Sample Question Paper - 10
Mathematics (041)
Class- XII, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 6 short answer type (SA1) questions of 2 marks each.
3. Section – B has 4 short answer type (SA2) questions of 3 marks each.
4. Section - C has 4 long answer-type questions (LA) of 4 marks each.
5. There is an internal choice in some of the questions.
6. Q 14 is a case-based problem having 2 sub-parts of 2 marks each.

Section A

1. Prove that: $\int\limits_{0}^{\pi/2} \frac{\cos^{1/4} x}{\left(\sin^{1/4} x+\cos^{1/4} x\right)} dx = \frac{\pi}{4}$ **[2]**

OR

Evaluate: $\int x^3 e^x dx$

2. Write the order and degree of the differential equation $y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$. **[2]**

3. For what value of' **a** the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear? **[2]**

4. Find the cartesian and vector equations of the planes through the line of intersection of the planes $\vec{r} \cdot (\hat{i} - \hat{j}) + 6 = 0$ and $\vec{r} \cdot (3\hat{i} + 3\hat{j} - 4\hat{k}) = 0$ which are at a unit distance from the origin. **[2]**

5. A can solve 90% of the problems given in a book and B can solve 70%. What is the probability that at least one of them will solve the problem, selected at random from the book? **[2]**

6. An electronic assembly consists of two sub-systems say A and B. From previous testing procedures, the following probabilities are assumed to be known: **[2]**

P (A fails) = 0.2

P (B fails alone) = 0.15

P (A and B fail) = 0.15

Evaluate the following probabilities.

(1) $P\left(\overline{A}|\overline{B}\right)$

(2)P(A fails alone).

Section B

7. Evaluate: $\int \frac{(x^2+1)}{(x-1)^2(x+3)} dx.$ **[3]**

8. Solve the following differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that y = 1, when x = **[3]**

0.

OR

Show that the family of curves for which the slope of the tangent at any point (x, y) on it is $\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$, is given by $x^2 - y^2 = cx$.

9. Find the value of λ so that the four points A, B, C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}, -\hat{j}-\hat{k}, 3\hat{i}+\lambda\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$, respectively are coplanar. **[3]**

10. Find the foot of perpendicular from the point (2, 3, -8) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also, find the perpendicular distance from the given point to the line. **[3]**

OR

Find the distance of the point $(-1,-5,-10)$ from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+\hat{2}k+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

Section C

11. Evaluate: $\int\frac{(3\sin x-2)\cos x}{5-\cos^2 x-4\sin x}dx$ **[4]**

12. Find the area common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6$ ax. **[4]**

OR

Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$

13. By computing the shortest distance determine whether the pairs of lines intersect or not: $\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j})+\mu(\hat{i}+\hat{j}-\hat{k})$ **[4]**

CASE-BASED/DATA-BASED

14. The probability that a certain person will buy a shirt is 0.2, the probability that he will buy a trouser is 0.3, and the probability that he will buy a shirt given that he buys a trouser is 0.4. **[4]**

i. Find the probability that he will buy both a shirt and a trouser.
ii. Find also the probability that he will buy a trouser given that he buys a shirt.

Solution

MATHEMATICS 041

Class 12 - Mathematics

Section A

1. Let $y=\int_0^{\pi/2} \frac{\cos^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x+\cos^{\frac{1}{4}} x} dx$ ..... (i)

   Use King theorem of definite integral

   $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

   $y=\int_0^{\pi/2} \frac{\cos^{\frac{1}{4}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{1}{4}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{1}{4}}\left(\frac{\pi}{2}-x\right)} dx$

   $y=\int_0^{\pi/2} \frac{\sin^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x+\cos^{\frac{1}{4}} x} dx$ .... (ii)

   Adding eq.(i) and eq.(ii), we get

   $2y=\int_0^{\pi/2} \frac{\cos^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x+\cos^{\frac{1}{4}} x} dx+\int_0^{\pi/2} \frac{\sin^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x+\cos^{\frac{1}{4}} x} dx$

