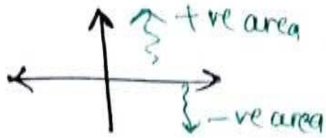
# Signals and system

- Area $[x(t)] = \int_{-\infty}^{\infty} x(t)\,dt$




Area $[x(n)] = \sum_{n=-\infty}^{\infty} x[n]$

Total No. of samples $= N_2 - N_1 + 1$

- Sampler

Something left.

- $t = nT_s = \dfrac{n}{f_s}$

- If time shifting is performed after time inversion then the nature of shift must change from delay to advance.

- Concept:

$$-Ax[-Kt \pm t_0]$$

Amplitude Scaling
- Area × (A)
- TW = same

Time inversion
- No change in Area & TW

Time scaling
- $\dfrac{\text{Area}}{K}$, $\dfrac{TW}{K}$

Time shifting
- No change in Ar & TW

Amplitude Inversion
- Area = -ve
- TW = same

- New Area $= \left(\dfrac{-A}{K}\right)$ Area

New TW $= \dfrac{\text{Old TW}}{K}$

- Interpolation: $x\left[\dfrac{n}{a}\right]$
  - Zero stuffing

- decimation: $x[an]$

start from ↑

$x[2n] = \{ \cancel{1}, \overset{0}{2}, \cancel{3}, 4 \}$

⊙ Known interval = new bracket form

$$-2t+3 = -3$$
$$-2t = -6$$
$$t = 3$$

** Properties of $u(t)$

⊙ $\boxed{u[K(t \pm t_0)] = u(t \pm t_0)}$

⊙ $u(t - \text{const.}) \rightarrow$ Amp = 1 from $t$ = const. to $+\infty$

$u(\text{const.} - t) \rightarrow$ Amp = 1 from $t$ = const. to $-\infty$

⊙ $u(\pm Kt \pm t_0)$

+ve sign with $t$ — Amp = 1 from $t$ = const. to $(\infty)$

−ve sign with $t$ — Amp = 1 from $t$ = const. to $(-\infty)$

⊙ $u(t) + u(-t) = u(t+t_0) + u(-(t+t_0)) = 1$

something left.

<i> Convolution Property

$$x(t) \circledast \delta'(t \pm a) = x'(t \pm a)$$

<ii> Sampling Property

$$f(t) \cdot \delta'(t \pm a) = f(\mp a)\,\delta'(t \pm a) - f'(\mp a)\,\delta(t \pm a)$$

Std. Results

- $t\,\delta'(t) = -\delta(t)$

  $t^2\,\delta'(t) = 0$

  $$\boxed{t^n\,\delta^n(t) = (-1)^n\, n!\,\delta(t)}$$

  Note $n$ on both $t^?$ & $n$th derivative of $\delta$

<iii> Generalised derivative

- $$\int_{-\infty}^{\infty} f(t)\,\delta'(t \pm a)\,dt = -f'(\mp a)$$
- $$\int_{-\infty}^{\infty} f(t)\,\delta^n(t \pm a)\,dt = (-1)^n f^n(\mp a)$$

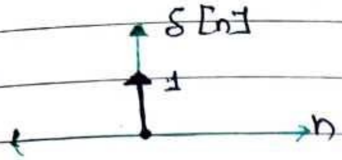
<iv> Time Scaling

$$\boxed{\delta(Kt) = \frac{1}{|K|}\,\delta(t)} \xrightarrow{\text{differentiating}} K\delta'(Kt) = \frac{1}{|K|}\,\delta'(t)$$

$$\boxed{\delta'(Kt) = \frac{1}{K|K|}\,\delta'(t)}$$

Same results for $(t \pm t_0)$ in place of $t$

# Impulse sequence or sample sequence




Amp = Area = 1

$$\delta(n) = \begin{cases} 1 & n=0 \\ 0 & n\neq 0 \end{cases}$$

**Properties:**

(i) Even signal $\delta[n] = \delta[-n]$

(ii) $\delta[Kn] = \delta[n]$

(iii) $x[n] \otimes \delta[n \pm a] = x[n \pm a]$    $a$ = integer

(iv) $f[n] \cdot \delta[n \pm a] = f(\mp a)\, \delta(n \pm a)$

(v) $\sum_{n=-\infty}^{\infty} f[n]\, \delta[n \pm a] = f[-a]$

Maybe something left → check it   why blank pages are there in Notes?

**Ramp waveform**

(i) LHS to RHS    (ii) w.r.t. currentline    ↑ : use +ve sign  ↓ : use −ve sign

(iii) Amp change = upper − lower    (iv) Time change = RHS − LHS

(v) Slope = $\frac{\text{Amp. change}}{\text{Time change}}$    (vi) 1 slope = 2 eq$^n$

(vii) Main slope and it's pair use same slope values and they have opposite signs.

# Ramp sequence :

$$r[n] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

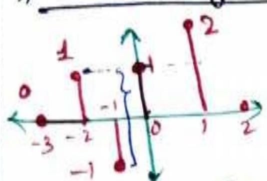

Relation b/w Ramp & step sequence:

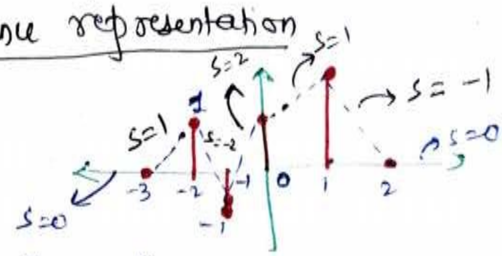
(i) $r[n] = n\,u[n]$

(ii) $u[n] = r[n+1] - r[n]$

$u[n \pm n_0] = r[n \pm n_0 + 1] - r[n \pm n_0]$

* Trick for step & ramp sequence representation




$x[n] = u[n+2] - 2u[n+1] + 2u[n] + u[n-1] - 2u[n-2]$

* dont forget

$x[n] = \sum [\text{slope change at } n_0]\, r(n-n_0)$

$= (1-0)\,r[n+3] + (-2-1)\,r[n+2] + [2-(-2)]\,r[n+1] + (1-2)\,r[n] +$

$= r[n+3] - 3r[n+2] + 4r[n+1] - r[n] - 2r[n-1] + 1\,r[n-2]$

*

## Classification of signal :

(1) ⟨i⟩ Causal

$x(t) = 0$ for $t < 0$

$x[n] = 0$ for $n < 0$

$x(t) / x[n] \neq 0$ for $(t, n \geq 0)$

⟨ii⟩ Anticausal

$x(t) = 0$ for $t > 0$

$x[n] = 0$ for $n > 0$

$x(t, n) \neq 0$ $(n, t \leq 0)$

(iii) Non Causal signal

exists for Both +ve & -ve time values.

Note: Causal & RH signal are not same.

eg :

signal is right sided but not Causal

- Causal $\xleftrightarrow{\text{Time Inversion}}$ Anticausal

NC × C ⟶ C or zero signal

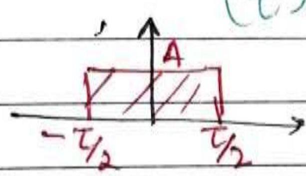
NC × AC ⟶ AC or zero signal

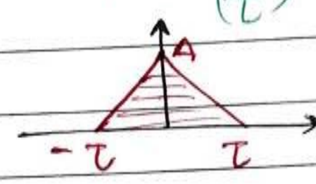
C + AC ⟶ NC

C × AC ⟶ $\begin{cases} A\,\delta[n] & n=0 \\ 0 & n \neq 0 \end{cases}$

(2.) ⟨i⟩ Even signal $x(t) = x(-t)$ ; $x[n] = x[-n]$

- Symmetrical about Y axis
- $A\,\text{rect}\left(\frac{t}{\tau}\right)$ , $A\,\text{tri}\left(\frac{t}{\tau}\right)$



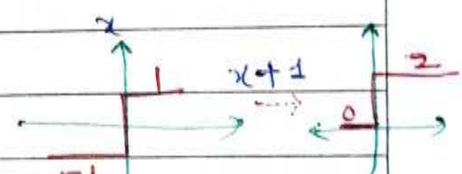




⟨ii⟩ odd signal $x(-t) = -x(t)$ , $x[-n] = -x[n]$

- Anti symmetrical about Y axis
- opposite quadrants are same
- Area or Avg/mean/dc value $= 0$

- Even ± odd = NENO
- Constant + Even = Even

  constant + odd = NENO ⟶ eg:




*** Energy of a signal $= \int_{-\infty}^{\infty} x^2(t)\,dt$

$$\int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} x_e^2(t)\,dt + \int_{-\infty}^{\infty} x_o^2(t)\,dt$$

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]\,dt$$

= Energy(E) + Energy [odd]

⊙ $\frac{d}{dt}[\text{odd}] = \text{even}$ ⊙ $\frac{d}{dt}[\text{even}] = \text{odd}$

$\int[\text{odd}]dt = \text{even}$ $\int \text{even} = \text{odd} + \text{constant}$

⊙ $x_e(t) = \frac{x(t) + x(-t)}{2}$ ⊙ $x_o(t) = \frac{x(t) - x(-t)}{2}$

→ ~~Doublet~~ ** $\boxed{u(t) = \frac{1 + \text{sgn}t}{2}}$

Something left

<i> $\delta[t^2 - a^2] = \frac{\delta(t-a) + \delta(t+a)}{2a}$

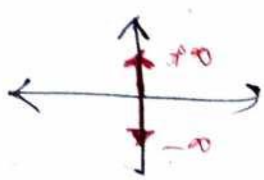
Doublet function:

$\delta'(t) = \frac{d}{dt}\delta(t)$ ⊙ Area of $\delta'(t) = 0$

$\int_{-\infty}^{\infty} \delta'(t)\,dt = 0$

⊙ odd signal

$\delta'(t) = \begin{cases} +\infty \text{ and } -\infty & \text{at } t=0 \\ 0 & \text{at } t \neq 0 \end{cases}$



# Conjugate Symmetric & conjugate Antisymmetric signal

1) $x_{cs}(t) = x^*_{cs}(-t)$ $\qquad$ $x_{CAS}(t) = -x^*_{CAS}(-t)$

$x_{cs}[n] = x^*_{cs}[-n]$ $\qquad$ $x_{CAS}[n] = -x^*_{CAS}[-n]$

2) $x_{cs} = \dfrac{x(t) + x^*(-t)}{2}$ $\qquad$ $x_{CAS} = \dfrac{x(t) - x^*(-t)}{2}$

3) Conjugate Symmetric :→ Real → Even
Imaginary → odd

Conjugate Antisymmetric :→ Real → odd
Imaginary → even

3.7 Periodic & Non periodic signals.

# Continuous time

$x(t \pm T_0) = x(t)$ ; $T_0 = \dfrac{2\pi}{\omega_0} = \dfrac{1}{f_0}$

Note :→ Only time scaling affects fundamental time period

$x(t) \rightarrow FTP = T_0$

$x(Kt) \rightarrow FTP = \dfrac{T_0}{|K|}$

1) for (Sub$^n$ or add$^n$) of 2 or more periodic signals with F.T.P. $T_1$, $T_2$ & $T_3$

Resultant = periodic if $T_1 : T_2 : T_3$ is rational

F.T.P. {resultant} = LCM $[T_1, T_2, T_3]$

freq. {resultant} = HCF $(f_1, f_2, f_3)$

(•) $$\text{LCM}\left[\frac{N_1}{D_1}, \frac{N_2}{D_2}, \frac{N_3}{D_3}\right] = \frac{\text{LCM}[N_1, N_2, N_3]}{\text{HCF}[D_1, D_2, D_3]}$$

$$\text{HCF}\left[\frac{N_1}{D_1}, \frac{N_2}{D_2}, \frac{N_3}{D_3}\right] = \frac{\text{HCF}[N_1, N_2, N_3]}{\text{LCM}[D_1, D_2, D_3]}$$

(•) Note always simplify $\frac{N_1}{D_1}$, they should be co-prime numbers

eg: $$\underbrace{\text{HCF}\left[\frac{12\pi}{10}, 2\pi, \frac{28}{10}\pi\right] = \frac{\text{HCF}[12\pi, 2\pi, 28\pi]}{\text{LCM}[10, 1, 10]} = \frac{2\pi}{10} = \frac{\pi}{5}}_{\text{WRONG}}$$

$$= \text{HCF}\left[\frac{6\pi}{5}, 2\pi, \frac{14}{5}\pi\right] = \frac{\text{HCF}[6\pi, 2\pi, 14\pi]}{\text{Lcm}[5, 1, 5]} = \frac{2\pi}{5}$$ correct

# Discrete time Periodic

(i) $x[n \pm N_0] = x[n]$

(ii) $\frac{m}{N_0} = \frac{\omega_0}{2\pi} = \frac{p}{q}$ = Ratio of integers

$N_0$ = F.T.P. = Integer (always) | $m = p$, $N_0 = q$
$m$ = order of harmonics | If periodic

# Avg value of continuous time signals

⟨i⟩ $x(t)$ = periodic

$$\boxed{\text{Avg value} = \frac{1}{T_0}\int_{T_0} x(t)\,dt}$$

⟨ii⟩ $x(t)$ = Non periodic

$$\boxed{\text{Avg. value} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt}$$

