

Gravitation

Quick Revision

1. **Universal Law of Gravitation** It states that, every body in this universe attracts every other body with a force whose magnitude is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

$$\text{Gravitational force, } F = G \frac{m_1 m_2}{r^2}$$

where, G is a constant of proportionality and is known as **universal gravitational constant**.

In CGS system, the value of G is $6.67 \times 10^{-8} \text{ dyne cm}^2 \text{g}^{-2}$ and its SI value is $6.67 \times 10^{-11} \text{ N-m}^2 \text{kg}^{-2}$.

Dimensional formula for G is $[\text{M}^{-1} \text{L}^3 \text{T}^{-2}]$.

2. **Vector Form of Newton's Law of Gravitation** In vector notation, Newton's law of gravitation is written as follows

$$\mathbf{F}_{12} = -G \frac{m_1 m_2}{r_{21}^2} \hat{\mathbf{r}}_{21} \quad \dots(i)$$

where, \mathbf{F}_{12} = gravitational force exerted on A by B and $\hat{\mathbf{r}}_{21}$ is a unit vector pointing towards A . Negative sign shows that the gravitational force is attractive in nature.

$$\text{Similarly, } \mathbf{F}_{21} = -G \frac{m_1 m_2}{r_{12}^2} \hat{\mathbf{r}}_{12} \quad \dots(ii)$$

where, $\hat{\mathbf{r}}_{12}$ is a unit vector pointing towards B .

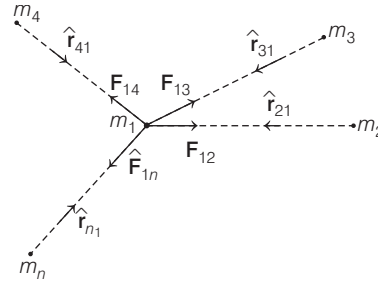
Equating Eqs. (i) and (ii), we have

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

As, \mathbf{F}_{12} and \mathbf{F}_{21} are directed towards the centres of the two particles, so gravitational force is a central force.

3. **Principle of Superposition** According to this principle, the resultant gravitational force \mathbf{F} can be expressed in vector addition of all forces, at a point (as shown below).

$$\text{i.e. } \mathbf{F} = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14} + \dots + \mathbf{F}_{1n}$$



Resultant force,

$$\mathbf{F} = -Gm_1 \left(\frac{m_2}{r_{21}^2} \hat{\mathbf{r}}_{21} + \frac{m_3}{r_{31}^2} \hat{\mathbf{r}}_{31} + \dots + \frac{m_n}{r_{n1}^2} \hat{\mathbf{r}}_{n1} \right)$$

4. **Acceleration due to gravity** The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity (g).

$$\text{At the surface of the earth, } g = \frac{GM}{R^2}$$

5. **Weight of a body** It is the gravitational force with which a body is attracted towards the centre of the earth $w = mg$

It is a vector quantity and its SI unit is newton (N).

6. **Factors Affecting Acceleration Due to Gravity**

- **Shape of Earth** Acceleration due to gravity,

$$g \propto \frac{1}{R^2}$$

Therefore, g is minimum at equator and maximum at poles.

- **Rotation of Earth about Its Own Axis** If ω is the angular velocity of rotation of earth about its own axis, then acceleration due to gravity at a place having latitude λ is given by

$$g' = g - R\omega^2 \cos^2 \lambda$$

At poles, $\lambda = 90^\circ$ and $g' = g$.

Therefore, there is no effect of rotation of earth about its own axis at poles.

At equator, $\lambda = 0^\circ$ and $g' = g - R\omega^2$

The value of g is minimum at equator.

If earth stops its rotation about its own axis, then g will remain unchanged at poles but increases by $R\omega^2$ at equator.

- **Effect of Altitude** The value of g at height h from earth's surface,

$$g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

Therefore, g decreases with altitude.

- **Effect of Depth** The value of g at depth h from earth's surface,

$$g' = g \left(1 - \frac{h}{R}\right)$$

Therefore, g decreases with depth from earth's surface.

The value of g becomes zero at earth's centre.

7. **Intensity of Gravitational Field at a**

Point The gravitational force acting per unit mass at any point in gravitational field is called intensity of gravitational field at that point.

Intensity of gravitational field at a distance r , from a body of mass M is given as

$$E = \frac{F}{m} = \frac{GM}{r^2}$$

It is a vector quantity and its direction is towards the centre of gravity.

8. **Gravitational Potential** Gravitational potential at a point in the gravitational field is defined as the amount of work done per unit mass in bringing a body of unit mass from infinity to that point without acceleration.

i.e.
$$V = - \frac{W}{m}$$

$$= - \int \frac{\mathbf{F} \cdot d\mathbf{r}}{m} = - \int \mathbf{E} \cdot d\mathbf{r} = \frac{-GM}{r}$$

It is a scalar quantity. The unit of gravitational potential in SI system is Jkg^{-1} and in CGS system is erg-g^{-1} .

Dimensional formula for gravitational potential is $[\text{M}^0\text{L}^2\text{T}^{-2}]$.

Special Cases

- When $r = \infty$, then $V = 0$, hence gravitational potential is maximum (zero) at infinity.
- At surface of the earth $r = R$, then

$$V = \frac{-GM}{R}$$

9. **Gravitational Potential Energy**

Gravitational potential energy of a body at a point is defined as the amount of work done in bringing the given body from infinity to that point against the gravitational force.

Gravitational potential energy,

$$U = \left(-\frac{GM}{r}\right) \times m$$

10. **Escape Speed** Escape speed on the earth (or any other planet) is defined as the minimum speed with which a body should be projected vertically upwards from the surface

of the earth, so that it just escapes out from gravitational field of the earth and never returns on its own.

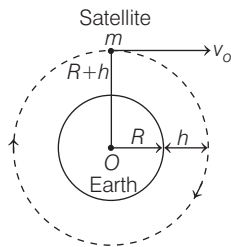
∴ Escape velocity, $v_e = \sqrt{2gR}$

where, R is the radius of the earth.

Also, escape velocity, $v_e = R\sqrt{\frac{8}{3}\pi G\rho}$

where, ρ is the mean density of the earth.

11. **Earth's Satellites** A satellite is a body which is constantly revolving in an orbit around a comparatively much larger body. e.g. The moon is a natural satellite while INSAT-1B is an artificial satellite of the earth. Condition for establishment of satellite is that the centre of orbit of satellite must coincide with centre of the earth or satellite must move around in greater circle of the earth.
12. **Orbital Velocity of a Satellite** Orbital velocity is the velocity required to put the satellite into its orbit around the earth or a planet.



Mathematically, it is given by $v_o = \sqrt{\frac{GM}{R+h}}$

13. **Energy of an Orbiting Satellite** When a satellite revolves around a planet in its orbit, it possesses both potential energy (due to its position against gravitational pull of the earth) and kinetic energy (due to orbital motion). If m is the mass of the satellite and v is its orbital velocity, then KE of the satellite,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{GM}{r} \quad (\because v = \sqrt{GM/r})$$

$$K = \frac{GMm}{2(R+h)} \quad (\because r = R+h)$$

PE of the satellite, $U = -mv^2 = -\frac{GMm}{R+h}$

Total mechanical energy of satellite,

$$E = K + U$$

$$E = -\frac{GMm}{2(R+h)}$$

Satellites are always at finite distance from the earth and hence their energies cannot be positive or zero.

14. **Geo-stationary Satellite** These satellites revolve in a circular orbits around the earth in the equatorial plane with period of revolution same as that of earth, i.e. $T = 24$ h and also known as geo-synchronous satellites.
 - It should revolve in an orbit concentric and coplanar with the equatorial plane of earth.
 - These satellites appears stationary due to its low relative velocity w.r.t. that place on earth.
 - It should be at a height around 36000 km.
 - These satellites are used for communication purpose like radio broadcast, TV broadcast, etc.
15. **Polar Satellite** They are low-altitude satellites ($h \approx 500$ to 800 km) which circle in a North-South orbit passing over the North and South poles. It is also known as sun synchronous satellite.
 - The time period is about 100 min.
 - These satellites are used for military purpose.
16. **Weightlessness** A body is said to be in a state of weightlessness when the relation of the supporting surface is zero or its apparent weight is zero. At one particular position, the two gravitational pulls may be equal & opposite and the net pull on the body becomes zero. This is zero gravity region or the null point where the body is said to be weightless. The state of weightlessness can be observed in the following situations
 - When objects fall freely under gravity.
 - When a satellite revolves in its orbit around the earth.
 - When bodies are at null points in outer space.

