

7. Limits

Limits of Exponential and Logarithmic Functions

The following results are useful in the evaluation of the limits of exponential and logarithmic functions:

1. $\lim_{x \rightarrow 0} a^x - 1 = \log_e a$, $a > 0$

2. $\lim_{x \rightarrow 0} e^x - 1 = x$

3. $\lim_{x \rightarrow 0} \log(1+x) = x$

4. $\lim_{x \rightarrow 0} (1+x)^{1/x} = e$

Limits at Infinity

A function f is said to tend to a limit L as $x \rightarrow \infty$ if for $\delta > 0$, however small it may be, there exists a positive number k such that $|f(x) - L| < \delta$ $\forall x$ in the domain of f , for which $x > k$. This can be mathematically written as $\lim_{x \rightarrow \infty} f(x) = L$.

1. $\lim_{x \rightarrow \infty} C = \lim_{x \rightarrow -\infty} C = C$, where C is a constant.

2. $\lim_{x \rightarrow \infty} Cx^n = 0$, $n > 0$

3. $\lim_{x \rightarrow -\infty} Cx^n = 0$, $n \in \mathbb{N}$

Infinite Limit

If for every $k > 0$, there exists $\delta > 0$ such that for all x in the domain of f and $x \in (a-\delta, a+\delta)$, we have $f(x) > k$, then the limit of $f(x)$ as x tends to a is infinity. This can be mathematically written as $\lim_{x \rightarrow a} f(x) = \infty$.

The following steps can help evaluate algebraic limits at infinity:

1. If the function is not in rational form, i.e., $\frac{f(x)}{g(x)}$, then first express it in rational form.

2. Divide the numerator and denominator by x^n , where n is the highest power of x .

3. Use the result $\lim_{x \rightarrow \infty} Cx^n = 0$, $n > 0$, and $\lim_{x \rightarrow \infty} C = C$.