Representing 3-D in 2-D

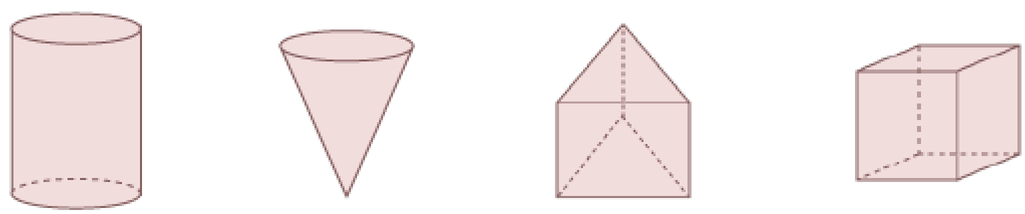
Identification of Three-dimensional Shapes

Similar to two-dimensional shapes, we have various types of three-dimensional objects, which are classified on the basis of the nature of arrangement and orientation of various faces of the shape.

Let us now look at some more examples to understand this concept better.

Example 1:

Identify the following shapes.



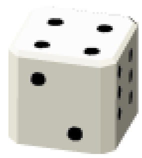
(i) **(ii)** **(iii)** **(iv)**

Solution:

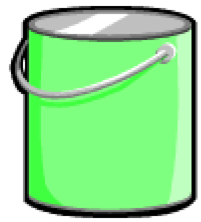
1. Cylinder
2. Cone
3. Prism
4. Cube

Example 2:

Identify the following three-dimensional shapes.



(a)



(b)

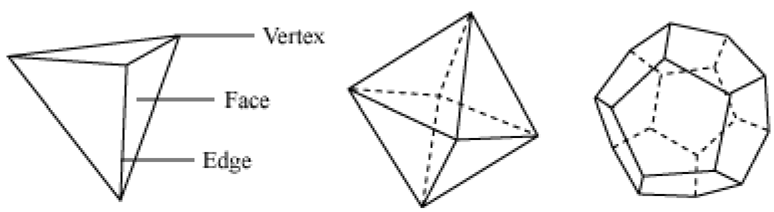
Solution:

1. This figure is cubical in shape. A dice has six sides and all of them are equal. Such types of shapes are known as cubes.

(b) This figure is cylindrical in shape.

Polyhedron

Let us look at the following solid figures.



Is there any similarity between the given solid figures?

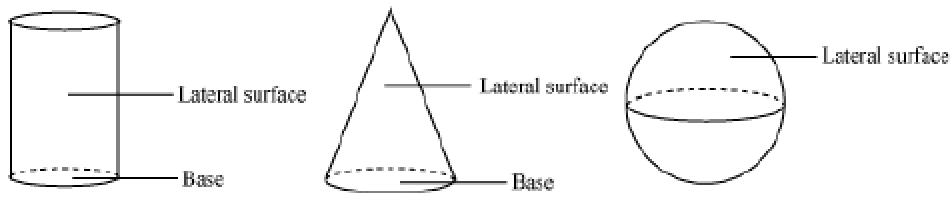
All of the above solid figures are made up of polygonal regions, lines and points. There is no curved surface in the given figures. Such solids are called **polyhedrons**.

The polygonal regions in polyhedrons are called the **faces**. The faces meet to form line segments which are known as **edges.** The edges meet at the points which are known as **vertices**.

Hence, **a polyhedron (plural polyhedra or polyhedrons) can be defined as a geometric object with flat faces and straight edges.**

Polyhedra are named according to the number of faces. For example: tetrahedron (4 faces), pentahedron (5 faces), hexahedron (6 faces) and so on. The first figure is the figure of a tetrahedron. The second figure is an octahedron.

Now, what can we say about the solids like cylinders, cones, spheres etc.?



These solids have lateral surfaces as well as curved edges. It means that these solids are not formed strictly with only flat surfaces as well as straight edges. Therefore, we can say that the solids like cylinders, cones and spheres are not polyhedrons.

We can classify polyhedrons into different categories. Let us discuss them one by one.

