Sample Paper 14

Class- X Exam - 2022-23

Mathematics - Standard

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A **20 marks**

(Section A consists of 20 questions of 1 mark each.)

1. The L.C.M. of ($2^3 \times 3 \times 5$) and ($2^4 \times 5 \times 7$) is:

(a) 1540 (b) 1680

(c) 1640 (d) 1200 1

2. A quadratic polynomial, whose zeros are -3 and 4 is:

(a) $x^2 + x + 5$ (b) $x^2 - x + 6$

(c) $x^2 - x - 12$ (d) $x^2 + 2x - 6$ 1

3. The 11^{th} term of the A.P.

$\sqrt{2}, 3\sqrt{2}, 5\sqrt{2},$ .... is:

(a) $21\sqrt{2}$ (b) $20\sqrt{2}$

(c) $17\sqrt{2}$ (d) $22\sqrt{2}$ 1

4. The nature of the quadratic equation $2x^2 + x + 4 = 0$ is:

(a) no real roots (b) real roots

(c) equal roots (d) None of these 1

5. The 2^{nd} term of the AP, if its $S_n = n^2 + 2n$ is:

(a) 4 (b) 2

(c) 8 (d) 5 1

6. The roots of $x + \frac{1}{x} = 2$ are:

(a) 5, 1 (b) 2, 3

(c) 1, 1 (d) 4, -2 1

7. A pair of linear equations, which has the unique solution $x = -1, y = 3$, is:

(a) $x - y = -4, x + 2y = 5$

(b) $2x + y = 0, x - 3y = 7$

(c) $x + y = 4, x + y = 7$

(d) $x + y = 0, x - y = 2$ 1

8. If the distance between the points (4, p) and (1, 0) is 5 units, then the value of p is:

(a) 3 (b) ± 4

(c) 5 (d) -3 1

9. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Then, the length of PQ is:

(a) $\sqrt{119}$ cm (b) 16 cm

(c) 15 cm (d) $\sqrt{211}$ cm 1

10. The distance between the points (0, 6) and (0, -2) is:

(a) 5 units (b) 6 units

(c) 8 units (d) 3 units 1

11. For the distribution given below, the median class is:

Marks	Frequency
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

(a) 20 – 30 (b) 60 – 70
(c) 30 – 40 (d) 40 – 50 1

12. The value of $(1 + \cos A)(1 - \cos A)\ \text{cosec}^2 A$ is:

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) $\frac{\sqrt{3}}{2}$ 1

13. Sarita buys a fish from a shop for her aquarium. The shopkeeper takes out a fish at random from a tank containing 10 male fishes and 12 female fishes. What is the probability that the fish taken out is a female fish?

(a) $\frac{3}{22}$ (b) $\frac{2}{22}$

(c) $\frac{5}{11}$ (d) $\frac{6}{11}$ 1

14. Write the formula used for calculating the median of a grouped frequency distribution.

(a) $l + \frac{\frac{N}{2} - cf}{f} \times h$ (b) $\frac{N}{2}$

(c) $\frac{N}{2}(l + cf)$ (d) $\frac{l + cf}{f}$ 1

15. The maximum value of $\frac{1}{\text{cosec}\,\theta}$ is:

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) 0 (d) 1 1

16. Find the number, if eight times of it is added to its square , the sum so obtained is –16.

(a) – 4 (b) 5
(c) 3 (d) 2 1

17. Write the formula used for calculating the mode of a grouped frequency distribution.

(a) $l + \frac{f_1 - f_0}{h}$

(b) $\frac{f_1 - f_0}{2f - f_0} \times h$

(c) $\frac{f_1 - f_0}{2} \times h$

(d) $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$ 1

18. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^2\theta + \cot^2\theta$ is:

(a) 2 (b) 3
(c) 4 (d) 5 1

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of reason (R).
Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

19. **Statement A (Assertion):** If a die is thrown, the probability of getting a number less than 3 and greater than 2 is zero.

Statement R (Reason): Probability of an impossible event is zero. 1

20. **Statement A (Assertion):** The radii of two cones are in the ratio 2:3 and their volumes in the ratio 1:3. Then the ratio of their heights is 3:2.

Statement R (Reason): The Volume of the cone $= \frac{1}{2}\pi r^2 h$. 1

SECTION - B

10 marks

(Section B consists of 5 questions of 2 marks each.)

21. Assuming that $\sqrt{2}$ is irrational, show that $5\sqrt{2}$ is an irrational number. 2

22. Find the greatest number that divides 338 and 59 and leaves remainders of 2 and 5 respectively.

OR

Find the coordinates of the point , where the perpendicular bisector of the line segment joining the points A(1,5) and B(4,6) cuts the y-axis. 2

23. Prove that the lengths of tangents drawn from an external point to a circle are equal. 2

24. Find the angle of elevation of the sun when the shadow of a pole 'h' metres high is $\sqrt{3}$ h metres long. 2

25. How many terms of AP : 18, 16, 14, ... make the sum zero?

OR

Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 7 cm. 2

SECTION - C **18 marks**

(Section C consists of 6 questions of 3 marks each.)

