Chapter

2

Straight Line

CONTENTS

Monge

A straight line is the simplest geometric curve.

Monge (1781 A.D.) gave the modern 'Point Slope' form of equation of a line as y - y' = m (x - x') and condition of perpendicularity of two lines as mm' + 1 = 0

S.f. Lacroix (1765-1843 A.D.) was a prolific text book writer; but his contributions to analytic geometry are found scattered. He gave the 'two point' form of equation of a line as $y-\beta=\frac{\beta'-\beta}{\alpha'-\alpha}(x-\alpha)$. *He also gave the formula for finding angles between two lines.*

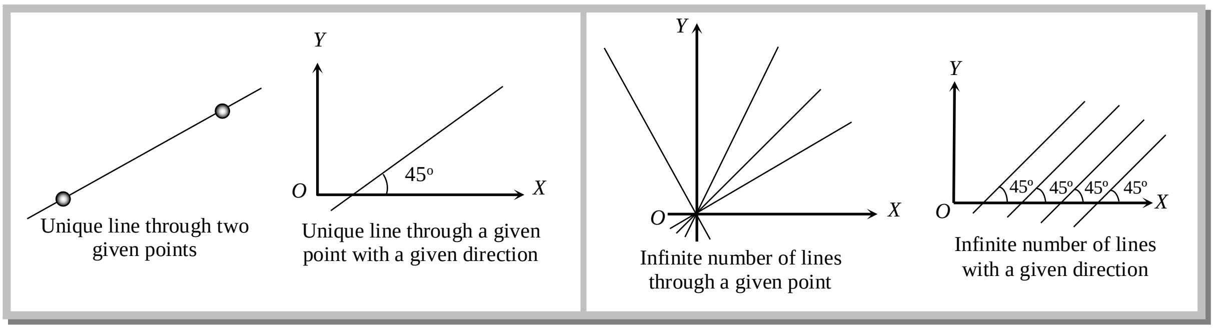
Straight Line

2.1 Definition

The straight line is a curve such that every point on the line segment joining any two points on it lies on it. The simplest locus of a point in a plane is a straight line. A line is determined uniquely by any one of the following:

(1) Two different points (because we know the axiom that one and only one straight line passes through two given points)

(2) A point and a given direction.



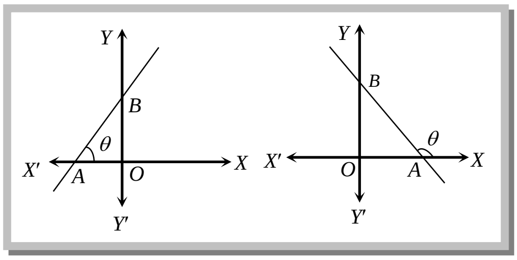
Unique line through two given points
Unique line through a given point with a given direction
Infinite number of lines through a given point
Infinite number of lines with a given direction

Thus, to determine a line uniquely, two geometrical conditions are required.

2.2 Slope (Gradient) of a Line

The trigonometrical tangent of the angle that a line makes with the positive direction of the *x*-axis in anticlockwise sense is called the slope or gradient of the line.

The slope of a line is generally denoted by *m*. Thus, $m = \tan\theta$



(1) Slope of line parallel to x – axis is $m = \tan 0^o = 0$.

(2) Slope of line parallel to y – axis is $m = \tan 90^o = \infty$.

(3) Slope of the line equally inclined with the axes is 1 or –1.

(4) Slope of the line through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\dfrac{y_2 - y_1}{x_2 - x_1}$ taken in the same order.

(5) Slope of the line $ax + by + c = 0, b \neq 0$ is $-\dfrac{a}{b}$.

(6) Slope of two parallel lines are equal.

(7) If m_1 and m_2 be the slopes of two perpendicular lines, then $m_1.m_2 = -1$.

Note : ❑ m can be defined as $\tan\theta$ for $0 < \theta \le \pi$ and $\theta \neq \dfrac{\pi}{2}$

❑ If three points *A*, *B*, *C* are collinear, then

Slope of *AB* = Slope of *BC* = Slope of *AC*

Example: 1 The gradient of the line joining the points on the curve $y = x^2 + 2x$, whose abscissae are 1 and 3, is **[MP PET 1997]**

(a) 6 (b) 5 (c) 4 (d) 3

Solution: (a) The points are (1, 3) and (3, 15)

Hence gradient is = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{12}{2} = 6$

Example: 2 Slope of a line which cuts intercepts of equal lengths on the axes is **[MP PET 1986]**

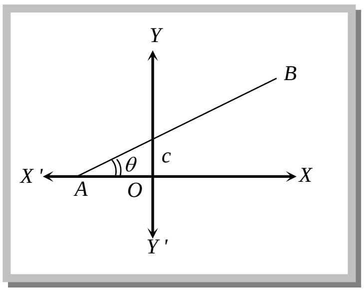
(a) – 1 (b) 0 (c) 2 (d) $\sqrt{3}$

Solution: (a) Equation of line is $\frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow \quad x + y = a \quad \Rightarrow \quad y = -x + a$. Hence slope of the line is – 1.

2.3 Equations of Straight line in Different forms

(1) **Slope form :** Equation of a line through the origin and having slope *m* is $y = mx$.

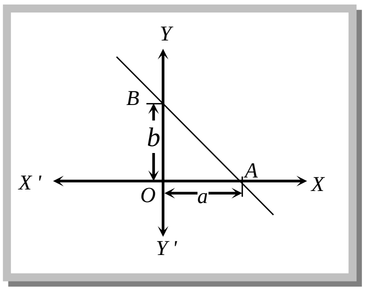


(2) **One point form or Point slope form :** Equation of a line through the point (x_1, y_1) and having slope *m* is $y - y_1 = m(x - x_1)$.

(3) **Slope intercept form :** Equation of a line (non-vertical) with slope *m* and cutting off an intercept *c* on the *y*-axis is $y = mx + c$.

The equation of a line with slope *m* and the *x*-intercept *d* is $y = m(x - d)$

(4) **Intercept form :** If a straight line cuts *x*-axis at *A* and the *y*-axis at *B* then *OA* and *OB* are known as the intercepts of the line on *x*-axis and *y*-axis respectively.



The intercepts are positive or negative according as the line meets with positive or negative directions of the coordinate axes.

In the figure, *OA* = *x*-intercept, *OB* = *y*-intercept.

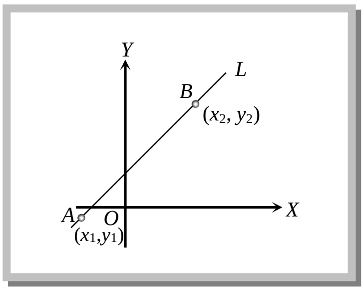
Equation of a straight line cutting off intercepts *a* and *b* on *x*–axis and *y*–axis respectively is $\frac{x}{a} + \frac{y}{b} = 1$.

Note: ❑ If given line is parallel to *X* axis, then *X*-intercept is undefined.

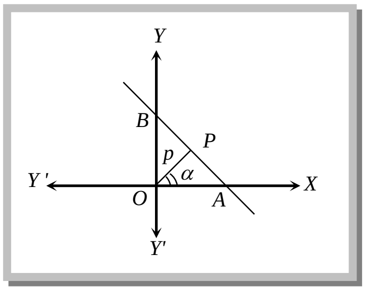
❑ If given line is parallel to *Y* axis, then *Y*-intercept is undefined.

(5)**Two point form:** Equation of the line through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$. In the determinant form it is gives as:

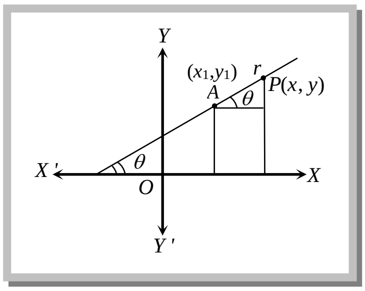
$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0 \text{ is the equation of line.}$$



(6) **Normal or perpendicular form :** The equation of the straight line upon which the length of the perpendicular from the origin is p and this perpendicular makes an angle α with x-axis is $x\cos\alpha + y\sin\alpha = p$.



(7) **Symmetrical or parametric or distance form of the line :** Equation of a line passing through (x_1, y_1) and making an angle θ with the positive direction of x-axis is $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$,

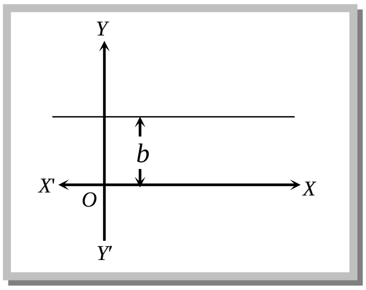


where r is the distance between the point $P(x, y)$ and $A(x_1, y_1)$.

The coordinates of any point on this line may be taken as $(x_1 + r\cos\theta, y_1 + r\sin\theta)$, known as parametric co-ordinates, 'r' is called the parameter.

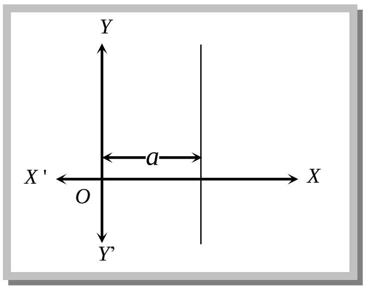
Note: ❑ Equation of x-axis $\Rightarrow y = 0$

Equation a line parallel to x-axis (or perpendicular to y-axis) at a distance 'b' from it $\Rightarrow y = b$



❑ Equation of y-axis $\Rightarrow x = 0$

Equation of a line parallel to y-axis (or perpendicular to x-axis) at a distance 'a' from it $\Rightarrow x = a$



Example: 3 Equation to the straight line cutting off an intercept 2 from the negative direction of the axis of y and inclined at 30° to the positive direction of x, is **[MP PET 2003]**

(a) $y + x - \sqrt{3} = 0$ (b) $y - x + 2 = 0$ (c) $y - \sqrt{3}x - 2 = 0$ (d) $\sqrt{3}y - x + 2\sqrt{3} = 0$

Solution: (d) Let the equation of the straight line is $y = mx + c$.

Here $m = \tan 30° = \dfrac{1}{\sqrt{3}}$ and $c = -2$

Hence, the required equation is $y = \dfrac{1}{\sqrt{3}}x - 2 \Rightarrow \sqrt{3}y - x + 2\sqrt{3} = 0$.

Example: 4 The equation of a straight line passing through (– 3, 2) and cutting an intercept equal in magnitude but opposite in sign from the axes is given by **[Rajasthan PET 1984; MP PET 1993]**

(a) $x - y + 5 = 0$ (b) $x + y - 5 = 0$ (c) $x - y - 5 = 0$ (d) $x + y + 5 = 0$

Solution: (a) Let the equation be $\frac{x}{a} + \frac{y}{-a} = 1 \Rightarrow x - y = a$

But it passes through (–3, 2), hence $a = -3 - 2 = -5$. Hence the equation of straight line is $x - y + 5 = 0$.

Example: 5 The equation of the straight line passing through the point (4, 3) and making intercept on the co-ordinates axes whose sum is – 1, is **[AIEEE 2004]**

(a) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{2} + \frac{y}{1} = 1$ (b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

(c) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$ (d) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

Solution: (d) Let the equation of line is $\frac{x}{a} + \frac{y}{-1-a} = 1$, which passes through (4, 3). Then $\frac{4}{a} + \frac{3}{-1-a} = 1 \Rightarrow a = \pm 2$

Hence equation is $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$.

Example: 6 Let *PS* be the median of the triangle with vertices $P(2, 2), Q(6, -1)$ and *R*(7, 3). The equation of the line passing through (1, – 1) and parallel to *PS* is **[IIT Screening 2000]**

(a) $2x - 9y - 7 = 0$ (b) $2x - 9y - 11 = 0$ (c) $2x + 9y - 11 = 0$ (d) $2x + 9y + 7 = 0$

Solution: (d) *S* = mid point of $QR = \left(\frac{6+7}{2}, \frac{-1+3}{2}\right) = \left(\frac{13}{2}, 1\right)$

$\therefore$ Slope (*m*) of *PS* = $\frac{2-1}{2-\frac{13}{2}} = \frac{-2}{9}$; $\therefore$ The required equation is $y + 1 = \frac{-2}{9}(x - 1) \Rightarrow 2x + 9y + 7 = 0$

2.4 Equation of Parallel and Perpendicular lines to a given Line

(1) Equation of a line which is parallel to $ax + by + c = 0$ is $ax + by + \lambda = 0$

(2) Equation of a line which is perpendicular to $ax + by + c = 0$ is $bx - ay + \lambda = 0$

The value of λ in both cases is obtained with the help of additional information given in the problem.

Example: 7 The equation of the line passes through (*a*, *b*) and parallel to the line $\frac{x}{a} + \frac{y}{b} = 1$, is **[Rajasthan PET 1986, 1995]**

(a) $\frac{x}{a} + \frac{y}{b} = 3$ (b) $\frac{x}{a} + \frac{y}{b} = 2$ (c) $\frac{x}{a} + \frac{y}{b} = 0$ (d) $\frac{x}{a} + \frac{y}{b} + 2 = 0$

Solution: (b) The equation of parallel line to given line is $\frac{x}{a} + \frac{y}{b} = \lambda$.

This line passes through point (*a*, *b*).

$\therefore \frac{a}{a} + \frac{b}{b} = \lambda \Rightarrow \lambda = 2$

Hence, required line is $\frac{x}{a} + \frac{y}{b} = 2$.

Example: 8 A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its *y*-intercept is **[IIT 1992]**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

Solution: (d) The equation of a line passing through (2, 2) and perpendicular to $3x + y = 3$ is

$y - 2 = \frac{1}{3}(x - 2)$ or $x - 3y + 4 = 0$. Putting $x = 0$ in this equation, we obtain $y = \frac{4}{3}$.

So *y*-intercept = $\frac{4}{3}$.

Example: 9 The equation of line passing through $\left(-1, \frac{\pi}{2}\right)$ and perpendicular to $\sqrt{3}\sin\theta + 2\cos\theta = \frac{4}{r}$ is **[EAMCET 2003]**

(a) $2 = \sqrt{3}\, r\cos\theta - 2r\sin\theta$ (b) $5 = -2\sqrt{3}\, r\sin\theta + 4r\cos\theta$

(c) $2 = \sqrt{3}\, r\cos\theta + 2r\sin\theta$ (d) $5 = 2\sqrt{3}\, r\sin\theta + 4r\cos\theta$

Solution: (a) Equation of a line, perpendicular to $\sqrt{3}\sin\theta + 2\cos\theta = \frac{4}{r}$ is $\sqrt{3}\sin\left(\frac{\pi}{2}+\theta\right) + 2\cos\left(\frac{\pi}{2}+\theta\right) = \frac{k}{r}$

It is passing through $\left(-1, \frac{\pi}{2}\right)$. Hence, $\sqrt{3}\sin\pi + 2\cos\pi = k/-1 \Rightarrow k = 2$

$\therefore\ \sqrt{3}\cos\theta - 2\sin\theta = \frac{2}{r} \Rightarrow 2 = \sqrt{3}\, r\cos\theta - 2r\sin\theta$.

