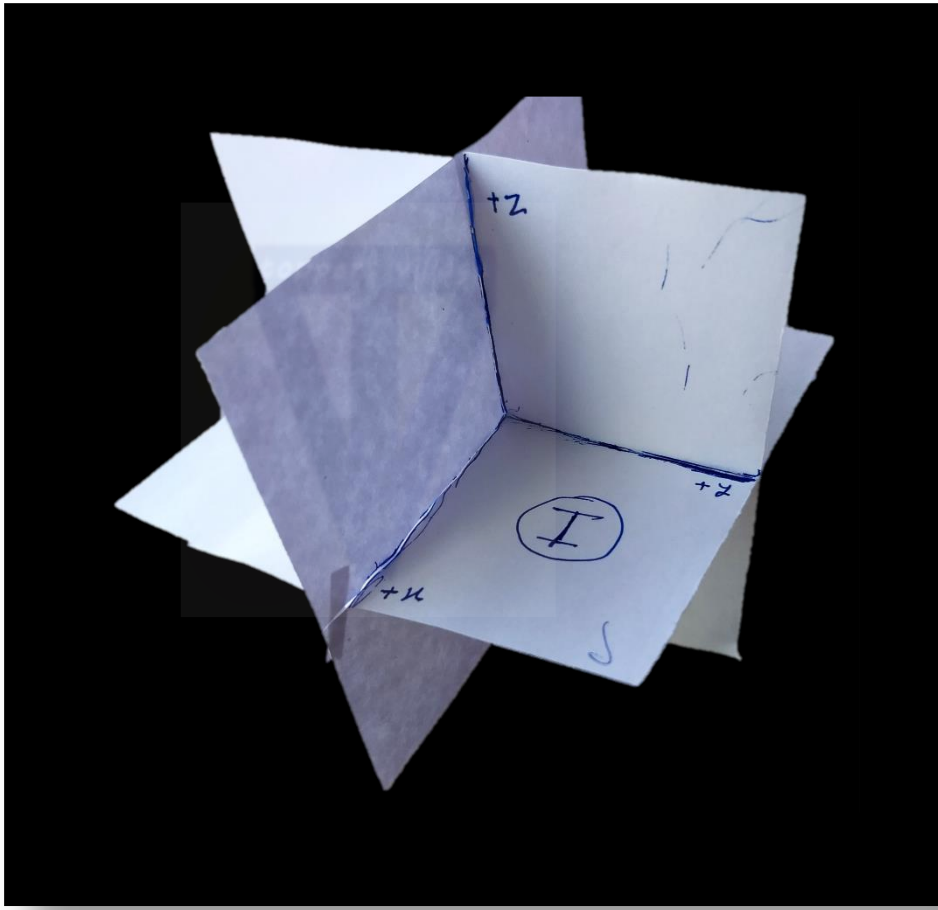
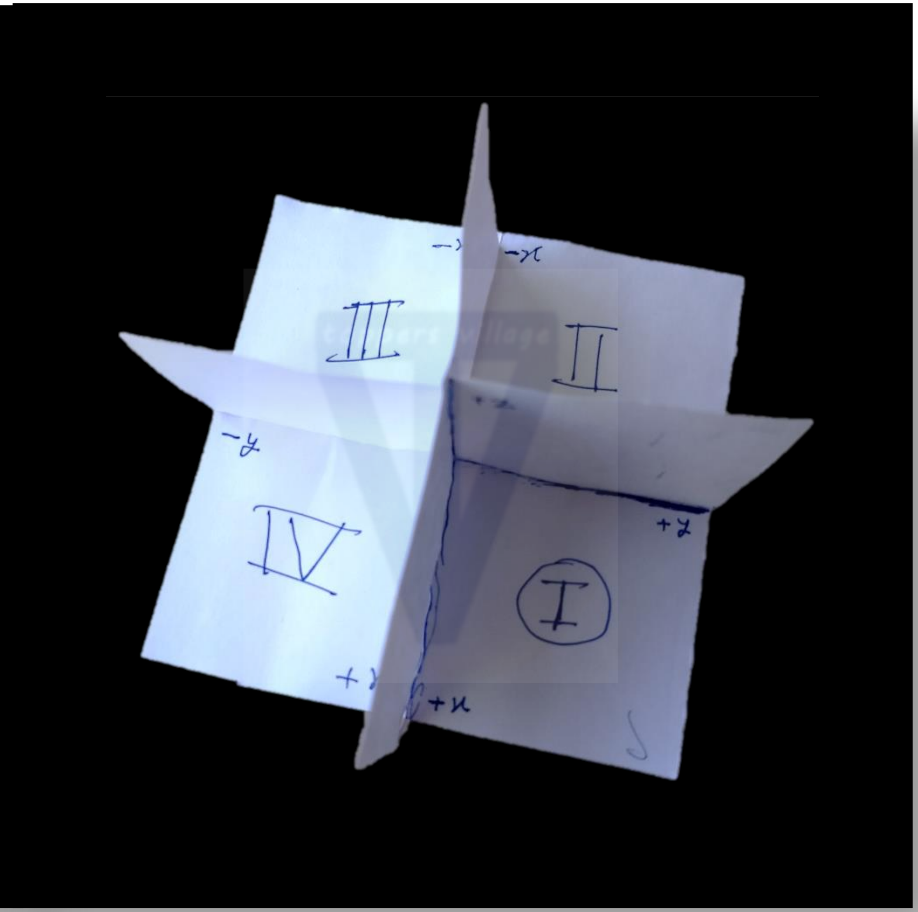
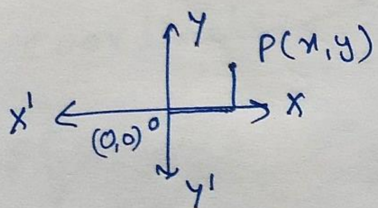