Objective Questions

Multiple Choice Questions

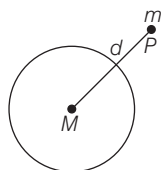
- The force of gravitation is
 - repulsive
 - electrostatic
 - conservative
 - non-conservative
- Newton's law of gravitation is universal because
 - it acts on all bodies in the universe
 - it acts on all the masses at all distances and not affected by the medium
 - it is a attractive force
 - it acts only when bodies are in contact
- Both the earth and the moon are subject to the gravitational force of the sun. As observed from the sun, the orbit of the moon (NCERT Exemplar)
 - will be elliptical
 - will not be strictly elliptical because the total gravitational force on it is not central
 - is not elliptical but will necessarily be a closed curve
 - deviates considerably from being elliptical due to influence of planets other than the earth
- Two sphere of masses m and M are situated in air and the gravitational force between them is F . The space around the masses is now filled with a liquid of specific gravity. The gravitational force will now be
 - F
 - $\frac{F}{3}$
 - $\frac{F}{9}$
 - $3F$
- A mass of 1g is separated from another mass of 1g by a distance of 1 cm . How many force (in g-wt) exists between them?
 - 7×10^{-11} g-wt
 - 7×10^{11} g-wt
 - 9×10^{-11} g-wt
 - 9×10^{11} g-wt
- If the distance between the sun and the earth is increased by three times, then attraction between two will
 - remain constant
 - decrease by 63%
 - increase by 63%
 - decrease by 89%
- If the gravitation force on body 1 due to 2 is given by \mathbf{F}_{12} and on body 2 due to 1 is given as \mathbf{F}_{21} , then
 - $\mathbf{F}_{12} = \mathbf{F}_{21}$
 - $\mathbf{F}_{12} = -\mathbf{F}_{21}$
 - $\mathbf{F}_{12} = \frac{\mathbf{F}_{21}}{4}$
 - None of the above
- Two equal point masses are separated by a distance d_1 . The force of gravitation acting between them is F_1 . If the separation is decreased to d_2 , then the new force of gravitation F_2 is given by
 - $F_2 = F_1$
 - $F_2 = F_1 \left(\frac{d_1}{d_2} \right)^2$
 - $F_2 = F_1 \left(\frac{d_2}{d_1} \right)^2$
 - $F_2 = F_1 \left(\frac{d_1}{d_2} \right)$
- Particles of masses $2M$, m and M are respectively at points A , B and C with $AB = \frac{1}{2}(BC)$, m is much-much smaller than M and at time $t = 0$, they are all at rest as given in figure. At subsequent times before any collision takes place. (NCERT Exemplar)

 - m will remain at rest
 - m will move towards M
 - m will move towards $2M$
 - m will have oscillatory motion

10. Two particles of equal masses go round a circle of radius R under the action of their mutual gravitational attraction. The speed v of each particle is

(a) $\sqrt{\left(\frac{GM}{2R}\right)}$ (b) $\frac{1}{2R}\sqrt{\left(\frac{1}{GM}\right)}$
 (c) $\frac{1}{2}\sqrt{\left(\frac{GM}{R}\right)}$ (d) $\sqrt{\left(\frac{4GM}{R}\right)}$

11. A point mass m is placed outside a hollow spherical shell of mass M and uniform density at a distance d from centre of the sphere as shown in figure. Gravitational force on point mass m at P is



(a) $\frac{GmM}{d^2}$ (b) zero
 (c) $\frac{2GmM}{d^2}$ (d) Data insufficient

12. Three equal masses of 2 kg each are placed at the vertices of an equilateral triangle and a mass of 4 kg is placed at the centroid of the triangle which is at a distance of $\sqrt{2}$ m from each of the vertices of the triangle. The force (in newton) acting on the mass of 4 kg is

(a) 2 (b) $\sqrt{2}$ (c) 1 (d) zero

13. During the free fall of an object,

- (a) acceleration due to gravity is zero
 (b) force on object is zero
 (c) force on object decreases with height
 (d) acceleration due to gravity is 9.8 m/s^2

14. What will happen to the weight of the body at the south-pole, if the earth stops rotating about its polar axis?

- (a) No change
 (b) Increases
 (c) Decreases but not become zero
 (d) Reduces to zero

15. If G is universal gravitational constant and g is acceleration due to gravity, then the unit of the quantity $\frac{G}{g}$ is

- (a) kg-m^2 (b) kgm^{-1}
 (c) kgm^{-2} (d) $\text{m}^2 \text{ kg}^{-1}$

16. The earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the earth, the acceleration due to gravity

(NCERT Exemplar)

- (a) will be directed towards the centre but not the same everywhere
 (b) will have the same value everywhere but not directed towards the centre
 (c) will be same everywhere in magnitude directed towards the centre
 (d) cannot be zero at any point

17. The height at which the weight of a body becomes 1/16th of its weight, on the surface of the earth (radius R), is

- (a) $5R$ (b) $15R$
 (c) $3R$ (d) $4R$

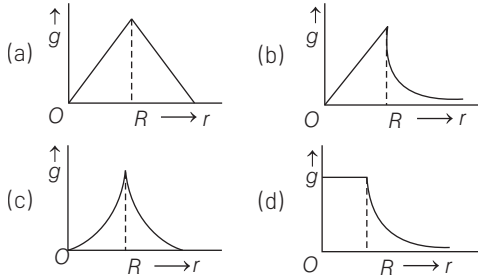
18. The radius of earth is R . Height of a point vertically above the earth's surface at which acceleration due to gravity becomes 1% of its value at the surface is

- (a) $8R$ (b) $9R$
 (c) $10R$ (d) $20R$

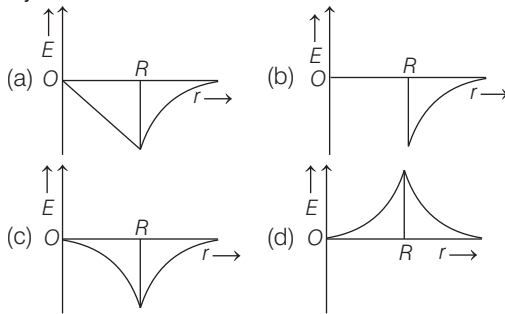
19. What will be the value of g at the bottom of sea 7 km deep? Diameter of the earth is 12800 km and g on the surface of the earth is 9.8 ms^{-2} .

- (a) 9.789 m/s^2 (b) 9.259 m/s^2
 (c) 97.89 m/s^2 (d) 0.987 m/s^2

20. Starting from the centre of the earth having radius R , the variation of g (acceleration due to gravity) is shown by which of the following option ?



21. Dependence of intensity of gravitational field (E) of earth with distance (r) from centre of earth is correctly represented by



22. A thin rod of length L is bent to form a semicircle. The mass of rod is M . What will be the gravitational potential at the centre of the circle?

(a) $-\frac{GM}{L}$ (b) $-\frac{GM}{2\pi L}$
(c) $-\frac{\pi GM}{2L}$ (d) $-\frac{\pi GM}{L}$

23. A particle is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done per unit mass against the gravitational force between them, to take the particle far away from the sphere (you may take $h = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$).

