CBSE Class 11 Mathematics
Sample Papers 09 (2020-21)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. Let A = {x : x $\in$N},and D = {x : x is a prime natural number} Find $A \cap D$.

OR

If A = {x : x $\in$ N}, B = {x : x $\in$ N and x is even}, C = {x : x $\in$ N and x is odd} and D = { x : x $\in$ N and x is prime} then find: A $\cap$ B

2. Name the octant in (-5, -3, -2) point lies.
3. Express as a product of sin and cos: sin 7x - sin 3x.

OR

Evaluate: $2\cos 22\frac{1^\circ}{2}\cdot\cos 67\frac{1^\circ}{2}$

4. Write the modulus of: -i
5. Three gentlemen and three ladies are candidates for two vacancies. A voter has to vote for two candidates. In how many ways can one cast his vote?

OR

If $^{n+1}C_3 = 2(^nC_2)$, find the value of n.

6. Is 184 a term of the sequence 3, 7 ,11,... ?
7. Let f : R $\rightarrow$ R : f(x) = x^2 + 3. Find the pre-images of the under f(x) = 2

OR

Find the domain of function given by $f(x) = \frac{1}{\sqrt{x+|x|}}$.

8. Find the equation of a circle whose centre is (2, -3) and radius 5.
9. If A and B be two events associated with a random experiment such that P(A) = 0.3, P(B) = 0.2 and P(A $\cap$ B) = 0.1, find $P(\bar{A}\cap B)$

OR

What is the probability that a randomly chosen two-digit positive integer is a multiple of 3?

10. If A, B and C are three arbitrary events. Find the expression for the event, in the context of A, B and C. None occurs.
11. A point is in the XZ-plane. What can you say about its y-coordinate?

12. Evaluate $^{10}C_3$

13. Find the radius of the circle in which a central angle of 60° intercepts an arc of length 37.4 cm (use $\pi = \frac{22}{7}$).

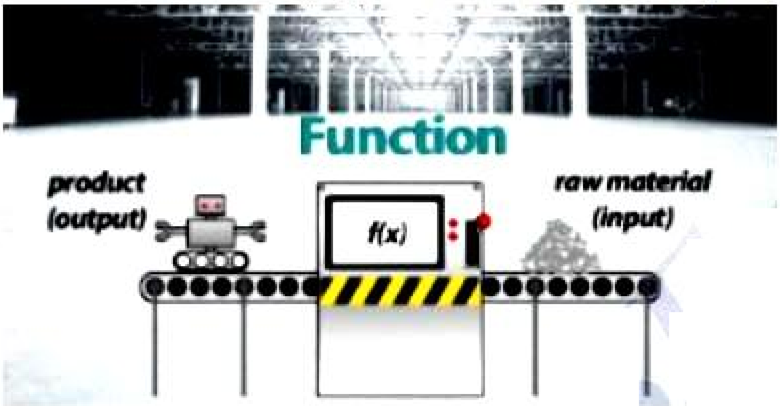
14. Evaluate: $\sin \frac{\pi}{12}$

15. Prove that $\frac{\sin x}{1+\cos x} = \tan \frac{x}{2}$.

16. Find the range of the function given by $f(x) = |x-3|$.

Section - II

17. **Read the Case study given below and attempt any 4 subparts:**



A company produces certain items. The manager in the company used to make a data record on daily basis about the cost and revenue of these items separately. The cost and revenue functions of a product are given by C(x) = 20 x + 4000 and R(x) = 60x + 2000, respectively, where x is the number of items produced and sold.

The company manager wants to know:

i. How many items must be sold to realize some profit
   a. $x < 50$
   b. $x > 50$
   c. $x \geq 50$
   d. $x \leq 50$

ii. Also if the cost and revenue functions of a product are given by C(x) = 2x + 400 and R(x) = 6x + 20 respectively, where x is the number of items produced by the manufacturer. The minimum number of items that the manufacturer must sell to realize some profit is
   a. 95
   b. 96
   c. 105
   d. 100

iii. solve for x : 12x+7< -11 OR 5x-8>40

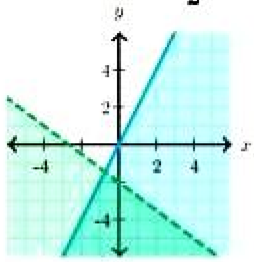
a. $x < \frac{-3}{2}$ or $x > \frac{48}{5}$

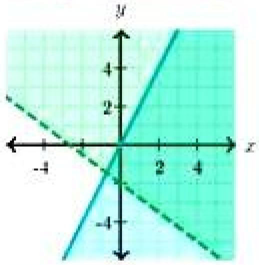
b. $\frac{-3}{2} < x < \frac{48}{5}$

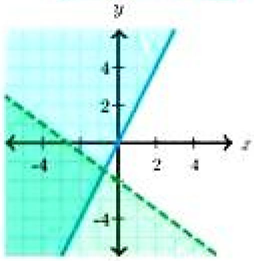
c. $x > \frac{3}{2}$ or $x < \frac{48}{5}$

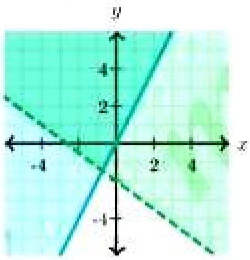
d. There are no solutions

iv. $y \leq 2x$ and $y < \frac{-3}{2}x - 2$ Which graph represents the system of inequalities

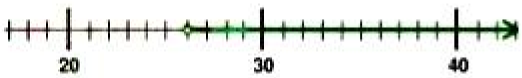
a. 

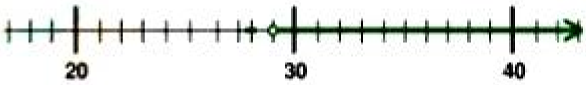
b. 

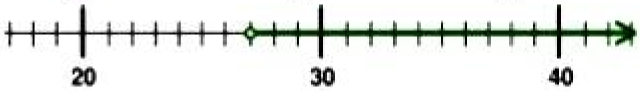
c. 

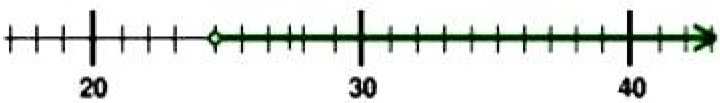
d. 

v. Graph the following inequality on the number line:x > 27

a. 