+z
+y
I
+x

-x
III
II
-y
IV
+y
I
+y
+x

Introduction to 3D Geometry

A Brief Revision of 2D



On X-axis → (x, 0)

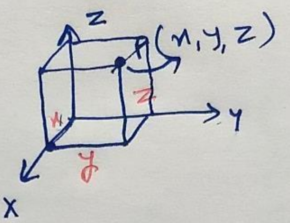
On y-axis → (0, y)

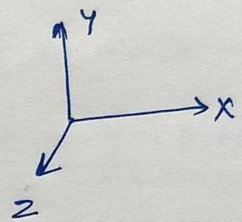
x-coordinate → ⊕ → Right; ⊖ → Left

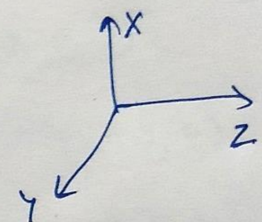
y-coordinate → ⊕ → Up; ⊖ → Down

$\boxed{\text{4 - Quadrants}}$

Three Dimensional Geometry (3D)

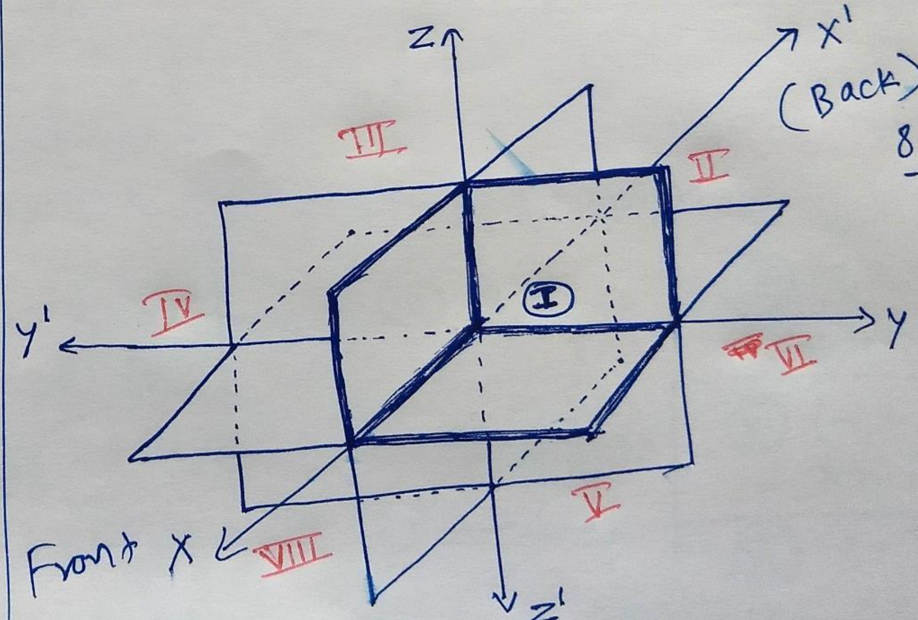






✓ ✓

Right Hand Thumb Rule ✓



8-octants

x-coordinate → ⊕ front, ⊖ Back

y-coordinate → ⊕ Right, ⊖ Left

z-coordinate → ⊕ UP, ⊖ Down

2D

-+	++
--	+-

$P(x, y, z)$

8-Octants

.	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−

Special Cases.

Origin O (0,0,0)

On X-axis → (x, 0, 0)
On Y-axis → (0, y, 0)
On Z-axis → (0, 0, z)

On XY-Plane → (x, y, 0)
On YZ-Plane → (0, y, z)
On ZX-Plane → (x, 0, z)

e.g. Identify the position of the following points.

