Chapter 16

Probability

Exercise 16.2

Q. 1 A die is rolled. Let, E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

Answer:

When the die is rolled the sample space will consist of following outcomes

S = (1, 2, 3, 4, 5, 6)

According to the given conditions

E = (4) and

(F) = (2, 4, 6)

$E \cap F = (4) \cap (2, 4, 6)$

$\Rightarrow (4) \neq \phi$ as there is a common element between E & F

Hence E and F are not mutually exclusive event.

Q. 2 A die is thrown. Describe the following events:

(i) A: a number less than 7

(ii) B: a number greater than 7

(iii) C: a multiple of 3

(iv) D: a number less than 4

(v) E: an even number greater than 4

(vi) F: a number not less than 3

Also find A B, A B, B C, E F, D E, A – C, D – E, E ∩ F’, F’

Answer:

When the die is rolled the sample space will consist of following outcomes

S = (1, 2, 3, 4, 5, 6)

According to the given conditions

(i) A: a number less than 7

As every number on a dice is less than 7,

A = (1, 2, 3, 4, 5, 6)

(ii) B: a number greater than 7

B= (ϕ) as there is no number greater than 7 on the die.

(iii) C: a multiple of 3

C = (3, 6)

Only two numbers are multiple of 3 in the given sample space.

(iv) D: a number less than 4

D = (1, 2, 3)

(v) E: an even number greater than 4

E = (6)

(vi) F: a number not less than 3

F = (3, 4, 5, 6)

Now we need to find A U B, A ∩ B, B U C, E ∩ F, D ∩ E, D - E, A - C, E ∩ F', F'

From above we have the sets as follows: A = (1, 2, 3, 4, 5, 6)

B= (ϕ)

C= (3, 6)

D= (1, 2, 3)

E = (6)

F= (3, 4, 5, 6)

Therefore, A U B = (1, 2, 3, 4, 5, 6) U (ϕ) = (1, 2, 3, 4, 5, 6)

A ∩ B = (1, 2, 3, 4, 5, 6) ∩ (ϕ) = (ϕ)

B U C = (ϕ) U (3, 6) = (3, 6)

E ∩ F = (6) ∩ (3, 4, 5, 6) = (3, 4, 5, 6)

D ∩ E = (1, 2, 3) ∩ (6) = (ϕ)

D - E = (1, 2, 3) - (6) = (1, 2, 3)

A - C = (1, 2, 3, 4, 5, 6) - (3, 6) = (1, 2, 4, 5) F' = (3, 4, 5, 6))' = (1, 2) E ∩ F' = (6) ∩ (1, 2) = (ϕ)

Q. 3 An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

A: the sum is greater than 8,

B: 2 occurs on either die

C: the sum is at least 7 and a multiple of 3.

Which pairs of these events are mutually exclusive?

Answer:

Since a pair of dice is thrown, all the possible outcomes or sample space (S) will be as follows

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)

S =

(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

A: the sum is greater than 8

Possible sum greater than 8 are 9, 10, 11 & 12

$\therefore$ A= $\begin{cases} (3,6)\ (4,5),\ \ (5,4),\ \ (6,3),\ \ (4,6) \\ (5,5),\ \ (6,4),\ \ (5,6),\ \ (6,5),\ \ (6,6) \end{cases}$

B: 2 occurs on either die

Here possibilities are there that the number 2 will come on first die or second die or both the die simultaneously

B= $\begin{cases} (2,1),\ \ (2,2),\ \ (2,3),\ \ (2,4),\ \ (2,5),\ \ (2,6) \\ \quad (1,2),\ \ (3,2),\ \ (4,2),\ \ (5,2),\ \ (6,2) \end{cases}$

C: The sum is at least 7 and multiple of 3

So the sum can be only 9 or 12

C= $\{(3,6),\ (4,5),\ (5,4)\ (6,3)\ (6,6)\}$

For 2 elements to be mutually exclusive, then there should not be any common element amongst them

(i) $A \cap B = \phi$

As there is no common element in A and B

So A& B are mutually exclusive

(ii) $B \cap C = \phi$

Since there is no common element between

So B and C are mutually exclusive.

(iii) A ∩ C

$\Rightarrow \{(3,6),\ (4,5),\ (5,4),\ (6,3),\ (6,6)\} \neq \phi$

Since A and C has common elements,

∴ A and C are mutually exclusive.

Q. 4 Three coins are tossed once. Let A denote the event ‘three heads show”, B denote the event “two heads and one tail show”, C denote the event” three tails show and D denote the event ‘a head shows on the first coin”. Which events are

(i) mutually exclusive?

(ii) simple?

(iii) Compound?