   $2y=\int_0^{\pi/2} \frac{\cos^{\frac{1}{4}} x+\sin^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x+\cos^{\frac{1}{4}} x} dx$

   $2y=\int_0^{\pi/2} 1dx$

   $2y=(x)_0^{\frac{\pi}{2}}$

   $y=\frac{\pi}{4}$

   OR

   Let I = $\int \underset{I}{x^3} \underset{II}{e^x} dx$, then we have

   $I=x^3 e^x-\int 3x^2 e^x dx = x^3 e^x - 3\int \underset{I}{x^3} \underset{II}{e^x} dx$

   $\Rightarrow I = x^3 e^x - 3\{x^2 e^x - \int 2x e^x dx\} = x^3 e^x - 3\{x^2 e^x - 2\int \underset{I}{x} \underset{II}{e^x} dx\}$

   $\Rightarrow I = x^3 e^x - 3[x^2 e^x - 2\{x e^x - \int 1\cdot e^x dx\}]$

   $\Rightarrow I = x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C$

   $\Rightarrow I = (x^3 - 3x^2 + 6x - 6) e^x + C$

2. We have

   y= $x\frac{dy}{dx}+a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

   $\left(y-x\frac{dy}{dx}\right)=a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

   Squaring both the sides,

   $y^2-x^2\left(\frac{dy}{dx}\right)^2-2xy\left(\frac{dy}{dx}\right)=a^2\left(1+\left(\frac{dy}{dx}\right)^2\right)$

   $x^2\left(\frac{dy}{dx}\right)^2-a^2\left(\frac{dy}{dx}\right)^2-2xy\left(\frac{dy}{dx}\right)+y^2-a^2=0$

   $\left(\frac{dy}{dx}\right)^2\left(x^2-a^2\right)-2xy\left(\frac{dy}{dx}\right)+y^2-a^2=0$

   The highest order differential coefficient is 2 so,

   Order of differential equation is 1

   Degree of differential equation is 2

3. Here,it is given that two vectors, let $\vec{p}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{q}=a\hat{i}+6\hat{j}-8\hat{k}$ are collinear

   Since the given vectors are collinear, we have, $\vec{p}=\lambda\vec{q}$

   $\Rightarrow 2\hat{i}-3\hat{j}+4\hat{k}=\lambda(a\hat{i}+6\hat{j}-8\hat{k})$

$\Rightarrow 2\hat{i} - 3\hat{j} + 4\hat{k} = a\lambda\hat{i} + 6\lambda\hat{j} - 8\lambda\hat{k}$
$\Rightarrow \lambda a = 2, 6\lambda = -3$ and $-8\lambda = 4$
$\Rightarrow \lambda = -\frac{1}{2}$ and $a = -4$

4. Given,equa equat of plane are$(x\hat{i} + y\hat{y} + z\hat{k}) \cdot (\hat{i} - \hat{j}) + 6 = 0$ and $(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} + 3\hat{j} - 4\hat{k}) = 0$
$\Rightarrow$ x - y + 6 = 0 and 3x + 3y - 4z = 0
Any plane through their intersection is (x - y + 6) + λ(3x + 3y - 4z) = 0
$\Rightarrow (1 + 3\lambda)x + (3\lambda - 1)y + 4\lambda x + 6 - 0$
$\therefore \frac{6}{\sqrt{(1+3\lambda)^2 + (3\lambda-1)^2 + (-4\lambda)^2}} = 1$
$\Rightarrow 34\lambda^2 + 2 = 36$
$\Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$
Therefore, the required planes are 2x + y - 2z + 3 = 0 and x + 2y - 2z - 3 = 0 In vector form they are
$\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) + 3 = 0$ and $\vec{r} \cdot (\hat{i} + 2\hat{j} - 2\hat{k}) - 3 = 0.$

5. Let E and F be the events defined as follows:
E = A solves the problem, F = B solves the problem.
Clearly, E and F are independent events such that
P(E) = $\frac{90}{100} = \frac{9}{10}$ and P(F) = $\frac{70}{100} = \frac{7}{10}$
Problem will be solved if atleast one of them solves it
Therefore, required probability = P$(E \cup F)$
= P(E) + P(F) - P$(E \cap F)$
= P(E) + P(F) - P(E)P(F) [as E and F are independent]
= P(E) + P(F) [1 - P(E)]
$= \frac{9}{10} + \frac{7}{10}[1 - \frac{9}{10}]$
$= \frac{9}{10} + \frac{7}{10} \times \frac{1}{10} = \frac{97}{100}$ = 0.97
Which is the required solution.

