6. Oscillatory Motion and Waves

6.1. At two moments in time the displacements of a harmonically oscillating point are the same. Can we state, on the basis of what we have just said, that the phases at these moments are also the same?

6.2. The oscillations depicted by curve *1* in the figure are expressed by the equation $x = A \sin \omega t$. What is the equation for the oscillations depicted by curve *2*?

6.3. Two material particles of equal mass are performing harmonic oscillations whose graphs are shown in the figure. What oscillation has a higher energy?

6.4. As a result of adding two mutually perpendicular oscillations of equal frequency, the motion of an object occurs along an ellipse; in one case the motion is clockwise, while in the other it is counterclockwise. Write the

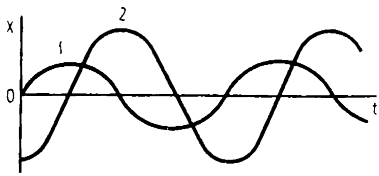

Fig. 6.2

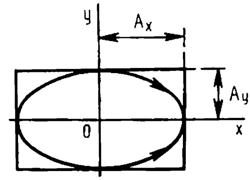

Fig. 6.4

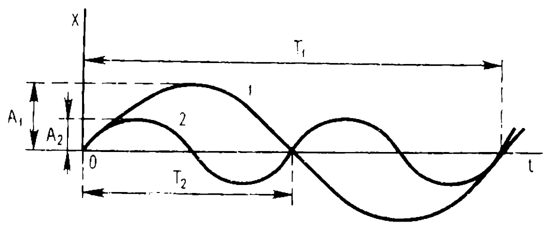

Fig. 6.3

equations of motion along each coordinate axis, assuming that the initial phase along the x axis is zero.

6.5. Two mutually perpendicular oscillations are added. In one case the graphs representing these oscillations are those shown in Figure (a) and in the other, those shown in Figure (b). In what respect do the resultant oscillations differ?

6.6. Suppose that the addition of two mutually perpendicular oscillations in which a material particle participates results in an ellipse, with the direction of motion indicated by the arrow in the figure. The equation of motion along the x axis can be written in the form $x = A_1 \sin \omega t$ and that along the y axis, in the form $y = A_2 \sin (\omega t + \varphi)$. Determine the condition that φ must meet.

6.7. Two mutually perpendicular oscillations obey the laws

$$x = A_1 \sin \omega_1 t \text{ and } y = A_2 \sin (\omega_2 t + \varphi).$$

The addition of these two oscillations leads to the Lissajous figure shown in the drawing accompanying the problem. Determine the relationship between ω_1 and ω_2 and the initial phase φ if the figure is traversed in the direction shown by the arrows.

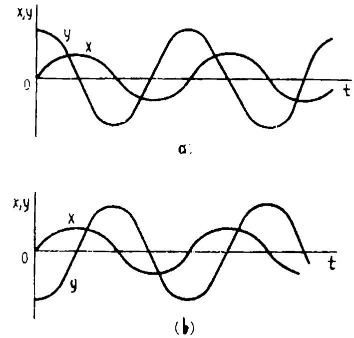

Fig. 6.5

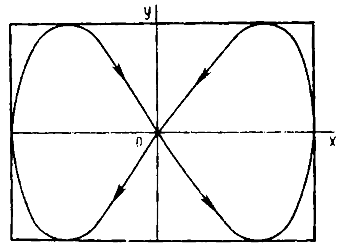

Fig. 6.7

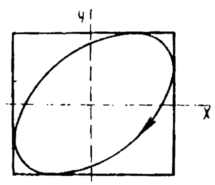

Fig. 6.6

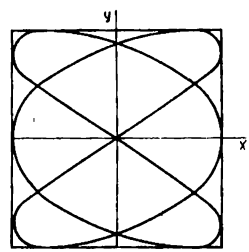

Fig. 6.8

6.8. Two mutually perpendicular oscillations are performed according to the laws

$$x = A_1 \sin \omega_1 t \quad \text{and} \quad y = A_2 \sin \omega_2 t.$$

Determine the relationship between ω_1 and ω_2 using the Lissajous figure shown in the drawing accompanying the problem.

6.9. A material particle oscillates according to the harmonic law. At which of the two moments, *1* or *2*, is the kinetic energy higher and in which, the potential energy? At which moment is the acceleration of the particle at its maximum (in absolute value)?

6.10. Two loads whose masses are m_1 and m_2 are suspended by springs ($m_1 > m_2$). When the loads were attached to the unloaded springs, it was found that the elongations of the springs were the same. Which of the two loads oscillates with a greater oscillation period and which of the two loads possesses a higher energy (provided that the oscillation amplitudes are equal)? The springs are considered massless.

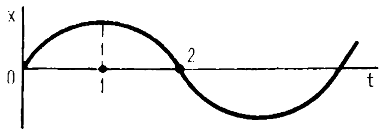

Fig. 6.9

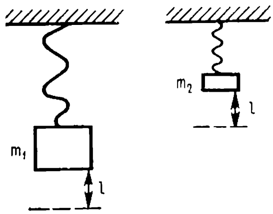

Fig. 6.10

6.11. A chemical test tube is balanced by a load at its bottom so that it does not tip when submurged in a liquid (the cross-sectional area of the tube is S). After submerging to a certain depth, the tube begins to oscillate about its position of equilibrium. The tube, whose mass together with the mass of the load is m, is in the state of equilibrium in a liquid with a density ρ when its bottom is below the level of liquid by a distance l. Determine the oscillation period of the tube assuming that the viscosity of the liquid is nil.

6.12. One way to measure the mass of an object in a space station at zero gravity is to use a device schematically

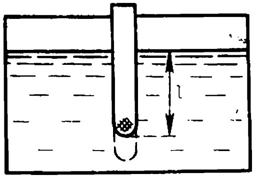

Fig. 6.11

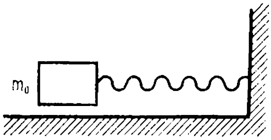

Fig. 6.12

shown in the figure. The principle of operation of this device is as follows. First the austronaut measures the oscillation frequency of an elastic system of known mass. Then the unknown mass is added to this system and a new measurement of the oscillation frequency is taken. How can one determine the unknown mass from the two measured values of frequency?

6.13. Two simple pendulums having equal masses but different lengths are in oscillatory motion with the same angular amplitudes. Which of the two pendulums has a higher oscillation energy?

6.14. Two pendulums, a physical one in the form of a homogeneous rod and a simple one, of equal mass and

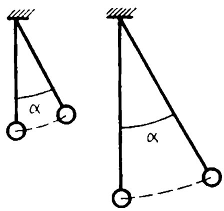

Fig. 6.13

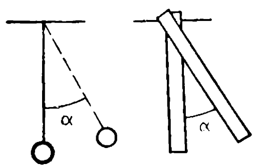

Fig. 6.14

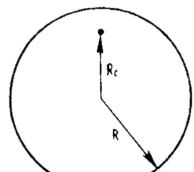

Fig. 6.15

length are in oscillatory motion with the same angular amplitudes. Which of the two pendulums has a higher oscillation energy?

6.15. An axis passes through a disk of radius R and mass m at a distance R_c from the disk's center. What will be the period of oscillations of the disk about this axis (which is fixed)?

