



Introduction

An algebraic expression is an expression in one or more variables having many terms. Depending on the number of terms it may be monomials, binomials, trinomials or polynomials. Like in the case of real numbers we can also operate the algebraic expression. Previously we have learnt to add and subtract the algebraic expression. In this chapter we will learn, how to multiply or divide the algebraic expression. We will also learn how to find the linear factors of the algebraic expression as in the case of real numbers. The constants multiplied with the variables in the algebraic expression are called the coefficient of the terms. The coefficient may be positive or negative.



Concept of Monomial, Binomial and Trinomial



Monomials

The polynomial having one term is called monomial.

Illustrative EXAMPLE



$x^5, 7x, 9xy$ are monomials as they contain only one term.



Binomial

The polynomial which contains two terms is called binomials.

Illustrative EXAMPLE



$4a + 3b, 2y + 3y$ etc. are binomials because they contain two terms.



Trinomial

A trinomial is a polynomial containing three terms.

Illustrative EXAMPLE



$3x + 5y + 7z, 2a + 6b + 7c$ are the polynomials containing three terms.



Addition and Subtraction of Algebraic Expressions

While adding or subtracting the algebraic expressions we add or subtract the like terms of the expression. While adding or subtracting the like terms of the algebraic expression we add or subtract the coefficients of the algebraic expression. But in case of multiplication or division we normally multiply or divide each term of one expression with the each term of the other expression.

Illustrative EXAMPLE



Add: $3x + 5y + 8z$ and $8x + 100y - 18z$

Solution:

$$\begin{aligned} &= 3x + 5y + 8z + 8x + 100y - 18z \\ &= (3+8)x + (5+100)y + (8-18)z \\ &= 11x + 105y - 10z \end{aligned}$$



Like Terms and Unlike Terms

The terms having same order of variables are called like terms and the terms which do not have same order of variables are called unlike terms.

Illustrative EXAMPLE



$7x, -14x, 25x$ are like terms while $8x, 9xy, 78zx$ are unlike terms.



Multiplication and Division of Algebraic Expressions

Various Algebraic Relations to be used in this chapter

1. $(A+B)^2 = A^2 + B^2 + 2AB$
2. $(A-B)^2 = A^2 + B^2 - 2AB$
3. $A^2 - B^2 = (A-B)(A+B)$
4. $(A+B+C)^2 = A^2 + B^2 + C^2 + 2AB + 2BC + 2CA$
5. $A^3 + B^3 + C^3 - 3ABC = (A+B+C)(A^2 + B^2 + C^2 - AB - BC - CA)$
6. $(A+B)^3 = A^3 + B^3 + 3A^2B + 3AB^2$
7. $(A-B)^3 = A^3 - B^3 - 3A^2B + 3AB^2$
8. $(A+B)^4 = A^4 + 4A^3B + 6A^2B^2 + 4AB^3 + B^4$
9. $(A-B)^4 = A^4 - 4A^3B + 6A^2B^2 - 4AB^3 + B^4$
10. $A^3 - B^3 = (A+B)(A^2 + B^2 - AB)$
11. $A^3 - B^3 = (A-B)(A^2 + B^2 + AB)$

Illustrative EXAMPLE



Find the product of $(2x^2 - 5x + 4)$ and $(x^2 + 7x - 8)$

- (a) $(2x^4 - 9x^3 - 47x^2 + 68x + 32)$
(b) $(2x^4 + 9x^3 - 47x^2 + 68x - 32)$
(c) $(2x^4 - 9x^3 - 47x^2 + 68x + 32)$
(d) $(2x^4 - 9x^3 - 47x^2 - 68x - 32)$
(e) None of these

Answer: (b)

Illustrative EXAMPLE



The product of $(3x + 5y)$ and $(5x - 7y)$ is.

- (a) $15x^2 + 4xy - 35y^2$ (b) $5x^2 - 4xy + 35y^2$
(c) $5x^5 + 4xy + 35y^2$ (d) $x^2 - 4xy - 35y$
(e) None of these

Answer: (a)

Explanation

$$\begin{aligned} &= (3x + 5y)(5x - 7y) \\ &= 3x(5x - 7y) + 5y(5x - 7y) \\ &= 15x^2 - 21xy + 25xy - 35y^2 \\ &= 15x^2 + 4xy - 35y^2 \end{aligned}$$

Illustrative EXAMPLE



The product of $(-3x^2y)(4x^2y - 3xy^2 + 4x - 5y)$ is

- (a) $12x^3y^2 + 9x^2y^2 - 12x^4y + 15x^3y^3$
- (b) $-12x^4y^2 + 9x^3y^3 - 12x^3y + 15x^2y^2$
- (c) $-12xy^2 + 9x^3y^2 - 12x^3y + 15xy$
- (d) $-12xy^3 + 9xy - 12xy^2 + 15x^2y$
- (e) None of these

Answer: (b)

Explanation

$$(-3x^2y)(4x^2y - 3xy^2 + 4x - 5y) = -12x^4y^2 + 9x^3y^3 - 12x^4y^2 + 9x^3y^3 - 12x^3y + 15x^2y^2$$

Therefore, option (b) is correct and rest of options is incorrect.