Example: 10 The equation of the line bisecting perpendicularly the segment joining the points (– 4, 6) and (8, 8) is **[Karnataka CET 2003]**

(a) $6x + y - 19 = 0$ (b) $y = 7$ (c) $6x + 2y - 19 = 0$ (d) $x + 2y - 7 = 0$

Solution: (a) Equation of the line passing through (–4, 6) and (8, 8) is

$y - 6 = \frac{8-6}{8+4}(x+4) \Rightarrow y - 6 = \frac{2}{12}(x+4) \Rightarrow 6y - x = 40$ ......(i)

Now equation of any line $\perp$ to it is $6x + y + \lambda = 0$ .....(ii)

This line passes through the midpoint of (–4, 6) and (8, 8) *i.e.,* (2, 7)

$\therefore$ From (ii) $12 + 7 + \lambda = 0 \Rightarrow \lambda = -19$, $\therefore$ Equation of line is $6x + y - 19 = 0$

2.5 General equation of a Straight line and its Transformation in Standard forms

General form of equation of a line is $ax + by + c = 0$, its

(1) **Slope intercept form:** $y = -\frac{a}{b}x - \frac{c}{b}$, slope $m = -\frac{a}{b}$ and intercept on y-axis is, $C = -\frac{c}{b}$

(2) **Intercept form :** $\frac{x}{-c/a} + \frac{y}{-c/b} = 1$, x intercept is $= \left(-\frac{c}{a}\right)$ and y intercept is $= \left(-\frac{c}{b}\right)$

(3) **Normal form :** To change the general form of a line into normal form, first take c to right hand side and make it positive, then divide the whole equation by $\sqrt{a^2+b^2}$ like

$-\frac{ax}{\sqrt{a^2+b^2}} - \frac{by}{\sqrt{a^2+b^2}} = \frac{c}{\sqrt{a^2+b^2}}$, where $\cos\alpha = -\frac{a}{\sqrt{a^2+b^2}}$, $\sin\alpha = -\frac{b}{\sqrt{a^2+b^2}}$ and $p = \frac{c}{\sqrt{a^2+b^2}}$

2.6 Selection of Co-ordinate of a Point on a Straight line

(1) If the equation of the straight line be $ax + by + c = 0$, in order to select a point on it, take the x co-ordinate according to your sweet will. Let $x = \lambda$; then $a\lambda + by + c = 0$ or $y = -\frac{a\lambda + c}{b}$;

$\therefore\ \left(\lambda, -\frac{a\lambda + c}{b}\right)$ is a point on the line for any real value of λ. If $\lambda = 0$ is taken then the point will be $\left(0, -\frac{c}{b}\right)$.

Similarly a suitable point can be taken as $\left(-\frac{c}{a}, 0\right)$.

(2) If the equation of the line be $x = c$ then a point on it can be taken as (c, λ) where λ has any real value.

In particular (c, 0) is a convenient point on it when $\lambda = 0$.

(3) If the equation of the line be $y = c$ then a point on it can be taken as (λ, c) where λ has any real value.

In particular (0, c) is a convenient point on it when $\lambda = 0$.

Example: 11 If we reduce $3x + 3y + 7 = 0$ to the form $x\cos\alpha + y\sin\alpha = p$, then the value of p is **[MP PET 2001]**

(a) $\frac{7}{2\sqrt{3}}$ (b) $\frac{7}{3}$ (c) $\frac{3\sqrt{7}}{2}$ (d) $\frac{7}{3\sqrt{2}}$

Solution: (d) Given equation is $3x+3y+7=0$, Dividing both sides by $\sqrt{3^2+3^2}$

$\Rightarrow \frac{3x}{\sqrt{3^2+3^2}}+\frac{3y}{\sqrt{3^2+3^2}}+\frac{7}{\sqrt{3^2+3^2}}=0 \Rightarrow \frac{3}{3\sqrt{2}}x+\frac{3}{3\sqrt{2}}y=\frac{-7}{3\sqrt{2}}$, $\therefore$ $p=\left|\frac{-7}{3\sqrt{2}}\right|=\frac{7}{3\sqrt{2}}$.

2.7 Point of Intersection of Two lines

Let $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ be two non-parallel lines. If (x',y') be the co-ordinates of their point of intersection, then $a_1x'+b_1y'+c_1=0$ and $a_2x'+b_2y'+c_2=0$

Solving these equation, we get $(x',y')=\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1},\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}\right)=\left(\frac{\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}},\frac{\begin{vmatrix} c_1 & c_2 \\ a_1 & a_2 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}\right)$

Note : ❑ Here lines are not parallel, they have unequal slopes, then $a_1b_2-a_2b_1\neq 0$.

❑ In solving numerical questions, we should not remember the co-ordinates (x',y') given above, but we solve the equations directly.

2.8 General equation of Lines through the Intersection of Two given Lines

If equation of two lines $P=a_1x+b_1y+c_1=0$ and $Q=a_2x+b_2y+c_2=0$, then the equation of the lines passing through the point of intersection of these lines is $P+\lambda Q=0$ or $a_1x+b_1y+c+\lambda(a_2x+b_2y+c_2)=0$; Value of λ is obtained with the help of the additional information given in the problem.

Example: 12 Equation of a line passing through the point of intersection of lines $2x-3y+4=0$, $3x+4y-5=0$ and perpendicular to $6x-7y+3=0$, then its equation is **[Rajasthan PET 2000]**

(a) $119x+102y+125=0$ (b) $119x+102y=125$ (c) $119x-102y=125$ (d) None of these

Solution: (b) The point of intersection of the lines $2x-3y+4=0$ and $3x+4y-5=0$ is $\left(\frac{-1}{17},\frac{22}{17}\right)$

The slope of required line = $\frac{-7}{6}$.

Hence, Equation of required line is, $y-\frac{22}{7}=\frac{-7}{6}\left(x+\frac{2}{34}\right) \Rightarrow 119x+102y=125$.

Example: 13 The equation of straight line passing through point of intersection of the straight lines $3x-y+2=0$ and $5x-2y+7=0$ and having infinite slope is **[UPSEAT 2001]**

(a) $x=2$ (b) $x+y=3$ (c) $x=3$ (d) $x=4$

Solution: (c) Required line should be, $(3x-y+2)+\lambda(5x-2y+7)=0$ .....(i)

$\Rightarrow (3+5\lambda)x-(2\lambda+1)y+(2+7\lambda)=0 \Rightarrow y=\left(\frac{3+5\lambda}{2\lambda+1}\right)x+\frac{2+7\lambda}{2\lambda+1}$ ......(ii)

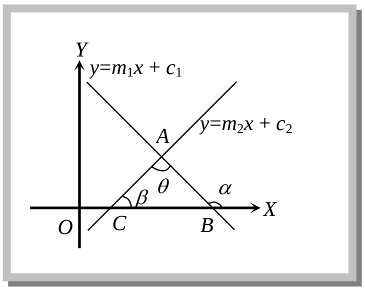
As the equation (ii) has infinite slope, $2\lambda+1=0 \Rightarrow \lambda=\frac{-1}{2}$

Putting $\lambda=\frac{-1}{2}$ in equation (i), We have $(3x-y+2)+\left(\frac{-1}{2}\right)(5x-2y+7)=0 \Rightarrow x=3$

2.9 Angle between Two non-parallel Lines

Let θ be the angle between the lines $y = m_1 x + c_1$ and $y = m_2 x + c_2$.

and intersecting at A.



where, $m_1 = \tan\alpha$ and $m_2 = \tan\beta$

$\because \quad \alpha = \theta + \beta \Rightarrow \theta = \alpha - \beta$

$$\Rightarrow \quad \tan\theta = \left|\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}\right|$$

$$\therefore \quad \theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|.$$

(1) **Angle between two straight lines when their equations are given :** The angle θ between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is given by, $\tan\theta = \left|\frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}\right|$.

(i) **Condition for the lines to be parallel :** If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel then, $m_1 = m_2 \Rightarrow \frac{a_1}{b_1} = \frac{a_2}{b_2} \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$.

(ii) **Condition for the lines to be perpendicular :** If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular then, $m_1m_2 = -1 \Rightarrow \frac{a_1}{b_1} \times \frac{a_2}{b_2} = -1 \Rightarrow a_1a_2 + b_1b_2 = 0$.

(iii) **Conditions for two lines to be coincident, parallel, perpendicular and intersecting :** Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are,

(a) Coincident, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (b) Parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (c) Intersecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(d) Perpendicular, if $a_1a_2 + b_1b_2 = 0$

Example: 14 Angle between the lines $2x - y - 15 = 0$ and $3x + y + 4 = 0$ is **[Rajasthan PET 2003]**

(a) $90°$ (b) $45°$ (c) $180°$ (d) $60°$

Solution: (b) $\tan\theta = \left|\frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}\right| = \left|\frac{(3)(-1)-(2)(1)}{(3)(2)+(-1)(1)}\right| \Rightarrow \tan\theta = \left|\frac{-3-2}{6-1}\right| = \left|\frac{-5}{5}\right| = |-1|$

$\theta = \tan^{-1}|-1| = \tan^{-1}1 = 45^o$.

Example: 15 To which of the following types the straight lines represented by $2x + 3y - 7 = 0$ and $2x + 3y - 5 = 0$ belongs **[MP PET 1982]**

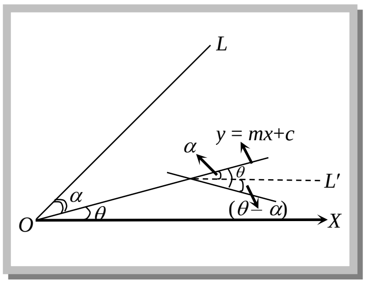
(a) Parallel to each other (b) Perpendicular to each other

(c) Inclined at 45° to each other (d) Coincident pair of straight lines

Solution: (a) Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$; $\frac{2}{3} = \frac{2}{3} \neq \frac{7}{5}$. Hence, lines are parallel to each other.

2.10 Equation of Straight line through a given point making a given Angle with a given Line

Since straight line L makes an angle $(\theta+\alpha)$ with x-axis, then equation of line L is $y-y_1=\tan(\theta+\alpha)(x-x_1)$ and straight line L' makes an angle $(\theta-\alpha)$ with x-axis, then equation of line L' is



$\Rightarrow \qquad y-y_1=\tan(\theta-\alpha)(x-x_1)$

where $\qquad m=\tan\theta$

Hence, the equation of the straight lines which pass through a given point (x_1, y_1) and make a given angle α with given straight line $y=mx+c$ are

$$y-y_1=\frac{m\pm\tan\alpha}{1\mp m\tan\alpha}(x-x_1)$$

Example: 16 The equation of the lines which passes through the point (3, – 2) and are inclined at 60^o to the line $\sqrt{3}x+y=1$

[IIT 1974; MP PET 1996]

(a) $y+2=0,\ \sqrt{3}x-y-2-3\sqrt{3}=0$ (b) $x-2=0,\ \sqrt{3}x-y+2+3\sqrt{3}=0$

(c) $\sqrt{3}x-y-2-3\sqrt{3}=0$ (d) None of these

Solution: (a) The equation of lines passing through (3, –2) is $(y+2)=m(x-3)$ .....(i)

The slope of the given line is $-\sqrt{3}$.

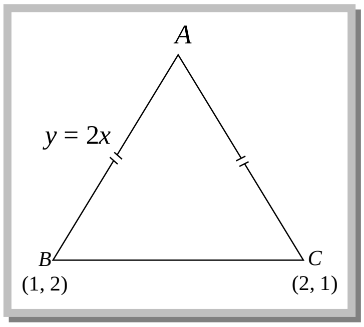
So, $\tan 60^o=\pm\frac{m-(-\sqrt{3})}{1+m(-\sqrt{3})}$. On solving, we get $m=0$ or $\sqrt{3}$

Putting the values of m in (i), the required equation is $y+2=0$ and $\sqrt{3}x-y-2-3\sqrt{3}=0$.

Example: 17 In an isosceles triangle ABC, the coordinates of the point B and C on the base BC are respectively (1, 2) and (2, 1). If the equation of the line AB is $y=2x$, then the equation of the line AC is **[Roorkee 2000]**

(a) $y=\frac{1}{2}(x-1)$ (b) $y=\frac{x}{2}$ (c) $y=x-1$ (d) $2y=x+3$

Solution: (b) Slope of $BC=\frac{1-2}{2-1}=-1$



$\because\ AB=AC$, $\therefore\ \angle ABC=\angle ACB$

$\Rightarrow \left|\frac{2+1}{1+2(-1)}\right|=\frac{m+1}{1+m(-1)} \Rightarrow \frac{m+1}{1-m}=|-3| \Rightarrow \frac{m+1}{1-m}=\pm 3 \Rightarrow m=2, \frac{1}{2}$.

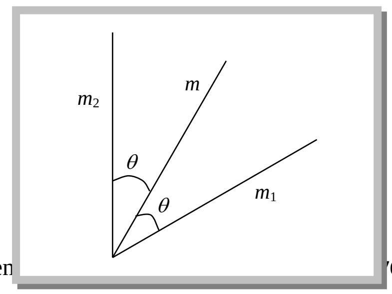
But slope of AB is 2; $\therefore\ m=\frac{1}{2}$ (Here m is the gradient of the line AC)

Equation of the line AC is $y-1=\frac{1}{2}(x-2) \Rightarrow x-2y=0$ or $y=\frac{x}{2}$.

2.11 A Line equally inclined with Two lines

Let the two lines with slopes m_1 and m_2 be equally inclined to a line with slope m

then, $\left(\frac{m_1-m}{1+m_1m}\right)=-\left(\frac{m_2-m}{1+m_2m}\right)$



Note : ❑ Sign of m in both brackets is same.

Example: 18 If the lines $y=3x+1$ and $2y=x+3$ are equally inclined to the line $y=mx+4$, then **6]**

(a) $\frac{1+3\sqrt{2}}{7}$ (b) $\frac{1-3\sqrt{2}}{7}$ (c) $\frac{1+3\sqrt{2}}{7}$ (d) $\frac{1\pm 5\sqrt{2}}{7}$

Solution: (d) If line $y = mx + 4$ are equally inclined to lines with slope $m_1 = 3$ and $m_2 = \frac{1}{2}$,then $\left(\frac{3-m}{1+3m}\right) = -\left(\frac{\frac{1}{2}-3}{1+\frac{1}{2}m}\right) \Rightarrow m = \frac{1\pm 5\sqrt{2}}{7}$

2.12 Equations of the bisectors of the Angles between two Straight lines

The equation of the bisectors of the angles between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are given by, $\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$ .....(i)

Algorithm to find the bisector of the angle containing the origin :

Let the equations of the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. To find the bisector of the angle containing the origin, we proceed as follows:

Step I : See whether the constant terms c_1 and c_2 in the equations of two lines positive or not. If not, then multiply both the sides of the equation by –1 to make the constant term positive.

Step II : Now obtain the bisector corresponding to the positive sign *i.e.*, $\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$.

This is the required bisector of the angle containing the origin.

Note : ❑ The bisector of the angle containing the origin means the bisector of the angle between the lines which contains the origin within it.

(1) To find the acute and obtuse angle bisectors

Let θ be the angle between one of the lines and one of the bisectors given by (i). Find $\tan\theta$. If $|\tan\theta| < 1$, then this bisector is the bisector of acute angle and the other one is the bisector of the obtuse angle.

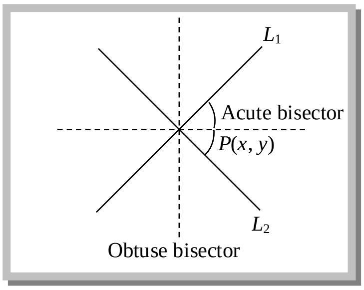
If $|\tan\theta| > 1$, then this bisector is the bisector of obtuse angle and other one is the bisector of the acute angle.