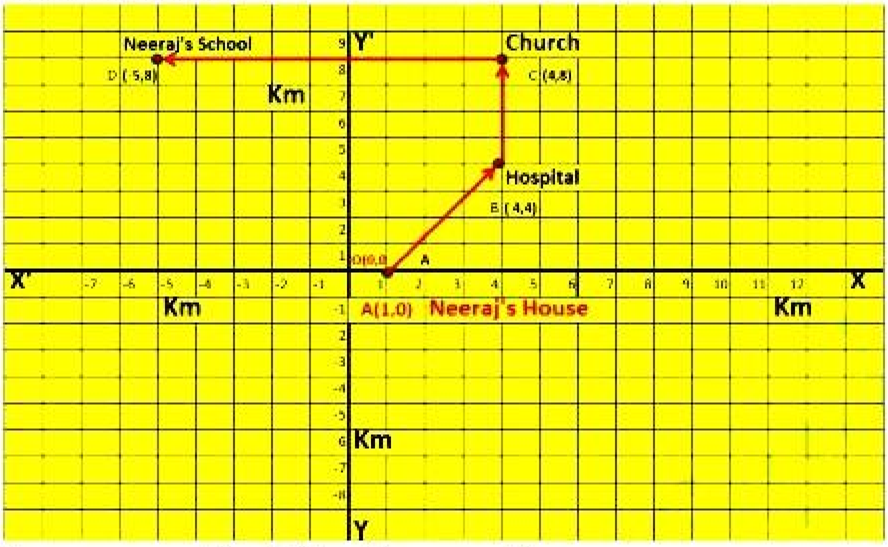
b. 

c. 

d. 

18. **Read the Case study given below and attempt any 4 sub parts:**

Neeraj's house is 1 km in the east of origin(0,0), While going to the school first he takes auto till hospital at B(4,4). From the hospital(4,4) to church (4,8) he travels by city bus. From Church C(4,8) he rides in a metro train and he reaches the school at D(-5,8). All the units are in km.



Now answer the following questions:

i. What is the slope of Neeraj's journey from home to Hospital?

a. $\frac{3}{4}$

b. $\frac{4}{3}$

c. $\frac{4}{5}$

d. $\frac{5}{4}$

ii. What is the distance of School from Hospital?

a. $\sqrt{97}$ km

b. 10 km

c. $\sqrt{145}$ km

d. 12 km

iii. What is the equation of the straight line joining the points A and D?

a. $4x - 3y = 4$ km

b. $5x + 4y = 10$

c. $4x + 3y = 4$

d. $6x + 7y = -15$

iv. What is the equation of the straight line joining church and hospital?

a. $y = 4$ km

b. $5x + 4y = 10$

c. $x = -4$

d. x = 4

v. What is the equation of the straight line joining the points A (House) and C (church)?

a. 4x - 3y = 4 km

b. 8x - 3y = 8

c. 4x + 3y = 4

d. 6x + 7y = -15

Part - B Section - III

19. Write the set in roster form: D = {x : x is an integer, $x^2 \leq 9$}

20. If f(x) = 1 - $\frac{1}{x^2}$ Then write the value of $f\left(f\left(\frac{1}{x}\right)\right)$.

OR

Let A = {1, 3, 5, 7} and B = {2, 4, 6, 8}. Let R = {(x, y): $x \in A, y \in B$ and x > y}.

i. Write R in roster form.

ii. Find dom (R) and range (R)

iii. Depict R by an arrow diagram.

21. If α and β are the solutions of the equation a tan x + b sec x = c, then show that $\tan(\alpha+\beta) = \frac{2ac}{a^2-c^2}$.

22. If the imaginary part of $\frac{2z+1}{iz+1}$ is -2, then show that the locus of the point representing z in the argand plane is a straight line.

23. For any complex number z, prove that $|\mathrm{Re}(z)| + |\mathrm{Im}(z)| \leq \sqrt{2}|z|$.

OR

If (1 + i)(1 + 2i)(1 + 3i)... (1 + ni) = (x + iy) then prove that $2 \times 5 \times 10 \times \ldots \times (1 + n^2) = (x^2 + y^2)$.

24. If f, g h are real function given by f(x) = $x^2, g(x)$ = tan x and h (x) = $\log_e$ x then write the value of (hogof) $\left(\sqrt{\frac{\pi}{4}}\right)$.

25. Evaluate: $\lim_{x\to 2}\left[\frac{1}{x-2} - \frac{2(2x-3)}{x^3-3x^2+2x}\right]$

26. Evaluate: $\lim_{x\to 0}\left(\frac{e^x-e^{-x}}{x}\right)$

27. Is E = {x : x $\in$ W, x + 3 $\leq$ 3} null set?

28. Express (7 + 5i).(7 - 5i) in the form of a + ib.

OR

If a and b are roots of the equation x^2 - px + q = 0, then write the value of $\frac{1}{a}+\frac{1}{b}$.

Section - IV

29. Differentiate kx^n from first principle.
30. In a certain lottery 10,000 tickets are sold and, ten equal prizes are awarded. What is the probability of not getting a prize if you buy (a) one ticket (b) two tickets (c) 10 tickets.
31. Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third term.

OR

If $\frac{a+bx}{a-bx}=\frac{b+cx}{b-cx}=\frac{c+dx}{c-dx}(x\neq 0)$, then show that a, b, c and d are in G.P.

32. Find the equation of the hyperbola with eccentricity $\frac{3}{2}$ and foci are $(\pm 2, 0)$.
33. How many different numbers of six digits each can be formed from the digits 4, 5, 6, 7, 8, 9 when repetition of digits is not allowed?
34. The number of a group of 400 people speak either Hindi or English or both. If 270 speak Hindi only and 50 speak both Hindi and English, then how many of them speak English only?

OR

In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all the three subjects. Find the number of students who had at least one of the three subjects.

35. If A = {1,2,3}, B = {2,3,4}, C = {1,3,4} and D = {2, 4, 5}, then verify that
$$(A\times B)\cap(C\times D)=(A\cap C)\times(B\cap D)$$

Section - V

36. If a and b are the roots x^2 - 3x + p = 0 and c, d are roots of x^2 - 12x + q = 0 where a, b, c, d form a G.P. Prove that (q + p):(q - p) = 17:15.