6.16. Consider a physical pendulum that is a homogeneous rod of length l. At what distance R_c from the center of

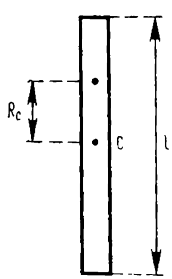

Fig. 6.16

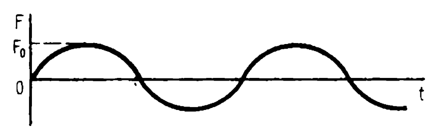

Fig. 6.17

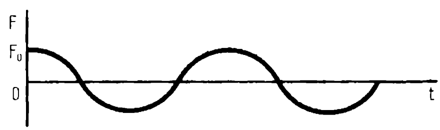

Fig. 6.18

gravity of the rod must the point of suspension lie for the oscillation period to be maximal?

6.17. A force acting on a material particle varies according to the harmonic law

$$F = F_0 \sin \omega t.$$

At time $t = 0$ the velocity v is zero. How do the velocity and position of the particle vary with time?

6.18. A force acting on a material particle varies according to the harmonic law

$$F = F_0 \cos \omega t.$$

At time $t = 0$ the velocity v is zero. How do the velocity and position of the particle vary with time?

6.19. The time dependence of the amplitude of damped oscillations is presented in the figures on a semilogarithmic scale, that is, the time is laid off on the horizontal axis on a linear scale and the amplitude, on the vertical

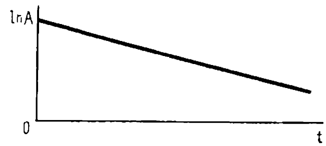

Fig. 6.19

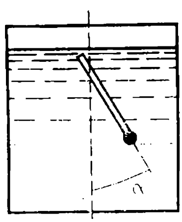

Fig. 6.21

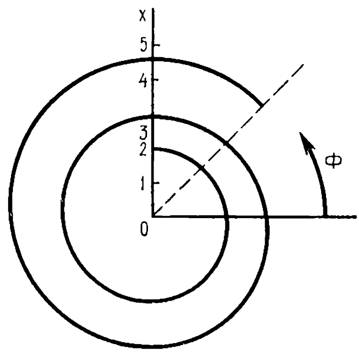

Fig. 6.20

axis on a logarithmic scale. Construct the time dependence of the energy of these oscillations using the semilogarithmic scale. Set the initial values of the logarithms of the amplitude and energy of the oscillations equal.

6.20. Suppose that certain damped oscillations are represented in polar coordinates. Depict these oscillations in Cartesian coordinates with the phase of the oscillations laid off on the horizontal axis and the displacement, on the vertical axis, assuming that the ratio of the sequential amplitudes of oscillations and the initial phase remain unchanged. Find the logarithmic decrement of the oscillations.

6.21. Suppose that a pendulum oscillates in a viscous medium. The viscosity of the medium and the mass and length of the pendulum are such that the oscillations are aperiodic. The pendulum is deflected from the position of equilibrium and released. How will the absolute value of the the pendulum's velocity vary with time: will it increase continuously, decrease continuously, pass through a maximum, or pass through a minimum?

6.22. A load suspended by a spring in a viscous medium performs damped oscillations. How should one change the length of the spring (preserving all the characteristics of the spring, i.e. the thickness of the wire, the density of the turns, etc.) so that the oscillations become aperiodic? The mass of the spring is assumed to be negligible compared to the mass of the load.

6.23. An oscillatory circuit consists of a capacitance C, an inductance L, and a resistance R. Damped oscillations set in in this circuit. (1) How should one change the distance between the plates of the capacitor for the discharge in the circuit to become aperiodic? (2) How should one change the capacitance and inductance (with the resistance remaining unchanged) for the damping in the contour to diminish provided that the natural frequency of free oscillations remains the same? How will this change the frequency of damped oscillations? (3) How will the logarithmic decrement of the oscillations change if the resistance and inductance change by the same factor?

6.24. Two spheres of the same diameter but of different masses are suspended by strings of equal length. If the

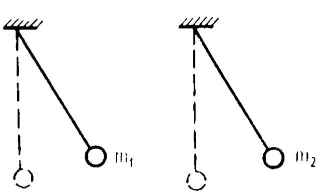

Fig. 6.24

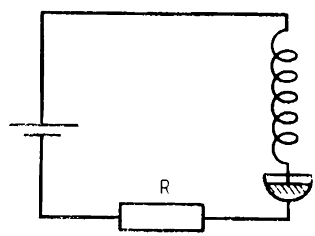

Fig. 6.25

spheres are deflected from their positions of equilibrium, which of the two will have a greater oscillation period and which will have a greater logarithmic decrement if their oscillations occur in a real medium with viscosity?

6.25. A "dancing spiral" is sometimes demonstrated at lectures. A spring fixed at its upper end is submerged by its lower end into mercury. Voltage supplied by a DC source is applied to the upper end and the mercury. When current flows in the spring, the rings of the spring tend to draw together, the spring gets shorter, and the lower end moves out of the mercury. The current ceases, and the lower end is again submerged in the mercury. The process repeats itself. What oscillations does the spring perform in the process: free, forced, damped, or self-oscillations?

6.26. Which of the two diagrams, Figure (a) or Figure (b), represents the dependence of the amplitude of displacements in forced oscillations on the frequency of the driving force and which represents the frequency dependence

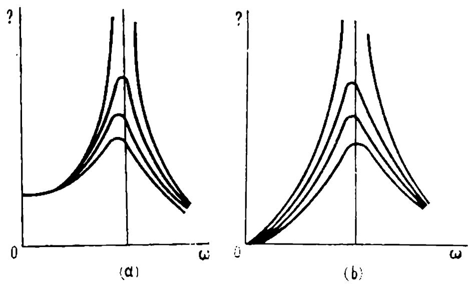

Fig. 6.26

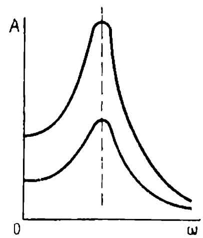

Fig. 6.29

of the velocity amplitude? In what parameter determining the oscillation conditions does each curve represented in Figures (a) and (b) differ? What parameters determine the intersection of each curve with the vertical axis in Figure (a) and the position of the maximum?

6.27. How will the displacement amplitude at $\omega = 0$ that is A_0, the maximal amplitude A_m, and the resonance frequency ω_{res} vary if the resistance of the medium in which the oscillations occur decreases provided that all the other parameters that determine the forced oscillations remain unchanged?

6.28. The curve depicting the dependence of the amplitude of forced oscillations on the frequency of the driving force in a medium with no resistance tends to infinity as $\omega = \omega_0$. Why is this situation meaningless not only from the physical standpoint but also from the mathematical standpoint? How does a system oscillate in a medium that has practically no resistance?

6.29. Two forced oscillations with the same natural frequencies have amplitudes that differ by a factor of 2 for all values of the frequency of the driving force. In what parameter, among the amplitude of the driving force, the mass of the oscillating object, the elasticity coefficient, and the resistance of the medium, do these systems differ? It is assumed that these systems may differ only in one parameter.

6.30. Waves on the surface of water in the form of parallel lines advance on a wall with an aperture much narrower than the wavelength. What will be the shape of the waves propagating on the surface behind the wall (and aperture)?

6.31. In the standing waves that form as a result of reflection of waves from an obstacle the ratio of the amplitude at a crest to the amplitude at a node is δ. What fraction of the energy passes past the obstacle?

