Chapter 2

Network Theorems

CHAPTER HIGHLIGHTS

- *Superposition Theorem*
- *Thevenin's Theorem*
- *Norton's Theorem*
- *Maximum Power Transfer Theorem*
- *Reciprocity Theorem*
- *Millman's Theorem*
- *Tellegen's Theorem*
- *Substitution Theorem*
- *Compensation Theorem*

Superposition Theorem

Statement

Whenever a linear bilateral circuit is excited by more than one independent source, the total response is the algebraic sum of individual responses due to all independent sources.

Steps to Apply Superposition Theorem

Step 1: Select a single source acting alone; short the other voltage sources and open the current sources (deactivate).

Step 2: Find the current through or the voltages across the required element, due to the sources under consideration.

Step 3: Repeat the above steps for all the independent sources

Step 4: Add all the individual effects produced by individual sources to obtain the total current through or voltage across the element.

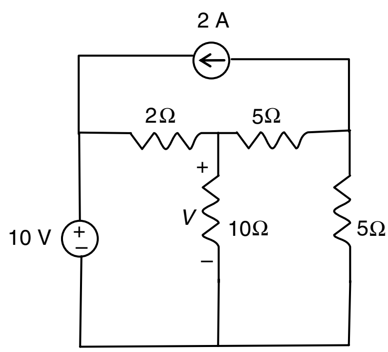
NOTES

1. Dependent sources are never deactivated.
2. When an independent voltage source is deactivated, it is set to zero and replaced by short circuit.
3. When an independent current source is deactivated, it is set to zero and replaced by open circuit.
   $\therefore I = 0, \Rightarrow$ open circuit.

Solved Examples

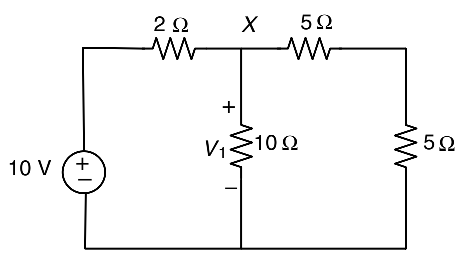
Example 1

Use superposition to find V.



Solution

Consider the independent voltage source acting alone



By applying KCL at node X

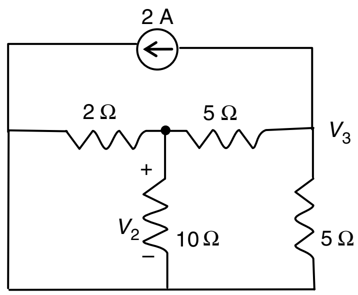
$$\frac{V_1-10}{2}+\frac{V_1}{10}+\frac{V_1}{10}=0$$

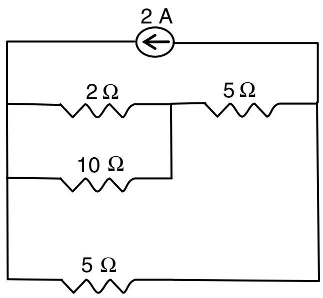
$$5\,(V_1-10)+2\,V_1=0$$

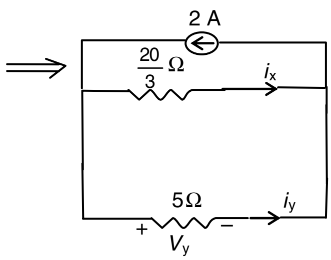
$$7V_1=50$$

$$V_1=\frac{50}{7}\text{ Volts}$$

2. Consider the independent current source acting alone:







$$i_x=\frac{2\times5}{\frac{20}{3}+5}=\frac{10\times3}{35}=\frac{30}{35}=\frac{6}{7}\text{A}$$

$$i_y=2-\frac{6}{7}=\frac{8}{7}\text{A}$$

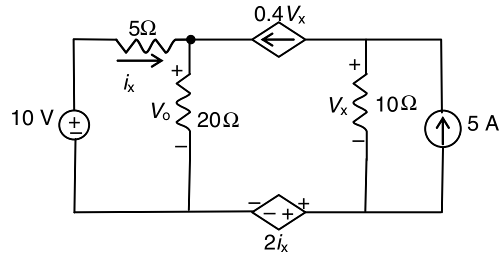
$$i_y=2-\frac{6}{7}=\frac{8}{7}\text{A}$$

$$V_y=\frac{5\times8}{7}=\frac{40}{7}\text{V}$$

$$V_2=V_{10\Omega}=\frac{40}{7}-5\times\frac{6}{7}=\frac{10}{7}\text{V}$$

$$V=V_1+V_2=\frac{50}{7}+\frac{10}{7}=\frac{60}{7}\text{V}$$

Example 2

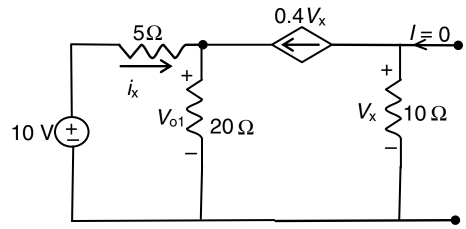


The value of V_o is

(A) −8 V (B) 16 V
(C) −16 V (D) 24 V

Solution

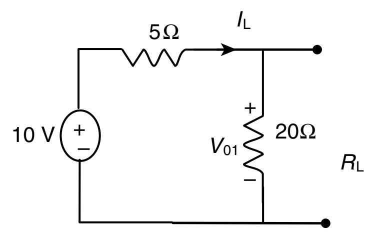
Activate independent voltage source only. Therefore, independent current source deactivated, that is, open circuit.



$$V_x=-\,10\times0.4\,V_x$$

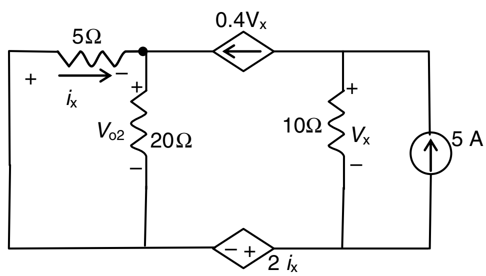
$$V_x=-\,4V_x\Rightarrow V_x=0$$

$\Rightarrow V_x=0$, and so dependent current 0.4 V_x equal to zero, so it acts like an open circuit.



$$V_{01}=10\times\frac{20}{25}=8\text{ V}$$

2. Activate dependent current source only



By applying KCL at node A,

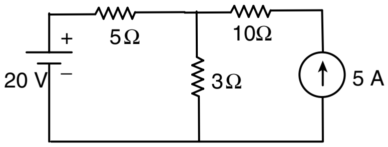
$$\frac{V_{02}}{20}+\frac{V_{02}}{5}0.4V_x=0$$

$$5\,V_{02}=8\,V_x \qquad (1)$$

At node B,

$$5 = \frac{V_x}{10} + 0.4V_x$$
$$50 = V_x + 4V_x$$
$$V_x = 10 \text{ V}$$
$$V_{02} = \frac{80}{5} = 16\text{V}$$
$$\therefore V_o = V_{01} + V_{02}$$
$$= 8 + 16 = 24 \text{ Volts}$$

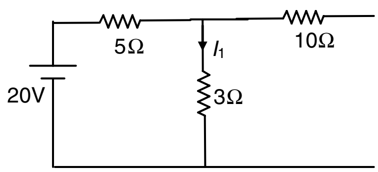
Example 3



Find the current through the 3 Ω resistor using Superposition Theorem.

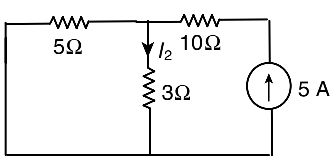
Solution

Considering 20 V source alone, that is, 5 A current source is open circuited.



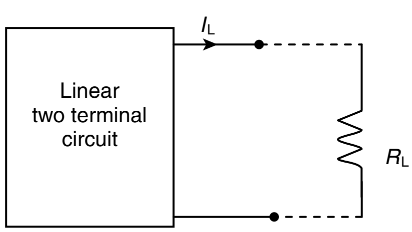
$$I_1 = \frac{20}{5+3} = \frac{20}{8} = 2.5 \text{ A}$$

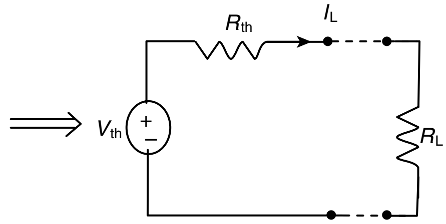
Considering 5 A source alone, that is, 20 V voltage source is short circuited.



$$I_2 = 5 \times \frac{5}{5+3}$$
$$I_2 = 3.125 \text{ A}$$
$$I = I_1 + I_2 = 2.5 + 3.125 = 5.625 \text{ A}$$

Thevenin's Theorem





Statement

Any two-terminal bilateral linear circuit can be replaced by an equivalent circuit consisting of a Thevenin's voltage source and Thevenin's series resistor. Thevenin's voltage source is the open-circuit voltage across the terminals and Thevenin's resistance is the equivalent resistance across the terminals.

NOTES

1. Circuit consisting only independent sources; V_{th} and R_{th} are calculated conventionally.
2. Circuit consisting both dependent and independent sources: V_{th} and I_{sc}

$$\Rightarrow R_{th} = \frac{V_{th}}{I_{sc}} \Omega$$

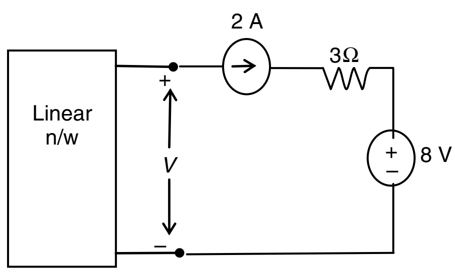
3. Circuit consisting only dependent sources.

$$R_{th} = \frac{V_{dc}}{I_{dc}} \Omega$$

$$V_{th} = 0 \text{ V and } I_{th} = 0 \text{ A}$$

Example 4

Consider the following circuit.



The linear network contains only DC sources and resistances. The value of V is

(A) 14 V (B) 2 V
(C) 8 V (D) indeterminate

Solution

From the given circuit

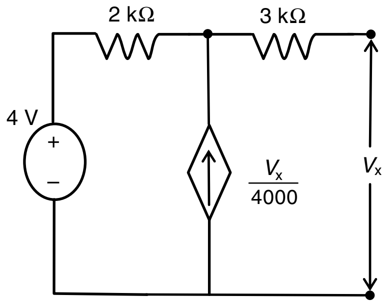
$$V = V_{2A} + 3 \times 2 + 8$$
$$V = V_{2A} + 14$$

Therefore, voltage across current source unknown.

$V = 14 +$ any value $\Rightarrow$ indeterminate

Example 5

Obtain the Thevenin's equivalent of the following network.