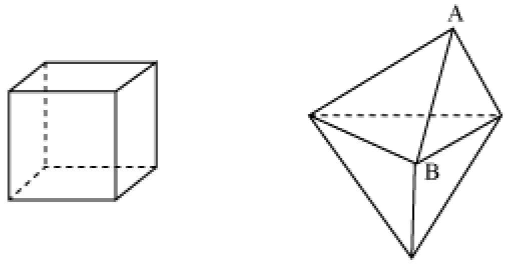
A polyhedron may be **a regular or an irregular polyhedron.**

A polyhedron is said to be **regular** if it satisfies two conditions which are given as follows.

(a) Its faces are made up of regular polygons.

(b) The same number of faces meets at each vertex.

If the polyhedron does not satisfy any one or both of the above conditions, then we can say that the polyhedron is **irregular**. To understand this concept, let us consider two solids, i.e. a cube and a hexahedron, as shown below.



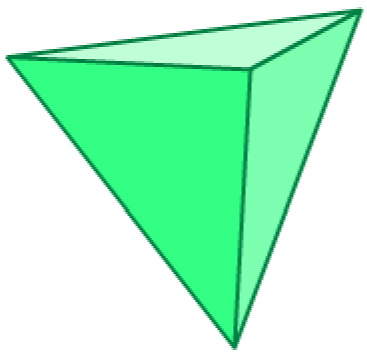
Here, we can see that the faces of the cube are congruent regular polygons (i.e. all the faces are squares of same dimension) and each vertex is formed by the same number of faces i.e. 3 faces. Therefore, a cube is a regular polyhedron.

For the above hexahedron, the faces are triangular in shape and they are congruent to each other. It means that the faces of the hexahedron are congruent regular polygons. If we look at the vertex A, we will notice that 3 faces meet at A. On the other hand, at point B, 4 faces meet. Thus, the vertices are not formed by equal number of faces. Therefore, the hexahedron is an irregular polygon.

There are only five types of regular polyhedra. They are given below.

1. Tetrahedron

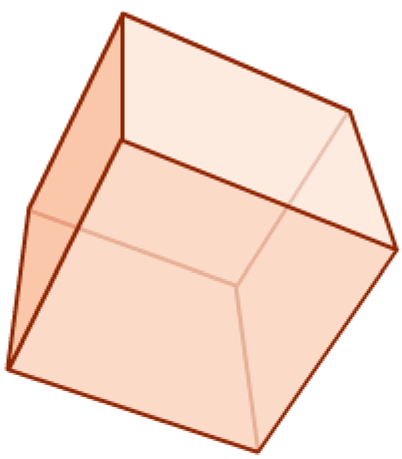
   Example:



Tetrahedron

2. Hexahedron

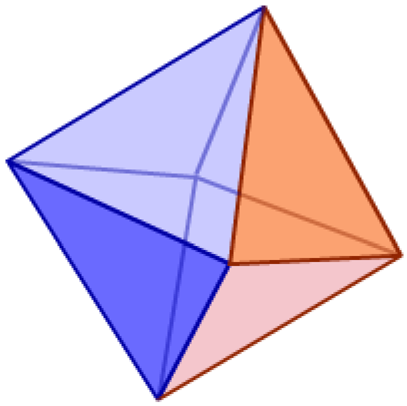
   Example:



Hexahedron

3. Octahedron

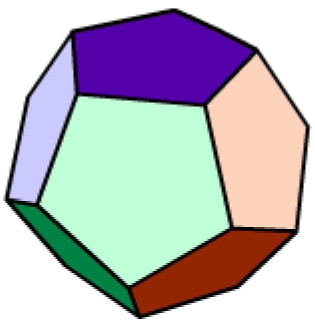
   Example:



Octahedron

4. Dodecahedron

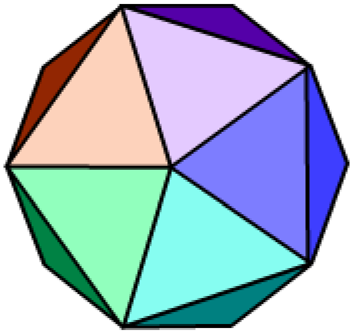
Example:



Dodecahedron

5. Icosahedron

Example:



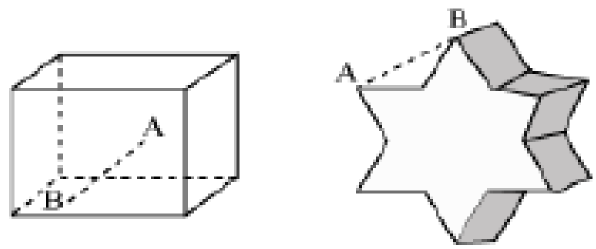
Icosahedron

These five polyhedra are known as ‘Platonic Solids’.