OR

The ratio of A M and G. M of two positive no. a and b is m : n show that
$a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right).$

37. An original frequency table with mean 11 and variance 9.9 was lost but the following table derived from it was found. Construct the original table.

Value of deviation (d)	-2	-1	0	1	2
Frequency (f)	1	6	7	4	2

OR

The mean and standard deviation of 20 observation are found to be 10 and 2 respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:

(i) If wrong item is omitted.

(ii) If it is replaced by 12

38. Solve graphically the following system of inequalities.

$x + 2y \leq 3$

$3x + 4y \geq 12$

$x \geq 0$

$y \geq 1$

OR

Solve: $\frac{|x-1|}{x+2} < 1.$

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Solution

Part - A Section - I

1. We have,
   $A = \{x : x \in N\}$
   ={1,2,3,...}, the set of natural numbers
   and D= {x : x is a prime natural number}
   ={2,3,5,7,...}
   $A \cap D = \{x : x \in A \text{ and } x \in D\}$
   = D [$\because D \in A$]

   OR

   Given: A = {x : x ∈ N}, B = {x : x ∈ N and x is even}, C = {x : x ∈ N and x is odd} and D = { x : x ∈ N and x is prime}
   Therefore ,A ∩ B = {x : x ∈ N and x is even}

2. x coordinate is -ve
   y coordinate is -ve
   z coordinate is -ve
   Therefore, this point lies in X'OY'Z' octant.
3. sin 7x - sin 6x $= 2\sin\frac{7x+3x}{2}\cos\frac{7x-3x}{2}$
   $= 2\sin\frac{10x}{2}\cos\frac{4x}{2}$ = 2 sin 5x sin 2x.

   OR

   Given, $2\cos 22\frac{1^\circ}{2}\cdot\cos 67\frac{1^\circ}{2}$
   [$\because 2\cos A\cdot\cos B =$ cos (A + B) + cos (A - B)]
   = cos $\left(22\frac{1}{2}+67\frac{1}{2}\right)^0$ + cos $\left(22\frac{1}{2}-67\frac{1}{2}\right)^\circ$
   $= cos90^o + cos45^o$ [$\because$ cos ($-\theta$) = cos θ]
   = 0 + $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

4. Let z = 0 - i. Then,

$|z|^2 = \{0^2 + (-1)^2\} = (0 + 1) = 1$

$\Rightarrow |z| = \sqrt{1} = 1$

5. Clearly, there are 6, candidates and a voter has to vote for any two of these candidates.

   So, the required number of ways is the number of ways of selecting 2, out of 6 i.e. ${}^6C_2$.

   Hence, the required number of ways = ${}^6C_2 = \frac{6!}{2!4!} = 15$.

OR

Given: ${}^{n+1}C_3 = 2({}^nC_2)$ Need to find: Value of n $\Rightarrow {}^{n+1}C_3 = 2({}^nC_2)$

$\Rightarrow \frac{(n+1)!}{3!(n+1-3)!} = 2\frac{n!}{2!(n-2)!}$

$\Rightarrow \frac{(n+1)n(n-1)}{6} = n(n-1)$

$\Rightarrow \frac{n+1}{6} = 1$

[As $n \neq 0$]

n = 5

6. Clearly, the given sequence is an A.P , as per the condition of A.P we can write , first term a = 3 and common difference d = 4. Let the nth term of the given sequence be 184. Then, $a_n = 184 \Rightarrow a + (n - 1) d = 184 \Rightarrow 3 + (n - 1) \times 4 = 184 \Rightarrow 4n = 185 \Rightarrow n = 46\frac{1}{4}$

   Since n is not a natural number. So, 184 is not a term of the given sequence.

7. Let x be the pre-image of 2. Therefore,we have,

   $f(x) = 2 \Rightarrow x^2 + 3 = 2 \Rightarrow x^2 = -1$

   But, no real value of x satisfies the equation, $x^2 = -1$

   $\therefore$ 2 does not have any pre-image under f.

OR

Here we have, $f(x) = \frac{1}{\sqrt{x+|x|}}$

If $x > 0$, $x + |x| = x + x = 2x > 0$

If $x < 0$, $x + |x| = x - x = 0$

Clearly, x = 0 is not possible.

$\therefore$ Domain of f = R^+

8. The required equation of circle is

$(x - 2)^2 + (y + 3)^2 = 5^2$

or, $x^2 + y^2 - 4x + 6y - 12 = 0$.

9. Given : P(A) = 0.3, P(B) = 0.2 , $P(A \cap B) = 0.1$

   To find : $P(\bar{A} \cap B)$

   We know : $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

   Substituting in the above formula we get,

   $P(\bar{A} \cap B) = 0.2 - 0.1$

   $P(\bar{A} \cap B) = 0.1$

OR

2-digit positive integers are 10, 11, 12, ... , 99. Thus, there are 90 such numbers. Since out of these, 30 numbers are multiple of 3, therefore, the probability that a randomly chosen positive 2-digit integer is a multiple of 3, is equal to $\frac{30}{90} = \frac{1}{3}$.

10. For none of A, B, C occurs we have. $\bar{A} \cap \bar{B} \cap \bar{C} = \overline{A \cup B \cup C}$.
11. Any point on the XZ-plane will have the coordinate (x, 0, z), so its y-coordinate is 0.
12. ${}^{10}C_3 = \frac{10\times9\times8}{3!} = \frac{10\times9\times8}{3\times2\times1} = 120$
13. Here length, l = 37.4 cm and $\theta = 60^\circ = \frac{60\pi}{180}$ radian $= \frac{\pi}{3}$

    Hence, by $r = \frac{l}{\theta}$, we have

    $r = \frac{37.4\times3}{\pi} = \frac{37.4\times3\times7}{22} = 35.7$ cm
14. Let y = $\sin\frac{\pi}{12}$, then

    $y = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

    $= \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

    $= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right) = \frac{(\sqrt{3}-1)}{2\sqrt{2}}$
15. To prove: $\frac{\sin x}{1+\cos x} = \tan\frac{x}{2}$

    Now, L.H.S $= \frac{\sin x}{1+\cos x} = \frac{2\cos\frac{x}{2}\sin\frac{x}{2}}{1+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}} = \frac{2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$