** Avg. value of finite duration signal is always zero.

$\because \frac{\text{Area}}{\infty} = \frac{\text{finite}}{\infty} = 0$

(i) $Avg[-x(t)] = -Avg[x(t)]$
$Avg[Kx(t)] = K\, Avg[x(t)]$
$Avg[\alpha + x(t)] = \alpha + Avg[x(t)]$

## Energy and Power Signals

Energy signals : (i) $E =$ finite , $P = 0$
(ii) Stable signal mostly
(iii) Absolutely integrable $\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$

(i) $E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$ = Area under $|x(t)|^2$ curve

$= \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2\, dt$

(ii) Results :
(1) $E[x(t)] = E[x_e(t)] + E[x_o(t)]$
(2) If $x(t)$ = purely even or purely odd
$E[x(t)] = 2\int_0^{\infty} x^2(t)\, dt$

Power signal : (i) $E = \infty$ , $P =$ finite
(ii) mostly periodic or $\infty$ duratn signal

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt = \lim_{T\to\infty} \left(\frac{1}{T}\right) E$$

$$P = \frac{1}{T_0} \int_{T_0} x^2(t)\, dt$$ for periodic



* $RMS = \sqrt{P}$

# Properties of Energy Signal

<i> Time inversion / Time shifting / Amplitude inversion has no effect on energy

<ii> Time scaling

$$x(Kt) \longrightarrow \frac{Area}{|K|}, \frac{E}{|K|}$$

<iii> Amplitude scaling

$$C\,x(t) \longrightarrow C(Area), \; |C|^2 E$$

$$Amp\, x[\alpha(t) \pm t_0] \longrightarrow (Area)\cdot\frac{Amp}{|\alpha|}, \; Energy \times \left(\frac{Amp^2}{|\alpha|}\right)$$

Note :- $\delta(t)$ is neither energy nor power.
$\because E = \infty, \; P = \frac{\infty}{\infty}$ (Indeterminate form

* Many thing left in blank pages.

| Signals | Power | RMS |
|---|---|---|
| $A\cos(\omega t \pm \phi)$<br>$A\sin(\omega t \pm \phi)$ | $P = \frac{A^2}{2}$ | $RMS = \frac{A}{\sqrt{2}}$ |
| $A\cos\omega t\, u(t)$<br>$A\sin\omega t\, u(t)$ | $P = \frac{A^2}{4}$ | $RMS = \frac{A}{2}$ |

$$P[x_1(t) \pm x_2(t)] = P[x_1(t)] + P[x_2(t)] \pm \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x_1(t)x_2(t)\,dt$$

$$P[DC] = P[x(t) = A] = A^2$$

$$P[DC + \text{sinusoid}] = P[DC] + P[\text{sinusoid}] = A^2 + \frac{B^2}{2}$$

To remember $RMS = \sqrt{A_{DC}^2 + \frac{A_{AC}^2}{2}}$ , $P = (RMS)^2$

$A\cos(\omega_1 t + \phi_1) + B\cos(\omega_2 t + \phi_2)$
or $A\sin(\omega_1 t + \phi_1) + B\sin(\omega_2 t + \phi_2)$

$\omega_1 \neq \omega_2$

$P = \frac{A^2}{2} + \frac{B^2}{2}$

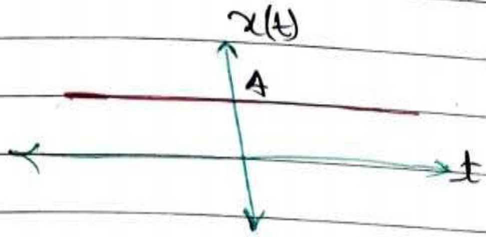
$\omega_1 = \omega_2$

$P = \frac{A^2}{2} + \frac{B^2}{2} + AB\cos(\phi_1 - \phi_2)$

Note Mean square value $= P$
root " " " $= RMS$

# Average, Power & RMS values of standard waveforms

(1)

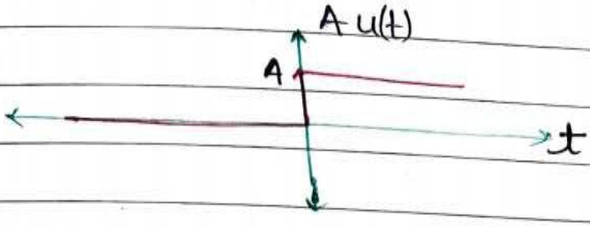



① Avg = A

RMS = A

Power = $A^2$

(2)

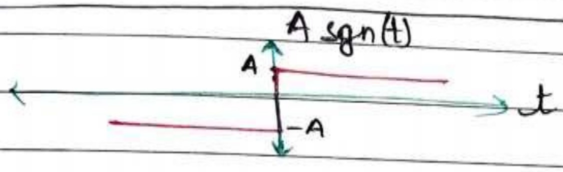



① Avg = $A/2$

$P = A^2/2$

RMS = $A/\sqrt{2}$

(3)

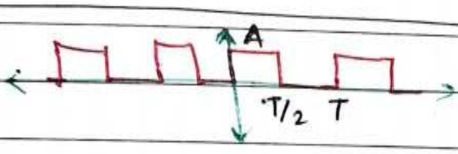



① Avg = 0

$P = A^2$

RMS = A

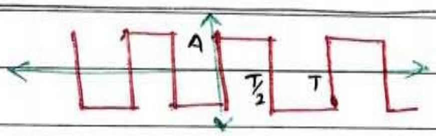
(4)




① Avg = $A/2$

$P = A^2/2$ , RMS = $A/\sqrt{2}$

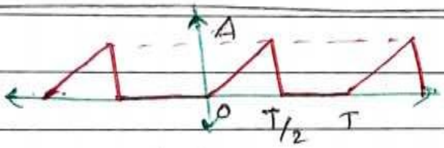
(5)




① Avg = 0

$P = A^2$ , RMS = A

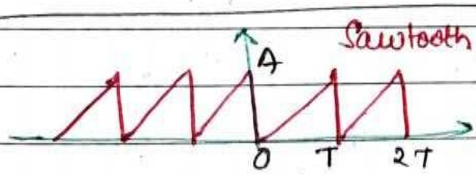
(6)




① Avg = $\dfrac{\frac{1}{2} \times A \times T/2}{T} = A/4$

$P = \dfrac{\frac{1}{3} A^2 \times T/2}{T} = \dfrac{A^2}{6}$

(7) Sawtooth waveform

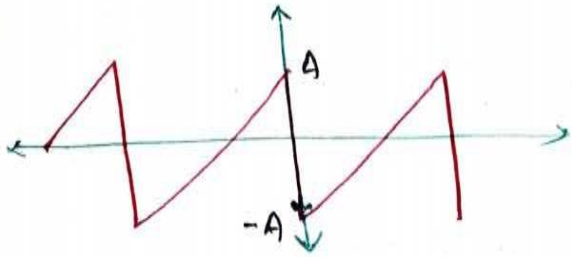



① Avg = $\dfrac{\frac{1}{2} \times A \times T}{T} = \dfrac{A}{2}$

$P = \dfrac{\frac{1}{3} A^2 \times T}{T} = \dfrac{A^2}{3}$

RMS = $\dfrac{A}{\sqrt{3}}$

(8)

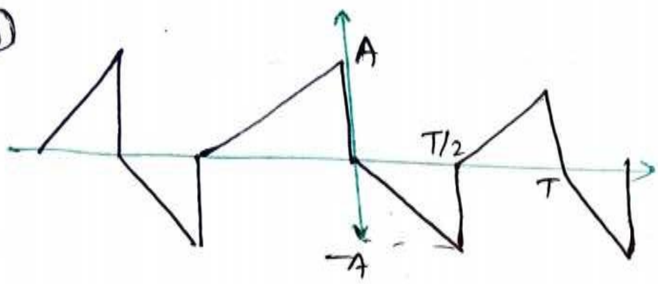



Avg $= 0$

$$P = \frac{2 \times \frac{A^2}{3} \times T/2}{T}$$

$$P = \frac{A^2}{3}, \quad RMS = \frac{A}{\sqrt{3}}$$

(9)

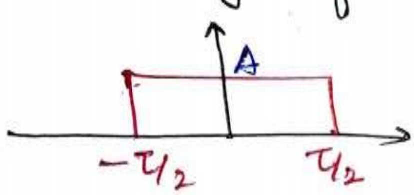



Avg $= 0$

$$P = \frac{2 \times \frac{1}{3} A^2 \frac{T}{2}}{T} = \frac{A^2}{3}$$

$$RMS = \frac{A}{\sqrt{3}}$$

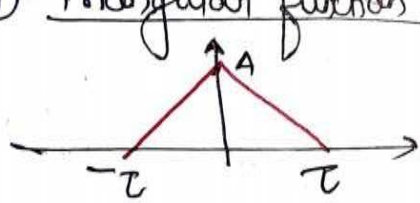
(10) Rectangular function




$$\boxed{E = A^2 \tau}$$

$A\,\text{rect}\left(\frac{t}{\tau}\right) \longrightarrow$ Area $= A\tau$
Energy $= A^2\tau$

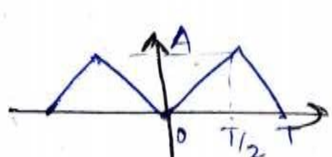
(11) Triangular function




$$E = \frac{2}{3} A^2 \tau$$

$$= \frac{1}{3} A^2 \text{ (Total width)}$$

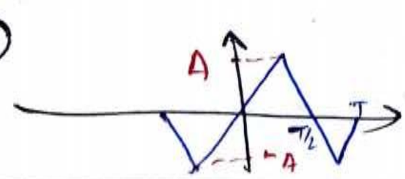
(12)




$$\text{Avg} = \frac{\frac{1}{2} \times T \times A}{T} = \frac{A}{2}$$

$$P_0 = \frac{\frac{2}{3} A^2 \times \frac{T}{2}}{T} = \frac{A^2}{3}, \quad RMS = \frac{A}{\sqrt{3}}$$

(13)




Avg $= 0$

$$P = \frac{\frac{1}{3} A^2 \times \frac{T}{2} \times 2}{T} = \frac{A^2}{3}$$

$$RMS = A/\sqrt{3}$$

* If $g(t) = -x\left(\frac{t-1}{2}\right)$ then $P\{g(t)\} = P\{x(t)\}$

* If $g(t) = 2x\left(\frac{t-1}{2}\right)$ then Let $h(t) = x\left(\frac{t-1}{2}\right) \Rightarrow g(t) = 2h(t)$

$$P[g(t)] = 4P[h(t)].$$
$$= 4P[x(t)]$$

Energy & Power conclusion:

# Energy & Power of discrete time signals

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 = \lim_{N\to\infty} \frac{E}{2N+1}$$

If $x(n)$ = Periodic, $P = \frac{1}{N_0} \sum_{N_0} |x(n)|^2$

- $c\,[x[n]] \Rightarrow$ New energy $= |c|^2$ Energy
- Interpolation doesn't affect Energy
  Decimation affects energy.

- $\delta[n] \rightarrow$ Energy signal {But $\delta(t) =$ NENP}

  $E = 1,\ P = 0$

  $A\,\delta[n] \rightarrow E = A,\ P = 0$

# floor and ceiling function

| |
|---|
| $[x]$ or $\lfloor x \rfloor$ $\rightarrow$ floor / Greatest integer $f^n$<br>Largest integer $\leq x$ |
| $]x[$ or $\lceil x \rceil$ $\rightarrow$ Ceiling / Least integer $f^n$<br>Smallest integer $\geq x$ |

System

<i> Static & Dynamic system
(memoryless) (memory)
↓
only present i/p dependency

eg: $y(t) = x(t)$ (S)
$y(t) = x(t) + x(t-1) + x(t+1)$ (D)

- differentiation / Integration / summation all are dynamic
- Resistor is static, Inductor / Capacitor are dynamic
- $x(\alpha t)$ or $x(-t)$ or $x(t \pm t_0) \rightarrow$ Dynamic

<ii> Causal & Non-Causal system
↓
(Present i/p + Past i/p dependency only)

↓
(O/p depends on future i/p also)

- Anticausal system → future i/p only
- Static ⇒ Causal but causal $\not\Rightarrow$ Static always
  Noncausal ⇒ Dynamic but Dynamic $\not\Rightarrow$ NC always

** Causal systems: - $\frac{dx(t)}{dt}$, $\int x(t)$, $y(t) = \begin{cases} x[2t] & t \leq 0 \\ x[t-1] & t > 0 \end{cases}$

Try to perform analysis for split sys: $y[2] = x[1]$ Past
$y[-2] = x[-4]$ Past
$y[0] = x[0]$ Present
} → Causal

- $\text{Real}[x(t)] = \frac{x(t) + x^*(t)}{2}$

$\text{Img}[x(t)] = \frac{x(t) - x^*(t)}{2}$

## ** Non-Causal system :

- $y(t) = \text{even}[x(t)] \,/\, \text{odd}[x(t)] = \dfrac{x(t) \pm x(-t)}{2}$

- $y(t) = C.S[x(t)] \,/\, C.A.S.[x(t)] = \dfrac{x(t) \pm x^*(-t)}{2}$

- $y[n] = \begin{cases} x[n+2] & n<0 \\ x[-n] & n \geq 0 \end{cases}$

$y(-1) = x(1) \rightarrow$ future

$y(2) = x(-2)$

$y(0) = x(0)$

→ Conditionally Causal system :

$$y[n] = \sum_{K=n_0}^{n} x[K]$$

① $n_0 < n$

$y[n] = x[n] + x[n-1] + \ldots + x[n_0]$

∴ Causal

② $n_0 > n$

$y[n] = x[n] + x[n+1] + \ldots + x[n_0]$

∴ Non Causal

### Time variant & Time invariant system / Shift variant or invar. sys.

- $y(t-t_0) = T[x(t-t_0)]$
  $y(n-n_0) = T\{x(n-n_0)\}$ } TIV

$\underbrace{y(n-n_0)}_{\text{Replace all } t \text{ by } t-t_0}$ , $\underbrace{T\{x(n-n_0)\}}$ → write $-t_0$ in given $f^n$ of $x()$

$x(\text{given time} - t_0)$

- eg : $y(t) = x[3(t)]$

$y \;\; x[3(t-t_0)] \neq x[3t - t_0]$

Hence TV

- for TIV System
  - → No time scaling & No time inversion
  - → constant coef. of $x(t)$
  - → $y[t^1]$ & $x[t^1]$ , No $t^2$, $\sqrt{t}$, sint etc.
  - → No extra 't' term or 'n' term added/sub/mul'
  - → No split system.