① (−2, 7, −13)

z = ⊖

VI | V
VII | VIII

VI - Octant

Ⓘ (−1, −2, −3)

z = ⊖

VI | V
VII | VIII

VII - octant

III (−2, 2, 4)

z = ⊕

II | I
III | IV

II - octant

IV (7, 0, −5)

y

XZ - Plane

V (−1, 2, 0)

z

XY - Plane

VI (0, −3, 0)

x z

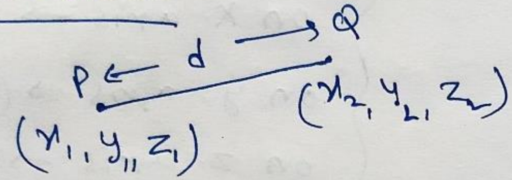
y - axis

VII (1, 2, −3)

Comment ?

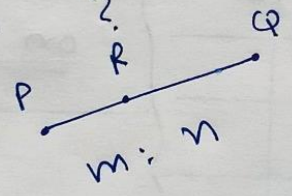
3D- Geometry - All Basic Formulas

(1) Distance Formula



$$\boxed{d = PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

(2) Section Formula



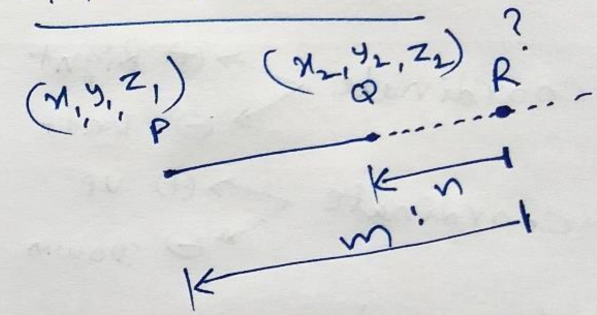
Internal Division

$P(x_1, y_1, z_1)$

$Q(x_2, y_2, z_2)$

$$R\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$$

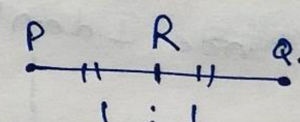
External Division



$PR : QR = m : n$

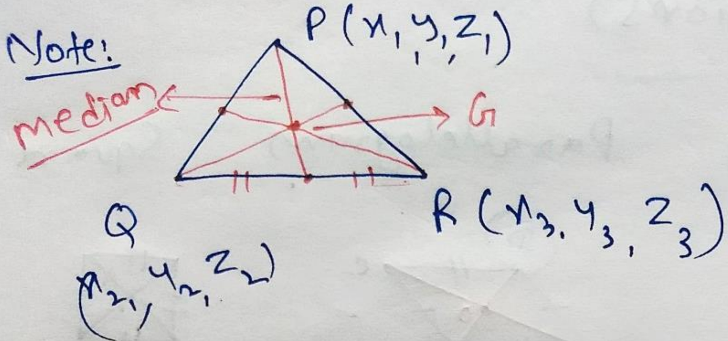
$$R\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$$

Note: Mid Point Formula



$$R\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

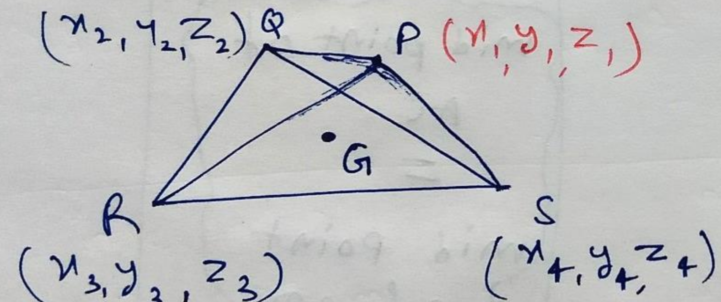
Note:

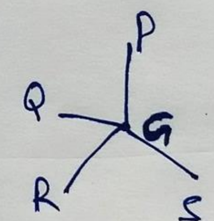


Centroid (G) of Triangle $\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}, \dfrac{z_1+z_2+z_3}{3}\right)$

Note.

Tetrahedron.

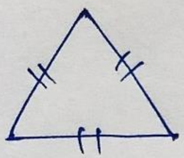




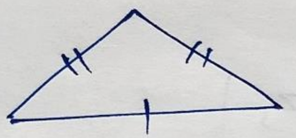
Centroid (G) : $\left(\dfrac{x_1+x_2+x_3+x_4}{4}, \dfrac{y_1+y_2+y_3+y_4}{4}, \dfrac{z_1+z_2+z_3+z_4}{4}\right)$

Basic Concepts (to Deal with Questions)

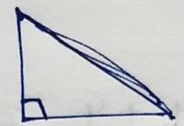
EQUILATERAL Triangle



Isosceles Triangle



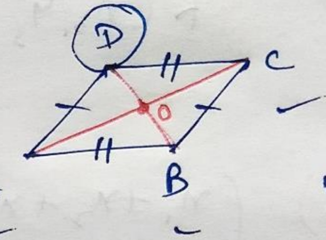
Right angled Triangle



(PGT)

Pythagoras Th.