(A) $V_{th} = 0$ V, $R_{th} = 5$ kΩ
(B) $V_{th} = 2.4$ V, $R_{th} = 1.2$ kΩ
(C) $V_{th} = 8$ V, $R_{th} = 10$ kΩ
(D) None of the above

Solution

Under open-circuit condition

$$I_{3\Omega} = 0$$

∴ The node equation is given by

$$\frac{V_x - 4}{2\text{ k}\Omega} - \frac{V_x}{4000} = 0$$

$$\frac{V_x - 4}{2\text{ k}\Omega} - \frac{V_x}{4000}$$

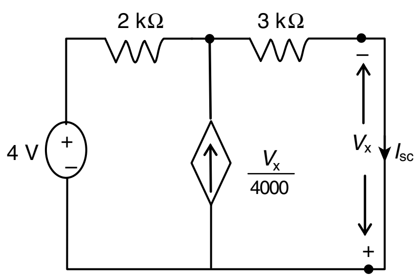
$$2\,(V_X - 4\,) = V_x$$

$$\Rightarrow V_X = V_{th} = 8 \text{ volts}$$

Therefore, we know

$$R_{th} = \frac{V_{th}}{I_{sc}} \quad \text{case (2)}$$

Compute the short-circuit current



Therefore, $V_x = 0$ from the abovementioned circuit

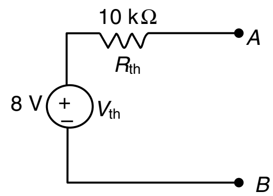
$$\text{dependent current source } \frac{V_x}{4000} = 0$$

⇒ open circuit

$$\text{i.e., } I_{sc} = \frac{4}{(2+3)}\text{mA} = 0.8 \text{ mA}$$

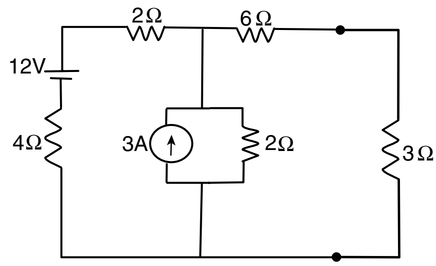
$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{8}{0.8}\text{k}\Omega = 10 \text{ k}\Omega$$

Thevenin's equivalent is



Example 6

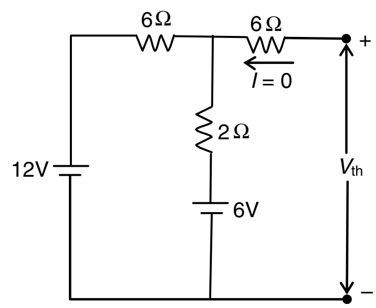
Find the current flowing through the 3 Ω resistor



(A) 1 A (B) 3 A (C) 5 A (D) 0.714 A

Solution

Apply source transformation to current source, it becomes



By applying nodal Analysis at node A

$$\frac{V_{th} - 12}{6} + \frac{V_{th} - 6}{2} = 0$$

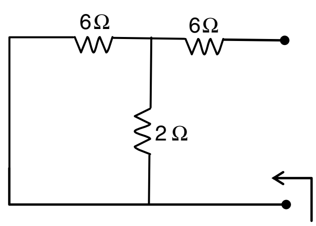
$$V_{th} - 12 + 3\,(V_{th} - 6) = 0$$

$$4V_{th} = 12 + 18$$

$$V_{th} = \frac{30}{4} = 7.5\,V$$

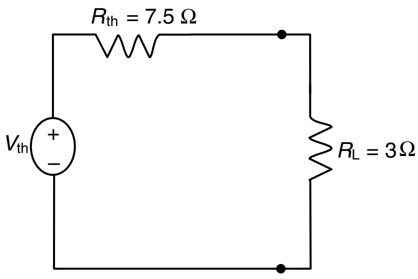
R_{th}:

1. All independent voltage sources are short circuited current sources are O.C
   Therefore, it becomes



$$R_{th} = 6 + 6||2$$
$$= 6 + 1.5$$
$$= 7.5\ \Omega$$

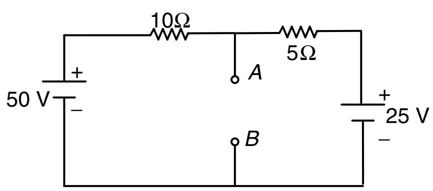
The following is the Thevenin's equivalent network.



$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{7.5}{7.5 + 3}$$

$$I_L = 0.71428\ \text{A}$$

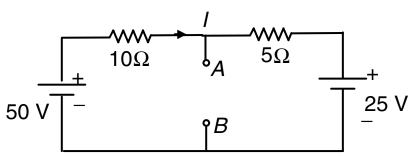
Example 7



Determine the Thevenin's equivalent circuit across *AB* for the abovementioned network, as shown in the figure.

Solution

To find V_{TH}.



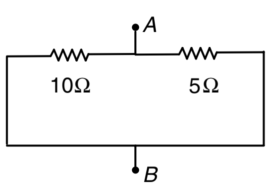
$$I = \frac{50 - 25}{10 + 5}$$

$$I = \frac{25}{15} = 1.67\ \text{A}$$

$$V_{AB} = 1.67 \times 5 + 25$$

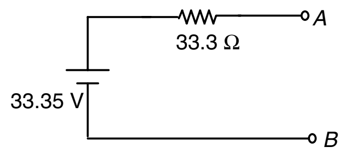
$$V_{TH} = 33.35\ \text{V}$$

To find R_{TH}, deactivate the voltage sources



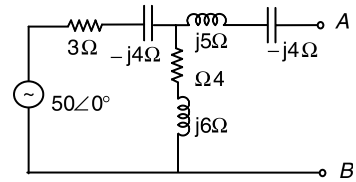
$$R_{TH} = 10 \,||\, 5 = 3.33\ \Omega$$

The Thevenin's equivalent circuit is shown in the figure.



Example 8

For the circuit shown in figure, determine the Thevenin's equivalent between the output terminals.



Solution

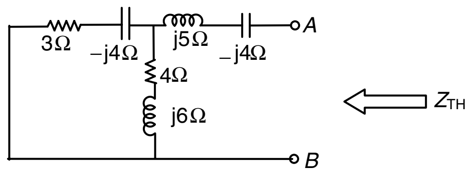
$$V_{TH} = 50 \angle 0° \times \frac{(4 + j6)}{(4 + j6) + (3 - j4)}$$

$$= 50 \angle 0° \times \frac{(4 + j6)}{7 + j2}$$

$$= 50 \angle 0° \times \frac{7.21\angle 56.3°}{7.28\angle 15.95°}$$

$$V_{TH} = 49.5 \angle 40.35°\ \text{V}$$

To find Z_{TH}, short circuit the source $50 \angle 0°$



$$Z_{TH} = (j5 - j4) + \frac{(3 - j4)(4 + j6)}{(3 - j4) + (4 + j6)}$$

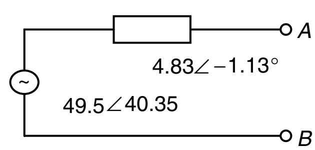
$$= j1 + \frac{5\angle 53.13° \times 7.21\angle 56.3°}{7.28\angle 15.95°}$$

$$= j1 + 4.95\angle -12.78°$$

$$= j1 + 4.83 - j1.095$$

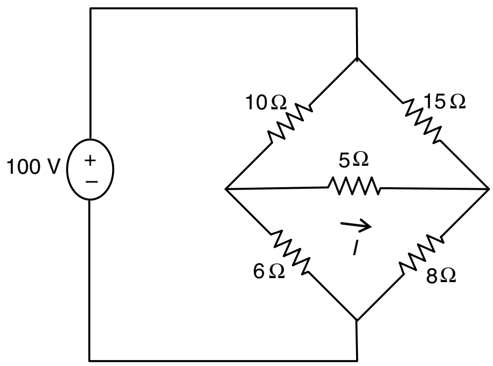
$$= 4.83 - j0.095$$

$$Z_{TH} = 4.83\angle -1.13°$$



Example 9

Find the current through the 5 Ω using Thevenin's theorem.



(A) $I = 0.194$ A (B) $I = 0.3$ A
(C) $I = 0$ A (D) None of the above

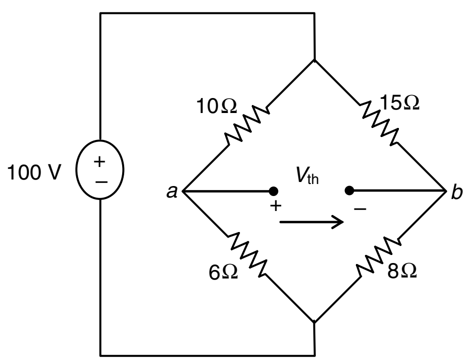
Solution

Balanced condition

$$R_1 R_4 = R_2 R_3$$

$$10 \times 8 \neq 6 \times 15$$

Bride is unbalanced, and therefore, current flowing through the 5 Ω resistor is not zero.



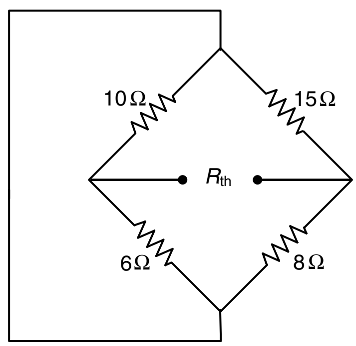
$$V_{th} = V_{ab} = V_a - V_b$$

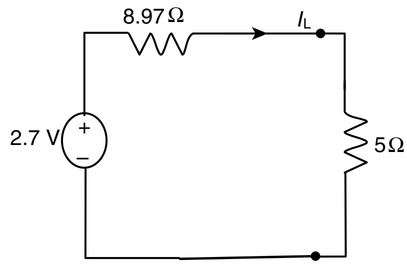
$$V_a = \frac{100 \times 6}{6 + 10} = 37.5 \text{ V}$$

$$V_b = 100 \times \frac{8}{15 + 8} = 34.78 \text{ V}$$

$$V_{th} = 37.5 - 34.78 = 2.717 \text{ V}$$

R_{th} :





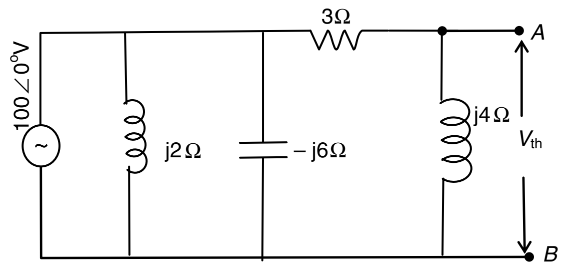
$$R_{th} = (10 \,//\, 6) + (15 \,//\, 8)$$

$$= 3.75 + 5.21 \approx 8.97 \text{ Ω}$$

$$I_L = \frac{2.717}{13.97} = 0.194 \text{ A}$$

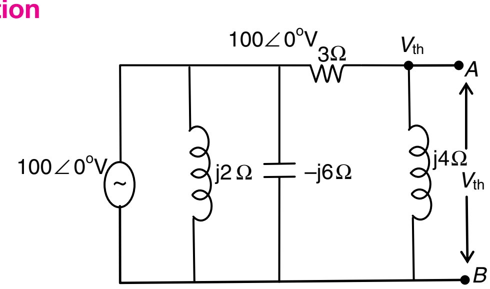
Example 10

The Thevenin's equivalent voltage V_{th} across the terminal A and B of the network shown in the figure is given by



