CBSE Class 10 Mathematics Standard

Sample Paper - 06 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. If the mean of the first n natural number is 15, then find n.

OR

State whether the following rational number will have a terminating decimal expansion or a nonterminating repeating decimal expansion. $\frac{64}{455}$

2. Find the values of p for which the quadratic equation $4x^2 + px + 3 = 0$ has equal roots.
3. For what value of a the following pair of linear equation has infinitely many solution?

$ax - 3y = 1$

$-12x + ay = 2$

4. Write the number of tangents to a circle which are parallel to a secant.
5. Does the sequence 11, 22, 33,.. form an AP? Justify your answer.

OR

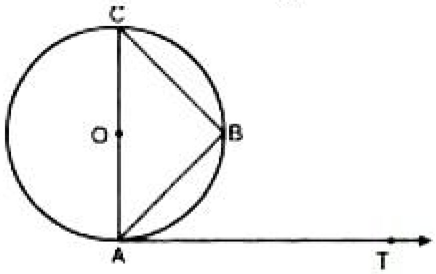
Find 10^{th} term of the A.P. 1, 4, 7, 10,...

6. Find the sum of first forty positive integers divisible by 6.
7. Find the roots of the quadratic equation $x^2 + 5x - (a + 1)(a + 6) = 0$, where a is a constant.

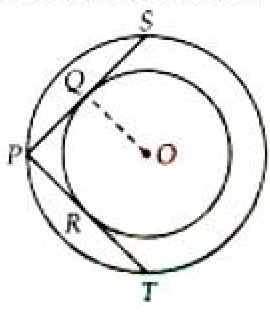
OR

Solve: $6x^2 + x - 12 = 0$

8. In the given figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^o$. If AT is the tangent to the circle at the point A, find $\angle BAT$



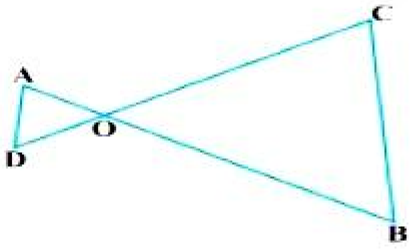
9. In figure, there are two concentric circles with centre O. PRT and PQS are tangents to the inner circle from a point P lying on the outer circle. If PR = 5 cm find the length of PS.



OR

What is the length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm?

10. In the given figure, OA × OB = OC × OD or $\frac{OA}{OC} = \frac{OD}{OB}$ prove that ∠ A = ∠ C and ∠ B = ∠ D



11. Find the 7^{th} term of the sequence whose n^{th} term is given by $a_n = (-1)^{n-1} \times n^3$

12. Evaluate ($cosec^2 45° \sec^2 30°$) ($\sin^2 30° + 4 \cot^2 45° - \sec^2 60°$)

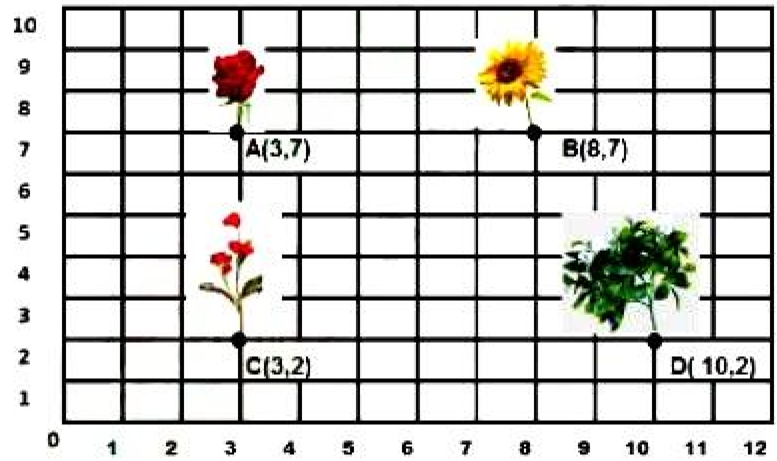
13. If $\cos \theta = \frac{2}{3}$, then find the value of $(4 + 4 \tan^2 \theta)$.

14. The height of a right circular cone is 12 cm and the radius of its base is 4.5 cm. Find its slant height.

15. Find the number of terms in the AP $17, 14\frac{1}{2}, 12, \ldots, -38$.

16. The king, queen and jack of clubs are removed from a deck of 52 playing cards and the well shuffled. One card is selected from the remaining cards. Find the probability of getting the 10 of hearts.

17. In the school garden Ajay(A), Brijesh(B), Chinki(C) and Deepak(D) planted their flower plants of Rose, Sunflower, Champa and Jasmine respectively as shown in the following figure. A fifth student Eshan wanted to plant her flower in this area. The teacher instructed Eshan to plant his flower plant at a point E such that CE: EB = 3 : 2.