Illustrative EXAMPLE



The product of $(3x^2 + y^2)$ and $(2x^2 + 3y^2)$ is----.

- (a) $x^3 + 10x^2y^3 + 3y^4$
- (b) $3x^2 + 5x^5y^2 + 3y^4$
- (c) $6x^5 + 10x^2y^3 + 3y^4$
- (d) $6x^4 + 11x^2y^2 + 30y^4$
- (e) None of these

Answer: (d)

Explanation

$$\begin{aligned}(3x^2 + y^2)(2x^2 + 3y^2) \\&= 6x^4 + 9x^2y^2 + 2x^2y^2 + 30y^4 \\&= 6x^4 + 11x^2y^2 + 30y^4\end{aligned}$$



Problems Based on Identities



Multiply: $(3x + 2y)(3x + 2y)$

- (a) $9x^2 + 4y^2 + 12xy$
- (b) $18x^3 + 2y^2 + 10xy$
- (c) $9x^2 + 4y^2 + 8xy$
- (d) $9x^2 + 6y^3 + 12xy$
- (e) None of these

Answer: (a)

Explanation

$$(3x + 2y)(3x + 2y) = 3x(3x + 2y) + 2y(3x + 2y) = 9x^2 + 6xy + 6xy + 4y^2 = 9x^2 + 12xy + 4y^2$$



Solve: $(4x^2 + 5)(4x^2 + 5)$

- (a) $16x^4 + 25 + 40x^2$
- (b) $16x^4 + 28 + 30x^2$
- (c) $16x^4 + 30 + 20x^2$
- (d) $16x^4 + 8 + 25x^2$
- (e) None of these

Answer: (a)

Explanation

$$(4x^2 + 5)(4x^2 + 5) = (4x^2 + 5)^2$$

$$(4x^2)^2 = 5^2 + 2(4x^2) \times 5$$

$$[\text{Using } (a + b)^2 = a^2 + b^2 + 2ab]$$

$$= 16x^4 + 25 + 40x^2$$

Multiplication of both the expressions is as same as option (a).



(4x - 7y)(4x - 7y) equal to:

- (a) $16x^2 + 56xy + 49y^2$ (b) $16x^2 - 8xy + 49y^2$
 (c) $16x^2 - 56xy - 49y^2$ (d) $16x^2 - 56xy + 49y^2$
 (e) None of these

Answer: (d)

Explanation

$$(4x - 7y)(4x - 7y) = (4x)^2 - 2 \cdot 4x \cdot 7y + (7y)^2 = 16x^2 - 56xy + 49y^2$$



Solve the expression: $\left(x - \frac{3}{x}\right)\left(x - \frac{3}{x}\right) =$

- (a) $x^2 + \frac{9}{x^2} - 18$ (b) $x^3 + \frac{9}{x^2} - 6$
 (c) $x^2 + \frac{9}{x^2} - 6$ (d) $x^2 + \frac{1}{x^2} - 6$
 (e) None of these

Answer: (c)

Explanation

$$\left(x - \frac{3}{x}\right)\left(x - \frac{3}{x}\right) = x^2 - 2 \times x \times \frac{3}{x} + \frac{9}{x^2} = x^2 + \frac{9}{x^2} - 6$$



Simplify: $(4x + 5y)(4x - 5y)$

- (a) $16x^2 - 25xy$ (b) $16x^2 - 5y^2$
 (c) $16x^2 - 25y^2$ (d) $16x^2 - y^2$
 (e) None of these

Answer: (c)

Explanation

$$(4x + 5y)(4x - 5y) = (4x)^2 - (5y)^2 = 16x^2 - 25y^2$$



Simplify: $(2x + 3y)(2x - 3y)$

- (a) $x^2 - 3y^2$ (b) $2x^2 - 3y^2$
 (c) $4x^2 - 9y^2$ (d) $x^2 - 9y^2$
 (e) None of these

Answer: (c)

Explanation

$$(2x + 3y)(2x - 3y) = (2x)^2 - (3y)^2 = 4x^2 - 9y^2$$

You Must KNOW

- ❖ An expression with one or more terms is called a polynomial.
- ❖ It can be used to find the sum of any order of the polynomials.
- ❖ A Strange Prime Number - The prime number 73,939,133 has a very strange; property. If you keep removing a digit from the right hand end of the number, each of the remaining numbers is also prime.
- ❖ At the rate of moving 1 disc per second around the clock, it is estimated to take about 585 billion years to complete the transfer of the gold discs in the tower of Brahma.