6. Event A fails and B fails denoted by $\overline{A}$ and $\overline{B}$ respectively.
$\therefore P\left(\overline{A}\right) = 0.2$ and P (A and B fails) = 0.15
$\Rightarrow P(A \cap B) = 0.15$
$\therefore$ P($\overline{B}$ above) = $P\left(\overline{B}\right) - P(A \cap B)$
$\Rightarrow 0.15 = P\left(\overline{B}\right) - 0.15$
$\Rightarrow P\left(\overline{B}\right) = 0.30$
i. $P\left(\overline{A}|\overline{B}\right) = \frac{P\left(\overline{A} \cap \overline{B}\right)}{P\left(\overline{B}\right)} = \frac{0.15}{0.30} = \frac{1}{2} = 0.5$
ii. P (A fails alone) = P ($\overline{A}$ alone) = $P\left(\overline{A}\right) - P\left(\overline{A} \cap \overline{B}\right)$ = 0.20 - 0.15 = 0.05

Section B

7. Let the given integral be, $I = \int \frac{x^2+1}{(x+3)(x-1)^2} dx$
Now using partial fractions by putting, $\frac{x^2+1}{(x+3)(x-1)^2} = \frac{A}{(x+3)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$ .... (1)
A(x - 1)^2 + B(x + 3)(x - 1) + C(x + 3) = x^2 + 1
Now put x - 1 = 0
Therefore, x = 1
A(0) + B(0) + C(4) = 2
$c = \frac{1}{2}$
Now put x + 3 = 0
Therefore, x = -3
A(-3 - 1)^2 + B(0) + C(0) = 9 + 1 = 10
$A = \frac{5}{8}$

By equating the coefficient of x^2, we get, A + B = 1

$\frac{5}{8}+B=1$

$B=1-\frac{5}{8}=\frac{3}{8}$

From equation (1), we get,

$\frac{x^2+1}{(x+3)(x-2)^2}=\frac{5}{8}\times\frac{1}{(x+3)}+\frac{3}{8}\times\frac{1}{(x-2)}+\frac{1}{(x-2)^2}$

$\int\frac{x^2+1}{(x+3)(x-2)^2}dx=\frac{5}{8}\int\frac{1}{(x+3)}dx+\frac{3}{8}\int\frac{1}{(x-2)}dx+\int\frac{1}{(x-2)^2}dx$

$=\frac{5}{8}\log|x+3|+\frac{3}{8}\log|x-1|-\frac{1}{2(x-1)}+c$

8. According to the question ,

Given differential equation is ,

$\frac{dy}{dx}=1+x^2+y^2+x^2y^2$

$\Rightarrow\quad\frac{dy}{dx}=1\left(1+x^2\right)+y^2\left(1+x^2\right)$

$\Rightarrow\quad\frac{dy}{dx}=\left(1+x^2\right)\left(1+y^2\right)$

$\Rightarrow\quad\frac{dy}{1+y^2}=\left(1+x^2\right)dx$

On integrating both sides, we get

$\int\frac{dy}{1+y^2}=\int\left(1+x^2\right)dx$

$\Rightarrow\quad\tan^{-1}y=x+\frac{x^3}{3}+C$ ...(i)

Given that y = 1, when x = 0.

On putting x = 0 and y = 1 in Eq. (i), we get

$\tan^{-1}1 = C$

$\Rightarrow\quad\tan^{-1}(\tan\pi/4)=C\quad\left[\because\tan\frac{\pi}{4}=1\right]$

$\Rightarrow\quad C=\frac{\pi}{4}$

On putting the value of C in Eq. (i), we get

$\tan^{-1}y=x+\frac{x^3}{3}+\frac{\pi}{4}$

$\therefore\quad y=\tan\left(x+\frac{x^3}{3}+\frac{\pi}{4}\right)$

which is the required solution of differential equation.