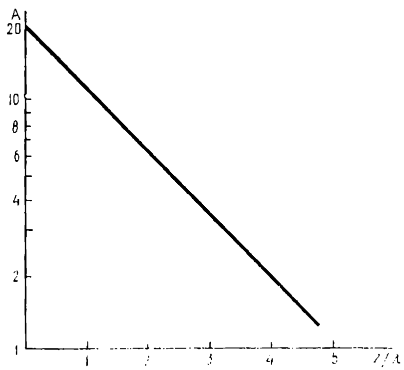

Fig. 6.32

6.32. A wave is propagating in a medium with damping. The distance from the source of oscillations (in units of the wavelength) is laid off on the horizontal axis and the common logarithm of the oscillation amplitude is laid off on the vertical axis. Using the graph shown in the figure accompanying the problem, write a formula that will link the amplitude with the distance.

6.33. The formula that expresses the speed of sound in a gas can be written in the following form:

$$c = \sqrt{\gamma p/\rho}. \qquad (6.33.1)$$

Here γ is the specific heat ratio (the ratio of the specific heat capacity of the gas at constant pressure to the specific heat capacity at constant volume), p is the pressure of the gas, and ρ is the density of the gas. Using this formula as a basis, can we stipulate that upon isothermal change of the state the speed of sound in the gas grows with pressure?

6.34. The figure demonstrates the temperature dependence of the speed of sound in neon and water vapor on the log-log scale. Which straight line corresponds to the lighter of the gases?

6.35. The dependence of the frequency of oscillations registered by a receiver when the receiver and the source of sound approach each other depends on whether the source moves and the receiver is fixed, or whether the source is fixed and the receiver is in motion. The curves in the figure represent the dependence of the ratio of the received

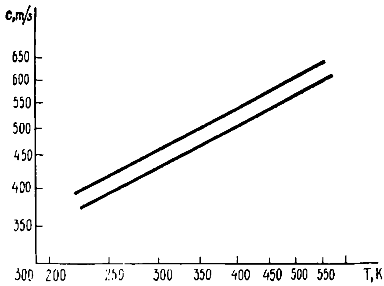

Fig. 6.34

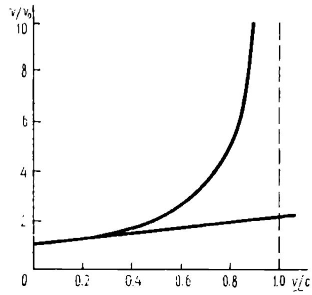

Fig. 6.35

frequency of oscillations to the frequency emitted by the source on the ratio of the rate of relative motion to the velocity of sound. Which of the two curves corresponds to a moving source and which, to a moving receiver? The medium where the propagation of sound takes place (air or water) is assumed fixed.

6.36. An observer standing at the bed of a railroad hears the whistle of the locomotive of the train that rushes past him. When the train is approaching the observer, the frequency of the whistle sound is ν_1, while when it has passed the observer, the frequency is ν_2. Determine the speed of the train and find the whistle frequency when the observer moves together with the train. The speed of sound is assumed to be known.

6.37. Two observers stand at different distances from the bed of a railroad. When a train passes them, each hears how the frequency of the train whistle changes, with the change occurring along curve *1* for one observer and along curve *2* for the other. Which of the two observers is standing closer to the roadbed?

6.38. A source of sound whose frequency is ν_0 is moving with a speed v. The waves travel to a fixed obstacle, are reflected by the obstacle, and are registed by a receiver that moves together with the source. What frequency is registed by the receiver if the speed of sound waves is c?
6.39. A source of oscillations S is fixed to the riverbed of a river whose waters flow with a velocity v. Up and down the stream there are fixed (also to the river bed)

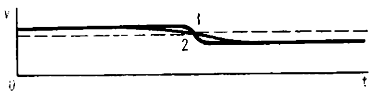

Fig. 6.37

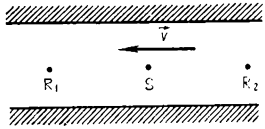

Fig. 6.39

two receivers, R_1 and R_2 (see the figure). The source generates oscillations whose frequency is ν_0. What frequencies do receivers R_1 and R_2 register?
6.40. Two boats are floating on a pond in the same direction and with the same speed **v**. Each boat sends, through the water, a signal to the other. The frequencies ν_0 of the generated signals are the same. Will the times

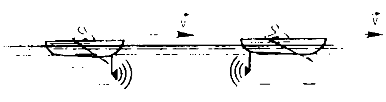

Fig. 6.40

Fig. 6.41

it takes the signals to travel from one boat to the other be the same? Are the wavelengths the same? Are the frequencies received by the boats the same?
6.41. An underground explosion at a point A generates vibrations. Seismographs that are capable of measuring longitudinal and transverse waves separately are placed at another point B. The time interval between the arrival of longitudinal and transverse waves is measured. How, knowing the velocities of propagation of longitudinal and transverse waves and the time difference between arrival, to determine the distance S between points A and B?
6.42. A sound wave travels in air and falls on the interface between air and water at an angle α_1. At what angle

α_2 will the wave propagate in the medium: greater than α_1 or smaller than α_1?

6.43. There are many documented cases when an explosion at a point A will be heard at a point B that is located far away from A while in a certain region, known as the zone of silence, located much closer to A than to B the explosion is not heard. Among the reasons for this is the deflection of sound waves caused by the presence of a vertical temperature gradient in the atmopshere. How should the air temperature change with altitude for the direction of propagation of sound waves to be as shown in the figure?

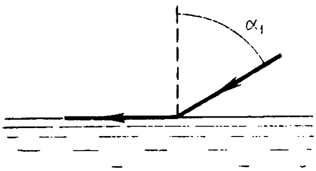

Fig. 6.42

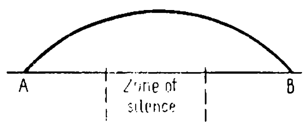

Fig. 6.43

6.44. At a depth h_1 below ground level there is a pocket of water of depth h_2. What type of artificial seismic waves, longitudinal or transverse, is needed to measure the depth of the water pocket?

6.45. An airlane is in supersonic flight at an altitude h. At what smallest distance a (along the horizontal) from the observer on the ground is there a point from which the sound emitted by the airplane motors travels to the observer faster than from point A that is directly above the observer?

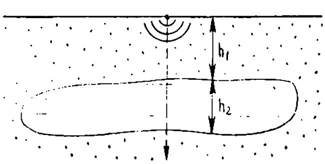

Fig. 6.44

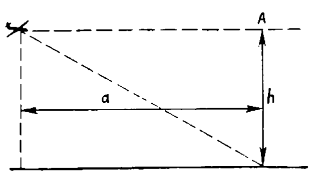

Fig. 6.45

6. Oscillatory Motion and Waves

6.1. Equal deflections from the position of equilibrium occur if

$$\sin \Phi_1 = \sin \Phi_2, \qquad (6.1.1)$$

where $\Phi_1 = \omega t_1$ and $\Phi_2 = \omega t_2$ (as shown in the figure, the initial phase is zero). The x vs. ωt curve shows that condition (6.1.1) is met if

$$\sin \Phi_2 = \sin (\pi - \Phi_1).$$

Here $\cos \Phi_2 = -\cos \Phi_1$, that is, phases Φ_1 and Φ_2 correspond to velocities of the oscillating point that are

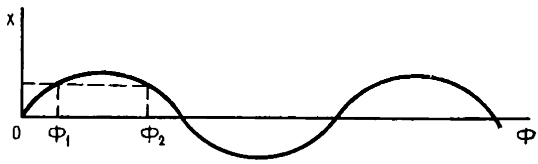

Fig. 6.1

opposite in direction. The phases of harmonic oscillations coincide if both the deflections and the velocities of the oscillations coincide (both in absolute value and in direction).