(2) Method to find acute angle bisector and obtuse angle bisector

(i) Make the constant term positive, if not. (ii) Now determine the sign of the expression $a_1a_2 + b_1b_2$.

(iii) If $a_1a_2 + b_1b_2 > 0$, then the bisector corresponding to "+" sign gives the obtuse angle bisector and the bisector corresponding to "–" sign is the bisector of acute angle between the lines.

(iv) If $a_1a_2 + b_1b_2 < 0$, then the bisector corresponding to "+" and "–" sign given the acute and obtuse angle bisectors respectively.



Note : ❑ Bisectors are perpendicular to each other.

❑ If $a_1a_2 + b_1b_2 > 0$, then the origin lies in obtuse angle and if $a_1a_2 + b_1b_2 < 0$, then the origin lies in acute angle.

Example: 19 The equation of the bisectors of the angles between the lines $|x| = |y|$ are **[Orissa JEE 2002]**

(a) $y = \pm x$ and $x = 0$ (b) $x = \frac{1}{2}$ and $y = \frac{1}{2}$ (c) $y = 0$ and $x = 0$ (d) None of these

Solution: (c) The equation of lines are $x + y = 0$ and $x - y = 0$.

$\therefore$ The equation of bisectors of the angles between these lines are $\frac{x+y}{\sqrt{1+1}} = \pm\frac{x-y}{\sqrt{1+1}} \Rightarrow x + y = \pm(x - y)$

Taking +*ve* sign, we get $y = 0$; Taking –*ve* sign, we get $x = 0$. Hence, the equation of bisectors are $x = 0,\ y = 0$.

Example: 20 The equation of the bisector of the acute angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$ is

[IIT 1975, 1983; Rajasthan PET 2003]

(a) $21x + 77y - 101 = 0$ (b) $11x - 3y + 9 = 0$ (c) $31x + 77y + 101 = 0$ (d) $11x - 3y - 9 = 0$

Solution: (b) Bisector of the angles is given by $\frac{3x - 4y + 7}{5} = \pm \frac{12x + 5y - 2}{13}$

$\Rightarrow 11x - 3y + 9 = 0$ .....(i) and $21x + 77y - 101 = 0$ ......(ii)

Let the angle between the line $3x - 4y + 7 = 0$ and (i) is α, then $\tan \alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{3}{4} - \frac{11}{3}}{1 + \frac{3}{4} \times \frac{11}{3}}\right| = \frac{35}{45} < 1 \Rightarrow \alpha < 45^o$

Hence $11x - 3y + 9 = 0$ is the bisector of the acute angle between the given lines.

2.13 Length of Perpendicular

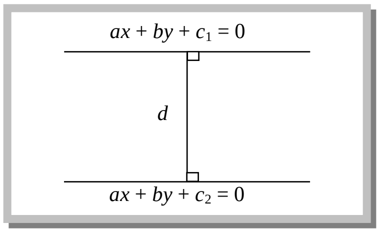
(1) **Distance of a point from a line :** The length p of the perpendicular from the point (x_1, y_1) to the line $ax + by + c = 0$ is given by $p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

Note : ❑ Length of perpendicular from origin to the line $ax + by + c = 0$ is $\frac{c}{\sqrt{a^2 + b^2}}$.

❑ Length of perpendicular from the point (x_1, y_1) to the line $x \cos \alpha + y \sin \alpha = p$ is $x_1 \cos \alpha + y_1 \sin \alpha - p$.

(2) **Distance between two parallel lines :** Let the two parallel lines be $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$.

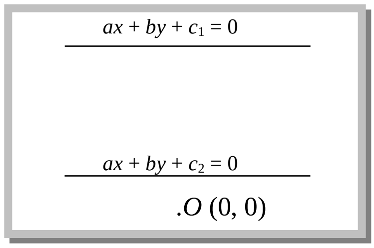
First Method : The distance between the lines is $d = \frac{|c_1 - c_2|}{\sqrt{(a^2 + b^2)}}$.



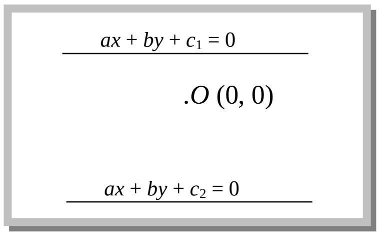
Second Method : The distance between the lines is $d = \frac{\lambda}{\sqrt{(a^2 + b^2)}}$, where

(i) $\lambda = |c_1 - c_2|$ if they be on the same side of origin.

(ii) $\lambda = |c_1| + |c_2|$ if the origin O lies between them.



Third method : Find the coordinates of any point on one of the given line, preferably putting $x = 0$ or $y = 0$. Then the perpendicular distance of this point from the other line is the required distance between the lines.



Note: ❑ Distance between two parallel lines $ax + by + c_1 = 0$ and $kax + kby + c_2 = 0$ is $\frac{\left|c_1 - \frac{c_2}{k}\right|}{\sqrt{a^2 + b^2}}$

❑ Distance between two non parallel lines is always zero.

2.14 Position of a Point with respect to a Line

Let the given line be $ax + by + c = 0$ and observing point is (x_1, y_1), then

(i) If the same sign is found by putting in equation of line $x = x_1, y = y_1$ and $x = 0$, $y = 0$ then the point (x_1, y_1) is situated on the side of origin.

(ii) If the opposite sign is found by putting in equation of line $x = x_1$, $y = y_1$ and $x = 0$, $y = 0$ then the point (x_1, y_1) is situated opposite side to origin.

2.15 Position of Two points with respect to a Line

Two points (x_1, y_1) and (x_2, y_2) are on the same side or on the opposite side of the straight line $ax + by + c = 0$ according as the values of $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the same sign or opposite sign.

Example: 21 The distance of the point (–2, 3) from the line $x - y = 5$ is **[MP PET 2001]**

(a) $5\sqrt{2}$ (b) $2\sqrt{5}$ (c) $3\sqrt{5}$ (d) $5\sqrt{3}$

Solution: (a) $p = \left|\frac{x_1 - y_1 - 5}{\sqrt{1^2 + 1^2}}\right| = \left|\frac{-2-3-5}{\sqrt{1^2+1^2}}\right| = \left|\frac{-10}{\sqrt{2}}\right| = 5\sqrt{2}$

Example: 22 The distance between the lines $4x + 3y = 11$ and $8x + 6y = 15$ is **[AMU 1979; MNR 1987; UPSEAT 2000; DCE 1999]**

(a) $\frac{7}{2}$ (b) 4 (c) $\frac{7}{10}$ (d) None of these

Solution: (c) Given lines $4x + 3y = 11$ and $8x + 6y = 15$, distance from the origin to both the lines are $\left|\frac{-11}{\sqrt{25}}\right|$ and $\left|\frac{-15}{\sqrt{100}}\right| \Rightarrow \frac{11}{5}, \frac{15}{10}$

Clearly both lines are on the same side of the origin.

Hence, distance between both the lines are, $\frac{11}{5} - \frac{15}{10} = \frac{7}{10}$.

Example: 23 If the length of the perpendicular drawn from origin to the line whose intercepts on the axes are *a* and *b* be *p*, then **[Karnataka CET 2003]**

(a) $a^2 + b^2 = p^2$ (b) $a^2 + b^2 = \frac{1}{p^2}$ (c) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{p^2}$ (d) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$

Solution: (d) Equation of line is $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow bx + ay - ab = 0$

Perpendicular distance from origin to given line is $p = \left|\frac{-ab}{\sqrt{a^2+b^2}}\right| \Rightarrow \frac{\sqrt{a^2+b^2}}{ab} = \frac{1}{p} \Rightarrow \frac{a^2+b^2}{a^2b^2} = \frac{1}{p^2} \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$

Example: 24 The point on the *x*-axis whose perpendicular distance from the line $\frac{x}{a} + \frac{y}{b} = 1$ is *a*, is **[Rajasthan PET 2001; MP PET 2003]**

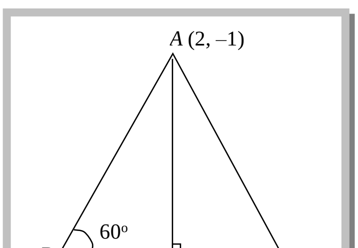
(a) $\left[\frac{a}{b}(b \pm \sqrt{a^2+b^2}), 0\right]$ (b) $\left[\frac{b}{a}(b \pm \sqrt{a^2+b^2}), 0\right]$ (c) $\left[\frac{a}{b}(a \pm \sqrt{a^2+b^2}), 0\right]$ (d) None of these

Solution: (a) Let the point be $(h, 0)$ then $a = \pm\frac{bh + 0 - ab}{\sqrt{a^2+b^2}} \Rightarrow bh = \pm a\sqrt{a^2+b^2} + ab \Rightarrow h = \frac{a}{b}(b \pm \sqrt{a^2+b^2})$

Hence the point is $\left[\frac{a}{b}(b \pm \sqrt{a^2+b^2}), 0\right]$

Example: 25 The vertex of an equilateral triangle is (2, –1) and the equation of its base is $x + 2y = 1$. The length of its sides is **[UPSEAT 2003]**

(a) $\frac{4}{\sqrt{15}}$ (b) $\frac{2}{\sqrt{15}}$



(c) $\frac{4}{3\sqrt{3}}$ (d) None of these

Solution: (b) $|AD| = \left|\frac{2-2-1}{\sqrt{1^2+2^2}}\right| = \frac{1}{\sqrt{5}}$

$\because \tan 60^o = \frac{AD}{BD} \Rightarrow \sqrt{3} = \frac{1/\sqrt{5}}{BD} \Rightarrow BD = \frac{1}{\sqrt{15}} \Rightarrow BC = 2BD = \frac{2}{\sqrt{15}}$

2.16 Concurrent Lines

Three or more lines are said to be concurrent lines if they meet at a point.

First method : Find the point of intersection of any two lines by solving them simultaneously. If the point satisfies the third equation also, then the given lines are concurrent.

Second method : The three lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ and $a_3x+b_3y+c_3=0$ are concurrent if,

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Third method : The condition for the lines $P=0$, $Q=0$ and $R=0$ to be concurrent is that three constants a, b, c (not all zero at the same time) can be obtained such that $aP+bQ+cR=0$.

Example: 26 If the lines $ax+by+c=0$, $bx+cy+a=0$ and $cx+ay+b=0$ be concurrent, then **[IIT 1985; DCE 2002]**

(a) $a^3+b^3+c^3+3abc=0$ (b) $a^3+b^3+c^3-abc=0$ (c) $a^3+b^3+c^3-3abc=0$ (d) None of these

Solution: (c) Here the given lines are, $ax+by+c=0$, $bx+cy+a=0$, $cx+ay+b=0$

The lines will be concurrent, *iff* $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \Rightarrow a^3+b^3+c^3-3abc=0$

Example: 27 If the lines $4x+3y=1$, $y=x+5$ and $5y+bx=3$ are concurrent, then b equals

[Rajasthan PET 1996; MP PET 1997; EAMCET 2003]

(a) 1 (b) 3 (c) 6 (d) 0

Solution: (c) If these lines are concurrent then the intersection point of the lines $4x+3y=1$ and $y=x+5$, is (–2, 3), which lies on the third line.

Hence, $\Rightarrow 5\times 3-2b=3 \Rightarrow 15-2b=3 \Rightarrow 2b=12 \Rightarrow b=6$

Example: 28 The straight lines $4ax+3by+c=0$ where $a+b+c=0$, will be concurrent, if point is **[Rajasthan PET 2002]**

(a) (4, 3) (b) $\left(\frac{1}{4}, \frac{1}{3}\right)$ (c) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (d) None of these

Solution: (b) The set of lines is $4ax+3by+c=0$, where $a+b+c=0$

Eliminating c, we get $4ax+3by-(a+b)=0 \Rightarrow a(4x-1)+b(3y-1)=0$

They pass through the intersection of the lines $4x-1=0$ and $3y-1=0$ *i.e.*, $x=\frac{1}{4}, y=\frac{1}{3}$ *i.e.*, $\left(\frac{1}{4}, \frac{1}{3}\right)$

2.17 Reflection on the Surface

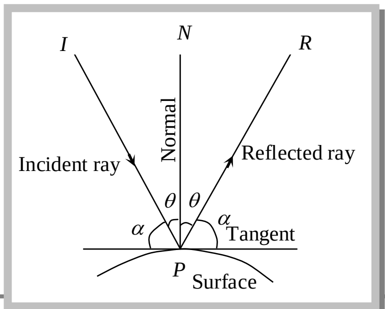
Here IP = Incident Ray

PN = Normal to the surface

PR = Reflected Ray

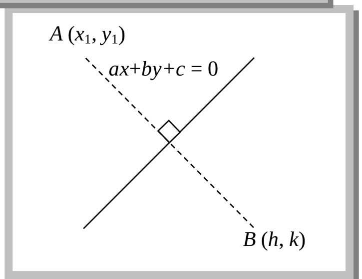
Then, $\angle IPN = \angle NPR$

Angle of incidence = Angle of reflection



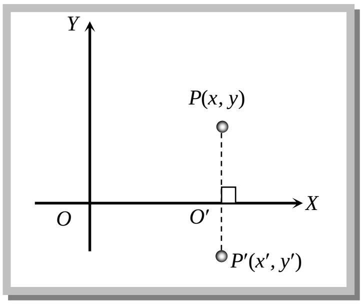
2.18 Image of a Point in Different cases

(1) **The image of a point with respect to the line mirror :** The image of $A(x_1, y_1)$ with respect to the line mirror $ax+by+c=0$ be $B(h, k)$ is given by,



$$\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}$$

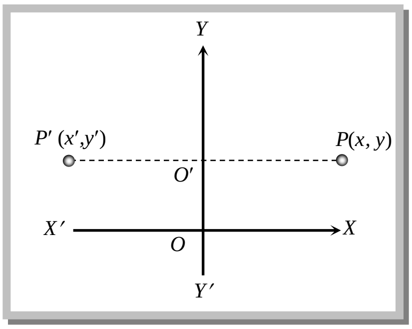
(2) **The image of a point with respect to x-axis :** Let $P(x,y)$ be any point and $P'(x',y')$ its image after reflection in the x-axis, then



$x' = x$ $\quad(\because\ O'$ is the mid point of P and $P')$

$y' = -y$

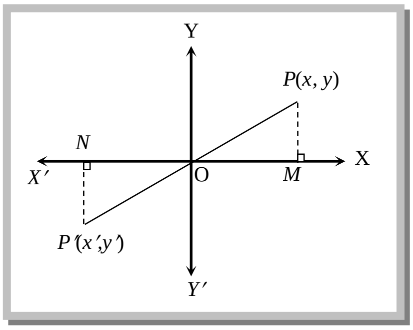
(3) **The image of a point with respect to y-axis :** Let $P(x,y)$ be any point and $P'(x',y')$ its image after reflection in the y-axis



then $x' = -x$ $\quad(\because\ O'$ is the mid point of P and $P')$

$y' = y$

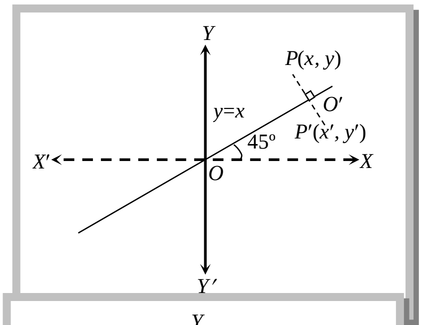
(4) **The image of a point with respect to the origin :** Let $P(x,y)$ be any point and $P'(x',y')$ be its image after reflection through the origin, then