    $= \frac{2\cos\frac{x}{2}\sin\frac{x}{2}}{2\cos^2\frac{x}{2}} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$ = R.H.S

    Since: L.H.S = R.H.S

    Hence proved.
16. Here we have

    We know that, $|x - 3| \geq 0$

$\Rightarrow f(x) \geq 0$

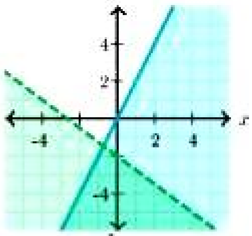
$\therefore$ Range of f = $[0, \infty)$

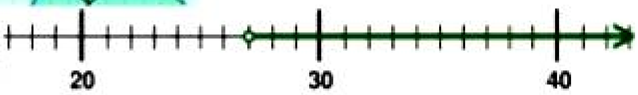
Section - II

17. i. (b) x > 50

ii. (b) 96

iii. (a) x< $\frac{-3}{2}$ or x> $\frac{48}{5}$

iv. 

v. (c) 

18. i. (b) 4/3

ii. (a) √ 97

iii. (c) 4x + 3y = 4

iv. (d) x = 4

v. (b) 8x - 3y = 8

Part - B Section - III

19. We have,the set of integers = {..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...}

$x = -4, x^2 = (-4)^2 = 16 > 9$

$x = -3, x^2 = (-3)^2 = 9$

$x = -2, x^2 = (-2)^2 = 4$

$x = -1, x^2 = (-1)^2 = 1$

$x = 0, x^2 = (0)^2 = 0$

$x = 1, x^2 = (1)^2 = 1$

$x = 2, x^2 = (2)^2 = 4$

$x = 3, x^2 = (3)^2 = 9$

$x = 4, x^2 = (4)^2 = 16$

The elements of this set are -3, -2, -1, 0, 1, 2, 3

Therefore, D = {-3, -2, -1, 0, 1, 2, 3}

20. According to the question,

$f(x) = 1 - \frac{1}{x}$

replace x by $\frac{1}{x}$

$\mathbf{f}\left(\frac{1}{x}\right) = \mathbf{1} - \frac{1}{\frac{1}{x}} = \mathbf{1} - \mathbf{x}$

Now, $f\left(f\left(\frac{1}{x}\right)\right) = 1 - \frac{1}{f\left(\frac{1}{x}\right)}$

$= 1 - \frac{1}{1-x} = \frac{1-x-1}{1-x}\mathrm{f}\left(\mathrm{f}\left(\frac{1}{x}\right)\right) = \frac{-x}{1-x} = \frac{x}{x-1}$

OR

Given that, A = {1, 3, 5, 7}, B = {2, 4, 6, 8} and R = {(x, y): $x \in A, y \in B$ and x > y}.

i. To write R is roster form

   R = {(x, y), : $x \in A, y \in B$ and x > y}

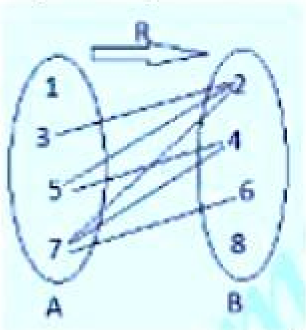
   So, R is Roster Form,

   R = {(3, 2), (5, 2), (5, 4), (7, 2), (7, 4), (7, 6)}

ii. Domain of R = {3, 5, 7}

   Range of R = {2, 4, 6}

iii. Depict R by an arrow diagram as below:



21. We have to prove: $\tan(\alpha + \beta) = \frac{2ac}{a^2 - c^2}$

It is given that

a tan x + b sec x = c ..... (i)

⇒ c - a tan x = b sec x

⇒ (c - a tan x) = b sec x

⇒ $c^2 + a^2 \tan^2 x - 2ac \tan x = b^2 (1 + \tan^2 x)$

⇒ $\tan^2 x (a^2 - b^2) - 2ac \tan x + (c^2 - b^2) = 0$ .... (ii)

It is given that a and P are the solutions of equation (i). Therefore, tan α and tan β are roots of

equation (ii).

$\therefore \tan\alpha + \tan\beta = \frac{2ac}{a^2-b^2}$ and $\tan\alpha\tan\beta = \frac{c^2-b^2}{a^2-b^2}$

Hence, $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{\frac{2ac}{(a^2-b^2)}}{1-\frac{c^2-b^2}{a^2-b^2}} = \frac{2ac}{a^2-c^2}$

22. Let z = x + iy . Then

$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{(2x+1)+i2y}{(1-y)+ix}$

= $\frac{\{(2x+1)+i2y\}}{\{(1-y)+ix\}} \times \frac{\{(1-y)-ix\}}{\{(1-y)-ix\}}$

= $\frac{(2x+1-y)+i(2y-2y^2-2x^2-x)}{1+y^2-2y+x^2}$

Thus Im$\left(\frac{2z+1}{iz+1}\right) = \frac{2y-2y^2-2x^2-x}{1+y^2-2y+x^2}$

But Im$\left(\frac{2z+1}{iz+1}\right)$ = -2 (Given)

So $\frac{2y-2y^2-2x^2-x}{1+y^2-2y+x^2}$ = -2

$\Rightarrow 2y - 2y^2 - 2x^2 - x = -2 - 2y^2 + 4y - 2x^2$

i.e., x + 2y – 2 = 0, which is the equation of straight line.

23. Let z = r $(\cos\theta + i\sin\theta)$. Then, $|z|$ = r and arg (z) = θ.

Now,

$|\text{Re}(z)| + |\text{Im}(z)| = |r\cos\theta| + |r\sin\theta|$

$\Rightarrow |\text{Re}(z)| + |\text{Im}(z)| = r\{|\cos\theta| + |\sin\theta|\}$ $[\because r = |z| > 0]$

$\Rightarrow \{|\text{Re}(z) + |\text{Im}(z)|\}^2 = r^2\{|\cos\theta| + |\sin\theta|\}^2$

$\Rightarrow \{|\text{Re}(z) + |\text{Im}(z)|\}^2 = r^2\{1 + |2\sin\theta\cos\theta|\}$

$\Rightarrow \{|\text{Re}(z) + |\text{Im}(z)|\}^2 = r^2\{1 + |\sin 2\theta|\}$

$\Rightarrow \{|\text{Re}(z) + |\text{Im}(z)|\}^2 \le r^2(1+1)$ $[\because |\sin 2\theta| \le 1]$

$\Rightarrow |\text{Re}(z)| + |\text{Im}(z)| \le \sqrt{2}r$

$\Rightarrow |\text{Re}(z)| + |\text{Im}(z)| \le \sqrt{2}|z|$

Hence proved.