(a) $13.34 \times 10^{-10} \text{ J}$ (b) $3.33 \times 10^{-10} \text{ J}$
(c) $6.67 \times 10^{-9} \text{ J}$ (d) $6.67 \times 10^{-8} \text{ J}$

24. The gravitational potential energy of a system consisting two particles separated by a distance r is

- (a) directly proportional to product of the masses of particles
(b) inversely proportional to the separation between them
(c) independent of distance r
(d) Both (a) and (b)

25. The mass of the earth is $6.00 \times 10^{24} \text{ kg}$ and that of the moon is $7.40 \times 10^{22} \text{ kg}$.

The constant of gravitation $G = 6.67 \times 10^{-11} \text{ N-m}^2 \text{ kg}^{-2}$. The potential energy of the system is $-7.79 \times 10^{28} \text{ J}$. The mean distance between the earth and the moon is

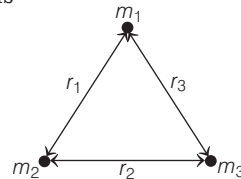
- (a) $3.80 \times 10^8 \text{ m}$ (b) $3.37 \times 10^8 \text{ m}$
(c) $7.60 \times 10^4 \text{ m}$ (d) $1.90 \times 10^2 \text{ m}$

26. Two point masses m_1 and m_2 are separated by a distance r . The gravitational potential energy of the system is G_1 . When the separation between the particles is doubled, the gravitational potential energy is G_2 .

Then, the ratio of $\frac{G_1}{G_2}$ is

- (a) 1 (b) 2 (c) 3 (d) 4

27. Gravitational potential energy of a system of particles as shown in the figure is



(a) $\frac{Gm_1m_2}{r_1} + \frac{Gm_2m_3}{r_2} + \frac{Gm_1m_3}{r_3}$
(b) $\left(\frac{-Gm_1m_2}{r_1} \right) + \left(\frac{-Gm_2m_3}{r_2} \right) + \left(\frac{-Gm_1m_3}{r_3} \right)$
(c) $\frac{-Gm_1m_2}{r_1} - \frac{Gm_2m_3}{r_2} + \frac{Gm_1m_3}{r_3}$
(d) $\frac{Gm_1m_2}{r_1} + \frac{Gm_2m_3}{r_2} - \frac{Gm_1m_3}{r_3}$

- 28.** Escape velocity of a body on the surface of earth is independent of
 (a) mass
 (b) radius of earth
 (c) direction of projection of body
 (d) Both (a) and (c)
- 29.** An object is thrown from the surface of the moon. The escape speed for the object is
 (a) $\sqrt{2g'R_m}$, where g' = acceleration due to gravity on the moon and R_m = radius of the moon
 (b) $\sqrt{2g'R_e}$, where g' = acceleration due to gravity on the moon and R_e = radius of the earth
 (c) $\sqrt{2gR_m}$, where g = acceleration due to gravity on the earth and R_m = radius of the moon
 (d) None of the above
- 30.** A body is projected vertically upwards from the surface of a planet of radius R with a velocity equal to $1/3$ rd of the escape velocity for the planet. The maximum height attained by the body is
 (a) $R/2$ (b) $R/3$
 (c) $R/5$ (d) $R/9$
- 31.** Two planets A and B have the same material density. If the radius of A is twice that of B , then the ratio of escape velocity $\frac{v_A}{v_B}$ is
 (a) 2 (b) $\sqrt{2}$
 (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$
- 32.** Escape velocity on earth is 11.2 kms^{-1} , what would be the escape velocity on a planet whose mass is 1000 times and radius is 10 times that of earth?
 (a) 112 kms^{-1} (b) 11.2 kms^{-1}
 (c) 1.12 kms^{-1} (d) 3.7 kms^{-1}
- 33.** The time period of a satellite in a circular orbit around a planet is independent of
 (a) the mass of the planet
 (b) the radius of the planet
 (c) the mass of the satellite
 (d) All the three parameters (a), (b) and (c)
- 34.** Satellites orbiting the earth have finite life and sometimes debris of satellites fall to the earth. This is because
 (NCERT Exemplar)
 (a) the solar cells and batteries in satellites run out
 (b) the laws of gravitation predict a trajectory spiralling inwards
 (c) of viscous forces causing the speed of satellite and hence height to gradually decrease
 (d) of collisions with other satellites
- 35.** An artificial satellite is revolving around the earth, close to its surface. Find the orbital velocity of artificial satellite? (Take, radius of earth = 6400 km)
 (a) 7.2 km/s (b) 7.9 km/s
 (c) 11.2 km/s (d) 9.5 km/s
- 36.** Two satellites A and B go around a planet P in circular orbits having radius $4R$ and R , respectively. If the speed of satellite A is $3v$, then the speed of satellite B will be
 (a) $6v$
 (b) $9v$
 (c) $3v$
 (d) None of the above
- 37.** An artificial satellite moving in a circular orbit around the earth has a total (kinetic + potential) energy E_0 . Its potential energy and kinetic energy respectively are
 (a) $2E_0$ and $-2E_0$ (b) $-2E_0$ and E_0
 (c) $2E_0$ and $-E_0$ (d) $-2E_0$ and $-E_0$

38. The kinetic energy of the satellite in a circular orbit with speed v is given as

$$\begin{aligned} \text{(a) } KE &= \frac{-GmM_e}{2(R_e + h)} & \text{(b) } KE &= \frac{GmM_e}{(R_e + h)} \\ \text{(c) } KE &= \frac{GmM_e}{2(R_e + h)} & \text{(d) } KE &= -\frac{1}{2}mv^2 \end{aligned}$$

39. The time period of geo-stationary satellite is

- (a) 6 h (b) 12 h (c) 24 h (d) 48 h

40. Geo-stationary satellites are placed in equatorial orbits at the height approximately

- (a) 1000 km (b) 15000 km
(c) 25000 km (d) 36000 km

41. An astronaut experiences weightlessness in a space satellite. It is because

- (a) the gravitational force is small at that location in space.
(b) the gravitational force is large at that location in space.
(c) the astronaut experiences no gravity
(d) the gravitational force is infinitely large at that location in space.

42. A pendulum beats sounds on the earth. Its time period on a stationary satellite of the earth will be

- (a) zero (b) 1s
(c) 2s (d) infinity

43. is defined to be numerically equal to the force of attraction between two bodies each of mass 1 kg and separated by a distance of 1 m.

- (a) Universal gravitational constant (G)
(b) Gravity (g)
(c) Force (F)
(d) Magnetic field (B)

44. Weight of a body is maximum at

- (a) poles (b) equator
(c) centre of earth (d) at latitude 45°

45. The ratio of the magnitude of potential energy and kinetic energy of a satellite is

- (a) 1:2 (b) 2:1
(c) 3:1 (d) 1:3

46. Weightlessness experienced while orbiting the earth in spaceship, is the result of

- (a) inertia
(b) acceleration
(c) zero gravity
(d) centre of gravity

47. Which of the following statement is incorrect? *(NCERT Exemplar)*

- (a) Acceleration due to gravity decreases with increasing altitude.
(b) Acceleration due to gravity increases with increasing depth (assume the earth to be a sphere of uniform density).
(c) Acceleration due to gravity increases with increasing altitude.
(d) None of the above

48. If the gravitational attraction of earth suddenly disappears, then which of the following statement is correct?

- (a) Both masses as well as the weight will be zero.
(b) Weight of the body will become zero but the mass will remain unchanged.
(c) Weight of the body will remain unchanged but the mass will become zero.
(d) Neither mass nor weight will be zero.

49. Study the following statements and choose the incorrect option.

I. G is not equal to $6.67 \times 10^{-11} \text{ Nm}^{-2} \text{ kg}^{-2}$ on the surface of earth.

II. The escape velocity on the surface of earth is lesser than the escape velocity from moon's surface.

III. The angular momentum of a satellite going around the earth remains conserved.

IV. The relation, $g = \frac{GM}{r^2}$ holds good for
all the celestial bodies.

- (a) Only I (b) Only III
(c) Both I and II (d) Both II and IV

50. Which of the following statement is correct? (NCERT Exemplar)

- (a) A polar satellite goes around the earth's pole in north-south direction.
(b) A geo-stationary satellite goes around the earth in east-west direction.
(c) A geo-stationary satellite goes around the earth in north-south direction.
(d) A polar satellite goes around the earth in east-west direction.

51. Match the Column I (quantities) with Column II (approximate values) and select the correct answer from the codes given below.