Answer the following questions:

i. Find the coordinates of point E where Eshan has to plant his flower plant.

a. (5, 6)

b. (6, 5)

c. (5, 5)

d. (6, 7)

ii. Find the area of △ECD.

a. 9.5 square unit

b. 11.5 square unit

c. 10.5 square unit

d. 12.5 square unit

iii. Find the distance between the plants of Ajay and Deepak.

a. 8.60 unit

b. 6.60 unit

c. 5.60 unit

d. 7.60 unit

iv. The distance between A and B is:

a. 5.5 units

b. 7 units

c. 6 units

d. 5 units

v. The distance between C and D is:

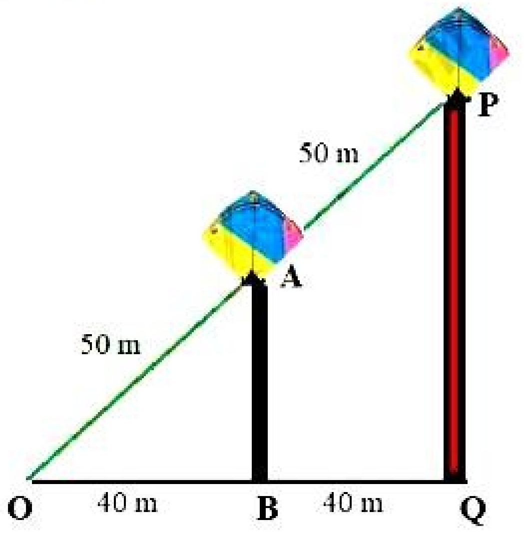
a. 5.5 units

b. 7 units

c. 6 units

d. 5 units

18.



As shown in the figure Harish is trying to measure the height of two towers AB and PQ. He is flying a kite He is having 100m thread with him, Harish found that when his half thread is open That time kite is just above the tower AB. Harish continues flying the kites, When his full thread is open that time kite reaches just above the tower PQ. Now answer the following questions:

i. What is the height of the tower AB?
   a. 40m
   b. 30 m
   c. 50 m
   d. 100m
ii. What is the height of the tower PQ?
   a. 40 m
   b. 30 m
   c. 60 m
   d. 100m
iii. What is the length of the hypotenuse in the triangle OAB?
   a. 40 m
   b. 50 m
   c. 100 m
   d. 80 m
iv. What is the length of the hypotenuse in the triangle OPQ?
   a. 40 m
   b. 50 m
   c. 100 m
   d. 80 m
v. What is the length of the Base in the triangle OPQ?
   a. 40 m
   b. 50 m
   c. 100 m
   d. 80 m

19. 

Thirty women were examined in a hospital by a doctor and the number of heartbeats per minute was recorded and summarised as follows:

Number of heartbeats per minute	65-68	68-71	71-74	74-77	77-80	80-83	83-86
Number of women	2	4	3	8	7	4	2

i. Find the mean heartbeats per minute for these women.

   a. 75.9
   b. 78.9
   c. 77.9
   d. 59.9

ii. Find the modal class of the given data.

   a. 74-77
   b. 77-80
   c. 65-68
   d. 68-71

iii. The abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives its:

   a. mean
   b. median
   c. mode
   d. all the three above

iv. The sum of the upper limit and lower limit of the median class is:

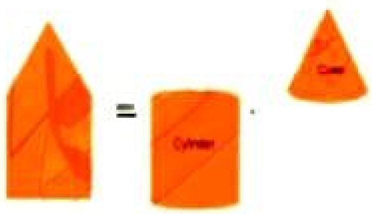
   a. 141
   b. 161

c. 151
d. 162

v. Formula for median is:

a. $M_d = L + \frac{\frac{N}{2} - cf}{f}$
b. $M_d = L + \frac{\frac{N}{2} - cf}{f} \times h$
c. $M_d = L + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$
d. $M_d = A + \frac{f_i u_i}{N} \times h$

20. Due to heavy floods in a state, thousands of people were homeless. 50 schools collectively offered to the state government to provide the place and the canvas for 1500 tent to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with the conical upper part of the same base radius but of height 2.1 m. [use $\pi = \frac{22}{7}$]



i. Area of canvas used to make the tent is

a. TSA of cylindrical portion + CSA of the conical portion
b. CSA of cylindrical portion + CSA of the conical portion
c. CSA of cylindrical portion + TSA of the conical portion

d. TSA of cylindrical portion + TSA of the conical portion

ii. The volume of the tent is

a. $\pi r^2(\frac{1}{3}r + h)$ cubic units

b. $\frac{1}{3}\pi r^2(r + h)$ cubic units

c. $\frac{4}{3}\pi r^2 h$ cubic units

d. none of these

iii. If the canvas used to make the tent cost ₹120 per sq.m, find the amount to be paid by the schools for making the tents.

a. ₹ 11098

b. ₹ 88889

c. ₹ 11088

d. ₹ 99998

iv. Amount shared by each school to set-up the tents.

a. ₹ 442640

b. ₹ 222640

c. ₹ 332640

d. ₹ 552640

v. According to the given information, what is the ratio of the curved surface area of the cylindrical portion to the conical portion:

a. 1:2

b. 2:3

c. 4:1

d. 2:1

Part-B

21. Without actual division, show that $\frac{19}{3125}$ is a terminating decimal number. Express this number in decimal form.

22. Find the coordinates of the point which divides the line segment joining (-1, 3) and (4, -7) internally in the ratio 3 : 4.