OR

(1 + i)(1 + 2i)(1 + 3i) ... (1 + ni) = (x + iy)

$\Rightarrow$ |(1 + i)(1 + 2i)(1 + 3i) ... (1 + ni)|^2 = |x + iy|^2 [taking modulas and squaring both the sides]

$\Rightarrow |1+i|^2 \times |1+2i|^2 \times |1+3i|^2 \times \ldots \times |1+ni|^2$ = $(x^2 + y^2)$

$\Rightarrow (\sqrt{2})^2 \times (\sqrt{5})^2 \times (\sqrt{10})^2 \times \ldots \times (\sqrt{1+n^2})^2 = (x^2 + y^2)$

$\Rightarrow 2 \times 5 \times 10 \times \ldots \times \left(1+n^2\right) = (x^2 + y^2)$

Hence $2 \times 5 \times 10 \times \ldots \times \left(1+n^2\right) = (x^2 + y^2)$

24. As per given question,

$f(x) = x^2$; $g(x) = \tan x$; $h(x) = \log_e x$

$f(\sqrt{\frac{\pi}{4}}) = (\sqrt{\frac{\pi}{4}})^2 = \frac{\pi}{4}$

$g(f(\sqrt{\frac{\pi}{4}})) = g\left(\frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1$

$(\text{hogof})\ (\sqrt{\frac{\pi}{4}}) = h(1) = \log_e 1 = 0$

25. $\lim_{x\to 2}[\frac{1}{x-2} - \frac{2(2x-3)}{x^3-3x^2+2x}] = \lim_{x\to 2}[\frac{1}{x-2} - \frac{2(2x-3)}{x(x-1)(x-2)}]$

$= \lim_{x\to 2}[\frac{x(x-1)-2(2x-3)}{x(x-1)(x-2)}]$

$= \lim_{x\to 2}[\frac{x^2-5x+6}{x(x-1)(x-2)}]$

$= \lim_{x\to 2}\frac{(x-2)(x-3)}{x(x-1)(x-2)}[x-2 \neq 0]$

$= \lim_{x\to 2}\frac{x-3}{x(x-1)} = \frac{-1}{2}$

26. $\lim_{x\to 0}\left(\frac{e^x-e^{-x}}{x}\right) = \lim_{x\to 0}\left\{\frac{(e^x-1)-(e^{-x}-1)}{x}\right\}$

$= \lim_{x\to 0}\left\{\left(\frac{e^x-1}{x}\right) - \left(\frac{e^{-x}-1}{x}\right)\right\}$

$= \lim_{x\to 0}\left(\frac{e^x-1}{x}\right) - \lim_{x\to 0}\left(\frac{e^{-x}-1}{x}\right)$

$= 1 - \lim_{y\to 0}\left(\frac{e^y-1}{-y}\right)$

where $-x = y$ $\left[\because \lim_{x\to 0}\left(\frac{e^x-1}{x}\right) = 1\right]$

$= 1 + \lim_{y\to 0}\left(\frac{e^y-1}{y}\right) = (1 + 1) = 2$ $\left[\because \lim_{y\to 0}\left(\frac{e^y-1}{y}\right) = 1\right]$

27. We know that ,Whole numbers = 0, 1, 2, 3, ...

If we take x = 0 then x + 3 = 0 + 3 = 3

If we take x = 1 then x + 3 = 1 + 3 = 4 > 3

Therefore, 0 is the element of set E because it satisfies the given equation.

∴ It is not a null set.

28. We have,

$(7 + 5i).(7 - 5i) = 7 \times 7 + 7 \times (-5i) + 5i \times 7 + 5i\,(-5i)$

$= 49 - 35i + 35i - 25i^2$

$= 49 + 25 = 74$ [$\because i^2 = -1$]

$= 74 + 0i$

$= a + ib$

where, a = 74 and b = 0

OR

Given:

$x^2 - px + q = 0$

Also, a and b are the roots of the given equation.

Sum of the roots = a + b = p ....(i)

Product of the roots = ab = q ....(ii)

Now,

$\frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab} = \frac{p}{q}$ [Using equation (i) and (ii)]

$\Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{p}{q}$

Hence, the value of $\frac{1}{a} + \frac{1}{b}$ is $\frac{p}{q}$.

Section - IV

29. We need to find the derivative of $f(x) = kx^n$

Derivative of a function f(x) from first principle is given by

$f'(x) = \lim_{h\to 0} = \frac{f(x+h)-f(x)}{h}$ {where h is a very small positive number}

$\therefore$ derivative of $f(x) = kx^n$ is given as

$f'(x) = \lim_{h\to 0} = \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{k(x+h)^n - kx^n}{h}$

$\Rightarrow f'(x) = k\lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$

Using binomial expansion we have:

$(x + h)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1}h + {}^nC_2\, x^{n-2}h^2 + \ldots\ldots + {}^nC_n\, h^n$

$\therefore f'(x) = k\lim_{h\to 0} \frac{x^n + {}^nC_1 x^{n-1}h + {}^nC_2 x^{n-2} + \ldots + {}^nC_n h^n - x^n}{h}$

$\Rightarrow f'(x) = k\lim_{h\to 0} \frac{{}^nC_1 x^{n-1}h + {}^nC_2 x^{n-2}h^2 + \ldots + {}^nC_n h^n}{h}$

$\Rightarrow f'(x) = k\lim_{h\to 0} \frac{h({}^nC_1x^{n-1}+{}^nC_2x^{n-2}h+\ldots+{}^nC_nh^{n-1})}{h}$

$\Rightarrow f'(x) = k\lim_{h\to 0} ({}^nC_1\, x^{n-1} + {}^nC_2\, x^{n-2}\, h + \ldots.. + {}^nC_n\, h^{n-1}$

$\Rightarrow f'(x) = k({}^nC_1\, x^{n-1} + {}^nC_2\, x^{n-2}\, 0 + \ldots.. + {}^nC_n\, 0^{n-1})$

$\Rightarrow f'(x) = k\times {}^nC_1\times x^{n-1} = k\, nx^{n-1}$

Hence,

Derivative of $f(x) = kx^n$ is $k\, nx^{n-1}$

30. Given, total number of tickets = 10,000

So out of 10,000 tickets one can buy 1 ticket in ${}^{10000}C_1 = 10,000\ ways$

Given, number of prize bearing tickets = 10

So number of tickets not bearing priz = 10,000-10 = 9990

(a) Let A represent the event that one bought ticket is not getting prize.