Column I	Column II
A. Escape velocity of earth	p. 1.6 m/s^2
B. Gravitational acceleration at moon's surface	q. 6400 km
C. Radius of earth	r. 11.2 km/s

Codes

- A B C A B C
(a) p q r (b) r q p
(c) q p r (d) r p q

52. A satellite of mass m revolving with a velocity v around the earth. With reference to the above situation, match the Column I (types of energy) with Column II (expression) and select the correct answer from the codes given below.

Column I	Column II
A. Kinetic energy of the satellite	p. $-\frac{1}{2}mv^2$
B. Potential energy of the satellite	q. $\frac{1}{2}mv^2$
C. Total energy of the satellite	r. $-mv^2$

Codes

- A B C
(a) p q r
(b) q r p
(c) r q p
(d) r p q

Assertion-Reasoning MCQs

For question numbers 53 to 66, two statements are given-one labelled **Assertion (A)** and the other labelled **Reason (R)**. Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below

- (a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

53. **Assertion** Newton's law of universal gravitation states that a particle attracts every other particle in the universe using a force of attraction that is directly proportional to the product of their masses and inversely proportional to the square of distance between them.

Reason Law of gravitation is analogous to magnetic force between the moving charges.

54. **Assertion** The value of acceleration due to gravity does not depend upon mass of the body on which force is applied.

Reason Acceleration due to gravity is a variable quantity.

55. **Assertion** As we go up the surface of the earth, we feel light weighed than on the surface of the earth.

Reason The acceleration due to gravity decreases on going up above the surface of the earth.

56. Assertion Work done by or against gravitational force in moving a body from one point to another is independent of the actual path followed between the two points.

Reason This is because gravitational forces are conservative in nature.

57. Assertion If gravitational potential at some point is positive, then the gravitational field strength at that point will also be zero.

Reason Except at infinity gravitational potential due to a system of point masses at some finite distance cannot be negative.

58. Assertion The force of attraction between a hollow spherical shell of uniform density and a point mass situated inside it, is zero.

Reason The value of G does not depend on the nature and size of the masses.

59. Assertion Moon has no atmosphere.

Reason The escape speed for the moon is much smaller.

60. Assertion The escape velocity for a planet is $v_e = \sqrt{2gR}$. If the radius of the planet is four times, the escape velocity becomes half (i.e. $v_e' = \frac{v_e}{2}$).

Reason In the relation for escape velocity, $v_e = \sqrt{2gR}$, the acceleration due to gravity g is inversely proportional to radius of the planet. Thus, $v_e \propto \frac{1}{\sqrt{R}}$.

61. Assertion The velocity of the satellite increases as its height above earth's surface increases and is minimum near the surface of the earth.

Reason The velocity of the satellite is directly proportional to square root of its height above earth's surface.

62. Assertion A satellite moves around the earth in a circular orbit under the action of gravity. A person in the satellite experience zero gravity field in the satellite.

Reason The contact force by the surface on the person is not zero.

63. Assertion The total energy of the satellite is always negative irrespective of the nature of its orbit, i.e. elliptical or circular and it cannot be positive or zero.

Reason If the total energy is negative the satellite would leave its orbit.

64. Assertion The geo-stationary satellite goes around the earth in west-east direction.

Reason Geo-stationary satellites orbits around the earth in the equatorial plane with $T = 24$ h same as that of the rotation of the earth around its axis.

65. Assertion In the satellite, everything inside it is in a state of free fall.

Reason Every part and parcel of the satellite has zero acceleration.

66. Assertion An object is weightless when it is in free fall and this phenomenon is called weightlessness.

Reason In free fall, there is upward force acting on the object.

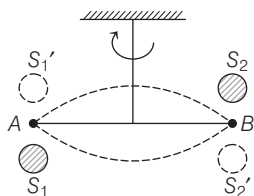
Case Based MCQs

Direction Answer the questions from 67-71 on the following case.

Cavendish's Experiment

The figure shows the schematic drawing of Cavendish's experiment to determine the value of the gravitational constant. The bar AB has two small lead spheres attached at its ends. The bar is suspended from a rigid support by a fine wire.

Two large lead spheres are brought close to the small ones but on opposite sides as shown. The value of G from this experiment came to be $6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$.



67. The big spheres attract the nearby small ones by a force which is

- (a) equal and opposite
- (b) equal but in same direction
- (c) unequal and opposite
- (d) None of the above

68. The net force on the bar is

- (a) non-zero
- (b) zero
- (c) Data insufficient
- (d) None of these

69. The net torque on the bar is

- (a) zero
- (b) non-zero
- (c) F times the length of the bar, where F is the force of attraction between a big sphere and its neighbouring
- (d) Both (b) and (c)

70. The torque produces twist in the suspended wire. The twisting stops when

- (a) restoring torque of the wire equals the gravitational torque
- (b) restoring torque of the wire exceeds the gravitational torque
- (c) the gravitational torque exceeds the restoring torque of the wire
- (d) None of the above

71. After Cavendish's experiment, there have been given suggestions that the value of the gravitational constant G becomes smaller when considered over very large time period (in billions of years) in the future. If that happens, for our earth,

- (a) nothing will change
- (b) we will become hotter after billions of years
- (c) we will be going around but not strictly in closed orbits
- (d) None of the above

Direction Answer the questions from 72-76 on the following case.

Acceleration due to gravity

The acceleration for any object moving under the sole influence of gravity is known as acceleration due to gravity. So, for an object of mass m , the acceleration experienced by it is usually denoted by the symbol g which is related to F by Newton's second law by relation $F = mg$. Thus,

$$g = \frac{F}{m} = \frac{GM_e}{R_e^2}$$

Acceleration g is readily measurable as R_e is a known quantity. The measurement of G by Cavendish's experiment (or otherwise), combined with knowledge of g and R_e enables one to estimate M_e from the above equation. This is the reason why there is a popular statement regarding Cavendish "Cavendish weighed the earth". The value of g decrease as we go upwards from the earth's surface or downwards, but it is maximum at its surface.

72. If g is the acceleration due to gravity at the surface of the earth, the force acting on the particle of mass m placed at the surface is

(a) mg (b) $\frac{GmM_e}{R_e^2}$
 (c) Data insufficient (d) Both (a) and (b)

73. The weight of a body at the centre of earth is

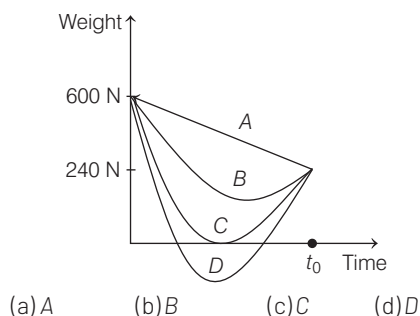
(a) same as on the surface of earth
 (b) same as on the poles
 (c) same as on the equator
 (d) None of the above

74. If the mass of the sun is ten times smaller and gravitational constant G is ten times larger in magnitude, then for earth,

(a) walking on ground would become more easy
 (b) acceleration due to gravity on the earth will not change
 (c) raindrops will fall much slower
 (d) airplanes will have to travel much faster

75. Suppose, the acceleration due to gravity at the earth's surface is 10 ms^{-2} and at the surface of mars, it is 4.0 ms^{-2} . A 60 kg passenger goes from the earth to the mars in a spaceship moving with a constant velocity. Neglect all other objects in the sky.

Which curve best represents the weight (net gravitational force) of the passenger as a function of time?



76. If the mass of the earth is doubled and its radius halved, then new acceleration due to the gravity g' is

(a) $g' = 4g$ (b) $g' = 8g$
 (c) $g' = g$ (d) $g' = 16g$

Direction Answer the questions from 77-81 on the following case.

Earth's Satellite

Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of planets around the Sun. In particular, their orbits around the earth are circular or elliptic. Moon is the only natural satellite of the earth with a near circular orbit with a time period of approximately 27.3 days which is also roughly equal to the rotational period of the moon about its own axis. Also, the speed that a satellite needs to be travelling to break free of a planet or moon's gravity well and leave it without further propulsion is known as escape velocity. For example, a spacecraft leaving the surface of earth needs to be going 7 miles per second or nearly 25000 miles per hour to leave without falling back to the surface or falling into orbit.

77. Gas escapes from the surface of a planet because it acquires an escape velocity. The escape velocity will depend on which of the following factors?