$x' = -x$ $\quad(\because\ O'$ is the mid point of P and $P')$

$y' = -y$

(5) **The image of a point with respect to the line $y = x$:** Let $P(x,y)$ be any point and $P'(x',y')$ be its image after reflection in the line $y = x$, then

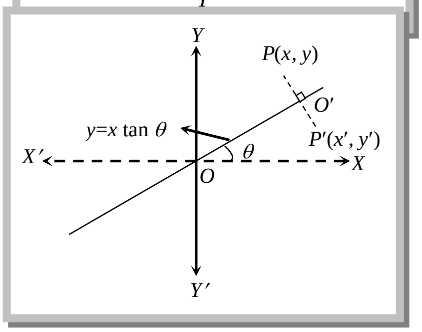


$x' = y$ $\quad(\because\ O'$ is the mid point of P and $P')$

$y' = x$

(6) **The image of a point with respect to the line $y = x\tan\theta$:**

Let $P(x,y)$ be any point and $P'(x',y')$ be its image after reflection in the line $y = x\tan\theta$ then



$x' = x\cos 2\theta + y\sin 2\theta$ $\quad(\because\ O'$ is the mid point of P and $P')$

$y' = x\sin 2\theta - y\cos 2\theta$

Example: 29 The reflection of the point (4, –13) in the line $5x + y + 6 = 0$ is **[EAMCET 1994]**

(a) (–1, –14) (b) (3, 4) (c) (1, 2) (d) (–4, 13)

Solution: (a) Let $Q(a,b)$ be the reflection of $P(4,-13)$ in the line $5x+y+6=0$. Then the point $R\left(\frac{a+4}{2},\frac{b-13}{2}\right)$ lies on $5x+y+6=0$

. $\therefore$ $5\left(\frac{a+4}{2}\right)+\left(\frac{b-13}{2}\right)+6=0 \Rightarrow 5a+b+19=0$ .....(i)

Also PQ is perpendicular to $5x+y+6=0$. Therefore $\left(\frac{b+13}{a-4}\right)\times\left(\frac{-5}{1}\right) \Rightarrow a-5b-69=0$ .....(ii)

Solving (i) and (ii), we get $a=-1, b=-14$.

Example: 30 The image of a point $A(3,8)$ in the line $x+3y-7=0$, is **[Rajasthan PET 1991]**

(a) (–1, –4) (b) (–3, –8) (c) (1, –4) (d) (3, 8)

Solution: (a) Equation of the line passing through (3, 8) and perpendicular to $x+3y-7=0$ is $3x-y-1=0$. The intersection point of both the lines is (1, 2). Now let the image of $A(3,8)$ be $A'(x_1,y_1)$.

The point (1, 2) will be the midpoint of AA'. $\frac{x_1+3}{2}=1 \Rightarrow x_1=-1$ and $\frac{y_1+8}{2}=2 \Rightarrow y_1=4$. Hence the image is (–1, –4).

2.19 Some Important Results

(1) Area of the triangle formed by the lines $y=m_1x+c_1$, $y=m_2x+c_2, y=m_3x+c_3$ is $\frac{1}{2}\left|\sum\frac{(c_1-c_2)^2}{m_1-m_2}\right|$.

(2) Area of the triangle made by the line $ax+by+c=0$ with the co-ordinate axes is $\frac{c^2}{2|ab|}$.

(3) Area of the rhombus formed by the lines $ax \pm by \pm c = 0$ is $\left|\frac{2c^2}{ab}\right|$

(4) Area of the parallelogram formed by the lines $a_1x+b_1y+c_1=0$; $a_2x+b_2y+c_2=0$, $a_1x+b_1y+d_1$ and $a_2x+b_2y+d_2=0$ is $\left|\frac{(d_1-c_1)(d_2-c_2)}{a_1b_2-a_2b_1}\right|$.

(5) The foot of the perpendicular (h,k) from (x_1,y_1) to the line $ax+by+c=0$ is given by $\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2}$. Hence, the coordinates of the foot of perpendicular is $\left(\frac{b^2x_1-aby_1-ac}{a^2+b^2},\frac{a^2y_1-abx_1-bc}{a^2+b^2}\right)$

(6) Area of parallelogram $A=\frac{p_1p_2}{\sin\theta}$, where p_1 and p_2 are the distances between parallel sides and θ is the angle between two adjacent sides.

(7) The equation of a line whose mid-point is (x_1,y_1) in between the axes is $\frac{x}{x_1}+\frac{y}{y_1}=2$

(8) The equation of a straight line which makes a triangle with the axes of centroid (x_1,y_1) is $\frac{x}{3x_1}+\frac{y}{3y_1}=1$.

Example: 31 The coordinates of the foot of perpendicular drawn from (2, 4) to the line $x+y=1$ is **[Roorkee 1995]**

(a) $\left(\frac{1}{3},\frac{3}{2}\right)$ (b) $\left(-\frac{1}{2},\frac{3}{2}\right)$ (c) $\left(\frac{4}{3},\frac{1}{2}\right)$ (d) $\left(\frac{3}{4},\frac{-1}{2}\right)$

Solution: (b) Applying the formula, the required co-ordinates is $\left(\frac{1^2\times 2-1\times 1\times 4+1}{1^2+1^2},\frac{1^2\times 4-1\times 1\times 2+1}{1^2+1^2}\right)=\left(\frac{-1}{2},\frac{3}{2}\right)$

Example: 32 The area enclosed within the curve $|x|+|y|=1$ is **[Rajasthan PET 1990, 97; IIT 1981; UPSEAT 2003]**

(a) $\sqrt{2}$ (b) 1 (c) $\sqrt{3}$ (d) 2

Solution: (d) The given lines are $\pm x\pm y=1$ *i.e.,* $x+y=1$, $x-y=1$, $x+y=-1$ and $x-y=-1$. These lines form a quadrilateral whose vertices are $A(-1,0)$, $B(0,-1)$, $C(1,0)$ and $D(0,1)$. Obviously $ABCD$ is a square. Length of each side of this square is $\sqrt{1^2+1^2}=\sqrt{2}$. Hence, area of square is $\sqrt{2}\times\sqrt{2}=2$ sq. units.

Example: 33 If x_1,x_2,x_3 and y_1,y_2,y_3 are both in G.P. with the same common ratio, then the point (x_1,y_1), (x_2,y_2) and (x_3,y_3) **[AIEEE 2003]**

(a) Lie on a straight line (b) Lie on an ellipse (c) Lie on a circle (d) Are vertices of a triangle

Solution: (a) Taking co-ordinates as $\left(\frac{x}{r},\frac{y}{r}\right)$, (x, y) and (x_r,y_r). Above co-ordinates satisfy the relation $y=mx$, $\therefore$ the three points lie on a straight line.

Example: 34 A square of side a lies above the *x*-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0<\alpha<\frac{\pi}{4}\right)$ with the positive direction of *x*-axis. The equation of its diagonal not passing through the origin is **[AIEEE 2003]**

(a) $y(\cos\alpha-\sin\alpha)-x(\sin\alpha-\cos\alpha)=a$ (b) $y(\cos\alpha+\sin\alpha)-x(\sin\alpha-\cos\alpha)=a$

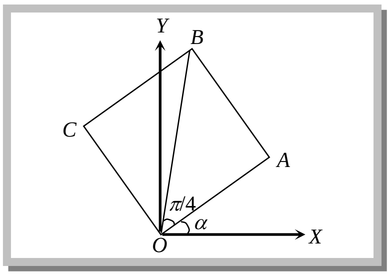
(c) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha+\cos\alpha)=a$ (d) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha-\cos\alpha)=a$

Solution: (b) Co-ordinates of $A=(a\cos\alpha,a\sin\alpha)$; Equation of OB $y=\tan\left(\frac{\pi}{4}+\alpha\right)x$

$\because$ $CA\perp$ to OB; $\therefore$ slope of $CA=-\cot\left(\frac{\pi}{4}+\alpha\right)$

Equation of *CA*, $y-a\sin\alpha=-\cot\left(\frac{\pi}{4}+\alpha\right)(x-a\cos\alpha)$

$\Rightarrow$ $y(\sin\alpha+\cos\alpha)+x(\cos a-\sin\alpha)=a$.



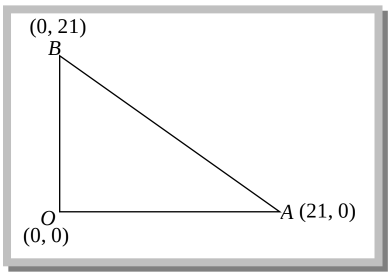
Example: 35 The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0) is **[IIT Screening 2003]**

(a) 133 (b) 190 (c) 233 (d) 105

Solution: (b) $x+y=21$

The number of integral solution to the equation $x+y<21$ *i.e.,* $x<21-y$

Number of integral co-ordinates $=19+18+..........+1=\frac{19\times 20}{2}=190$.



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Assignment

Equation in Different forms and Slope of Line

Basic Level

1. The equation of the straight line which passes through the point $(1, -2)$ and cuts off equal intercepts from axes, is [MNR 1978]

(a) $x+y=1$ (b) $x-y=1$ (c) $x+y+1=0$ (d) $x-y-2=0$

2. Equation of the straight line making equal intercepts on the axes and passing through the point (2, 4) is [Karnataka CET 2004]

(a) $4x-y-4=0$ (b) $2x+y-8=0$ (c) $x+y-6=0$ (d) $x+2y-10=0$

3. In the equation $y-y_1=m(x-x_1)$ if m and x_1 are fixed and different lines are drawn for different values of y_1, then [MP PET 1986]

(a) The lines will pass through a single point (b) There will be a set of parallel lines

(c) There will be one line only (d) None of these

4. The equation of the straight line passing through the point (3, 2) and perpendicular to the line $y=x$ is [MNR 1979; MP PET 2002]

(a) $x-y=5$ (b) $x+y=5$ (c) $x+y=1$ (d) $x-y=1$

5. The equation of the line perpendicular to the line $\frac{x}{a}-\frac{y}{b}=1$ and passing through the point at which it cuts x-axis, is [Rajasthan PET 1996]

(a) $\frac{x}{a}+\frac{y}{b}+\frac{a}{b}=0$ (b) $\frac{x}{b}+\frac{y}{a}=\frac{b}{a}$ (c) $\frac{x}{b}+\frac{y}{a}=0$ (d) $\frac{x}{b}+\frac{y}{a}=\frac{a}{b}$

6. The equation of the line passing through the point (1, 2) and perpendicular to the line $x+y+1=0$ is [MNR 1981]

(a) $y-x+1=0$ (b) $y-x-1=0$ (c) $y-x+2=0$ (d) $y-x-2=0$

7. If the equations $y=mx+c$ and $x\cos\alpha+y\sin\alpha=p$ represent the same straight line, then

(a) $p=c\sqrt{1+m^2}$ (b) $c=p\sqrt{1+m^2}$ (c) $cp=\sqrt{1+m^2}$ (d) $p^2+c^2+m^2=1$

8. A line passes through the point of intersection of $2x+y=5$ and $x+3y+8=0$ and parallel to the line $3x+4y=7$ is [Rajasthan PET 1984; MP PET 1991]

(a) $3x+4y+3=0$ (b) $3x+4y=0$ (c) $4x-3y+3=0$ (d) $4x-3y=3$

9. The equation of straight line passing through the intersection of the lines $x-2y=1$ and $x+3y=2$ and parallel to $3x+4y=0$ is [MP PET 2000]

(a) $3x+4y+5=0$ (b) $3x+4y-10=0$ (c) $3x+4y-5=0$ (d) $3x+4y+6=0$

10. The equation of the line joining the origin to the point (–4, 5) is [MP PET 1984]

(a) $5x+4y=0$ (b) $3x+4y=2$ (c) $5x-4y=0$ (d) $4x-5y=0$

11. The equation of the line which cuts off an intercept 3 units on *OX* and an intercept –2 unit on *OY*, is

(a) $\frac{x}{3}-\frac{y}{2}=1$ (b) $\frac{x}{3}+\frac{y}{2}=1$ (c) $\frac{x}{2}+\frac{y}{3}=1$ (d) $\frac{x}{2}-\frac{y}{3}=1$

12. The equation of a line through (3, – 4) and perpendicular to the line $3x+4y=5$ is [Rajasthan PET 1981, 84, 86; MP PET 1984]

(a) $4x+3y=24$ (b) $y-4=x+3$ (c) $3y-4x=24$ (d) $y+4=\frac{4}{3}(x-3)$

13. Equation of the line passing through (1, 2) and parallel to the line $y=3x-1$ is [MP PET 1984]

(a) $y+2=x+1$ (b) $y+2=3(x+1)$ (c) $y-2=3(x-1)$ (d) $y-2=x-1$

14. Equation of the line passing through (–1, 1) and perpendicular to the line $2x+3y+4=0$ is [MP PET 1984]

(a) $2(y-1)=3(x+1)$ (b) $3(y-1)=-2(x+1)$ (c) $y-1=2(x+1)$ (d) $3(y-1)=x+1$

15. The equation of line passing through (*c*, *d*) and parallel to $ax+by+c=0$ is [Rajasthan PET 1987]

(a) $a(x+c)+b(y+d)=0$ (b) $a(x+c)-b(y+d)=0$ (c) $a(x-c)+b(y-d)=0$ (d) None of these

16. The equation of a line through the intersection of lines $x=0$ and $y=0$ and through the point $(2,2)$ is [MP PET 1984]

(a) $y=x-1$ (b) $y=-x$ (c) $y=x$ (d) $y=-x+2$

17. Equation of a line through the origin and perpendicular to the line joining (*a*, 0) and $(-a,0)$ is [MP PET 1984]

(a) $y=0$ (b) $x=0$ (c) $x=-a$ (d) $y=-a$

18. For what values of *a* and *b* the intercepts cut off on the coordinate axes by the line $ax+by+8=0$ are equal in length but opposite in signs to those cut off by the line $2x-3y+6=0$ on the axes

(a) $a=\frac{8}{3},b=-4$ (b) $a=-\frac{8}{3},b=-4$ (c) $a=\frac{8}{3},b=4$ (d) $a=-\frac{8}{3},b=4$

19. For specifying a straight line how many geometrical parameters should be known [MP PET 1982]

(a) 1 (b) 2 (c) 4 (d) 3

20. The equation of line passing through point of intersection of line $3x-2y-1=0$ and $x-4y+3=0$ and the point $(\pi,0)$ is [Rajasthan PET 1987]

(a) $x-y=\pi$ (b) $x-y=\pi(y+1)$ (c) $x-y=\pi(1-y)$ (d) $x+y=\pi(1-y)$

21. A line perpendicular to the line $ax+by+c=0$ and passes through (a,b). The equation of the line is [Rajasthan PET 1988; MP PET 1995]

(a) $bx-ay+(a^2-b^2)=0$ (b) $bx-ay-(a^2-b^2)=0$ (c) $bx-ay=0$ (d) None of these

22. If the line passing through (4, 3) and (2, *k*) is perpendicular to $y=2x+3$, then *k*= [Rajasthan PET 1985; MP PET 1999]