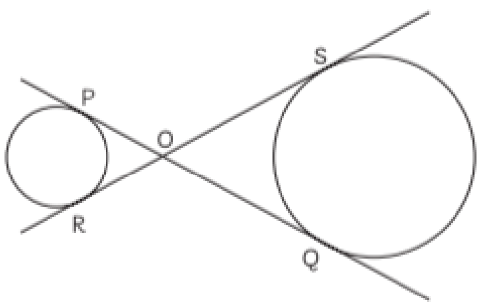
26. Determine the zeros of the polynomial $p(x) = x^3 - 2x^2$. Also verify the relationship between the zeros and the coefficient. 3

27. A sum of ₹ 250 was divided equally among a certain number of children. If there were 25 more children, each would have received 50 paise less. Find the number of children.

OR

The centre of a circle is C(2a, a, −7). Find the values of 'a' if the circle passes through the point P(11, −9) and has diameter $10\sqrt{2}$ units. 3

28. In the figure, PQ and RS are the common tangents of two circles intersecting at O.



Prove that: PQ = RS. 3

29. In ΔABC, $\angle A$ is acute. BD and CE are perpendiculars on AC and AB respectively. Prove that AB × AE = AC × AD. 3

30. If $x = a\cos^3\theta$ and $y = b\sin^3\theta$, then prove that $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3}$ 3

31. For the following frequency distribution, find the median marks :

Marks	Number of students
0-20	7
20-40	12
40-60	23
60-80	18
80-100	10

OR

I toss three coins together. The possible outcomes are no heads, 1 head, 2 heads and 3 heads. So, I can say that the probability of no heads is $\frac{1}{4}$. What is wrong with this conclusion? 3

SECTION - D **20 marks**

(Section D consists of 4 questions of 5 marks each.)

32. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio. 5

33. From the top of a building 60 m high, the angle of depression of the top and bottom of a vertical lamp-post are observed to be 30° and 60° respectively. Find the height of the lamp-post, and the distance between the top of building and the top of lamp-post.

OR

Prove that $\frac{1 + \sec A - \tan A}{1 + \sec A + \tan A} = \frac{1 - \sin A}{\cos A}$ 5

34. Solve the following pair of linear equations by the substitution method.

(A) $x + y = 14, x - y = 4$

(B) $s - t = 3, \frac{s}{3} + \frac{t}{2} = 6$

(C) $3x - y = 3, 9x - 3y = 9$

(D) $0.2x + 0.3y = 1.3, 0.4x + 0.5y = 2.3$

(E) $\sqrt{2}x + \sqrt{3}y = 0, \sqrt{3}x - \sqrt{8}y = 0$

OR

Find the mean and mode for the following frequency distribution:

Monthly consumption (in units)	Number of consumers
65-85	4
85-105	5
105-125	13
125-145	20
145-165	14
165-185	8
185-205	4

5

35. An arc subtends an angle of 60 at the centre of a circle with a radius of 21 cm. Find:

(A) the arc's length.

(B) the area of the sector the arc formed.

(C) the area of the segment that the corresponding chord forms. 5

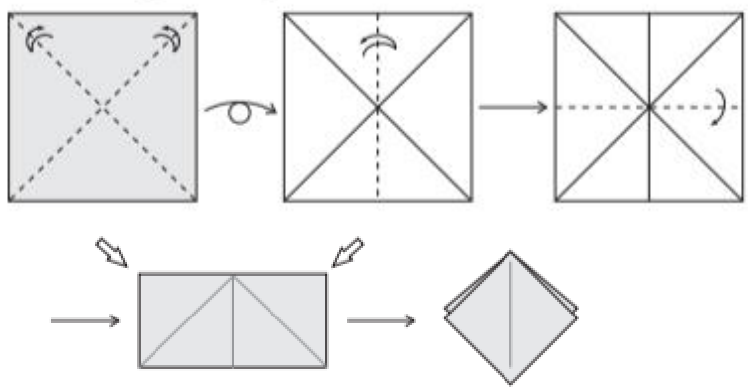
SECTION - E

12 marks

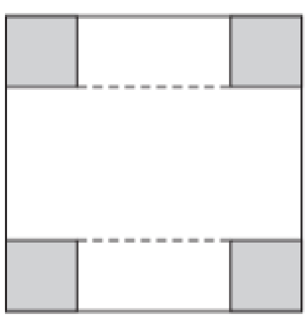
(Case Study Based Questions)

(Section E consists of 3 questions. All are compulsory.)

36. *'Origami' is the art of paper folding, which is often associated with Japanese culture. Gurmeet is trying to learn Origami using paper cutting and folding technique. A square base is sometimes referred to as a "preliminary" base or preliminary fold.*



Here is a 20 cm × 20 cm square. Gurmeet wants to first cut the squares of integral length from the corners and by folding the flaps along the sides.



On the basis of the above information, answer the following questions:

(A) Find the dimensions of the box with maximum volume. 1

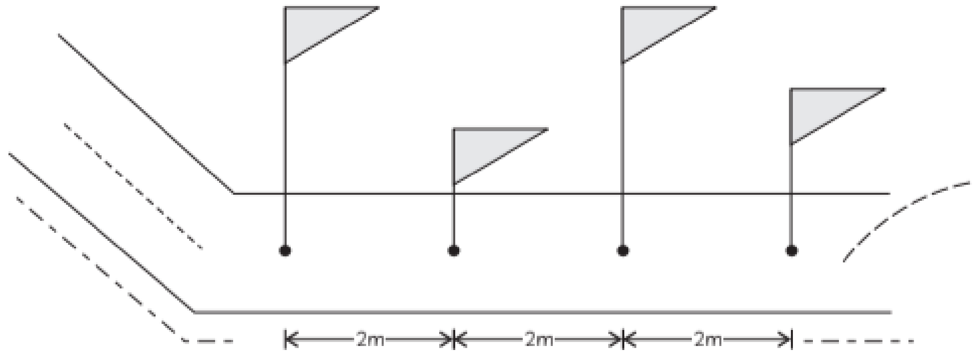
(B) Find the dimensions of the box with minimum volume. 1

(C) Find the equation relating the size of the square cut out and volume of the box.