Parallelogram

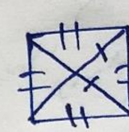


↓

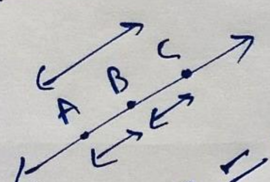
Find 4^{th} vertex

mid point of AC = mid point of BD

Square



Collinear Points.



$AB + BC = AC$

Exercise 11.1 (3D)

Q.1 A point on X-axis.

$(x, 0, 0)$

y-coordinate $= 0$

z- ——— $= 0$

Q.2 A point in the XZ-plane

$(x, 0, z)$ y-coordinate $= 0$

Q.3 Name the octants.

$(1, 2, 3)$ → $z = +$

II | I
III | IV

I-octant

$(4, -2, 3)$ → $z = +$

II | I
III | IV

IV-octant

$(4, -2, -5)$ → $z = -$

VI | V
VII | VIII

VIII-octant

$(4, 2, -5)$ → $z = -$

VI | V
VII | VIII

V-octant

$(-4, 2, -5)$ → $z = -$

VI | V
VII | VIII

VI-octant

$(-4, 2, 5)$ → $z = +$

II | I
III | IV

II-octant

$(-3, -1, 6)$ → $z = +$

II | I
III | IV

III-octant

$(2, -4, -7)$ → $z = -$

VI | V
VII | VIII

VIII-octant

Q.4 Fill in the Blanks:

(i) The x-axis and y-axis taken together determine a plane known as XY-Plane

(ii) The coordinates of points in the XY-plane are of the form $(x, y, 0)$.

(iii) Coordinate planes divide the space into 8 octants.

Exercise 11.2 3D

Distance Formula.

P ——— Q

(x_1, y_1, z_1) (x_2, y_2, z_2)

$$\boxed{PQ = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}}$$

Q. 1 (i) (2, 3, 5) & (4, 3, 1)

$\xleftrightarrow{\quad d \quad}$

$$d = \sqrt{(2-4)^2 + (3-3)^2 + (5-1)^2}$$

$$d = \sqrt{4 + 0 + 16}$$

$$d = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

(ii) (−3, 7, 2) & (2, 4, −1)

$$d = \sqrt{(-3-2)^2 + (7-4)^2 + (2+1)^2}$$

$$d = \sqrt{25 + 9 + 9}$$

$$d = \sqrt{43}$$

(iii) (−1, 3, −4) & (1, −3, 4)

$$d = \sqrt{(-2)^2 + (6)^2 + (-8)^2}$$

$$d = \sqrt{4 + 36 + 64}$$

$$d = \sqrt{104} = \sqrt{4 \times 26}$$

$$d = 2\sqrt{26}$$

(IV) $(2,-1,3)$, $(-2,1,3)$

$$d = \sqrt{4^2 + (-2)^2 + \underbrace{(3-3)^2}_{0}}$$

$$d = \sqrt{16 + 4 + 0}$$

$$d = \sqrt{20} = 2\sqrt{5}$$

$PQ = \sqrt{14}$

$QR = 2\sqrt{14}$

$RP = 3\sqrt{14}$

$\boxed{PQ + QR = RP}$

$\therefore$ P, Q, R → Collinear

Q.2 Show that P$(-2,3,5)$, Q$(1,2,3)$ & R$(7,0,-1)$ are collinear.

Proof:

$$PQ = \sqrt{(-3)^2 + (1)^2 + 2^2} = \sqrt{14}$$

$$QR = \sqrt{(-6)^2 + (2)^2 + (3+1)^2} = \sqrt{56} = \sqrt{4 \times 14} = 2\sqrt{14}$$

$$RP = \sqrt{(9)^2 + (-3)^2 + (-6)^2} = \sqrt{126} = \sqrt{9 \times 14} = 3\sqrt{14}$$

③ Verify.

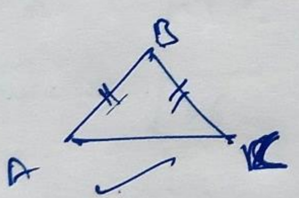
(i) (0, 7, −10) A
(1, 6, −6) B
(4, 9, −6) C
→ Isosceles Triangle

$AB = \sqrt{(-1)^2 + (1)^2 + (-4)^2} = \sqrt{18}$

$BC = \sqrt{(-3)^2 + (-3)^2 + (0)^2} = \sqrt{18}$

$CA = \sqrt{16 + 4 + 16} = \sqrt{36} = 6$

$\underline{AB = BC}$



$\underline{\text{Isosceles } \Delta}$.

