Design of Rigid Pavement

(lecture - 9 / 24/04/19)

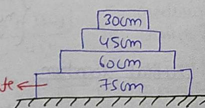
① **Design Parameter** → ① Modulus of subgrade reaⁿ $\boxed{K = \frac{p}{\Delta}}$

It is define as pressure required per unit deflection of (0.125 cm)

$$\boxed{K = \frac{p}{\Delta} = \frac{p\ \text{kg/cm}}{0.125\ \text{cm}}}$$

② Measure by = Plate load test

Standard Plate



$$K = \frac{p}{\Delta} = \frac{p}{\left(1.18 \cdot \frac{p \cdot a}{E_s}\right)} = \frac{E_s}{1.18\,a}$$

$$\Rightarrow K \cdot a = \frac{E_s}{1.18} = \text{constant}$$

** $\boxed{K_1 a_1 = K_2 a_2}$ Plate correction

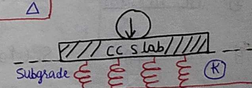
(How to find K)

Example $K = 21\ \text{kg/cm}^2 \rightarrow$ Plate size = 60cm then $K_1 a_1 = K_2 a_2$

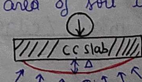
$$\left(K_1 \cdot \frac{75}{2} = 21 \times \frac{60}{2}\right) \quad K_1 = 16.8\ \text{kg/cm}^3$$

$$\frac{F}{A}^{P} = \frac{K}{A}^{K}\Delta \quad \text{then } p = K \cdot \Delta, \quad \boxed{K = \frac{p}{\Delta}} ***$$

{Modulus of subgrade reaⁿ can be considered as a spring constant per unit area of material subgrade as spring}



- ***[2017] **Radius of relative stiffness: (l)** the radius of area of soil which is effective in resisting deformation of slab



h = thickness of slab
K = subgrade Reaⁿ modulus
μ = Poisson's ratio
E = modulus of elasticity

$$\boxed{l = \left\{\frac{Eh^3}{12K(1-\mu^2)}\right\}^{\frac{1}{4}}} **$$

(b)

- **Equivalent radius of resisting section:** Radius of area of slab effecting in resisting bending moment

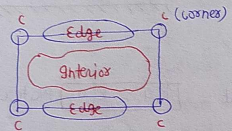
$$\boxed{\begin{aligned} b &= \sqrt{1.6a^2 + h^2} - 0.675h \quad \text{if } a < 1.724h \\ &= a \quad \text{if } a \geq 1.724h \end{aligned}}$$

E = modulus of elascity of slab
h = thickness of slab
K = Modulus of subgrade reaⁿ
μ = Poisson's ratio
a = Radius of contact area

I] Analysis of Stresses :

Stress
- load Stress
- temperature stress
  - Due to day-night temp variation → Warping Stress
  - Frictional Stress {due to seasonal temp. variation}

critical location



[Edge = 4
Interior = 2
Corner = 8]

- **Load stress** – Stress developed in longitudinal dirn of slab due to wheel load (edge pe maximum)

Magnitude

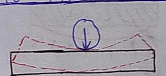
1] At interior $S_i\,(Kg/cm^2) = \frac{0.316P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 1.069\right]$

2] At Edge $S_e\,(Kg/cm^2) = \frac{0.572P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 0.359\right]$

3] At corner $S_c\,(Kg/cm^2) = \frac{3P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right]$

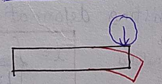
- Nature

At interior / edge



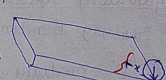
Top : compression
bottom : tension

At corner



Top : tension
bottom : compression

Note Position of crack due to corner load $\boxed{x = 2.58\sqrt{al}}$ **



Where: P = Wheel load (Kg)
h = thickness of slab (cm)
l = Radius of relative stiffness
b = Equivalent radius of resisting secn (cm)
a = Radius of contact area

② Warping Stress

It is type of temperature stress developed in longitudinal dirn of slab due to day night temperature variation

I] Magnitude

1] At interior → $S_{ti}\,(Kg/cm^2) = \frac{E\alpha T}{2}\left[\frac{C_x + \mu C_y}{(1-\mu^2)}\right]$