(A) $j16(3 - j4)$ (B) $j16\,(3 + 4)$
(C) $16\,(3 + j4)$ (D) $16\,(3 - j4)$

Solution

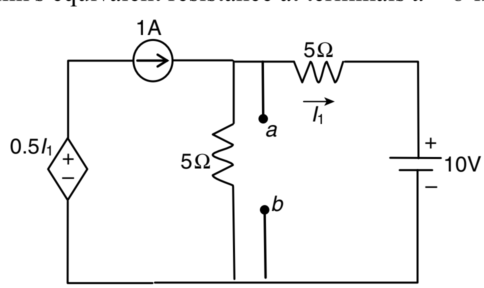


$$V_{th} = V_{j4} = 100 \times \frac{j4}{j4 + 3}$$

$$= \frac{100 \times 4(3 - j4)}{25} = j16(3 - j4)$$

Example 11

For circuit shown in the figure, Thevenin's voltage and Thevenin's equivalent resistance at terminals $a - b$ is



(A) 5 V and 2 Ω (B) 7.5 V and 2.5 Ω
(C) 4 V and 2 Ω (D) 3 V and 2.5 Ω

Solution

$$V_{th} = V_{ab}$$

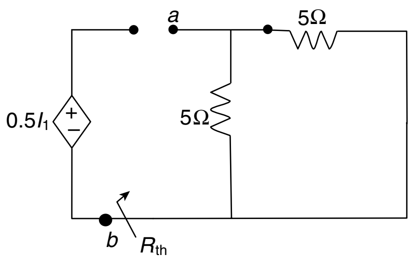
For V_{th}:

$$\frac{V_{th}-10}{5}+\frac{V_{th}}{5}-1=0$$

$$2\,V_{th} - 10 - 5 = 0$$

$$V_{th} = 7.5\text{ V.}$$

For R_{th}, deactivate the independent sources

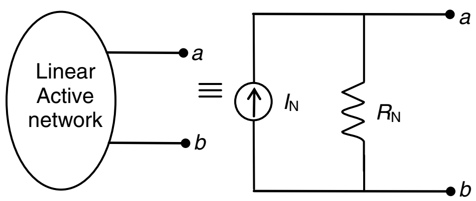


$$R_{th} = R_{ab} = 5||5 = 2.5\ \Omega$$

$$\therefore\ V_{th} = 7.5\text{ V and } R_{th} = 2.5\ \Omega$$

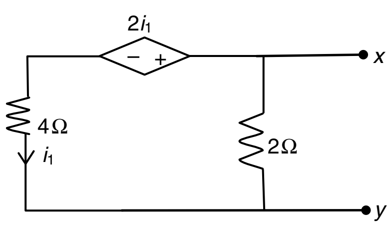
Norton's Theorem

A one-port linear, active, and resistive network, which contains one or more voltage or current sources can be replaced by a single current source in parallel with a resistance.



$$I_n = \frac{V_{th}}{R_{th}} \text{ and } R_N = R_{th}$$

Example 12



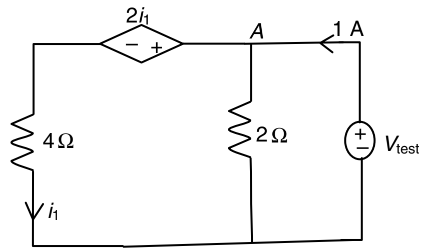
The Norton's equivalent of the network is

(A) $I_{SC} = 0, R_N = 1.5\ \Omega$ (B) $V_{th} = 0\text{ V}, R_{th} = \frac{4}{3}\ \Omega$
(C) $I_{SC} = 0, R_N = \frac{4}{3}\ \Omega$ (D) None of the above

Solution

The network does not have any active independent sources

$$I_{sc} = 0$$



By applying KCL at node A

$$1\text{ A} = \frac{V_{test}}{2} + i_1$$

However,
$$i_1 = \frac{V_{test} - 2i_1}{4}$$

$$4i_1 + 2i_1 = V_{test}$$

$$i_1 = \frac{V_{test}}{6}\text{ amp}$$

$$1\text{ A} = \frac{V_{test}}{2} + \frac{V_{test}}{6}$$

$$6 = 3V_{test} + V_{test}$$

$$V_{test} = 1.5\text{ Volts}$$

$$R_{th} = \frac{V_{test}}{1A} = 1.5\Omega$$

Example 13

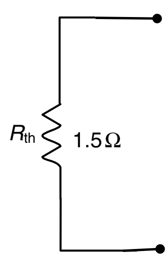
For the circuit in example 12, find the Thevenin's equivalent circuit.

Solution

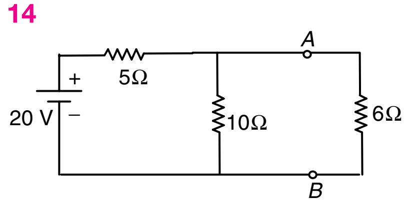
We know $R_{th} = R_N = 1.5\ \Omega$

The network does not have any active independent sources.

Therefore, $V_{th} = 0$ V



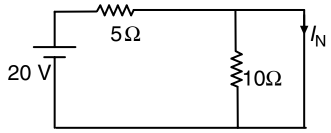
Example 14



Using Norton's Theorem, find the current through the 6 Ω load.

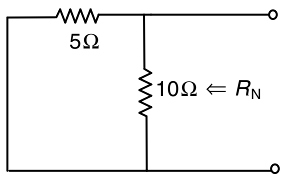
Solution

To find I_N, short circuit load terminals.



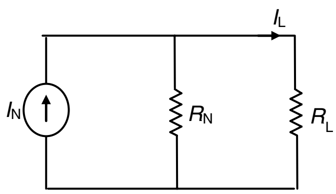
$$I_N = \frac{20}{5} = 4\text{ A}$$

To find R_N, remove load, short circuit the voltage source 20 V



$$R_N = 5 \parallel 10 = \frac{5\times 10}{5+10} = 3.33\ \Omega$$

The Norton equivalent circuit is

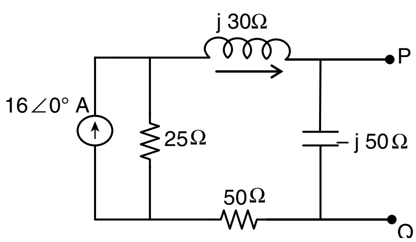


$$I_L = I_N \frac{R_N}{R_N + R_L} = 4 \times \frac{3.33}{3.33+6}$$

$$I_L = 1.43\text{ A}$$

Example 15

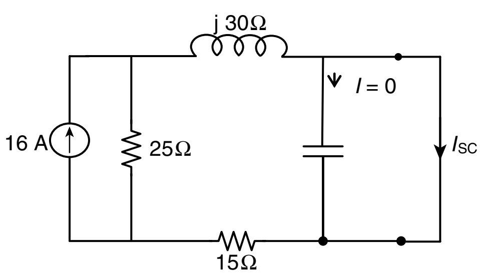
In the following circuit, the Norton's equivalent current, with respect to the terminals P and Q is



(A) $(6.4+j4.8)$ A (B) $(16+j0)$ A
(C) $(6.4-j4.8)$ A (D) $(6.56-j7.87)$ A

Solution

For Norton equivalent current, short circuiting the terminals PQ



$$I_{sc} = 16\angle 0° \times \frac{25}{25+15+j30}$$

$$\frac{25}{40+j30} \times 16\angle 0°$$

$$= \frac{25\times 16}{\sqrt{(40)^2+(30)^2}}\cdot\frac{1}{\angle 36.86^\circ}$$

$$= 8\angle -36.86° = 8\,[\cos 36.86° - j\sin 36.86°]$$

$$I_{sc} = (6.4 - j4.8)\text{ A}$$

Maximum Power Transfer Theorem

This theorem is used to find the value of load resistance for which there would be maximum amount of power transfer from source to load.

Statement

A resistance load connected to a DC network receives maximum power when the load resistance is equal to the internal resistance (Thevenin's equivalent resistance) of the source network, as seen from the load terminals.

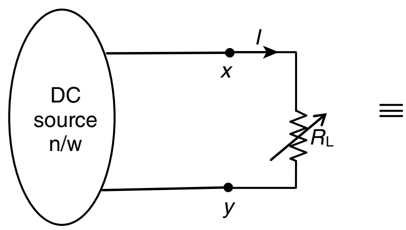


Figure 1 Load connected to the DC source network.

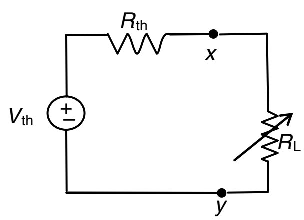
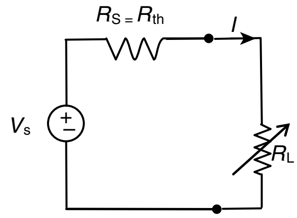


Figure 2 Equivalent network.

⇒ This theorem is applicable only for linear networks and when load is variable.

Case 1: Load is variable resistance R_L



$$I = \frac{V_S}{R_S + R_L}$$

$$P_L = V I = I^2 . R_L \text{ Watts}$$

Maximum power deliver to R_L is

$$P_L = \frac{V_s^2}{(R_S + R_L)^2}.R_L \text{ Watts}$$

To determine the value of R_L for maximum power transferred to the load.

$$\therefore \frac{dP_L}{dR_L} = 0$$

$$\frac{dP_L}{dR_L}. = \frac{v_s^2[(R_S+R_L)^2.1 - 2R_L(R_S+R_L)]}{(R_S+R_L)^2} = 0$$

$$R_s^2 + R_L^2 + 2R_s R_L - 2R_L R_s - 2R_L^2 = 0$$

$$R_S = R_L$$

For maximum power transfer condition.

$$P_{\max} = V_s^2 \cdot \frac{R_s}{(R_s+R_s)^2}$$

$$\boxed{P_{\max} = \frac{V_s^2}{2Rs} \text{ watts}}$$

$$P_{\text{total}} = I^2 R_S + I^2 R_L$$

$$= 2I^2 R_s \Rightarrow \frac{V_s^2}{2R_s} \text{W}$$

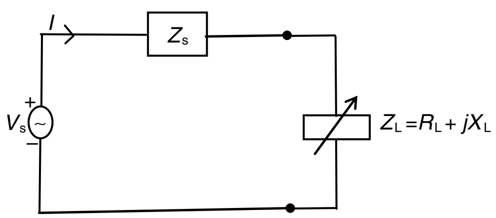
$$\text{Efficiency of MPT } \eta = \frac{\text{useful power (load power)}}{\text{Total power}}$$

$$\eta\% = \frac{\dfrac{V_s^2}{4R_s}}{\dfrac{V_s^2}{2R_s}} \times 100$$

$$\eta_{\max} = 50\%$$

The efficiency of a circuit at maximum power transfer condition is 50% only.