OR

How many different sixes of boxes Gurmeet can make? If sides of the square are not integral length then find the number of boxes? 2

37. *The students of a school decided to beautify the school on the Annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag.*



Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flag were stored. She could carry only one flag at a time.

On the basis of the above information, answer the following questions:

(A) What is the position of the middle flag? 1

(B) Find total distance travelled for placing all the flags. 1

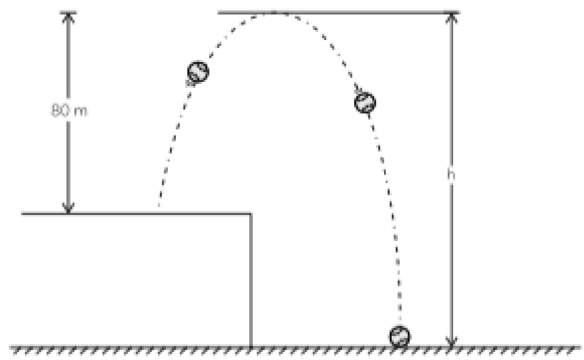
(C) Find total distance travelled for placing 13 flags on left.

OR

Find the maximum distance she travelled carrying a flag. 2

38. *Soumya throws a ball upwards, from a rooftop, 80 m above. It will reach a maximum height and then fall back to the ground. The height of the ball from the ground at time 't' is 'h', which is given by,*

$$h = -16t^2 + 64t + 80$$



On the basis of the above information, answer the following questions:

(A) What is the height reached by the ball after 1 second? 1

(B) What is the maximum height reached by the ball? 1

(C) What are the two possible times to reach the ball at the same height of 128 m?

OR

How long will the ball take to hit the ground? 2

SOLUTION

SECTION - A

1. *(b) 1680*

Explanation: LCM of $(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$

$\text{LCM} = 2^4 \times 3 \times 5 \times 7$

$= 1,680$

⚠ Caution

➥ *While calculating LCM, take the highest power of each multiple from the given numbers.*

2. *(c)* $x^2 - x - 12$

Explanation: A quadratic polynomial with zeros –3 and 4 is:

$$(x + 3)(x - 4)$$

i.e. $\quad x^2 - x - 12$

⚠ Caution

➥ *Remember that a quadratic polynomial cannot have more than two zeros.*

3. *(a)* $21\sqrt{2}$

Explanation: Here, 11^{th} term, $a_{11} = a + 10d$

$$= \sqrt{2} + 10(2\sqrt{2}) = 21\sqrt{2}$$

4. *(a) no real roots*

Explanation: For the equation $2x^2 + x + 4 = 0$

$$D = b^2 - 4ac$$
$$= 1 - 32 = -31 < 0$$

∴ The given equation has no real roots.

5. *(d) 5*

Explanation: Here, $a_2 = S_2 - S_1 = (2^2 + 2 \times 2) - (1^2 + 2 \times 1)$

$$= (4 + 4) - (1 + 2) = 5$$

6. *(c) 1, 1*

Explanation: The equation is, $x + \frac{1}{x} = 2$

$$x^2 - 2x + 1 = 0, (x - 1)^2 = 0$$

So, roots are 1, 1.

7. *(a)* $x - y = -4, x + 2y = 5$

Explanation: As $x = -1$, $y = 3$ is a point.
So many lines can pass through a point. Therefore, infinitely many pairs are possible in (a)

$$x - y = -4 \qquad x + 2y = 5$$
$$-1 - 3 = -4 \qquad -1 + 2 \times 3 = 5$$
$$-4 = -4 \qquad -1 + 6 = 5$$
$$5 = 5$$

8. *(b)* ± 4

Explanation:

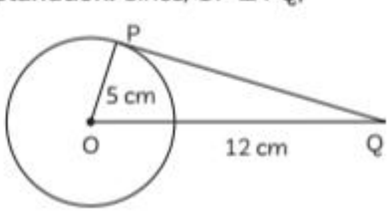
Here, $\sqrt{(4-1)^2 + (p-0)^2} = 5$

$\Rightarrow \quad 9 + p^2 = 25$

$\Rightarrow \quad p = \pm 4$

9. *(a)* $\sqrt{119}$ cm

Explanation: Since, OP ⊥ PQ.



$$PQ = \sqrt{OQ^2 - OP^2} = \sqrt{12^2 - 5^2}$$
$$= \sqrt{144 - 25} = \sqrt{119} \text{ cm}$$

10. *(c) 8 units*

Explanation:

Given, points are P(0, 6) and Q(0, –2)

By distance formula, distance between PQ is

$$PQ = \sqrt{(0-0)^2 + (-2-6)^2}$$
$$= \sqrt{0^2 + (-8)^2}$$
$$= \sqrt{0 + 64}$$
$$= 8 \text{ units}$$

11. *(c) 30 – 40*

Explanation: Here, $\frac{N}{2} = \frac{80}{2} = 40$

Then, median class is 30-40.