OR

We have, $\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$

Clearly, each of the function $x^2 + y^2$ and 2xy is a homogeneous function of degree 2, so the given equation is homogeneous.

Put y = vx and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

The given equation becomes

$v+x\frac{dv}{dx}=\frac{x^2+v^2x^2}{2vx^2}$

$\Rightarrow v+x\frac{dv}{dx}=\frac{v^2+1}{2v}$

$\Rightarrow\quad x\frac{dv}{dx}=\left(\frac{v^2+1}{2v}-v\right)$

$\Rightarrow\quad x\frac{dv}{dx}=\frac{v^2+1-2v^2}{2v}=\frac{1-v^2}{2v}$

$\Rightarrow\quad x\frac{dv}{dx}=\frac{-\left(v^2-1\right)}{2v}\Rightarrow-\frac{2v}{v^2-1}dv=\frac{dx}{x}$ [using variable separable form]

On integrating both sides, we get

$-\log|v^2 - 1| = \log x - \log C_1$

$\Rightarrow-\log\left|v^2-1\right|-\log x=-\log C_1$

$\Rightarrow\quad\log\left|x\left(v^2-1\right)\right|=\log C_1\Rightarrow x\left(v^2-1\right)=C_1$

$\Rightarrow\quad x\left(\frac{y^2}{x^2}-1\right)=C_1\Rightarrow x\left(\frac{y^2-x^2}{x^2}\right)=C_1$

$\Rightarrow \quad \frac{y^2-x^2}{x} = C_1 \Rightarrow x^2 - y^2 = -C_1 x$

$\Rightarrow \quad x^2 - y^2 = Cx\ [\because C = -C_1]$

9. According to the question,

Given, $\overrightarrow{OA} = 4\hat{i} + 5\hat{j} + \hat{k}$,

$\overrightarrow{OB} = -\hat{j} - \hat{k}$,

$\overrightarrow{OC} = 3\hat{i} + \lambda\hat{j} + 4\hat{k}$ and

$\overrightarrow{OD} = -4\hat{i} + 4\hat{j} + 4\hat{k}$.

Now, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -\hat{j} - \hat{k} - (4\hat{i} + 5\hat{j} + \hat{k})$

$= -4\hat{i} - 6\hat{j} - 2\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 3\hat{i} + \lambda\hat{j} + 4\hat{k} - (4\hat{i} + 5\hat{j} + \hat{k})$

$= -\hat{i} + (\lambda - 5)\hat{j} + 3\hat{k}$

and $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$= -4\hat{i} + 4\hat{j} + 4\hat{k} - (4\hat{i} + 5\hat{j} + \hat{k})$

$= -8\hat{i} - \hat{j} + 3\hat{k}$

Since, vectors $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ and $\overrightarrow{OD}$ are coplanar.

$\therefore \quad [\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

$$\therefore \begin{vmatrix} -4 & -6 & -2 \\ -1 & (\lambda - 5) & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$$

$-4(3\lambda - 15 + 3) + 6(-3 + 24) - 2(1 + 8\lambda - 40) = 0$

$\Rightarrow \quad -4(3\lambda - 12) + 6(21) - 2(8\lambda - 39) = 0$

$\Rightarrow \quad -12\lambda + 48 + 126 - 16\lambda + 78 = 0$

$\Rightarrow \quad -28\lambda + 252 = 0$

$\Rightarrow \lambda = 9$

10. We have equation of line is $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

$\Rightarrow \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda$

$\Rightarrow x = -2\lambda + 4, y = 6\lambda$ and $z = -3\lambda + 1$

Let the coordinates of L be $(4 - 2\lambda, 6\lambda, 1 - 3\lambda)$, then, direction ratios of PL are proportional to $(4 - 2\lambda - 2, 6\lambda - 3, 1 - 3\lambda + 8)$ i.e., $(2 - 2\lambda, 6\lambda - 3, 9 - 3\lambda)$.