- eg: $\text{Real}[x(t)] = \frac{x(t) + x^*(t)}{2}$ TIV $\qquad \text{Even}[x(t)] = \frac{x(t) + x(-t)}{2}$ TV

- for TV system
  - → $y(t) = x(t) \ {+ \atop \times}$ [Time dependent Basic signal]
  - → $y(t)$ = Basic signal$[x(t)]$
  - → $y(t) = x$ [Basic signal]

- Summation, Integration, differentiation are TIV.

# Linear and Non-Linear System

For Linear:

(i) Additivity: $y_1(t) + y_2(t) = T\{x_1(t)\} + T\{x_2(t)\}$

$$y'(t) = T\{x_1(t) + x_2(t)\}$$

$\boxed{y'(t) = y_1(t) + y_2(t)}$ ⇒ Additive

(ii) Homogenity: $K\,y(t) = K\,T\{x(t)\}$

$$y'(t) = T\{K\,x(t)\}$$

$\boxed{y'(t) = K\,y(t)}$ ⇒ Homogeneous

Additive + Homogenous ⇓ Linear sys.

* $y(t) = x(t) \pm \text{constant}$
  $y(t) = x(t) \pm \text{time dependent term}$ } Non-linear sys.

* $y(t) = \pm \underset{\text{constant}}{m}\, x(t)$ — Linear

* $y(t) = x[t^2]$ — Linear

  $y^1(\ ) = m x^1(\ )$ Linear
  Time operatⁿ doesn't affect Linearity.

* $y[n] = x[n] \cdot x[n-1]$
  Trick → $x(\ ) \cdot x(\ ) = x^2(\ )$ → Nonlinear

* Linear ± Linear → Linear
  NL + L → NL

* Split system

$$y[n] = \begin{cases} 3x[n] & n < 0 \Rightarrow n \leq 1 \\ 0 & n = 0 \\ x[n+1] & n > 0 \Rightarrow n \geq 1 \end{cases}$$

Time range specified ⇒ Linear

$y[n] = 3x[n]\, u[n-1] + x[n+1]\, u[n-1]$ → Linear

$$y(t) = \text{sgn}[x(t)] = \begin{cases} 1 & x(t) < 0 \\ 0 & x(t) = 0 \\ -1 & x(t) > 0 \end{cases}$$

function range specified ⇒ Non Linear

- $y(t) = x$ [Basic signal]
  $y(t) =$ Basic signal $\times x(t)$ } Linear

- $y(t) =$ Basic signal $[x(t)]$ } NL

- $y(t) = \frac{d^n}{dx^n} x(t)$ → linear

  Summatⁿ → linear

  Integratⁿ → linear

- $y(t) = |x(t)|$ → NL

eg: (i) NL system which are homogeneous but not Additive

$y(t) = \frac{x^2(t)}{x(t-1)}$, $y[n] = \frac{x[n]\ x[n+2]}{x[n-3]}$, $y[n] = \frac{x^2[n-1]}{x[\cos \omega_0 n]}$

$y(t) = \frac{x(t) \frac{d x(t)}{dt}}{x(t-1)}$, $y(t) = \frac{x^2(t-1)}{\frac{dx}{dt}}$, $y(t) = \frac{x^3(t)}{x(t)\ x(t-1)}$

(ii) NL system that additive but non-homogeneous

- $y(t) = x^*(t)$ eg { Take $K = j$ $[Kx(t)]^* = K^* x^*(t)$ }
- $y(t) = x^*(-t)$ or $x^*(t+1)$
- $y(t) = x^*(3t)$ or $3x^*(t)$
- $y(t) =$ Real $[x(t)]$ / Img $[x(t)]$
- $y(t) =$ CS $[x[n]]$ or C.A.S. $[x[n]]$

# Memoryless $\times$ static $\times$ LTI $\times$ system

$y(t) = K\,x(t)$ $\quad$ $y[n] = K\,x[n]$ $\quad$ Static LTI

① Impulse response for static LTI is an impulse at origin

$h(t) = K\,\delta(t)$

$h(t) = K\,\delta(t) \quad = K\,\delta[n]$

$h(t) = 0$ for $t \neq 0$, $h[n] = 0$ for $n \neq 0$

② Transfer $f^n$ of static LTI is a constant $\quad H(s) = K$

① Causal LTI system

$h(t) = 0$ for $t < 0$

$h[n] = 0$ for $n < 0$

Anticausal LTI system

$h(t) = 0$ for $t > 0$

$h[n] = 0$ for $n > 0$

# Invertible & Non-Invertible system:

→ different i/p gives different o/ps

→ Always has an inverse sys.

→ $H_{inv}(s) = \dfrac{1}{H(s)} = \dfrac{X(s)}{Y(s)}$ $\quad$ $H_{inv}(z) = \dfrac{1}{H(z)} = \dfrac{X(z)}{Y(z)}$

$h(t) \otimes h_{inv}(t) = \delta(t)$ $\quad$ $h(n) \otimes h_{inv}(n) = \delta[n]$

Something left.

08/03/19 from LTI system

**Q1** If $I = \int \left(e^{e^t + e^{-t} + t}\right) dt$

$$J = \int e^{(e^t + e^{-t} - t)}\, dt$$

$I(0) - J(0) = e^2 + 1$, then find the value of $(I - J)$ at $t = (\log 2)$

**Ans:** $$I - J = \int \left[ e^{(e^t + e^{-t} + t)} - e^{(e^t + e^{-t} - t)} \right] dt$$

$$= \int e^{e^t + e^{-t}} \{ e^t - e^{-t} \} dt$$

Let $u = e^t + e^{-t} \Rightarrow du = (e^t - e^{-t})\, dt$

$$I - J = \int e^u\, du = e^u + C = e^{e^t + e^{-t}} + C$$

Putting $t = 0$

$$(I - J)\big|_{t=0} = e^2 + C$$

$$1 + e^2 = e^2 + C \Rightarrow \boxed{C = 1}$$

Putting $t = (\log 2)$

$$I - J = e^{e^{\log 2} + e^{-\log 2}} + 1 = e^{2 + \frac{1}{2}} + 1$$

$$= 1 + e^{5/2}$$

Q.2 Evaluate the integral

$$\int \cos x \left(\log x + \frac{1}{x^2}\right) dx$$

Ans: $I = \int \cos x \log x \, dx + \int \frac{\cos x}{x^2} dx$

$I = I_1 + I_2$

$I_1 = \int \underset{\text{II}}{\cos x} \, \underset{\text{I}}{\log x} \, dx = \log x (\sin x) - \int \frac{\sin x}{x} dx$

$= (\log x) \sin x - \left[ \int \underset{\text{II}}{(\sin x)} \underset{\text{I}}{\left(\frac{1}{x}\right)} dx \right]$

$I_1 = \log x \sin x + \frac{\cos x}{x} - \int \frac{\cos x}{x^2}$

$= \log x \sin x + \frac{\cos x}{x} - I_2$

$I = I_1 + I_2 = \log x (\sin x) + \frac{\cos x}{x}$

$\boxed{Q.3}$ Evaluate $\int \frac{1}{x\{\ln x^{1/2} + \ln x^{3/2}\}} dx = ?$

Ans: Let $t = \ln x \Rightarrow dt = \frac{1}{x} dx$

$$I = \int \frac{1}{t^{1/2} + t^{3/2}} dt$$

Put $t^{1/2} = u = (\ln x)^{1/2}$

$$\frac{1}{2t^{1/2}} dt = du$$

$$\cancel{I = \int \frac{1}{t^{1/2}(1+t)} dt}$$

$$I = \int \frac{1}{t^{1/2}(1+t)} dt = \int \frac{2}{1+u^2} du$$

$$= 2\tan^{-1} u + C$$

$$I = 2\tan^{-1}\{(\ln x)^{1/2}\} + c$$

**Q.4** $\displaystyle\int_0^{\pi/4} \frac{\tan x}{\log|\sec x|}\,dx = ?$

Ans: for $x \in [0, \pi/4)$ $\sec x > 0$

$\therefore |\sec x| = \sec x$

$$I = \int_0^{\pi/4} \frac{\tan x}{\log \sec x}\,dx$$

$$= \int_0^{\pi/4} \underset{\mathrm{II}}{(\tan x)} \underset{\mathrm{I}}{\left(\frac{1}{\log \sec x}\right)} dx$$

$$= \left[\frac{1}{\log \sec x} \times \log \sec x - \int \log \sec x \times \frac{-1}{(\log \sec x)^2} \times \frac{1}{\sec x} \times \sec x \tan x\right]_0^{\pi/4}$$

$$= \left[1 + \int \frac{\tan x}{\log \sec x}\right]_0^{\pi/4}$$

$$= \left[1 + \frac{1}{\log \sec x} \times \log \sec x - \int \log \sec x \times \frac{-1}{(\log \sec x)^2} \times \frac{1}{s}\right.$$

Damping factor ~~Attenuation~~

## Laplace transform

- $s = \sigma + j\omega$, frequency ~~of damping~~
- Laplace transform is valid for both stable and unstable signals
- B.L.T. $\boxed{X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt}$ for Causal, Anti ~~or~~ & Non causal signals
- Non invertible → for non causal — U.L.T. $\boxed{X(s) = \int_{0^-}^{\infty} x(t)\, e^{-st}\, dt}$ for Causal & Causal part of Non-causal signals

→ only for Causal * $\boxed{B.L.T = U.L.T.}$

- For convergence or existence of laplace transform:
  (i) $x(t)\, e^{-\sigma t}$ should be absolutely integrable.
  (ii) $x(t)$ should be continuos or piecewise continuos [countable discontinuity] in the given closed interval
  (iii) A piece wise continuos except ~~at~~ at finite no. of pnts.
  ex : ~~stable~~ step signal

- Region of convergence: (ROC)

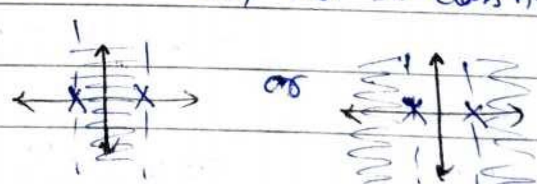
$$\int_{-\infty}^{\infty} |x(t)\, e^{-\sigma t}|\, dt < \infty$$

$\sigma$ : Re(S) ROC

Note

$$\text{Area of } x(t) = X(s)\Big|_{s=0} = X(0)$$

- Properties of ROC [only imp. mentioned here, rest in class notes]

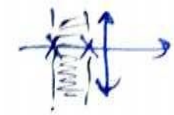
(i) for rational L.T. ROC: 

ROC doesn't contains poles, $X(s) = \infty$ at poles

(ii) If $x(t)$ = (finite duration + Absolutely integrable) then ROC: entire s-plane

(iii) If $X(s)$ = (Rational) ~~+ right sided~~ & $x(t)$ = (right sided) then ROC: region to right of rightmost pole

(iv) If $X(s)$ = Rational & $x(t)$ = leftsided, ROC: left of leftmost pole

(v) If $X(s)$ = Rational & $x(t)$ = bothsided, ROC: 