12. *(c) 1*

Explanation: Here,

$(1 + \cos A)(1 - \cos A)\,\text{cosec}^2 A$

$$= (1 - \cos^2 A)\,\text{cosec}^2 A$$
$$= \sin^2 A\,\text{cosec}^2 A \qquad [\sin^2 A + \cos^2 A = 1]$$
$$= 1 \qquad \left[\because \sin A = \frac{1}{\text{cosec } A}\right]$$

⚠ Caution

➥ *Use trigonometric identities wherever necessary.*

13. *(d)* $\frac{6}{11}$

Explanation: Total number of fishes = 10 + 12 = 22

Probability (female fish)

$$= \frac{\text{Total number of female fishes}}{\text{Total number of fishes}}$$
$$= \frac{12}{22}$$
$$= \frac{6}{11}$$

14. *(a)* $l + \frac{\frac{N}{2} - cf}{f} \times h$

Explanation: Median = $l + \frac{\frac{N}{2} - cf}{f} \times h$

15. *(d) 1*

Explanation: Since, $\frac{1}{\text{cosec}\,\theta} = \sin\theta$

And maximum value of $\sin\theta$ is 1 *i.e.* when

$$\theta = 90°$$

16. *(a)* −4

Explanation: Let the number be x

Then, according to the question

$$8x + x^2 = -16$$

$\Rightarrow \quad x^2 + 8x + 16 = 0$

$\Rightarrow \quad x^2 + 4x + 4x + 16 = 0$

$\Rightarrow \quad (x + 4)(x + 4) = 0$

$\Rightarrow \quad x = -4, -4$

Hence, the number is −4.

17. *(d)* $l+\left(\dfrac{f_1-f_2}{2f_1-f_0-f_2}\right)\times h$

Explanation:

$$\text{Median} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

18. *(a)* 2

Explanation: Given, $\tan\theta + \cot\theta = 2$

Then, on squaring both sides, we get

$\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 4$

$\Rightarrow \quad \tan^2\theta + \cot^2\theta = 4 - 2\tan\theta\cot\theta$

$\Rightarrow \quad \tan^2\theta + \cot^2\theta = 4 - 2\tan\theta \times \dfrac{1}{\tan\theta}$,

$= 4 - 2 = 2$

19. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)*

Explanation: The probability of getting a number less than 3 and greater than 2 is 0. Event given in Assertion is an impossible event.

20. *(d) Assertion (A) is false but reason (R) is true.*

Explanation: Ratio of volume

$$= \frac{\frac{1}{3}\pi\times(2x)^2\times h_1}{\frac{1}{3}\pi\times(3x)^2\times h_2}$$

$$\frac{1}{3} = \frac{4}{9}\times\frac{h_1}{h_2}$$

$$\frac{h_1}{h_2} = \frac{3}{4}$$

$$h_1 : h_2 = 3 : 4$$

Caution

➥ *Students should learn the formulae of all 3-D shapes.*

SECTION - B

21. Let $5\sqrt{2}$ be rational. Then,

$5\sqrt{2} = \dfrac{p}{q}$, where p and q are co-prime.

$\Rightarrow \quad \sqrt{2} = \dfrac{p}{5q}$

Here, $\dfrac{p}{5q}$ is rational, which implies $\sqrt{2}$ is rational, which is a contradiction, as $\sqrt{2}$ is irrational.

$\therefore \quad 5\sqrt{2}$ is an irrational number.

22. Since 338 and 59 divided by the required number leave remainder of 2 and 5 respectively.

$$338 - 2 = 336;\ 59 - 5 = 54$$

are completely divisible by the number

Now, we have to find HCF of 336 and 54.

3	336
2	112
2	56
2	28
2	14
7	7
	1

2	54
3	27
3	9
3	3
	1

$$336 = 2^4 \times 3 \times 7$$

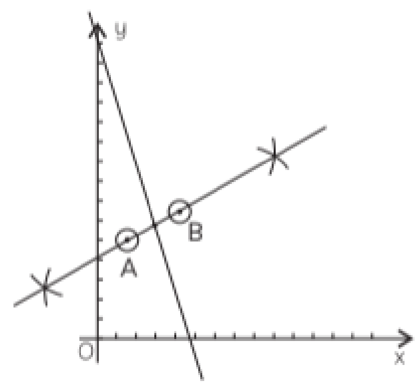
$$54 = 2 \times 3^3$$

$$\text{HCF} = 2 \times 3 = 6$$

Hence, the required number is 6.

OR

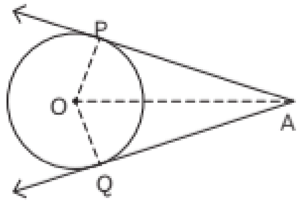
Since, the ⊥r bisector of line segment AB bisect the segment AB, i.e., ⊥r bisector of line segment AB passes through the mid-point of AB.



$$\text{Mid-point of AB} = \left(\frac{1+4}{2}, \frac{5+6}{2}\right)$$

$$P = \left(\frac{5}{2}, \frac{11}{2}\right)$$

23. Let AP and AQ be the two tangents drawn to the circle from the external point A.



We need to show that AP = AQ.