6.2. The amplitude of the oscillations depicted by curve *2* in the figure accompanying the question is twice as large as that of the oscillations depicted by curve *1*.

The periods of the two oscillations coincide. Oscillations *2* lag in phase behind oscillations *1* by a quarter of one period. Hence, oscillations *2* are represented by the equation

$$x = 2A \sin (\omega t - \pi/2).$$

6.3. Oscillations *1* have a period that is twice as large as that of oscillations *2*, so that the frequency of oscillations *1* is one-half of that of oscillations *2*. Amplitude A_1 is twice as large as amplitude A_2. The energies of these oscillations are

$$W_1 = \frac{1}{2} m\omega_1^2 A_1^2$$

and $$W_2 = \frac{1}{2} m\omega_2^2 A_2^2 = \frac{1}{2} m \left(\frac{\omega_1}{2}\right)^2 (2A_1)^2 = W_1,$$

that is, coincide.

6.4. The equation of the motion projected on the x axis is

$$x = A_x \sin \omega t.$$

In the case where the object moves clockwise, the deflection along the y axis at time zero ($t = 0$) is $y = A_y$, and then it decreases to zero when the maximum on the x axis is attained. The sine decreases from unity to zero as the angle changes from $\pi/2$ to π. In this case the initial phase of oscillations along the y axis is $\pi/2$, and the equation of motion projected on the y axis is

$$y = A_y \sin (\omega t + \pi/2).$$

In the case where the object moves counterclockwise, the deflection along the y axis is zero when the phase of motion along the x axis becomes $\pi/2$ and, hence, the initial value of this deflection is $y = -A_y$ and increases to zero in the course of a quarter of the period. In the case at hand the equation of motion projected on the y axis can be written in the form

$$y = A_y \sin (\omega t - \pi/2).$$

6.5. In the first case the oscillations along the y axis begin $\pi/2$ earlier in phase than along the x axis, while in the second case they lag behind by the same quantity.

In both cases the motion takes place along an ellipse described by the equation

$$\frac{x^2}{A_x^2}+\frac{y^2}{A_y^2}=1.$$

The two motions differ in direction. In the first case the motion is clockwise while in the second it is counterclockwise. The equations of motion have the form

$$x=A_x \sin \omega t,\ y_1=A_y \sin\left(\omega t+\frac{\pi}{2}\right),\ y_2=A_y \sin\left(\omega t-\frac{\pi}{2}\right).$$

6.6. When the deflection along the x axis is zero and the velocity is positive, the deflection along the y axis is greater than zero but smaller than A_2, with y continuing to increase according to the direction designated by the arrow and reaching the value A_2 (i.e. when $\omega t + \varphi = \pi/2$) for $0 < \omega t < \pi/2$. Hence,

$$0<\varphi<\pi/2.$$

6.7. In the course of one period the oscillating point attains each of its maximal (but opposite) values once (i.e. in the motion along an axis). For this reason the complete Lissajous figure touches the sides of the rectangle limiting the motion exactly the same number of times as there are periods in the motion of the point in a certain direction. Along the x axis the figure touches the sides of the rectangle twice, while along the y axis four times. Hence

$$\omega_2=2\omega_1 \quad \text{and} \quad y=A_2 \sin(2\omega_1 t+\varphi).$$

To determine φ, we assign to $\omega_1 t$ the values that correspond to points where the Lissajous figure touches the limiting rectangle. For instance, if we take $\omega_1 t = \pi/2$, then $2\omega_1 t + \varphi = \pi/2 + \varphi$. Here

$$\sin(2\omega_1 t+\varphi)=-1.$$

Hence,

$$\pi/2+\varphi=-\pi/2, \quad \text{or} \quad \varphi=-\pi.$$

6.8. Just like in the previous problem, the number of periods it takes to traverse completely the Lissajous figure in either direction is determined by the number of points where the Lissajous figure touches the rectangle that limits the motion. There are three such points in the positive direction of x and two points in the positive direction of y. Thus, the entire figure is traversed in the direction x in the course of three periods and in the direction y in the course of two periods. Hence,

$$\omega_1/\omega_2 = 3/2.$$

6.9. The kinetic energy is maximal when the velocity is maximal in absolute value. Being the time derivative of displacement, the velocity is maximal at moment *2*. The maximal potential energy is determined by the maximal displacement, that is, the amplitude, and is equal to $kA^2/2$. Hence, it is maximal at moment *1*. At this moment the kinetic energy is zero, while the potential energy is zero at moment *2*. The acceleration of the particle is at its maximum when the second time derivative of the displacement is maximal. This corresponds to moment *1*. Since at this moment the second derivative is negative, so is the acceleration.

6.10. The period of harmonic oscillations that take place due to a quasielastic force ($F = -kx$) is determined from the formula

$$\tau = 2\pi \sqrt{m/k}. \qquad (6.10.1)$$

The spring constant k is defined as the force that is required to stretch the spring in such a manner that the spring elongation becomes equal to its initial length. In the case at hand the elongations occur because of the weight of the loads, with the result that

$$k_1 = m_1 g/l \quad \text{and} \quad k_2 = m_2 g/l.$$

Susbtituting k into (6.10.1), we see that the masses cancel out and in both cases the period is

$$\tau = 2\pi \sqrt{l/g}.$$

The same result can be obtained (to within a constant coefficient) from dimensional reasoning. There are three quantities that appear in the problem: mass, elongation, and time (the sought period). In addition, since forces equal to the weights of the loads are applied to the springs, we may assume that the acceleration of gravity g will enter into the solution. Bearing in mind that the dimensions of the left- and right-hand sides of any equation must be the same, we can write

$$T = M^a L^b [LT^{-2}]^c,$$

where a, b, and c are the exponents of the corresponding quantities. We have the following equations for the exponents:

$$a = 0, \quad b + c = 0, \quad c = -1/2.$$

Hence,

$$\tau = \mathscr{K} l^{1/2} g^{1/2},$$

where $\mathscr{K}$ is a dimensionless coefficient, which cannot be found using solely dimensional considerations. Above it was shown that this coefficient is equal to 2π.

The energy of the oscillations of a load can be written in the form

$$W = mA^2\omega^2/2.$$

Since the periods of oscillations (and hence the frequencies) are equal and so are the amplitudes (by hypothesis), the load with the higher energy is the one whose mass is m_1.

6.11. In the case at hand the quasielastic force is Archimedes' force. When the bottom of the test tube lies above or below the position of equilibrium by a distance x, this force is

$$F = -Sx\rho g. \qquad (6.11.1)$$

The mass of the test tube together with the mass of the load is equal to the mass of the displaced water, or

$$m = lS\rho. \qquad (6.11.2)$$

Using (6.11.1), we can find the "spring constant"

$$k = |F|/x = S\rho g. \qquad (6.11.3)$$

Substituting (6.11.2) and (6.11.3) into the expression for the period of oscillations (6.10.1), we get

$$\tau = 2\pi\sqrt{m/k} = 2\pi\sqrt{l/g}.$$

We see that τ depends neither on the mass and cross-sectional area of the tube nor on the density of the liquid. The same result can be obtained from dimensional considerations, just like it was done in Problem 6.10.

