Mathematical Modelling

2

Mechanical Systems

All mechanical systems are classified of two types.

(i) Mechanical translation system.

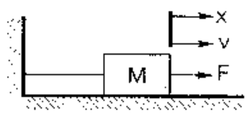
(ii) Mechanical rotational system.

1. Mechanical Translational System

Input : Force

Output : Linear displacement (x) OR Linear velocity (v)

(a) Intertia force



$$F = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$$

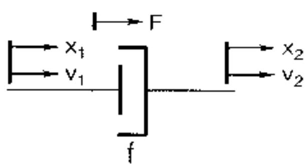
where, F = Force on block M

x = Displacement of block M

v = Velocity of block M

M = Mass of block M

(b) Damping force



$$F = f\frac{d(x_1 - x_2)}{dt} = f(v_1 - v_2)$$

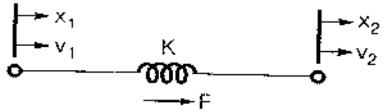
where, F = Damper force

x_1, x_2 = Displacement at side 1 and 2 of damper

v_1, v_2 = Velocity at side 1 and 2

f = Damper constant

(c) Spring force



$$F = K(x_1 - x_2) = K\int (v_1 - v_2)\,dt$$

where F = Spring force

x_1, x_2 = Compression or expansion of spring

v_1, v_2 = Velocity at side 1 and 2

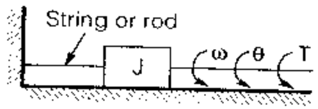
K = Spring constant

2. Mechanical Rotational System

Input : Torque (T)

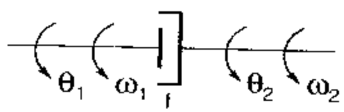
Output : Angular displacement (θ) OR Angular velocity (ω)

(a) Inertia torque



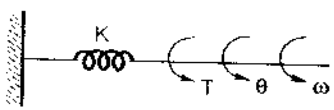
$$T = J\frac{d^2\theta}{dt^2} = J\frac{d\omega}{dt}$$

(b) Damper torque



$$T = f\frac{d\theta}{dt} = f(\omega_1 - \omega_2)$$

(c) Spring torque

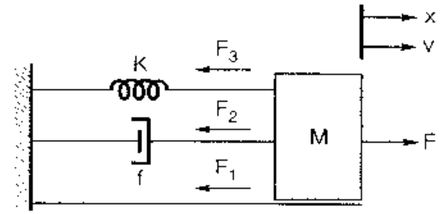


$$T = K\theta = K\int \omega\,dt$$

Analogous System

Electrical equivalent of mechanical system called analogous system

1. Mechanical Translation Systems

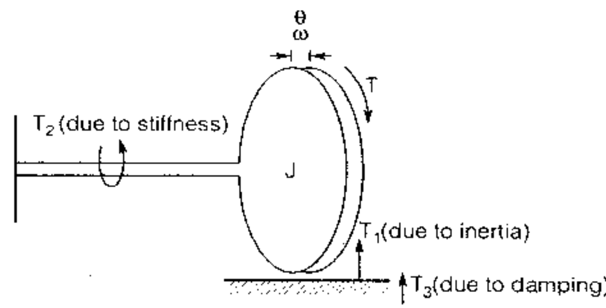


$$F = F_1 + F_2 + F_3$$

$$F = M\frac{dv}{dt} + fv + K\int v\,dt$$

$$F = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + Kx$$

2. Mechanical Rotational Systems

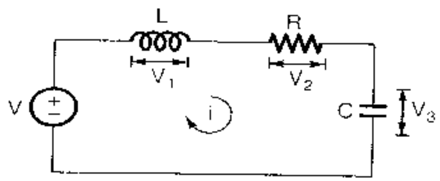


$$T = T_1 + T_2 + T_3$$

$$T = J\frac{d\omega}{dt} + f\omega + K\int \omega\,dt$$

$$T = J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} + K\theta$$

3. Electrical Series RLC Systems

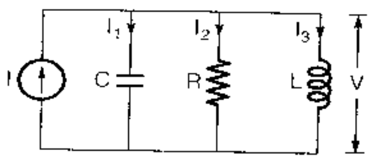


$$V = V_1 + V_2 + V_3$$

$$V = L\frac{di}{dt} + iR + \frac{1}{C}\int i dt$$

$$V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C}$$

Electrical Parallel RLC Systems



$$I = I_1 + I_2 + I_3$$

$$I = C\frac{dv}{dt} + \frac{V}{R} + \frac{1}{L}\int V dt$$

$$I = C\frac{d^2\phi}{dt} + \frac{I}{R}\frac{d\phi}{dt} + \frac{\phi}{L}$$

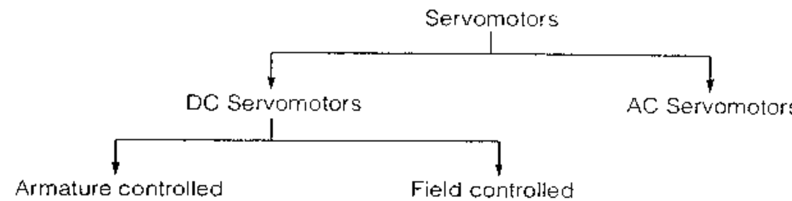
Force Voltage Analogy & Force Current Analogy