(a) Mass of the planet
 (b) Mass of the particle escaping
 (c) Temperature of the planet
 (d) None of the above

78. The escape velocity of a satellite from the earth is v_e . If the radius of earth contracts to $(1/4)$ th of its value, keeping the mass of the earth constant, escape velocity will be

(a) doubled
 (b) halved
 (c) tripled
 (d) unaltered

79. The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p), whose radius and mean density are twice as that of earth is

(a) $1:2\sqrt{2}$ (b) 1: 4
(c) $1:\sqrt{2}$ (d) 1: 2

80. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small as compared to the mass of the earth, then

(a) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant

(b) the total mechanical energy of S varies periodically with time
(c) the linear momentum of S remains constant in magnitude
(d) the acceleration of S is always directed towards the centre of the earth

81. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is v_o . The orbital velocity of a satellite orbiting at an altitude of half of the radius, is

(a) $\frac{3}{2}v_o$ (b) $\frac{2}{3}v_o$ (c) $\sqrt{\frac{3}{2}}v_o$ (d) $\sqrt{\frac{2}{3}}v_o$

ANSWERS

Multiple Choice Questions

1. (c) 2. (b) 3. (b) 4. (a) 5. (a) 6. (d) 7. (b) 8. (b) 9. (c) 10. (c)
11. (a) 12. (d) 13. (d) 14. (a) 15. (d) 16. (d) 17. (c) 18. (b) 19. (a) 20. (b)
21. (a) 22. (d) 23. (d) 24. (d) 25. (a) 26. (b) 27. (b) 28. (d) 29. (a) 30. (d)
31. (a) 32. (a) 33. (c) 34. (c) 35. (b) 36. (a) 37. (c) 38. (c) 39. (c) 40. (d)
41. (c) 42. (d) 43. (a) 44. (a) 45. (b) 46. (c) 47. (b) 48. (b) 49. (c) 50. (a)
51. (d) 52. (b)

Assertion-Reasoning MCQs

53. (c) 54. (a) 55. (a) 56. (a) 57. (d) 58. (b) 59. (a) 60. (a) 61. (d) 62. (c)
63. (c) 64. (a) 65. (c) 66. (c)

Case Based MCQs

67. (a) 68. (b) 69. (d) 70. (a) 71. (c) 72. (d) 73. (d) 74. (d) 75. (c) 76. (b)
77. (a) 78. (a) 79. (a) 80. (d) 81. (d)

SOLUTIONS

1. As the work done by the gravitational force F in closed path is zero. So, it is conservative in nature, i.e. work done by the body is independent of path followed.
2. According to universal law of gravitation, gravitational force is given by

$$F = \frac{Gm_1m_2}{r^2}$$

It depends on all the masses at all distances but does not depend on medium between them.

3. As observed from the sun, two types of forces are acting on the moon one is due to gravitational attraction between the sun and the moon and the other is due to gravitational attraction between the earth and the moon. Therefore, total force on the moon is not central.
Hence, the orbit of the moon will not be strictly elliptical.

4. Gravitational force does not depend on the medium between masses.

So, it will remain same, i.e. F .

5. Gravitational force is given by

$$\begin{aligned}
 F &= G \frac{m_1 m_2}{r^2} \\
 &= (6.67 \times 10^{-8}) \left(\frac{1 \times 1}{1^2} \right) \text{ dyne} \\
 &= 6.67 \times 10^{-8} \text{ dyne} = \frac{6.67 \times 10^{-8}}{980} \text{ g-wt} \\
 &\quad (\because 1 \text{ dyne} = \frac{1}{980} \text{ g-wt}) \\
 &= 7 \times 10^{-11} \text{ g-wt}
 \end{aligned}$$

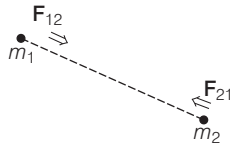
6. As we know, $F = \frac{Gm_1 m_2}{r^2}$

If distance increased by 3 times, then gravitational force is given by

$$F' = \frac{Gm_1 m_2}{(3r)^2} = \frac{F}{9}$$

$$\begin{aligned}
 \text{Per cent decrease in force} &= \left(\frac{F - F'}{F} \right) \times 100 \\
 &= \frac{8}{9} \times 100 = 88.88\% \approx 89\%
 \end{aligned}$$

7. Since, gravitational forces are attractive, \mathbf{F}_{12} is directed opposite to \mathbf{F}_{21} and they are also equal in magnitude.



$$\begin{aligned}
 \text{Hence,} \quad \mathbf{F}_{21} &= -\mathbf{F}_{12} \\
 \text{or} \quad \mathbf{F}_{12} &= -\mathbf{F}_{21}
 \end{aligned}$$

8. From Newton's law of gravitation,

$$\begin{aligned}
 &\text{Diagram: Two masses } m \text{ separated by distance } d_1. \\
 F_1 &= \frac{Gm \cdot m}{d_1^2} = \frac{Gm^2}{d_1^2} \quad \dots(i)
 \end{aligned}$$

Similarly,

$$\begin{aligned}
 &\text{Diagram: Two masses } m \text{ separated by distance } d_2. \\
 F_2 &= \frac{Gm^2}{d_2^2} \quad \dots(ii)
 \end{aligned}$$

From Eqs. (i) and (ii), we get

$$\frac{F_1}{F_2} = \left(\frac{d_2}{d_1} \right)^2$$

$$\therefore F_2 = F_1 \left(\frac{d_1}{d_2} \right)^2$$

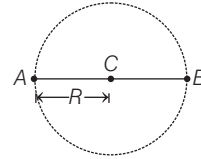
9. Force on B due to A , $F_{BA} = \frac{G(2Mm)}{(AB)^2}$ (towards BA)

$$\begin{aligned}
 \text{Force on } B \text{ due to } C, F_{BC} &= \frac{GMm}{(BC)^2} \quad (\text{towards } BC) \\
 \text{As, } BC &= 2AB \quad (\text{given})
 \end{aligned}$$

$$\Rightarrow F_{BC} = \frac{GMm}{(2AB)^2} = \frac{GMm}{4(AB)^2} < F_{BA}$$

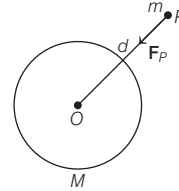
Hence, m will move towards BA , i.e. $2M$.

10. Two particles A and B each of mass m move in a circular path of radius R . Then, gravitational force between them provides the necessary centripetal force,



$$\text{i.e. } \frac{mv^2}{R} = \frac{GMm}{(2R)^2} \Rightarrow v = \frac{1}{2} \sqrt{\left(\frac{GM}{R} \right)}$$

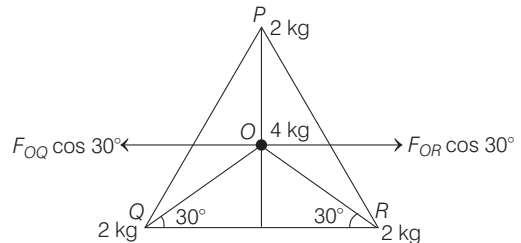
11. For a point outside the spherical shell as shown below



According to Newton's gravitational law, gravitational force on point mass m at P is

$$|\mathbf{F}_p| = \frac{GmM}{d^2}$$

12. Consider the equilateral triangle as PQR with centroid at O . Given, $OP = OQ = OR = \sqrt{2} \text{ m}$



The gravitational force on mass 4 kg due to mass 2 kg at point P is

$$F_{Op} = G \frac{4 \times 2}{(\sqrt{2})^2} = 4G, \text{ along } OP$$

Similarly, $F_{OQ} = G \frac{4 \times 2}{(\sqrt{2})^2} = 4G$, along OQ

and $F_{Or} = G \frac{4 \times 2}{(\sqrt{2})^2} = 4G$, along OR

$F_{OQ} \cos 30^\circ$ and $F_{Or} \cos 30^\circ$ are equal and acting in opposite directions, hence cancel out each other. Then, the resultant force on the mass 4 kg at point O , $F = 0$ (zero).

13. Force on the object is given by $F = \frac{Gm_1m_2}{r^2}$

It is not zero under free fall. As, the height decreases, force will increase on object. The acceleration due to gravity is constant and equal to 9.8 m/s^2 during the free fall.