OR

Points P, Q, R and S divide the line segment joining the points A (1,2) and B (6, 7) in 5 equal parts. Find the coordinates of the points P, Q and R.

23. Find the zeroes of the quadratic polynomial $9t^2 - 6t + 1$ and verify the relationship between the zeroes and the coefficients.

24. Draw a pair of tangents to a circle of radius 6 cm which are inclined to each other at 60°. Also write steps of construction.

25. If 3 cot A = 4, find the value of $\frac{cosec^2 A+1}{cosec^2 A-1}$.

OR

Verify that, $4\left(\sin^4 30^\circ + \cos^4 60^\circ\right) - 3\left(\cos^2 45^\circ - \sin^2 90^\circ\right) = 2.$

26. Two tangents PA and PB are drawn from an external point P to a circle inclined to each other at an angle of 70°, then what is the value of $\angle$PAB?

27. Prove that $\sqrt{5} + \sqrt{3}$ is irrational.

28. Solve: $\frac{16}{x} - 1 = \frac{15}{x+1}; x \neq 0, -1$

OR

Solve the following quadratic equation by factorisation method .

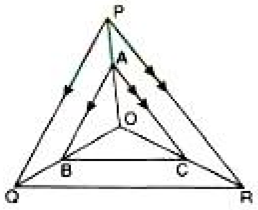
i. $x^2 - 14x + 24 = 0$
ii. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

29. Find the zeroes of the quadratic polynomial $x^2 - 2\sqrt{2}\,x$ and verify the relationship between the zeroes and the coefficients.

30. The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus.

OR

In the given figure A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Prove that BC || QR.



31. A box contains 19 balls bearing numbers 1, 2 3, ... 19. A ball is drawn at random from the

box. Find the probability that the number on the ball is

i. a prime number
ii. divisible, by 3 or 5
iii. neither divisible by 5 nor 10
iv. an even number.

32. Two poles of equal heights are standing opposite to each other on either side of a road, which is 80 m wide. From a point between them on the road, angles of elevation of their top are 30^{0} and 60^{0}. Find the height of the poles and distance of point from poles.
33. If the median of the distribution given below is 28.5, find the value of x and y.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
No. of students:	5	x	20	15	y	5

34. The difference between the sides at right angles in a right-angled triangle is 14 cm. The area of the triangle is 120 cm^2. Calculate the perimeter of the triangle.
35. Solve for x and y: bx + ay = a + b;
$ax\left(\frac{1}{a-b}-\frac{1}{a+b}\right)+by\left(\frac{1}{b-a}-\frac{1}{b+a}\right)=\frac{2a}{a+b}$
36. On the same side of a tower, two objects are located. When observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 150 m, find the distance between the objects.

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Solution

Part-A

1. The first n natural numbers are 1, 2, 3, ..., n

   Given that mean of n natural numbers is 15

   $\Rightarrow \frac{1+2+3+\ldots+n}{n}$ = 15 ...(1)

   We know that sum of first n natural numbers is $\frac{n(n+1)}{2}$

   $\Rightarrow$ 1 + 2 + 3 + ... + n= $\frac{n(n+1)}{2}$ ...(2)

   Substituting (2) in (1) we get

   $\Rightarrow \frac{n(n+1)}{2n}$ = 15

   $\Rightarrow$ n + 1 = 30 $\Rightarrow$ n = 30 - 1 = 29

OR

   The given number can be expressed as

   $\frac{64}{455} = \frac{64}{5\times7\times13}$,

   Here, $q = 5 \times 7 \times 13$

   Which is not of the form, $2^n \times 5^m$

   So, the rational number $\frac{64}{455}$

   has a non-terminating repeating decimal expansion.

2. $4x^2 + px + 3 = 0$

   $a = 4, b = p$ and $c = 3$

   As the equation has equal roots

   $\therefore D = 0$

   $D = b^2 - 4ac = 0$

   or, $p^2 - 4 \times 4 \times 3 = 0$

   or, $p^2 - 48 = 0$

   or, $p^2 = 48$

   or, $p = \pm 4\sqrt{3}$

3. For infinitely many solutions,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{a}{-12} = \frac{-3}{a} = \frac{1}{2}$

$\Rightarrow \frac{a}{-12} = \frac{-3}{a}$ and $\frac{-3}{a} = \frac{1}{2}$

$\Rightarrow a^2 = 36 \; and \; a = -6$

$\Rightarrow a = \pm 6 \; and \; a = -6$

For $a = -6$, pair of given linear equations has infinitely many solutions.

4. A tangent is a line that intersects a circle at only one point on its circumference. On the other hand, a secant is a line that cuts through a circle such that it touches at two points of the circumference.

   Therefore, a circle can have a maximum o**f two tangents parallel to a secant.**

5. We have $a_1 = 11$, $a_2 = 22$ and $a_3 = 33$

   $a_2 - a_1 = 11$

   $a_3 - a_2 = 11$

   Clearly, the difference of successive terms is same, therefore given list of numbers form an AP.