Series RLC	Parallel RLC	Mechanical Translation System	Mechanical Rotational System
V	I	F	T
q	ϕ	x	θ
R	1/R	f	f
I	V	Linear velocity (v)	Angular velocity (ω)
1/C	1/L	K	K
L	C	M	J

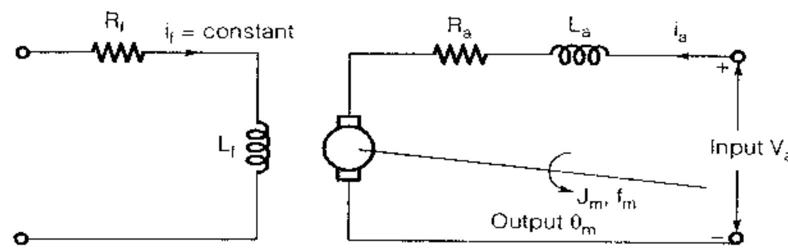
Nodal Method

1. Number of nodes = Number of displacement.
2. Take an additional node which is reference node.
3. Connect the mass or inertia element between principle node and reference node only.
4. Connect the spring and damping elements either between the principle nodes or between the principal node and reference node, depending on their position.
5. Obtain the nodal diagram and write the differential equation at each node.

Servomotors

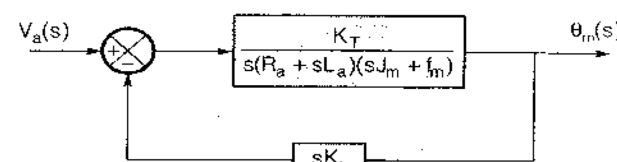


1. Armature Controlled DC Servo Motors

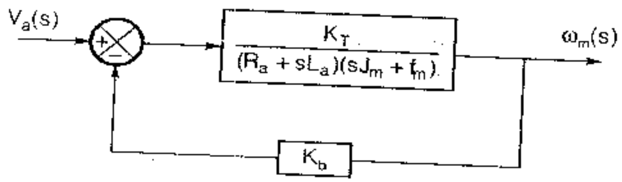


❑ Block diagram

(a) Relating $\theta_m(s)$ and $V_a(s)$



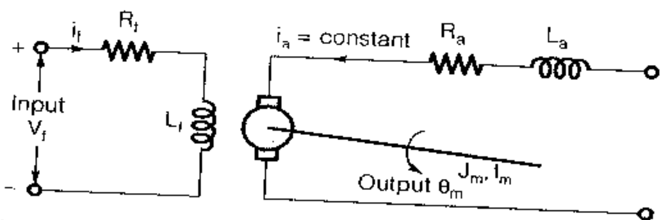
(b) Relating $\omega_m(s)$ and $V_a(s)$



❑ Dynamic equations

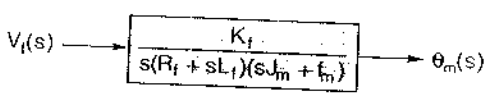
Dynamic equations in time domain	Corresponding equation in s-domain
$V_a - e_b = R_a i_a + L_a \frac{di_a}{dt}$	$V_a(s) - E_b(s) = R_a I_a(s) + sL_a I_a(s)$
$e_b = K_b \frac{d\theta_m}{dt} = K_b \omega_m$	$E_b(s) = sK_b\theta_m(s) = K_b\omega_m(s)$
$T_M = K_T i_a$	$T_M(s) = K_T I_a(s)$
$T_M = J_m \frac{d^2\theta_m}{dt^2} + f_m \frac{d\theta_m}{dt}$	$T_M(s) = s^2 J_m\theta_m(s) + sf_m\theta_m(s)$
$T_M = J_m \frac{d\omega_m}{dt} + f_m\omega_m$	$T_M(s) = sJ_m\omega_m(s) + f_m\omega_m(s)$