6.12. If m_0 is the known mass and m is the unknown mass and if ω_0 and ω are the angular frequencies of oscillations of the systems with the known mass and the known mass plus the unknown, then

$$\omega_0 = \sqrt{k/m_0}, \qquad (6.12.1)$$

$$\omega = \sqrt{k/(m_0 + m)}, \qquad (6.12.2)$$

where k is the spring constant. Combining (6.12.1) with (6.12.2), we arrive at a formula for the unknown mass:

$$m = m_0 \left(\frac{\omega_0^2}{\omega^2} - 1 \right).$$

6.13. The total energy of oscillations of a material particle can be made equal to the maximal kinetic energy or maximal potential energy of the particle. In the case at hand it proves expedient to compare the maximal potential energies, which are specified by the maximal deflections. When the deflection is at its maximum, the load (or particle) is at a height h above the position of equilibrium:

$$h = l\,(1 - \cos\alpha).$$

Since the expression inside the parentheses is the same for both pendulums, the pendulum with the greater length is raised to the greater height and, hence, has the higher energy.

6.14. Just like in the previous problem, the total energy can be made equal to the maximal potential energy. Since the center of gravity of the physical pendulum is higher than that of the simple pendulum, the physical pendulum can be thought of as a simple pendulum of smaller length. Thus, the given simple pendulum has a higher energy.

6.15. In the case at hand the disk constitutes a physical pendulum. The period of oscillations of a physical pendulum is given by the formula

$$T = 2\pi \left(\frac{J}{mgR_c} \right)^{1/2}.$$

The moment of inertia of the disk about the center is $J = mR^2/2$. According to Steiner's theorem,

$$J = m\,(R^2/2 + R_c^2),$$

whence

$$T = 2\pi \left[\frac{(R^2/2 + R_c^2)}{gR_c} \right]^{1/2}.$$

As expected, the period does not depend on the mass of the pendulum.

6.16. The angular frequency of oscillations for a physical pendulum is

$$\omega = (mgR_c/J)^{1/2},$$

where m is the mass of the pendulum, and J is the pendulum's moment of inertia. If the distance from the center of gravity to the point of suspension is R_c, then, according to Steiner's theorem, the moment of inertia of the rod about the suspension point is equal to the moment of inertia of the rod about the center of gravity plus the moment of inertia of a material particle whose mass is that of the rod about the point of suspension:

$$J = \frac{ml^2}{12} + mR_c^2.$$

Thus,

$$\omega = \left(\frac{12gR_c}{l^2+12R_c^2}\right)^{1/2}.$$

To find the extremum, we nullify the derivative of ω with respect to R_c:

$$\frac{d\omega}{dR_c} = \frac{6g\,(l^2-12R_c^2)}{R^{1/2}\,(l^2+12R_c^2)^{3/2}} = 0.$$

Whence

$$R_c = \frac{l}{2\sqrt{3}} = 0.29l.$$

6.17. The acceleration varies according to the same law as the force. Thus,

$$v = \frac{F_0}{m}\int_0^t \sin\omega t\,dt = \frac{F_0}{m\omega}(1-\cos\omega t) = v_m(1-\cos\omega t).$$

The v vs. t curve is depicted in Figure (a) accompanying the answer. If the initial position of the point is taken as the origin, then

$$x = v_m\int_0^t (1-\cos\omega t)\,dt = v_m t - \frac{v_m}{\omega}\sin\omega t.$$

Thus, we have found that the particle is in translational motion with a velocity that periodically increases from zero to its maximum, equal to $2v_m$, and then drops off to

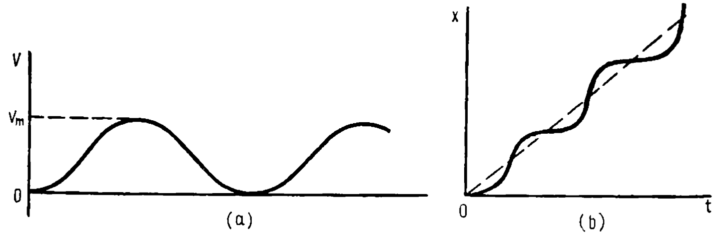

Fig. 6.17

zero. The motion is depicted schematically in Figure (b) accompanying the answer.

6.18. The solution to this problem is similar to that of Problem 6.17, the difference being that the initial phase

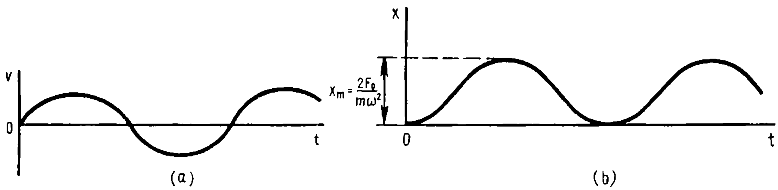

Fig. 6.18

of the driving force is different. In the case at hand, initially the force is maximal. The time dependence of the velocity is

$$v = \frac{F_0}{m}\int_0^t \cos\omega t = \frac{F_0}{m\omega}\sin\omega t = v_m \sin\omega t.$$

In contrast to the previous case, the velocity changes its direction during motion (Figure (a) accompanying the answer). The displacement of the particle can be found after integration:

$$x = v_m \int_0^t \sin\omega t = \frac{v_m}{\omega}(1 - \cos\omega t).$$

Thus, in the case at hand the motion is purely harmonic, as shown by the curve in Figure (b).

A comparison of the results of the previous problem with those of the present problem demonstrates that the motion of a material particle under a force that varies according to the harmonic law depends on the initial phase of the force. The motion may vary from purely translational to purely oscillatory. These features of a periodic force manifest themselves in various phenomena, say, in high-frequency electric discharge in gases, where the moments of collision of electrons, ions, and atoms accompanied by changes in velocities occur at different phases of the applied variable electric field.

6.19. If the amplitude decreases with the passage of time according to the law

$$A = A_0 e^{-\beta t},$$

then, since the oscillation energy is proportional to the square of the amplitude, the decrease in energy occurs according to the law

$$W = W_0 e^{-2\beta t}, \quad \text{or} \quad \ln W = \ln W_0 - 2\beta t.$$

The slope of the straight line that expresses the decrease in energy on the semilogarithmic scale must be double

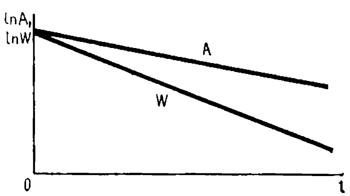

Fig. 6.19

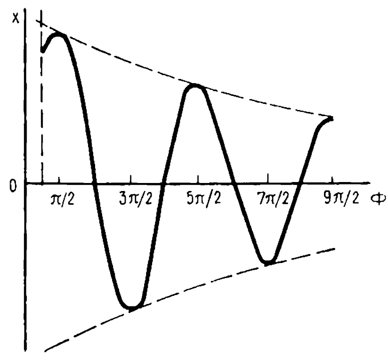

Fig. 6.20

the slope of the straight line that expresses the decrease in amplitude.

6.20. The figure accompanying the problem shows that the initial phase is $\pi/4$ while the ratio of the amplitude whose phases differ by 2π is equal to 1.5. This means that the logarithmic decrement $\ln (A_{n+1}/A_n)$ is approximately equal to 0.4.