The polyhedron may be **a concave or a convex polyhedron.**

A polyhedron is said to be **convex,** if the line segment joining any two points of the polyhedron is contained in the interior and surface of the polyhedron. A polyhedron is said to be **concave,** if the line segment joining any two points of the polyhedron is not contained in the interior and surface of the polyhedron.

It can be understood easily by taking two solids, i.e. a cube and the star shaped polyhedron, as shown below.



For the cube, the line segment AB joining the two points A and B of the polyhedron is contained either in the polyhedron or on the surface which is clearly shown in the figure. Thus, a cube is a convex polyhedron.

For the star shaped polyhedron, the line segment AB joining the two points A and B of the polyhedron is neither contained in the polyhedron nor on the surface. Thus, the star shaped polyhedron is a concave polyhedron.

In this way, we can easily identify a polyhedron and classify it as concave or convex and regular or irregular.

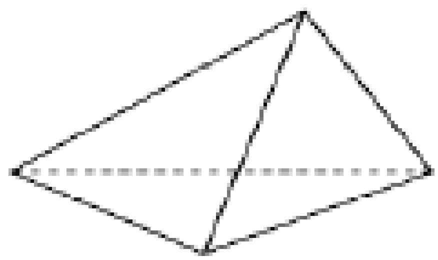
Let us discuss one more example using the concept of polyhedron.

Example 1:

How many faces are at least required to make a polyhedron?

Solution:

At least 4 triangular faces are required to make a polyhedron. For the base of the polyhedron, we require at least a three sided closed figure or a triangle (at least three sides are required to form a closed figure). Let us take another point which is not on the previous triangle. If we join the line segments from that point to each of the vertices of the base triangle, then we will have three triangles. In this way, we require 4 triangular faces to make a polyhedron as shown below.



Thus, at least four faces are required to make a polyhedron.

Prisms and Pyramids

Prisms and pyramids are both 3D figures, but have few basic differences in their structures. Let us understand these differences with the help of the given video.

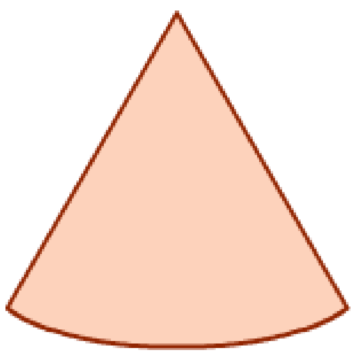
Formation of Cone from the Sector of a Circle

There exists a variety of three dimensional shapes in our surroundings. Cone is one such example of those shapes.

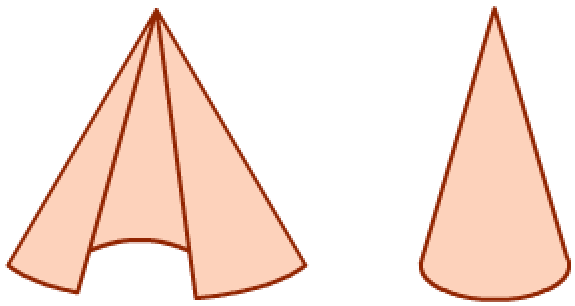
A cone is actually a pyramid having a circular base. We can make a cone by using a sector of a circle.

Let us try to make a cone.

Consider a sector of a circle (as shown below).



Let us try to roll this sector so that a circle is formed at the base. The shape obtained by doing so is an open cone (as shown below).