2. Field Controlled DC Servomotor

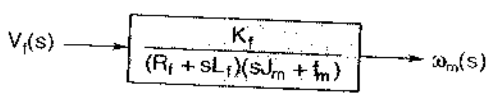


❑ Block diagram

(a) Relating $\theta_m(s)$ and $V_f(s)$



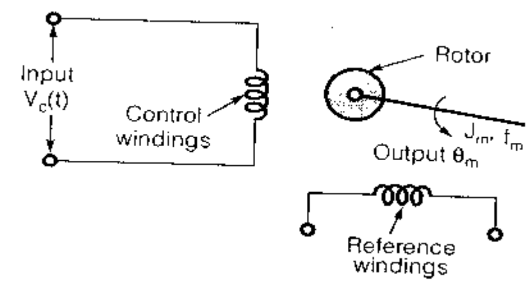
(b) Relating $\omega_m(s)$ and $V_f(s)$



❑ Dynamic equations

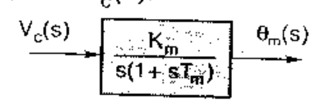
Dynamic equations in time domain	Corresponding equation in s-domain
$V_f = R_f i_f + L_f \frac{di_f}{dt}$	$V_f(s) = R_f I_f(s) + sL_f I_f(s)$
$T_M = K_f i_f$	$T_M(s) = K_f I_f(s)$
$T_M = J_m \frac{d^2\theta_m}{dt^2} + f_m \frac{d\theta_m}{dt}$	$T_M(s) = s^2 J_m\theta_m(s) + sf_m\theta_m(s)$
$T_M = J_m \frac{d\omega_m}{dt} + f_m\omega_m$	$T_M(s) = sJ_m\omega_m(s) + f_m\omega_m(s)$

3. Two Phase AC Servomotor

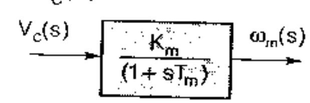


❑ Block diagram

(a) Relating $\theta_m(s)$ and $V_c(s)$.



(b) Relating $\omega_m(s)$ and $V_c(s)$.



❑ Dynamic equations

Dynamic equations in time domain	Corresponding equations in s-domain
$T_M = m\frac{d\theta_m}{dt} + KV_c$	$T_M(s) = sm\theta_m(s) + KV_c(s)$
$T_M = J_m \frac{d^2\theta_m}{dt^2} + f_m \frac{d\theta_m}{dt}$	$T_M(s) = s^2 J_m\theta_m(s) + sf_m\theta_m(s)$

where, $m = \frac{T_o}{\omega_o}$ (slope of torque speed characteristic)

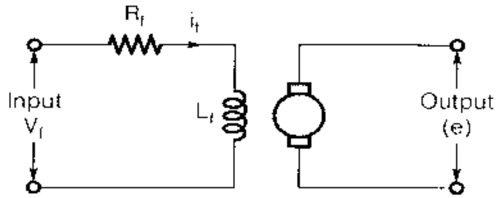
$K = \frac{T_o}{V_c}$

T_o = Stalling torque

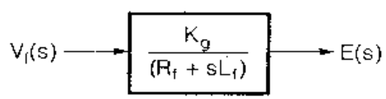
ω_o = No load speed

Generators

1. Separately Excited DC Generator



❑ Block diagram

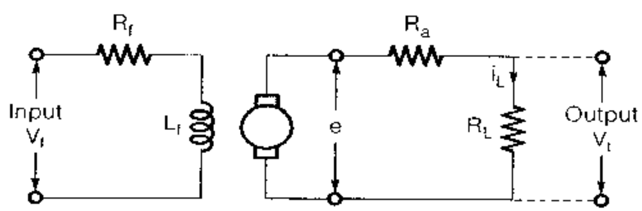


❑ Dynamic equations

Related equations in time domain	Corresponding equations in s-domain
$V_f = R_f i_f + L_f \frac{di_f}{dt}$	$V_f(s) = R_f I_f(s) + sL_f I_f(s)$
$e = K_g i_f$	$E(s) = K_g I_f(s)$

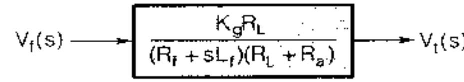
where, K_g = Generator constant

2. Separately Excited DC Generator Connected to Load



❑ Block diagram

Relating $V_t(s)$ and $V_f(s)$



❑ Dynamic equations

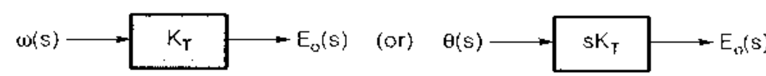
Related equations in time domain	Corresponding equations in s-domain
$V_t = e - R_a i_L$	$V_t(s) = E(s) - R_a I_L(s)$
$I_L = \frac{V_t}{R_L}$	$I_L(s) = \frac{V_t(s)}{R_L}$

Note:

- Servomechanism is electromechanical system whose input is electrical ***voltage*** and output is mechanical ***position*** or its time derivative.
- Field control DC servo motor is used for low power applications while armature control DC servomotor is used for high power applications.
- AC servomotor is generally used for low power application.
- Torque speed characteristic is linear for AC servomotor whereas, it is nonlinear for general purpose induction motor.
- For the fast response of the servomotor system, the inertia of the rotor should be kept to minimum. This is achieved by reducing the diameter and increasing the length of the rotor.

Tachometer

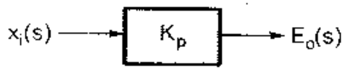
Tachometer is speed transducer, used as feedback element in control system.



where, sK_T = Tachometer constant

Potentiometer

It is variable resistive displacement transducer. A pair of potentiometer act as error detectors in control system application.



Input = Wiper displacement (x_i)

Output = Voltage (e_o)

$$K_P = \frac{e_i}{x_t} = \text{Potentiometer gain}$$