Chapter 14

Factorisation

Introduction to Factorisation

Factors of Natural Numbers

A number which is a product of two or more numbers, then each of these numbers is called the factors of the given number.

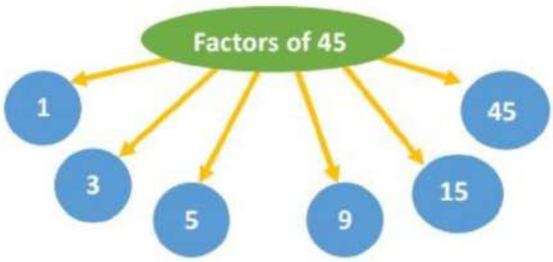
A number may be made by multiplying two or more numbers together. The numbers that are multiplied together are called factors of the final number.

Consider the number 45

$45 = 3 \times 3 \times 5$

The factors of 45 are 1, 3, 5, 9, 15, 45.

So, 3 and 5 are prime factors of 45.

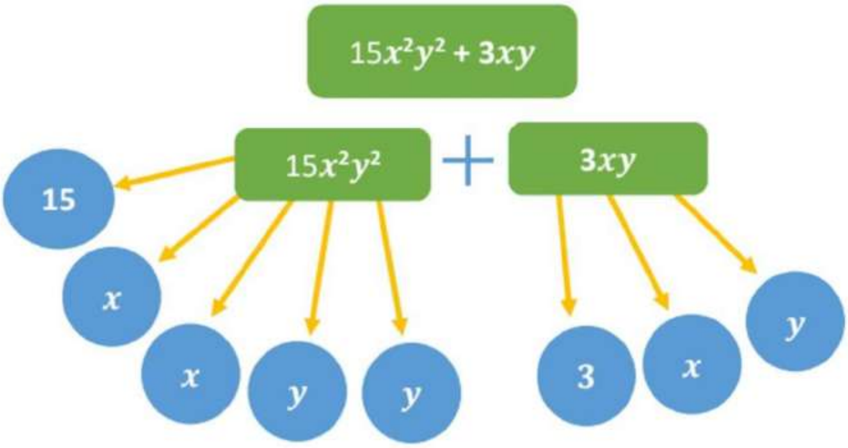


Factors of Algebraic Expressions

When an algebraic expression can be written as the product of two or more expressions, then each of these expressions is called a factor of the given expression.

In an Algebraic Expression, the terms are formed as a product of factors.

Consider the expression, $15x^2y^2 + 3xy$



Expression	Terms	Factors
$15x^2y^2 + 3xy$	$15x^2y^2$, $3xy$	$15, x, x, y, y$; $3, x, y$
The expression consists of constants and variables	Terms are separated by '+' or '-' sign	Terms are formed as a product of factors

Factorisation

The process of writing an algebraic expression as the product of two or more factors is called Factorisation. 1 is the factor of every algebraic term.

Example: $35\ x^2y^2 = 5 \times 7 \times x \times x \times y \times y$

Methods of Factorization of an algebraic expression are as follows:

1. Factorization by taking out a common factor

2. Factorization by regrouping terms

3. Factorization using Identities

4. Factorization of the form $(x + a)(x + b)$

Methods of Factorisation

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1. Factorisation by Method of Common Factor

Case 1:

When each term of the given expression contains a common monomial factor, then we will take out the common multiplier and use the distributive property.

Factorise:
$15\ a^2b + 27\ ab^2$

$15\ a^2b = 5 \times 3 \times a \times a \times b$

$27ab^2 = 3 \times 3 \times 3 \times a \times b \times b$

$15a^2b + 27ab^2$

$= (5 \times 3 \times a \times a \times b) + (3 \times 3 \times 3 \times a \times b \times b)$

$= 3 \times a \times b \times (5 \times a + 3 \times 3 \times b)$

$= 3ab(5a + 9b)$

The common factor in the two terms is 3, a and b.

Case 2:

When a polynomial is a common multiplier of each term of the given expression, then we will take out the common multiplier and use the distributive property.

Factorise:

$3(3x - 4y) - 4(3x - 4y)^2$

$3(3x - 4y) - 4(3x - 4y)^2$

$= (3x - 4y)[3 - 4(3x - 4y)]$

$= (3x - 4y)[3 - 12x + 16y]$

(3x-4y) is the common factor in both the terms.

What Is Regrouping?

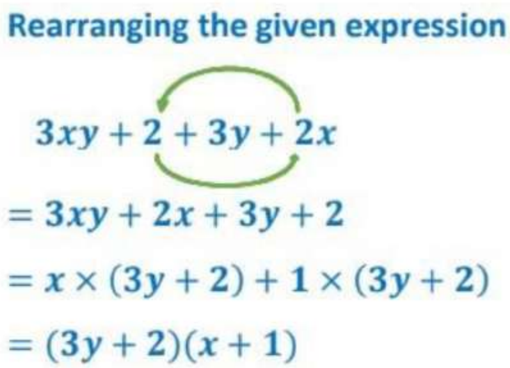
We can rearrange the terms of the expression into two pairs of terms

which share a common factor. This is called Regrouping.