14. As, weight of the body at pole is mg and g is not affected by the rotation of earth at poles. So, there is no change in weight of body.

15. As, we know, $g = \frac{GM}{R^2}$ or $\frac{G}{g} = \frac{R^2}{M}$

Hence, the unit of the quantity $\frac{G}{g} = \frac{\text{m}^2}{\text{kg}}$

16. If we assume the earth as a sphere of uniform density, then it can be treated as point mass placed at its centre. In this case, acceleration due to gravity $g = 0$, at the centre.

It is not so, if the earth is considered as a sphere of non-uniform density, in that case value of g will be different at different points and cannot be zero at any point.

17. According to the question,

$$\frac{GMm}{(R+h)^2} = \frac{1}{16} \frac{GMm}{R^2}$$

where, m = mass of the body

and $\frac{GM}{R^2}$ = gravitational acceleration.

$$\text{So, } \frac{1}{(R+h)^2} = \frac{1}{16R^2}$$

$$\text{or } \frac{R}{R+h} = \frac{1}{4} \text{ or } \frac{R+h}{R} = 4$$

$$h = 3R$$

18. Given, $g' = \left(\frac{1}{100}\right)g$ or $g'/g = \frac{1}{100}$

For height h above the surface of the earth,

$$g' = g \left(\frac{R}{R+h} \right)^2 \Rightarrow \frac{g'}{g} = \left(\frac{R}{R+h} \right)^2$$

$$\Rightarrow \left(\frac{1}{100} \right) = \left(\frac{R}{R+h} \right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{10}$$

$$\therefore h = 10R - R = 9R$$

19. Given, depth of sea, $d = 7 \text{ km}$ and $g = 9.8 \text{ ms}^{-2}$

Radius of the earth,

$$R = \frac{D}{2} = \frac{12800}{2} \text{ km} = 6400 \text{ km}$$

Value of g at bottom of sea, $g_d = g \left(1 - \frac{d}{R} \right)$

$$= 9.8 \left(1 - \frac{7}{6400} \right) = \frac{9.8 \times 6393}{6400}$$

$$= 9.789 \text{ m/s}^2$$

20. Acceleration due to gravity at a depth d below the surface of the earth is given by

$$g_{\text{depth}} = g_{\text{surface}} \left(1 - \frac{d}{R} \right)$$

$$= g_{\text{surface}} \left(\frac{R-d}{R} \right) = g_{\text{surface}} \left(\frac{r}{R} \right)$$

Also, for a point at height h above surface,

$$g_{\text{height}} = g_{\text{surface}} \left[\frac{R^2}{(R+h)^2} \right]$$

Therefore, we can say that value of g increases from centre to maximum at the surface and then decreases as depicted in graph (b).

21. Dependence of gravitational field (E) with distance is depicted properly in option (a) because at centre $r = 0$,

$$\therefore E = 0$$

For a point outside the earth ($r > R$),

$$E = -\frac{GM}{r^2} \Rightarrow E \propto \frac{1}{r^2}$$

and at the surface of earth ($r = R$),

$$E = -\frac{GM}{R^2}$$

Inside the earth ($r < R$),

$$E = -\frac{GMr}{R^3} \Rightarrow E \propto r$$

22. Since, length of rod is equal to the circumference of semicircle

$$\pi R = L \Rightarrow R = \frac{L}{\pi}$$

Therefore, the gravitational at the centre of circle will be

$$V = -\frac{GM}{R} = -\frac{\pi GM}{L}$$

23. Gravitational potential, $V_i = -\frac{GM}{r}$

$$V_i = -\frac{6.67 \times 10^{-11} \times 100}{0.1}$$

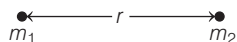
$$V_i = -\frac{6.67 \times 10^{-9}}{0.1} = -6.67 \times 10^{-8} \text{ J}$$

$$\therefore V_f = 0$$

\therefore Work done per unit mass,

$$W = \Delta V = (V_f - V_i) = 6.67 \times 10^{-8} \text{ J}$$

24. Two point masses m_1 and m_2 are separated by a distance r is shown as



Gravitational potential energy (U) of the above system is given as

$$U = -\frac{Gm_1m_2}{r}$$

i.e. $U \propto m_1m_2$ and $U \propto \frac{1}{r}$ or gravitational

potential energy is directly proportional to the product of the masses of particles and inversely proportional to the separation between them.

25. Given, $U = -7.79 \times 10^{28} \text{ J}$

$$G = 6.67 \times 10^{-11} \text{ N-m}^2\text{kg}^{-2}$$

$$m = 6 \times 10^{24} \text{ kg}$$

$$\text{and } M = 7.40 \times 10^{22} \text{ kg}$$

Potential energy of the system,

$$U = \frac{-GMm}{R}$$

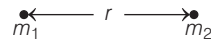
$$\Rightarrow -7.79 \times 10^{28}$$

$$= \frac{-6.67 \times 10^{-11} \times 7.4 \times 10^{22} \times 6 \times 10^{24}}{R}$$

$$\Rightarrow R = \frac{-6.67 \times 10^{-11} \times 7.4 \times 10^{22} \times 6 \times 10^{24}}{-7.79 \times 10^{28}}$$

$$\Rightarrow R = 3.8 \times 10^8 \text{ m}$$

26. As we know, $G_1 = \frac{-Gm_1m_2}{r}$... (i)



$$\text{and } G_2 = -\frac{Gm_1m_2}{2r} \quad \dots (ii)$$

$$\therefore \frac{G_1}{G_2} = 2 \text{ [dividing Eq. (i) by Eq. (ii)]}$$

27. For a system of particles, all possible pairs are taken and total gravitational potential energy is the algebraic sum of the potential energies due to each pair, applying the principle of superposition.

Total gravitational potential energy

$$\begin{aligned} &= U_{12} + U_{23} + U_{31} \\ &= \frac{-Gm_1m_2}{r_1} - \frac{Gm_2m_3}{r_2} - \frac{Gm_1m_3}{r_3} \\ &= \left(\frac{-Gm_1m_2}{r_1} \right) + \left(\frac{-Gm_2m_3}{r_2} \right) + \left(\frac{-Gm_1m_3}{r_3} \right) \end{aligned}$$

28. Escape velocity on the surface of earth is given by

$$v = \sqrt{2gR_e}$$

$$\text{i.e. } v \propto \sqrt{R_e}$$

Hence, escape velocity does not depend on the mass and direction of projection of body, it depends on the radius of earth.

29. Escape speed from the moon = $\sqrt{2g'R_m}$

where, g' = acceleration due to gravity on the surface of moon

and R_m = radius of the moon.

30. Let h be the maximum height attained. Then from equation of the motion $v^2 = u^2 + 2gh$

$$\text{When } u = 0, v = \sqrt{2gh}$$

$$\text{Given, } v = \frac{v_e}{3}, \text{ where } v_e = \sqrt{2gR}$$

$$\Rightarrow \sqrt{2gh} = \frac{1}{3} \sqrt{2gR}$$

On squaring both sides, we get

$$h = \frac{R}{9}$$

31. Escape velocity is given by

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R} \times \frac{4}{3} \pi R^3 \rho}$$

$$\Rightarrow v_e = R\sqrt{\frac{8}{3}\pi G\rho}$$

$$\Rightarrow v \propto R \quad [\because \rho \text{ is same for } A \text{ and } B]$$

$$\therefore \frac{v_A}{v_B} = \frac{R_A}{R_B} = 2 \quad [\because R_A = 2R_B, \text{ given}]$$

$$32. v_e = \sqrt{2gR} = \sqrt{2 \frac{GM}{R^2} \cdot R} \text{ or } v_e \propto \sqrt{\frac{M}{R}}$$

Mass is 1000 times and radius is 10 times.

$$\therefore v'_e = \sqrt{\frac{(1000M)G}{10R}}$$

$$\Rightarrow v'_e = 10\sqrt{\frac{GM}{R}} \Rightarrow v'_e = 10v_e$$

$$\Rightarrow v'_e = 10 \times 11.2 \Rightarrow v'_e = 112 \text{ kms}^{-1}$$

33. The time period of satellite in a circular orbit around a planet is independent of the mass of satellite.

34. As the total energy of the earth satellite bounded system is negative $\left(\text{i.e. } -\frac{GM}{2a}\right)$,

where a is radius of the satellite and M is mass of the earth.

