) $a = b = -\frac{1}{2}$

27. The set of points where the function $f(x) = |x - 1| e^x$ is differentiable is

(a) R (b) $R - \{1\}$ (c) $R - \{-1\}$ (d) $R - \{0\}$

28. Let $f(x)$ be defined on R such that $f(1) = 2, f(2) = 8$ and $f(u + v) = f(u) + kuv - 2v^2$ for all $u, v \in R$ and $u \neq v$ (k is a fixed constant). Then

(a) $f'(x) = 8x$ (b) $f(x) = 8x$ (c) $f'(x) = x$ (d) None of these

29. Let $f : R \to R$ be a function defined by $f(x) = \max\ \{x, x^3\}$. The set of all points where $f(x)$ is not differentiable is

[IIT Screening 2001]

(a) $\{-1, 1\}$ (b) $\{-1, 0\}$ (c) $\{0, 1\}$ (d) $\{-1, 0, 1\}$

30. Let $f(x) = \begin{cases} 0, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ then for all values of x **[MP PET 2002]**

(a) f is continuous but not differentiable (b) f is differentiable but not continuous
(c) f' is continuous but not differentiable (d) f' is continuous and differentiable

31. If $u(x) = \sin x$, $v(x) = \operatorname{sgn}(x) = \begin{cases} 1 \text{ for } x > 0 \\ 0 \text{ for } x = 0 \\ -1 \text{ for } x < 0 \end{cases}$ then $u(x).v(x)$ has a derivative at $x = 1$ is

(a) cos 1 (b) sin 1 (c) Not continuous at $x = 1$ (d) None of these

32. The coefficient a and b that make the function $f(x) = \begin{cases} \frac{1}{|x|} & \text{for } |x| \geq 1 \\ ax^2 + b & \text{for } |x| < 1 \end{cases}$ continuous and differentiable at any point are given by

(a) $a = -1/2, b = 3/2$ (b) $a = 1/2, b = -3/2$ (c) $a = 1, b = -1$ (d) None of these

33. If $f(x) = \int_{-1}^{x} |t|\ dt$, $x \geq -1$, then **[UPSEAT 1994]**

(a) f and f' are continuous for $x + 1 > 0$ (b) f is continuous but f' is not for $x + 1 > 0$
(c) f and f' are continuous at $x = 0$ (d) f is continuous at $x = 0$ but f' is not so

34. Let $f(x) = \begin{cases} x^n \sin\frac{1}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$, then $f(x)$ is continuous but not differentiable at $x = 0$ if

(a) $n \in (0, 1]$ (b) $n \in [1, \infty)$ (c) $n \in (-\infty, \infty)$ (d) $n = 0$

35. Which of the following is differentiable at $x = 0$ **[IIT Screening 2001]**

(a) $\cos(|x|) + |x|$ (b) $\cos(|x|) - |x|$ (c) $\sin(|x|) + |x|$ (d) $\sin(|x|) - |x|$

36. If $x + 4|y| = 6y$, then y as a function of x is

(a) Continuous at $x=0$ (b) Derivable at $x=0$ (c) $\frac{dy}{dx}=\frac{1}{2}$ for all x (d) None of these

37. The set of point where the function $f(x)=x|x|$ is differentiable is

(a) $(-\infty,\infty)$ (b) $(-\infty,0)\cup(0,\infty)$ (c) $(0,\infty)$ (d) $[0,\infty)$

38. If $f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ then $f(x)$ is differentiable on

(a) [−1, 1] (b) R− $\{-1,1\}$ (c) R − (−1, 1) (d) None of these

39. Let $f(x)=|x|$ and $g(x)=|x^3|$, then

(a) $f(x)$ and $g(x)$ both are continuous at $x=0$ (b) $f(x)$ and $g(x)$ both are differentiable at $x=0$

(c) $f(x)$ is differentiable but $g(x)$ is not differentiable at $x=0$ (d) $f(x)$ and $g(x)$ both are not differentiable at $x=0$