(ii) (0, 7, 10) A
(−1, 6, 6) B
(−4, 9, 6) C
→ Right angled Triangle

PGT

$AB = \sqrt{(1)^2 + (1)^2 + (4)^2} = \sqrt{18}$

$1 + 1 + 16$

$BC = \sqrt{(3)^2 + (-3)^2 + 0} = \sqrt{18}$

$AC = \sqrt{(4)^2 + (-2)^2 + (4)^2} = \sqrt{36}$

$16 + 4 + 16$

$AB^2 = 18$
$BC^2 = 18$

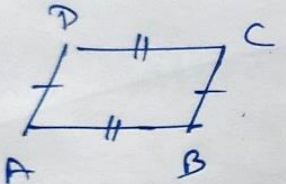
$AC^2 = 36$

$\underline{AB^2 + BC^2} = 36 = \underline{AC^2}$

PGT ∴ Right angled triangle

(3) (iii) $(-1,2,1)$ A
$(1,-2,5)$ B
$(4,-7,8)$ C
$(2,-3,4)$ D

→ Vertices of Parallelogram



AB = CD & BC = AD

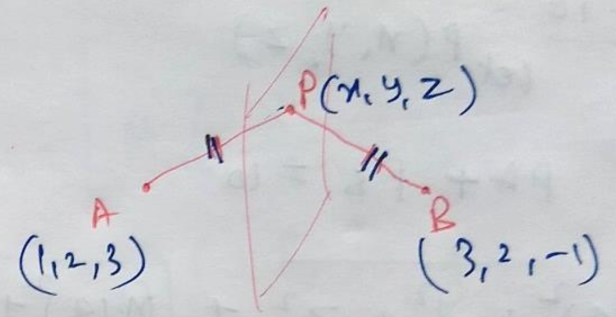
$AB = \sqrt{4+16+16} = \sqrt{36}$

$BC = \sqrt{9+25+9} = \sqrt{43}$

$CD = \sqrt{4+16+16} = \sqrt{36}$

$AD = \sqrt{9+25+9} = \sqrt{43}$

Q.4 set of Points equidistant from the points A(1,2,3) & B(3,2,−1).



Equidistant. $PA = PB$

$$\Rightarrow \sqrt{(x-1)^2+(y-2)^2+(z-3)^2} = \sqrt{(x-3)^2+(y-2)^2+(z+1)^2}$$

$$\Rightarrow \cancel{x^2}-2x+\cancel{1}+\cancel{y^2}-\cancel{4y}+\cancel{4}+\cancel{z^2}-6z+\cancel{9}$$
$$= \cancel{x^2}-6x+\cancel{9}+\cancel{y^2}-\cancel{4y}+\cancel{4}+\cancel{z^2}+2z+\cancel{1}$$

$$\Rightarrow 6x-2x = 2z+6z$$

$$\Rightarrow 4x = 8z \Rightarrow \boxed{x = 2z}$$

Q.5 A(4, 0, 0)

B(−4, 0, 0)

sum = 10

let P(x, y, z)

ATQ. PA + PB = 10

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$$

$$\Rightarrow \left(\sqrt{x^2 - 8x + 16 + y^2 + z^2}\right)^2 = \left(10 - \sqrt{x^2 + 8x + 16 + y^2 + z^2}\right)^2 \quad (a-b)^2$$

$$\Rightarrow \cancel{x^2} - 8x + \cancel{16} + \cancel{y^2} + \cancel{z^2} = 100 + \cancel{x^2} + 8x + \cancel{16} + \cancel{y^2} + \cancel{z^2} - 20\sqrt{x^2 + 8x + 16 + y^2 + z^2}$$

$$\Rightarrow \frac{-100 - 16x}{-4} = \frac{-20\sqrt{x^2 + y^2 + z^2 + 8x + 16}}{-4}$$

$$\Rightarrow [25 + 4x]^2 = \left(5 \cdot \sqrt{x^2 + y^2 + z^2 + 8x + 16}\right)^2$$

$$\Rightarrow 625 + 16x^2 + \cancel{200x} = 25x^2 + 25y^2 + 25z^2 + \cancel{200x} + 400$$

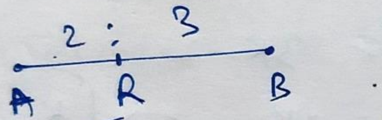
$$\Rightarrow \boxed{9x^2 + 25y^2 + 25z^2 - 225 = 0}$$