*** (vi) If ROC contains $j\omega$ axis, then signal = stable (Absolutely int)

(vii) ROC ⇒ (causality + stability) Information

| S.NO. | $x(t)$ | $X(s)$ | ROC |
|---|---|---|---|
| 1 | $\delta(t)$ | $1$ | entire s-plane |
| 2 | $u(t)$ | $\frac{1}{s}$ | $Re(s) > 0$ |
| 3 | $-u(-t)$ | $1/s$ | $Re(s) < 0$ |
| 4 | $r(t) = t\,u(t)$ | $\frac{1}{s^2}$ | $Re(s) > 0$ |
| 5. | $t^n u(t)$ | $\frac{n!}{s^{n+1}} = \frac{\overline{\lvert n+1}}{s^{n+1}}$ | $Re(s) > 0$ |
| 6. | $\cos\omega_0 t\, u(t)$ | $\frac{s}{s^2+\omega_0^2}$ | $Re(s) > 0$ |
| 7. | $-\cos\omega_0 t\, u(-t)$ | $\frac{s}{s^2+\omega_0^2}$ | $Re(s) < 0$ |
| 8. | $\sin\omega_0 t\, u(t)$ | $\frac{\omega_0}{s^2+\omega_0^2}$ | $Re(s) > 0$ |
| 9. | $\cosh\omega_0 t\, u(t)$ | $\frac{s}{s^2-\omega_0^2}$ | $Re(s) > \omega_0$ |
| 10 | $\sinh\omega_0 t\, u(t)$ | $\frac{\omega_0}{s^2-\omega_0^2}$ | $Re(s) > \omega_0$ |
| * 11. | $Sa(t) = \frac{\sin t}{t} u(t)$ | $\cot^{-1}(s)$ | |

12. $\frac{\sin at}{at} u(t) \longleftrightarrow \frac{1}{|a|} \cot^{-1}\left(\frac{s}{a}\right)$



* Amplitude inversion & scaling doesn't affect ROC
* Time inversion:

$$x(t) \longleftrightarrow X(s) \quad ROC: R$$
$$x(-t) \longleftrightarrow X(-s) \quad ROC: -R$$

valid only for BLT

* Time shifting property:

$$x(t) \longleftrightarrow X(s) \quad ROC: R$$
$$x(t-t_0) \longleftrightarrow e^{-st_0} X(s) \quad ROC: R$$
$$x(t+t_0) \longleftrightarrow e^{st_0} X(s) \quad ROC: R$$

* Time Scaling property:

If $x(t) \longleftrightarrow X(s) \quad ROC: R$
$$x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{s}{a}\right) \quad ROC: aR$$

* frequency shifting property:

If, $x(t) \longleftrightarrow X(s) \quad ROC: R$
$$e^{s_0 t} x(t) \longleftrightarrow X(s-s_0) \quad ROC: R + Re(s_0)$$
$$e^{-s_0 t} x(t) \longleftrightarrow X(s+s_0) \quad ROC: R - Re(s_0)$$

* Linearity property:

$$a x_1(t) \pm b x_2(t) \longleftrightarrow a X_1(s) \pm b X_2(s)$$
ROC : $R_1 \cap R_2$

common ROC of $R_1$ & $R_2$

If no common ROC, L.T. doesn't exist.

eg: (i) $sgn(t) = u(t) - u(-t)$

BLT doesn't exist as no common ROC

ULT $\Rightarrow$ $L[sgn(t)] = L[u(t)] = 1/s \quad ROC: Re(s) > 0$

(ii) $1 = u(t) + u(-t)$

(iii) $e^{-a|t|} = e^{-at} u(t) + e^{at} u(-t)$

No common ROC hence L.T. doesn't exist

** Use Gamma f^n when it's format in Question: $\int_0^\infty e^{-x} x^{m-1}\,dx = \Gamma m = (m-1)!$

**Note:** (i) Area $x(t) = [X(s)]_{s=0}$

(ii) Energy $x(t)$ = Area$[x^2(t)] = L[x^2(t)]\big|_{s=0}$

* $e^{-at}x(t) \longleftrightarrow X(s+a)$ ROC: $R + a$

$e^{at}x(t) \longleftrightarrow X(s-a)$ ROC: $R - a$

eg: $e^{-at}\cos\omega t\, u(t) \longleftrightarrow \dfrac{s+a}{(s+a)^2+\omega^2}$ $Re(s) > -a$

* Multiplication by 't' property

$t\,x(t) \longleftrightarrow -\dfrac{d}{ds}X(s)$ ROC: R

$t^n x(t) \longleftrightarrow (-1)^n \dfrac{d^n}{ds}X(s)$ ROC: R

* Time integration property

If $x(t) \longleftrightarrow X(s)$ ROC: R

ULT: $\displaystyle\int_0^t x(\tau)\,d\tau \longleftrightarrow \frac{X(s)}{s}$

BLT: $\displaystyle\int_{-\infty}^t x(\tau)\,d\tau \longleftrightarrow \frac{X(s)}{s} + \frac{1}{s}\Big[\underbrace{\int_{-\infty}^0 x(\tau)\,d\tau}_{\text{LHS Area}}\Big]$

ROC: $R \cap [Re(s) > 0]$

* Time differentiation property

ULT: $\dfrac{d}{dt}x(t) \longleftrightarrow sX(s) - x(0)$

$\dfrac{d^2}{dt^2}x(t) \longleftrightarrow s^2X(s) - s\,x(0) - \dot{x}(0)$

BLT: $\dfrac{d}{dt}x(t) \longleftrightarrow sX(s)$

$\dfrac{d^n x(t)}{dt^n} \longleftrightarrow s^n X(s)$

$\left[ f(t) \otimes \delta(t-t_0) \right] = f(t-t_0)$

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## * Division by t property

$$\boxed{\frac{x(t)}{t} \longleftrightarrow \int_s^\infty X(s)\, ds}$$

## # Inverse laplace transform

$$\boxed{x(t) = \frac{1}{2\pi j} \int_{\sigma - j\omega}^{\sigma + j\omega} X(s)\, e^{st}\, ds}$$

⊙ Partial fraction is only valid for proper fraction [deg $N^r$ < deg $D^r$]

* Conjugate Property $\boxed{x^*(t) \longleftrightarrow X^*(s) \quad ROC: R}$

* Convolution property $\boxed{x_1(t) \otimes x_2(t) \longleftrightarrow X_1(s) \cdot X_2(s) \quad ROC: R_1 \cap R_2}$

* Multiplication property $\boxed{x_1(t) \cdot x_2(t) \longleftrightarrow \frac{1}{2\pi j} \left[ X_1(s) \otimes X_2(s) \right] \quad ROC: R_1 \cap R_2}$

**Initial value theorem**

$$[x(t)]_{t=0} = x(0) = \lim_{s \to \infty} sX(s)$$

cond$^n$ *

⊙ Causal $x(t)$ i.e. $x(t) = 0$ for $t < 0$

⊙ $X(s)$ = Proper function

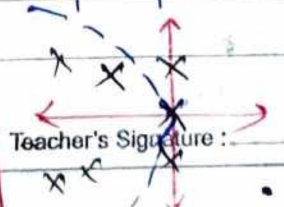
⊙ If $X(s)$ is improper function then signal $x(t)$ contains $\delta(t)$ & $\delta(0) = \infty$ at $t = 0$

**final value theorem**

$$[x(t)]_{t=\infty} = x(\infty) = \lim_{s \to 0} sX(s)$$

⊙ Causal $x(t)$

⊙ FVT valid only if all poles have negative real parts except a simple pole at $s = 0$



FVT not valid for

- RHS poles
- Multiple poles at $s=0$
- $s = \pm j\omega$ {one img. pole pair}
- Multiple img. pole pair

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Date ______________

Impulse response : $x(t) = \delta(t) \Rightarrow y(t) = h(t)$ = Impulse response

Step response : $x(t) = u(t) \Rightarrow y(t) = s(t)$ = Step response

$$\boxed{\begin{aligned} y(t) &= s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau \\ h(t) &= \frac{d}{dt} y(t) = \frac{d}{d(t)} s(t) \end{aligned}}$$

for a continuos time LTI system,

- Step response is the running integration of impulse response
- Impulse response is the ~~running~~ first derivative of step response

<u>Note</u>:- Some-times in exam it takes s(t) as impulse response, so be careful

Impulse response:

(i) Causal

$h(t) = 0$ for $t < 0$

$\neq 0$ for $t > 0$

(ii) Anticausal

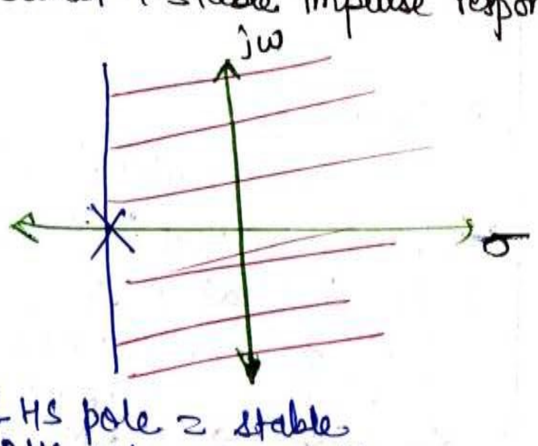
$h(t) = 0$ for $t > 0$

$\neq 0$ for $t < 0$

Stable impulse response / Stable LTI system:

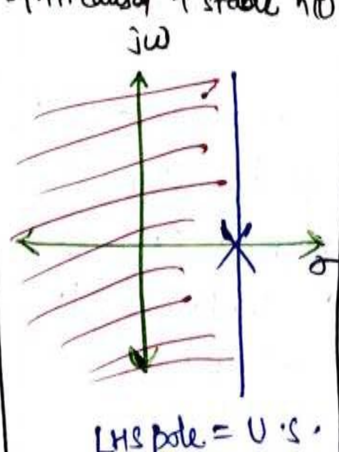
- $\int_{-\infty}^{\infty} |h(t)| < \infty$
- ROC must include $j\omega$ axis

Causal + stable impulse response




Causal $\left[\begin{array}{l}\text{LHS pole} = \text{stable} \\ \text{RHS pole} = \text{unstable}\end{array}\right.$

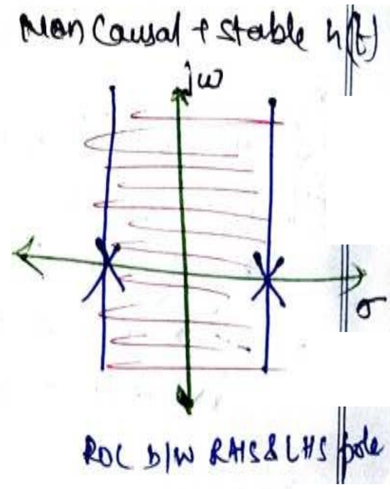
Anticausal + stable h(t)




LHS pole = U.S.

RHS pole = S

Non Causal + stable h(t)




ROC b/w RHS & LHS pole

** $f(t)\delta(t) = f(0)\delta(t)$

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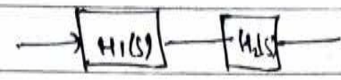
① Inverse system: $H_{inv}(s) = \frac{X(s)}{Y(s)} = \frac{1}{H(s)}$

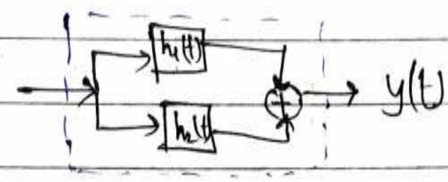
① Cascade connection




$h(t) = h_1(t) \otimes h_2(t)$

$H(s) = H_1(s) \cdot H_2(s)$

① Parallel connection



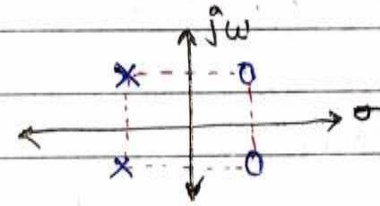

$h(t) = h_1(t) + h_2(t)$

$H(s) = H_1(s) + H_2(s)$

① Minimum phase system : All poles & zeroes to the left of jω axis

Non-minimum phase system : Some poles/zeroes or all of them to right of jω axis

All pass system :




pole-zero mirror image w.r.t. IM axis.