Case 2: Load is variable impedance Z_L and source impedance Z_s.



$$I = \frac{V_S}{Z_L + Z_S} = \frac{V_S}{(R_S + jX_S) + (R_L + jX_L)}$$

$$P_L = |I^2| R_L \text{ W}$$

$$|I| = \frac{V_S}{\sqrt{(R_S+R_L)^2 + (X_S+X_L)^2}} \text{A}$$

$$\boxed{P_L = \frac{V_S^2 R_L}{(R_S+R_L)^2 + (X_S+X_L)^2}}$$

1. Only R_L is variable:

For maximum power transfer, $\dfrac{dP_L}{dR_L} = 0$

Condition for MPT

$$R_L = \sqrt{R_S^2 + (X_S + X_L)^2} \quad \Omega$$

$$\boxed{P_{\max} = \frac{P_L}{R_L} = \sqrt{R_S^2 + (X_S+X_L)^2}}$$

2. Only 'X_L' is variable:

For MPT, $\dfrac{dP_L}{dX_L} = 0$

Condition for maximum power transfer

$\Rightarrow X_L = -X_S$

$\Rightarrow \boxed{X_L + X_S = 0}$

Case 3: Both R_L and X_L are varied simultaneously.
In this case, consider abovementioned two conditions.

$$\therefore Z_L = R_L + jX_L$$

$$= R_s - jX_s = Z_s^*$$

$$\therefore \boxed{Z_L = Z_S^* \Omega}$$

$$P_{\max} = P_L \text{ at } R_L = R_s \text{ and at } X_L = -X_S$$

$$P_{\max} = P_L/Z_L = Z_s^*$$

$$P_{\max} = \frac{V_s^2}{4R_L} = \frac{V_s^2}{4R_s} \text{W}$$

NOTE

In $Z_L = R_L + jX_L$

If $X_L = 0$, then $R_L = \sqrt{R_S^2 + (X_S+X_L)^2}\Omega$

subjected to $X_L = 0$ in the above expression

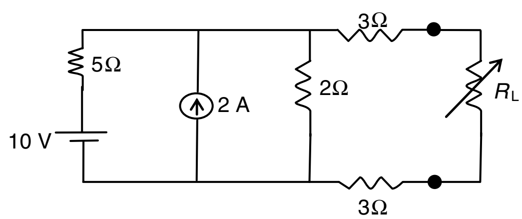
$$R_L = \sqrt{R_s^2 + X_s^2}\Omega$$

$$\boxed{\therefore R_L = |Z_S|\Omega}$$

$$P_{\max} = P_L \text{ at } R_L = |Z_s|\Omega$$

Example 16

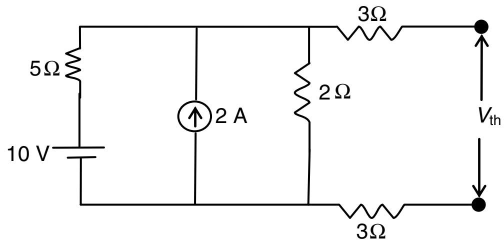
Find the maximum power that can be transferred to R_L



Solution

Find the Thevenin's equivalent circuit

Case 1: V_{th}:



By applying nodal analysis

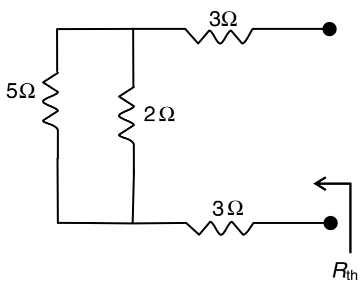
$$\frac{V_{th}-10}{5}+\frac{V_{th}}{2}-2=0$$

$$2(V_{th}-10)+5\,V_{th}=20$$

$$7V_{th}=40$$

$$V_{th}=\frac{40}{7}\ \text{Volts}$$

Case 2: R_{th}:



$$R_{th}=(5//2)+3+3$$

$$=6+\frac{10}{7}$$

$$=7.423\ \Omega$$

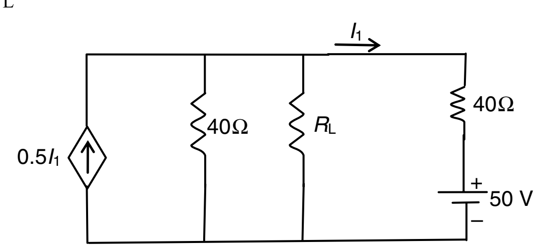
$$P_{max}=\frac{V_{th}^{2}}{4R_{th}}=\frac{36.65}{29.69}$$

$$=1.099\ \text{Watts}$$

$$\approx 1\ \text{Watt}$$

Example 17

In the network of the figure, the maximum power is delivered to R_L if its value is

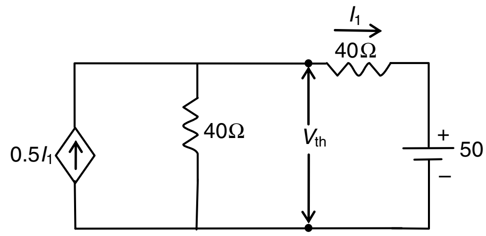


(A) $R_L = 16.66\ \Omega$ (B) $R_L = 20\ \Omega$
(C) $R_L = 26.66\ \Omega$ (D) None of these

Solution

$$R_{th}=R_L=\frac{V_{th}}{I_{sc}}$$

V_{th}:



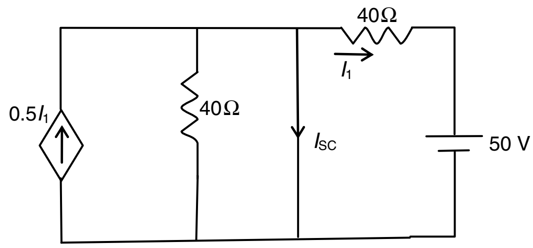
$$\frac{V_{th}-50}{40}+\frac{V_{th}}{40}=0.5\left[\frac{V_{th}-50}{40}\right]$$

$$\frac{1}{2}\left[\frac{V_{th}-50}{40}\right]+\frac{V_{th}}{40}=0$$

$$V_{th}-50+2\,V_{th}=0$$

$$3V_{th}=50\Rightarrow V_{th}=\frac{50}{3}\ \text{V}$$

I_{sc}:



$$I_{SC}=-0.5I_1$$

$$=-0.5\left[\frac{0-50}{40}\right]$$

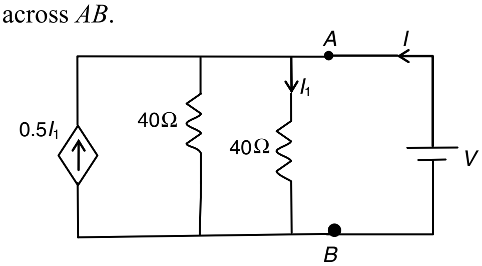
$$=0.625\ \text{A}$$

$$\therefore\ R_{th}=\frac{V_{th}}{I_{sc}}=\frac{50}{3\times 0.625}\,\Omega$$

$$=26.66\ \Omega$$

Second method:

For maximum power delivered to R_L open circuit R_L.
R_{th} across AB.



All independent sources deactivated KCL at node A.

$$0.5I_1+I=\frac{V}{40}+\frac{V}{40}$$

Therefore, $I_1 = \dfrac{V}{40}$

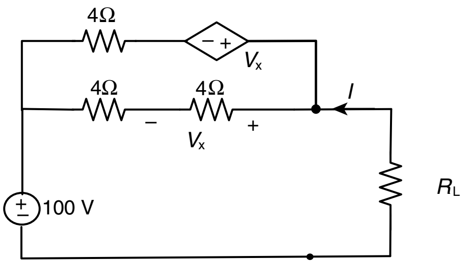
$$0.5\,\frac{V}{40} + I = \frac{2V}{40}$$

$$I = \frac{1.5V}{40}$$

$$\frac{V}{I} = \frac{40}{1.5} = 26.66\ \Omega$$

Solution

In the following circuit, what value of R_L maximize the power delivered to R_L?

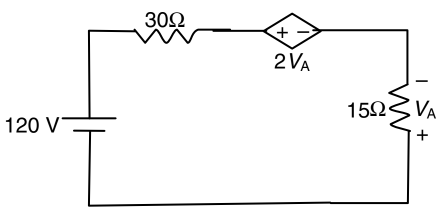


NOTE

The value of R_L maximizes the power delivered to the load is equal to Thevenin's resistance. Find the Thevenin's resistance by using case 3.

Example 18

In the circuit shown in figure, the power absorbed by each element is



Solution

By applying KVL for the loop,

$$120 = 30\,I + 2V_A = +\,15I$$

$$V_A = -15I$$

$$120 = 30\,I - 2V_A + 15I = 0$$

$$I = 8\ A$$

$$P_{120v} = VI = -\,8 \times 120 = -\,960\ \text{W}$$

Delivered by 960 W

$$P_{30\Omega} = 30 \times 8^2 = 1920\ \text{W} \Rightarrow \text{absorbed}$$

$$P_{2VA} = +\,8 \times (-\,2 \times 15 \times 8) = -1{,}920\ \text{W}$$

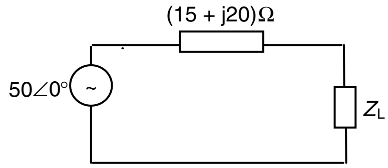
$$P_{15\Omega} = 82 \times 15 = 960\ \text{W}$$

$$\text{Total power} = -960 + 1920 + 960 - 1{,}920$$

$$= 0\ \text{W.}$$

∴ Total power delivered = total power absorbed.

Example 19



For the circuit shown in figure, find the value of load impedance for which the source delivers maximum power. Calculate the value of the maximum power.

(A) 41.33 W (B) 50 W
(C) 75 W (D) −30W

Solution

For maximum power transfer,

$$Z_L = Z_S^*$$

$$Z_L = (15 - j20)\ \Omega$$

When $Z_L = (15 - j20)\ \Omega$, the current passing through the circuit is

$$I = \frac{V_S}{Z_S + Z_L} = \frac{50\angle 0^\circ}{15 + j20 + 15 - j20}$$

$$I = \frac{50\angle 0^\circ}{30\angle 0^\circ} = 1.66\ \angle 0^\circ$$

The maximum power transferred to load is

$$P_{max} = I^2 R_L$$

$$= (1.66)^2 \times 15 = 41.33\ \text{W}$$

Reciprocity Theorem

Statement

In a linear, passive, bilateral, and time invariant network, the ratio of output to input (source) is constant even though the source is interchanged from input terminals to output terminals.

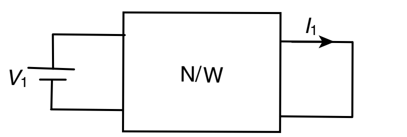


Figure 1

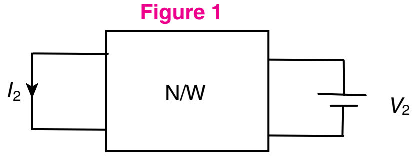


Figure 2

i.e., $\dfrac{I}{V}$ = constant

$$\Rightarrow \frac{I_1}{V_1} = \frac{I_2}{V_2}$$

NOTE

The presence of the dependent sources makes the network active, and hence, the Reciprocity Theorem is not applicable to active networks.

Example 20

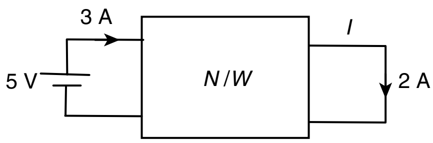


Figure 1

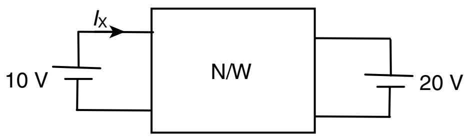


Figure 2

The network contains only resistances. Use the data given in Figure 5 and find the current I_x in Figure 6.