(a) –1 (b) 1 (c) –4 (d) 4

23. The line passes through (1,0) and $(-2,\sqrt{3})$ makes an angle of ....with *x*-axis [Rajasthan PET 1985]

(a) 60^o (b) 120^o (c) 150^o (d) 135^o

24. If *a* and *b* are two arbitrary constants, then the straight line $(a-2b)x+(a+3b)y+3a+4b=0$ will pass through [Rajasthan PET 1990]

(a) $(-1,-2)$ (b) $(1,2)$ (c) $(-2,-3)$ (d) $(2,3)$

25. The equation of line passing through the point of intersection of the lines $4x-3y-1=0$ and $5x-2y-3=0$ and parallel to the line $2y-3x+2=0$, is [Rajasthan PET 1985,86, 88]

(a) $x-3y=1$ (b) $3x-2y=1$ (c) $2x-3y=1$ (d) $2x-y=1$

26. The equation of line passing through (4, –6) and makes an angle 45^o with positive *x*-axis, is [Rajasthan PET 1984]

(a) $x-y-10=0$ (b) $x-2y-16=0$ (c) $x-3y-22=0$ (d) None of these

27. The straight line passes through the point of intersection of the straight lines $x+2y-10=0$ and $2x+y+5=0$, is [IIT 1983]

(a) $5x-4y=0$ (b) $5x+4y=0$ (c) $4x-5y=0$ (d) $4x+5y=0$

28. The equation to the straight line passing through the point $(a\cos^3\theta, a\sin^3\theta)$ and perpendicular to the line $x\sec\theta+y\operatorname{cosec}\theta=a$, is

[AMU 1975]

(a) $x\cos\theta-y\sin\theta=a\cos 2\theta$ (b) $x\cos\theta+y\sin\theta=a\cos 2\theta$

(c) $x\sin\theta+y\cos\theta=a\cos 2\theta$ (d) None of these

29. Equation of the right bisector of the line segment joining the points (7, 4) and (–1, –2) is [AMU 1979]

(a) $4x-3y=15$ (b) $3x+4y=15$ (c) $4x+3y=15$ (d) None of these

30. Equations of lines which passes through the points of intersection of the lines $4x-3y-1=0$ and $2x-5y+3=0$ and are equally inclined to the axes are [AMU 1981]

(a) $y\pm x=0$ (b) $y-1=\pm 1(x-1)$ (c) $x-1=\pm 2(y-1)$ (d) None of these

31. Equation of line passing through (1, 2) and perpendicular to $3x+4y+5=0$ is [Rajasthan PET 1995]

(a) $3y=4x-2$ (b) $3y=4x+3$ (c) $3y=4x+4$ (d) $3y=4x+2$

32. The equation of a straight line passing through the points (–5, –6) and (3, 10) is [MNR 1974]

(a) $x-2y=4$ (b) $2x-y+4=0$ (c) $2x+y=4$ (d) None of these

33. A straight line through *P*(1, 2) is such that its intercept between the axes is bisected at *P*. Its equation is [EAMCET 1994]

(a) $x+2y=5$ (b) $x-y+1=0$ (c) $x+y-3=0$ (d) $2x+y-4=0$

34. The equation to the straight line passing through the point of intersection of the lines $5x-6y-1=0$ and $3x+2y+5=0$ and perpendicular to the line $3x-5y+11=0$ is [MP PET 1994]

(a) $5x+3y+8=0$ (b) $3x-5y+8=0$ (c) $5x+3y+11=0$ (d) $3x-5y+11=0$

35. The opposite vertices of a square are (1, 2) and (3, 8), then the equation of a diagonal of the square passing through the point (1, 2) is

[Roorkee 1981]

(a) $3x-y-1=0$ (b) $3y-x-1=0$ (c) $3x+y+1=0$ (d) None of these

36. If the straight line $ax+by+c=0$ always passes through (1, –2), then *a*,*b*,*c*, are [AMU 2000]

(a) In A.P. (b) In H.P. (c) In G.P. (d) None of these

37. The equation of the straight line joining the origin to the point of intersection of $y-x+7=0$ and $y+2x-2=0$ is [MP PET 2001]

(a) $3x+4y=0$ (b) $3x-4y=0$ (c) $4x-3y=0$ (d) $4x+3y=0$

38. A straight line makes an angle of 135^o with the *x*-axis and cuts y -axis at a distance –5 from the origin. The equation of the line is

[MP PET 1998]

(a) $2x+y+5=0$ (b) $x+2y+3=0$ (c) $x+y+5=0$ (d) $x+y+3=0$

39. If line $y=mx$ meets the lines $x+2y-1=0$ and $2x-y+3=0$ at the same point, then *m* equals

(a) 1 (b) –1 (c) 2 (d) –2

40. Equation of a line passing through (1, – 2) and perpendicular to the line $3x-5y+7=0$ is [Rajasthan PET 2003]

(a) $5x+3y+1=0$ (b) $3x+5y+1=0$ (c) $5x-3y-1=0$ (d) $3x-5y+1=0$

41. The line $\frac{x}{a}-\frac{y}{b}=1$ cuts the *x*-axis at P. The equation of the line through *P* perpendicular to the given line is [Kerala (Engg.) 2002]

(a) $x+y=ab$ (b) $x+y=a+b$ (c) $ax+by=a^2$ (d) $bx+ay=b^2$

42. The equation of line perpendicular to $x=c$ is [Rajasthan PET 2001]

(a) $y=d$ (b) $x=d$ (c) $x=0$ (d) None of these

43. The inclination of the straight line passing through the point $(-3,6)$ and the midpoint of the line joining the point (4, –5) and (–2,9) is

[Kerala (Engg.) 2002]

(a) $\pi/4$ (b) $\pi/6$ (c) $\pi/3$ (d) $3\pi/4$

44. If the intercept made by the line between the axis is bisected at the point (5, 2), then its equation is

(a) $5x+2y=20$ (b) $2x+5y=20$ (c) $5x-2y=20$ (d) $2x-5y=20$

45. The equation of the line passing through (1, 1) and parallel to the line $2x+3y-7=0$ is [Rajasthan PET 1993, 96]

(a) $2x+3y-5=0$ (b) $3x+2y-5=0$ (c) $3x-2y-7=0$ (d) $2x+3y+5=0$

46. The equation of a straight line passing through origin and through the point of intersection of lines $x+y-2=0$ and $2x-y+1=0$ is

[Rajasthan PET 1993]

(a) $5x-y=0$ (b) $5x+y=0$ (c) $x+5y=0$ (d) $x-5y=0$

47. The equations $(b-c)x+(c-a)y+a-b=0$ and $(b^3-c^3)x+(c^3-a^3)y+a^3-b^3=0$ will represent the same line, if

(a) $b+c=0$ (b) $b=c$ and $c=a$ and $a=b$ or $a+b+c=0$

(c) $a+b=0$ (d) $a+b+c\neq 0$

48. The straight line passing through the point of intersection of the straight lines $x-3y+1=0$ and $2x+5y-9=0$ and having infinite slope and at a distance of 2 units from the origin, has the equation

(a) $x=2$ (b) $3x+y-1=0$ (c) $y=1$ (d) None of these

49. The equation of the line whose slope is 3 and which cuts off an intercept 3 from the positive *x*-axis is

(a) $y=3x-9$ (b) $y=3x+3$ (c) $y=3x+9$ (d) None of these

50. The equations of the lines which cuts off an intercept –1 from *y*-axis and are equally inclined to the axes are

(a) $x-y+1=0, x+y+1=0$ (b) $x-y-1=0, x+y-1=0$

(c) $x-y-1=0, x+y+1=0$ (d) None of these

51. If the line segment joining (2,3) and (–1, 2) is divided internally in the ratio 3:4 by the line $x+2y=k$, then k is

(a) $\frac{41}{7}$ (b) $\frac{5}{7}$ (c) $\frac{36}{7}$ (d) $\frac{31}{7}$

52. If $A(1,1), B(\sqrt{3}+1,2)$ and $C(\sqrt{3},\sqrt{3}+2)$ be three vertices of a square, then the diagonal through *B* is

(a) $y=(\sqrt{3}-2)x+(3-\sqrt{3})$ (b) $y=0$ (c) $y=x$ (d) None of these

53. In what ratio the line $y-x+2=0$ divides the line joining the points $(3,-1)$ and $(8,9)$ [Karnataka CET 2002]

(a) 1: 2 (b) 2: 1 (c) 2: 3 (d) 3: 4

Advance Level

54. For the straight lines given by the equation $(2+k)x+(1+k)y=5+7k$, for different values of *k* which of the following statements is true

[IIT 1971]

(a) Lines are parallel
(b) Lines pass through the point (–2, 9)
(c) Lines pass through the point $(2,-9)$
(d) None of these

55. The line joining two points $A(2,0)$, $B(3,1)$ is rotated about A in anti-clockwise direction through an angle of 15^o. The equation of the line in the new position, is

(a) $\sqrt{3}x-y-2\sqrt{3}=0$
(b) $x-\sqrt{3}y-2=0$
(c) $\sqrt{3}x+y-2\sqrt{3}=0$
(d) $x+\sqrt{3}y-2=0$

56. If the slope of a line passing through the point $A(3,2)$ be 3/4, then the points on the line which are 5 units away from A, are [IIT 1965]

(a) $(5,5),(-1,-1)$
(b) $(7,5),(-1,-1)$
(c) $(5,7),(-1,-1)$
(d) $(7,5),(1,1)$

57. The equation of a line passing through the point of intersection of the lines $x+5y+7=0$, $3x+2y-5=0$ and perpendicular to the line $7x+2y-5=0$ is given by [Rajasthan PET 1987; MP PET 1993]

(a) $2x-7y-20=0$
(b) $2x+7y-20=0$
(c) $-2x+7y-20=0$
(d) $2x+7y+20=0$

58. Equations of diagonals of square formed by lines $x=0, y=0, x=1$ and $y=1$ are [MP PET 1984]

(a) $y=x, y+x=1$
(b) $y=x, x+y=2$
(c) $2y=x, y+x=\frac{1}{3}$
(d) $y=2x, y+2x=1$

59. If the middle points of the sides BC, CA and AB of the triangle ABC be (1, 3), (5, 7) and (–5, 7), then the equation of the side AB is

(a) $x-y-2=0$
(b) $x-y+12=0$
(c) $x+y-12=0$
(d) None of these

60. Given the four lines with equations $x+2y=3$, $3x+4y=7, 2x+3y=4$ and $4x+5y=6$, then these lines are [IIT 1980]

(a) Concurrent
(b) Perpendicular
(c) The sides of a rectangle
(d) None of these

61. The equation of straight line passing through (–*a*, 0) and making the triangle with axes of area '*T*', is

(a) $2Tx+a^2y+2aT=0$
(b) $2Tx-a^2y+2aT=0$
(c) $2Tx-a^2y-2aT=0$
(d) None of these

62. The points $A(1,3)$ and $C(5,1)$ are the opposite vertices of rectangle. The equation of line passing through other two vertices and of gradient 2, is [Rajasthan PET 1991]

(a) $2x+y-8=0$
(b) $2x-y-4=0$
(c) $2x-y+4=0$
(d) $2x+y+7=0$

63. The intercept cut off from *y*-axis is twice that from *x*-axis by the line and line is passes through (1, 2) then its equation is

[AMU 1972; Rajasthan PET 1985]

(a) $2x+y=4$
(b) $2x+y+4=0$
(c) $2x-y=4$
(d) $2x-y+4=0$

64. The equation of line, which bisect the line joining two points (2, –19) and (6, 1) and perpendicular to the line joining two points (–1, 3) and (5, –1), is [Rajasthan PET 1987]

(a) $3x-2y=30$
(b) $2x-y-3=0$
(c) $2x+3y=20$
(d) None of these

65. The vertices of a triangle *OBC* are $(0,0)$, $(-3,-1)$ and $(-1,-3)$ respectively. Then the equation of line parallel to *BC* which is at $\frac{1}{2}$ unit distant from origin and cuts *OB* and *OC*, is **[IIT 1976]**

(a) $2x+2y+\sqrt{2}=0$ (b) $2x+2y-\sqrt{2}=0$ (c) $2x-2y+\sqrt{2}=0$ (d) None of these

66. The equation of line whose mid point is (x_1, y_1) in between the axes, is

(a) $\frac{x}{x_1}+\frac{y}{y_1}=2$ (b) $\frac{x}{x_1}+\frac{y}{y_1}=\frac{1}{2}$ (c) $\frac{x}{x_1}+\frac{y}{y_1}=1$ (d) None of these

67. The intercept of a line between the coordinate axes is divided by the point (–5, 4) in the ratio 1:2. The equation of the line will be **[IIT 1986]**

(a) $5x-8y+60=0$ (b) $8x-5y+60=0$ (c) $2x-5y+30=0$ (d) None of these

68. The diagonal passing through origin of a quadrilateral formed by $x=0, y=0, x+y=1$ and $6x+y=3$, is **[IIT 1973]**

(a) $3x-2y=0$ (b) $2x-3y=0$ (c) $3x+2y=0$ (d) None of these

69. Equation of one of the sides of an isosceles right angled triangle whose hypotenuse is $3x+4y=4$ and the opposite vertex of the hypotenuse is (2, 2), will be **[MNR 1986]**

(a) $x-7y+12=0$ (b) $7x+y-12=0$ (c) $x-7y+16=0$ (d) $7x+y+16=0$

70. A line $4x+y=1$ passes through the point $A(2,-7)$ meets the line *BC* whose equation is $3x-4y+1=0$ at the point *B*. The equation to the line *AC* so that $AB=AC$, is **[IIT 1971]**

(a) $52x+89y+519=0$ (b) $52x+89y-519=0$ (c) $89x+52y+519=0$ (d) $89x+52y-519=0$

71. Equation of the line which passes through the point (–4, 3) and the portion of the line intercepted between the axes is divided internally in the ratio 5:3 by this point, is **[AMU 1973; Dhanbad Engg. 1971]**

(a) $9x+20y+96=0$ (b) $20x+9y+96=0$ (c) $9x-20y+96=0$ (d) None of these

72. A line is such that its segment between the straight lines $5x-y-4=0$ and $3x+4y-4=0$ is bisected at the point (1, 5), then its equation is **[Roorkee 1988]**

(a) $83x-35y+92=0$ (b) $35x-83y+92=0$ (c) $35x+35y+92=0$ (d) None of these

73. $A(-1,1)$, $B(5,3)$ are opposite vertices of a square in *xy*-plane. The equation of the other diagonal (not passing through *A*, *B*) of the square is given by **[EAMCET 1993]**

(a) $x-3y+4=0$ (b) $2x-y+3=0$ (c) $y+3x-8=0$ (d) $x+2y-1=0$

74. The point $P(a,b)$ lies on the straight line $3x+2y=13$ and the point $Q(b,a)$ lies on the straight line $4x-y=5$, then the equation of line PQ is **[MP PET 1999]**

(a) $x-y=5$ (b) $x+y=5$ (c) $x+y=-5$ (d) $x-y=-5$

75. If $P(1+t/\sqrt{2}, 2+t/\sqrt{2})$ be any point on a line then the range of values of *t* for which the point *P* lies between the parallel lines $x+2y=1$ and $2x+4y=15$ is

(a) $-\frac{4\sqrt{2}}{3}<t<\frac{5\sqrt{2}}{6}$ (b) $0<t<\frac{5\sqrt{2}}{6}$ (c) $-\frac{4\sqrt{2}}{3}<t<0$ (d) None of these