* $|H(j\omega)|$ = constant for all pass system

*** ① Eigen function & Eigen value

An eigen function is any non-zero $x(t)$ such that, operation (function) = constant (function) ; $S(x(t)) = \lambda x(t)$

eg: take $S = \frac{d}{dt}$ , $x(t) = e^{at}$ $S\{x(t)\} = ae^{at}$, $\lambda = a$

# Zero state response (ZSR) : $H(s) = \frac{Y(s)}{X(s)}\Big|_{\text{initial cond}^n = 0}$

{Steady state o/p or forced response or Particular integral}

Zero input response (ZIR) : response with all initial cond$^n$ but i/p = 0

{Transient response or Natural response or complementary f$^n$}

Date ______ Teacher's Signature : ______

$$\boxed{y(t) = y_{ZSR}(t) + y_{ZIR}(t)}$$

## Convolution Integral $x(t) \otimes h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau$

(1) Laplace transform: $\boxed{Y(s) = X(s) \cdot H(s)}$

(2) Commutative property: $\boxed{x(t) \otimes h(t) = h(t) \otimes x(t)}$

(3) Associative property: $\boxed{x(t) \otimes [h(t) \otimes g(t)] = [x(t) \otimes h(t)] \otimes g(t)}$

(4) Distributive property: $\boxed{x(t) \otimes [h(t) + g(t)] = x(t) \otimes h(t) + x(t) \otimes g(t)}$

(5) Convolution with impulse:

* • $x(t) \otimes \delta(t) = x(t)$
• $x(t) \otimes \delta(t-a) = x(t-a)$
• $x(t) \otimes \delta(t+a) = x(t+a)$

• $x(t) \otimes A\delta(t \pm a) = Ax(t \pm a)$

$[\ \delta(t) = h(t) \longleftrightarrow H(s) = 1\ ]$
$X(s)\, H(s) = X(s)$

(6) Convolution with step

⊙ $x(t) \otimes u(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

⊙ $x(t) \otimes u(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau$

(7) Time differentiation property

$y(t) = x(t) \otimes h(t)$ , $\boxed{\dfrac{dy(t)}{dt} = \dfrac{d[x(t)]}{dt} \otimes h(t) = x(t) \otimes \dfrac{d[h(t)]}{dt}}$

$$\boxed{\frac{d^2y(t)}{dt^2} = \frac{d^2x(t)}{dt^2} \otimes h(t) = x(t) \otimes \frac{d^2h(t)}{dt} = \frac{dx(t)}{dt} \otimes \frac{dh(t)}{dt}}$$

** (8) If $y(t) = x(t) \otimes h(t)$

* $x(t-t_1) \otimes h(t-t_2) = y(t-t_1-t_2)$

* * $x(t-1) \otimes h(t-1) = y(t-2) \neq y(t-1)$

* In convolution do not change t directly on both sides of equations

(9) $x(kt) \circledast h(kt) = \frac{1}{|k|} y(kt)$

(10) $x(-t) \circledast h(-t) = y(-t)$

| | | |
|---|---|---|
| Even | Even | Even |
| odd | odd | even |
| Even | odd | odd |

(11) $Area[y(t)] = Area[x(t)] \cdot Area[h(t)]$

** for steady state output with sinusoidal i/p proceed as follows:

Let $x(t) = \sin t$ , $G(s) = \frac{s-1}{s+1}$ $\Rightarrow G(j\omega_0) = \frac{\sqrt{2}\angle 135}{\sqrt{2}\angle 45} = 1\angle 90$

$\omega_0 = 1$ put $j\omega_0 = j1$

$y(t) = x(t)\, G(j\omega_0)$

$y(t) = \sin(t+90^\circ)$

eg: 2 $\frac{Y(s)}{R(s)} = \frac{s}{s+2}$ , $x(t) = \cos 2t$

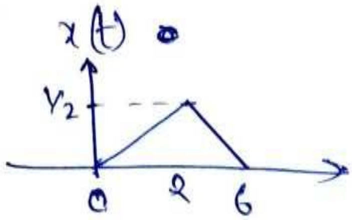
$\omega_0 = 2$

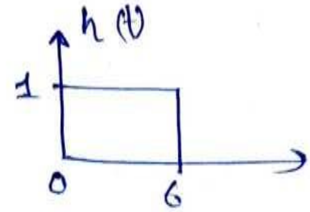
$H(s) = \frac{2j}{2j+2} = \frac{1\angle 90'}{\sqrt{2}\angle 45^\circ} = \frac{1}{\sqrt{2}}\angle 45^\circ$

Steady state o/p, $y(t) = \cos 2t \times \frac{1}{\sqrt{2}}\angle 45^\circ = \frac{1}{\sqrt{2}}\cos(2t + 45^\circ)$

* If $y(t) = x(t) \circledast h(t)$ & $y(2), y(1), y(5) \dots$ are asked in question

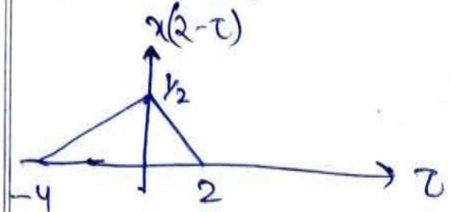
eg:




$y(2) = ?$

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau \Rightarrow y(2) = \underbrace{\int_{-\infty}^{\infty} x(2-\tau)\, h(\tau)\, d\tau}_{\text{Area}}$$

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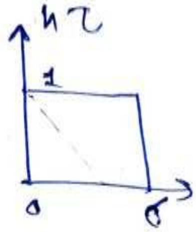


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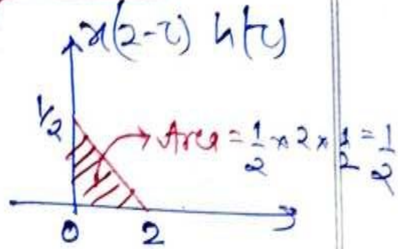
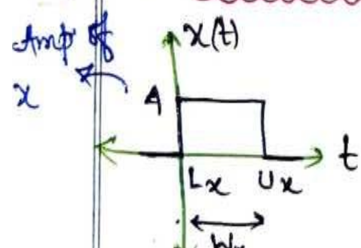


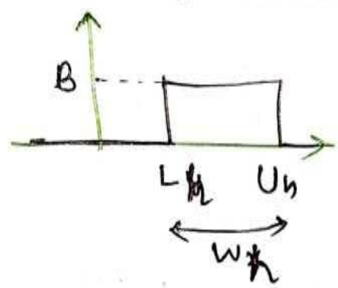

$$y(2) = \text{Area}\,[x(2-\tau)\,h(\tau)] = \frac{1}{2}$$

* Hence if t is mentioned, use $y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$

# Convolution of 2 rectangular waveforms




$\circledast$

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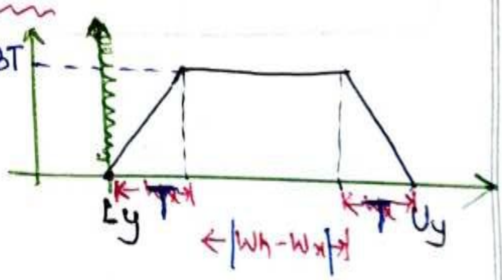
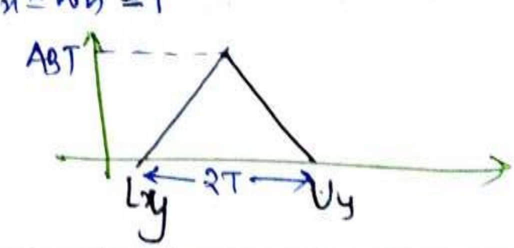



(i) $T = \min[W_x, W_h]$

(ii) $L_y = L_x + L_h$
$U_y = U_x + U_h$
$W_y = W_x + W_h$

Special Case: If $W_x = W_h = T$



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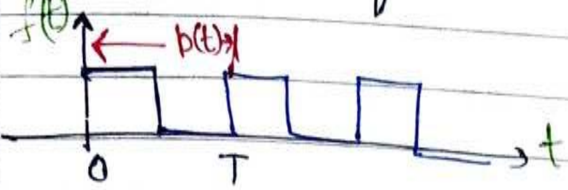
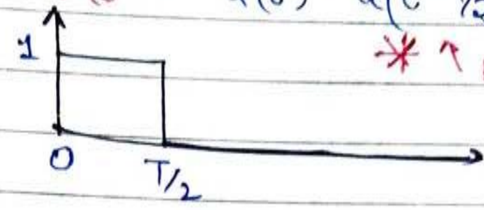
## Periodic waveforms



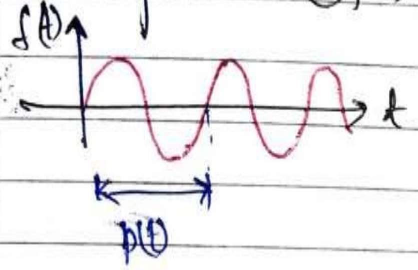

$\rightarrow$ $p(t) = u(t) - u(t - T/2)$

* ↑ Don't forget

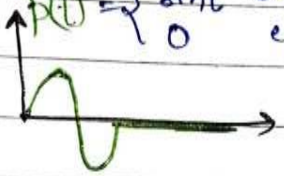


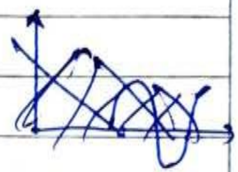

**
$$\boxed{F(s) = \frac{P(s)}{1 - e^{-sT_0}}}$$

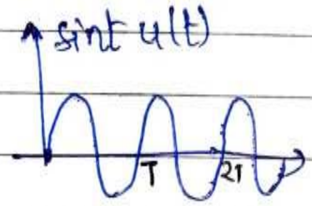
* Imp. ex: ① $f(t) = \sin t\, u(t)$

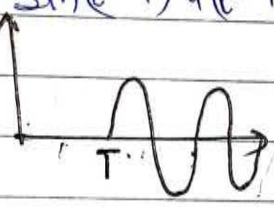



$p(t) = \begin{cases} \sin t & 0 < t < T \\ 0 & \text{else} \end{cases}$



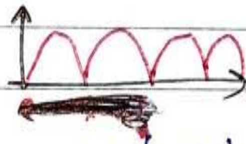





$p(t) = \sin t\, u(t) - \sin(t - T)\, u(t - T)$

② $f(t) =$



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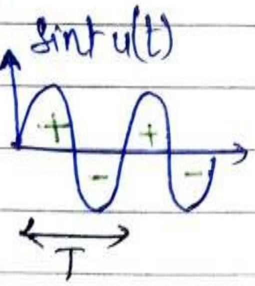


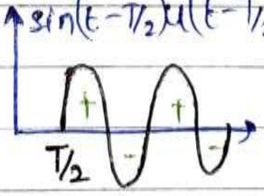


$p(t) = \sin t\, u(t) + \sin(t - T/2)\, u(t - T/2)$ **

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## Fourier transform

- $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$
- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$

Dirichlet cond$^n$

- $\int_{-\infty}^{\infty} |x(t)|\,dt < \infty$ Absolutely integrable [Energy & impulse signal]
- finite no. of maxima/minima in a finite interval
- finite no. of discontinuities in a finite interval
- finite energy or square integrable $\int_{-\infty}^{\infty} |x(t)|^2\,dt < \infty$
- single valued function

$x(t)\Big|$ all dirichlet cond$^n$ are satisfied $\longrightarrow$ $\boxed{X(s)\big|_{s=j\omega} = X(\omega)}$

L.T. F.T.

Absolutely integrable

(1) Linearity property: $\boxed{a\,x_1(t) + b\,x_2(t) \longleftrightarrow a\,X_1(\omega) + b\,X_2(\omega) \text{ or } a\,X_1(f) + b\,X_2(f)}$

(2) frequency differentiation: Both changes, Mag. & phase spectrum

If $x(t) \longleftrightarrow X(\omega)$

then $\boxed{t\,x(t) \longleftrightarrow j\frac{d}{d\omega}X(\omega)}$

If $x(t) \longleftrightarrow X(f)$

then $\boxed{t\,x(t) \longleftrightarrow \frac{j}{2\pi}\frac{d}{df}X(f)}$

(3) Time shifting property:

$$\boxed{\begin{aligned} x(t-t_0) &\longleftrightarrow e^{-j\omega t_0}X(\omega) \text{ or } e^{-j2\pi f t_0}X(f) \\ x(t+t_0) &\longleftrightarrow e^{j\omega t_0}X(\omega) \text{ or } e^{j2\pi f t_0}X(f) \end{aligned}}$$

Magnitude spectrum $\rightarrow$ same

phase spectrum $\rightarrow$ Linear phase shift of $\pm\omega t_0$

(4) Time differentiation property:

$$\boxed{\frac{d\,x(t)}{dt} \longleftrightarrow j\omega X(\omega) \qquad \frac{d^n}{dt^n}x(t) \longleftrightarrow (j\omega)^n X(\omega)}$$

or $j2\pi f\, X(f)$ or $(j2\pi f)^n X(f)$

| S.No. | $x(t)$ | $X(f)$ | $X(\omega)$ |
|---|---|---|---|
| 1. | $\delta(t)$ | $1$ | $1$ |
| 2. | $e^{-at}u(t)$ | $\frac{1}{a+j2\pi f}$ | $\frac{1}{a+j\omega}$ |
| 3. | $e^{at}u(-t)$ | $\frac{1}{a-j2\pi f}$ | $\frac{1}{a-j\omega}$ |
| 4. | $A\,rect(t/\tau)$ | $A\tau\,sinc(f\tau)$ | $A\tau\,Sa(\omega\tau/2)$ |
| 5. | $A\,tri(t/\tau)$ | $A\tau\,sinc^2(f\tau)$ | $A\tau\,Sa^2(\omega\tau/2)$ |
| 6. | $sgn(t)$ | $1/j\pi f$ | $2/j\omega$ |
| 7. | $e^{-a\lvert t\rvert}sgn(t)$ | $\frac{-j4\pi f}{a^2+(2\pi f)^2}$ | $\frac{-j2\omega}{a^2+\omega^2}$ |
| 8. | $1$ | $\delta(f)$ | $2\pi\,\delta(\omega)$ |
| 9. | $u(t)$ | $\frac{\delta(f)}{2}+\frac{1}{j2\pi f}$ | $\pi\delta(\omega)+\frac{1}{j\omega}$ |
| 10. | $\cos 2\pi f_0 t$ | $\frac{\delta(f-f_0)+\delta(f+f_0)}{2}$ | $\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$ |
| 11. | $\sin 2\pi f_0 t$ | $\frac{\delta(f-f_0)-\delta(f+f_0)}{2j}$ | $\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$ |
| 12. | $t^n e^{-at}u(t)$ | $\frac{n!}{(a+j2\pi f)^{n+1}}$ | $\frac{n!}{(a+j\omega)^{n+1}}$ |
| 13. | $e^{-\lvert at\rvert}$ | $\frac{2a}{a^2+\omega^2}$ | |

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- LTI system preserves frequency, ∴ Harmonic distortion is possible only in non-LTI system.