Join OA, OP and OQ.

Consider ΔOPA and ΔOQA.

Here, OQ = OP (radii of the circle)

OA = OA (common)

$\angle$OPA = $\angle$OQA [Each 90°]

So, $\Delta OPA \cong \Delta OQA$

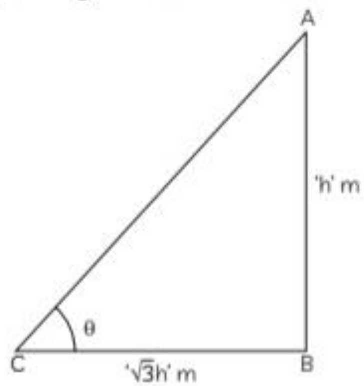
$\Rightarrow$ PA = QA or AP = AQ

Caution

Remember that the point, where tangent touches the circle, is perpendicular to the radius.

24. Here, AB is a pole of height 'h' m and its shadow BC of length '$\sqrt{3}h$' m

Let, the angle of elevation be θ.



Then, $\tan\theta = \dfrac{AB}{BC} = \dfrac{h}{\sqrt{3}h}$

$\Rightarrow$ $\tan\theta = \dfrac{1}{\sqrt{3}}$

$\Rightarrow$ $\tan\theta = \tan 30°$

$\therefore$ $\theta = 30°$

Caution

Draw the figure, after reading the question keenly and carefully.

25. Let n term of the given AP make the sum zero.

Then,

$$\frac{n}{2}[18\times2+(n-1)(-2)] = 0$$

$\Rightarrow$ $36 - 2(n-1) = 0$

$\Rightarrow$ $2n - 2 = 36$

$\Rightarrow$ $2n = 38$

$\Rightarrow$ $n = 19$

OR

The base radius and the vertical height of the largest cone that can be carved out of a cube are $\frac{7}{2}$ cm and 7 cm respectively.

So, volume $= \dfrac{1}{3}\pi\left(\dfrac{7}{2}\right)^2(7)\ \text{cm}^2$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2} \times 7\ \text{cm}^2$

$= 89.83\ \text{cm}^2$

SECTION - C

26. Here, $p(x) = x^3 - 2x^2$

For calculating zeros, put p (x) = 0

$x^3 - 2x^2 = 0$

$\Rightarrow$ $x^2(x-2) = 0$

$\Rightarrow$ $x = 0, 0, 2$

The required zeros are 0, 0 and 2.

Here, $\alpha = 0$, $\beta = 0$ and $\gamma = 2$

$$\alpha + \beta + \gamma = 0 + 0 + 2 = 2 = \frac{-\text{ coefficient of } x^2}{\text{coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 0\times0 + 0\times0 + 2\times0 = 0 = \frac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$$

$$\alpha\beta\gamma = 0\times0\times2 = 0 = \frac{-\text{constant term}}{\text{coefficient of } x^3}$$

27. Let the number of children be 'x'. Then,

amount received by each child = ₹$\left(\dfrac{250}{x}\right)$

When there are $(x + 25)$ children,

amount received by each child = ₹$\left(\dfrac{250}{x+25}\right)$

As per the question,

$$\frac{250}{x} - \frac{250}{x+25} = \frac{1}{2}$$

[$\because$ 50 paise = ₹ $\frac{1}{2}$]

$$250\left[\frac{x + 25 - x}{x(x + 25)}\right] = \frac{1}{2}$$

$\Rightarrow \quad x(x + 25) = 12500$

$\Rightarrow \quad x^2 + 25x - 12500 = 0$

$\Rightarrow \quad x^2 + 125x - 100x - 12500 = 0$

$\Rightarrow \quad x(x + 125) - 100\,(x + 125) = 0$

$\Rightarrow \quad (x + 125)\,(x - 100) = 0$

$\Rightarrow \quad x + 125 = 0 \quad \text{or} \quad x - 100 = 0$

$\Rightarrow \quad x = 100$

$(\because\ x \neq -125)$

Hence, there were 100 children in all.

OR

Let, the given point $P(x_1, y_1) = (11, -9)$ lie on a circle with centre $C(x_2, y_2) = (2a, a - 7)$ and radius 'r'.

Then, $\quad PC = r \quad$ .... (i)

Given, that the diameter of circle is $10\sqrt{2}$ units.

$$r = \frac{\text{diameter}}{2}$$

$$= \frac{10\sqrt{2}}{2} = 5\sqrt{2} \text{ units} \quad .....(ii)$$

By distance formula,

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$P = r = \sqrt{(11 - 2a)^2 + (-9 - a + 7)^2}$$

$$= \sqrt{5a^2 - 40a + 125}$$

From (i) and (ii), we have

$5a^2 - 40a + 125 = \left(5\sqrt{2}\right)^2$

$\Rightarrow 5a^2 - 40\,a + 75 = 0$

$\Rightarrow \quad (a - 5)\,(5a - 3) = 0$

$\Rightarrow \quad a = 5, \frac{3}{5}$

Hence, the value of 'a' are 5 and $\frac{3}{5}$

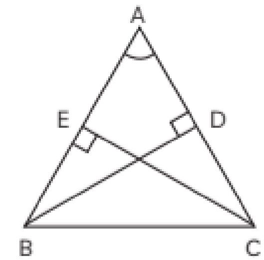
28. From the figure, OP = OR and OS = OQ