Exercise 11.3

3D

$\boxed{Q.1}$ $A(-2, 3, 5)$ $B(1, -4, 6)$

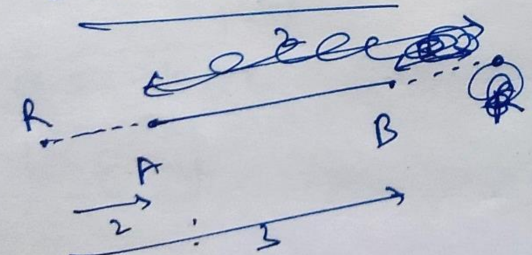
(i) 2:3 internally



$$R\left(\frac{2\times1 + 3\times(-2)}{2+3}, \frac{-8+9}{5}, \frac{12+15}{5}\right)$$

$$R\left(\frac{-4}{5}, \frac{1}{5}, \frac{25}{7}\right)$$

(ii) 2:3 externally.



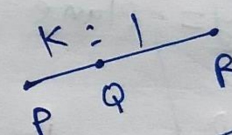
externally

$$R\left(\frac{2-(-6)}{2-3}, \frac{-8-9}{2-3}, \frac{12-15}{2-3}\right)$$

$$R\left(\frac{8}{-1}, \frac{-17}{-1}, \frac{-3}{-1}\right)$$

$$R(-8, 17, 3)$$

$\boxed{Q-2}$ $P(3, 2, -4)$ $Q(5, 4, -6)$ $R(9, 8, -10)$ collinear



Let $\frac{PQ}{QR} = ? = K:1$

By Section Formula,

$$\underset{\text{(Given)}}{5} = \frac{K\cdot 9 + 1\times 3}{(K+1)} \Rightarrow 5K+5 = 9K+3$$

$\Rightarrow 5K + 5 = 9K + 3$

$\Rightarrow 5 - 3 = 9K - 5K$

$\Rightarrow 2 = 4K$

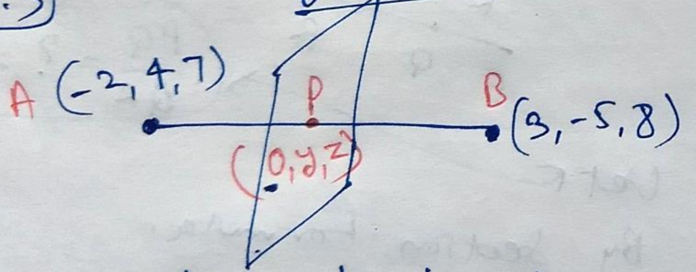
$\Rightarrow 1 = 2K$

$\Rightarrow K = \frac{1}{2}$

$K:1 = \left(\frac{1}{2}\right):1 = \frac{\left(\frac{1}{2}\right)}{(1)} = \frac{1}{2}$

Ratio $= 1:2$

[Q.3] yz-plane



$K : 1$ (let)

By Section Formula,

x-coordinate of $P = \frac{3K + 1(-2)}{K+1}$

$\Rightarrow 0 = \frac{3K - 2}{K+1}$

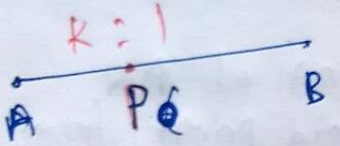
$\Rightarrow 0 = 3K - 2$

$\Rightarrow 2 = 3K \Rightarrow \boxed{K = \frac{2}{3}}$

$\therefore$ Required Ratio $= \frac{K}{1} = \frac{2}{3}$

$\boxed{Q.4}$ $A(2,-3,4)$, $B(-1,2,1)$, $\boxed{C\left(0,\frac{1}{3},2\right)}$ are collinear

(Using Section Formula)



Let P divides AB in ratio K:1

$$\boxed{P\left(\frac{-K+2}{K+1}, \frac{2K-3}{K+1}, \frac{K+4}{K+1}\right)}$$

Let P & C coincide.

x-coordinate of C = x-coord. of P

$\Rightarrow 0 = \frac{-K+2}{K+1}$

$\Rightarrow 0 = -K+2$

$\Rightarrow \boxed{K=2}$

Put K = 2, in coordinates of P.

$$P\left(0, \frac{1}{3}, \frac{6}{3}\right)$$

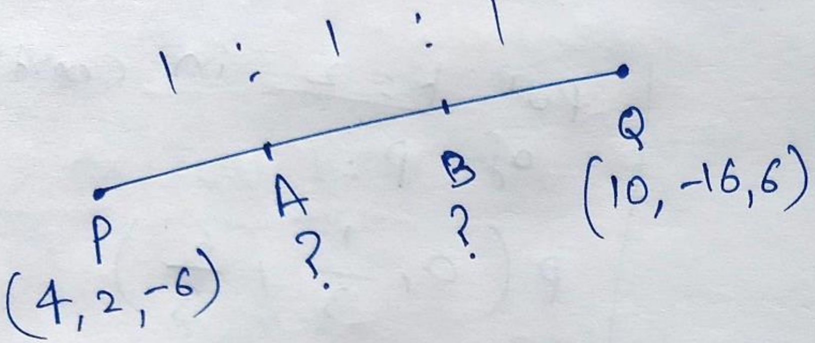
$$P\left(0, \frac{1}{3}, 2\right) \longleftrightarrow C$$

$\therefore$ C lies on the line segment AB.