# Distortion in LTI system

(1) Magnitude spectrum

$$x(t) = \sin(\omega_1 t) + \sin\omega_2(t) \longrightarrow \boxed{LTI} \longrightarrow y(t) = A\sin\omega_1 t + B\sin\omega_2 t$$

Different amplification to different freq.

for eliminating mag. distortion, $A = B$

(2) Phase distortion or delay distortion

$$\sin(\omega_1 t) + \sin(\omega_2 t) \longrightarrow \boxed{LTI} \longrightarrow y = \sin\omega_1(t-t_1) + \sin\omega_2(t-t_2)$$

$$t_1 \neq t_2$$

To eliminate phase distortion, $t_1 = t_2$

# Distortionless LTI System

- $y(t) = K\,x(t - t_d)$ $\quad$ $t_d$ = timedelay, $K$ = Gain constant $(K>0)$
- $H(\omega) = \dfrac{Y(\omega)}{X(\omega)} = K e^{-j\omega t_d}$ $\quad$ $h(t) = K\,\delta(t - t_d)$
- $|H(\omega)| = K =$ constant for all $\omega$ ; $\angle H(\omega) = -\omega t_d$
- Phase delay and group delay

$$x(t) = \underbrace{m(t)}_{\text{Group of freq.}}\ \underbrace{\cos\omega_c t}_{\text{single freq. (carrier wave)}} \longrightarrow \boxed{LTI} \longrightarrow y(t) = m(t-t_g)\cos[\omega_c(t-t_p)]$$

$t_p$ : Phase delay
$t_g$ : group delay

→ Phase delay, $t_p = -\dfrac{\angle H(\omega)}{\omega}$

→ Group delay, $t_d = -\dfrac{d}{d\omega}(\angle H(\omega))$

for distortionless LTI System : $\boxed{t_p = t_g}\ (= t_d)$

* Zero crossing pnt for rect. $f^n$

$$X(\omega) = A\tau\, Sa\left(\frac{\omega\tau}{2}\right) = 0$$

$$\Rightarrow \frac{\omega\tau}{2} = n\pi \Rightarrow \omega = \frac{2n\pi}{\tau} \quad n = \pm 1, \pm 2 \ldots$$

(5) frequency shifting property

$$x(t) \longleftrightarrow X(\omega)$$

$$e^{j\omega_0 t} x(t) \longleftrightarrow X(\omega - \omega_0) \;/\; X(f - f_0)$$

$$e^{-j\omega_0 t} x(t) \longleftrightarrow X(\omega + \omega_0)$$

(8) Time Integration property

$$\int_{-\infty}^{t} x(\tau)\, d\tau \longleftrightarrow \frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$$

$$\text{or } \frac{X(f)}{j2\pi f} + \frac{1}{2} X(0)\, \delta(f)$$

(6) Time Scaling property

$$x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right) \quad \text{or } \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

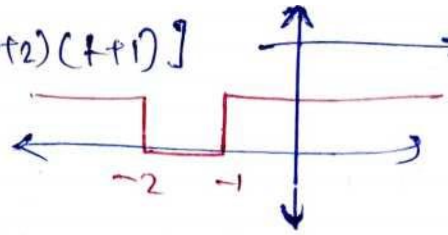
(7) Time Scaling + time shifting

$$x[a(t \pm t_0)] \longleftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right) e^{\pm j\omega t_0} \text{ or } \frac{1}{|a|} X\left(\frac{f}{a}\right) e^{\pm j2\pi f t_0}$$

Note: – ① for absolutely integrable $f^n$ or signals, we can use both mathematical definition & derivative of basic signals $\frac{d^2 r(t)}{dt^2} = \delta(t)$ or $\frac{d}{dt} u(t) = \delta(t)$

② Absolutely integrable signals are finite duration & finite area signals

③ If $x(t)$ is not absolutely integrable, we can't use $\frac{d}{dt}$ property

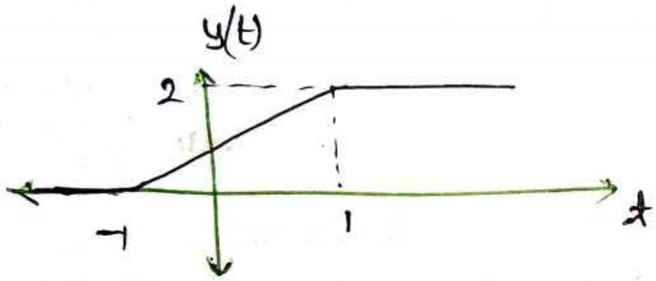
* $x(t) = u[t^2+3t+2] = u[(t+2)(t+1)]$  $= u(t+1) + u(-t-2)$

then calculate $X(\omega)$

## Modulation theorem

(i) $\cos(\omega_0 t)\, x(t) \longleftrightarrow \dfrac{X(\omega-\omega_0)+X(\omega+\omega_0)}{2}$ or $\dfrac{X(f-f_0)+X(f+f_0)}{2}$

(ii) $\sin(\omega_0 t)\, x(t) \longleftrightarrow \dfrac{X(\omega-\omega_0)-X(\omega+\omega_0)}{2j}$

* 

- not absolutely Integrable, so $\frac{d}{dt}$ can't be used.
- $y(t) = r(t+1) - r(t-1)$

- $y(t) = \int [u(t+1) - u(t-1)]\, dt$

$$= \frac{2\sin\omega}{j\omega^2} + \pi\, 2\, Sa(0)\, \delta(\omega)$$

$$= \frac{2\sin\omega}{j\omega^2} + 2\pi\, \delta(\omega)$$

## Duality property

f domain:

$x(t) \longleftrightarrow X(f)$

$X(t) \longleftrightarrow x(-f)$ $(t=-f)$

$(f=t)$

ω-domain:

$x(t) \longleftrightarrow X(\omega)$

$X(t) \longleftrightarrow 2\pi\, x(-\omega)$

$(\omega=t)$ $(t=-\omega)$

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examples:

(1) $A\, rect\left(\frac{t}{\tau}\right) \longleftrightarrow A\tau\, Sa\left(\frac{\omega\tau}{2}\right)$ | $A\, rect(t/\tau) \longleftrightarrow A\tau\, sinc(f\tau)$

$A\tau\, Sa\left(\frac{t\tau}{2}\right) \longleftrightarrow 2\pi A\, rect\left(\frac{\omega}{\tau}\right)$ | $A\tau\, sinc(t\tau) \longleftrightarrow A\, rect\left(\frac{f}{\tau}\right)$

(2) $rect(t) \longleftrightarrow Sa(\omega/2)$ | $rect(t) \longleftrightarrow sinc(f)$

$Sa\left(\frac{t}{2}\right) \longleftrightarrow 2\pi\, rect(\omega)$ | $sinc(t) \longleftrightarrow rect(f)$

(3) $A\, tri(t/\tau) \longleftrightarrow A\tau\, Sa^2(\omega\tau/2)$ | $tri(t) \longleftrightarrow Sa^2(\omega/2)$

$A\tau\, Sa^2\left(\frac{t\tau}{2}\right) \longleftrightarrow 2\pi A\, tri\left(\frac{\omega}{\tau}\right)$ | $Sa^2(t/2) \longleftrightarrow 2\pi\, tri(\omega)$

(4) $A\, tri(t/\tau) \longleftrightarrow A\tau\, sinc^2(f\tau)$ | $tri(t) \longleftrightarrow sinc^2(f)$

$A\tau\, sinc^2(t\tau) \longleftrightarrow A\, tri\left(\frac{f}{\tau}\right)$ | $sinc^2(t) \longleftrightarrow tri(f)$

## Total Area Integral

① Area in t-domain $\quad Area[x(t)] = X(f)\big|_{f=0} = X(\omega)\big|_{\omega=0}$

② Area in f-domain $\quad Area[X(f)] = x(t)\big|_{t=0}$

③ Area in ω-domain $\quad Area[X(\omega)] = 2\pi\, x(t)\big|_{t=0}$

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(i) Area [sinc(t)] = Area [sinc(f)] = 1

(ii) Area [Sa(t)] = Area [Sa(ω)] = π = Area [Sa(nω)]

(iii) E [sinc(t)] = 1

E [Sa(t)] = π

# Convolution Property (Time convolution)

$$\boxed{x_1(t) \circledast x_2(t) \longleftrightarrow X_1(f) \cdot X_2(f) \text{ or } X_1(\omega)\, X_2(\omega)}$$

$$x_1(t) \circledast x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

**Result:**

① $\text{sinc}(t) \circledast \text{sinc}(t) = \text{sinc}(t)$

② $A\,\text{sinc}(t) \circledast B\,\text{sinc}(t) = AB\,\text{sinc}(t)$

③ $\text{sinc}(Kt) \circledast \text{sinc}(Kt) = \frac{1}{K}\,\text{sinc}(Kt)$

④ $A\,\text{sinc}(Kt) \circledast B\,\text{sinc}(Kt) = \frac{AB}{K}\,\text{sinc}(Kt)$

⑤ Put $K = 1/\pi$

$$\text{sinc}\left[\frac{t}{\pi}\right] \circledast \text{sinc}\left[\frac{t}{\pi}\right] = \pi\,\text{sinc}\left[\frac{t}{\pi}\right]$$

$$\boxed{Sa(t) \circledast Sa(t) = \pi\, Sa(t)}$$

⑥ $\text{sinc}(\alpha t) \circledast \text{sinc}(\beta t) \longleftrightarrow \frac{1}{\max(\alpha,\beta)}\,\text{sinc}[\min(\alpha,\beta)\, t]$

# frequency convolution

$$x_1(t) \cdot x_2(t) \longleftrightarrow X_1(f) \circledast X_2(f) \text{ or } \frac{1}{2\pi}\left[X_1(\omega) \circledast X_2(\omega)\right]$$

## Steady state output

$x(t) = A\cos(\omega_0 t + \phi)$ or $A\sin(\omega_0 t + \phi)$

$$y(t) = |H(\omega_0)|\, A\cos(\omega_0 t + \phi + \angle H(\omega_0))$$

**Note:-** Convolution of the signal with cos or sine can be solved using both fourier transform & steady state.

** Except all pass filter, all other filters are non invertible, $\because$ they are zero for more than one value of $\omega$.

# Rayleigh Energy theorem

$$\boxed{\text{Energy}[x(t)] = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega}$$

# Gaussian function

$$\boxed{e^{-\pi t^2} \longleftrightarrow e^{-\pi f^2}}$$

① $x(t) = e^{-\pi t^2}$

Step 1 $\frac{dx(t)}{dt} = -2\pi t e^{-\pi t^2} = -2\pi\,[t\,x(t)]$

Step 2 $(j2\pi f)\cdot X(f) = -2\pi\left[\frac{j}{2\pi}\frac{d}{df}X(f)\right] \Rightarrow \left[-2\pi f X(f) = \frac{d}{df}X(f)\right]$

Step 3 $X(f) = e^{-\pi f^2}$

# Concept of even, odd, real, Imaginary.

| x(t) | X(f) | Example |
|---|---|---|
| EVEN | EVEN | $rect(t) \leftrightarrow sinc(f)$ |
| ODD | ODD | $sgn(t) \leftrightarrow \frac{1}{j\pi f}$ |
| Real & Even | Real & Even | $rect(t) \leftrightarrow sinc(f)$ |
| Img & Even | Img & Even | $j\,rect(t) \leftrightarrow j\,sinc(f)$ |
| Real & odd | Img & odd | $sgn(t) \leftrightarrow 1/j\pi f$ |
| Img & odd | Real & odd | $j\,sgn\,t \leftrightarrow \frac{1}{\pi f}$ |

* even ⇒ Real $x(t) \rightarrow$ Real $X(f)$ , Img $x(t) \rightarrow$ Img $X(f)$
odd ⇒ Real $x(t) \rightarrow$ Img $X(f)$ , Img $x(t) \rightarrow$ Real $X(f)$

| | | |
|---|---|---|
| Complex & even | Complex & even | $(1+j)\,rect(t) \leftrightarrow (1+j)\,sinc(f)$ |
| complex & odd | Complex & odd | $(1+j)\,sgn(t) \leftrightarrow \frac{1+j}{j\pi f} = \frac{1-j}{\pi f}$ |
| Real even & Img(odd) | Real (NENO) | later |
| Real odd & Img even | Img (NENO | later |