For example,

Consider the expression,

$3xy + 2 + 3y + 2x$



2. Factorisation by Regrouping Terms

Consider ac + ad + bc + bd
We cannot factorise the expression by taking out the HCF of all the terms as there is no common factor except 1.

$$ac + ad + bc + bd = a\,(c + d) + b\,(c + d)$$
$$= (c + d)\,(c + d)$$

In this expression, we grouped the first and second terms and the third and fourth terms.

Sometimes, we have to make some arrangements of terms to have a common polynomial.

$x^2 - xy + y^3 - xy^2 = x^2 - xy - xy^2 + y^3$

$= x(x - y) - y^2(x - y)$

$= (x - y)(x - y^2)$ [taking $(x - y)$ common]

For example,

Factorise: $15xy + 15 + 9y + 25x$

$15xy + 15 + 9y + 25x$

$15xy + 25x + 9y + 15$

$5x \times (3y + 5) + 3 \times (3y + 5)$

$(5x + 3)(3y + 5)$

Arrange the terms of the expression in groups in such a way that the groups have a common factor.

3. Factorisation using Identities

Standard Identities:
$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b)(a - b) = a^2 - b^2$

Since Factorisation is the reverse process of multiplication, we can regard these standard identities as a special case of factorisation.

Note: For factorisation, first of all, we must take out whatever is common throughout and arrange the terms either in ascending or descending order of the power of the variables. Then, we check the following two conditions:

a) The first and last terms must be the square of some monomials.

b) The middle term must be twice the product of these monomials; whose squares are the first and last terms.

For example,

i) $4y2 + 20y + 25$

This expression is of the form $(a + b)^2 = a^2 + 2ab + b^2$

Where, $a = 2y$, $b = 5$, $2ab = 2 \times 2y \times 5$

$(2y)^2 + (2 \times 2y \times 5) + 5^2$ $[4y^2 = (2y)^2, 25 = 5^2]$

$= (2y + 5)^2$

$4y^2 = (2y)^2$

ii) $4y^2 - 12y + 9$

This expression is of the form

$a^2 - 2ab + b^2 = (a - b)^2$ $[4y^2 = (2y)^2, 9 = 3^{2,}\ 12y = 2 \times 2y \times 3]$

Where, $a = 2y$, $b = 3$, $2ab = 2 \times 2y \times 3$
$(2y)^2 - (2 \times 2y \times 3) + 3^2$

$= (2y - 3)^2$

iii) $49y^2 - 16x^2$

This expression is of the form $(a^2 - b^2) = (a + b)(a - b)$
Where, $a = 7y$, $b = 4x$ $[49y^2 = (7y)^2\ 16x^2 = (4x)^2]$

$49y^2 - 16x^2 = (7y)^2 - (4x)^2$

$= (7y + 4x)(7y - 4x)$

4. Factorisation of the form $(x + a)(x + b)$

We know that

$(x + a)\ (y + b) = x^2 + (a + b)x + ab$

So, we can also factorise the algebraic expression using the above identity.

It can also be generalized as

$(x + a)\ (y + b) = x^2 +$ (sum of the expression)x +Products of the constants

Factorise: $x^2 + 9x + 20$

$a + b = 9, ab = 20$

$20 = 1 \times 20$ or 2×10 or 4×5

As $a + b = 9$, therefore the numbers are 4 and 5

Comparing the expression with
$(x + a)(x + b) = x^2 + (a + b)x + ab$
Sum of the constants = 9

$x^2+9x + 20$

$= x^2+(5 + 4)x + 20$

$= x^2+5x + 4x + 20$

$= x(x + 5) + 4(x + 5)$

$= (x + 5)(x + 4)$

The factors of $x^2 + 9x + 20$ are $(x + 4)$ and $(x + 5)$

Factorise: $x^2 + 8x + 15$

$= (x^2 + 8x + 16) - 1$

$= \{(x)^2 + 2 \times x \times x + (4)^2\} - 1$

$= (x + 4)^2 - 1$

$= (x + 4)^2 - 1^2$

$= \{(x + 4) + 1\}\{(x + 4) - 1\}$

$= \{x + 4 + 1\}\{x + 4 - 1\}$

$= (x + 5)(x + 3)$

Factorise: $16x^4 + 7x^2 + 1$

$= (16x^4 + 8x^2 + 1) - x^2$

$= \{(4x^2)^4 + 2 \times 4x^2 \times 1 + 12\} - x^2$

$= (4x^2 + 1)^2 - x^2$

$= \{(4x^2 + 1) + x\}\{(4x^2 + 1) - x\}$

$= (4x^2 + x + 1)(4x2 - x + 1)$

Division of Algebraic Expressions

Division of Algebraic Expressions

We know that division is the inverse operation of multiplication.