SUMMARY

- ❖ While adding or subtracting the algebraic expression we always add or subtract; the like terms.
- ❖ While multiplying we multiply each term of the expression with each term of the other expression.
- ❖ The constants multiplied with the variables are called the coefficient of the term.
- ❖ If the variables and powers of the terms are same then they are called like terms otherwise they are called unlike terms.
- ❖ An identity is an equality, which is true for all values of the variables in the equality.
- ❖ An algebraic expression consists of variables and constants.

Commonly Asked

QUESTIONS



Find the sum of the given algebraic expression.

$$2p^2q^2 - 3pq + 4, 4 + 7p^2q^2 - 8pq, 5p^2q^2 - 8 + 9pq$$

- (a) $1p^2q^2 - 5pq + 5$ (b) $14p^2q^2 - 2pq + 1$
 (c) $11p^2q^2 + 5pq + 10$ (d) $14p^2q^2 + 19pq + 17$
 (e) None of these

Answer: (b)



Find the sum of the given expressions: $-5x^2 + 3x - 8$, $4x + 7x - 2x^2$ **and** $6 - 2x + 2x^2$

- (a) $2x + 3y + z$ (b) $x + 2y + 3z$
 (c) $9x + 6y + 4z$ (d) $-5x^2 + 5x + 5$
 (e) None of these

Answer: (d)

Explanation

Writing the given expression in descending powers of x in the form of rows with the like terms as shown below and adding them column wise we get,

$$-5x^2 + 3x - 8$$

$$-2x^2 + 4x + 7$$

$$2x^2 - 2x + 6$$

$$-5x^2 + 5x + 5$$



Subtract: $3a - 3b + c$ from $4a + 5b - 3c$

- (a) $2a + 3b + c$ (b) $a + 2b + 3c$
 (c) $a + 8b - 4c$ (d) $-5x^2 + 5x + 5$
 (e) None of these

Answer: (c)

Explanation

$$\begin{array}{r} 4a + 5b - 3c \\ \pm 3a + 3b \pm c \\ \hline a + 8b - 4c \end{array}$$



What must be subtracted from $3a^2 - 6ab - 3b^2 - 1$ **to get** $4a^2 - 7ab - 4b^2 + 1$?

- (a) $-a^2 + ab + b^2 - 2$ (b) $a^2 + ab + b^2 + 2$
 (c) $a^2 - ab - b^2 + 2$ (d) $a^2 - ab - 4b^2 - 2$
 (e) None of these

Answer: (a)

Explanation

$$\begin{array}{r} 3a^2 - 6ab - 3b^2 - 1 \\ \pm 4a^2 \mp 7ab \mp 4b^2 \pm 1 \\ \hline -a^2 + ab + b^2 - 2 \end{array}$$



Find the product of $5m^2n$, $-3mnp$ **and** $-5n^2p$

- (a) $75m^3n^3p^2$ (b) $75m^2n^4p^2$
 (c) $75mn^3p^3$ (d) $75m^3np^2$
 (e) None of these

Answer: (b)

Explanation

$$5m^2n \times (-3mnp) \times (-5n^2p) = 75m^3n^4p^2$$



Simplify $3x^2y^2(5x^2 - 4xy + 6y^2)$

- (a) $15x^4y^2 - 12x^3y^3 + 18y^4x^2$
 (b) $15x^2y^2 - 12x^3y^3 + 18y^2x^2$
 (c) $15x^2y^3 - 12x^2y^2 + 6y^2y^2$
 (d) $15xy^3 - 12x^3y^3 + 18x^2y^4$
 (e) None of these

Answer: (a)

Explanation

$$3x^2y^2(5x^2 - 4xy + 6y^2) = 15x^4y^2 - 12x^3y^3 + 18y^4x^2$$



The product of $(5x^2 - 6x + 9)$ and $(2x - 3)$ is-----.

- (a) $10x^3 - 27x^2 + 36x - 27$ (b) $10x^2 - 26x^2 + 36x - 27$
 (c) $10x^3 - 27x^2 - 36x - 27$ (d) $10x^3 - 27x^2 + 36x + 27$
 (e) None of these

Answer: (a)

Explanation

$$\begin{aligned} & (5x^2 - 6x + 9)(2x - 3) \\ &= 10x^3 - 27x^2 + 36x - 27 \end{aligned}$$



The product of $(2x^2 - 5x^2 - x + 7)$ and $(3 - 2x + x^2)$ is-----.