$\therefore P(A) = \frac{9990}{10000} = \frac{999}{1000}$

Hence, probability of not getting a prize if one ticket is bought $= \frac{999}{1000}$

(b) Let B be the event that two bought tickets are not prize bearing tickets.

Now, out of 10,000 tickets one can buy 2 ticket in ${}^{10000}C_2$ ways,

and out of 9990 tickets not bearing any prize, one can buy 2 tickets in ${}^{9990}C_2$ ways.

$\therefore P(B) = \frac{{}^{9990}C_2}{{}^{10000}C_2}$

Hence, probability of not getting a prize if two tickets are bought $= \frac{{}^{9990}C_2}{{}^{10000}C_2}$

(c) Let C be the event that ten bought tickets are not prize bearing tickets.

Now,out of 10,000 tickets one can buy 2 ticket in ${}^{10000}C_{10}$ ways

and out of 9990 tickets not bearing any prize, one can buy 10 tickets in ${}^{9990}C_{10}$ ways.

$\therefore P(C) = \frac{{}^{9990}C_{10}}{{}^{10000}C_{10}}$

Hence, probability of not getting a prize if ten tickets are bought $= \frac{{}^{9990}C_{10}}{{}^{10000}C_{10}}$.

31. Let a be the first term and r be the common ratio of given G.P.

Given: a + ar = -4

$\Rightarrow a(1 + r) = -4$ ....(i)

And $a_5 = 4a_3$

$\Rightarrow ar^4 = 4ar^2$

$\Rightarrow r^2 = 4$

$\Rightarrow r = \pm 2$

Putting r = 2 $r = 2$ in eq. (i), we get a(1 + 2) = -4

$\Rightarrow a = \frac{-4}{3}$

Therefore, required G.P. is $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots$

Putting r = -2 in eq. (i), we get a(1 - 2) = -4

$\Rightarrow$ a = 4

Therefore, required G.P. is 4, -8, 16, -32,.......

OR

Taking $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx}$

$\Rightarrow$ (a + bx)(b - cx) = (b + cx) (a - bx)

$\Rightarrow ab - acx + b^2x - bcx^2 = ab - b^2x + acx - bcx^2$

$\Rightarrow 2b^2x = 2acx$

$\Rightarrow b^2 = ac$

$\Rightarrow \frac{b}{a} = \frac{c}{b}$ ....(i)

Taking $\frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$

$\Rightarrow$ (b + cx) (c - dx) = (c + dx) (b - cx)

$\Rightarrow 2c^2x = 2bdx$

$\Rightarrow c^2 = bd$

$\Rightarrow \frac{c}{b} = \frac{d}{c}$ ....(ii)

From eq. (i) and (ii), $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

32. Let the equation of hyperbola be

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ...(i)

Given, e = $\frac{3}{2}$ and foci = $(\pm ae, 0) = (\pm 2, 0)$

$\therefore$ e = $\frac{3}{2}$ and ae = 2

$\Rightarrow a \times \frac{3}{2} = 2 \Rightarrow a^2 = \frac{16}{9}$

$\because b^2 = a^2 (e^2 - 1)$

$\therefore b^2 = \frac{16}{9}\left(\frac{9}{4} - 1\right) = \frac{16}{9} \times \frac{5}{4} = \frac{20}{9}$

On putting the values of a^2 and b^2 in Eq. (i), we get

$\frac{x^2}{16/9} - \frac{y^2}{20/9} = 1$

$\Rightarrow \frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$

33. In the question, we have to find the possible number of 6 digit numbers formed by the numbers 4, 5, 6, 7, 8, 9 when repetition of digits is not allowed.

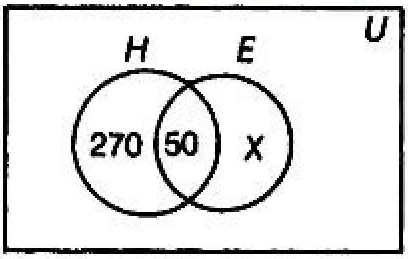
We will use the concept of multiplication because there are six sub jobs dependent on each other because a number appearing on any one place will not appear in any other place.

The number of ways in which we can form six digit numbers with the help of given numbers is $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = 720$

The numbers occurring on first place from the left have 6 choices and when one number is placed then number occurring on the second place from the left will have 5 choices and so on one fewer choice will be available to every next place till one occurs

34. Let H denotes the number of people who speaks Hindi and E denotes the number of people who speaks English.

Let x be the number of people who speak English only. The Venn diagram is shown.



Given, Total no. of people, $n(H \cup E) = 400$

and no. of people who speak both Hindi and English, $n(H \cap E) = 50$

Now,

$n(H \cup E) = n(H) + n(E) - n(H \cap E)$

Here, no. of people who speak Hindi, $n(H) = 270 + 50 = 320$ and no. of people who speak English, $n(E) = x + 50$

Thus,

$400 = (270 + 50) + (x + 50) - 50$

$\Rightarrow x = 400 - 270 - 50$

$\therefore x = 80$

Hence, number of people who speaks English only is 80.

OR

Let M be the set of students who had taken mathematics, P be the set of students who had taken physics and C be the set of students who had taken chemistry.