2] At Edge → $S_{te}\,(Kg/cm^2) = \left\{\frac{C_x E\alpha T}{2}, \frac{C_y E\alpha T}{2}\right\}$

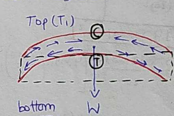
3] At corner → $S_{tc}\,(Kg/cm^2) = \frac{E\alpha T}{3(1-\mu)}\sqrt{\frac{a}{l}}$

E = modulus of elasticity of slab
α = coeff. of thermal expansion
T = temp. difference b/w top & bottom of slab
μ = Poisson ratio
l = Radius of relative stiffness
a = radius of contact area

[C_x & C_y are the factor which depend upon $\frac{L_x}{l}$ and $\frac{L_y}{l}$ resp]

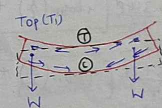
Nature

At day time ($T_1 > T_2$)



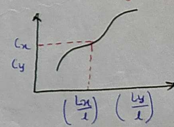
Top : compression
bottom : tension

At night time ($T_2 > T_1$)

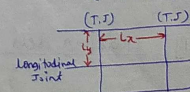


Top : tension
bottom : compression

[Stress developed due to resisting of weight downward dirn]
[not due to increasing temp]



(L_x & L_y are the spacing b/w transverse joint and longitudinal joint respect.)



③ Frictional Stress

Due to seasonal temp. variation slab expand or contract linearly

- this movement of slab is resisted by friction at bottom due to which stress developed in concrete which is called frictional stress.
- In summer season compressive stress developed and in winter season tensile.

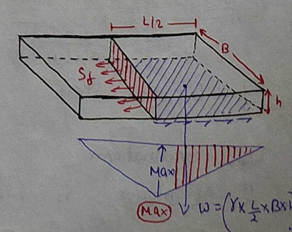
In winter season

Frictional force developed by half slab
= Resistive force by concrete

$f(\gamma \cdot \frac{L}{2} \times B \times h) = S_f \times B \times h$

$\boxed{S_f = \frac{1}{2}\gamma \cdot f \cdot L}$

$S_f \propto x$
at $x = 0$ (corner)
$S_f = 0$
At $x = \frac{L}{2}$ | $S_f = max$



Worst combination of stress

★★★★★

		Load Stress	Warping stress Day	Warping stress Night	Frictional stress Summer	Frictional stress Winter	Worst combination
Interior	Top	C	C	T	C	T	$S_I = (+S_i + S_{ti} + S_f)$
	bottom	$\boxed{T}$	$\boxed{T}$	C	C	$\boxed{T}$	At bottom during day time
Edge	Top	C	C	T	C	T	$S_E = (+S_e + S_{te} + S_f)$
	bottom	$\boxed{T}$	$\boxed{T}$	C	C	$\boxed{T}$	At bottom during day time
Corner	Top	$\boxed{T}$	C	$\boxed{T}$	~~C~~ 0	~~T~~ 0	$S_c = (+S_c + S_{tc})$
	bottom	C	T	C	~~C~~ 0	~~T~~ 0	At top during night

Pb (38) Design parameter $K = 15\,kg/cm^3$, $l = \left[\frac{Eh^3}{12K(1-\mu^2)}\right]^{\frac{1}{4}} = \left[\frac{3\times10^5\times20^3}{12\times15\times(1-0.15^2)}\right]^{\frac{1}{4}} = \boxed{60.77\,cm}$

$$b = \sqrt{1.6a^2 + h^2} - 0.675h$$
$$= \sqrt{1.6\times(15)^2 + 20^2} - 0.675\times(20) = 14\,cm$$