Answer:

Here, three coins are tossed once so the possible outcomes or sample space (S) consist of

S= $\{HHH,\ HHT,\ HTH,\ THH,\ HTT,\ THT,\ TTH,\ TTT\}$

Now,

A: ‘three heads’

A= (HHH)

B: “two heads and one tail”

B= (HHT, THH, HTH)

C: ‘three tails’

C= (TTT)

D: a head shows on the first coin

D= (HHH, HHT, HTH, HTT)

(i) Mutually exclusive

$A \cap B = (HHH) \cap (HHT, THH, HTH)$

$= \phi$

Since there is no common element between A&B,

they are mutually exclusive

$A \cap C = (HHH) \cap (TTT) = \phi$

Since here is no common element,

So, A and C are mutually exclusive.

$A \cap D = (HHH) \cap (HHH, HHT, HTH, HTT)$

$= (HHH) \neq \phi$

Since there is a common element in A and D

So they are not mutually exclusive

$B \cap C = (HHT, HTH, THH) \cap (TTT) = \phi$

Since there is no common element in B & C, so they are mutually exclusive.

$B \cap D = (HHT, THH, HTH) \cap (HHH, HHT, HTH, HTT)$

$= (HHT, HTH) \neq \phi$

Since there are common elements in B & D,

so, they not mutually exclusive.

$C \cap D = (TTT) \cap (HHH, HHT, HTH, HTT) = \phi$

Since there is no common element in C & D,

So they are not mutually exclusive.

(ii) Simple event

An event is said to be simple if it has only one sample point in its sample space.

Here A = (HHH)

C = (TTT)

Both A & C have only one element,

so they are simple events.

(iii) Compound events

Is an event has more than one sample point of a sample space, it is called as compound event.

Here B= (HHT, HTH, THH)

D= (HHH, HHT, HTH, HTT)

Both B & D have more than one element,

So, they are compound events.

Q. 5 Three coins are tossed. Describe

(i) Two events which are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive.

(iii)Two events, which are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive.

(v) Three events which are mutually exclusive but not exhaustive.

Answer:

since 3 coins are tossed, the sample space will consist of following possibilities

S= (HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)

(i) Two events which are mutually exclusive.

Let A be the event of getting only head

A= (HHH)

And let B be the event of getting only Tail

B=(TTT)

So $A \cap B = \phi$

Since there is no common element in A& B so these two are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive

Now,

Let A be the event of getting exactly two tails

A= (HTT, TTH, THT)

Let B be the event of getting at least two heads

B = (HHT, HTH, THH, HHH)

Let C be the event of getting only one tail

C= (TTT)

$A \cap B$ = (HTT, TTH, THT) $\cap$ (HHT, HTH, THH, HHH)

$= \phi$

Since there is no common element in A and B hence they are mutually exclusive

$B \cap C$ = (HHT, HTH, THH, HHH) $\cap$ (TTT)

$= \phi$

Since there is no common element in B and C

So they are mutually exclusive.

$A \cap C = (HTT, TTH, THT) \cap (TTT) = \phi$

Since there is no common element in A and C,

So they mutually exclusive

Now, since A & B, B& C and A& C are mutually exclusive

∴ A, B, and C are mutually exclusive.

Also,

$A \cup B \cup C = (HTT, TTH, THT, HHT, HTH, THH, HHH, TTT) = S$

Hence A, B and C are exhaustive events.

(iii) Two events, which are not mutually exclusive

Let a be the event of getting at least two heads

A = (HHH, HHT, THH, HTH)

Let B be the event of getting only head

B= (HHH)

Now $A \cap B = (HHH) \neq \phi$

Since there is a common element in A & B,

So they are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive

Let A be the event of getting only Head

A= (HHH)

Let b be the event of getting only tail

B = (TTT)

$A \cap B = \phi$

Since there is no common element in A & B, These are mutually exclusive events.

But

A ⋃ B = (HHH) ∪ (TTT)

= $\{(HHH),\ (TTT)\}$ ≠ S

Since A ∪ B ≠ S these are not exhaustive events.

(v) Three events which are mutually exclusive but not exhaustive

Let A be the event of getting only head

A = (HHH)

Let B be the event of getting only tail

B = (TTT)

Let C be the event of getting exactly two heads

C= (HHT, THH, HTH)

Now,

A ∩ B = (HHH) ∩ (TTT) = ϕ

A ∩ C = (HHH) ∩ (HHT, THH, HTH) = ϕ

B ∩ C = (TTT) ∩ (HHT, THH, HTH) = ϕ

⇒ A ∩ B = ϕ, A ∩ C = ϕ and B ∩ C = ϕ

i.e. they are mutually exclusive

also

A ∪ B ∪ C = (HHH TTT, HHT, THH, HTH) ≠ S

Since A ∪ B ∪ C ≠ S,

So, A, B and C are not exhaustive.

It`s is hence proved that A, B and C are mutually exclusive but not exhaustive.