$\Rightarrow \quad PQ = PO + OQ$

$= OR + OS$

$= RS$

29. In Δs AEC and ADB,



$\angle A = \angle A$ [common angle]

$\angle AEC = \angle ADB$ [each 90°]

So, by AA similarity criterion, $\Delta AEC \sim \Delta ADB$

$\Rightarrow \quad \frac{AE}{AD} = \frac{AC}{AB}$

$\Rightarrow \quad AB \times AE = AC \times AD$

30. $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = \left(\frac{a\cos^3\theta}{a}\right)^{\frac{2}{3}} + \left(\frac{b\cos^3\theta}{b}\right)^{\frac{2}{3}}$

$= \cos^2\theta + \sin^2\theta$

$= 1$

31.

Marks	Number of Students (f_s)	C.f.	
0-20	7	7	
20-40	12	19	
40-60	23	42	Median class
60-80	18	60	
80-100	10	70	
Total	70		

Here, $\quad N = 70$

Then, $\quad \frac{N}{2} = 35$

median class is 40-60

Then, $\quad M_e = l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times h$

$= 40 + \left(\frac{35 - 19}{23}\right) \times 20$

$= 40 + \frac{16}{23} \times 20$

$= 40 + 13.91$

$= 53.91$

$\simeq 54$

Hence, the median of the given data is 54.

OR

Toss 3 coins getting heads in one = $\frac{1}{2}$, so in all

three = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

So outcomes of 1 head is $\frac{3}{8}$ and 2 head is $\frac{3}{8}$

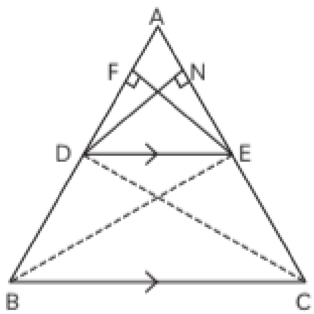
and remaining 3 heads = $1 - \frac{1}{8} - \frac{3}{8} - \frac{3}{8}$

$= \frac{1}{8}$

So, probability of getting no heads is $\frac{1}{8}$ not $\frac{1}{4}$

SECTION - D

32. ABC is a triangle and DE is a line parallel to side BC which cuts AB at D and AC at E.



We need to prove that:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Join BE and CD and draw EF ⊥ AB and DN ⊥ AC.

Now, $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times BD \times EF} = \frac{AD}{BD}$ ...(i)

$\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN} = \frac{AE}{EC}$ ...(ii)

But ΔBDE and ΔCDE are on the same base DE and between the same parallels DE and BC.

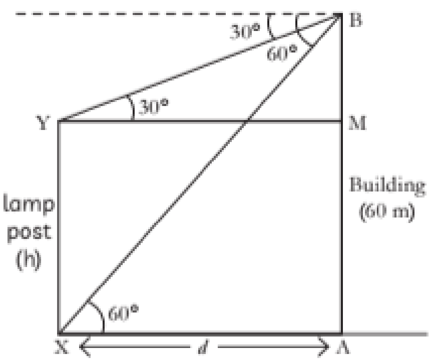
So, $ar(\Delta BDE) = ar(\Delta CDE)$ ...(iii)

$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{ar(\Delta ADE)}{ar(\Delta CDE)}$

Hence, by (i) and (ii), we have $\frac{AD}{BD} = \frac{AE}{EC}$, or

$\frac{AD}{DB} = \frac{AE}{EC}$

33. Let 'h' metres be the height of the lamp-post and 'd' metres be the distance between feet of the lamp post and the building.



Then,

From right ΔBMY, we have:

$$\frac{BM}{YM} = \tan 30°$$

$\Rightarrow \quad \frac{60 - h}{d} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad d = \sqrt{3}(60 - h)$ ...(i)

From right ΔBAX, we have:

$$\frac{BA}{XA} = \tan 60°$$

$\Rightarrow \quad \frac{60}{d} = \sqrt{3}$

$\Rightarrow \quad d = 20\sqrt{3}$ ...(ii)

From (i) and (ii),

$20\sqrt{3} = \sqrt{3}(60 - h)$

$\Rightarrow \quad h = 40$ m and $d = 20\sqrt{3}$ m

Now: $BY = \sqrt{YM^2 + BM^2} = \sqrt{d^2 + 20^2}$

$= \sqrt{1200 + 400} = \sqrt{1600}$

$= 40$ m

Thus, the distance between the top of the building and the top of lamp-post is 40 m.