# Conjugate property of Fourier transform.

$$x(t) \leftrightarrow X(f)$$
$$x^*(t) \leftrightarrow X^*(-f)$$
$$x^*(-t) \leftrightarrow X^*(f)$$

FT [Real] $\longrightarrow$ C.S.
FT [C.S.] $\longrightarrow$ Real
FT [Img] $\longrightarrow$ C.A.S.
FT [C.A.S.] $\longrightarrow$ Img

C.S. : Real even & Im odd
C.A.S. : Real odd & Im even

Real : $x^*(t) = x^*(t)$ Img: $x(t) = -x^*(t)$

C.S. : $X(f) = X^*(-f)$ C.A.S.: $X(f) = -X^*(-f)$

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**Q.1** Fourier transform of following in terms of $X(\omega)$ & $X f$ are

$\langle i \rangle$ $t\,x(t) \leftrightarrow$ ______ In terms of $X(\omega)$ & $X(f)$

$\langle ii \rangle$ $x(t \pm t_0) \leftrightarrow$

$\langle iii \rangle$ $\frac{d}{dt} x(t) \leftrightarrow$

$\langle iv \rangle$ $e^{\pm j\omega_0 t} x(t) \leftrightarrow$

$\langle v \rangle$ $x(at) \leftrightarrow$

$\langle vi \rangle$ $x[a(t \pm t_0)] \leftrightarrow$

$\langle vii \rangle$ $\cos(\omega_0 t)\, x(t) \leftrightarrow$

$\langle viii \rangle$ $\sin(\omega_0 t)\, x(t) \leftrightarrow$

$\langle ix \rangle$ $x_1(t) \circledast x_2(t) \leftrightarrow$

$\langle x \rangle$ $x_1(t) \cdot x_2(t) \leftrightarrow$

$\langle xi \rangle$ $\int_{-\infty}^{t} x(\tau)\, d\tau \leftrightarrow$

$\langle xii \rangle$ $x^*(t) \leftrightarrow$

**Q.2** Duality property

$x(t) \longleftrightarrow X(f)$ | $x(t) \longleftrightarrow X(\omega)$

$X(t) \longleftrightarrow$ | $X(t) \longleftrightarrow$

**Q.3** If $y(t) = K x(t - t_d)$, then $H(\omega) = ?$, $h(t) = ?$

**Q.4** Area $[x(t)] = ?$  Area $[X(\omega)] = ?$

Area $[X(f)] = ?$

**Q.5** Area $[\text{sinc}(t) / \text{sinc}(f)] = ?$  $E[\text{Sinc}(t)] = ?$

Area $[Sa(t) / Sa(\omega)] = ?$  $E[Sa(t)] = ?$

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Q.6 Rayleigh energy theorem

Energy $[x(t)] = ?$ in terms of

(a) $x(t)$ (c) $X(\omega)$

(b) $X(f)$

Q.7

$$x(t) \longrightarrow \boxed{H(\omega_0)} \longrightarrow y(t)$$

$x(t) = A\cos(\omega_0 t + \phi)$ / $A\sin(\omega_0 t + \phi)$

$y(t) = ?$ steady state output.

Q.8 $A\,\text{sinc}(Kt) \otimes B\,\text{sinc}(Kt) = ?$

$\text{sinc}(t) \otimes \text{sinc}(t) = ?$

$Sa(t) \otimes Sa(t) = ?$

$\text{sinc}(\alpha t) \otimes \text{sinc}(\beta t) = ?$

Q.9 $X(\omega) = ?$ , $X(f) = ?$

(a) $\delta(t)$

(b) $e^{-at}u(t)$

(c) $e^{at}u(-t)$

(d) $A\,\text{rect}(t/\tau)$

(e) $A\,\text{tri}(t/\tau)$

(f) $\text{sgn}(t)$

(g) $e^{-a|t|}\,\text{sgn}\,t$

(h) $1$

(i) $u(t)$

(j) $\cos 2\pi f_0 t$ , $\sin 2\pi f_0 t$

(K) $t^n e^{-at}u(t)$

(l) $e^{-\pi t^2}$

# Continuos time fourier series

## Orthogonal signal :

Two signals $x_1(t)$ & $x_2(t)$ are orthogonal if $\int_{-\infty}^{\infty} x_1(t) \cdot x_2(t) = 0$ [N·P·]

$\int_{T_0} x_1(t) \cdot x_2(t) = 0$ [P – energy/periodic]

(1) DC and sinusoidal signals are orthogonal always.
(2) Harmonics of different frequencies are always orthogonal

$$\left.\begin{array}{l} \int_{T_0} \sin[n\omega_0 t + \phi_1] \; \cos[m\omega_0 t + \phi_2] \; dt \\ \text{or } \sin(\;''\;) \;\; \sin[\;''\;] \\ \cos(\;''\;) \;\; \cos \;\; '' \end{array}\right\} m \neq n$$

(3) Sine and cosine of same phase & freq· are orthogonal
$\int \sin(n\omega_0 t + \phi) \; \cos(m\omega_0 t + \phi)$

(4) Effect on energy & Power

[* $x(t) = x_1(t) + x_2(t)$ ]

orthogonal signals

$$\boxed{\begin{aligned} E[x(t)] &= E[x_1(t)] + E[x_2(t)] \\ P[x(t)] &= P[x_1(t)] + P[x_2(t)] \end{aligned}}$$

$\left[\because 2\int_{-\infty}^{\infty} x_1(t)\, x_2(t)\, dt = 0\right]$

for Nonorthogonal signals

$$\boxed{E[x(t)] = E[x_1(t)] + E[x_2(t)] - 2\int_{-\infty}^{\infty} x_1(t)\, x_2(t)\, dt}$$

## ① Trigonometric Fourier series (T·F·S·)

⊙ $x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(\omega_0 n t) + b_n \sin(\omega_0 n t)]$

⊙ → $a_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt$

→ $b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin(\omega_0 n t)\, dt$

→ $a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos(\omega_0 n t)\, dt$

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① $\boxed{a_0 = \frac{\text{Area in 1 period}}{\text{Time period}}}$

DC or avg. value = $a_0$

Area above X axis = +ve area
Area below X axis = −ve area

Total Area = (+ve area) + (−ve area)

| | |
|---|---|
| $a_0 = 0$ | +ve area = −ve area |
| $a_0 > 0$ | +ve area > −ve area |
| $a_0 < 0$ | +ve area < −ve area |

$x(t)$ = Even $\quad a_0 \underset{\neq}{=} 0, \; a_n \neq 0, \; b_n = 0$

$a_0 = \frac{2}{T_0}\int_0^{T_0/2} x(t)\,dt$

$a_n = \frac{4}{T_0}\int_0^{T_0/2} x(t)\cos(n\omega_0 t)\,dt$

$x(t)$ = odd $\quad a_0 = a_n = 0, \; b_n \neq 0$

$b_n = \frac{4}{T_0}\int_0^{T/2} x(t)\sin(n\omega_0 t)\,dt$

② Polar form Fourier series (P.F.S.)

$$x(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos[\omega_0 n t - \theta_n]$$

$$A_n = \sqrt{a_n^2 + b_n^2} \qquad \theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

③ Exponential Fourier series (E.F.S.)

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j\omega_0 n t}$$

$$C_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-j\omega_0 n t}\,dt$$

→ $X(f) = \sum_{n=-\infty}^{\infty} C_n\, \delta(f - n f_0)$

→ $C_n = \frac{X(\omega_0 n)}{T_0}$

→ $X(\omega) = \sum_{n=-\infty}^{\infty} C_n\, \delta(\omega - n\omega_0)$

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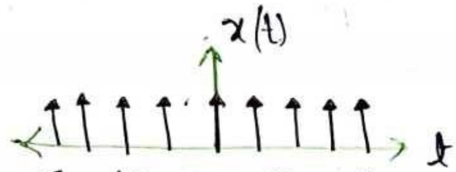
## Inter-relⁿ

- $C_0 = a_0$
- $C_n = \frac{a_n - jb_n}{2}$ $\quad C_{-n} = \frac{a_n + jb_n}{2}$
- $|C_n| = |C_{-n}| = \frac{\sqrt{a_n^2 + b_n^2}}{2}$
- $a_n = C_n + C_{-n}$
  $b_n = jC_n - jC_{-n}$

- $\angle C_n = -\angle C_{-n}$
- $C_n = C_{-n}^*$
  $C_{-n} = C_n^*$

- $A_n = \sqrt{a_n^2 + b_n^2} = 2|C_n|$
  $\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right) = -\angle C_n$

## ** Periodic Impulse train




$$\left[ x(t) = \sum_{K=-\infty}^{\infty} \delta(t - Kt_0) \right]$$

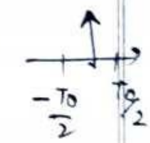
- $a_0 = \frac{1}{T_0}$, $a_n = \frac{2}{T_0}$, $b_n = 0$
- $A_n = a_n$, $\theta_n = 0$, $C_n = 1/T_0$

$$\boxed{X(f) = \sum_{n=-\infty}^{\infty} \frac{1}{T_0} \delta(f - f_0)}$$

$$\boxed{X(\omega) = \sum_{n=-\infty}^{\infty} 2\pi C_n \, \delta[\omega - n\omega_0]}$$

$\rightarrow C_n = \frac{X(\omega_0 n)}{T_0} = \frac{1}{T_0}$

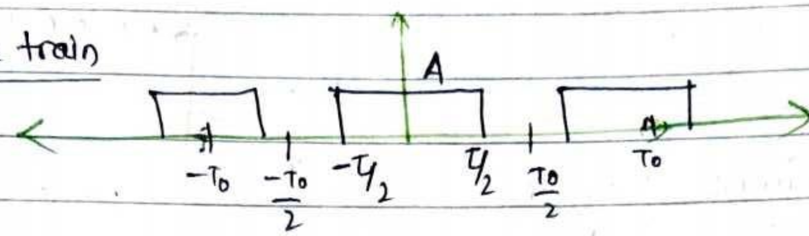
$\because$ In one period $x(t) = \delta(t)$
$X(\omega) = 1$




$\rightarrow C_n = \frac{a_n - jb_n}{2} = \frac{1}{T_0} \Rightarrow a_n = \frac{2}{T_0}, b_n = 0$

$\rightarrow C_0 = \frac{1}{T_0} = a_0$

# Periodic pulse train




ⓐ $a_0 = c_0 = \frac{A\tau}{T_0}$ ← Area

ⓑ $c_n = \frac{X(\omega_0 n)}{T_0} = \frac{A\tau\, \mathrm{sa}(\omega_0 n\tau/2)}{T_0}$

# Half wave symmetry

For H.W.S. ** $\boxed{x(t) = -x\left(t \pm \frac{T_0}{2}\right)}$

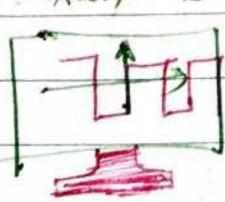
$\boxed{x(t) = -x\left(t \pm \frac{\pi}{\omega_0}\right)}$

Even + H.W.S. $a_0 = 0$, $b_n = 0$, $a_n \begin{cases} \neq 0 & n \text{ odd} \\ = 0 & n \text{ even} \end{cases}$

odd + H.W.S. $a_0 = 0$, $a_n = 0$, $b_n \begin{cases} \neq 0 & n = \text{odd} \\ = 0 & n = \text{even} \end{cases}$

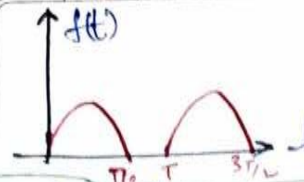
} odd harmonic in case of H.W.S.