OR

First term(a) $= 1$

Common difference(d) $= 4 - 1 = 3$

We have,

n^{th} term $(a_n) = a + (n - 1)d$

Then, 10^{th} term$(a_{10}) = 1 + (10 - 1)3$

$= 1 + 9 \times 3$

$= 1 + 27$

$= 28$

6. We know that, first forty positive integers divisible by 6 are as follows:

   6,12,18,24,....,240. We also know that,

   $n = 40$, $a = 6$ and $l = 240$

   $S_n = \frac{n}{2}[a + l]$

   Therefore, $S_{40} = \frac{40}{2}[6 + 240]$

   $= 20 \times 246$

   $= 4920.$

7. The given quadratic Equation is:

$x^2 + 5x - (a + 1)(a + 6) = 0$

By middle term splitting,

$x^2 + [(a + 6) - (a + 1)]x - (a + 1)(a + 6) = 0$

$x^2 + (a + 6)x - (a + 1)x - (a + 1)(a + 6) = 0$

$x[x + a + 6] - (a + 1) [x + (a + 6)] = 0$

$[x - (a + 1)] [x + (a + 6)] = 0$

$\therefore x_1 = (a + 1)$ and $x_2 = -(a + 6)$

Hence, the two roots are (a + 1) and -(a + 6)

OR

$6x^2 + x - 12 = 0$
$\Rightarrow 6x^2 + 9x - 8x - 12 = 0$
$\Rightarrow 3x(2x + 3) - 4(2x + 3) = 0$
$\Rightarrow (3x - 4)(2x + 3) = 0$
$\Rightarrow 3x - 4 = 0$ or $2x + 3 = 0$
$\Rightarrow x = \frac{4}{3}$ or $x = \frac{-3}{2}$

8. $\because \angle ACB = 50^\circ$
$\angle CBA = 90^\circ$ (Angle in a semi-circle)
$\therefore \angle OAB = 90^\circ - 50^\circ$
$= 40^\circ$
$\angle BAT = 90^\circ - \angle OAB$
$= 90^\circ - 40^\circ$
$= 50^\circ$

9. PQ = PR = 5 cm (Length of Tangents from the same external point are always equal) and PQ = QS (perpendicular from the centre of the circle to the chord bisects the chord)

$\therefore$ PS = 2PQ$= 2 \times 5 = 10\text{cm}$

OR

We have to find the length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm.

Length of the tangent = $\sqrt{d^2 - r^2}$

$= \sqrt{(8)^2 - (6)^2}$
$= \sqrt{64 - 36}$
$= \sqrt{28} = 2\sqrt{7}\text{cm}$

10. In $\triangle$AOD and $\triangle$BOC,

    OA $\times$ OB = OC $\times$ OD

    i.e $\frac{OA}{OC} = \frac{OD}{OB}$

    And $\angle$ AOD = $\angle$ BOC [Vertically opposite Angles]

    $\therefore$ $\triangle$AOD $\sim$ $\triangle$BOC [By SAS]

    $\therefore$ $\angle$A = $\angle$C and $\angle$B = $\angle$D [Corresponding angles of similar $\triangle$]

11. we have to find the 7^{th} term

    i.e. n = 7

    given general term $a_n = (-1)^{n-1} \times (n)^3$

    $a_7 = (-1)^{7-1} \times (7)^3$

    $a_7 = 343$

12. We know that, cosec45° = $\sqrt{2}$, sec30° = ($\frac{2}{\sqrt{3}}$), sin30° = ($\frac{1}{2}$), cot45° = 1 & sec60° = 2, putting these values in the given expression, we get:-

    $(cosec^2 45^\circ \sec^2 30^\circ)\left(\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ\right)$
    $= \left[(\sqrt{2})^2 \times \left(\frac{2}{\sqrt{3}}\right)^2\right]\left[(\frac{1}{2})^2 + 4 \times (1)^2 - (2)^2\right]$
    $= \left[2 \times \frac{4}{3}\right]\left[\frac{1}{4} + 4 - 4\right]$
    $= \frac{8}{3} \times \frac{1}{4}$
    $= \frac{2}{3}$

13. cosθ = $\frac{2}{3}$

    $\therefore$cos$^2\theta$ = $\frac{4}{9}$.

    Now, 4 + 4tan$^2\theta$ = 4(1+tan$^2\theta$) = 4 sec$^2\theta$ = 4(1/cos$^2\theta$)

    = 4 $\times$ $\frac{1}{4/9}$

    = 9

14. h = 12 cm, r = 4.5 cm

    Slant height $l = \sqrt{r^2 + h^2} = \sqrt{(4.5)^2 + 12^2}$
    $= \sqrt{20.25 + 144} = \sqrt{164.25}$
    = 12.816 (approx)

15. Here a = 17, d= $14\frac{1}{2}$ - 17 = $-\frac{5}{2}$

Let the number of terms in AP = n

$\therefore a_n = -38$

$\Rightarrow a + (n-1)d = -38$

$\Rightarrow 17 + (n-1)(-\frac{5}{2})$ = -38

$\Rightarrow$ (n - 1) $(-\frac{5}{2})$ = -55

$\Rightarrow$ (n - 1) = -55$\times \left(-\frac{2}{5}\right)$ = 22

$\therefore$ n = 23

16. After removing king, queen and jack of clubs from a deck of 52 playing cards there are 49 cards left in the deck. Out of these 49 cards one card can be chosen in 49 ways.