- (a) $-3x^4 + 5x^3 - 17x + 21$ (b) $x^5 + 24x^4 + 5x^2 + x + 21$
 (c) $8x^5 + x^4 - 12 + 7x + 1$ (d) $3x^5 - 4x^4 + 1x^3 - 5x^2 - 7x + 2$
 (e) None of these

Answer: (a)

Explanation

$$\begin{aligned} &= (2x^2 - 5x^2 - x + 7)(3 - 2x + x^2) \\ &= 6x^2 - 4x^3 + 2x^4 - 15x^2 + 10x^3 - 5x^4 - 3x^2 + 21 - 14x + 7x^2 \\ &= -3x^4 + 5x^3 - 17x + 21 \end{aligned}$$



Find the quotient of polynomial if $y^3 - 6y^2 + 9y - 2$ is divided by $y - 2$.

- (a) $y^2 + 4y + 1$ (b) $y^2 - 4y + 1$
 (c) $y^2 + 4y - 1$ (d) $y^2 - 4y - 1$
 (e) None of these

Answer: (b)

Explanation

$$\begin{array}{r} y^2 - 4y + 1 \\ y - 2 \overline{) y^3 - 6y^2 + 9y - 2} \\ \underline{\pm y^3 \mp 2y^2} \\ -4y^2 + 9y \\ \underline{\mp 4y^2 \pm 8y} \\ y - 2 \\ \underline{\pm y \mp 2} \\ 0 \end{array}$$



Find the remainder when $5m^3 - 13m^2 + 15m + 7$ is divided by $4 - 3m + m^2$.

- (a) $m - 1$ (b) $m + 1$ (e) None of these
 (c) $2m + 1$ (d) $2m - 1$

Answer: (a)

Explanation

$$\begin{array}{r} 5m+2 \\ 4-3m+m^2 \overline{) 5m^3-13m^2+15m+7} \\ \underline{\pm 5m^3 \mp 15m^2 \pm 20m} \\ 2m^2-5m+7 \\ \underline{\mp 2m^2 \mp 6m+8} \\ m-1 \\ \underline{\pm m \pm 1} \\ 0 \end{array}$$



Divide $2x^2 + 3x + 1$ **by** $(x+1)$ **then quotient is:**

- (a) $2x+1$ (b) $5x+1$
(c) $4x+1$ (d) $6x+1$
(e) None of these

Answer: (a)

Explanation

$$\begin{array}{r} 2x+1 \\ x+1 \overline{) 2x^2+3x+1} \\ \underline{\pm 2x^2 \pm 2x} \\ x+1 \\ \underline{\pm x \pm 1} \\ 00 \end{array}$$

Quotient = $2x+1$

Remainder = 0



Divide $(29x - 6x^2 - 28)$ **by** $(3x - 4)$ **and find the quotient**

- (a) $2x+7$ (b) $-2x+7$
(c) $2x-7$ (d) $2x+6$
(e) None of these

Answer: (b)

Explanation

$$\begin{array}{r} -2x+7 \\ 3x-4 \overline{) -6x^2+29x-28} \\ \underline{\mp 6x^2 \pm 8x} \\ 21x-28 \\ \underline{\pm 21x \mp 28} \\ 00 \end{array}$$

Self Evaluation TEST



Duration
10 Minutes

1. If $x + \frac{1}{x} = 5$ then the value of $x^2 + \frac{1}{x^2} =$
- (a) 21 (b) 22
(c) 23 (d) 24
(e) None of these

2. If $x + \frac{1}{x} = 6$ then the value of $x^4 + \frac{1}{x^4} =$
- (a) 526 (b) 1154
(c) 1528 (d) 529
(e) None of these

3. If $ab = 6$ and $a + b = 5$ then the value of $a^2 + b^2$ is:
- (a) 11 (b) 12
(c) 13 (d) 16
(e) None of these

4. If the value of $a + \frac{1}{a} = 14$, then find the value of $a^3 + \frac{1}{a^3}$.
- (a) 2027 (b) 2072
(c) 2772 (d) 2702
(e) None of these

5. The remainder obtained on dividing $x^3 + 3x^2 - 5x + 4$ by $(x - 1)$ is:
- (a) -1 (b) 1
(c) 2 (d) 3
(e) None of these