$\therefore$ A, B, C $\rightarrow$ Collinear.

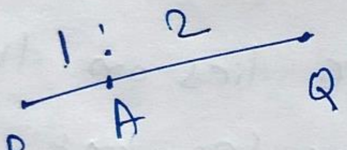
(C divides AB in 2:1 internally)

(5)



$PA = AB = BQ$

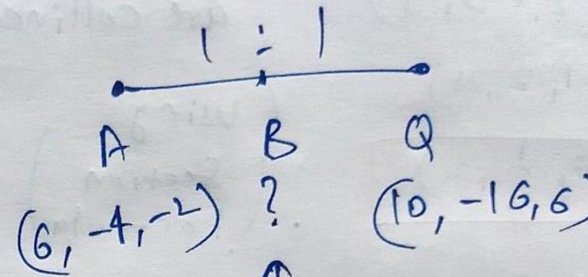
A



By section Formula

$$A\left(\frac{10+8}{1+2}, \frac{-16+4}{1+2}, \frac{6-12}{1+2}\right)$$

$$\Rightarrow A(6, -4, -2)$$

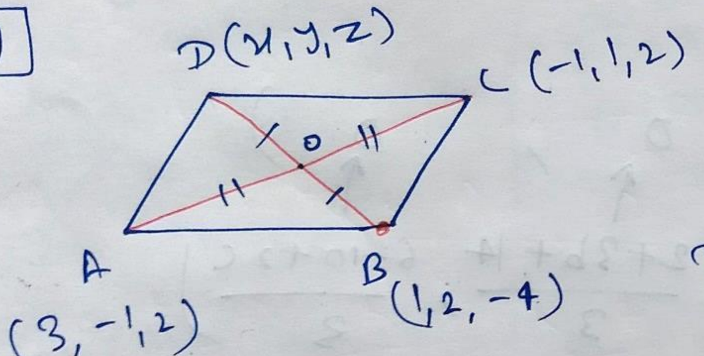


mid point of AQ
(mid point formula)

$$B\left(\frac{16}{2}, \frac{-20}{2}, \frac{4}{2}\right)$$

$$B(8, -10, 2)$$

Q.1



$D(1, -2, 8)$ ✓

Diagonals bisects each other.

$\therefore$ mid point of AC

= mid point of BD.

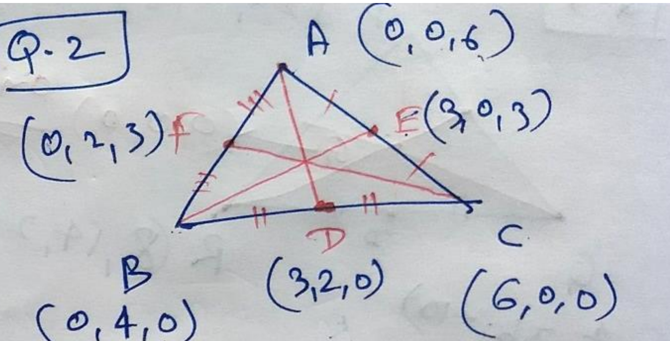
$$\Rightarrow \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) \equiv \left(\frac{1+x}{2}, \frac{2+y}{2}, \frac{-4+z}{2}\right)$$

$2 = 1 + x$ $\quad$ $\boxed{x = 1}$

$0 = 2 + y$ $\quad$ $\boxed{y = -2}$

$4 = -4 + z$ $\Rightarrow \boxed{z = 8}$

Q.2



mid point formula

$$D\left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right) \equiv D(3, 2, 0)$$

$E(3, 0, 3)$

$F(0, 2, 3)$

median (Distance Formula)