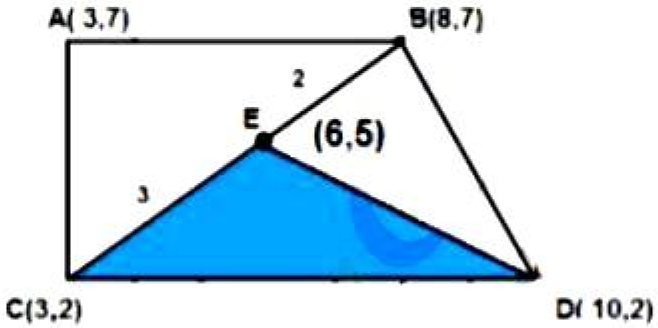
$\therefore$ Total number of elementary events = 49

There is only one '10' of hearts.

$\therefore$ Favourable number of elementary events = 1

Hence, P (Getting the '10' to hearts) = $\frac{1}{49}$

17. i. (b) Let us redraw the figure as follows:



Now, Coordinates of C and B are C(3, 2) and B(8, 7).

E divides CB internally in the ratio 3 : 2.

Therefore, the coordinates of E, by applying the section formula, are

$\left(\frac{3\times8+2\times3}{3+2}, \frac{3\times7+2\times2}{3+2}\right)$ = (6, 5)

ii. (c) Area of the $\triangle$ECD $= \frac{1}{2}[6(2-2)+3(2-5)+10(5-2)]$

$= \frac{1}{2}[3\times -3 + 10\times 3]$ = 10.5 square unit

iii. (a) The coordinates of the flower plants of Ajay and Deepak are (3, 7) and (10, 2) respectively.

Therefore, the distance between plants of Ajay and Deepak

$= \sqrt{(10-3)^2 + (7-2)^2}$

$= \sqrt{74}$ = 8.60 unit

iv. (d) 5 units

v. (b) 7 units

18. i. (a) 30 m

ii. (c) 60 m

iii. (b) 50 m

iv. (d) 100 m

v. (d) 80 m

19. i. (a) 75.9

ii. (a) 74-77

iii. (b) Median

iv. (c) 151

v. (b) $M_d = L + \frac{\frac{N}{2} - cf}{f} \times h$

20. i. (b) CSA of cylindrical portion + CSA of the conical portion

ii. (a) $\pi r^2(\frac{1}{3}r + h)$ cubic units

iii. (c) ₹ 11088

iv. (c) ₹ 332640

v. (d) 2:1

Part-B

21. The number = $\frac{19}{3125}$

Here the denominator = $3125 = 5^5$

This can be written as $2^0 \times 5^5$

Clearly, $2^0 \times 5^5$ is in the form of $2^m \times 5^n$.

So, the given number is a terminating decimal.

Now, $\frac{19}{2^0 \times 5^5} = \frac{19 \times 2^5}{2^5 \times 5^5} = \frac{19 \times 32}{100000}$

$= \frac{608}{100000} = 0.00608$

22. Let AB be a line and P(x,y) divide in m:n ratio, we have to find P(x,y).

Then by section formula,

$P(x, y) = (\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n})$,

Here, x_1 = -1, y_1 = 3, x_2 = 4, y_2 = -7, m =3, n=4

p(x,y) =($\frac{3 \times 4 + 4 \times (-1)}{3+4}$, $\frac{3 \times (-7) + 4 \times (3)}{3+4}$)

p(x,y) =($\frac{12-4}{7}$, $\frac{-21+12}{7}$)

p(x,y) = ($\frac{8}{7}$, $\frac{-9}{7}$)

$\Longrightarrow x = \frac{8}{7}$ and $y = \frac{-9}{7}$

$\therefore$ The coordinates of P are $\left(\frac{8}{7}, \frac{-9}{7}\right)$

OR

The difference between the x-coordinates of A and B is 6 - 1 =5

Similarly, the difference between the y-coordinates of A and B is 7 - 2 = 5

Hence, if the line segment joining A(1, 2) and B(6, 7) is divided into 5 equal parts by the points P, Q, R and S, then the coordinates of P, Q, R and S can be found out by increasing the x and the y coordinates of A by 1 successively.

Hence, the coordinates of P are (1 + 1, 2 + 1) = (2, 3)

The coordinates of Q are (2 + 1, 3 + 1) = (3 , 4)

The coordinates of R are (3 + 1, 4 + 1) = (4, 5)

23. The given polynomial is

$p(t) = 9t^2 - 6t + 1$

For zeroes of p(t)

$9t^2 - 6t + 1 = 0$

$\Rightarrow 9t^2 - 3t - 3t + 1 = 0$

$\Rightarrow 3t(3t - 1) - 1(3t - 1) = 0$

$\Rightarrow (3t - 1)(3t - 1) = 0$

$\Rightarrow t = \frac{1}{3}, \frac{1}{3}$

$\therefore$ zeroes are $\frac{1}{3}, \frac{1}{3}$

Now, a = 9, b = -6 , c = 1

$\frac{-b}{a} = \frac{-(-6)}{9} = \frac{2}{3}$ ...... (1)