6.21. Initially the velocity of the pendulum is zero and tends to zero as the pendulum approaches its position of equilibrium, so that it first grows and then, after passing through its maximum, decreases. We can arrive at the same conclusion after analyzing qualitatively the differential equation of the motion of the pendulum written

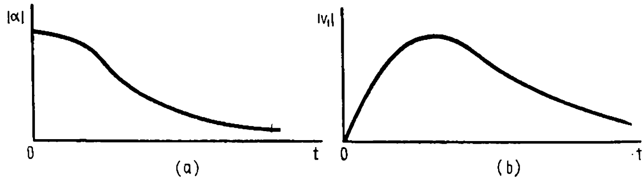

Fig. 6.21

in polar coordinates in the common approximation of small deflections:

$$I\ddot{\alpha} = -q\alpha - r\dot{\alpha}.$$

We select a system of coordinates in which the positive direction is the one in which the pendulum was initially deflected from the point of equilibrium. Initially, when the velocity was zero and the deflection was the largest, the acceleration was the highest. The curve depicting the time dependence of the deflection has at this point the greatest curvature. In the process of motion, the first term on the right-hand side of the equation decreases in numerical value, while the second term (which is positive since $\dot{\alpha} < 0$) grows, and because of this the absolute value of the acceleration decreases. There finally comes a moment when the acceleration vanishes and the velocity reaches its maximum. After that the acceleration grows, that is, becomes positive and increases in numerical value, which in the system of coordinates employed here implies deceleration, and the pendulum asymptotically approaches the position of equilibrium. The time dependences of the absolute values of the deflection and the velocity of the pendulum are shown in Figures (a) and (b) accompanying the answer.

6.22. In damped oscillations the damping factor is smaller than the natural frequency of free oscillations of the system: $\beta < \omega_0$. In aperiodic motion the situation is reversed: $\beta > \omega_0$. The damping factor is defined as the ratio $\beta = r/2m$, where r is the resistance of the medium, and m is the mass of the load. Both quantities remain unchanged, and so does β. To go over to the aperiodic mode, we must make ω_0 smaller. Since $\omega_0 = \sqrt{k/m}$, we must diminish k since m is fixed. At a given elongation force, the elongation of the spring is proportional to the initial length of the spring. Hence, the spring constant is inversely proportional to the length of the spring, with the result that we must increase the length of the spring if we wish to diminish k.

6.23. (1) The condition for an aperiodic discharge is $\beta > \omega_0$. The damping factor

$$\beta = R/2L \tag{6.23.1}$$

does not depend on the capacitance. To make the process aperiodic, we must diminish the natural frequency, which for a fixed inductance means increasing the capacitance, and the easiest way to do this is to bring the plates of the capacitor closer together.

(2) According to (6.23.1), to decrease the damping factor for a fixed resistance, we must increase the inductance. To preserve the value of the natural frequency $\omega_0 = 1/\sqrt{LC}$, the capacitance must be decreased by the same factor. The frequency of the damped oscillations,

$$\omega = \sqrt{\omega_0 - \beta^2},$$

increases in the process, approaching ω_0.

(3) When the resistance and inductance are decreased simultaneously, the damping factor remains unchanged, but for a fixed capacitance the oscillation period $T = 2\pi/\sqrt{\omega_0^2 - \beta^2}$ decreases and, hence, so does the logarithmic decrement.

6.24. Both the logarithmic decrement and the period depend on the damping factor:

$$\theta = \beta T \tag{6.24.1}$$

$$T = 2\pi/\sqrt{\omega_0^2 - \beta^2}. \tag{6.24.2}$$

Since the lengths of the pendulums are equal, the natural frequencies of free oscillations (that is, without resistance) are equal, too. The damping factor is

$$\beta = r/2m, \tag{6.24.3}$$

where r is the resistance of the medium, which is the same for the two pendulums. Substituting (6.24.3) into (6.24.1) and (6.24.2), we see that both the period and the logarithmic decrement of the sphere with the smaller mass are greater.

6.25. There is no periodic driving force in the system; hence, the oscillations are not forced. The oscillation frequency is determined by the mass and by the elastic properties of the spring, and since the amplitude of the oscillations remains unchanged, the oscillations are undamped although, of course, loss of energy is inevitable. This loss is compensated by the energy stored in the DC source. Thus, the oscillations belong to the type that occur with a natural frequency but with replenishing the energy from an external nonperiodic source, that is, self-oscillations.

6.26. The frequency dependence of the displacement amplitude in forced oscillations is given by the formula

$$A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}},$$

while the frequency dependence of the velocity amplitude is given by the formula

$$v_m = \frac{F_0\omega}{m\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}}.$$

In the first case, at $\omega = 0$ the amplitude A does not vanish but becomes equal to $F_0/m\omega_0^2$, or F/k, so that the curve cuts off a segment on the vertical axis, which segment is the displacement under a constant force. The velocity, of course, is zero in this case. Thus, the curves in Figure (a) correspond to the frequency dependence of the displacement amplitudes, while the curves in Figure (b) correspond to the frequency dependence of the velocity amplitudes. The smaller the damping factor β, the higher the curve in the respective diagrams. The damping factor also determines the position of the maxima of the displacement amplitudes:

$$\omega_{res} = \sqrt{\omega_0^2 - 2\beta^2}.$$

The maximal velocity amplitude for all damping factors is achieved at $\omega = \omega_0$.

6.27. The displacement A_0 at $\omega = 0$ is determined by the ratio of the maximal force F to the elastic constant

k (the spring constant), or $A = F/k$. By hypothesis, both F_0 and k remain unchanged, whereby A does not depend on the resistance of the medium. The resonance frequency, defined as

$$\omega_{res} = \sqrt{\omega_0^2 - 2\beta^2},$$

is the closer to the natural frequency the smaller the values of the damping factor β. Since the latter is defined as $\beta = r/2m$ and the mass of the oscillating object remains unchanged (by hypothesis), β decreases and ω_{res} grows as r drops. The amplitude at the resonance frequency,

$$A_{res} = \frac{F_0}{2\beta m \sqrt{\omega_0^2 - \beta^2}},$$

is the higher the smaller the resistance of the medium.

6.28. The differential equation describing the behavior of the system is

$$m\ddot{x} + r\dot{x} + kx = F_0 \sin \omega t, \qquad (6.28.1)$$

and it has two solutions, a steady-state and a transient. The latter describes the process of setting in of forced oscillations. Usually only the steady-state solution is considered. However, at $r = 0$ and $\omega = \omega_0$ this equation has no steady-state solution, and because of this the amplitude continuously increases and so does the energy of the system, which energy is taken from the source of oscillations. In reality, a system in which the resistance of the medium is negligible for all practical purposes either behaves in such a manner that the amplitude reaches values at which Hooke's law ceases to be valid (and, respectively, Eq. (6.28.1) loses meaning) or is destructed. One must bear in mind also that the fact that we ignore the resistance of the medium, which at low velocities is a valid assumption, cannot be justified as the velocity grows higher and higher.

6.29. The resonance frequency is the same for both oscillations:

$$\omega_{res} = \sqrt{\omega_0^2 - 2\beta^2}.$$

Since the natural frequencies also coincide, so do the damping factors β. The resonance amplitude is

$$A_{res} = \frac{F_0}{2\beta m \sqrt{\omega_0^2 - \beta^2}}.$$

Only two quantities in this formula can vary: the mass of the oscillating object and the amplitude of the driving force. However, from the fact that the natural frequencies are the same and the damping factors are the same, it follows that for different masses only the elasticity coefficients and the resistances differ:

$$\omega_0 = \sqrt{k/m}, \quad \beta = r/2m.$$

But by hypothesis, the systems are supposed to differ only in one parameter. This parameter, therefore, can only be the amplitude of the driving force, which for one system is twice as high as for the other.