Q. 6 Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice 5.

Describe the events

(i) A'

(ii) not B

(iii) A or B

(iv) A and B

(v) A but not C

(vi) B or C

(viii) A B' C'

Answer:

Here 2 dice are thrown

So the sample space (S) will have following events

$$\left.\begin{matrix} (1, 1) & (1, 2) & (1, 3) & (1, 4) & (1, 5) & (1, 6) \\ (2, 1) & (2, 2) & (2, 3) & (2, 4) & (2, 5) & (2, 6) \\ (3, 1) & (3, 2) & (3, 3) & (3, 4) & (3, 5) & (3, 6) \\ (4, 1) & (4, 2) & (4, 3) & (4, 4) & (4, 5) & (4, 6) \\ (5, 1) & (5, 2) & (5, 3) & (5, 4) & (5, 5) & (5, 6) \\ (6, 1) & (6, 2) & (6, 3) & (6, 4) & (6, 5) & (6, 6) \end{matrix}\right] = S$$

According to the given conditions

A: getting an even number on the first die.

A =

(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

B: getting an odd number on the first die.

B =

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)

C: getting the sum of the numbers on the dice ≤ 5

C = {(1, 1) (1, 2) (1, 3) (1, 4) (2, 1) (2, 2) (3, 1) (3, 2) (2, 3) (4, 1)}

Now

(ii) = A

(iv) A and B (A ∩ B) = ϕ

∴ A ∩ B’ ∩ C’ = A ∩ A ∩ C’ = A ∩ C’

Q. 7 Refer to question 6 above, state true or false: (give reason for your answer)

(i) A and B are mutually exclusive

(ii) A and B are mutually exclusive and exhaustive

(iii) A = B’

(iv) A and C are mutually exclusive

(v) A and B’ are mutually exclusive.

(vi) A’, B’, C are mutually exclusive and exhaustive.

Answer:

since 2 dice are thrown the sample space will consist of following outcomes

S =

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Here A =

(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

B =

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)

And

C = {(1, 1) (1, 2) (1, 3) (1, 4) (2, 1) (2, 2) (3, 1) (3, 2) (2, 3) (4, 1)}

(i) A and B are mutually exclusive $(A \cap B) = \phi$

Since A and B has no common element,

So, A & B are mutually exclusive.

Thus, the given statement is true.

(ii) A and B are mutually exclusive and exhaustive

(1, 1)(1, 2)(1, 3)(1, 4)(1, 5)(1, 6)

(2, 1)(2, 2)(2, 3)(2, 4)(2, 5)(2, 6)

(3, 1)(3, 2)(3, 3)(3, 4)(3, 5)(3, 6) = S

(4, 1)(4, 2)(4, 3)(4, 4)(4, 5)(4, 6)

(5, 1)(5, 2)(5, 3)(5, 4)(5, 5)(5, 6)

(6, 1)(6, 2)(6, 3)(6, 4)(6, 5)(6, 6)

⇒ A ∪ B = S

And from (i) we have A and b are mutually exclusive

∴ A and B are mutually exclusive and exhaustive.

So, the statement is true.

(iii) A = B

B' = (2, 1) (2, 2)(2, 3)(2, 4)(2, 5)(2, 6)

(4, 1) (4, 2)(4, 3)(4, 4)(4, 5)(4, 6) = A

(6, 1) (6, 2)(6, 3)(6, 4)(6, 5)(6, 6)

Thus, the statement is true.

(iv) A and C are mutually exclusive

Here A ∩ C = {(2, 1) (2, 2) (2, 3) (4, 1)} ≠ ϕ

Since A and C have common element,

So, they are not mutually exclusive

So, the given statement is false.

(v) A and B' are mutually exclusive.

A ∩ B' = A ∩ A = A

∴ A ∩ B' ≠ ϕ

So, A and B' not mutually exclusive.

Hence, the given statement is false.

(vi) A', B', C are mutually exclusive and exhaustive.

Here

A’ = (1, 1) (1, 2)(1, 3)(1, 4)(1, 5)(1, 6)

(5, 1)(5, 2)(5, 3)(5, 4)(5, 5)(5, 6)

(3, 1)(3, 2)(3, 3)(3, 4)(3, 5)(3, 6)

B = (1, 1) (1, 2) (1, 3) (1, 4)(1, 5)(1, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

And C = {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (3,1), (3,2), (2,3), (4,1)}

A’ ∩ B’ = ϕ

Hence there is no common element in A’ and B’

So they are mutually exclusive.

B’ ∩ C = {(2, 1) (2, 2) (2, 3) (4, 1)} ≠ ϕ

Since there is common element between B’ and C

They are not mutually exclusive.

Now, since B’ and C are not mutually exclusive,

So A’, B’ and C are not mutually exclusive and exhaustive.

Hence the given statement is false.