6. What must be subtracted from $4px^2 - 2pq - 6q^2 - v + 5$ to get $-q^2 - pq - 8q^2 - 2v + 6$?
- (a) $4px^2 + 3q^2 - 3pq + v - 1$ (b) $5q^2 - 3pq + 8pq + 2q^2 + v - 1$
(c) $-5p^2 + 3pq - 2q^2 - v + 1$ (d) $-3p^2 + pq + 14q^2 + 3v - 11$
(e) None of these

7. **Divide** $(2x^3 - 5x^2 + 8x - 5)$ **by** $(2x^2 - 3x + 5)$ **and find the quotient.**

- (a) $x + 5$ (b) $x + 4$
(c) $x - 1$ (d) $x + 2$
(e) None of these
-

8. **Solve:** $(9x^2 - x + 15) \times (x^2 - x)$.

- (a) $9x^2 - 10x^3 + 16x^2 - 15x$ (b) $9x^4 + 10x^3 + 16x^2 - 15x$
(c) $9x^4 + 10x^3 - 16x^2 - 15x$ (d) $9x^4 + 10x^3 - 16x^2 + 15x$
(e) None of these
-

9. **Which one of the following expressions is equal to** $25x^2 + 4y^2 - 10xy$?

- (a) $(5x + 2)^2$ (b) $(5x - 2y)^2$
(c) $(x + 4)^2$ (d) $(9x + 3)^2$
(e) None of these
-

10. **Which one of the following expressions is equal to** $64x^2 + 9y^2 - 48xy$?

- (a) $(8x + 3y)^2$ (b) $(5x + y)^2$
(c) $(8x - 3y)^2$ (d) $(5x - 2y)^2$
(e) None of these
-

Answers – Self Evaluation Test

1. C	2. B	3. C	4. D	5. D	6. A	7. C	8. A	9. B	10. C
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Self Evaluation Test

SOLUTIONS

1. $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 \times x \times \left(\frac{1}{x}\right) = 5^2 - 2 = 23$

2. $x^4 + \frac{1}{x^4} = (x^2)^2 + \left(\frac{1}{x^2}\right)^2 = \left\{x^2 + \left(\frac{1}{x^2}\right)\right\}^2 - 2 \times x^2 \times \frac{1}{x^2} = \left\{\left(x + \frac{1}{x}\right)^2 - 2 \times x \times \frac{1}{x}\right\}^2 - 2 = \{6^2 - 2\}^2 - 2 = \{36 - 2\}^2 - 2 = \{34\}^2 - 2 = 1154$

3. $(a+b)^2 = 5^2$
 $\Rightarrow a^2 + b^2 + 2ab = 25 \quad \Rightarrow a^2 + b^2 + 2 \times 6 = 25 \quad \Rightarrow a^2 + b^2 = 25 - 12 = 13 \quad \Rightarrow a^2 + b^2 = 13$

5.

$$\begin{array}{r}
 x^2 + 4x - 1 \\
 x-1 \overline{) x^3 + 3x^2 - 5x + 4} \\
 \underline{\pm x^3 \mp x^2} \\
 4x^2 - 5x \\
 \underline{\pm 4x^2 \mp 4x} \\
 -x + 4 \\
 \underline{\mp x \pm 1} \\
 3
 \end{array}$$

We get remainder 3 in above division,

6. Let us consider, x be an expression subtracted from the following expressions to get a result as given.
 $\Rightarrow (4px^2 - 2pq - 6q^2 - v + 5) - (-q^2 + pq - 8q^2 - 2v + 6)$
 $= 4px^2 - 2pq - 6q^2 - v + 5 + q^2 - pq + 8q^2 + 2v - 6 = 4px^2 + 3q^2 - 3pq + v - 1$

7.

$$\begin{array}{r}
 x-1 \\
 2x^2 - 3x + 5 \overline{) 2x^3 - 5x^2 + 8x - 5} \\
 \underline{\pm 2x^3 \mp 3x^2 \pm 5x} \\
 -2x^2 + 3x - 5 \\
 \underline{\mp 2x^2 \pm 3x \mp 5} \\
 0
 \end{array}$$

8. $(9x^2 - x + 15)(x^2 - x) = 9x^4 - x^3 + 15x^2 - 9x^3 + x^2 - 15x = 9x^4 - 10x^3 + 16x^2 - 15x$

9. $25x^2 + 4y^2 - 10xy = (5x)^2 + (2y)^2 - 2 \cdot 5x \cdot 2y$
 $\Rightarrow (5x - 2y)^2 = 25x^2 + 4y^2 - 10xy$

10. $64x^2 + 9y^2 - 48xy = (8x)^2 + (3y)^2 - 2 \cdot 8x \cdot 3y = (8x - 3y)^2$