For Example:

$6 \times 8 = 48$

Therefore, $48 \div 6 = 8$ Or $48 \div 8 = 6$

Similarly, in the case of Algebraic Expressions

$3x \times 2x^2 = 3 \times 2 \times x^{2+1} = 6x^3$

Therefore, $6x^3 \div 2x^2 = 3x$ Or $6x^3 \div 3x = 2x^2$

Polynomials

An algebraic expression in which the variables involved have only non- negative integral powers is called a polynomial.

For example,

$3 - 2x^2 + 4x^2y + 8y - \frac{5}{3}xy^2$ is a polynomial in two variables x and y.

Degree of a Polynomials in two variables

In a polynomial of more than one variable, the sum of the powers of the variables in each term is computed and the highest sum so obtained is called the degree of the polynomial.

For example,

$3x^4 - 2x^3y^3 + 7xy^3 - 9x + 5y - 4$ is a polynomial in x and y of degree 6, whereas

$\frac{1}{2} - 3x + 7x^2y - \frac{3}{4}x^2y^2$ is a polynomial of degree 4 in x and y.

Division of a Monomial by another Monomial

To divide a monomial by another monomial, follow these steps,

Step 1: Find the quotient of the numerical coefficients

Step 2: Find the quotient of the variables.

Step 3: Find the product of the results obtained in step 1 and step 2.

Solve:
i) $56x^4 \div 28x$ ii) $7x^2y^2 \div 14xy$

i) $56x^4 \div 28x$

$56x^4 = 2 \times 2 \times 2 \times 7 \times x \times x \times x \times x$

$28x = 2 \times 2 \times 7 \times x$

$56x^4 \div 28x = \frac{2X2X2X7XxXxXxXx}{2X2X7Xx}$

$= 2 \times x \times x \times x$

$= 2x^3$

$56x^4 \div 28x = 2x^3$

ii) $7x^2y^2 \div 14xy$

$7x^2y^2 = 7 \times x \times x \times y \times y$

$14xy = 2 \times 7 \times x \times y$

$7x^2y^2 \div 14xy = \frac{7XxXxXyXy}{2X7XxXy}$

$= \frac{xXy}{2}$

$7x^2y^2 \div 14xy = \frac{xy}{2}$

Division of a polynomial by a Monomial

Step 1: Split the polynomial to be divided into separate terms.

Step 2: Divide each term by the given monomial.

Solve:

i) $(3pq + 2pr + 4p) \div 2p$ ii) $(x^3 + 2x^2 + 3x) \div 2x$

i) $(3pq + 2pr + 4p) \div 2p$

$= \frac{(3pq + 2pr + 4p)}{2p}$

$= \frac{(3XpXqp)}{2Xp} + \frac{(2XpXr)}{2Xp} + \frac{(4Xp)}{2Xp}$

$= \frac{(3q)}{2} + r + 2$

ii) $(x^3 + 2x^2 + 3x) \div 2x$

$= \frac{((x3 + 2x2 + 3x))}{2x}$

$= (\frac{xXxXx}{2Xx}) + (\frac{2XxXx}{2Xx}) + (\frac{3Xx}{2Xx})$

$= \frac{xXx}{2x} + x + \frac{3}{2}$

$= \frac{x^2}{2} + x + \frac{3}{2}$

Division of a polynomial by a Polynomial

Solve:
i) $(10x - 25) \div (2x - 5)$
ii) $5(2x + 1)(3x + 5) \div (2x + 1)$
iii) $5xy(x^2 - y^2) \div 2x(x + y)$

i) $(10x - 25) \div (2x - 5)$

$= \frac{(10x - 25)}{(2x - 5)}$

$= 5\frac{(2x - 5)}{(2x - 5)}$

$= 5$

ii) $5(2x + 1)(3x + 5) \div (2x + 1)$

$= 5\frac{(2x + 1)(3x + 5)}{(2x + 1)}$

$= 5(3x + 5)$

iii) $5xy(x^2 - y^2) \div 2x(x + y)$
$= 5xy(x + y)(x - y) \div 2x(x + y)$

$$= \frac{5XxXy(x+y)(x-y)}{2Xx(x+y)}$$

$$= \frac{5Xy(x-y)}{2}$$

Can you find the Error?

Find and correct the errors in the following mathematical statements.

i) $x(2x + 1) = 2x^2 + 1$

ii) $(3x)^2 + 2x = 9x + 2x = 11x$

i) $x(2x + 1)$
$= 2x^2 + x$

When an expression enclosed in a bracket is multiplied by a constant (or a variable), each term of the expression has to be multiplied by that constant (or variable).

ii) $(3x)^2 + 2x$
$= 9x^2 + 2x$

When a monomial is squared, the numerical coefficient and each factor has to be squared.