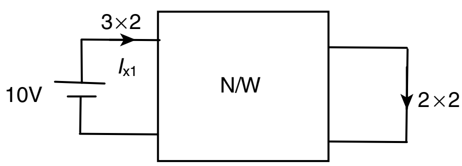
(A) 2 A (B) 6 A
(C) −2 A (D) 1 A

Solution

From the Reciprocity Theorem

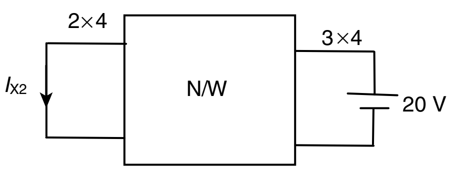
$$\frac{I_1}{V_1} = \frac{I_2}{V_2} = \text{ constant.}$$

Case 1: Consider the 10 V is activated



$$I_{x1} = 6\text{ A.}$$

Case 2: let us consider that 20 V source is activated.



$$I_{x2} = 8\text{ A}$$

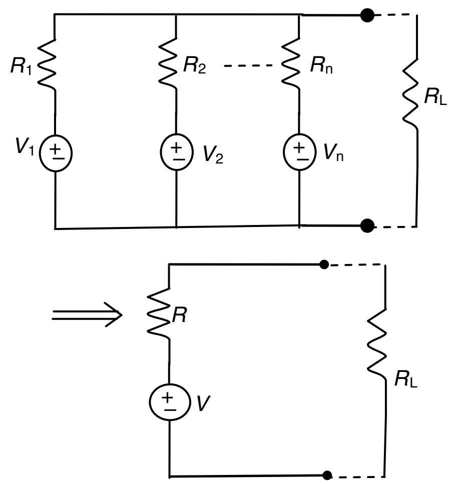
By using Superposition Theorem,

$$I_X = I_{X1} - I_{X2}$$
$$= 6 - 8 = -2\text{ A}$$

Millman's Theorem

Statement

When a number of voltage sources ($V_1, V_2 \ldots V_n$) are in parallel having internal resistances ($R_1, R_2, \ldots R_n$), respectively. The arrangement can be replaced by a single equivalent voltage source V in series with an equivalent series resistance R, as shown in the following figure.



As per Millman's theorem

$$V = \frac{\pm V_1G_1 \pm V_2G_2 \pm \ldots\ldots\ldots\ldots \pm V_nG_n}{G_1 + G_2 + \ldots\ldots\ldots G_n}$$

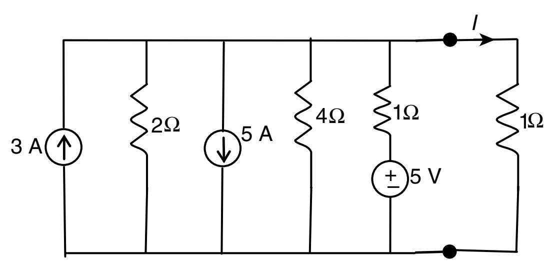
$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \ldots\ldots\ldots + G_n}$$

i.e. $$\boxed{V = \frac{\sum_{k=1}^{n} V_K G_k}{\sum_{k=1}^{n} G_k}}$$

This theorem is applicable to only linear networks.

Example 21

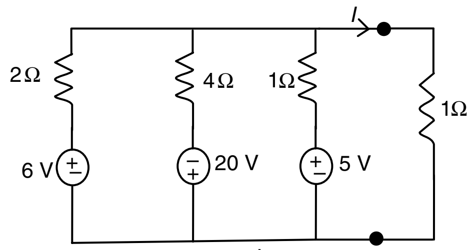
Find the current through the 1 Ω resistor using Millman's Theorem.

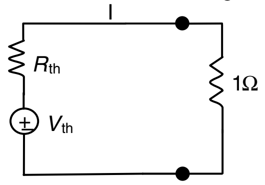


(A) $I = 3$ A (B) $I = 1.09$ A
(C) $I = 7$ A (D) $I = -1$ A

Solution

Converting current source to an equivalent voltage source.





$$I = \frac{V_{th}}{R_{th}+1}\text{ A}$$

$$\Rightarrow V_{\text{th}} = \frac{\Sigma V_k G_k}{G_k}$$

$$V_{\text{th}} = \frac{\frac{6}{2}-\frac{20}{4}+\frac{5}{1}}{\frac{1}{2}+\frac{1}{4}+\frac{1}{1}}$$

$$= \frac{3-5+5}{0.5+0.25+1} = \frac{3}{1.75} = 1.714\text{ V}$$

$$R = \frac{1}{\frac{1}{2}+\frac{1}{4}+\frac{1}{1}} = \frac{1}{1.75} = 0.5714\Omega$$

$$\Rightarrow I = \frac{1.714}{1+0.57} = 1.09\text{ A}$$

Tellegen's Theorem

Statement

In an arbitrary network, the algebraic sum of powers at any given instant is zero. The power delivered by some elements is equal to power absorbed by other elements present in the network.

$$\therefore \sum_{j=1}^{n} V_j \cdot i_j = 0$$

where n is the total number of branches.

NOTES

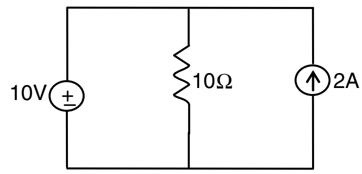
1. When current enters at the positive terminals of an element, then that element will absorb the power, otherwise it will deliver the power.
2. Sources can deliver power or absorb power, whereas the passive elements will always absorb power since current will enter at the positive terminal in the respective R, L, C's.

Properties

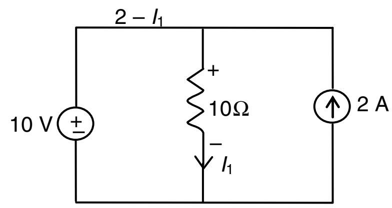
This theorem is independent of the nature of the elements.

Example 22

For the circuit shown in figure, verify the Tellegen's theorem.



Solution



$$I_1 = \frac{10}{10} = 1$$

$$\therefore P_{10\,\Omega} = VI = I^2R$$

$$= 10\text{ Watts (absorbed)}$$

$$V_{2A} = 10\text{ V}$$

$$\Rightarrow P_{2A} = -2 \times 10 = -20\text{ Watts (delivered)}$$

$$I_{10v} = 2 - 1 = 1\text{ A} \Rightarrow P_{10v} = 10 \times 1 = 10\text{ Watts (absorbed)}$$

$$\therefore \Sigma VI = 0$$

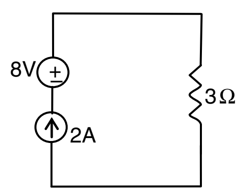
Total absorbed power = total delivered power

$$P_{2A} + P_{10V} + P_{10\Omega} = 0$$

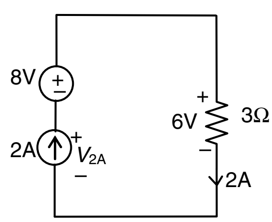
∴ Tellegen's Theorem is verified.

Example 23

For the circuit shown in figure, verify the Tellegen's theorem.



Solution



$$[V_{2A} = 6 - 8 = -2 \text{ V}]$$
$$P_{8V} = VI = -8 \times 2 = -16 \text{ W (delivers)}$$
$$P_{2A} = 2 \times 2 = 4 \text{ W (absorbed)}$$
$$P_{3\Omega} = VI = 12 \text{ W (absorbed)}$$
$$\therefore \Sigma\text{power} = 0$$
$$-16 + 4 + 12 = 0$$

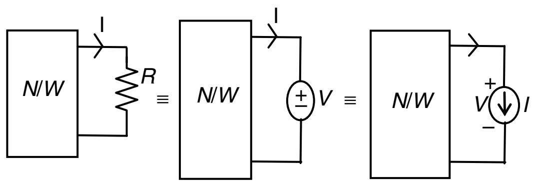
$\therefore$ Tellegen's Theorem is verified.

Substitution Theorem

The voltage across the current through any branch of a DC bilateral network can be replaced by any combination of elements that will make the same voltage across and current through the chosen branch.

In a linear network, any passive element can be equivalently substituted by an ideal voltage source or an ideal current source, provided all the other branch currents and voltages are kept constant.

Any branch in a linear network can be substituted by a different branch without disturbing the voltages and currents in the entire network, provided the new branch has the same set of terminal voltages and currents as those of the original network. This theorem is applicable for any LTI and bilateral networks.



where $V = RI$ and $P = VI = I^2 R(W)$.

For example, consider the following figure.

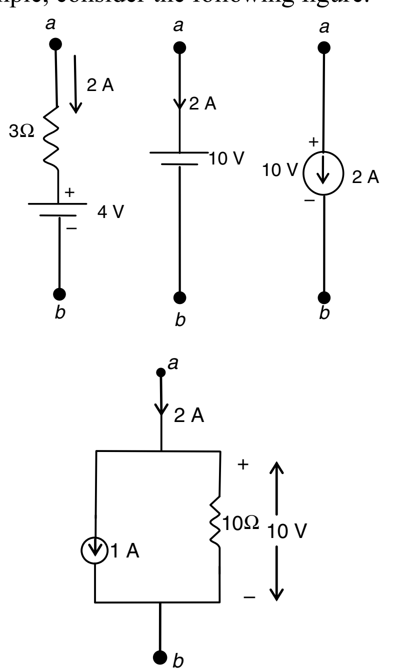


Figure 7 Equivalent circuits of branch $a - b$.

Compensation Theorem

Statement

In an LTI network, when the resistance 'R' of an uncoupled branch carrying a current (I) is changed by ΔR, the current in all the branches would change and can be obtained by assuming that an ideal voltage source of (V_s) has been connected in series with ($R + \Delta R$) when all other sources in the network are replaced by their internal resistances ($V_s = I\Delta R$)

This theorem is useful in determining the current and voltage changes in circuit element when the value of its impedance is changed. For example, bridge and potentiometer circuit.

Duality of Circuits

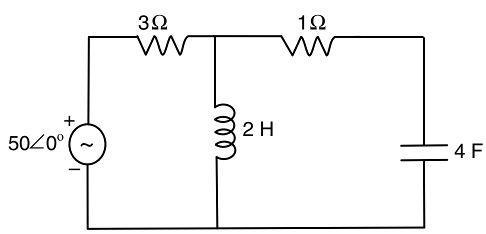
Two linear circuits are said to be duals of one another if they are described by the same characteristic equations with dual quantities interchanged.

Network and its dual are same only with respect to the performance, but the elements and connecting points of view are not equal.

Dual Pairs

R	$\leftrightarrow$	G
L	$\leftrightarrow$	C
Z	$\leftrightarrow$	Y
V	$\leftrightarrow$	I
Voltage Source	$\leftrightarrow$	Current Source
KCL	$\leftrightarrow$	KVL
Star	$\leftrightarrow$	Delta
Node	$\leftrightarrow$	Mesh
Series	$\leftrightarrow$	Parallel
Open circuit	$\leftrightarrow$	Short circuit
$L.\frac{di(t)}{dt}$	$\leftrightarrow$	$C.\frac{dv(t)}{dt}$
$\frac{1}{C}\int i(t)dt$	$\leftrightarrow$	$\frac{1}{L}\int v(t).dt$
Thevenin's	$\leftrightarrow$	Norton's
$Ri(t)$	$\leftrightarrow$	$GV(t)$

Example 24

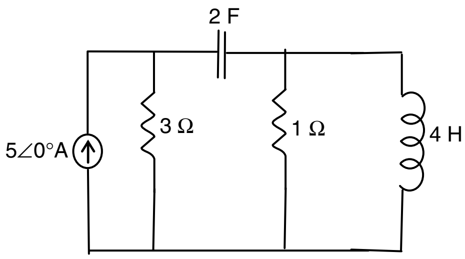


The dual of the network is as follows.