Also, sum of the zeroes = $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ .... (2)

From (1) and (2)

Sum of zeroes = $\frac{-b}{a}$

Also, $\frac{c}{a} = \frac{1}{9}$ .... (3)

and product of the zeroes = $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ .... (4)

From (3) and (4)

Product of zeroes = $\frac{c}{a}$

24. Steps of construction:

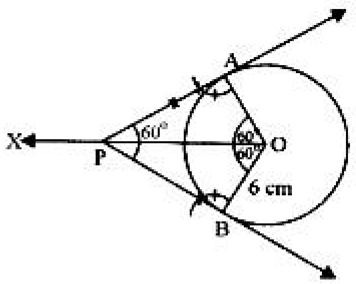
i. Draw a circle with centre O and radius = 6 cm.

ii. Take a ray OP.

iii. Construct $\angle POA$ and $\angle POB$ = 60° such that $\angle AOB = 120^\circ$.

iv. Construct $\angle OAP = \angle OBP = 90^\circ$ with radii OA and OB respectively such that their arms AP and BP intersect ray OP at P, then $\angle APB = 60^\circ$

Such that tangents PA and PB are inclined at 60° with each other.



25. Given,

3 cot A = 4

$\Rightarrow$ cot A = $\frac{4}{3}$

We know that,

cosec^{2}A - cot^{2}A = 1

cosec^{2}A - $\left(\frac{4}{3}\right)^2$ = 1

cosec^{2}A = 1 + $\frac{16}{9} = \frac{9+16}{9} = \frac{25}{9}$

Thus, $\frac{cosec^2A+1}{cosec^2A-1} = \frac{25/9+1}{25/9-1} = \frac{34}{16}$

OR

We have,

LHS = $4\left(\sin^4 30^\circ + \cos^4 60^\circ\right) - 3\left(\cos^2 45^\circ - \sin^2 90^\circ\right)$

$= 4\left\{\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right\} - 3\left\{\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right\}$

$= 4\left(\frac{1}{16} + \frac{1}{16}\right) - 3\left(\frac{1}{2} - 1\right)$

$= 4 \times \frac{2}{16} - 3\left(-\frac{1}{2}\right)$

$= \frac{1}{2} + \frac{3}{2} = 2 = \text{RHS}.$

Hence verified.

26. $\angle OPA = \frac{1}{2}\angle APB$

$= \frac{1}{2} \times 70^\circ = 35^\circ$

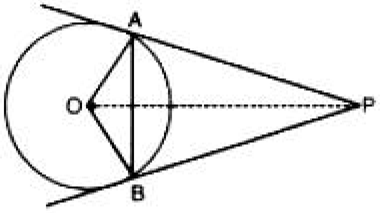
$\angle OAP = 90^\circ$ [radius $\perp$ tangent]

$\therefore \quad \angle AOP = 180^\circ - (90^\circ + 35^\circ) = 55^\circ$

$\therefore \angle OAB = 90^\circ - 55^\circ = 35^\circ$

$\angle PAB = 90^\circ - 35^\circ$

$= 55^\circ$



27. Let assume that $\sqrt{5} + \sqrt{3}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $\sqrt{5} = \frac{p}{q} - \sqrt{3}$

$(\sqrt{5})^2 = \left(\frac{p}{q} - \sqrt{3}\right)^2$

$5 = \frac{p2}{q2} - \frac{2p\sqrt{3}}{q} + 3$

$5 - 3 = \frac{p^2}{q^2} - \frac{2p\sqrt{3}}{q}$

$\frac{p^2}{q^2} - 2 = \frac{2p\sqrt{3}}{q}$

$\frac{p2-2q2}{qp} = \sqrt{3}$

$\frac{p2-2q2}{qp}$ is a rational number as p and q are integers. This contradicts the fact that √3 is irrational, so our assumption is incorrect. Therefore $\sqrt{5} + \sqrt{3}$ is irrational.

28. The given equation is: $\frac{16}{x} - 1 = \frac{15}{x+1}$

Or $\frac{16}{x} - \frac{15}{x+1} = 1$

$16(x + 1) - 15x = x^2 + x$

$16x + 16 - 15x = x^2 + x$

$x + 16 = x^2 + x$

$x^2 - 16 = 0$

$(x + 4)(x - 4) = 0$

$x + 4 = 0, x - 4 = 0$

$x = -4, x = +4$

OR

i. We have, $x^2 - 14x + 24 = 0$

Now factorise the equation,

$\Rightarrow x^2 - (12 + 2)x + 24 = 0$

$\Rightarrow x^2 - 12x - 2x + 24 = 0$

$\Rightarrow x(x - 12) - 2(x - 12) = 0$

$\Rightarrow (x - 2)(x - 12) = 0$

$\Rightarrow (x - 2) = 0$ or $(x - 12) = 0$

$\Rightarrow x = 2$ or $x = 12$

Hence, roots of the equation $x^2 - 14x + 24 = 0$ are 2 and 12.

ii. The given quadratic equation is $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Factorise the equation,