Due to the viscous force acting on satellite, energy decreases continuously and radius of the orbit or height decreases gradually.

35. Here, $R_e = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$, $g = 9.8 \text{ m/s}^2$

Hence, orbital velocity of artificial satellite near the earth's surface,

$$v_o = \sqrt{gR_e} = \sqrt{9.8 \times 6.4 \times 10^6} \\ = 7.9 \times 10^3 \text{ m/s} = 7.9 \text{ km/s}$$

36. Velocity of satellite varies inversely as the square root of the orbit of radius R ,

$$v \propto \frac{1}{\sqrt{R}} \\ \therefore \frac{v_A}{v_B} = \sqrt{\frac{R_B}{R_A}} = \sqrt{\frac{R}{4R}} \\ \Rightarrow \frac{3v}{v_B} = \frac{1}{2} \Rightarrow v_B = 6v$$

37. \therefore Total energy, $E_0 = -\frac{GMm}{2r}$

$$\text{Potential energy, } U = -\frac{GMm}{r} = 2E_0$$

$$\text{Kinetic energy, } K = +\frac{GMm}{2r} = -E_0$$

$$38. \text{ KE of satellite} = \frac{1}{2}mv^2 = \frac{1}{2}m \left(\sqrt{\frac{GM_e}{(R_e + h)}} \right)^2 \\ = \frac{1}{2} \frac{GmM_e}{(R_e + h)}$$

39. Geo-stationary satellite has an orbital period equal to earth's rotational period of 23 h and 56 min, i.e. approx. 24 h.

40. A geo-stationary satellite is an earth's orbiting satellite. It is placed at an altitude of approximately 36000 km directly over the equator and it revolves in the same direction the earth rotates.

41. An astronaut experiences weightlessness in a space satellite. It is because the astronaut experiences no gravity.

42. Inside a satellite, every object experiences weightlessness. Therefore, time period of a pendulum inside a satellite is $T = 2\pi\sqrt{\frac{L}{g}}$

$$\text{As, } g = 0$$

$$\therefore T = \infty \text{ (infinity)}$$

43. According to universal law of gravitation,

$$F = \frac{Gm_1m_2}{r^2} \Rightarrow G = \frac{Fr^2}{m_1m_2}$$

Here, $m_1 = m_2 = 1 \text{ kg}$ and $r = 1 \text{ m}$

$$\therefore G = F$$

44. At poles, value of g is maximum. So, there is no effect of rotation of earth.

45. The ratio of magnitude of PE and KE of a satellite is 2 : 1.

$$\therefore \left| \frac{\text{PE}}{\text{KE}} \right| = \left| \frac{-mv^2}{\frac{1}{2}mv^2} \right| = \left| \frac{2}{1} \right| = 2:1$$

46. Weightlessness experienced while orbiting the earth in spaceship is the result of zero gravity because the surface does not exert any force on the body.

$$\text{By Newton's law, } \frac{GMm}{r^2} - R = ma$$

$$\Rightarrow \frac{GMm}{r^2} - R = m \left(\frac{GM}{r^2} \right) \Rightarrow R = 0$$

Since, reaction force by the surface of spaceship is zero, so body will experience weightlessness. This happens in zero gravity region and also from, $w = mg$, when $g = 0$, then $w = 0$.

47. Acceleration due to gravity at altitude h ,

$$g_h = \frac{g}{(1 + h/R)^2} \approx g \left(1 - \frac{2h}{R} \right)$$

At depth d , $g_d = g \left(1 - \frac{d}{R} \right)$

In both cases, with increase in h and d , g decreases.

At latitude ϕ , $g_\phi = g - \omega^2 R \cos^2 \phi$

As ϕ increases g_ϕ increases.

So, option (b) is incorrect.

48. As, $w = mg$ and $g = 0$.

So, weight of the body will become zero but the mass will remain unchanged.

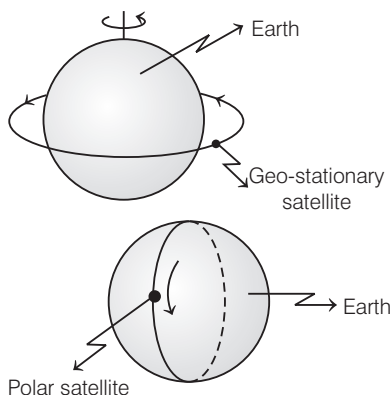
49. The statements I and II are incorrect and these can be corrected as,

As, $G = 6.67 \times 10^{-11} \text{ N-m}^2\text{kg}^{-2}$ on the surface of earth.

The escape velocity on the surface of earth is greater than the escape velocity from moon's surface because the moon has no atmosphere while earth has a very draws one.

50. A geo-stationary satellite is having same sense of rotation as that of earth, i.e. west-east direction.

A polar satellite goes around the earth's pole in north-south direction.



51. A. Escape velocity of earth is 11.2 km/s while for moon is 2.4 km/s.
B. Gravitational acceleration of earth is 9.8 ms^{-2} while for moon's surface is 1.6 m/s^2 .

C. Radius of moon is 1740 km while radius of earth is 6400 km.

Hence, $A \rightarrow r$, $B \rightarrow p$ and $C \rightarrow q$.

52. A. If the velocity of satellite is v and mass m , then

$$\text{KE} = \frac{1}{2} mv^2$$

B. Since, potential energy of the satellite
 $= -2$ kinetic energy of satellite
 $\Rightarrow \text{PE} = -mv^2$

C. Also, total energy = KE + PE

$$= \frac{1}{2} mv^2 - mv^2 = -\frac{1}{2} mv^2$$

Hence, $A \rightarrow q$, $B \rightarrow r$ and $C \rightarrow p$.

53. According to universal law of gravitation,

$$F = \frac{Gm_1m_2}{r^2}$$

This force of attraction between two bodies is directly proportional to products of their masses and inversely proportional to the square of distance between them.

Law of gravitation is not analogous to magnetic force between the moving charges.

Therefore, A is true but R is false.

54. Acceleration due to gravity is given by

$$g = \frac{GM}{R^2}$$

Thus, it doesn't depend on mass of the body on which it is acting. Also, it is a variable quantity, it changes with change in value of both M and R .

Therefore, both A and R are true and R is the correct explanation of A.

55. Variation of acceleration due to gravity at height is given by

$$g_h = g \left(1 - \frac{2h}{R} \right)$$

Since, acceleration due to gravity decreases above the surface of the earth and weight is directly proportional to the acceleration due to gravity, so as we go up, we feel light weighted than on the surface of the earth.

Therefore, both A and R are true and R is the correct explanation of A.

- 56.** As, work done by or against gravitational force in moving a body from one point to another is independent of the actual path followed because it is conservative force of nature which only depends on the initial and final positions.

Therefore, both A and R are true and R is the correct explanation of A.

- 57.** Gravitational potential due to a point mass at some finite distance is always negative.

Gravitational potential at infinity is zero.

Therefore, A is false and R is also false.

- 58.** The force of attraction due to a hollow spherical shell of uniform density on a point mass situated inside it, is zero because gravitational force possesses spherical symmetry.

Gravitational force is conservative in nature and the value of G does not depend on the nature and size of the masses.

Therefore, both A and R are true but R is not the correct explanation of A.

- 59.** The escape speed for the moon is much smaller and hence any gas molecule formed having thermal velocity larger than escape speed will escape from the gravitational pull of the moon.

So, moon has no atmosphere.

Therefore, both A and R are true and R is the correct explanation of A.

- 60.** Escape velocity, $v_e = \sqrt{2gR}$

$$\text{where, } g = \frac{GM}{R^2} \Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

$$\text{i.e. } v_e \propto \frac{1}{\sqrt{R}}$$

So, if radius is four times, i.e. $R' = 4R$

$$v'_e = \sqrt{\frac{2GM}{(4R)}} = \frac{1}{2} \sqrt{\frac{2GM}{R}} = \frac{v_e}{2}$$

Therefore, both A and R are true and R is the correct explanation of A.