Here n(U) = 25, n(M) = 15, n(P) = 12, n(C) = 11, $n(M \cap C) = 5,$

$n(M \cap P) = 9, n(P \cap C) = 4, n(M \cap P \cap C) = 3,$

From the Venn diagram, we have

n(M) = a + b + d + e = 15

n(P) = b + c + e + f = 12

n(C) = d + e + f + g = 11

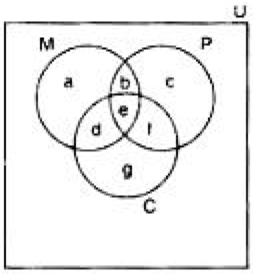
$n(M \cap C) = d + e = 5,$

$n(M \cap P) = b + e = 9,$

$n(P \cap C) = e + f = 4$

$n(M \cap P \cap C) = e = 3$

Now e = 3



$d + e = 5 \Rightarrow d + 3 = 5 \Rightarrow d = 5 - 3 \Rightarrow d = 2$

b +e = 9⇒b + 3 = 9 ⇒b = 9 - 3 ⇒ b = 6

e + f = 4 ⇒ 3 + f = 4 ⇒ f = 4 - 3 ⇒ f = 1

a +b + d + e = 15 ⇒ a + 6 + 2 + 3 = 15 ⇒ a = 15 - 14 = 4

b + c + e + f = 12 ⇒ 6 + c + 3 + 1 = 12 ⇒ c = 12 - 10 = 2

d + e + f + g = 11 ⇒ 2 + 3+ 1+ g = 11 ⇒g = 11 - 6 = 5

∴ a + b + c + d + e + f + g = 4 + g + 2 + 2+ 3 + 1+ 5 = 23

35. $A \times B = \{1,2,3\} \times \{2,3,4\}$

= {(1, 2), (1,3), (1,4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}

$C \times D = \{1,3,4\} \times \{2,4,5\}$

= {(1, 2), (1,4), (1,5), (3, 2), (3, 4), (3, 5), (4, 2), (4, 4), (4, 5)}

$\therefore (A \times B) \cap (C \times D)$= {(1, 2), (1, 4), (3, 2), (3, 4)} ...... (i)

Also, $(A \cap C) = \{1,2,3\} \cap \{1,3,4\} = \{1,3\}$

and $(B \cap D) = \{2,3,4\} \cap \{2,4,5\} = \{2,4\}$

Therefore, $(A \cap C) \times (B \cap D) = \{1,3\} \times \{2,4\}$

= {(1,2), (1,4), (3, 2), (3, 4)} ...... (ii)

From Eqs. (i) and (ii),

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

Section - V

36. Let $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = k$

$\therefore \frac{b}{a} = k$

$\Rightarrow$ b = ak

And $\frac{c}{b} = k$

$\Rightarrow$ c = bk = (ak)k = ak^2

Also $\frac{d}{c} = k$

$\Rightarrow$ d = ck = (ak^2)k = ak^3

$\because \because$ a a and b b are the roots $x^2 - 3x + p = 0$

$\therefore a + b = \frac{-(-3)}{1} = 3$

$\Rightarrow$ a + ak = 3

$\Rightarrow$ a(1 + k) = 3 ....(i)

And $ab = \frac{p}{1}$

$\Rightarrow$ a(ak) = p

$\Rightarrow$ a^2k = p ......(ii)

Also c, d are roots of $x^2 - 12x + q = 0$

$\therefore \quad c + d = \frac{-(-12)}{1} = 12$

$\Rightarrow$ $ak^2 + ak^3 = 12$

$\Rightarrow$ $ak^2(1 + k) = 12$ ....(iii)

And $cd = \frac{q}{1}$

$\Rightarrow$ $ak^2(ak^3) = q$

$\Rightarrow$ $a^2k^5 = q$ ....(iv)

Dividing eq. (iii) by eq. (i), $\frac{ak^2(1+k)}{a(1+k)} = \frac{12}{3}$

$\Rightarrow$ $k^2 = 4$

$\Rightarrow k = \pm 2$

Now $\frac{q+p}{q-p} = \frac{a^2k^5 + a^2k}{a^2k^5 - a^2k} = \frac{a^2k(k^4+1)}{a^2k(k^4-1)}$

$= \frac{(\pm 2)^4 + 1}{(\pm 2)^4 - 1} = \frac{16+1}{16-1} = \frac{17}{15}$

Therefore, (q + p):(q - p) = 17:15

OR

$\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{m}{n}$

$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$

by C and D

$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{m+n}{m-n}$

$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{m+n}{m-n}$

$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$

by C and D

$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$

Sq both side

$\frac{a}{b} = \frac{m+n+m-n+2\sqrt{m^2-n^2}}{m+n+m-n-2\sqrt{m^2-n^2}}$

$\frac{a}{b} = \frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$

37.

d	f	d^2	fd	fd^2
-2	1	4	-2	4
-1	6	1	-6	6
0	7	0	0	0
1	4	1	4	4
2	2	4	4	8
Total	$\sum f = 20$		$\sum fd = 0$	$\sum fd^2 = 22$

As we know, $\overline{x} = a + h\frac{\sum fd}{\sum f}$

$\Rightarrow 11 = a + h \times \frac{0}{20} \Rightarrow a = 11$

Also, variance, $\sigma^2 = h^2\left[\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2\right]$

$\Rightarrow 9 = h^2 \left[\frac{22}{20}\right] \Rightarrow h = 3$

Mid value is given by, $d = \frac{x-a}{h} \Rightarrow x = a + dh$

$\therefore$ Different values of x for different values of d are:

$11 - 2 \times 3, 11 - 1 \times 3, 11 - 0 \times 3, 11 + 1 \times 3, 11 + 2 \times 3$

i.e., $5, 8, 11, 14, 17.$

$\therefore$ The original frequency table is as follows:

Class	3.5-6.5	6.5-9.5	9.5-12.5	12.5-15.5	15.5-18.5
Frequency	1	6	7	4	2

OR

Here n = 20, $\bar{x} = 10$ and $\sigma = 2$

$\therefore\ \bar{x} = \frac{1}{n}\Sigma x_i \Rightarrow n \times \bar{x} = \Sigma x_i$

$\Rightarrow \Sigma x_i = 20 \times 10 = 200$

$\therefore$ Incorrect $\Sigma x_i = 200$

Now $\frac{1}{n}\Sigma x_i^2 - (\bar{x})^2 = \sigma^2$

$\Rightarrow \frac{1}{20}\Sigma x_i^2 - (10)^2 = 4 \Rightarrow \Sigma x_i^2 = 2080$

(i) If wrong item is omitted.

When wrong item 8 is omitted from the data then we have 19 observations.