$\Rightarrow 4\sqrt{3}x^2 + (8 - 3)x - 2\sqrt{3} = 0$

$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

$\Rightarrow (4x - \sqrt{3})(\sqrt{3}x + 2) = 0$

$\Rightarrow (4x - \sqrt{3}) = 0$ or $(\sqrt{3}x + 2) = 0$

$\Rightarrow (4x) = \sqrt{3}$ or $(\sqrt{3}x) = -2$

$\Rightarrow x = \frac{\sqrt{3}}{4}$ or $x = \frac{-2}{\sqrt{3}}$

Hence, roots of the equation $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$ are $\frac{\sqrt{3}}{4}$ and $\frac{-2}{\sqrt{3}}$

29. $p(x) = x^2 - 2\sqrt{2}x = 0$

Now, $p(x) = 0$

$\Rightarrow x(x - 2\sqrt{2}) = 0$

$\Rightarrow$Zeroes are 0 and $2\sqrt{2}$

Sum of zeroes = $2\sqrt{2} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

and Product of zeroes = 0 = $\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

30. Let ABCD is a rhombus with AC and BD as its diagonals.

We know that the diagonals of a rhombus bisect each other at right angles.

Let O be the intersecting point of both the diagonals.

Let AC = 30 cm and BD = 40 cm

OA = AC/2

OA = 30/2

= 15 cm

OB = BD/12

OB = 40/2

= 20 cm

In right $\triangle$AOB by pythagoras theorem we have

$AB^2 = OA^2 + OB^2$

$= (15)^2 + (20)^2$

= 225 + 400

= 625

AB = 25 cm

Hence, each side of the rhombus is length 25 cm.

OR

Proof : In $\triangle POQ, AB||PQ$,(Given)

$\frac{AO}{AP} = \frac{OB}{BQ}$.......(i) (BPT)

In $\triangle$OPR $AC \| PR$

$\frac{OA}{AP} = \frac{OC}{CR} \ldots\ldots (ii)$

From eqn (I) and (ii)

$\frac{OB}{BQ} = \frac{OC}{CR}$

Hence BC | | Q R (By converse of BPT)

31. Total number of balls = 19 (bearing numbers 1, 2, 3, .,. . . ., 19).

∴ Number of all possible outcomes = 19

$Proabibilty\ of\ the\ event = \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes}$

i. Let E be the event that the number on the ball is a prime number.

   Then, the outcomes favourable to E are 2, 3, 5, 7, 11, 13, 17.,19

   Therefore, the number of outcomes favourable to E is 8.

   So, P(E) = P (a prime number) = $\frac{8}{19}$

ii. Let E be the event that the number on the ball is divisible by 3 or 5.

   Then, the outcomes favourable to E are 3, 5, 6, 9, 10, 12, 15, 18.

   Therefore, the number of outcomes favourable to E is 8.

   So, P(E) = P (divisible by 3 or 5) = $\frac{8}{19}$

iii. Let E be the event that the number on the ball is neither divisible by 5 nor 10.

Then, the outcomes favourable to E are 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19.

Therefore, the number of outcomes favourable to E is 16.

So, P(E) = P (neither divisible by 5 nor by 10)= $\frac{16}{19}$

iv. Let E be the event that the number on the ball is an even number.

Then, the outcomes favourable to E are 2, 4, 6, 8, 10, 12, 14, 16, 18.

Therefore, the number of outcomes favourable to E is 9.

So, P(E) = P (an even number) = $\frac{9}{19}$

32. Let the distance between pole AB and man be x

∴ Distance between pole CD and man = 80 - x

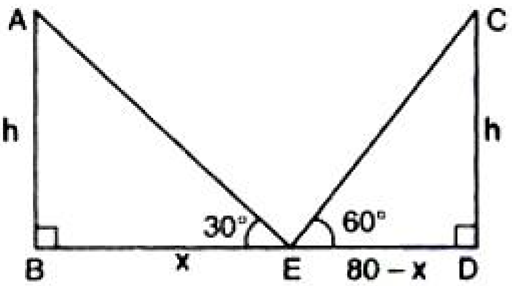
In △ABE, $\angle AEB = 30^\circ$

$\tan 30^\circ = \frac{h}{x}$ (using Pythagoras theorem)

$\sqrt{3} = \frac{h}{x}$

$h = \frac{x}{\sqrt{3}}$ ....(I)

In △CDE, $\angle CED = 60^\circ$



$\tan 60^\circ = \frac{h}{80-x}$

$\Rightarrow \quad \sqrt{3} = \frac{h}{80-x}$

$\Rightarrow \quad h = 80\sqrt{3} - x\sqrt{3}$ ....(ii)

Comparing (i) and (ii) we get

$\Rightarrow \quad \frac{x}{\sqrt{3}} = 80\sqrt{3} - x\sqrt{3}$

$\Rightarrow \quad x = 80 \times \sqrt{3} \times \sqrt{3} - x\sqrt{3} \times \sqrt{3}$

$\Rightarrow$ 4x = 240

$= \frac{240}{4} = 60\text{m}$

Substituting this value of x in (i)

$h = \frac{60}{\sqrt{3}} = 20\sqrt{3}$

Hence, height of the pole = 60m

Distance between pole CD and man = 80 - x

= 80 - 60 = 20m

33.