A] Load stress

$$S_i = \frac{0.316P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 1.069\right] = \frac{0.316\times5100}{(20)^2}\left[4\log_{10}\left(\frac{60.77}{14}\right) + 1.069\right] = \boxed{14.58\,kg/cm^2}$$

$$S_e = \frac{0.572P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 0.359\right] = \frac{0.572\times5100}{20^2}\left[4\log_{10}\left(\frac{60.77}{14}\right) + 0.359\right]$$

$$S_e = \boxed{21.21\,kg/cm^2}$$

$$S_c = \frac{3P}{h^2}\left(1 - \left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right) = \frac{3\times5100}{20^2}\left(1 - \left(\frac{15\sqrt{2}}{60.77}\right)^{0.6}\right) = \boxed{17.90\,kg/cm^2}$$

B] Warping stress

$$\frac{L_x}{l} = \frac{4.5m}{0.6077} = 7.4 \rightarrow C_x = 1$$
$$\frac{L_y}{l} = \frac{3.5}{0.6077} = 5.75 \rightarrow C_y = 0.84$$

After linear interpolation

$$S_{ti} = \frac{E\alpha T}{2}\left(\frac{C_x + \mu C_y}{1-\mu^2}\right) = \frac{3\times10^5\times10\times10^{-6}\times18}{2} = \frac{(1+0.15+0.84)}{(1-0.15^2)} = \boxed{31.10\,kg/cm^2}$$

$$S_{te} = \frac{C_x E\alpha T}{2} = \left(\frac{1\times3\times10^5\times10\times10^{-6}\times18}{2}\right) = \boxed{27\,kg/cm^2}$$

$$S_{tc} = \frac{E\alpha T}{3(1-\mu)}\sqrt{\frac{a}{l}} = \frac{3\times10^5\times10\times10^{-6}\times18}{3(1-0.15)}\sqrt{\frac{15}{60.77}} = \boxed{10.52\,kg/cm^2}$$

C] Frictional stress

$$S_f = \frac{1}{2}\gamma.f.L = \frac{1}{2}\times2400\times1.5\times4.5\times10^{-4} = 0.81\,kg/cm^2$$

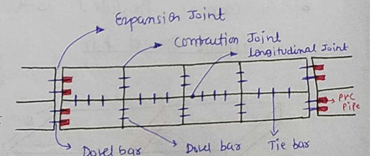
Worst combination →

1] At interior $S_I = (S_i + S_{ti} + S_f) = 14.58 + 31.1 + 0.81 = 46.49\,kg/cm^2$

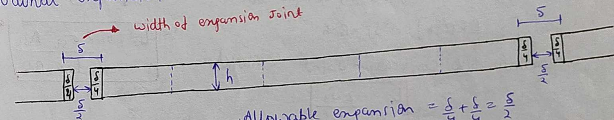
2] At edge $S_E = (S_e + S_{te} + S_f) = 21.21 + 27 + 0.81 = \cancel{46.49\,kg/cm^2}$ $\boxed{49.02\,kg/cm^2}$

3] At corner $S_c = (S_c + S_{tc}) = 17.90 + 10.52 = \boxed{28.42\,kg/cm^2}$

- **Design of Joints**
  - 1. Transverse Joint
    - Expansion Joint
    - contraction Joint
  - 2. Longitudinal Joint



1. Expansion Joint → are provided in transverse dirⁿ to allow longitudinal expansion of slab due to increase in environment temp.



Allowable expansion $= \frac{\delta}{4} + \frac{\delta}{4} = \frac{\delta}{2}$

$$\frac{\delta}{2} = L\alpha\Delta T \rightarrow (T_{max} - T_{construction})$$

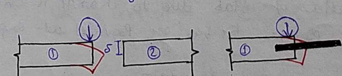
As per IRC: Width $= \delta \not> 2.5\,cm$
Spacing $= L \not> 140\,m$

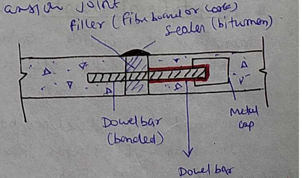
2007 ★★★ $\boxed{L = \frac{\delta}{2\alpha(T_m - T_c)}}$

problem (39) Spacing $= \frac{\delta}{2\alpha(T_{max} - T_{const})} = \frac{2\times10^{-2}}{2\times12\times10^{-6}(45-20)} = 33.33\,m \not> 140\,m$ (safe)

- **Objective to Provide dowel bar** – to transfer the wheel load from one slab to another and to reduce the differential deflection b/w the slab at joint