OR

$$LHS = \frac{1 + \sec A - \tan A}{1 + \sec A + \tan A}$$

$$= \frac{1 + \frac{1}{\cos A} - \frac{\sin A}{\cos A}}{1 + \frac{1}{\cos A} + \frac{\sin A}{\cos A}} = \frac{\frac{\cos A + 1 - \sin A}{\cos A}}{\frac{\cos A + 1 + \sin A}{\cos A}}$$

$$= \frac{(\cos A + 1) - \sin A}{(\cos A + 1) + \sin A} \times \frac{\cos A + 1 - \sin A}{\cos A + 1 - \sin A}$$

$$= \frac{(\cos A + 1 - \sin A)^2}{(\cos A + 1)^2 - \sin^2 A}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\cos A - 2\sin A - 2\sin A\cos A}{\cos^2 A + 1 + 2\cos A - \sin^2 A}$$

$$= \frac{2 + 2\cos A - 2\sin A - 2\sin A\cos A}{\cos^2 A + (1 - \sin^2 A) + 2\cos A}$$

$$= \frac{2(1 + \cos A) - 2\sin A(1 + \cos A)}{2\cos^2 A + 2\cos A}$$

$$= \frac{(1 + \cos A)\,2(1 - \sin A)}{2\cos A(1 + \cos A)}$$

$$= \frac{1 - \sin A}{\cos A} = \text{RHS}$$ Hence, Proved

34. (A) $x + y = 14$ ...(i)

$x - y = 4$ ...(ii)

$x = 4 + y$ from equation (ii)

Putting this in equation (i), we get

$4 + y + y = 14$

$\Rightarrow 2y = 10$

$\Rightarrow y = 5$

Putting value of y in equation (i), we get

$\Rightarrow x + 5 = 14$

$\Rightarrow x = 14 - 5 = 9$

Therefore, $x = 9$ and $y = 5$

(B) $s - t = 3$ ...(i)

$\frac{s}{3} + \frac{t}{2} = 6$ ...(ii)

Putting $s = 3 + t$ in equation (ii), we get

$\frac{3+t}{3} + \frac{t}{2} = 6$

$\Rightarrow \frac{6+2t+3t}{6} = 6$

$\Rightarrow 5t + 6 = 36$

$\Rightarrow 5t = 30$

$\Rightarrow t = 6$

Putting value of t in equation (i), we get

$\Rightarrow s - 6 = 3$

$\Rightarrow s = 3 + 6 = 9$

Therefore, $t = 6$ and $s = 9$

(C) $3x - y = 3$ ...(i)

$9x - 3y = 9$ ...(ii)

Comparing equation $3x - y = 3$ with $a_1x + b_1y + c_1 = 0$ and equation $9x - 3y = 9$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 3$, $b_1 = -1$, $c_2 = -3$, $a_2 = 9$, $b_2 = -3$ and $c_2 = -9$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, we have infinite many solutions of x and y.

(D) $0.2x + 0.3y = 1.3$ ...(i)

$0.4x + 0.5y = 2.3$ ...(ii)

Using equation (i), we can say that

$0.2x = 1.3 - 0.3y$

$\Rightarrow x = \frac{1.3 - 0.3y}{0.2}$

Putting this in equation (ii), we get

$0.4\left(\frac{1.3 - 0.3y}{0.2}\right) + 0.5y = 2.3$

$\Rightarrow 2.6 - 0.6y + 0.5y = 2.3$

$\Rightarrow -0.1y = -0.3$

$\Rightarrow y = 3$

Putting value of y in (i), we get

$\Rightarrow 0.2x + 0.3(3) = 1.3$

$\Rightarrow 0.2x + 0.9 = 1.3$

$\Rightarrow 0.2x = 0.4$

$\Rightarrow x = 2$

Therefore, $x = 2$ and $y = 3$

(E) $\sqrt{2}x + \sqrt{3}y = 0$ ...(i)

$\sqrt{3}x - \sqrt{8}y = 0$ ...(ii)

Using equation (i), we can say that

$x = \frac{-\sqrt{3}y}{\sqrt{2}}$

Putting this in equation (ii), we get

$\sqrt{3}\left(\frac{-\sqrt{3}y}{\sqrt{2}}\right) - \sqrt{8}y = 0$

$\Rightarrow \frac{-3y}{\sqrt{2}} - \sqrt{8}y = 0$

$\Rightarrow y\left(\frac{-3}{\sqrt{2}} - \sqrt{8}\right) = 0$

$\Rightarrow y = 0$

Putting value of y in (i), we get $x = 0$

Therefore, $x = 0$ and $y = 0$

OR

Mode: Here, modal class is 125 – 145

For this class,

$l = 125,\ f_1 = 20,\ f_0 = 13,\ f_2 = 14,\ h = 20$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

So, $\text{Mode} = 125 + \frac{20 - 13}{40 - 13 - 14} \times 20$

$= 135.77$

Mean:

Monthly consumption	No. of consumers (f_i)	Class mark (x_i)	$u_i = \frac{x_i - A}{h}$ where **A = 135; h = 20**	$f_i u_i$
65–85	4	75	–3	–12
85–105	5	95	–2	–10
105–125	13	115	–1	–13
125–145	20	135	0	0
145–165	14	155	1	14
165–185	8	175	2	16
185–205	4	195	3	12
	Σf_i **= 68**			$\Sigma f_i u_i$ **= 7**

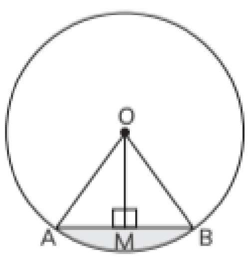
$$\text{Mean} = 135 + \frac{7}{68} \times 20$$

$$= 135 + \frac{140}{68}$$

$$= 135 + 2.06$$

$$= 137.06$$

35.