- 61.** Orbital velocity of satellite, $v_o = \sqrt{\frac{GM_e}{(R_e + h)}}$

$$\Rightarrow v_o \propto \frac{1}{\sqrt{R_e + h}}$$

Thus, v_o is maximum near the surface of the earth for $h = 0$.

$$(v_o)_{\max} = \sqrt{\frac{GM_e}{R_e}}$$

Therefore, A is false and R is also false.

- 62.** The person experiences zero net force as the force of gravity is balanced by the centrifugal force inside the satellite. So, person experience no gravity.

The contact force by the surface on the person is zero.

Therefore, A is true but R is false.

- 63.** Total energy of a satellite is always negative irrespective of the nature of its orbit. It indicates that the satellite is bound to the earth. At infinity, the potential energy and kinetic energy of satellite is zero.

Hence, total energy at infinity is zero, therefore only negative energy of satellite is possible when it is revolved around the earth. If it is positive or zero, the satellite would leave its definite orbit and escape to infinity.

Therefore, A is true but R is false.

- 64.** The geo-stationary satellite goes around the earth in west-east direction.

It is because it orbits around earth in the equatorial plane with a time period of 24 h same as that of rotation of the earth around its axis.

Therefore, both A and R are true and R is the correct explanation of A.

- 65.** In a satellite around the earth, every part and parcel of the satellite has an acceleration towards the centre of the earth which is exactly the value of earth's acceleration due to gravity at that position.

Thus, in the satellite, everything inside it is in a state of free fall.

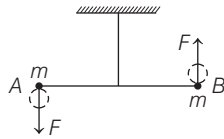
Therefore, A is true but R is false.

- 66.** An object is weightless when it is in free fall as during free fall, there is no upward force acting on the body and this phenomenon is called weightlessness.

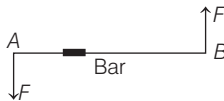
Therefore, A is true but R is false.

- 67.** The force of attraction on small spheres due to big sphere are equal and opposite in

direction. Hence, equal and opposite force separated by a fixed distance forms a couple.



68. $|\mathbf{F}_{\text{net}}| = \text{zero}$



Since, the force are equal and opposite, net force on the bar is zero.

69. Magnitude of torque due to a couple
 $= (\text{Either Force}) \times (\text{Distance between of forces})$
 $= F \times l$

where, l = length of the bar

and F = force of attraction between a big sphere and its neighbouring small sphere.

70. The torque produces a twist in the suspended wire. The twisting stops when the restoring torque of the wire equal the gravitational torque.

71. We know that, gravitational force between the earth and the sun.

$$F_G = \frac{GMm}{r^2}, \text{ where } M \text{ is mass of the sun and}$$

m is mass of the earth.

When G decreases with time, the gravitational force F_G will become weaker with time. As F_G is changing with time. Due to it, the earth will be going around the sun not strictly in closed orbit and radius also increases, since the attraction force is getting weaker. Hence, after long time the earth will leave the solar system.

72. The force acting on the particle of mass m at surface of the earth,

$$F = mg \quad \dots(i)$$

where, g = acceleration due to gravity at the earth's surface.

$$\text{Also, } g = \frac{GM_e}{R_e^2} \quad \dots(ii)$$

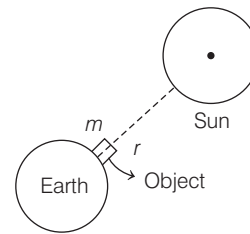
Then, from Eqs. (i) and (ii), we get

$$\Rightarrow F = mg = \frac{GmM_e}{R_e^2}$$

Hence, options (a) and (b) are correct.

73. Gravitational acceleration (g) at the centre of earth is zero, hence weight of body ($w = mg$) at the centre of earth becomes zero.

74. Consider the given diagram



Force on the object due to the earth,

$$F = \frac{G' M_e m}{R^2} = \frac{10 GM_e m}{R^2} \quad (\because G' = 10 G)$$

$$= 10 \left(\frac{GM_e m}{R^2} \right) = (10 g) m = 10 mg \quad \dots(i)$$

$$\left(\because g = \frac{GM_e}{R^2} \right)$$

Now, force on the object due to the sun ,

$$F' = \frac{GM_s m}{r^2}$$

$$= \frac{G (M_s) m}{10 r^2} \quad \left(\because M_s' = \frac{M_s}{10} \right)$$

As, $r \gg R$ (radius of the earth)

$\Rightarrow F'$ will be very small, so the effect of the sun will be neglected.

Now, as $g' = 10 g$

Hence, weight of person $= mg' = 10 mg$

[from Eq. (i)]

i.e. Gravity pull on the person will increase.

Due to it, walking on ground would become more difficult.

Escape velocity v_e is proportional to g , i.e.

$$v_e \propto g.$$

$$\text{As, } g' > g \Rightarrow v_e' > v_e$$

Hence, rain drops will fall much faster.

To overcome the increased gravitational force of the earth, the airplanes will have to travel much faster.

75. Initially, the weight of the passenger at the earth's surface, $w = mg = 60 \times 10 = 600 \text{ N}$.

Finally, the weight of the passenger at the surface of the mars $= 60 \times 4 = 240 \text{ N}$ and during the flight in between somewhere its weight will be zero because at that point, gravitational pull of earth and mars will be equal.

Only the curve (c) represents the weight $= 0$. So, (c) is correct option.

76. As we know that, acceleration due to gravity,

$$g = \frac{GM}{R^2}$$

Given, $M' = 2M$ (\because mass gets doubled)

$\Rightarrow R' = (R/2)$ (\because radius gets halved)

Then, acceleration becomes

$$\Rightarrow g' = \frac{GM'}{R'^2} = \frac{G(2M)}{(R/2)^2} = \frac{8GM}{R^2}$$

$\therefore g' = 8g$

Thus, the new acceleration due to gravity g' is 8 times that of g .

77. As we know that, escape velocity,

$$v_e = \sqrt{\frac{2GM}{R}} \quad \dots(i)$$

where, M is mass of planet.

So, on the basis of Eq. (i), it can be said that escape velocity will depend upon the mass of the planet (M).

78. Given, escape velocity on the surface of earth,

$$v_e = \sqrt{\frac{2GM_e}{R_e}}$$

where, M_e = mass of the earth

and R_e = radius of the earth.

Now, according to the question, radius of earth,

$$R' = R_e / 4$$

$$\begin{aligned} \Rightarrow v_e' &= \sqrt{\frac{2GM_e}{R'}} = \sqrt{4 \left(\frac{2GM_e}{R_e} \right)} \\ &= 2 \sqrt{\frac{2GM_e}{R_e}} \end{aligned}$$

or $v_e' = 2v_e$

Hence, the escape velocity will be doubled.

79. Since, the escape velocity of earth can be given as

$$v_e = \sqrt{2gR}$$

$$\Rightarrow v_e = R \sqrt{\frac{8}{3} \pi G \rho} \quad (\rho = \text{density of earth}) \dots(i)$$

As it is given that, the radius and mean density of planet are twice as that of earth.

So, escape velocity at planet will be

$$v_p = 2R \sqrt{\frac{8}{3} \pi G 2\rho} \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{v_e}{v_p} = \frac{R \sqrt{\frac{8}{3} \pi G \rho}}{2R \sqrt{\frac{8}{3} \pi G 2\rho}}$$

$$\Rightarrow \frac{v_e}{v_p} = \frac{1}{2\sqrt{2}} = 1 : 2\sqrt{2}$$

80. As, we know that, force on satellite is only gravitational force which will always be towards the centre of earth. Thus, the acceleration of S is always directed towards the centre of the earth.

81. Orbital velocity is given by $v_o = \sqrt{\frac{GM}{r}}$

where, $r = R + h$.

If $h = \frac{R}{2}$, then $r = R + \frac{R}{2} = \frac{3}{2}R$

Then orbital velocity of satellite orbiting at half altitude becomes,

$$\therefore v = \sqrt{\frac{GM \times 2}{3R}} = \sqrt{\frac{2}{3}} v_o$$