Rational Numbers

- The numbers, which can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are called rational numbers. Rational numbers can be positive as well as negative. Rational numbers include all integers and fractions.

  For example

  $-\frac{2}{7}, \frac{41}{366}, 2 = \frac{2}{1},$ etc.

All the operations on rational numbers are performed as in fractions.

Example: Solve

1. $-\frac{3}{4} + \frac{5}{6}$

2. $\frac{2}{7} - \frac{3}{5}$

Solution:

1. $-\frac{3}{4} + \frac{5}{6}$

   $= \frac{-3\times3+5\times2}{12} = \frac{-9+10}{12} = \frac{1}{12}$

2. $\frac{2}{7} - \frac{3}{5}$

$$= \frac{2\times5-3\times7}{35} = \frac{10-21}{35} = -\frac{11}{35}$$

- When 0 is added to any rational number, say $\frac{p}{q}$, the same rational number is obtained. Therefore, 0 is the additive identity of rational numbers.

$$\frac{p}{q} + 0 = \frac{p}{q} = 0 + \frac{p}{q}$$

- $-\frac{p}{q}$ is the additive inverse of the rational number $\frac{p}{q}$.

  Example: $-\frac{4}{7}$ is the additive inverse of the rational number $\frac{4}{7}$.

Example: Solve

1. $\frac{2}{9} \times \left(-\frac{4}{3}\right)$
2. $-\frac{3}{7} \div \frac{11}{21}$

Solution:

1. $\frac{2}{9} \times \left(-\frac{4}{3}\right)$

   $= \frac{2\times(-4)}{9\times3} = -\frac{8}{27}$

2. $-\frac{3}{7} \div \frac{11}{21}$

   $= -\frac{3}{7} \times \frac{21}{11} = -\frac{9}{11}$

- Natural numbers are a collection of all positive numbers starting from 1.
- Whole numbers are a collection of all natural numbers including 0.
- Integers are the set of numbers comprising of all the natural numbers 1, 2, 3 ... and their negatives −1, −2, −3 ..., and the number 0.
- Rational numbers are the numbers that can be written in $\frac{p}{q}$ form, where *p* and *q* are integers and $q \neq 0$

- **Closure property**
  - Whole numbers are closed under addition and multiplication. However, they are **not** closed under subtraction and division.
  - Integers are also closed under addition, subtraction and multiplication. However, they are **not** closed under division.
  - Rational numbers:
    1. Rational numbers are closed under addition.

       Example: $\frac{2}{5} + \frac{3}{2} = \frac{19}{10}$ is a rational number.

    2. Rational numbers are closed under subtraction.

       Example: $\frac{1}{5} - \frac{3}{4} = \frac{-11}{20}$ is rational number.

    3. Rational numbers are closed under multiplication.

       Example: $\frac{2}{3} \times \left(\frac{-3}{5}\right) = \frac{-2}{5}$ is a rational number.

    4. Rational numbers are **not** closed under division.

       Example: 2 ÷ 0 is not defined.

- **Commutativity**
  - Whole numbers are commutative under addition and multiplication. However, they are not commutative under subtraction and division.
  - Integers are commutative under addition and multiplication. However, they are not commutative under subtraction and division.
  - Rational numbers:
    1. Rational numbers are commutative under addition.

       Example:

       $\frac{2}{3} + \left(\frac{-3}{2}\right) = \left(\frac{-3}{2}\right) + \left(\frac{2}{3}\right) = \frac{-5}{6}$

    2. Rational numbers are not commutative under subtraction.

       Example :
       $\left(\frac{3}{4}\right) - \left(\frac{5}{2}\right) = \left(\frac{-7}{4}\right)$ and $\frac{5}{2} - \frac{3}{4} = \frac{7}{4}$
       $\therefore \left(\frac{3}{4}\right) - \left(\frac{5}{2}\right) \neq \left(\frac{5}{2}\right) - \left(\frac{3}{4}\right)$

3. Rational numbers are commutative under multiplication.

   Example:
   $\left(\frac{3}{4}\right) \times \left(\frac{-2}{6}\right) = \left(\frac{-2}{6}\right) \times \left(\frac{3}{4}\right) = \frac{-1}{4}$

4. Rational numbers are not commutative under division.

   $2 \div 5 \neq 5 \div 2$

- **Associativity**
  - Whole numbers are associative under addition and multiplication. However, they are **not** associative under subtraction and division.
  - Integers are associative under addition and multiplication. However, they are **not** associative under subtraction and division.
  - Rational numbers:
    1. Rational numbers are associative under addition.