76. The equations of the sides AB, BC and CA of the ΔABC are $y-x=2$, $x+2y=1$ and $3x+y+5=0$ respectively. The equation of the altitude through *B* is

(a) $x-3y+1=0$ (b) $x-3y+4=0$ (c) $3x-y+2=0$ (d) None of these

77. One side of a square of length *a* is inclined to the *x*-axis at an angle α with one of the vertices of the square at the origin. The equation of a diagonal of the square is

(a) $y(\cos\alpha - \sin\alpha) = x(\cos\alpha + \sin\alpha)$ (b) $y(\cos\alpha + \sin\alpha) = x(\cos\alpha - \sin\alpha)$

(c) $y(\sin\alpha + \cos\alpha) - x(\sin\alpha - \cos\alpha) = a$ (d) $y(\sin\alpha + \cos\alpha) + x(\sin\alpha - \cos\alpha) = a$

78. Straight lines $3x + 4y = 5$ and $4x - 3y = 15$ intersect at the point A. Points *B* and *C* are chosen on these lines such that $AB = AC$. Determine the possible equations of the line *BC* passing through the point (1, 2) [IIT 1990]

(a) $x - 7y + 13 = 0$ and $7x + y = 9$ (b) $x + 7y + 13 = 0$ and $6x - y = 9$

(c) $x - 7y + 12 = 0$ and $4x + 3y = 9$ (d) $x - 6y + 11 = 0$ and $7x - y = 9$

79. The base *BC* of a triangle *ABC* is bisected at the point (*p*, *q*) and the equations to the sides *AB* and *AC* are respectively $px + qy = 1$ and $qx + py = 1$. Then the equation to the median through *A* is

(a) $(2pq - 1)(px + qy - 1) = (p^2 + q^2 - 1)(qx + py - 1)$ (b) $(p^2 + q^2 - 1)(px + qy - 1) = (2p - 1)(qx + py - 1)$

(c) $(pq - 1)(px + qy - 1) = (p^2 + q^2 - 1)(qx + py - 1)$ (d) None of these

80. If a variable line drawn through the point of intersection of straight lines $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ and $\frac{x}{\beta} + \frac{y}{\alpha} = 1$ meets the coordinate axes in *A* and *B*, then the locus of the mid-point of *AB* is

(a) $\alpha\beta(x + y) = xy(\alpha + \beta)$ (b) $\alpha\beta(x + y) = 2xy(\alpha + \beta)$ (c) $(\alpha + \beta)(x + y) = 2\alpha\beta xy$ (d) None of these

81. Equation of the hour hand at 4 O' clock is

(a) $x - \sqrt{3}y = 0$ (b) $\sqrt{3}x - y = 0$ (c) $x + \sqrt{3}y = 0$ (d) $\sqrt{3}x + y = 0$

82. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$, then the other vertices and *c* are

(a) $(1, 1), (2, 3)$ and $c = 4$ (b) $(4, 4), (2, 0)$ and $c = -4$ (c) $(0, 0), (5, 4)$ and $c = 3$ (d) None of these

Angle between two Straight lines

Basic Level

83. The angle between the lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$ is

(a) 30^o (b) 60^o (c) 45^o (d) 90^o

84. The angle between the lines $x\cos\alpha_1 + y\sin\alpha_1 = p_1$ and $x\cos\alpha_2 + y\sin\alpha_2 = p_2$ is

(a) $(\alpha_1 + \alpha_2)$ (b) $(\alpha_1 \sim \alpha_2)$ (c) $2\alpha_1$ (d) $2\alpha_2$

85. Angle between the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is [MP PET 1995]

(a) $2\tan^{-1}\frac{b}{a}$ (b) $\tan^{-1}\frac{2ab}{a^2 + b^2}$ (c) $\tan^{-1}\frac{a^2 - b^2}{a^2 + b^2}$ (d) None of these

86. The angle between the two lines $y - 2x = 9$ and $x + 2y = -7$, is [Rajasthan PET 1981, 85, 86; MP PET 1984]

(a) 60^o (b) 30^o (c) 90^o (d) 45^o

87. The obtuse angle between the lines $y=-2$ and $y=x+2$ is [Rajasthan PET 1984]

(a) 120^o (b) 135^o (c) 150^o (d) 160^o

88. The acute angle between the lines $y=3$ and $y=\sqrt{3}x+9$ is [Rajasthan PET 1984, 87, 88]

(a) 30^o (b) 60^o (c) 45^o (d) 90^o

89. Angle between $x=2$ and $x-3y=6$ is [MNR 1988]

(a) ∞ (b) $\tan^{-1}(3)$ (c) $\tan^{-1}\left(\frac{1}{3}\right)$ (d) None of these

90. The angle between the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ is [MP PET 1994]

(a) $\tan^{-1}\frac{a_1b_2+a_2b_1}{a_1a_2-b_2b_1}$ (b) $\cot^{-1}\frac{a_1a_2+b_1b_2}{a_1b_2-a_2b_1}$ (c) $\cot^{-1}\frac{a_1b_1-a_2b_2}{a_1a_2+b_1b_2}$ (d) $\tan^{-1}\frac{a_1b_1-a_2b_2}{a_1a_2+b_1b_2}$

91. If the lines $2x+3ay-1=0$ and $3x+4y+1=0$ are mutually perpendicular, then the value of '*a*' will be [MNR 1975]

(a) $\frac{1}{2}$ (b) 2 (c) $-\frac{1}{2}$ (d) None of these

92. The lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are perpendicular to each other if [MP PET 1996]

(a) $a_1b_2-b_1a_2=0$ (b) $a_1a_2+b_1b_2=0$ (c) $a_1^2b_2+b_1^2a_2=0$ (d) $a_1b_1+a_2b_2=0$

93. The angle between the straight lines $x-y\sqrt{3}=5$ and $\sqrt{3}x+y=7$ is

(a) 90^o (b) 60^o (c) 75^o (d) 30^o

94. The angle between the lines $2x-y+3=0$ and $x+2y+3=0$ is [Kerala (Engg.) 2002]

(a) 90^o (b) 60^o (c) 45^o (d) 30^o

95. The lines $y=2x$ and $x=-2y$ are [MP PET 1993]

(a) Parallel (b) Perpendicular (c) Equally inclined to axes (d) Coincident

96. The line which is parallel to *x*-axis and crosses the curve $y=\sqrt{x}$ at an angle of 45^o is [Roorkee 1993]

(a) $x=1/4$ (b) $y=1/4$ (c) $y=1/2$ (d) $y=1$

97. The angle between the lines whose intercepts on the axes are $a,-b$ and $b,-a$ respectively, is

(a) $\tan^{-1}\frac{a^2-b^2}{ab}$ (b) $\tan^{-1}\frac{b^2-a^2}{2}$ (c) $\tan^{-1}\frac{b^2-a^2}{2ab}$ (d) None of these

98. The line $3x+2y=9$ intersects the axes in *A* and *B*. If *O* is the origin, then $\angle OAB$ equals

(a) $\tan^{-1}(1/3)$ (b) 45^o (c) $\tan^{-1}(2/3)$ (d) $\tan^{-1}(3/2)$

99. The angle between two lines is $\frac{\pi}{4}$. If the slope of one of them be $\frac{1}{2}$, then the slope of the other line is

(a) $1,-\frac{1}{3}$ (b) $-1,\frac{1}{2}$ (c) $-\frac{1}{3},3$ (d) None of these

Advance Level

100. A vertex of equilateral triangle is (2, 3) and equation of opposite side is $x+y=2$, then the equation of one side from rest two is

[IIT 1975]

(a) $y-3=2(x-2)$ (b) $y-3=(2-\sqrt{3})(x-2)$ (c) $y-3=(\sqrt{3}-1)(x-2)$ (d) None of these

101. Coordinates of the vertices of a quadrilateral are (2, –1),(0, 2),(2, 3) and (4, 0) . The angle between its diagonals will be [IIT 1986]

(a) 90^o (b) 0^o (c) $\tan^{-1}(2)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

102. In what direction a line be drawn through the point (1, 2) so that its point of intersection with the line $x+y=4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point [IIT 1966; MNR 1987]

(a) 30^o (b) 45^o (c) 60^o (d) 75^o

103. The line passing through the points $(3,-4)$ and $(-2,6)$ and a line passing through $(-3,6)$ and $(9,-18)$, are [AMU 1974]

(a) Perpendicular (b) Parallel

(c) Makes an angle 60^o with each other (d) None of these

104. Equation of the two straight lines passing through the point (3, 2) and making an angle of 45^o with the line $x-2y=3$, are [AMU 1978]

(a) $3x+y+7=0$ and $x+3y+9=0$ (b) $3x-y-7=0$ and $x+3y-9=0$

(c) $x+3y-7=0$ and $x+3y-9=0$ (d) None of these

105. The diagonals of the parallelogram whose sides are $lx+my+n=0$, $lx+my+n'=0$, $mx+ly+n=0$, $mx+ly+n'=0$ include an angle

[EAMCET 1994]

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\tan^{-1}\left(\frac{l^2-m^2}{l^2+m^2}\right)$ (d) $\tan^{-1}\left(\frac{2lm}{l^2+m^2}\right)$

106. The sides AB, BC, CD and DA of a quadrilateral are $x+2y=3, x=1, x-3y=4, 5x+y+12=0$ respectively. The angle between diagonals AC and BD is [Roorkee 1993]

(a) 45^o (b) 60^o (c) 90^o (d) 30^o

107. One diagonal of a square is along the line $8x-15y=0$ and one of its vertex is (1, 2). Then the equation of the sides of the square passing through this vertex, are [IIT 1962]

(a) $23x+7y=9,\ 7x+23y=53$ (b) $23x-7y+9=0$, $7x+23y+53=0$

(c) $23x-7y-9=0$, $7x+23y-53=0$ (d) None of these

108. The parallelism condition for two straight lines one of which is specified by the equation $ax+by+c=0$ the other being represented parametrically by $x=\alpha t+\beta,\ y=\gamma t+\delta$ is given by

(a) $a\gamma-b\alpha=0, \beta=\delta=c=0$ (b) $a\alpha-b\gamma=0, \beta=\delta=0$ (c) $a\alpha+b\gamma=0$ (d) $a\gamma=b\alpha=0$

109. If straight lines $ax+by+p=0$ and $x\cos\alpha+y\sin\alpha-p=0$ include an angle $\frac{\pi}{4}$ between them and meet the straight line $x\sin\alpha-y\cos\alpha=0$ in the same point, then the value of a^2+b^2 is equal to

(a) 1 (b) 2 (c) 3 (d) 4

110. The ends of the base of an isosceles triangle are at (2*a*, 0) and $(0, a)$. The equation of one side is $x=2a$. The equation of the other side is

(a) $x+2y-a=0$ (b) $x+2y=2a$ (c) $3x+4y-4a=0$ (d) $3x-4y+4a=0$

111. If a,b,c are in harmonic progression, then straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point, that point is **[MP PET 1999]**

(a) $(-1,-2)$ (b) $(-1, 2)$ (c) $(1,-2)$ (d) $(1,-1/2)$

112. Angles made with the x-axis by two lines drawn through the point (1, 2) and cutting the line $x+y=4$ at a distance $\frac{1}{3}\sqrt{6}$ from the point (1, 2) are **[DCE 1995]**

(a) $\frac{\pi}{6}$ and $\frac{\pi}{3}$ (b) $\frac{\pi}{8}$ and $\frac{3\pi}{8}$ (c) $\frac{\pi}{12}$ and $\frac{5\pi}{12}$ (d) None of these

Bisectors of Angle between two Lines

Basic Level

113. The equation of the line which bisects the obtuse angle between the lines $x-2y+4=0$ and $4x-3y+2=0$ is **[IIT 1979]**

(a) $(4-\sqrt{5})x-(3-2\sqrt{5})y+(2-4\sqrt{5})=0$ (b) $(4+\sqrt{5})x-(3+2\sqrt{5})y+(2+4\sqrt{5})=0$

(c) $(4+\sqrt{5})x+(3+2\sqrt{5})y+(2+4\sqrt{5})=0$ (d) None of these

114. Equation of angle bisectors between x and y-axes are **[MP PET 1984]**

(a) $y=\pm x$ (b) $y=\pm 2x$ (c) $y=\pm\frac{1}{\sqrt{2}}x$ (d) $y=\pm 3x$

115. Equation of angle bisector between the lines $3x+4y-7=0$ and $12x+5y+17=0$ are **[Rajasthan PET 1995]**

(a) $\frac{3x+4y-7}{\sqrt{25}}=\pm\frac{12x+5y+17}{\sqrt{169}}$ (b) $\frac{3x+4y+7}{\sqrt{25}}=\frac{12x+5y+17}{\sqrt{169}}$

(c) $\frac{3x+4y+7}{\sqrt{25}}=\pm\frac{12x+5y+17}{\sqrt{169}}$ (d) None of these

116. The equation of the bisector of the acute angle between the lines $2x-y+4=0$ and $x-2y=1$ is

(a) $x+y+5=0$ (b) $x-y+1=0$ (c) $x-y=5$ (d) None of these

117. The vertices of a triangle are $A(-1,-7)$, $B(5,1)$ and $C(1,4)$. The equation of the internal bisector of the angle $\angle ABC$ is

(a) $3x-7y-8=0$ (b) $x-7y+2=0$ (c) $3x-3y-7=0$ (d) None of these

118. The equation (s) of the bisector (s) of that angle between the lines $x+2y-11=0$, $3x-6y-5=0$, which contains the point (1, –3) is

(a) $3x=19$ (b) $3y=7$ (c) $3x=19$ and $3y=7$ (d) None of these

Advance Level

119. The equations of two equal sides of an isosceles triangle are $7x-y+3=0$ and $x+y-3=0$ and the third side passes through the point (1, –10). The equation of the third side is **[IIT 1984]**

(a) $x-3y-31=0$ but not $3x+y+7=0$ (b) $3x+y+7=0$ but not $x-3y-31=0$

(c) $3x+y+7=0$ or $x-3y-31=0$ (d) Neither $3x+y+7=0$ nor $x-3y-31=0$

120. Given vertices $A(1,1); B(4,-2)$ and $C(5,5)$ of a triangle, then the equation of the perpendicular dropped from *C* to the interior bisector of the angle *A* is [Roorkee 1994]

(a) $y-5=0$ (b) $x-5=0$ (c) $y+5=0$ (d) $x+5=0$

121. The equation of bisectors of the angles between the lines $|x|=|y|$ are

(a) $y=\pm x$ and $x=0$ (b) $x=\frac{1}{2}$ and $y=\frac{1}{2}$ (c) $y=0$ and $x=0$ (d) None of these