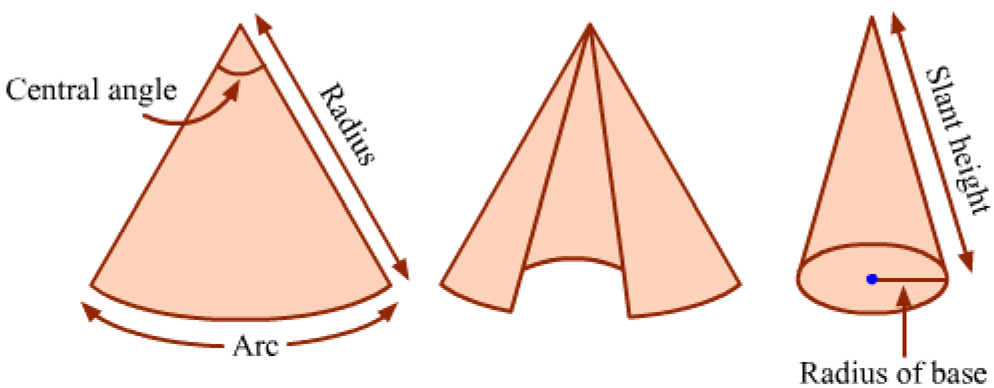


We can make a closed cone by pasting a circle at the base of the cone such that the radius of the circle is equal to the radius of the base of the cone.

Relations between the dimensions of the sector and that of the cone:

As the cone is obtained from the sector, there must be some relation between their dimensions.

Look at the images given below.



These images depict the various dimensions of a sector and a cone obtained from it.

It can be observed that if we roll a sector to get the cone, then the radius of the sector becomes the slant height of the cone and the length of the arc, of the sector, becomes the circumference of the base of the cone.

Finding the radius of the circular base of the cone, if radius of the circle from which the sector is cut off and the central angle of the sector is known:

We are quite familiar with the fact that the size of the sector of a circle is depicted in terms of its central angle.

We know that measure of the complete angle is 360°. Since the angle formed at the centre of a circle is also a complete angle, its measure is 360°.

Now, suppose we have a sector of a circle whose central angle is 30° and radius 24 cm.

$$\frac{30°}{360°} = \frac{1}{12}$$

Thus, we can say that 30° is $\frac{1}{12}$ of 360°.

We know that **measure of the central angle of the sector is proportional to the length of the arc.**

As the central angle of the sector is 1/12 times the central angle of the circle, the length of the arc of the sector is 1/12 times the circumference of the circle from which the sector is cut off.

As the length of the arc of the sector is equal to the circumference of the base of the cone, the circumference of the circular base of the cone is 1/12 times the circumference of the circle from which the sector is cut off.

As the **radius of a circle is proportional to its circumference**, the radius of the circular base of the cone is 1/12 times the radius of the circle from which the sector is cut off.

As the radius of the circle from which the cone is made (or the sector is cut off) is 24 cm, the radius of the circular base of the cone is $\frac{1}{12} \times 24 \text{ cm} = 2 \text{ cm}$

Finding the central angle of the circle if the base radius of the cone and the slant height is known:

The method to find the central angle is quite analogous to the above discussed method.

Suppose, we are required to make a cone of slant height 10 cm and base radius 2 cm, then we need to find the central angle of the sector from which the cone is made.

We know that the slant height of a cone is equal to the radius of the circle from which the sector is made.

∴ Radius of the larger circle = 10 cm

Let the radius of the smaller circle be 1/x times the radius of the larger circle.

∴ 2 cm = $\frac{1}{x} \times 10 \text{ cm}$

⇒ $x = 5$

Thus, the radius of the smaller circle is 1/5 times the radius of the larger circle.

We know that radius of a circle is proportional to its circumference, the circumference of the smaller circle is 1/5 times the circumference of the larger circle.

Circumference of the smaller circle is equal to the length of the arc of the sector.

Also, the measure of the central angle is directly proportional to the length of the arc. Thus, the measure of the central angle is 1/5 times the circle from which the sector is cut off.

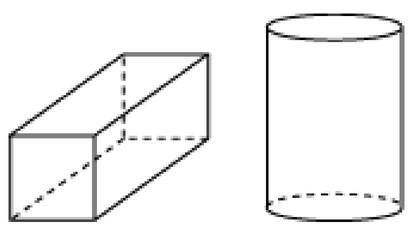
∴ Central angle = $\frac{1}{5} \times 360° = 72°$

Thus, the required cone is made by using the sector of a circle of radius 10 cm and central angle 72°.

Note: The small circle refers to the circle of the base of the cone and the large circle refers to the circle from which the sector is cut off.