Given, $r = 21$ cm and $\theta = 60°$

(A) Length of arc $= \frac{\theta}{360°} \times 2\pi r$

$$= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21$$

$$= 22 \text{ cm}$$

(B) Area of the sector $= \frac{\theta}{360°} \times \pi r^2$

$$= \frac{60°}{360°} \times \frac{22}{7} \times 21 \times 21$$

$$= 231 \text{ cm}^2$$

(C) Area of segment by corresponding chord

$$= \frac{\theta}{360°} \times \pi r^2 - \text{Area of } \Delta OAB$$

$\Rightarrow$Area of segment = 231 – Area of ΔOAB ...(i)

In right angled triangle OMA and OMB,

OM = OB [Radii of the same circle]

OM = OM [Common]

$\therefore$ $\Delta OMA \cong \Delta OMB$ [RHS congruency]

$\therefore$ $AM \cong BM$ [By CPCT]

$\therefore$ M is the mid-point of AB and $\angle AOM = \angle BOM$

$\Rightarrow \angle AOM = \angle BOM = \frac{1}{2} \angle AOB = \frac{1}{2} \times 60° = 30°$

Therefore, in right angled triangle OMA,

$$\cos 30° = \frac{OM}{OA}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{21}$$

$$\Rightarrow OM = \frac{21\sqrt{3}}{2} \text{cm}$$

Also, $\sin 30° = \frac{AM}{OA}$

$$\Rightarrow \frac{1}{2} = \frac{AM}{21}$$

$$\Rightarrow AM = \frac{21}{2} \text{cm}$$

$$\therefore AB = 2\,AM = 2 \times \frac{21}{2} = 21 \text{ cm}$$

$$\therefore \text{Area of } \Delta OAB = \frac{1}{2} \times AB \times OM$$

$$= \frac{1}{2} \times 21 \times \frac{21\sqrt{3}}{2}$$

$$= \frac{441\sqrt{3}}{4}$$

Using eq. (i),

Area of segment formed by corresponding chord $= \left(231 - \frac{441\sqrt{3}}{4}\right)$ cm^2

SECTION - E

36. (A) On Calculating the volume of the boxes given in the options. The box with dimension $14 \times 14 \times 3$ has maximum volume as 588.

(B) On Calculating the volume of the boxes given in the option. The box with dimensions $18 \times 18 \times 1$ has minimum volume.

(C) Let the width of square of each side be 'x'

Then, sides of box are $20 - 2x$, $20 - 2x$ and x

$$\text{Volume} = lbh$$
$$= (20 - 2x)(20 - 2x)x$$
$$= (400 - 40x - 40x + 4x^2)x$$
$$= 4x^3 - 80x^2 + 400x$$

OR

Different size of squares are = $18 \times 18 \times 1$, $16 \times 16 \times 2$, $14 \times 14 \times 3$, $12 \times 12 \times 4$, $10 \times 10 \times 5$, $8 \times 8 \times 6$....

As the side length of any value could be cut out from the square and it could be infinite in number.

37. (A)

There are 27 flags. So the middle most flag is 14th flag.

(B) Total distance travelled = 13 flags on left side + 13 flags on right side

$$= 364 + 364$$
$$= 728 \text{ m}$$

(C) For placing first placing she go 2 m and come back 2 m. Then for second flag, she goes 4 m and come back 4 m and so one ....

Distance travelled = 4 , 8 , 12, ...

Then it forms an A.P. with $a = 4$, $d = 4$ and $n = 13$

$$\text{Then } S_{13} = \frac{13}{2}[8 + 12 \times 4]$$

$$= \frac{13}{2}(56) = 364 \text{ m}$$

Caution

➥ *Read such types of questions which are based on situation atleast twice and match it with the figure before attempting it. It helps to understand, what is being asked.*

OR

The maximum distance that she covered in placing will be the 13th flag on both side

$$\text{Then, } a_{13} = a + (n-1)d$$
$$= 4 + (13 - 1) \times 4$$
$$= 4 + 48 = 52$$

∴ From carrying the flag to its position,

$$\text{the covered distance} = \frac{52}{2}$$
$$= 26 \text{ m}$$

38. (A) On the basis of given equation,

$h = -16t^2 + 64t + 80$

when, $t = 1$ second

$h = -16(1)^2 + 64(1) + 80$

$= -16 + 144 = 128$ m

(B) Rearrange the given equation, by completing the square, we get

$h = -16(t^2 - 4t - 5)$

$= -16[(t - 2)2 - 9]$

$= -16(t - 2)2 + 144$

Height is maximum, when $t = 2$

∴ Maximum height = 144 m

(C) Since, $h = -16t^2 + 16t^2 + 80$

$\Rightarrow \quad 128 = -16t^2 + 64t^2 + 80$

$\Rightarrow \quad 16t^2 + 64t + 80 - 128 = 0$

$\Rightarrow \quad 16t^2 + 64t - 48 = 0$

$\Rightarrow \quad t^2 - 4t + 3 = 0$

$\Rightarrow \quad t^2 + 3t - t + 3 = 0$

$\Rightarrow \quad (t - 3)(t - 1) = 0$

$\Rightarrow \quad t = 3, 1$

OR

When ball hits the ground, $h = 0$

$-16t^2 + 64t + 80 = 0$

∴ $\quad t^2 - 4t - 5 = 0$

$(t - 5)(t + 1) = 0$

$t = 5$ or $t = -1$

Since, time cannot be negative, so the time = 5 seconds.

$$= \frac{1}{2} \times 21 \times \frac{21\sqrt{3}}{2}$$