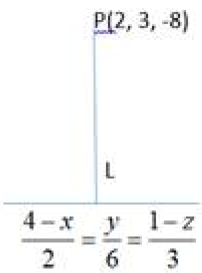
Also, direction ratios are proportional to -2, 6, -3. Since, PL is perpendicular to give line.

$\therefore -2(2 - 2\lambda) + 6(6\lambda - 3) - 3(9 - 3\lambda) = 0$

$\Rightarrow -4 + 4\lambda + 36\lambda - 18 - 27 + 9\lambda = 0$

$\Rightarrow 49\lambda = 49 \Rightarrow \lambda = 1$

So, the coordinates of L are $(4 - 2\lambda, 6\lambda, 1 - 3\lambda)$ i.e., (2, 6, -2).



Also, length of PL $= \sqrt{(2-2)^2 + (6-3)^2 + (-2+8)^2}$

$= \sqrt{0 + 9 + 36} = 3\sqrt{5} units$

OR

Given: A point P (say) $(-1, -5, -10)$

and equation of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$...(i)

equation of the plane is $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

Putting the value of $\vec{r}$ from eq. (i) in eq. (ii),

$\left[\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)\right].\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

$\Rightarrow\left(2\hat{i}-\hat{j}+2\hat{k}\right).\left(\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right).\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

$\Rightarrow 2+1+2+\lambda(3-4+2)=5$

$\Rightarrow 5+\lambda=5$

$\Rightarrow\lambda=0$

Putting $\lambda=0$ in eq. (i), $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+0\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$

$\Rightarrow\vec{r}=2\hat{i}-\hat{j}+2\hat{k}$

Therefore, Point of intersection is $(-2,1,2)$

$\therefore$ Distance of the given point $P(-1,-5,-10)$from the point of intersection is

$\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$

$=\sqrt{9+16+144}$

$=\sqrt{169}=13$ units

Section C

11. Let the given integral be

I = $\int\frac{(3\sin x-2)\cos x}{5-\cos^2 x-4\sin x}dx$

$\therefore\ I=\int\frac{(3\sin x-2)\cos x}{5-\left(1-\sin^2 x\right)-4\sin x}dx$

$\Rightarrow 1=\int\frac{(3\sin x-2)\cos x}{5-1+\sin^2 x-4\sin x}dx$

Substitute sin x = t

=> cos x dx = dt

Thus,

$I=\int\frac{(3t-2)}{4+t^2-4t}dt$

$\Rightarrow I=\int\frac{(3t-2)}{t^2-4t+4}dt$

$\Rightarrow \mathrm{I}=\int\frac{(3t-2)}{(t-2)^2}dt$

Now let us separate the integrand into the simplest form using partial fractions.