6.30. According to Huygens' principle, each point of a wavefront is an independent source of oscillations. An ap-

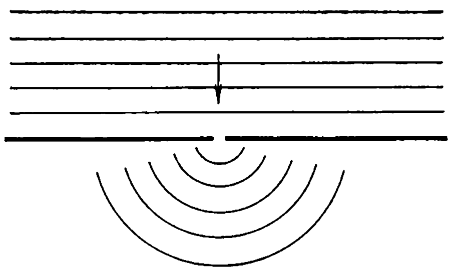

Fig. 6.30

erture whose width is much smaller than the wavelength limits a section of the wavefront (a line in the present case) that can be considered as a point source. This source emits approximately semispherical waves that propagate in space; in the case at hand these are approximately semicircles with differences in radii between the neighboring waves equal to one wavelength.

6.31. Since the frequency of the oscillations remains constant, the energy carried by the wave is determined uniquelly by the amplitude, that is, is proportional to the square of the amplitude. The amplitude at a crest A_l is equal to the sum of the amplitudes of the incident and reflected waves, A_1 and A_2, while the amplitude at a node, A_n, is equal to the difference between A_1 and A_2:

$$A_l = A_1 + A_2, \quad A_n = A_1 - A_2.$$

Hence, the amplitudes of the incident and reflected waves are

$$A_1 = \frac{A_l + A_n}{2}, \quad A_2 = \frac{A_l - A_n}{2}.$$

Hence,

$$\frac{A_2}{A_1} = \frac{A_l - A_n}{A_l + A_n} = \frac{A_l/A_n - 1}{A_l/A_n + 1} = \frac{\delta - 1}{\delta + 1}.$$

The ratio of the energy of the reflected wave to that of the incident wave is equal to the ratio of the squares of the amplitudes:

$$\frac{W_2}{W_1} = \left(\frac{\delta - 1}{\delta + 1}\right)^2.$$

Hence, the ratio of the energy that has passed the obstacle to the energy of the waves incident on the obstacle is

$$\frac{W_3}{W_1} = 1 - \left(\frac{\delta - 1}{\delta + 1}\right)^2 = \frac{4\delta}{(\delta + 1)^2}.$$

When the amplitudes are equal ($\delta = 1$) no standing waves are formed and the entire energy passes the obstacle.

In the theory and practice of propagation of waves (say, electromagnetic waves) a common notion is that of the standing-wave ratio, which is the ratio of the energies (or squares of amplitudes) at crest and node. Obviously, in an ideal standing wave this ratio is infinite.

6.32. The figure accompanying the problem shows that the amplitude decreases ten-fold over a distance equal to four wavelengths. Denoting the amplitude near the source by A_0 and the amplitude at a distance of four wavelengths from the source by A_4, we can write

$$A_0/A_4 = 10, \quad \text{or} \quad \log (A_0/A_4) = 1.$$

In natural logarithms,

$$\ln (A_0/A_4) = 2.3.$$

For the amplitude at a distance of one wavelength from the source we have

$$\ln (A_0/A_1) = 2.3/4 = 0.575,$$

while for the amplitude at a distance of z from the source we have

$$\ln (A_0/A_z) = 0.575z/\lambda.$$

Whence

$$A_z = A_0 \exp(-0.575z/\lambda).$$

This dependence is often expressed in terms of the wave number k, which is related to the wavelength as follows: $k = 2\pi/\lambda$. Thus,

$$A_z = A_0 \exp(-0.0916\, kz).$$

6.33. The statement is false. The density of the gas, which is in the denominator of formula (6.33.1), is determined by the ideal-gas law thus:

$$\rho = pM/RT, \tag{6.33.1}$$

where M is the molecular mass (weight) of the gas, and R is the universal gas constant. If we substitute this value of the density into (6.33.1), the pressure cancels out and we get the formula

$$c = \sqrt{\gamma RT/M}, \tag{6.33.2}$$

according to which for given gas the speed of sound depends only on the temperature of the gas. Actually, the temperature dependence is somewhat more complicated than simple proportionality to $T^{1/2}$, since in diatomic and especially multiatomic gases the specific heat capacity at constant volume grows noticeably with temperature.

6.34. According to formula (6.33.2), the speed of sound in a gas is proportional to the square root of γ and inversely proportional to the molecular mass. At a fixed temperature the difference in speeds of sound is determined by the ratio γ/M. For water vapor (six degrees of freedom) $\gamma = 1.33$ and for neon (three degrees of freedom) $\gamma = 1.67$. The molecular mass of water is 1.8×10^{-2} kg/mol and that of neon is 2.02×10^{-2} kg/mol. The ratios γ/M is 74.1 for water vapor and 82.5 for neon.

Thus, the upper straight line depicts the temperature dependence of the speed of sound in neon and the lower one depicts the temperature dependence of the speed of sound in water vapor. Both straight lines have the same slope equal to 0.5. A calculation via formula (6.33.2) yields 454 m/s for neon at 300 K and 430 m/s for water vapor at the same temperature.

6.35. When the source is moving and the receiver is fixed, the registered frequency is

$$\nu_1 = \nu_0 \frac{1}{1-v/c},$$

while when the source is fixed and the receiver is moving,

$$\nu_2 = \nu_0 (1 + v/c).$$

The first formula implies that ν_1 grows without limit as v/c tends to unity (curve *1* in the figure accompanying the problem), while ν_2 increases linearly as v/c tends to unity (curve *2* in the same figure).

6.36. When the train is moving with a speed v and the speed of sound is c and the frequency measured by an observer on the train is ν_0 (better to say, when the train is at rest), the frequency registered when the train approaches the observer standing at the roadbed is

$$\nu_1 = \frac{\nu_0}{1-v/c}, \qquad (6.36.1)$$

while the frequency registered when the train is moving away from the observer is

$$\nu_2 = \frac{\nu_0}{1+v/c}. \qquad (6.36.2)$$

For the sake of brevity we introduce the notation $\nu_1/\nu_2 = \delta$ and $v/c = \beta$. Then

$$\delta = \frac{1+\beta}{1-\beta},$$

whence $\beta = (\delta - 1)/(\delta + 1)$, or

$$v = \frac{\nu_1 - \nu_2}{\nu_1 + \nu_2} c. \qquad (6.36.3)$$

Substituting (6.36.3) into (6.36.1) or (6.36.2), we get

$$\nu_0 = \nu_1 (1 - v/c) = \nu_2 (1 + v/c) = \frac{2\nu_1\nu_2}{\nu_1 + \nu_2}.$$

6.37. When the observer stands far from the line along which the source of sound is moving, the equation that describes the Doppler effect contains not the velocity of the sound proper but its projection on the direction of propagation of the wave. For the observer that stands very near to the moving train this velocity is practically that of the train and varies suddenly, and so does the pitch of the sound heard by that observer (curve *1* in the figure accompanying the problem). For the observer that stands at a rather big distance from the moving train, the projection of the velocity varies more smothly, dropping to zero when the train is closest to that observer and then increasing. For this reason the time it takes the registered frequency to change is greater (curve *2*).