Class	Frequency (f)	Cumulative frequency (cf)
0-10	5	5
10-20	x	5 + x
20-30	20	25 + x
30-40	15	40 + x
40-50	y	40 + x + y
50-60	5	45 + x + y
	$N = \Sigma f = 60$	

N = 60

45 + x + y = 60

⇒ x + y =15 ... (i)

Median = 28.5

Clearly, it lies in the class interval 20 - 30.

So, 20 - 30 is the median class.

$\therefore l = 20, h = 10, f = 20, F = 5 + x \; and \; N = 60$

we know that,

$$Median = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow \quad 28.5 = 20 + \frac{30-(5+x)}{20} \times 10$$

$$\Rightarrow \quad 28.5 = 20 + \frac{25-x}{2}$$

$$\Rightarrow \quad 8.5 = \frac{25-x}{2} \Rightarrow 25 - x = 17 \Rightarrow x = 8$$

Putting x = 8 in Eq(i), we get

8 + y = 15

y = 7

Hence, x = 8 and y = 7

34. Let the sides containing the right angle be x cm and (x -14) cm.

Then, its area = $\left[\frac{1}{2} \times x \times (x - 14)\right]$ cm^2.

But according to question, it is given that area of a triangle is 120 cm^2

$\therefore \quad \frac{1}{2}x(x-14)=120 \Rightarrow x^2-14x-240=0$

$\Rightarrow \quad x^2-24x+10x-240=0 \Rightarrow x(x-24)+10(x-24)=0$

$\Rightarrow x=24, -10$

$\therefore$ one side = 24 cm, and other side = (24 -14) cm = 10 cm.

hypotenuse $= \sqrt{(24)^2+(10)^2}\text{cm}$

$= \sqrt{576+100}\text{cm}$

$=\sqrt{676}\ cm = 26\ cm$

$\therefore$ perimeter of the triangle = (24 +10 + 26) cm = 60 cm.

35. $ax\left(\frac{1}{a-b}-\frac{1}{a+b}\right)+by\left(\frac{1}{b-a}-\frac{1}{b+a}\right)=\frac{2a}{a+b}$

$\Rightarrow \text{ax}\left(\frac{a+b-a+b}{(a-b)(a+b)}\right)+by\left(\frac{b+a-b+a}{(b-a)(b+a)}\right)=\frac{2a}{a+b}$

$\Rightarrow \frac{2abx}{a^2-b^2}-\frac{2aby}{a^2-b^2}=\frac{2a}{a+b}$

$\frac{2ab}{a^2-b^2}(x-y)=\frac{2a}{a+b}$

$x-y=\frac{2a}{a+b}\times\frac{a^2-b^2}{2ab}$

$\Rightarrow \quad x-y=\frac{a-b}{b}$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow \quad x=\frac{a-b}{b}+y$ ..(i)

and $bx+ay=a+b$ ..(ii)

Using eq. (i), we get

$b\left(\frac{a-b}{b}+y\right)+ay=a+b$

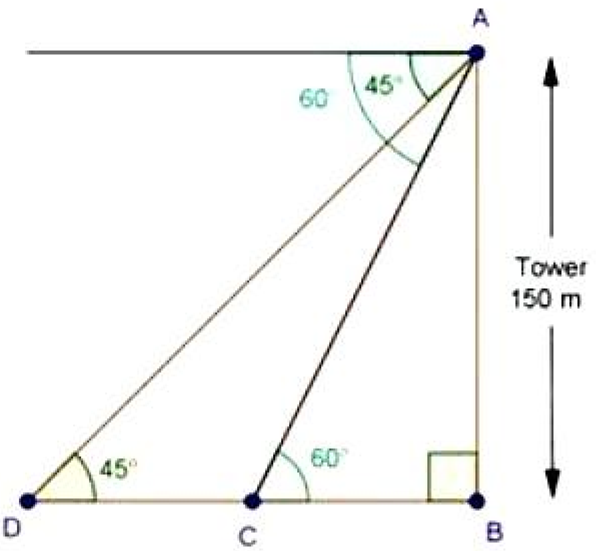
$\Rightarrow a-b+by+ay=a+b$

$\Rightarrow$ y = $\frac{2b}{a+b}$

When y = $\frac{2b}{a+b}$ in (i) becomes

x = $\frac{a-b}{b}+\frac{2b}{a+b}$

= $\frac{a^2-b^2+2b^2}{b(a+b)}=\frac{a^2+b^2}{b(a+b)}$

36. 

Let AB be the tower of height 150 m and two objects are located when the top of the tower are observed, makes an angle of depression from the top and bottom of the tower are 45^{o} and 60^{0}

In ΔABD

$\tan 45^{0} = \frac{AB}{BD}$

$\Rightarrow 1 = \frac{150}{BD}$

$\Rightarrow BD = 150m$

In ΔABC

$\tan 60^{o} = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{150}{BC}$

$\Rightarrow BC = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow BC = \frac{150\sqrt{3}}{3}$

$\Rightarrow BC = 50\sqrt{3} = 50 \times 1.732 = 86.6m$

$\therefore$ Distance between two objects = DC

$= BD - BC$

= 150 - 86.6

= 63.4 m