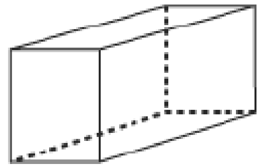
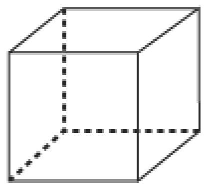
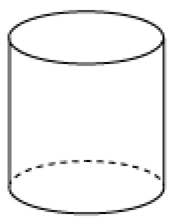
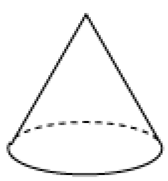
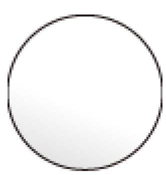
Attributes of Three-dimensional Shapes

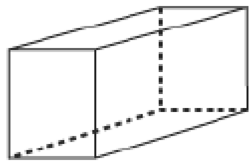
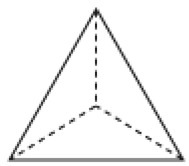
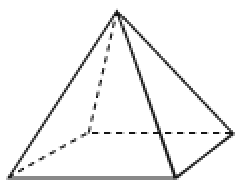
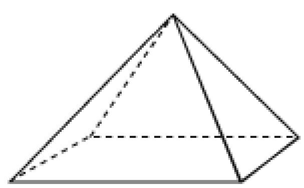
Consider the following figures.



The following table helps us to understand the attributes of three-dimensional figures.

Name	Shape	No. of straight edges	No. of faces	No. of Vertices	Example

Cuboid		12	6	8	Pencil box, notebook
Cube		12	6	8	Dice
Cylinder		None	Two flat faces and one curved surface	None	Can, cooking cylinder
Cone		None	One flat face and one curved surface	1	Softy cone, birthday cap
Sphere		None	None	None	Ball
Triangular prism		9	5	6	Laboratory prisms

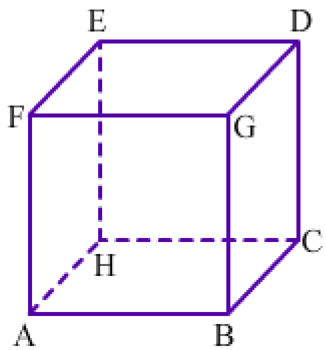
Rectangular prism		12	6	8	A rectangular glass slab
Triangular pyramid		6	4	4	
Square pyramid		8	5	5	The great pyramids of Egypt
Rectangular pyramid		8	5	5	

We know about the top and base of the solid, let us learn about its lateral face(s).

The faces that join the bases of a solid are called **lateral faces**.

We know that a cube has six square faces. Any face of the cube can be taken as its base.

Consider the cube shown below.



Here, ABCH is the base of the cube and EFGD is the top of the cube.

Rest four faces of the cube, namely ABGF, BGCD, CDEH and AHEF are the lateral faces of the cube as these faces meet the base as well as the top of the cube.

Let us now look at some examples.

Example 1:

Find the number of faces and vertices of the following three-dimensional shapes.

(i) Cuboid (ii) Cube (iii) Cylinder (iv) Cone

(v) Sphere

Solution:

1. A cuboid has six faces and eight vertices.
2. A cube has six faces and eight vertices.
3. A cylinder has two flat faces and one curved surface. It has no vertices.
4. A cone has one flat face and one curved surface. It has one vertex.
5. A sphere has no flat face. Also, it has no vertex.

Example 2:

Find the number of faces and edges of the following three-dimensional shapes.

1. **Triangular prism**
2. **Rectangular prism**
3. **Triangular pyramid**
4. **Rectangular pyramid**

Solution:

1. A triangular prism has five faces and nine edges.

2. A rectangular prism has six faces and twelve edges.
3. A triangular pyramid has four faces and six edges.
4. A rectangular pyramid has five faces and eight edges.

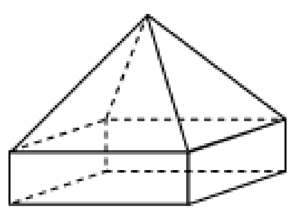
Euler's Formula

Every polyhedron has a specific number of faces, edges, and vertices (depending upon the type of polyhedron it is). However, is there any relation that can be applied to the number of faces, edges, and vertices of any polyhedron irrespective of the type of polyhedron?

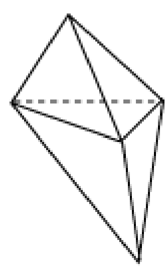
Let us discuss some examples based on Euler’s formula.