Solution

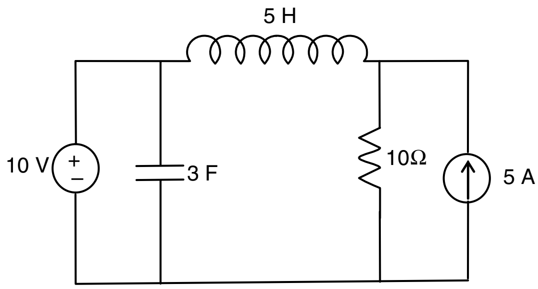
Dual of the given network

$V \leftrightarrow I$, $R \leftrightarrow G$; $L \leftrightarrow C$ and series $\leftrightarrow$ parallel



Example 25

Obtain the dual of the network shown in figure.

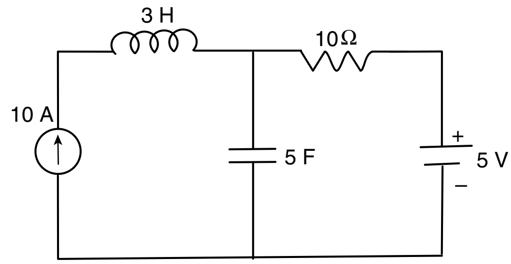


Solution

Dual of the abovementioned network

Series $\leftrightarrow$ parallel, $V \leftrightarrow I$, $L \leftrightarrow C$

$R \leftrightarrow G$

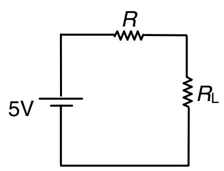


Exercises

Practice Problems I

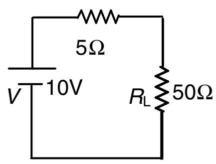
Direction for questions 1 to 28: Select the correct alternative from the given choices.

1. The maximum power transferred to the load in the circuit is given as 0.5 W. Get the values of R and R_L.



(A) 15 Ω, 10 Ω (B) 12.5 Ω, 12.5 Ω
(C) 10 Ω, 15 Ω (D) 10 kΩ, 10 kΩ

2. Find the efficiency of the circuit given for $R_L = 50\ \Omega$.

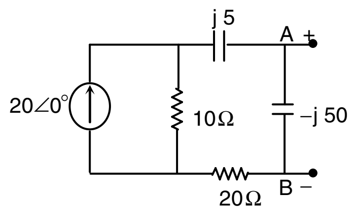


(A) 99% (B) 91% (C) 80% (D) 87%

3. Current in the circuit is given by the equation $i(t) = 10\cos(20\pi t + 50)$ and the impedance of the load is given as $Z_L = 5 + j3$. Find the average power delivered to the load.

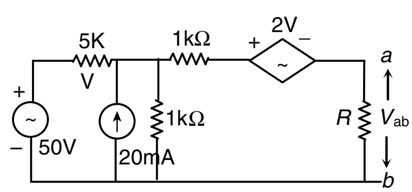
(A) 353.5 W (B) 291.5 W
(C) 250 W (D) 176.7 W

4. In the following circuit, the Norton equivalent current (in A) across $A - B$ is



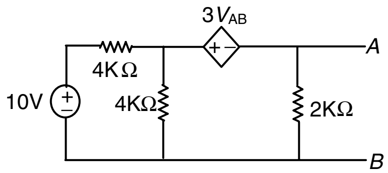
(A) $19.45 + j3.24$ (B) $6.48 - j1.08$
(C) $12.97 - j2.16$ (D) $20 + j0$

5. Find the Thevenin's equivalent voltage external to the load R_L.



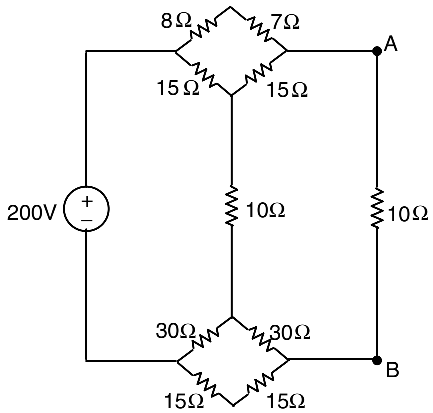
(A) 25 V (B) 50 V (C) 49 V (D) 45 V

6. Find the Thevenin's resistance associated with the circuit.



(A) 1 kΩ (B) 0.45 kΩ
(C) 2 kΩ (D) 0.22 kΩ

Direction for questions 7 and 8: Select the correct alternative from the given choices.



7. Find Thevenin's equivalent voltage of the circuit.
(A) 100 V (B) 120 V (C) 125 V (D) 150 V

8. The resistance across $A - B$ is 10 Ω. Find the current through the 10 Ω resistor.
(A) 5.1 A (B) 6.45 A (C) 3.35 A (D) 13.9 A

9. When a resistor R is fed from an electrical network, 'N' consumes a power of 'P' W, as shown in the Figure (a). If an identical network is added as shown in Figure (b) the power consumed by R will be _____.

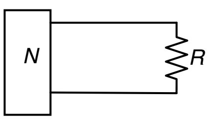


Figure (a)

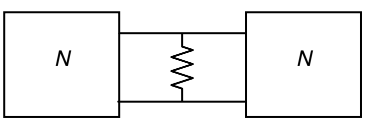
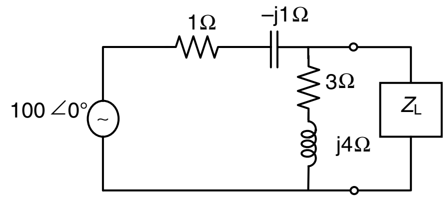


Figure (b)

(A) 2P (B) P
(C) $\dfrac{P}{2}$ (D) between P and 4P

Direction for questions 10 and 11: Select the correct alternative from the given choices.

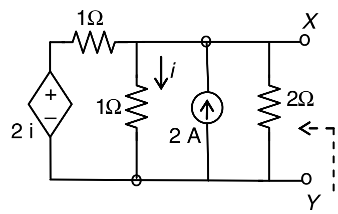


10. Find the value of Z_L at which maximum power is transferred to Z_L
(A) $(1.24 - j0.676)$ Ω
(B) $(1.24 + j0.676)$ Ω
(C) 1.31 Ω
(D) 1.24 Ω

11. The maximum power transferred is
(A) 201.6 W (B) 617 W
(C) 2016 W (D) 6170 W

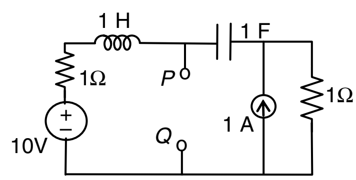
Direction for questions 12 to 15: Select the correct alternative from the given choices.

12. For the circuit shown in the figure, the Thevenin's voltage and resistance looking into $x - y$ are _____.



(A) 4/3 V, 2 Ω (B) 4 V, 2/3 Ω
(C) 4/3 V, 2/3 Ω (D) 4 V, 2 Ω

13. The Thevenin's equivalent impedance Z_{TH} between the nodes P and Q in the following circuit is _____.



(A) 1 (B) $1 + S + \dfrac{1}{S}$

(C) $2 + S + \dfrac{1}{S}$ (D) $\dfrac{S^2 + S + 1}{S^2 + 2S + 1}$

14. The short-circuit test of a two-port π network is shown in Figure (a). The voltage across the terminals 11^1 in the network shown in Figure (b) will be

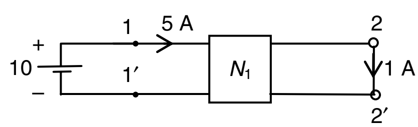


Figure (a)

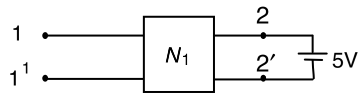
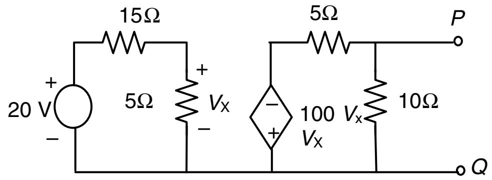


Figure (b)

(A) 2 V (B) 5 V (C) 10 V (D) 1 V

15. The Norton's equivalent circuit at terminals PQ has a current source and a Norton's resistance of ______.

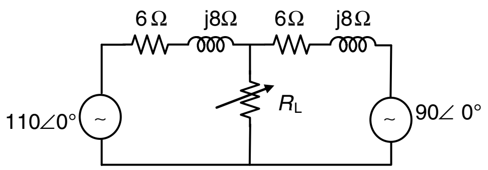


(A) 10 A, 3.33 Ω (B) 100 A, $\frac{85}{12}$ Ω

(C) 100 A, 3.33 Ω (D) 500 A, 3.33 Ω

Direction for questions 16 and 17: Select the correct alternative from the given choices.

16.



In the circuit shown in figure, under the maximum power transfer condition, the value of R_L is ______.

(A) 5 Ω (B) 20 Ω (C) $\frac{25}{3}$ Ω (D) 6 Ω

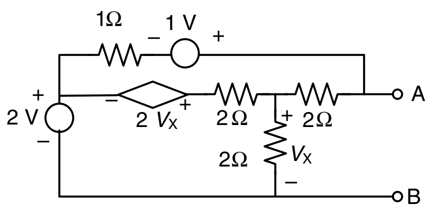
17. The power absorbed by R_L at maximum power transfer condition is ______.

(A) 1,000 W (B) 500 W

(C) 625 W (D) 2,000 W

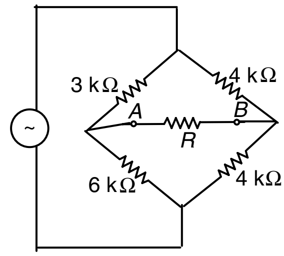
Direction for questions 18 to 22: Select the correct alternative from the given choices.