6.38. If for an immobile source the wavelength is λ_0, the wavelength λ when the source moves with a velocity v is shorter than λ_0 by vT_0. The waves will arrive at the obstacle having the frequency

$$\nu_1 = \frac{c}{\lambda} = \frac{c}{\lambda_0 - vT} = \nu_0 \frac{1}{1 - v/c}.$$

The waves will reflect from the obstacle but will retain their frequency and wavelength. Since the receiver is moving toward the waves with a velocity v with respect to the medium, the relative velocity of the receiver and waves is $c + v$ and the registered frequency is

$$\nu_2 = \frac{c+v}{\lambda} = \frac{c+v}{c/\nu_0 - v/\nu_0} = v_0 \frac{c+v}{c-v} = \nu_0 \frac{1+v/c}{1-v/c}.$$

6.39. At frequency ν_0 the wavelength in still water is $\lambda_0 = c/\nu_0$. In a river whose waters flow with a velocity v, the wavelength downstream is by vT longer than λ_0 and the wavelength upstream is by vT shorter, that is,

$$\lambda = \lambda_0 \pm vT.$$

In relation to the receiver that is down the stream, the velocity of the received waves is the sum of the velocity of waves in still water and the velocity of the river waters (as if the receiver was moving against the waves). For the receiver that is up the stream the velocities are subtracted from each other, with the result that

$$c = c_0 \pm v.$$

The frequency ν registered by a receiver is the ratio of the speed of sound to the wavelength, or

$$\nu = \frac{c_0 \pm v}{\lambda_0 \pm vT} = \frac{c_0 \pm v}{c_0/\nu_0 \pm v/\nu_0} = \nu_0.$$

We see that ν is equal to the frequency of the oscillations generated by the source.

6.40. The wavelength of waves generated by a source moving in a stationary medium is

$$\lambda = \lambda_0 \pm vT,$$

where the minus sign corresponds to the propagation of waves from the source forward, while the plus sign corresponds to waves propagating backward. When the receiver is in motion, its velocity with respect to the waves is

$$c = c_0 \pm v.$$

Here the plus corresponds to motion against the waves, while the minus corresponds to motion in the same direction as the waves propagate. Since the velocities of the boats in relation to waves are different and the distance between the boats remains unchanged, the time it takes a signal to travel from one boat to the other depends on which boat is the receiver and which boat is the source:

$$t = \frac{l}{c_0 \pm v}.$$

If the boats could move with a speed equal to the speed of waves, then the boat moving ahead of the other one would cease to receive any signal, since the signal could not catch up with it. The frequency of the signal received by each boat is defined as the ratio of the velocity with respect to the waves to the receiver wavelength. For the boat floating at the rear,

$$\nu = \frac{c_0 + v}{\lambda_0 + vT} = \frac{1 + v/c_0}{(1 + v/c)\,\nu_0^{-1}} = \nu_0,$$

and for the boat floating in front,

$$\nu = \frac{c_0 - v}{\lambda_0 - vT} = \frac{1 - v/c_0}{(1 - v/c)\,\nu_0^{-1}} = \nu_0.$$

In both cases the frequency of the received signal is equal to that of the sent signal.

6.41. The times of arrival of longitudinal and transverse waves are, respectively,

$$t_{\parallel} = S/v_{\parallel} \text{ and } t_{\perp} = S/v_{\perp},$$

where $v_{\parallel}$ and $v_{\perp}$ are the velocities of propagation of the longitudinal and transverse waves, and S is the distance between A and B. The time interval between the arrival of longitudinal and transverse waves is

$$\Delta t = t_{\perp} - t_{\parallel} = S\left(\frac{1}{v_{\perp}} - \frac{1}{v_{\parallel}}\right),$$

whence

$$S = \frac{v_{\parallel} v_{\perp}}{v_{\parallel} - v_{\perp}} \Delta t.$$

If the seismographs are placed at two points, then by measuring the distances S_1 and S_2 (see the figure accompanying the answer) we can establish at which point the source of explosion is located. In fact, in this way the epicenters of earthquakes are located.

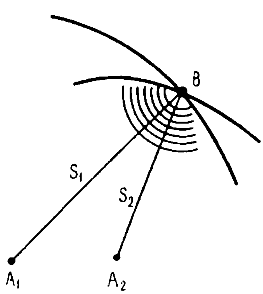

Fig. 6.41

6.42. The speed of sound waves in air is $c_1 \approx 330$ m/s and in water it is $c_2 \approx 1500$ m/s. According to Snell's law,

$$\sin \alpha_1 / \sin \alpha_2 = c_1 / c_2.$$

Accordingly, when the "sound beam" enters the water, it will be deflected from the perpendicular line still stronger and angle α_2 becomes greater than angle α_1. The velocity ratio determines the maximal angle at which sound waves can go "into" water. The maximal angle of incidence α_m satisfies the condition ($\alpha_2 = 90°$)

$$\sin \alpha_m = c_1 / c_2.$$

At $c_1 = 330$ m/s and $c_2 = 1500$ m/s we have $\sin \alpha_m = 0.22$ and $\alpha_m \approx 13°$. At an angle greater than 13° total reflection occurs. Such a situation is depicted in the figure accompanying the problem.

The perturbation caused by the incident wave penetrates the surface of the water but dies out exponentially, and this happens the faster the greater the angle of incidence of the wave. The wave dies out practically at a depth of the order of one wavelength. Sometimes one can hear a fisherman whisper: "Keep quiet! The fish is here!" The above estimate shows that a person standing at a distance away from the riverbank can never "scare" the fish.

6.43. Imagine a plane that is parallel to the surface of the earth. The sound that an explosion generates and that propagates at a certain angle α to the normal to this plane will be deflected still greater. As Snell's law shows, this happens when the speed of sound increases with altitude. Thus, the curve that represents the path along which the sound wave propagates suggests that the speed of sound increases continuously with altitude. Since the speed of propagation of waves in a gas is proportional to the square root of the temperature, then, hence, the behavior of the curve of sound propagation (see the figure accompanying the problem) can be explained by the fact that the air temperature increases with altitude.

6.44. Both longitudinal and transverse waves can travel in the earth. The first are partially reflected by water and partially transmitted through water, while the second are completely reflected by water. The reflection of the longitudinal and transverse waves can be used to estimate the upper boundary of the water pocket. The longitudinal waves will be partially reflected by the bottom of the pocket. Thus, to measure the depth of the pocket one can use only longitudinal waves.

6.45. For the observer to hear the sound of the airplane from a distance a earlier than the sound arrives from point A that is directly above the observer, the time it takes the sound to travel from airplane to observer must be shorter than the time it takes the airplane to fly the distance a plus the time it takes the sound to travel from point A to the observer. The first time is

$$t_1 = \sqrt{a^2 + h^2}/c,$$

while the second is

$$t_2 = a/v + h/c,$$

where c is the speed of sound. The above-stated condition can be written thus:

$$\frac{\sqrt{a^2+h^2}}{c} < \frac{a}{v} + \frac{h}{c}.$$

If we square both sides of this inequality and carry out the necessary manipulations, we get

$$a\left(\frac{v^2}{c^2} - 1\right) < \frac{2v}{c}\, h. \qquad (6.45.1)$$

The ratio $v/c = M$ is known as the Mach number. Then (6.45.1) can be written thus:

$$a < 2\frac{M}{M^2-1}h.$$

If, say, the airplane is flying with a speed double the speed of sound, the maximal distance from which the sound will arrive sooner than when the airplane appears overhead is equal to $(4/3)h$.