       Example:
       $\left(\frac{2}{3} + \frac{1}{3}\right) + 1 = \frac{2}{3} + \left(\frac{1}{3} + 1\right) = 2$

    2. Rational numbers are **not** associative under subtraction.

       Example:
       $\left(\frac{2}{3} - \frac{1}{3}\right) - 1 = \frac{-2}{3}$
       $\frac{2}{3} - \left(\frac{1}{3} - 1\right) = \frac{4}{3}$
       $\therefore \left(\frac{2}{3} - \frac{1}{3}\right) - 1 \neq \frac{2}{3} - \left(\frac{1}{3} - 1\right)$

    3. Rational numbers are associative under multiplication.

       Example:
       $\left(\frac{2}{3} \times \frac{1}{3}\right) \times 1 = \frac{2}{3} \times \left(\frac{1}{3} \times 1\right) = \frac{2}{9}$

    4. Rational numbers are **not** associative under division.

       Example:

$$\left\{\frac{2}{7} \div \left(\frac{-1}{14}\right)\right\} \div \frac{3}{7} = \frac{-28}{3}$$
$$\frac{2}{7} \div \left\{\left(\frac{-1}{14}\right) \div \frac{3}{7}\right\} = \frac{-12}{7}$$
$$\therefore \frac{2}{7} \div \left\{\left(\frac{-1}{14}\right)\right\} \div \frac{3}{7} \neq \frac{2}{7} \div \left\{\left(\frac{-1}{14}\right) \div \frac{3}{7}\right\}$$

- 0 is the additive identity of whole numbers, integers, and rational numbers.

  $\therefore$ $0 + a = a + 0 = a$, where a is a rational number

- 1 is the multiplicative identity of whole numbers, integers, and rational numbers.

  $a \times 1 = 1 \times a = a$

- Additive inverse of a number is the number, which when added to a number, gives 0. It is also called the negative of a number.

  a + (– a) = (– a) + = 0

  $\therefore$Additive inverse of $\frac{2}{5}$ is $\left(\frac{-2}{5}\right)$

- Reciprocal or multiplicative inverse of a number is the number, which when multiplied by the number, gives 1. Therefore, the reciprocal of a is $\frac{1}{a}$ $\left[a \times \frac{1}{a} = 1\right]$.

  $\therefore$ Reciprocal of $\frac{-2}{3}$ is $\frac{-3}{2}$

- Rational numbers are distributive over addition and subtraction.

  i.e., for any rational numbers a, b, and c, $a(b + c) = ab + ac$, $a(b - c) = ab - ac$

- **Rational numbers on number line**

  Rational numbers can be represented on number line in the similar manner like fractions and integers.

Negative rational numbers are marked to the left of 0 while positive rational numbers are marked to the right of 0.

Example:

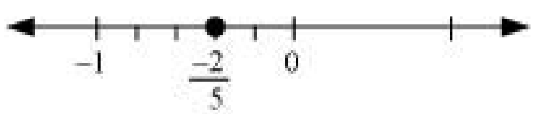
Represent $-\frac{2}{5}$ on number line.

Solution:

The given rational number is negative. Therefore, it will lie to the left of 0.

The space between –1 and 0 is divided into 5 equal parts. Therefore, each part represents $-\frac{1}{5}$.

Marking $-\frac{2}{5}$ at 2 units to the left of 0, we obtain the number line as shown below.



- To find rational numbers between any two given rational numbers, firstly we have to make their denominators same and then find the respective rational numbers.

Example:

Find some rational numbers between $\frac{1}{6}$ and $\frac{7}{8}$.

Solution:

The L.C.M. of 6 and 8 is 24.

Now, we can write

$$\frac{1}{6} = \frac{1 \times 4}{6 \times 4} = \frac{4}{24}$$

$$\frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}$$

Therefore, some of the rational numbers between $\frac{4}{24}\left(\frac{1}{6}\right)$ and $\frac{21}{24}\left(\frac{7}{8}\right)$ are

$$\frac{5}{24}, \frac{6}{24}, \frac{7}{24}, \frac{8}{24}, \frac{9}{24}, \frac{10}{24}, \frac{11}{24}, \frac{12}{24}, \frac{13}{24}, \frac{14}{24}, \frac{15}{24}, \frac{16}{24}, \frac{17}{24}, \frac{18}{24}, \frac{19}{24}, \frac{20}{24}$$