18. The Thevenin's voltage at the terminals AB of the network shown in the figure is



(A) 4 V (B) 2 V (C) $\frac{3}{2}$ V (D) $\frac{1}{2}$ V

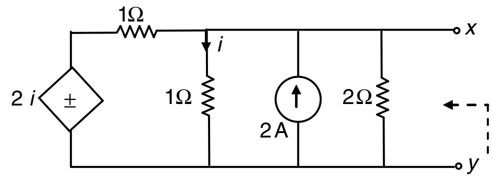
19. The value of the resistance R, connected across the terminals A and B, which will absorb the maximum power, is



(A) 4 kΩ (B) 4.11 kΩ

(C) 8 kΩ (D) 9 kΩ

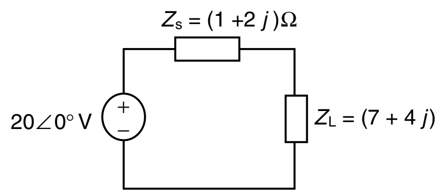
20. For the circuit shown in figure, the Thevenin's voltage and resistance looking into X - Y are



(A) $\frac{4}{3}$ V, 2 Ω (B) 4 V, $\frac{2}{3}$ Ω

(C) $\frac{4}{3}$ V, $\frac{2}{3}$ Ω (D) 4 V, 2 Ω

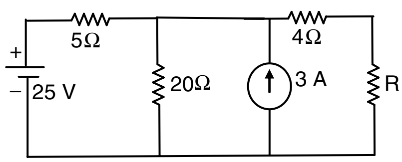
21. An AC source of RMS voltage 20 V with internal impedance $Z_S = (1 + 2j)$ Ω feeds a load of impedance $Z_L = (7 + 4j)$ Ω shown in the figure. The reactive power consumed by the load is



(A) 8 VAR (B) 16 VAR

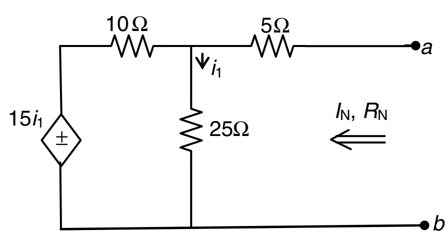
(C) 28 VAR (D) 35 VAR

22. The value of R (in Ohms) required for maximum power transfer in the following network is



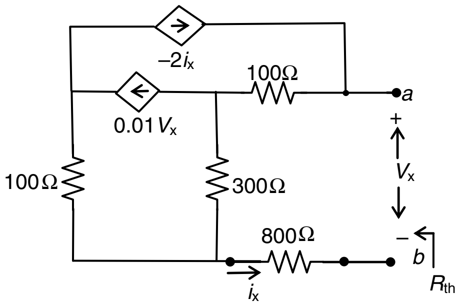
(A) 2 (B) 4 (C) 8 (D) 16

23. For the following circuit, the value of i_N and R_N are



(A) 2 A, 12 Ω (B) 0 A, 20 Ω
(C) 0.5 A, 20 Ω (D) 0 A, 12 Ω

24. For the following circuit, the values of R_{th} and V_{th} are



(A) 0 V, 100 Ω (B) 0 V, 1200 Ω
(C) 5 V, 100 Ω (D) None of the above

25. Consider the following circuits

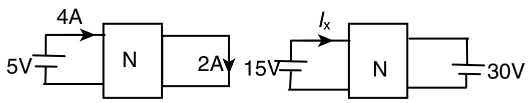
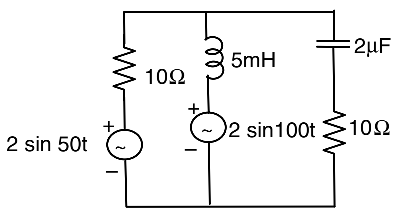


Figure (a) Figure (b)

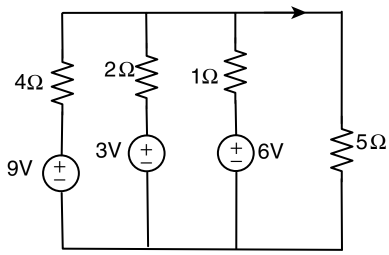
The network '*N*' contains only resistances. Use the data given in Figure (a) and find the current i in Figure (b).
(A) 0 A (B) 12 A (C) –6 A (D) 6 A

26. In the circuit shown in figure, which one of the following theorem can be more conveniently used to evaluate the responses in the 10 Ω resistors?



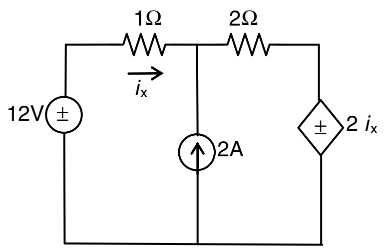
(A) Thevenin's Theorem
(B) MPTT (Maximum Power Transfer Theorem)
(C) Milliman's Theorem
(D) Superposition Theorem

27. Consider the following network



The current I is
(A) 0.23 A (B) –0.23 A
(C) 2.25 A (D) –0.5 A

28. Consider the following circuit

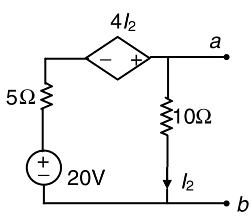


The current i_x would be
(A) $i_x = 1.6$ A (B) $i_x = 1.3$ A
(C) $i_x = -1.5$ A (D) $i_x = 0.8$ A

Practice Problems 2

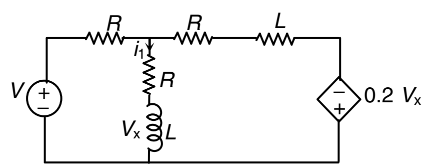
Direction for questions 1 to 16: Select the correct alternative from the given choices.

1. Find the Thevenin's equivalent of the circuit given



(A) $V_{TH} = 20$ V, $R_{TH} = 3.3$ Ω
(B) $V_{TH} = 16$ V, $R_{TH} = 5$ Ω
(C) $V_{TH} = 20$ V, $R_{TH} = 5$ Ω
(D) $V_{TH} = 4$ V, $R_{TH} = 10$ Ω

2. Find the state equation for the circuit given.



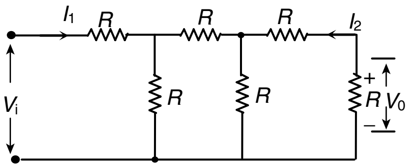
(A) $L\dfrac{di_2}{dt} = 0.7V_x - 1.5i_2R + .5V$

(B) $L\dfrac{di_2}{dt} = -0.7V_x + 1.5i_2R - .5V$

(C) $L\dfrac{di_2}{dt} = 0.7V_x - 1.5i_2R + .5V$

(D) $L\dfrac{di_2}{dt} = 0.7V_x - 1.5i_2R - .5V$

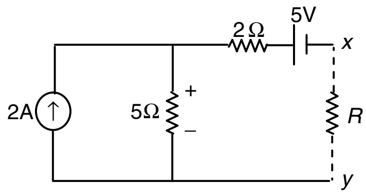
3. Find the transfer function of the following network.



(A) $\frac{V_0}{V_i} = \frac{1}{5}$ (B) $\frac{V_0}{V_i} = \frac{1}{13}$

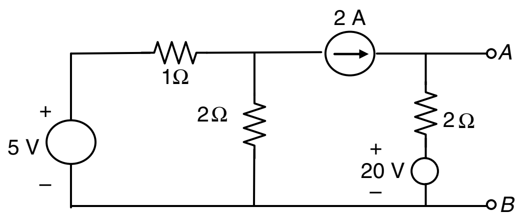
(C) $\frac{V_0}{V_i} = 0$ (D) $\frac{V_0}{V_i} = \frac{-1}{13}$

4. A network is shown in the following figure with an unknown load R. Find the value of R so that maximum power is delivered to the load.



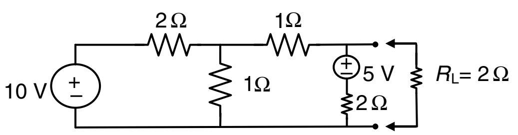
(A) 5Ω (B) 7Ω (C) 1.43Ω (D) 2Ω

5. The Thevenin's resistance across the terminals AB of the figure is ______.



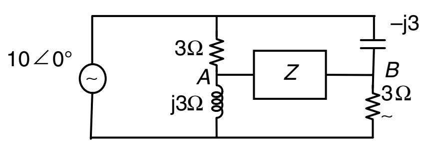
(A) $\frac{2}{3}\Omega$ (B) 2 Ω (C) $\frac{7}{3}\Omega$ (D) 5 Ω

6. In the following circuit, the power consumed by R_L is



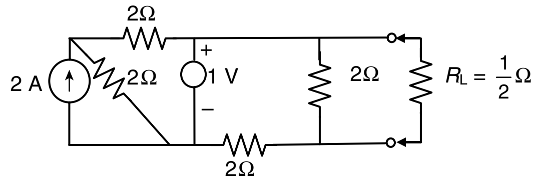
(A) $\frac{25}{16}$ W (B) $\frac{25}{3}$ W (C) $\frac{25}{8}$ W (D) $\frac{5}{2}$ W

7. In the following circuit, the Thevenin's impedance between terminals A and B is ______.



(A) 6 Ω (B) 3 Ω (C) $\frac{3}{2} - \frac{j3}{2}$ (D) 6 + j6

8. In the following circuit, the current through resistance R_L is ______.

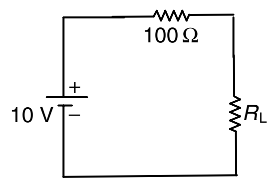


(A) $\frac{2}{3}$A (B) $\frac{3}{2}$A (C) $\frac{4}{3}$A (D) $\frac{1}{3}$A

9. A source of angular frequency 1 rad/s has source impedance consisting of 1 Ω resistance in series with 1 H inductance. The load that will obtain the maximum power transfer is
(A) 1 Ω resistance
(B) 1 Ω resistance in parallel with 1 H inductance
(C) 1 Ω resistance in series with 1 F capacitor
(D) 1 Ω resistance in parallel with 1 F capacitor

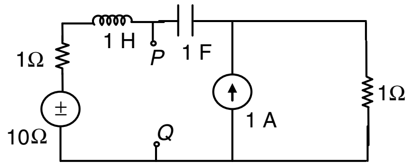
10. Superposition Theorem is not applicable to networks containing
(A) non-linear elements
(B) dependent voltage sources
(C) dependent current sources
(D) transformers

11. The maximum power that can be transferred to the load resistor R_L from the voltage source in the figure is



(A) 1 W (B) 10 W (C) 0.25 W (D) 0.5 W

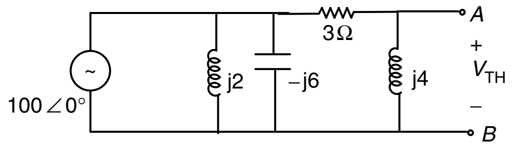
12. The Thevenin's equivalent impedance Z_{TH} between the nodes P and Q in the following circuit is



(A) 1 (B) $1 + s + \frac{1}{s}$

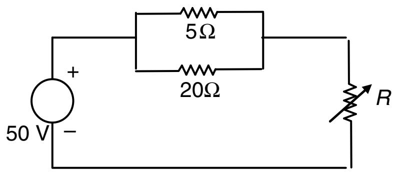
(C) $2 + s + \frac{1}{s}$ (D) $\frac{s^2 + s + 1}{s^2 + 2s + 1}$

13. The Thevenin's equivalent voltage V_{TH} appearing between the terminals A and B of the following network is given by



(A) $j16\,(3 - j4)$
(B) $j16\,(3 + j4)$
(C) $16\,(3 + j4)$
(D) $16\,(3 - j4)$

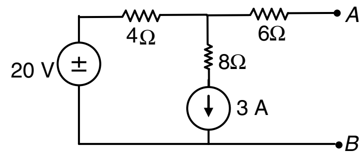
14.



In the following circuit, the adjustable resistor R is set such that the power in the 5 Ω resistor is 20 W. The value of R is

(A) 6 Ω
(B) 25 Ω
(C) 4 Ω
(D) 16 Ω

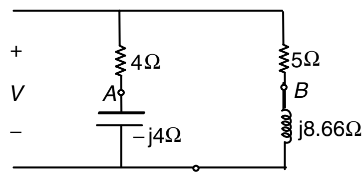
15.