40. The function $f(x)=\sin^{-1}(\cos x)$ is

(a) Discontinuous at $x=0$ (b) Continuous at $x=0$ (c) Differentiable at $x=0$ (d)

41. Let $f(x)=(x+|x|)|x|$. Then for all x

(a) f is continuous (b) f is differentiable for some x (c) f' is continuous (d)

42. The set of all those points, where the function $f(x)=\frac{x}{1+|x|}$ is differentiable, is

(a) $(-\infty,\infty)$ (b) $[0,\infty]$ (c) $(-\infty,0)\cup(0,\infty)$ (d) $(0,\infty)$

43. $f(x)$ and $g(x)$ are two differentiable function on $[0,2]$ such that $f''(x)-g''(x)=0, f'(1)=2, g'(1)=4, f(2)=3, g(2)=9$, then $f(x)-g(x)$ at $x=\frac{3}{2}$ is

(a) 0 (b) 2 (c) 10 (d) -5

44. The set of points of differentiability of the $f(x)=\begin{cases}\frac{\sqrt{x+1}-1}{\sqrt{x}}, & \text{for } x\neq 0\\ 0, & \text{for } x=0\end{cases}$ is

(a) R (b) $[0,\infty)$ (c) $(0,\infty)$ (d) $R-\{0\}$

45. If $f(x)=a|\sin x|+be^{|x|}+c|x|^3$ and if $f(x)$ is differentiable at $x=0$, then

(a) $a=b=c=0$ (b) $a=0,\ b=0,\ c\in R$ (c) $b=c=0,\ a\in R$ (d) $c=0,\ a=0,\ b\in R$

46. If $f(x)=\begin{cases}\frac{x-1}{2x^2-7x+5}, & x\neq 1\\ -\frac{1}{3}, & x=1\end{cases}$, then $f'(1)$ equals **[IIT 1979]**

(a) $\frac{2}{9}$ (b) $\frac{-2}{9}$ (c) 0 (d) Does not exist

47. Function $f(x)=1+|\sin x|$ is

(a) Continuous no where (b) Differentiable no where (c) Every where continuous (d)

48. Function $f(x)=\begin{cases}x^2, & x\leq 0\\ 1, & 0<x\leq 1\\ 1/x, & x>1\end{cases}$ is

(a) Differentiable at $x=0,1$ (b) Differentiable only at $x=0$ (c) Differentiable at only $x=1$ (d) Not differentiable at $x=0, 1$

49. Function $f(x)=\begin{cases}-x, & \text{if } x<0\\ x^2, & \text{if } 0\leq x\leq 1\\ x^3-x+1, & \text{if } x>1\end{cases}$, is differentiable at

(a) $x=0$ but not at $x=1$ (b) $x=1$ but not at $x=0$ (c) $x=0$ and $x=1$ (d) Neither $x=0$ nor $x=1$

50. If $g(x) = x\,f(x)$ where $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ then at $x = 0$

(a) g is differentiable but g' is discontinuous function (b) Both f and g are differentiable

(c) g is differentiable and g' is continuous function (d) None of these

51. The set of points where $f(x) = x\,|\,x\,|$ is differentiable two times is

(a) R_0 (b) R_+ (c) R (d) None of these

52. If $f(x) = \begin{cases} \frac{x(3e^{1/x} + 4)}{2 - e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then **[Roorkee 1995]**

(a) $\lim_{x \to 0} f(x) = 1$ (b) $f(x)$ is continuous at $x = 0$ (c) $f(x)$ is differentiable at $x = 0$

(d) $f'(0+0) = 3$

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Answer Sheet

Assignment (Basic and Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d	d	b	d	d	b	a	b	d	d	b	b	a,d	a	b	a	c	b	d	c
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
c	b,d	c	c	a	b	b	d	d	c	a	a	a	a	d	a	a	b	a	b
41	42	43	44	45	46	47	48	49	50	51	52								
a,c	a	d	c	b	b	d	b	b	b	a	b								