Distance between two lines, Perpendicular length on the Line

Basic Level

122. The distance between the lines $3x+4y=9$ and $6x+8y=15$ is [MNR 1982; Rajasthan PET 1995; MP PET 2002]

(a) $3/2$ (b) $3/10$ (c) 6 (d) None of these

123. The perpendicular distance of the straight line $12x+5y=7$ from the origin is given by [MP PET 1993]

(a) $\frac{7}{13}$ (b) $\frac{12}{13}$ (c) $\frac{5}{13}$ (d) $\frac{1}{13}$

124. The length of perpendicular from (3, 1) on line $4x+3y+20=0$, is [Rajasthan PET 1989; MP PET 1984]

(a) 6 (b) 7 (c) 5 (d) 8

125. The distance between two parallel lines $3x+4y-8=0$ and $3x+4y-3=0$, is given by [MP PET 1984]

(a) 4 (b) 5 (c) 3 (d) 1

126. The equations of two lines through (0, *a*) which are at a distance '*a*' from the point (2*a*, 2*a*) are

(a) $y-a=0$ and $4x-3y-3a=0$ (b) $y-a=0$ and $3x-4y+3a=0$

(c) $y-a=0$ and $4x-3y+3a=0$ (d) None of these

127. The vertices of a triangle are (2, 1), (5, 2) and (4, 4). The lengths of the perpendiculars from these vertices on the opposite sides are [IIT 1962]

(a) $\frac{7}{\sqrt{5}},\frac{7}{\sqrt{13}},\frac{7}{\sqrt{6}}$ (b) $\frac{7}{\sqrt{6}},\frac{7}{\sqrt{8}},\frac{7}{\sqrt{10}}$ (c) $\frac{7}{\sqrt{5}},\frac{7}{\sqrt{8}},\frac{7}{\sqrt{15}}$ (d) $\frac{7}{\sqrt{5}},\frac{7}{\sqrt{13}},\frac{7}{\sqrt{10}}$

128. A point moves such that its distance from the point (4, 0) is half that of its distance from the line $x=16$. The locus of this point is [AMU 1980]

(a) $3x^2+4y^2=192$ (b) $4x^2+3y^2=192$ (c) $x^2+y^2=192$ (d) None of these

129. The locus of a point so that sum of its distance from two given perpendicular lines is equal to 2 units, is [Bihar CEE 1994]

(a) $x+y+2=0$ (b) $x+y=2$ (c) $x-y=2$ (d) None of these

130. Distance between the two parallel lines $y=2x+7$ and $y=2x+5$ is

(a) $\frac{\sqrt{5}}{2}$ (b) $\frac{2}{5}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{5}}$

131. The length of the perpendicular drawn from origin upon the straight line $\frac{x}{3}-\frac{y}{4}=1$ is [MP PET 1997]

(a) $2\frac{2}{5}$ (b) $3\frac{1}{5}$ (c) $4\frac{2}{5}$ (d) $3\frac{2}{5}$

132. Distance between the parallel lines $3x + 4y + 7 = 0$ and $3x + 4y - 9 = 0$ is [Rajasthan PET 2003]

(a) $\frac{2}{5}$ (b) $\frac{12}{5}$ (c) $\frac{5}{12}$ (d) $\frac{3}{5}$

133. The equation of the line joining the point (3, 5)to the point of intersection of the lines $4x + y - 1 = 0$ and $7x - 3y - 35 = 0$ is equidistant from the points $(0, 0)$ and $(8, 34)$ [Roorkee 1984]

(a) True (b) False (c) Nothing can be said (d) None of these

134. Distance between the lines $5x + 3y - 7 = 0$ and $15x + 9y + 14 = 0$ is [Kerala (Engg.) 2002]

(a) $\frac{35}{\sqrt{34}}$ (b) $\frac{1}{3\sqrt{34}}$ (c) $\frac{35}{3\sqrt{34}}$ (d) $\frac{35}{2\sqrt{34}}$

135. The distance between the lines $3x - 2y = 1$ and $6x + 9 = 4y$ is [MP PET 1998]

(a) $\frac{1}{\sqrt{52}}$ (b) $\frac{11}{\sqrt{52}}$ (c) $\frac{4}{\sqrt{13}}$ (d) $\frac{6}{\sqrt{13}}$

136. The distance of the line $2x - 3y = 4$ from the point (1, 1) measured parallel to the line $x + y = 1$ is [Orissa JEE 2002]

(a) $\sqrt{2}$ (b) $\frac{5}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) 6

137. The distance between the parallel lines $y = 2x + 4$ and $6x = 3y + 5$ is

(a) $17/\sqrt{3}$ (b) 1 (c) $3/\sqrt{5}$ (d) $7\sqrt{5}/15$

138. The position of the point $(8, -9)$ with respect to the lines $2x + 3y - 4 = 0$ and $6x + 9y + 8 = 0$ is

(a) Point lies on the same side of the lines (b) Point lies on the different sides of the line

(c) Point lies on one of the lines (d) None of these

139. Consider the lines $2x + 3y = 7$, $2x + 3y = 12$ and point $A(3, -5)$. Then

(a) Point '*A*' lies between the lines (b) Sum of perpendicular distance from *A* to the lines $= 5/\sqrt{13}$

(c) Distance between lines is $19/\sqrt{13}$ (d) None of these

Advance Level

140. A point moves so that square of its distance from the point (3, –2) is numerically equal to its distance from the line $5x - 12y = 13$. The equation of the locus of the point is [Roorkee 1974]

(a) $13x^2 + 13y^2 - 83x + 64y + 182 = 0$ (b) $x^2 + y^2 - 11x + 16y + 26 = 0$

(c) $x^2 + y^2 - 11x + 16y = 0$ (d) None of these

141. The points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$, are [IIT 1976]

(a) $(3, 1), (-7, 11)$ (b) $(3, 1), (7, 11)$ (c) $(-3, 1), (-7, 11)$ (d) $(1, 3), (-7, 11)$

142. A variable line passes through a fixed point *P*. The algebraic sum of the perpendiculars drawn from (2, 0),(0, 2) and (1, 1) on the line is zero, then the coordinates of the *P* are [IIT 1991]

(a) (1, –1) (b) (1, 1) (c) (2, 1) (d) (2, 2)

143. A line L passes through the points (1, 1) and (2, 0) and another line L' passes through $\left(\frac{1}{2}, 0\right)$ and perpendicular to L. Then the area of the triangle formed by the lines L, L' and y-axis, is [Rajasthan PET 1991]

(a) $\frac{15}{8}$ (b) $\frac{25}{4}$ (c) $\frac{25}{8}$ (d) $\frac{25}{16}$

144. Equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120° with the x-axis, is [MNR 1986]

(a) $x\sqrt{3} + y + 8 = 0$ (b) $x\sqrt{3} - y = 8$ (c) $x\sqrt{3} - y = 8$ (d) $x - \sqrt{3}y + 8 = 0$

145. Locus of the points which are at equal distance from $3x + 4y - 11 = 0$ and $12x + 5y + 2 = 0$ and which is near the origin is [MNR 1987]

(a) $21x - 77y + 153 = 0$ (b) $99x + 77y - 133 = 0$ (c) $7x - 11y = 19$ (d) None of these

146. The equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is (2, –1). The length of the side of the triangle is [IIT 1973, 1983; MP PET 1995; Rajasthan PET 1999, 2000]

(a) $\sqrt{3/2}$ (b) $\sqrt{2}$ (c) $\sqrt{2/3}$ (d) None of these

147. If the straight line through the point $P(3,4)$ makes an angle $\frac{\pi}{6}$ with the x-axis and meets the line $12x + 5y + 10 = 0$ at Q, then the length PQ is

(a) $\frac{132}{12\sqrt{3} + 5}$ (b) $\frac{132}{12\sqrt{3} - 5}$ (c) $\frac{132}{5\sqrt{3} + 12}$ (d) $\frac{132}{5\sqrt{3} - 12}$

148. The equations of the lines through the point of intersection of the lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ and whose distance from the point (3, 2) is $\frac{7}{5}$, is [IIT 1963]

(a) $3x - 4y - 6 = 0$ and $4x + 3y + 1 = 0$ (b) $3x - 4y + 6 = 0$ and $4x - 3y - 1 = 0$

(c) $3x - 4y + 6 = 0$ and $4x - 3y + 1 = 0$ (d) None of these

149. A point equidistant from the lines $4x + 3y + 10 = 0$, $5x - 12y + 26 = 0$ and $7x + 24y - 50 = 0$ is [EAMCET 1994]

(a) (1, –1) (b) (1, 1) (c) (0, 0) (d) (0, 1)

150. A line through $A(-5,-4)$ meets the lines $x + 3y + 2 = 0$, $2x + y + 4 = 0$ and $x - y - 5 = 0$ at B, C and D respectively. If $\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$, then the equation of the line is [IIT 1993]

(a) $2x + 3y + 22 = 0$ (b) $5x - 4y + 7 = 0$ (c) $3x - 2y + 3 = 0$ (d) None of these

151. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of '$c$' is [IIT Screening 2003]

(a) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ (b) $a_1^2 - a_2^2 + b_1^2 - b_2^2$ (c) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$ (d) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

152. If p_1, p_2 and p_3 be the perpendiculars from the points $(m^2, 2m), (mm', m + m')$ and $(m'^2, 2m')$ respectively on the line $x\cos\alpha + y\sin\alpha + \frac{\sin^2\alpha}{\cos\alpha} = 0$, then p_1, p_2 and p_3 are in

(a) A.P. (b) G.P. (c) H.P (d) None of these

153. If p and p' be perpendiculars from the origin upon the straight lines $x \sec\theta + y \operatorname{cosec}\theta = a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$ respectively, then the value of the expression $4p^2 + p'^2$ is

(a) a^2 (b) $3a^2$ (c) $2a^2$ (d) $4a^2$

154. A family of lines is given by $(1+2\lambda)x + (1-\lambda)y + \lambda = 0, \lambda$ being the parameter. The line belonging to this family at the maximum distance from the point (1, 4) is

(a) $4x - y + 1 = 0$ (b) $33x + 12y + 7 = 0$ (c) $12x + 33y = 7$ (d) None of these

155. If the point (a, a) falls between the lines $|x + y| = 2$, then

(a) $|a| = 2$ (b) $|a| = 1$ (c) $|a| < 1$ (d) $|a| = \frac{1}{2}$

Concurrency of Three lines

Basic Level

156. The value of k for which the lines $7x - 8y + 5 = 0$, $3x - 4y + 5 = 0$ and $4x + 5y + k = 0$ are concurrent is given by **[MP PET 1993]**

(a) – 45 (b) 44 (c) 54 (d) –54

157. For what value of 'a' the lines $x = 3, y = 4$ and $4x - 3y + a = 0$ are concurrent **[Rajasthan PET 1984]**

(a) 0 (b) –1 (c) 2 (d) 3

158. The lines $15x - 18y + 1 = 0, 12x + 10y - 3 = 0$ and $6x + 66y - 11 = 0$ are **[AMU 1978]**

(a) Parallel (b) Perpendicular (c) Concurrent (d) None of these

159. The lines $2x + y - 1 = 0$, $ax + 3y - 3 = 0$ and $3x + 2y - 2 = 0$ are concurrent for **[EAMCET 1994]**

(a) All a (b) $a = 4$ only (c) $-1 \le a \le 3$ (d) $a > 0$ only

160. The value of λ for which the lines $3x + 4y = 5, 5x + 4y = 4$ and $\lambda x + 4y = 6$ meet at a point is

(a) 2 (b) 1 (c) 4 (d) 3

161. Three lines $3x - y = 2, 5x + ay = 3$ and $2x + y = 3$ are concurrent, then $a =$ **[MP PET 1996]**

(a) 2 (b) 3 (c) –1 (d) –2

162. If the lines $x + q = 0, y - 2 = 0$ and $3x + 2y + 5 = 0$ are concurrent, then the value of q will be **[DCE 2002]**

(a) 1 (b) 2 (c) 3 (d) 5

163. The equation of the line with gradient $-3/2$ which is concurrent with the lines $4x + 3y - 7 = 0$ and $8x + 5y - 1 = 0$ is **[DCE 1999]**

(a) $3x + 2y - 2 = 0$ (b) $3x + 2y - 63 = 0$ (c) $2y - 3x - 2 = 0$ (d) None of these

164. If lines $y = mx$, $x + 2y - 1 = 0$ and $2x - y + 3 = 0$ are concurrent, then value of m is **[Rajasthan PET 1994]**

(a) 1 (b) 0 (c) –1 (d) 2

Advance Level

165. The lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$ are concurrent at the point [IIT 1982]

(a) $(1/2, 3/4)$ (b) $(1, 3)$ (c) $(3, 1)$ (d) $(3/4, 1/2)$

166. The equations $(b-c)x + (c-a)y + (a-b) = 0$ and $(b^3 - c^3)x + (c^3 - a^3)y + a^3 - b^3 = 0$ will represent the same line, if

(a) $b = c$ (b) $c = a$ (c) $a = b$ (d) $a + b + c = 0$

167. If the lines $ax + 2y + 1 = 0$, $bx + 3y + 1 = 0$ and $cx + 4y + 1 = 0$ are concurrent, then *a, b, c* are in

(a) A.P (b) G.P (c) H.P (d) None of these

168. If the lines $ax + y + 1 = 0$, $x + by + 1 = 0$ and $x + y + c = 0$ (a, b, c being distinct and different from 1) are concurrent, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} =$

(a) 0 (b) 1 (c) $\frac{1}{a+b+c}$ (d) None of these

169. The three straight lines $ax + by = c, bx + cy = a$ and $cx + ay = b$ are collinear, if [MP PET 2004]

(a) $a + b + c = 0$ (b) $b + c = a$ (c) $c + a = b$ (d) $a + b = c$

170. The three lines $3x + 4y + 6 = 0; \sqrt{2}x + \sqrt{3}y + 2\sqrt{2} = 0$ and $4x + 7y + 8 = 0$ are [Rajasthan PET 1992]

(a) Sides of a triangle (b) Concurrent (c) Parallel (d) None of these

Miscellaneous problems

Basic Level

171. The coordinate of the foot of perpendicular from the point (2, 3) on the line $x + y - 11 = 0$ are

(a) (– 6, 5) (b) (5, 6) (c) (–5, 6) (d) (6, 5)

172. The coordinate of the foot of the perpendicular from the point (2,3) on the line $y = 3x + 4$ are given by [MP PET 1984]

(a) $\left(\frac{37}{10}, \frac{-1}{10}\right)$ (b) $\left(\frac{-1}{10}, \frac{37}{10}\right)$ (c) $\left(\frac{10}{37}, -10\right)$ (d) $\left(\frac{2}{3}, \frac{-1}{3}\right)$

173. If the coordinates of the middle point of the portion of a line intercepted between coordinate axes (3, 2), then the equation of the line will be [Rajasthan PET 1985; MP PET 1984]

(a) $2x + 3y = 12$ (b) $3x + 2y = 12$ (c) $4x - 3y = 6$ (d) $5x - 2y = 10$

174. Coordinates of the foot of the perpendicular drawn from (0, 0) to the line joining $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$, are [IIT 1982]

(a) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (b) $\left[\frac{a}{2}(\cos\alpha + \cos\beta), \frac{a}{2}(\sin\alpha + \sin\beta)\right]$

(c) $\left(\cos\frac{\alpha+\beta}{2}, \sin\frac{\alpha+\beta}{2}\right)$ (d) None of these

175. The pedal points of a perpendicular drawn from origin on the line $3x + 4y - 5 = 0$,is [Rajasthan PET 1990]

(a) $\left(\frac{3}{5}, 2\right)$ (b) $\left(\frac{3}{5}, \frac{4}{5}\right)$ (c) $\left(-\frac{3}{5}, -\frac{4}{5}\right)$ (d) $\left(\frac{30}{17}, \frac{19}{17}\right)$

176. The coordinates of the foot of the perpendicular from (x_1, y_1) to the line $ax + by + c = 0$ are [Dhanbad Engg. 1973]