Example 1:

Verify Euler’s formula for the following solids.



(a)



(b)

Solution:

(a) The number of faces (F), edges (E), and vertices (V) for the given solid are 9, 16, and 9 respectively.

Now, $F + V - E = 9 + 9 - 16 = 18 - 16 = 2$

Thus, Euler’s formula is verified for the given solid.

(b) The number of faces (F), edges (E), and vertices (V) for the given solid are 6, 9, and 5 respectively.

Now, $F + V - E = 6 + 5 - 9$

$= 11 - 9$

$= 2$

Thus, Euler's formula is verified for the given solid.

Example 2:

Is a polyhedron with 12 faces, 21 edges, and 13 vertices possible?

Solution:

The number of faces (F), edges (E), and vertices (V) is given as 12, 21, and 13 respectively.

Now, $F + V - E = 12 + 13 - 21 = 25 - 21 = 4$

However, according to Euler's formula, the relation between the number of faces (F), the number of edges (E), and the number of vertices (V) for any polyhedron is $F + V - E = 2$

Thus, a polyhedron with 12 faces, 21 edges, and 13 vertices is not possible.

Example 3:

Find the unknown values for polyhedrons in the following table.

Face	Edge	Vertex
8	12	?
12	?	20
?	18	12

Solution:

Let the number of vertices for the first polyhedron be V.

It is given that the number of faces (F) and edges (E) for this polyhedron are 8 and 12 respectively.

Using Euler's formula, we obtain

$F + V - E = 2$

$8 + V - 12 = 2$

$V = 2 + 12 - 8$

$V = 6$

Thus, the number of vertices for the first polyhedron is 6.

Let the number of edges for the second polyhedron be E.

It is given that the number of faces (F) and vertices (V) for this polyhedron are 12 and 20 respectively.

Using Euler's formula, we obtain

$F + V - E = 2$

$12 + 20 - E = 2$

$E = 12 + 20 - 2$

$E = 30$

Thus, the number of edges for the second polyhedron is 30.

Let the number of faces for the third polyhedron be F.

It is given that the number of edges (E) and vertices (V) for this polyhedron are 18 and 12 respectively.

Using Euler's formula, we obtain

$F + V - E = 2$

$F + 12 - 18 = 2$

$F = 2 + 18 - 12$

$F = 8$

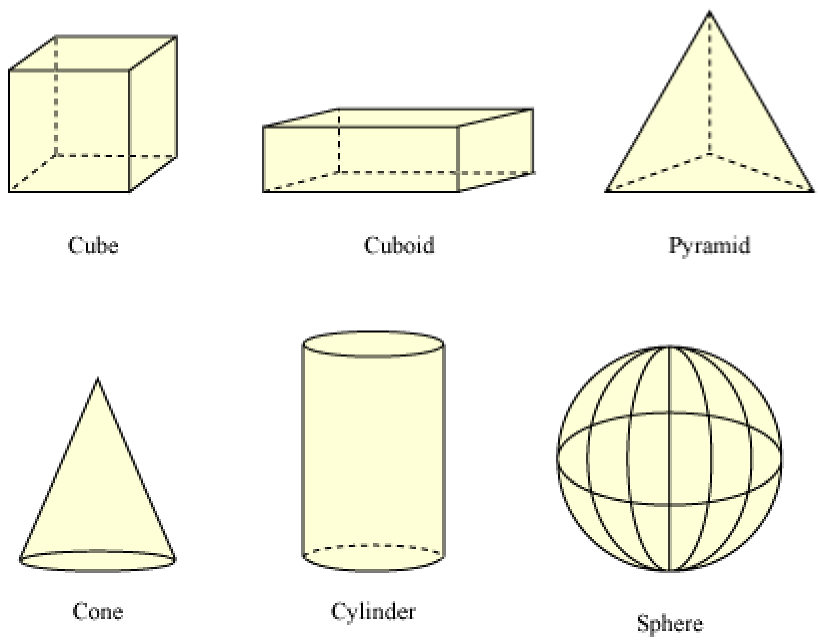
Thus, the number of faces for the third polyhedron is 8.

Nets of Three-Dimensional Figures

Look at the following figures.

What do you think about the shapes of the objects shown in the figure above?