$\therefore$ Correct Σx_i = Incorrect $\Sigma x_i - 8$

Correct $\Sigma x_i = 200 - 8 = 192$

$\therefore$ Correct mean = $\frac{192}{19}$ = 10.1

Also correct Σx_i^2 = Incorrect $\Sigma x_i^2 - (8)^2$

$\Rightarrow$ Correct $\Sigma x_i^2 = 2080 - 64$ = 2016

$\therefore$ Correct variance $= \frac{1}{19}\left(correct\ \Sigma x_i^2\right)$ - (correct mean)^2

$= \frac{1}{19} \times 2016 - \left(\frac{192}{19}\right)^2$

$= \frac{2016}{19} - \frac{36864}{361} = \frac{38304-36884}{361} = \frac{1440}{361}$

Correct S.D. $= \sqrt{\frac{1440}{361}} = \sqrt{3.99}$ = 1.997

(ii) If it is replaced by 12

When wrong item 8 is replaced by 12

$\therefore$ Correct Σx_i = Incorrect Σx_i - 8 + 12

= 200 - 8 + 12 = 204

$\therefore$ Correct mean = $\frac{204}{20}$ = 10.2

Also correct Σx_i^2 = Incorrect Σx_i^2 - $(8)^2$ + $(12)^2$

= 2080 - 64 + 144 = 2160

$\therefore$ Correct variance $= \frac{1}{20}(correct\ \Sigma x_1^2)$ - (correct mean)^2

$= \frac{2160}{20} - \left(\frac{204}{20}\right)^2$

$= \frac{2160}{20} - \frac{41616}{400} = \frac{43200-41616}{400} = \frac{1584}{400}$

Correct S.D. $= \sqrt{\frac{1584}{400}} = \sqrt{3.96}$ = 1.989

38. We have,

x + 2y $\leq$ 3 ... (i)

3x + 4y $\geq$ 12 ... (ii)

x $\geq$ 0 ... (iii)

y $\geq$ 1 .. (iv)

Take inequality (i) x + 2y $\leq$ 3

Corresponding linear equation is

x + 2y = 3

x	0	3
y	$\frac{3}{2}$	0

Thus, the line x + 2y = 3 passes through points $\left(0, \frac{3}{2}\right)$ and (3, 2)

Now, on putting x = 0 and y = 0 in inequality (i),we get 0 $\leq$ 3, which is true.

$\therefore$ Shaded region for inequality x + 2y $\leq$ 3 contains origin.

Take inequality (ii) 3x + 4y $\geq$ 12

Corresponding linear equation is

3x + 4y = 12

x	0	4
y	3	0

Thus, the line 3x + 4y = 12 passes through points (0, 3) and (4, 0).

Now, on putting x = 0 and y = 0 in inequality (ii), we get

0 $\geq$ 12, which is false.

$\therefore$ Shaded region for inequality 3x + 4y $\geq$ 12 does not contain origin.

Take inequality (iii)

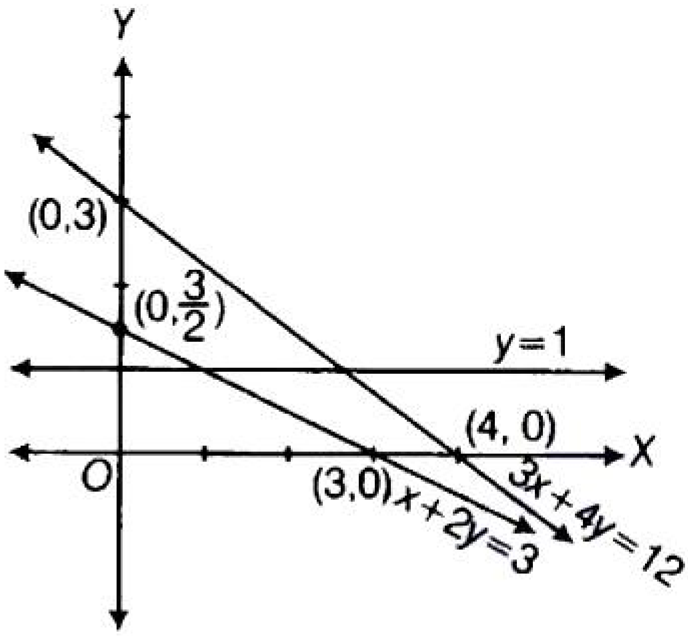
$x \geq 0$ represent the region in first quadrant.

Take inequality (iv)

$y \geq 1$

Corresponding linear equation is y = 1, which is a line parallel to X-axis at distance of 1 unit from

X-axis and this shaded region does not contain origin.



We observe that, there is no common region represented by these inequalities. We thus conclude that, there is no solution for given system of inequalities.

OR

We have,

$\frac{|x-1|}{x+2} < 1 \Rightarrow \frac{|x-1|}{x+2} - 1 < 0 \Rightarrow \frac{|x-1|-(x+2)}{x+2} < 0$

Now the following cases arise.

CASE I When x - 1 > 0 i.e. x > 1: In this case, we have |x - 1| = (x - 1)

$\therefore \frac{|x-1|-(x+2)}{x+2} < 0$

$\Rightarrow \frac{(x-1)-(x+2)}{x+2} < 0$

$\Rightarrow \frac{-3}{x+2} < 0$

$\Rightarrow x + 2 > 0$

But, $x \geq 1$. Therefore, $x > -2$ and $x \geq 1$. implies that $x \geq 1$.

For case I the solution set of the given inequation is $[1, \infty)$.

CASE II When x - 1 < 0 i.e. x < 1: In this case, we have |x - 1| = - (x - 1).

$\therefore \frac{|x-1|-(x+2)}{x+2} < 0$

$\Rightarrow \frac{-(x-1)-(x+2)}{x+2} < 0$

$\Rightarrow -\frac{2x+1}{x+2} < 0$

$\Rightarrow \frac{2x+1}{x+2} > 0$

$\Rightarrow x \in (-\infty - 2) \cup (\frac{-1}{2}, \infty)$

But, x < 1. Therefore, x $\in(-\infty$, -2) $\cup(\frac{-1}{2}, -\infty)$ and x < 1 implies that x $\in(-\infty$, -2) $\cup$ $\left(-\frac{1}{2}, 1\right)$.

For case II the solution set of the given inequation is $(-\infty$, -2) $\cup(\frac{-1}{2}$,1) Combining Case I and Case II, we obtain that the solution set of the given inequation is

$(-\infty$, -2) $\cup$ $(\frac{-1}{2}, \infty)$