$\frac{(3t-2)}{(t-2)^2}=\frac{A}{(t-2)}+\frac{B}{(t-2)^2}$

$=\frac{A(t-2)+B}{(t-2)^2}$

$=\frac{At-2A+B}{(t-2)^2}$

= 3t - 2 = At - 2A + B

Comparing the coefficients, we have,

A = 3 and -2A + B = -2

Substituting the value of A = 3 in the above equation, we have,

$\Rightarrow$-2 $\times$ 3 + B = -2

$\Rightarrow$-6 + B = -2

$\Rightarrow$ B = 6 -2

$\Rightarrow$ B = 4

Therefore we have, $I=\int\frac{(3t-2)}{(t-2)^2}dt$ becomes

$I=\int\frac{3}{(t-2)}dt+\int\frac{4}{(t-2)^2}dt$

= 3 log |t - 2| - 4 $\left(\frac{1}{t-2}\right)+C$

= 3 log |2 - t| + 4 $\left(\frac{1}{2-t}\right)+C$

Now substituting t = sin x, we have,

I = 3 log |2 - sin x| +4 $\left(\frac{1}{2-\sin x}\right)+C$

12. To find: Area enclosed by

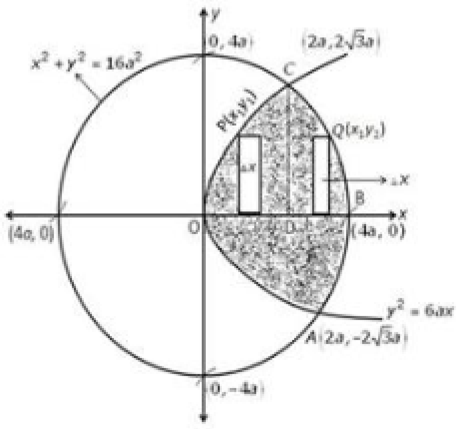
$x^2 + y^2 = 16$ ...(i)

and $y^2 = 6ax$ ...(ii)

Equation (i) represents a circle with centre (0, 0) and meets X-axis $(\pm 4a, 0)$.

Equation (ii) represents a parabola with vertex (0, 0) and axis as x-axis, Points of intersection of circle and parabola are $(2a, 2\sqrt{3}a), (2a, -2\sqrt{3}a)$.

A rough sketch of curves is given as:-



Required ODCO is sliced into rectangles of area $y_1 \triangle x$ and it slides from x = 0 to x = 2a.

Region BCDB is sliced into rectangle of area $y_2 \triangle x$ it slides from x = 2a to x = 4a. So,

Required area = 2 [Region ODCO + Region BCDB]

$= 2\left[\int_0^{2a} y_1 dx + \int_2^{4a} y_2 dx\right]$

$= 2\left[\int_0^{2a} \sqrt{6ax} dx + \int_{2a}^{4a} \sqrt{16a^2 - x^2}\, dx\right]$

$= 2\left[\sqrt{6a}\left(\frac{2}{3}x\sqrt{x}\right)\right]_0^{2a} + \left[\frac{x}{2}\sqrt{16a^2 - x^2} + \frac{16a^2}{2}\sin^{-1}\left(\frac{x}{4a}\right)\right]_{2a}^{4a}$

= $\frac{2\sqrt{2}}{\sqrt{3}}$ a. $2a \cdot \sqrt{2a} + aa^2 \cdot \sin^{-1}(1) - \frac{2a}{2} \cdot \sqrt{12a^2} - 8a^2 \cdot \sin^{-1}\left(\frac{1}{2}\right)$

$= 2\left[\left(\sqrt{6a} \cdot \frac{2}{3} 2a\sqrt{2a}\right) + \left[\left(0 + 8a^2 \cdot \frac{\pi}{2}\right) - \left(a\sqrt{12a^2} + 8a^2 \cdot \frac{\pi}{6}\right)\right]\right]$

$= 2\left[\frac{8\sqrt{3}a^2}{3} + 4a^2\pi - 2\sqrt{3}a^2 - \frac{4}{3}a^2\pi\right]$

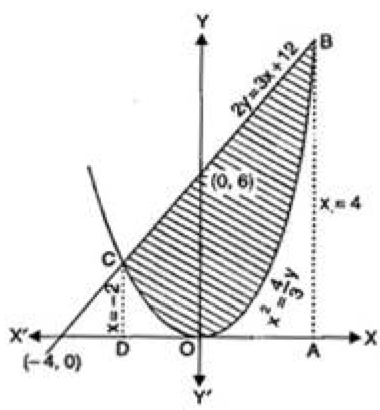
$= 2\left[\frac{2\sqrt{3}a^2}{3} + \frac{8a^2\pi}{3}\right]$

$A = \frac{4a^2}{3}(4\pi + \sqrt{3})$ sq. units.

OR

Equation of the parabola is

$4y = 3x^2$ ...(i)



$\Rightarrow x^2 = \frac{4}{3}y$

Equation of the line is 2y = 3x + 12 ...(ii)

$\Rightarrow y = \frac{3x+12}{2} = \frac{3x}{2} + 6$

In the graph, points of intersection are B (4, 12) and C (-2, 3).