The Norton equivalent of the abovementioned circuit is

(A) $I_N = 8$ A, $R_N = 10\ \Omega$
(B) $I_N = 0.8$ A $R_N = 10\ \Omega$
(C) $I_N = 3$ A $R_N = 8\ \Omega$
(D) $I_N = 8$ A $R_N = 3\ \Omega$

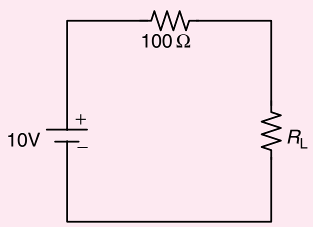
16. In the following circuit, $V_{AB} = 48.3\angle 30°$. The applied voltage V is



(A) $40 \angle 90°$
(B) $100 \angle 130°$
(C) $50 \angle 135°$
(D) $100 \angle 135°$

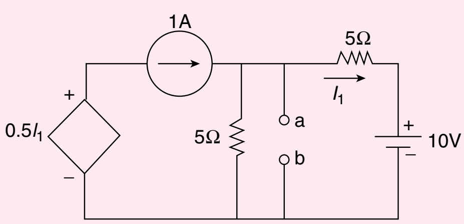
Previous Years' Questions

1. The maximum power that can be transferred to the load resistor R_L from the voltage source in figure is **[2005]**



(A) 1 W
(B) 10 W
(C) 0.25 W
(D) 0.5 W

2. For the following circuit, Thevenin's voltage and Thevenin's equivalent resistance at terminals $a - b$ is **[2005]**

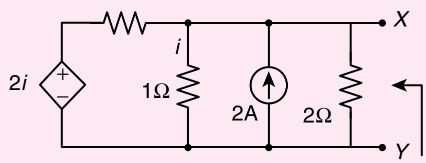


(A) 5 V and 2 Ω
(B) 7.5 V and 2.5 Ω
(C) 4 V and 2 Ω
(D) 3 V and 2.5 Ω

3. An independent voltage source in series with an impedance $Z_S = R_S + jX_S$ delivers a maximum average power to a load impedance Z_L when **[2007]**

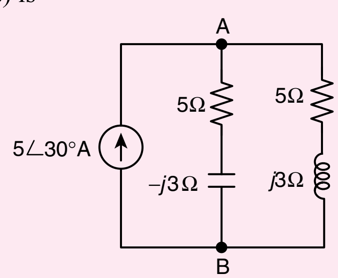
(A) $Z_L = R_S + jX_S$
(B) $Z_L = R_S$
(C) $Z_L = jX_s$
(D) $Z_L = R_S - jX_s$

4. For the following circuit, the Thevenin's voltage and resistance looking into $X - Y$ are **[2007]**



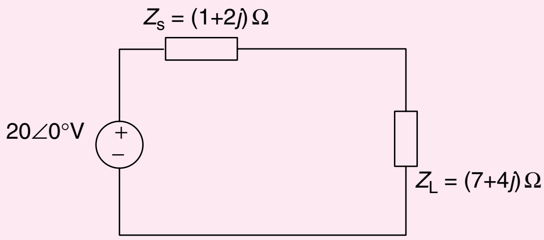
(A) $\frac{4}{3}$ V, 2Ω
(B) 4V, $\frac{2}{3}$ Ω
(C) $\frac{4}{3}$ V, $\frac{2}{3}$ Ω
(D) 4 V, 2Ω

5. In the following AC network, the phasor voltage V_{AB} (in Volts) is **[2007]**



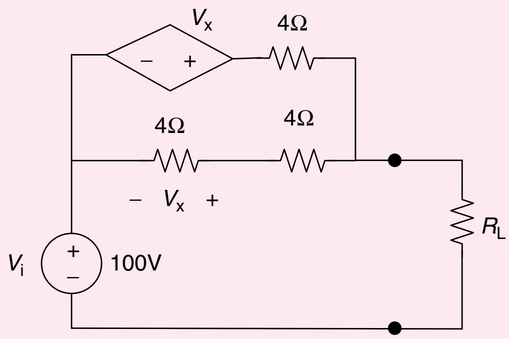
(A) 0
(B) $5 \angle 30°$
(C) $12.5 \angle 30°$
(D) $17 \angle 30°$

6. An AC source of RMS voltage 20 V with internal impedance $Z_s = (1 + 2j)\ \Omega$ feeds a load of impedance $Z_L = (7 + 4j)\ \Omega$ in the following figure. The reactive power consumed by the load is **[2009]**



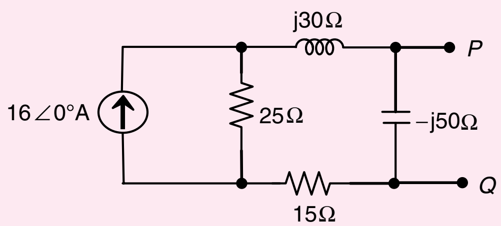
(A) 8 VAR (B) 16 VAR
(C) 28 VAR (D) 32 VAR

7. In the following circuit, what value of R_L maximizes the power delivered to R_L? **[2009]**



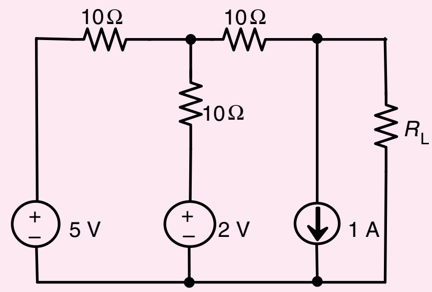
(A) 2.4 Ω (B) $\frac{8}{3}\ \Omega$ (C) 4 Ω (D) 6 Ω

8. In the following circuit, the Norton equivalent current in amperes with respect to the terminals P and Q is **[2011]**



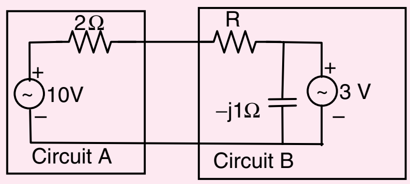
(A) $6.4 - j4.8$ (B) $6.56 - j7.87$
(C) $10 + j0$ (D) $16 + j0$

9. In the following circuit, the value of R_L such that the power transferred to R_L is maximum. **[2011]**



(A) 5 Ω (B) 10 Ω
(C) 15 Ω (D) 20 Ω

10. Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is **[2011]**

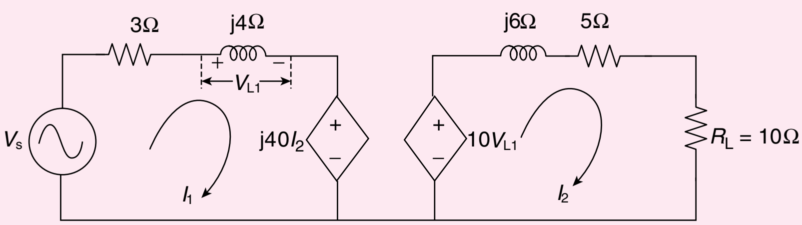


(A) 0.8 Ω (B) 1.4 Ω
(C) 2 Ω (D) 2.8 Ω

11. A source $V_s(t) = V \cos 100\pi t$ has internal impedance of $(4 + j3)\ \Omega$. If a purely resistive load connected to this source has to extract the maximum power out of source, its value in Ω should be **[2013]**

(A) 3 (B) 4
(C) 5 (D) 7

12. In the following circuit, if the source voltage $V_s = 100 \angle 53.13°$V, then the Thevenin's equivalent voltage (in Volts) as seen by the load resistance R_L is **[2013]**



(A) 100 ∠ 90° (B) 800 ∠ 0° (C) 800 ∠ 90° (D) 100 ∠ 60°

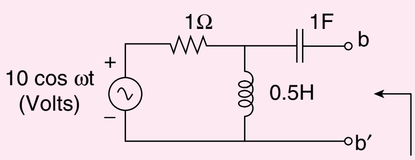
13. For maximum power transfer between two cascaded sections of an electrical network, the relationship between the output impedance Z_1 of the first section to the input impedance Z_2 of the second section is **[2014]**

(A) $Z_2 = Z_1$ (B) $Z_2 = -Z_1$
(C) $Z_2 = Z^*_1$ (D) $Z_2 = -Z^*_1$

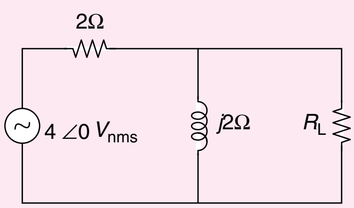
14. Norton's Theorem states that a complex network connected to a load can be replaced with an equivalent impedance **[2014]**

(A) in series with a current source
(B) in parallel with a voltage source
(C) in series with a voltage source
(D) in parallel with a current source

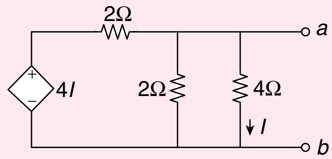
15. In the following circuit, the angular frequency ω (in rad/s), at which the Norton equivalent impedance as seen from terminals b–b' is purely resistive, is ______________. **[2014]**



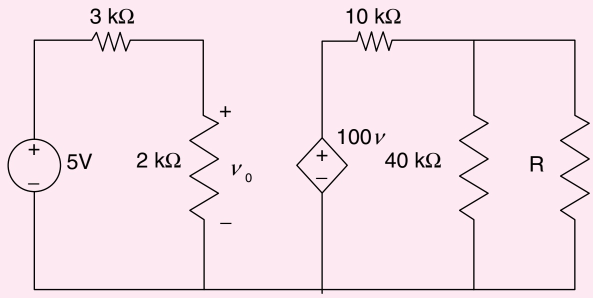
16. In the given circuit, the maximum power (in watts) that can be transferred to the load R_L is ________. **[2015]**



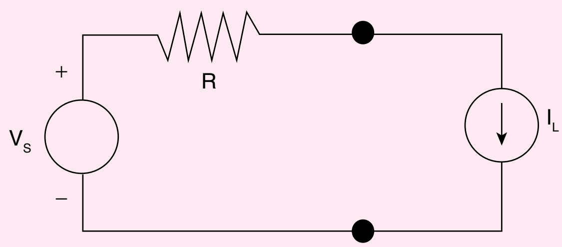
17. In the circuit shown, the Norton equivalent resistance (in Ω) across terminals $a - b$ is ______. **[2015]**



18. In the circuit shown in the figure, the maximum power (in watt) delivered to the resistor R is ___________ . **[2016]**



19. In the circuit shown below, V_S is a constant voltage source and I_L is a constant current load



The value of I_L that maximizes the power absorbed by the constant current load is: **[2016]**

(A) $\frac{V_{ref}}{2R}$ (B) $\frac{V_s}{2R}$
(C) $\frac{V_s}{R}$ (D) ∞

Answer Keys

Exercises

Practice Problems 1

1. B	2. B	3. C	4. B	5. A	6. D	7. C	8. B	9. D	10. B
11. C	12. D	13. A	14. D	15. C	16. A	17. B	18. A	19. A	20. D
21. A	22. C	23. B	24. A	25. A	26. C	27. B	28. A		

Practice Problems 2

1. A	2. A	3. B	4. B	5. B	6. C	7. B	8. D	9. C	10. A
11. C	12. A	13. A	14. D	15. B	16. C				

Previous Years' Questions

1. C	2. B	3. D	4. D	5. D	6. B	7. C	8. A	9. C	10. A
11. C	12. C	13. C	14. D	15. 1.9 to 2.1		16. 1.6 to 1.7		17. 1.3 to 1.35	
18. 0.8	19. B								