- arrangement of dowel bar at expansion joint





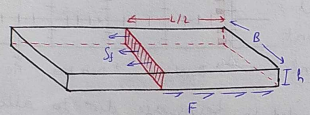
- **Specification of dowel bars**
  - Diameter = 25 m
  - length = 50 cm (60% unbonded and 40% bonded)
  - Depth = $\frac{h}{2}$
  - spacing = 300 mm (center to center)

- **Contraction Joint**

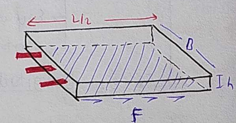
Contraction Joint are Provided in transverse dirⁿ to allow longitudinal contraction of slab due to decrease in environment temp

- It can be Provided with or without drowel bar

Case ① **Without Dowel bar**



Case-② **With Dowel bar**



Friction force developed by half slab = Resistance force by concrete

$$f \cdot \left[\gamma \cdot \frac{L}{2} \times B \times h\right] = S_f \times B \times h$$

Spacing b/w contraction Joint

$$= \boxed{L = \frac{2 S_f}{\gamma \cdot f}}$$

$$f\left[\gamma \cdot \frac{L}{2} \cdot B \times h\right] = \sigma_{st} \cdot A_{st}$$

$$\boxed{L = \frac{2\sigma_{st} \cdot A_{st}}{f \cdot \gamma \cdot B h}} \;**$$

As per IRC

spacing b/w contraction Joint ≯ 4.5m for PCC

Problem Ques- 40

$B = 4.52\,m$,
$h = 0.25\,m$
$f = 1.5$
$\sigma_{st} = 1400\,kg/cm^2$
$\phi = 12\,mm$
Center to center spacing $= 300\,mm$ (c/c)

No. of bars

$$n = \frac{B}{\text{spacing}} = \left(\frac{4.52}{0.8}\right) = 15.06 \simeq 16 \text{ bars}$$

$$A_{st} = n\frac{\pi}{4}\phi^2 = 16 \times \frac{\pi}{4}(12)^2 = 18.09\,mm^2 \simeq 18.09\,cm^2$$

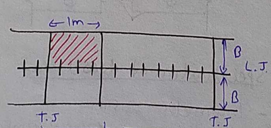
$$L = \frac{2\sigma_{st} \cdot A_{st}}{\gamma \cdot f \cdot B \cdot h}$$

$$\left(\frac{2 \times (1400 \times 18.09)\,kg}{(2400\,kg/m^3 \times 1.5 \times 4.52\,m \times 0.25\,m)}\right) = \boxed{12.45\,m}$$

kg/m

- **Longitudinal Joints**: is Provided in longitudinal dirⁿ to allow transverse expansion, contraction of slab due to change in environment temp.
- to hold slab in Position tie bar are Provided at expansion Joi longitudinal Joint.

Frictional force developed by ~~tie bars~~ 1m slab = Resistance force by tie bars per m slab.



$$f(\gamma \cdot 1(m) \times B \times h) = \sigma_{st} \cdot A_{st}$$

Per m slab

Area of steel required Per m slab

$$\boxed{A_{st} = \frac{f \cdot \gamma \cdot B \cdot h}{\sigma_{st}}}$$

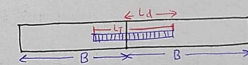
Assume diameter of bar $= \phi$

No. of tie bar per m slab $(n) = \left(\frac{A_{st}}{\frac{\pi}{4}\phi^2}\right)$

c/c spacing b/w tie bars $(S) = \left(\frac{1000}{n}\right)$

As Per IRC
Dia $= \phi$ ≯ 20mm
Spacing $= S$ ≯ 75cm

Length of tie bar (L_T)



$$L_T = 2L_d = 2 \times \left[\frac{\phi\,\sigma_{st}}{4\,\tau_{bd}}\right]$$

(length of tie bar)

2013

$$\boxed{L_T = \frac{\phi\,\sigma_{st}}{2\,\tau_{bd}}} \;**$$

[σ_{st} = Allowable stress in steel
τ_{bd} = Bound stress