These figures are three-dimensional in shape. Let us see some more three-dimensional figures.

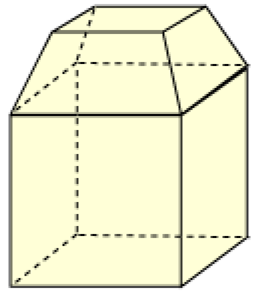


As the paper is two-dimensional, we cannot draw these figures on paper very easily. If we try to draw them on paper, then the back edges will not be shown. But we can draw the net of the three-dimensional solids.

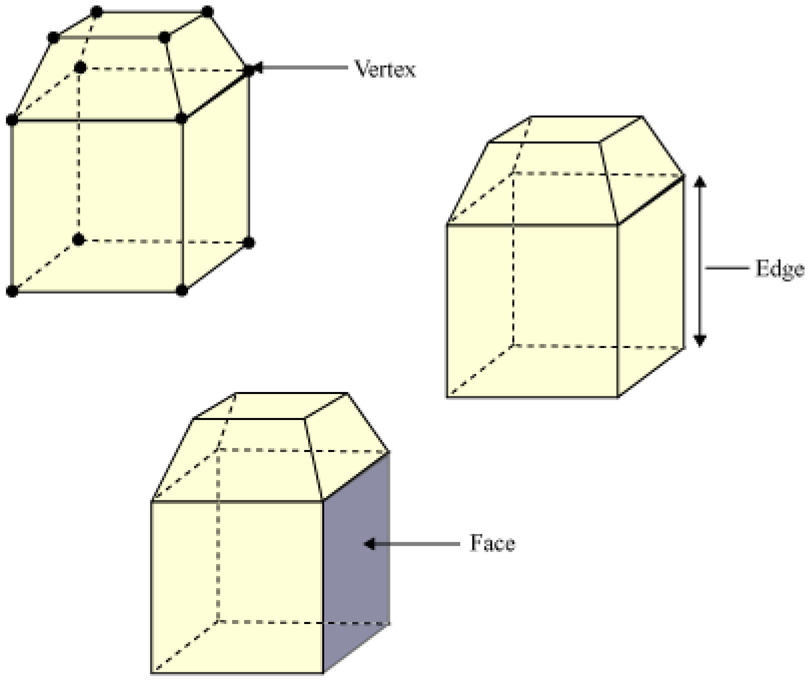
Let us learn more about three-dimensional shapes through various illustrative examples.

Example 1:

Find the number of vertices, edges, and faces in the following figure.



Solution:

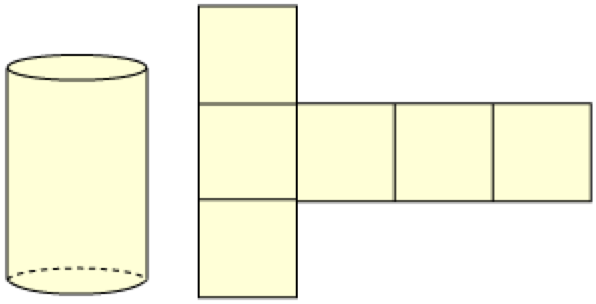


The given figure has 12 vertices, 20 edges, and 10 faces.

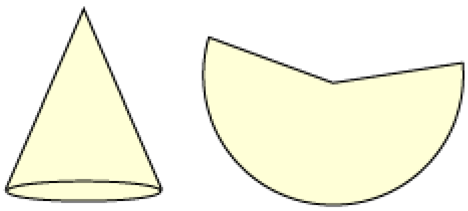
Example 2:

Match the following shapes with their appropriate nets.

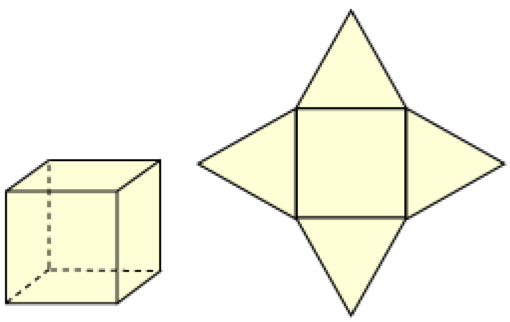
(i) (a)