Now, Area ABCD $= \left|\int_{-2}^{4} \left(\frac{3}{2}x + 6\right) dx\right|$

$= \left[\frac{3}{4}x^2 + 6x\right]_{-2}^{4}$

= (12 + 24) - (3 - 12)

= 45 sq units

Again, Area CDO + Area OAB $= \left|\int_{-2}^{4} \left(\frac{3}{4}x^2\right) dx\right|$

$= \left[\frac{3}{4} \cdot \frac{x^3}{3}\right]_{-2}^{4}$

$= \frac{1}{4}[64 - (-8)] = 18$ sq. units

$\therefore$ Required area = Area ABCD - (Area CDO + Area OAB)

= 45 - 18 = 27 sq. units

13. Equation of line in vector form

Line I: $\vec{\mathbf{r}} = (\hat{\imath} - \hat{\jmath} + 0\hat{\mathrm{k}}) + \lambda(2\hat{\imath} + 0\hat{\jmath} + \hat{k})$

Line II: $\vec{\mathrm{r}} = (2\hat{\mathrm{f}} - \hat{\mathrm{j}}) + \mu(\hat{\mathrm{i}} + \hat{\mathrm{j}} - \hat{\mathrm{k}})$

Here,

$\overrightarrow{a_1} = \hat{\imath} - \hat{\jmath} + 0\hat{k}$

$\overrightarrow{a_2} = 2\hat{\imath} - \hat{\jmath}$

$\overrightarrow{\mathrm{b}_1} = 2\hat{\mathrm{i}} + 0\hat{\mathrm{j}} + \hat{\mathrm{k}}$

$\overrightarrow{\mathrm{b}_2} = \hat{\mathrm{i}} + \hat{\mathrm{j}} - \hat{\mathrm{k}}$

We know that the shortest distance between lines is

d = $\frac{|(\overrightarrow{\mathrm{a}_2} - \overrightarrow{\mathrm{a}_1})(\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2})|}{|\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2}|}$

$(\overrightarrow{a_2} - \overrightarrow{a_1})$ = $(2\hat{\imath} - \hat{\jmath}) - (\hat{\imath} - \hat{\jmath} + 0\hat{k})$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = \hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

$\vec{b}_1 \times \vec{b}_2$ = $\begin{vmatrix} \hat{1} & \hat{\jmath} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$\vec{b}_1 \times \vec{b}_2 = (0-1)\hat{\mathrm{i}} - (-2-1)\hat{\mathrm{j}} + (2-0)\hat{\mathrm{k}}$

$\Rightarrow \vec{b}_1 \times \vec{b}_2 = -\hat{1} + 3\hat{\mathrm{j}} + 2\hat{\mathrm{k}}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-1)^2 + 3^2 + 2^2}$

$\Rightarrow |\vec{b}_1 \times \vec{b}_2| = \sqrt{14}$

$|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|$ = $|(\hat{\imath} + 0\hat{\jmath} + 0\hat{\mathrm{k}})(-\hat{\imath} + 3\hat{\jmath} + 2\hat{\mathrm{k}})|$

$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = 1$

Substituting these values in the expression,

d = $\frac{|(\vec{a}_2 - \vec{a}_1)(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$

d = $\frac{1}{\sqrt{14}}$

d = $\frac{1}{\sqrt{14}}$ units

Shortest distance d between the lines is not 0. Hence the given lines are not intersecting.

CASE-BASED/DATA-BASED

14. Let S = Shirt, T = Trouser

P(S) = 0.2 , P(T) = 0.3 and $P\left(\frac{S}{T}\right)$ = 0.4

We need to find P(S⋂T) and $\mathrm{P}\left(\frac{\mathrm{T}}{\mathrm{S}}\right)$

We know, $P\left(\frac{S}{T}\right)=\frac{P(S\cap T)}{P(T)}$

From given data, 0.4 = P (S⋂T) / 0.3

P (S⋂T) =0.4×0.3 = 0.12

Also,we have,

$P\left(\frac{T}{S}\right)=\frac{P(T\cap S)}{P(S)}=\frac{0.12}{0.2}=\frac{12}{20}=\frac{6}{10}=\frac{3}{5}=0.6$