(a) $\left(\frac{b^2x_1 - aby_1 - ac}{a^2+b^2}, \frac{a^2y_1 - abx_1 - bc}{a^2+b^2}\right)$ (b) $\left(\frac{b^2x_1 + aby_1 + ac}{a^2+b^2}, \frac{a^2y_1 + abx_1 + bc}{a^2+b^2}\right)$

(c) $\left(\frac{ax_1 + by_1 + ab}{a+b}, \frac{ax_1 - by_1 - ab}{a+b}\right)$ (d) None of these

177. The area of the triangle bounded by the straight line $ax + by + c = 0$, $(a, b, c \neq 0)$ and the coordinate axes is [AMU 2000]

(a) $\frac{1}{2}\frac{a^2}{|bc|}$ (b) $\frac{1}{2}\frac{c^2}{|ab|}$ (c) $\frac{1}{2}\frac{b^2}{|ac|}$ (d) 0

178. The image of the point (4, –3)with respect to the line $y = x$ is [Rajasthan PET 2002]

(a) $(-4, -3)$ (b) $(3, 4)$ (c) $(-4, 3)$ (d) $(-3, 4)$

179. The triangle formed by the lines $x + y = 0,\ 3x + y = 4, x + 3y = 4$ is [Rajasthan PET 2002]

(a) Isosceles (b) Equilateral (c) Right -angled (d) None of these

180. The diagonals of a parallelogram $PQRS$ are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then *PQRS* must be a [IIT 1998]

(a) Rectangle (b) Square (c) Cyclic quadrilateral (d) Rhombus

181. Two points *A* and *B* have coordinates (1, 1) and (3, –2) respectively. The coordinates of a point distant $\sqrt{85}$ from *B* on the line through *B* perpendicular to *AB* are [AMU 2000]

(a) (4, 7) (b) (7, 4) (c) (5, 7) (d) (–5, –3)

182. The line $3x + 2y = 24$ meets *y*-axis at *A* and *x*-axis at *B*. The perpendicular bisector of *AB* meets the line through (0, –1) parallel to *x*-axis at *C*. The area of the triangle *ABC* is

(a) 182 *sq.*units (b) 91 *sq.* units (c) 48 *sq.* units (d) None of these

183. The area of a parallelogram formed by the lines $ax \pm by \pm c = 0$, is [IIT 1973]

(a) $\frac{c^2}{ab}$ (b) $\frac{2c^2}{ab}$ (c) $\frac{c^2}{2ab}$ (d) None of these

184. The area of triangle formed by the lines $x = 0, y = 0$ and $\frac{x}{a} + \frac{y}{b} = 1$, is [Rajasthan PET 1984]

(a) *ab* (b) *ab*/2 (c) 2*ab* (d) *ab*/3

185. A line *L* is perpendicular to the line $5x - y = 1$ and the area of the triangle formed by the line *L* and coordinate axes is 5. The equation of the line *L* is [IIT 1980;Rajasthan PET 1997]

(a) $x + 5y = 5$ (b) $x + 5y = \pm 5\sqrt{2}$ (c) $x - 5y = 5$ (d) $x - 5y = 5\sqrt{2}$

186. The point (4, 1) undergoes the following two successive transformations

(i) Reflection about the line $y = x$ (ii) Translation through a distance 2 units along the positive *x*-axis

Then the final coordinates of the point are [MNR 1987; UPSEAT 2000]

(a) (4, 3) (b) (3, 4) (c) (1, 4) (d) $\left(\frac{7}{2}, \frac{7}{2}\right)$

Advance Level

187. A straight line moves so that the sum of the reciprocals of its intercepts on two perpendicular lines is constant, then the line passes through [IIT 1977]

(a) A fixed point (b) A variable point (c) Origin (d) None of these

188. The line $2x+3y=12$ meets the *x*-axis at *A* and *y*-axis at *B*. The line through (5, 5) perpendicular to *AB* meets the *x*-axis, *y*-axis and the *AB* at *C*, *D* and *E* respectively. If *O* is the origin of coordinates, then the area of *OCEB* is [IIT 1976]

(a) 23 sq. units (b) $\frac{23}{2}$ sq.units (c) $\frac{23}{3}$ sq. units (d) None of these

189. The locus of a point *P* which divides the line joining (1, 0) and $(2\cos\theta, 2\sin\theta)$ internally in the ratio 2 : 3 for all θ, is a [IIT 1986]

(a) Straight line (b) Circle (c) Pair of straight lines (d) Parabola

190. Line *L* has intercepts *a* and *b* on the coordinate axes. When the axes are rotated through a given angle keeping the origin fixed, the same line *L* has intercepts *p* and *q*, then [IIT 1990; Kurukshetra CEE 1998]

(a) $a^2+b^2=p^2+q^2$ (b) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$ (c) $a^2+p^2=b^2+q^2$ (d) $\frac{1}{a^2}+\frac{1}{p^2}=\frac{1}{b^2}+\frac{1}{q^2}$

191. One side of a rectangle lies along the line $4x+7y+5=0$. Two of its vertices are (–3, 1) and (1, 1). Then the equations of other sides are

(a) $7x-4y+25=0, 4x+7y=11$ and $7x-4y-3=0$ (b) $7x+4y+25=0, 7y+4x-11=0$ and $7x-4y-3=0$

(c) $4x-7y+25=0, 7x+4y-11=0$ and $4x-7y-3=0$ (d) None of these

192. Two consecutive sides of a parallelogram are $4x+5y=0$ and $7x+2y=0$. If the equation to one diagonal is $11x+7y=9$, then the equation of the other diagonal is

(a) $x+2y=0$ (b) $2x+y=0$ (c) $x-y=0$ (d) None of these

193. If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is [IIT 1992; Karnataka CET 1999; DCE 2000, 01]

(a) Square (b) Circle (c) Straight line (d) Two intersecting lines

194. A pair of straight lines drawn through the origin form with the line $2x+3y=6$ an isosceles right angled triangle, then the lines and the area of the triangle thus formed is

(a) $x-5y=0, 5x+y=0, \Delta=\frac{36}{13}$ (b) $3x-y=0, x+3y=0, \Delta=\frac{12}{17}$

(c) $5x-y=0, x+5y=0, \Delta=\frac{13}{5}$ (d) None of these

195. *P* is a point on either of the two lines $y-\sqrt{3}|x|=2$ at a distance of 5 units from their point of intersection. The coordinates of the foot of the perpendicular from *P* on the bisector of the angle between them are [Roorkee 1992]

(a) $\left(0, \frac{4+5\sqrt{3}}{2}\right)$ or $\left(0, \frac{4-5\sqrt{3}}{2}\right)$ depending on which the point *P* is taken (b) $\left(0, \frac{4+5\sqrt{3}}{2}\right)$

(c) $\left(0, \frac{4-5\sqrt{3}}{2}\right)$ (d) $\left(\frac{5}{2}, \frac{5\sqrt{2}}{2}\right)$

196. A ray of light passing through the point (1, 2) is reflected on the *x*-axis at a point *P* and passes through the point (5, 3). Then the abscissa of the point *P* is [Orissa JEE 2003]

(a) –3 (b) 13/3 (c) 13/5 (d) 13/4

197. The point moves such that the area of the triangle formed by it with the points (1, 5) and (3, –7) is 21 *sq*. unit. The locus of the point is

(a) $6x + y - 32 = 0$ (b) $6x - y + 32 = 0$ (c) $x + 6y - 32 = 0$ (d) $6x - y - 32 = 0$

198. If for a variable line $\frac{x}{a} + \frac{y}{b} = 1$ the condition $a^{-2} + b^{-2} = c^{-2}$ (*c* is a constant) is satisfied, then locus of foot of perpendicular drawn from origin to the straight line is **[Rajasthan PET 1999]**

(a) $x^2 + y^2 = c^2/2$ (b) $x^2 + y^2 = 2c^2$ (c) $x^2 + y^2 = c^2$ (d) $x^2 - y^2 = c^2$

199. Let *L* be the line $2x + y = 2$. If the axes are rotated by 45^o, then the intercepts made by the line *L* on the new axes are respectively **[Roorkee 1998]**

(a) $\sqrt{2}$ and 1 (b) 1 and $\sqrt{2}$ (c) $2\sqrt{2}$ and $2\sqrt{2}/3$ (d) $2\sqrt{2}/3$ and $2\sqrt{2}$

200. The graph of the function $\cos x \cos(x+2) - \cos^2(x+1)$ is **[IIT 1997 Re-Exam]**

(a) A straight line passing through $(0, -\sin^2 1)$ with slope 2

(b) A straight line passing through (0,0)

(c) A parabola with vertex $(1, -\sin^2 1)$

(d) A straight line passing through the point $\left(\frac{\pi}{2}, -\sin^2 1\right)$ and parallel to the *x*-axis

201. Two lines are drawn through (3, 4), each of which makes angle of 45^o with the line $x - y = 2$, then area of the triangle formed by these lines is **[Rajasthan PET 2000]**

(a) 9 (b) 9/2 (c) 2 (d) 2/9

202. A point starts moving from (1, 2) and its projections on *x* and y-axes are moving with velocities of 3 *m*/s and 2 *m/s* respectively. Its locus is **[Roorkee 1999]**

(a) $2x - 3y + 4 = 0$ (b) $3x - 2y + 1 = 0$ (c) $3y - 2x + 4 = 0$ (d) $2y - 3x + 1 = 0$

203. If (–2, 6) is the image of the point (4, 2) with respect to line *L*= 0, then *L*= **[EAMCET 2002]**

(a) $3x - 2y + 5$ (b) $3x - 2y + 10$ (c) $2x + 3y - 5$ (d) $6x - 4y - 7$

204. The area of the parallelogram formed by the lines $y = mx, y = mx + 1, y = nx$ and $y = nx + 1$ equals

(a) $\frac{|m+n|}{(m-n)^2}$ (b) $\frac{2}{|m+n|}$ (c) $\frac{1}{|m+n|}$ (d) $\frac{1}{|m-n|}$

205. A line *AB* makes zero intercept on *x*-axis and *y*-axis and it is perpendicular to another line *CD*, $3x + 4y + 6 = 0$. The equation of line *AB* is **[Karnataka CET 2001]**

(a) $y = 4$ (b) $4x - 3y + 8 = 0$ (c) $4x - 3y = 0$ (d) $4x - 3y + 6 = 0$

206. Area of the parallelogram whose sides are $x\cos\alpha + y\sin\alpha = p$, $x\cos\alpha + y\sin\alpha = q$, $x\cos\beta + y\sin\beta = r$ and $x\cos\beta + y\sin\beta = s$ is

(a) $\pm(p-q)(r-s)\text{cosec}(\alpha - \beta)$ (b) $(p+q)(r-s)\text{cosec}(\alpha + \beta)$

(c) $(p+q)(r+s)\text{cosec}(\alpha - \beta)$ (d) None of these

207. If the transversal $y = m_r x; r = 1, 2, 3$ cut off equal intercepts on the transversal $x + y = 1$, then $1 + m_1, 1 + m_2, 1 + m_3$ are in

(a) A.P (b) G.P. (c) H.P. (d) None of these

208. If the extremities of the base of an isosceles triangle are the points (2*a*, 0) and (0, *a*) and the equation of one of the sides is $x=2a$, then the area of the triangle is

(a) $5a^2 sq.$ units (b) $\frac{5a^2}{2} sq.$ units (c) $\frac{25\,a^2}{2} sq.$ units (d) None of these

209. The coordinates of the four vertices of a quadrilateral are (–2, 4), (–1, 2),(1, 2) and (2, 4) taken in order. The equation of the line passing through the vertex (–1, 2) and dividing the quadrilateral in two equal areas is

(a) $x+1=0$ (b) $x+y=1$ (c) $x-y+3=0$ (d) None of these

210. If a ray travelling along the line $x=1$ gets reflected from the line $x+y=1$, then the equation of the line along which the reflected ray travels is

(a) $y=0$ (b) $x-y=1$ (c) $x=0$ (d) None of these

211. If $bx+cy=a$, where *a*, *b*, *c* are of the same sign, be a line such that the area enclosed by the line and the axes of reference is $\frac{1}{8}$ unit^2, then

(a) b, a, c are in G.P. (b) $b, 2a, c$ are in G.P. (c) $b, \frac{a}{2}, c$ are in A.P. (d) $b, -2a, c$ are in G.P.

212. Determine all values of α for which the point (α, α^2) lies inside the triangle formed by the lines $2x+3y-1=0$, $x+2y-3=0, 5x-6y-1=0$ [IIT 1992]

(a) $-3/2<\alpha<-1$ and $1/2<\alpha<1$ (b) $-3/2<\alpha<1$ and $-1/2<\alpha<1$

(c) $-3<\alpha<-1$ and $2<\alpha<1$ (d) None of these

213. The symmetry in curve $x^3+y^3=3axy$ along

(a) *x*-axis (b) *y*-axis (c) Line $y=x$ (d) Opposite quadrants

214. If m_1, m_2 are the roots of the equation $x^2-ax-a-1=0$, then the area of the triangle formed by the three straight lines $y=m_1x, y=m_2x$ and $y=a(a\neq -1)$ is

(a) $\frac{a^2(a+2)}{2(a+1)}$, if $a>-1$ (b) $\frac{-a^2(a+2)}{2(a+1)}$, if $a<-1$

(c) $\frac{-a^2(a+2)}{2(a+1)}$, if $-2<a<-1$ (d) $\frac{a^2(a+2)}{2(a+1)}$, if $a<-2$

215. A line which makes an acute angle θ with the positive direction of *x*-axis is drawn through the point $P(3,4)$ to meet the line $x=6$ at *R* and $y=8$ at *S*, then

(a) $PR=3\sec\theta$ (b) $PS=4\operatorname{cosec}\theta$

(c) $PR+PS=\frac{2(3\sin\theta+4\cos\theta)}{\sin 2\theta}$ (d) $\frac{9}{(PR)^2}+\frac{16}{(PS)^2}=1$

216. $P(m,n)$ (where *m*, *n* are natural numbers) is any point in the interior of the quadrilateral formed by the pair of lines $xy=0$ and the two lines $2x+y-2=0$ and $4x+5y=20$. The possible number of positions of the point *P* is

(a) Six (b) Five (c) Four (d) Eleven

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Answer Sheet

Straight Line

Assignment (Basic and Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	c	b	b	d	b	b	a	c	a	a	d	c	a	c	c	b	d	b	c
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
c	d	c	a	b	a	b	a	c	b	d	b	d	a	a	a	d	c	b	a
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
c	a	d	b	a	a	b	a	a	c	a	d	c	b	a	b	a	a	b	d
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
b	b	a	a	a	a	b	a	a	a	c	a	c	b	a	b	a,c	a	a	b
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
c	b	b	b	a	c	b	b	b	b	c	b	a	a	b	c	c	d	c	b
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
c	d	b	b	b	c	c	c	b	d	c	c	a	a	a	b	b	a	c	b
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
c	b	a	b	d	c	d	a	b	c	a	b	a	c	b	a	d	a	d	a
141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
a	b	d	a	b	c	a	c	c	a	a	b	a	c	c	a	a	c	a	b
161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
d	c	a	c	d	a,b,c,d	a	b	a	b	b	b	a	b	b	a	b	d	a	d
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200

c	b	b	b	b	b	a	c	b	b	a	c	a	a	b	c	a	c	c	d
201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216				
b	a	a	d	c	a	c	b	c	a	b,d	a	c	a,c,d	a,b,c,d	a				