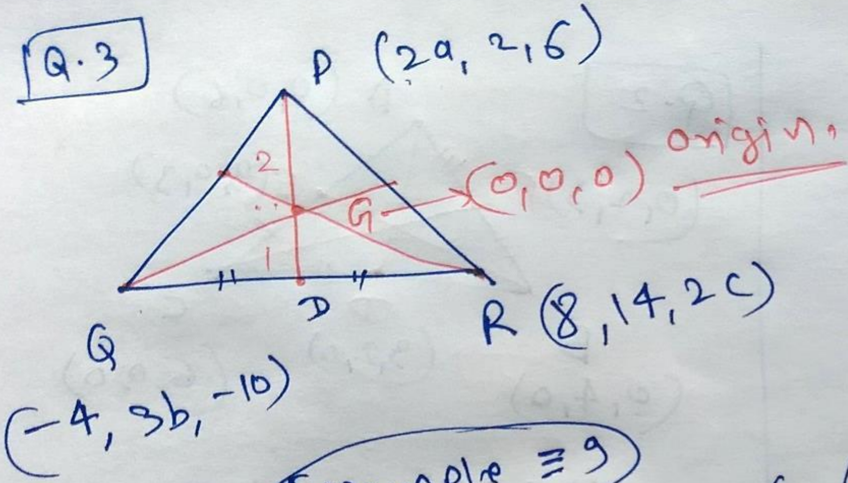
$AD = \sqrt{9 + 4 + 36}$

$= \sqrt{49} = 7$ ✓

$BE = \sqrt{9 + 16 + 9} = \sqrt{34}$ ✓

$CF = \sqrt{36 + 4 + 9} = 7$ ✓

Q.3



Example ≡ 9

$$G\left(0,\ 0,\ 0\right)$$

$$\text{Centroid of } \Delta PQR \equiv G\left(\frac{2a-4+8}{3},\ \frac{2+3b+14}{3},\ \frac{6-10+2c}{3}\right)$$

$$\Rightarrow \frac{2a+4}{3}=0 \qquad \frac{3b+16}{3}=0 \qquad \frac{2c-4}{3}=0$$

$$\Rightarrow \boxed{a=-2} \qquad \boxed{b=\frac{-16}{3}} \qquad \boxed{c=2}$$

Q. 4

$P(3,-2,5)$

A point on y-axis $A(0,y,0)$

$d = 5\sqrt{2}$

Answer

$A(0,2,0)$ ✓

$A(0,-6,0)$ ✓

ATQ. $AP = d$

$\Rightarrow AP^2 = d^2$

$\Rightarrow (3)^2 + (-2-y)^2 + (5)^2 = (5\sqrt{2})^2$

$\Rightarrow 9 + 4 + y^2 + 4y + \cancel{25} = \cancel{50}\ 25$

$\Rightarrow y^2 + 4y - 12 = 0$

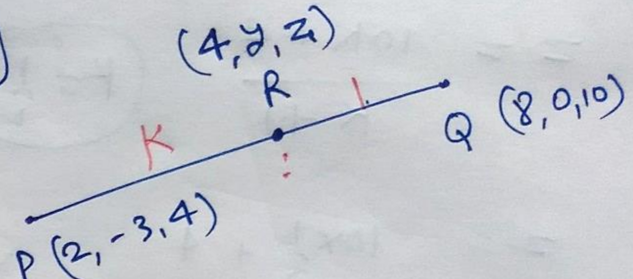
$\Rightarrow \underbrace{y^2 + 6y} - \underbrace{2y - 12} = 0$

$\Rightarrow \cancel{6}\ y(y+6) - 2(y+6) = 0$

$\Rightarrow (y-2)(y+6) = 0$

$y = 2, \quad y = -6$

Q. 5



Section formula,

$$R\left(\frac{8K+2}{K+1}, \frac{-3}{K+1}, \frac{10K+4}{K+1}\right)$$

But $R(4, y, z)$

Comparison:

$$\frac{8K+2}{K+1} = 4 \Rightarrow 8K+2 = 4K+4$$

$\Rightarrow 4K = 2 \Rightarrow K = \frac{1}{2}$

$$y = \frac{-3}{K+1} = \frac{-3}{\frac{1}{2}+1} = \frac{-\cancel{3}^{1}}{(\cancel{3}/2)}$$

$y = -2$

$$z = \frac{10K+4}{K+1} \quad K = \frac{1}{2}$$

$$z = \frac{10 \times \frac{1}{2} + 4}{\frac{1}{2}+1} = \frac{5+4}{\left(\frac{3}{2}\right)} = \frac{\cancel{9}^{3}}{\left(\frac{\cancel{3}}{2}\right)}$$

$z = 6$

$R(4, -2, 6)$

$\boxed{Q.6}$ $A(3,4,5)$ & $B(-1,3,-7)$

$PA = \sqrt{}$ $PB = \sqrt{}$

$P(x, y, z)$

Given: $(PA)^2 + (PB)^2 = K^2$

$$\Rightarrow (x-3)^2 + (y-4)^2 + (z-5)^2$$

$$+ (x+1)^2 + (y-3)^2 + (z+7)^2 = K^2$$

$$\Rightarrow x^2 + 9 - 6x + y^2 - 8y + 16$$

$$+ z^2 + 25 - 10z +$$

$$x^2 + 2x + 1 + y^2 - 6y + 9$$

$$+ z^2 + 49 + 14z = K^2$$

$$\Rightarrow \boxed{2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = K^2}$$