

Topics : Circle, Straight Lines

Type of Questions

M.M., Min.

Comprehension (no negative marking) Q.1 to Q.3

(3 marks, 3 min.)

[9, 9]

Subjective Questions (no negative marking) Q.4,5,6,7

(4 marks, 5 min.)

[16, 20]

COMPREHENSION (For Q.No. 1 to 3)

Let $f(x) \equiv x^2 + px + q = 0$ have real roots α, β and $g(x) \equiv x^2 + rx + s = 0$ have real roots γ, δ

1. The area of the quadrilateral formed by points $(\gamma, 0), (\alpha, 0), (0, \beta), (0, \delta)$ taken in order is

(A) $\frac{|q-s|}{2}$

(B) $\frac{|q+s|}{2}$

(C) $\frac{|r+p|}{2}$

(D) $\frac{|p-r|}{2}$

2. The centre of the circle passing through the points of intersection of pairs of lines $f(x) = 0$ and $g(y) = 0$ is

(A) $\left(\frac{p}{2}, \frac{r}{2}\right)$

(B) $\left(\frac{q}{2}, \frac{s}{2}\right)$

(C) $\left(-\frac{q}{2}, -\frac{s}{2}\right)$

(D) $\left(-\frac{p}{2}, -\frac{r}{2}\right)$

3. Equation of the director circle of the circle $f(x) + g(y) = 0$ is

(A) $f(x) + g(y) = p^2 + r^2 - q - s$

(B) $f(x) + g(y) = q^2 + s^2$

(C) $f(x) + g(y) = \frac{p^2 + r^2}{4} - q - s$

(D) $f(x) + g(y) = p + r - \frac{(q^2 + s^2)}{4}$

4. Two circles touch the x-axis and the line $y = mx$ ($m > 0$). They meet at $(9, 6)$ and at another point and the product of their radii is 68. Find 'm'.

5. Show that the common tangents to the circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 + 2x = 0$ form an equilateral triangle.

6. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + k\sqrt{x^2 + y^2} = 0$, find k.

7. Let A, B, C be real numbers such that

(1) $(\sin A, \cos B)$ lies on a unit circle centred at origin.

(2) $\tan C$ and $\cot C$ are defined.

If the minimum value of $(\tan C - \sin A)^2 + (\cot C - \cos B)^2$ is $a + b\sqrt{2}$, where $a, b \in \mathbb{I}$, find the value of $a^3 + b^3$.

Answers Key

1. A 2. D 3. C 4. $m = \frac{12\sqrt{221}}{49}$ 6. $k = 1$

7. 19