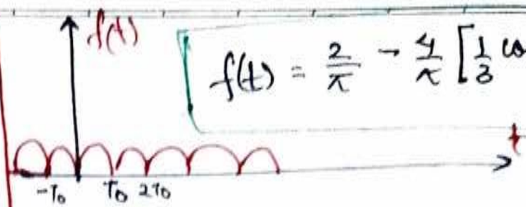
→ In exam shift Y axis in screen by puttin & then check



* 2 imp fourier coef. in pg 5-3 of B-4, To remember by Mam

** Imp. {to remember by mom}




$$f(t) = \frac{2}{\pi} - \frac{4}{\pi}\left[\frac{1}{3}\cos\omega_0 t + \frac{1}{15}\cos 2\omega_0 t \dots\right]$$

$$f(t) = \frac{A}{\pi} + \frac{A}{2}\sin\omega_0 t - \frac{2A}{3}\left[\frac{1}{3}\cos 2\omega_0 t + \frac{1}{15}\cos 4\omega_0 t + \dots\right]$$

* **Conjugate property:** If $x(t) \longleftrightarrow C_n$, then $x^*(t) \longleftrightarrow C^*_{-n}$

* **Time Inversion:** If $x(t) \longleftrightarrow C_n$, then $x(-t) \longleftrightarrow C_{-n}$

| $x(t)$ | $C_n = \frac{a_n - jb_n}{2}$ & $C_{-n} = a_n + jb_n$ |
|---|---|
| Real<br>$x(t) = x^*(t)$<br>$a_0, a_n, b_n \to$ Real | conjugate symmetric<br>$C_n = C^*_{-n}$<br>Real$[C_n]$ = Even, Img$[C_n]$ = odd |
| Real & even<br>$x(t) = x^*(t)$ {Real}<br>$x(t) = x(-t)$ {Even}<br>$\therefore b_n = 0$ | Real & Even + C.S.<br>$C_n = C^*_{-n}$ {C.S.}<br>$C_n = C_{-n}$ {Even}<br>$C_n = \frac{a_n}{2}$ {∵ Real} |
| Real & odd<br>$x(t) = x^*(t)$<br>$x(t) = x(-t)$ | Imaginary & odd + C.S.<br>$C_n = C^*_{-n}$ {C.S.}<br>$C_n = -C_{-n}$ {odd}<br>$C_n = -j\frac{b_n}{2}$ {Img} $b_n$ = Real |
| Imaginary<br>$x(t) = -x^*(t)$ $[a_0, a_n, b_n \to$ Img$]$ | conjugate antisymmetric<br>$C_n = -C^*_{-n}$ {C.A.S.} |
| Imaginary & Even<br>$x(t) = x^*(t)$ {Img}<br>$x(t) = x(-t)$ {Even} $\therefore b_n = 0$ | Imaginary & Even + C.A.S.<br>$C_n = -C^*_{-n}$ {C.A.S.}<br>$C_n = C_{-n}$ {Even}<br>$C_n = \frac{a_n}{2}$ {Img} |
| Imaginary & odd<br>$x(t) = -x^*(t)$ {Img}<br>$x(t) = -x(-t)$ {odd}<br>$\therefore a_n = 0$ | Real & odd + C.A.S.<br>$C_n = -C^*_{-n}$ {C.A.S.}<br>$C_n = -C_{-n}$ {odd}<br>$C_n = -\frac{jb_n}{2}$ {Real} $b$ = Img |

Polar form Fourier series : ⊙ $x(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(\omega_0 n t - \theta_n)$

⊙ $A_n = \sqrt{a_n^2 + b_n^2} = 2|C_n|$ , $\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right) = -\angle C_n$

⊙ $x(t) = a_0 + \sum_{n=1}^{\infty} 2|C_n| \cos[\omega_0 n t + \angle C_n]$

Properties of $C_n$ : $x(t) \longleftrightarrow C_n$ , $y(t) \longleftrightarrow d_n$

(1) Linearity : $\alpha x(t) + \beta y(t) \longleftrightarrow \alpha C_n + \beta d_n$

(2) Time shifting : $x(t \pm t_0) \longleftrightarrow e^{\pm(j\omega_0 n t_0)} C_n$

(3) time inversion : $x(-t) \longleftrightarrow C_{-n}$

(4) Conjugate property : $x^*(t) \longleftrightarrow C^*_{-n}$

(5) Frequency shifting : $e^{\pm jK\omega_0 t} x(t) \longleftrightarrow C_{n \mp K}$

(6) Time Scaling : $x(Kt) \longleftrightarrow C_n$

$\left\{ T_0' = \frac{T_0}{K} , \omega_0' = K\omega_0 \right\}$

* In time scaling property, both time period and fundamental angular frequency change but no change in E.F.S. coefficient

(7) Time differentiation : $\frac{d\,x(t)}{dt} \longleftrightarrow (j\omega_0 n) C_n$

$\frac{d^2 x(t)}{dt^2} \longleftrightarrow (j\omega_0 n)^2 C_n$

(8) Time integration :

If $x(t) \leftrightarrow C_n$ with $C_0 = 0$ , then , $\boxed{\int_{-\infty}^{t} x(\tau)\, d\tau \xleftrightarrow{E.F.S.} \frac{C_n}{j\omega_0 n}}$

$C_0 = 0$ , means $a_0 = 0$ $\xrightarrow{\text{cond}^n}$ (1) odd signal

(2) Even signal with zero area in one period.

(9) Time Convolution : $x(t) \circledast y(t) \longleftrightarrow T_0 C_n d_n$

$x(t) \longleftrightarrow C_n$

To : fundamental Time period.

(10) Time multiplication : $x(t) \cdot y(t) \longleftrightarrow C_n \circledast d_n$

$$= \sum_{K=-\infty}^{\infty} C_K d_{n-K}$$

$$= \sum_{K=-\infty}^{\infty} d_K C_{n-K}$$

# Parseval's theorem :

- $$P = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |C_n|^2$$

$$\sum_{n=-\infty}^{\infty} |C_n|^2 = |C_0|^2 + |C_{-1}|^2 + |C_{+1}|^2 + \dots\dots$$

$$= |C_0|^2 + 2|C_1|^2 + 2|C_2|^2 + \dots$$

$\left[\because |C_n|^2 = \frac{a_n^2 + b_n^2}{4} \right]$
$|C_1|^{*} = |C_{-1}|^{*}$

- $$\boxed{\begin{aligned} P &= \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = |C_0|^2 + 2|C_1|^2 + 2|C_2|^2 + \dots\dots \\ &= (a_0)^2 + \sum_{n=1}^{\infty} 2|C_n|^2 \\ &= (a_0)^2 + \sum_{n=1}^{\infty} \frac{a_n^2 + b_n^2}{2} \end{aligned}}$$

- $$\int_{T_0} |x(t)|^2 dt = T_0 \sum_{n=-\infty}^{\infty} |C_n|^2 = T_0 \left\{ |C_0|^2 + 2\sum_{n=1}^{\infty} |C_n|^2 \right\}$$

# Response of an LTI system to periodic signal

$$x(t) \longrightarrow \boxed{\begin{matrix}\text{LTI system} \\ H(\omega)\end{matrix}} \longrightarrow y(t)$$

or $[H(S)]_{s=j\omega}$

$x(t) \longleftrightarrow C_n$

$y(t) \longleftrightarrow d_n$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$y(t) = \sum_{n=-\infty}^{\infty} d_n e^{jn\omega_0 t}$$

** $\boxed{d_n = [H(\omega)]_{\omega = n\omega_0} C_n}$

Put $n=0$ to check $C_0 = 0$, don't consider $C_0 = 0$ if option doesn't have contant term

eg: $\cos\left[\frac{3\pi n}{2}\right]$ $C_0 = \cos 0 = 1 \neq 0$

# Z- transform

- $ZT\{x(n)\} = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$
- $z = re^{j\omega} = |z| e^{j\omega}$
- $ZT\{x(n)\} = DTFT\{x(n) r^{-n}\} = \sum_{n=-\infty}^{\infty} x(n) r^{-n} e^{-j\omega n}$

  for $(r=1)$ $ZT\{x(n)\} = DTFT\{x n\}$

- All values that satisfies $\sum_{n=-\infty}^{\infty} |x(n) r^{-n}| < \infty$, defines the ROC
- Causal $|z| > |z_{max}|$ (pole)
  - → System stable if ROC contains $r=1$ circle
  - → System stable if all poles inside $r=1$ circle

## Important z-transforms

(i) $a^n u(n) \longleftrightarrow \frac{z}{z-a}$ ; $|z| > |a|$

(ii) $-a^n u(-n-1) \longleftrightarrow \frac{z}{z-a}$ ; $|z| < |a|$

(iii) $\delta(n) \longleftrightarrow 1$ ; entire z plane

(iv) $n a^n u(n) \leftrightarrow \frac{az}{(z-a)^2}$ ; $|z| > |a|$

$n a^{n-1} u(n) \leftrightarrow \frac{z}{(z-a)^2}$ ; $|z| > |a|$

(v) for $\cos\omega_0 n\, u(n) / \sin(\omega_0 n)\, u(n)$ split in terms of $e^{\pm j\omega_0 n}$ & apply $a^n u(n)$

$\cos\omega_0 n\, u(n) \leftrightarrow \frac{z^2 - z\cos\omega_0}{z^2 - 2z\cos\omega_0 + 1}$ $|z| > 1$

$\sin\omega_0 n\, u(n) \leftrightarrow \frac{z\sin\omega_0}{z^2 - 2z\cos\omega_0 + 1}$ $|z| > 1$

## Properties of Z-transform

(i) Linearity

(ii) $\sum_{n=-\infty}^{\infty} f(n)\,\delta(n) = f(n)\Big|_{n=0} = f(0)$

(iii) Time shifting property

$x(n) \longleftrightarrow X(z)$ ROC : R

$x(n-n_0) \longleftrightarrow X(z)\,z^{-n_0}$ ROC : R with $|z| = 0$ or $\infty$ may be excluded or included

eg: $\delta(n-1) \leftrightarrow z^{-1}$ entire z-plane with $|z| = 0$ excluded

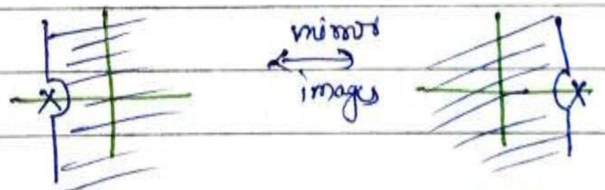
$\delta(n+1) \leftrightarrow z^{1}$ entire z-plane with $|z| = \infty$ excluded

(iv) $x(-n) \longleftrightarrow X(z^{-1})$ ROC : $R^{-1}$

eg:- $2^{-n}u(-n) \leftrightarrow \frac{z^{-1}}{z^{-1}-2} = \frac{1}{1-2z}$ $|z|^{-1} > 2$

$|z| < \frac{1}{2}$

Note: In laplace xform




In z- transform


(v) $n\,x(n) \longleftrightarrow -z\frac{d}{dz}\{X(z)\}$ ROC : R

(vi) $a^n x(n) \longleftrightarrow X\left(\frac{z}{a}\right)$ ROC : $\frac{R}{a}$ {means $z \to \frac{z}{a}$}

Note:- ① If any confusion in ROC, then go by def$^n$ (100% correct ROC)

③ if $X_1(z) + X_2(z)$ resultant signal is infinite → Common ROC concept Applicable

is finite → Go by def$^n$

④ Stability:

→ BI ⇒ BO

→ $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

→ ROC contains $r=1$ circle

(viii) $e^{j\omega_0 n} x[n] \longleftrightarrow \sum_{n=-\infty}^{\infty} x(n)\, e^{j\omega_0 n} z^{-n}$

$$= X\left(z e^{-j\omega_0}\right)$$

$$\neq X(z - z_0)$$

(vii) $e^{a/z} = 1 + az^{-1} + \frac{(az^{-1})^2}{2!} + \frac{(az^{-1})^3}{3!} + \cdots$

$\updownarrow$

$$x(n) = \frac{a^n}{n!} u(n)$$

## Residue Method

Simple pole $\quad X(z)\,(z - z_i)\, z^{n-1} \Big|_{z = i}$

Multiple pole $\quad \dfrac{1}{m-1} \dfrac{d^{m-1}}{dz^{m-1}} \left\{ X(z) \cdot (z - z_i)^m z^{n-1} \right\} \Big|_{z = z_i}$

eg: $\dfrac{z}{z-2} \longleftrightarrow \left( \dfrac{z}{z-2} \times (z-2) \cdot z^{n-1} \right) \Big|_{z=2} = 2^n\, u(n)$

$$x(n) \circledast u(n) = \sum_{k=-\infty}^{n} x(k)$$

$$x(t) \circledast u(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

IVT : $\quad x(0) = \lim_{z \to \infty} X(z)$

FVT : $\quad x(\infty) = \lim_{z \to 1} \underbrace{(1 - z^{-1})\, X(z)}$

, all poles of combⁿ must lie inside $|z| = 1$

| $h(n) \backslash x(n)$ | | 2 | 4 | -3 | 1 |
|---|---|---|---|---|---|
| 3 | 6 | 6 | 12 | -9 | 3 |
| 6 | 24 | 12 | 24 | -18 | 6 |
| → 1 | 17 | 2 | 4 | -3 | 1 |
| 4 | 23 | 8 | 16 | -12 | 4 |
| 5 | 29 | 10 | 20 | -15 | 5 |
| | | 9 | -11 | 5 | |

$y(n) = \{6, 24, 17, -3, 29 \ldots 5\}$

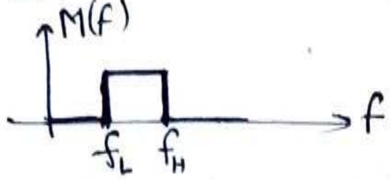
$x(n) \quad -1 \text{ to } 2$

$h(n) \quad -2 \text{ to } 2$

$y(n) \quad -3 \text{ to } 4$

# Sampling

- $\cancel{f_s \leq 2f_m}$ $f_s \geq 2f_m$
- for BPF




$$B = f_H - f_L$$

$$K = \left[\frac{f_H}{B}\right] \leftarrow \text{G.I.f}^n$$

$$f_s = \frac{2f_H}{K}$$

- Output contains
  $f_m$, $f_s \pm f_m$, $2f_s \pm f_m$
- (Gaps b/w samples) / (Guard bands)

  ★ $G = f_s - 2f_m$

- $T_s = \frac{1}{f_s}$ put $t = nT_s$ in $x(t)$

  from there $\omega_0 = ?$

  $N = \frac{2\pi}{\omega_0}$