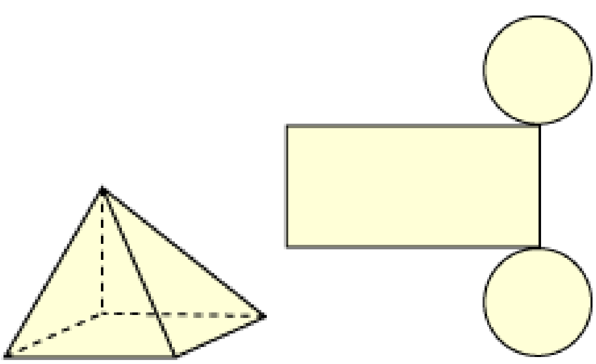
(ii) (b)



(iii) (c)

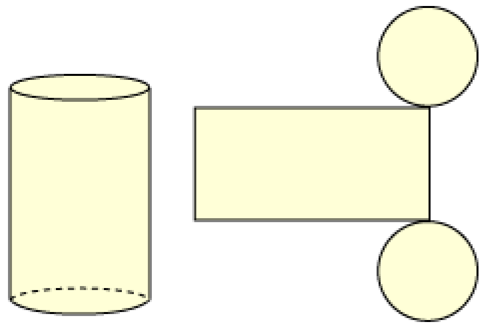


(iv) (d)

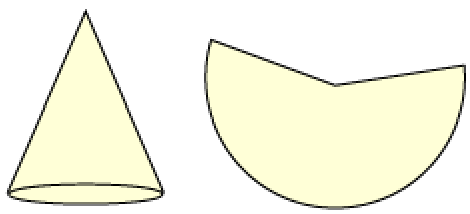


Solution:

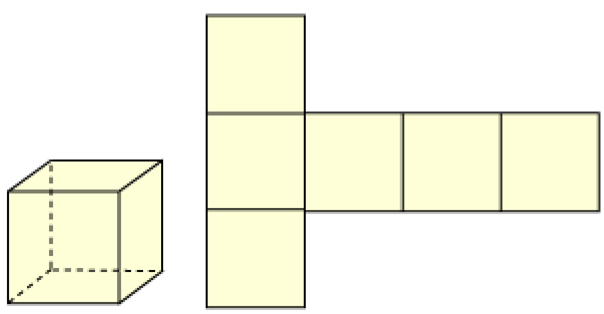
(i) (d)



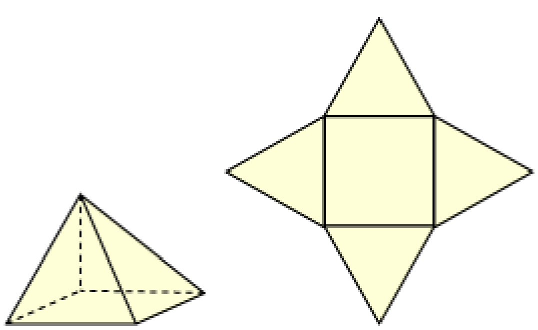
(ii) (b)



(iii) (a)



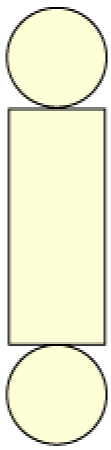
(iv) (c)



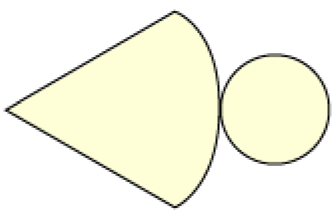
Example 3:

Draw the 3-D shapes that can be obtained from the following 2-D nets.

(i)

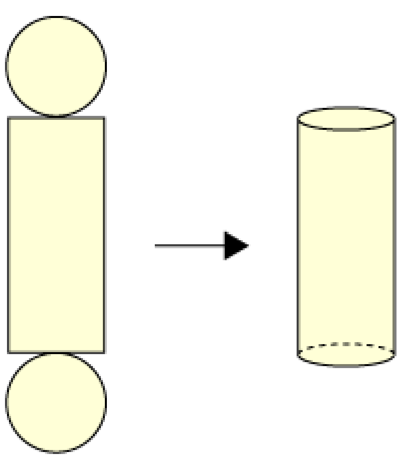


(ii)



Solution:

(i) From the given net, a cylinder will be obtained.



(ii) From the given net, a cone will be obtained.