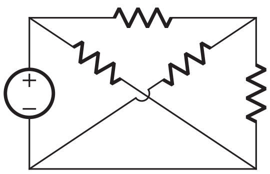
CHAPTER

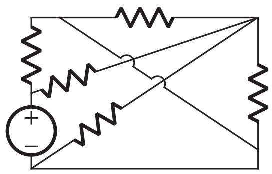
1.2

GRAPH THEORY

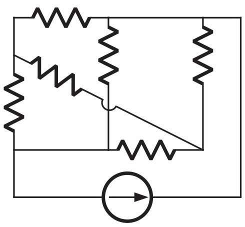
1. Consider the following circuits :



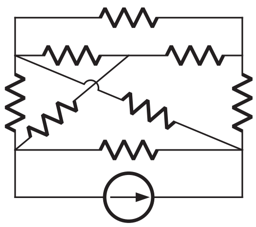
(1)



(2)



(3)



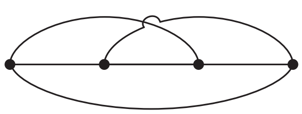
(4)

The planner circuits are

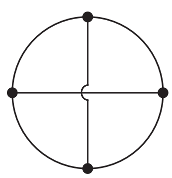
(A) 1 and 2 (B) 2 and 3

(C) 3 and 4 (D) 4 and 1

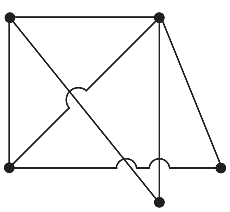
2. Consider the following graphs



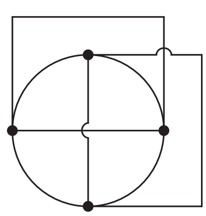
(1)



(2)



(3)



(4)

Non-planner graphs are

(A) 1 and 3 (B) 4 only

(C) 3 only (D) 3 and 4

3. A graph of an electrical network has 4 nodes and 7 branches. The number of links l, with respect to the chosen tree, would be

(A) 2 (B) 3

(C) 4 (D) 5

4. For the graph shown in fig. P.1.1.4 correct set is

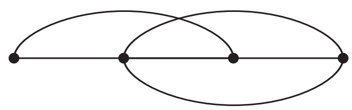


Fig. P.1.1.4

	Node	Branch	Twigs	Link
(A)	4	6	4	2
(B)	4	6	3	3
(C)	5	6	4	2
(D)	5	5	4	1

5. A tree of the graph shown in fig. P.1.2.5 is

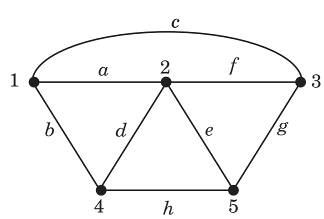


Fig. P.1.2.5

(A) $a\ d\ e\ h$ (B) $a\ c\ f\ h$

(C) $a\ f\ h\ g$ (D) $a\ e\ f\ g$

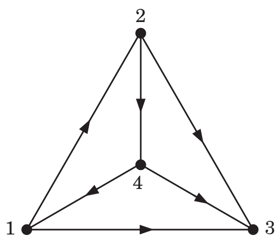
(A) $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & -1 \\ -1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$

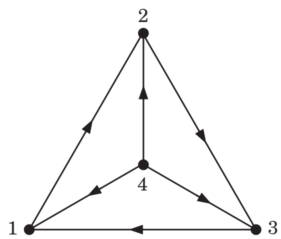
13. The incidence matrix of a graph is as given below

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

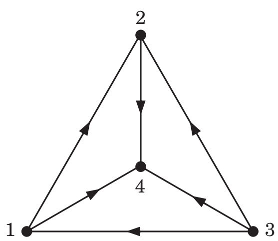
The graph is



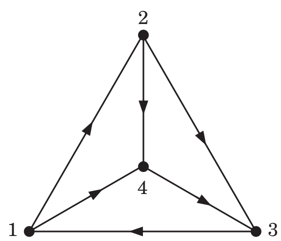
(A)



(B)



(C)

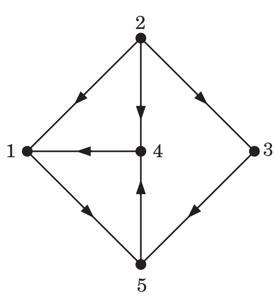


(D)

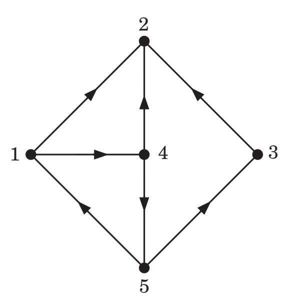
14. The incidence matrix of a graph is as given below

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

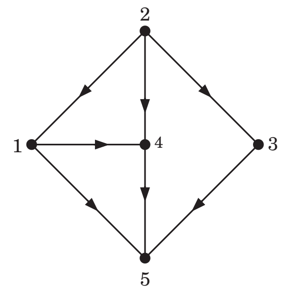
The graph is



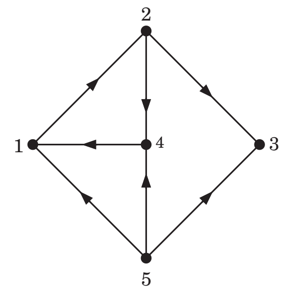
(A)



(B)



(C)

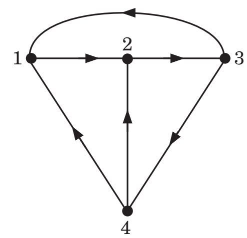


(D)

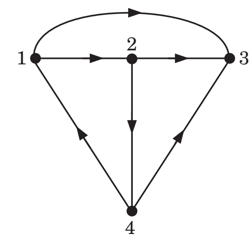
15. The incidence matrix of a graph is as given below

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

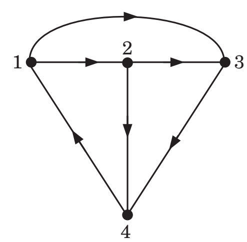
The graph is



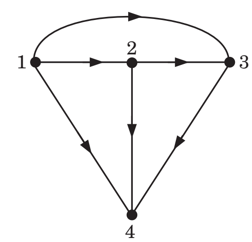
(A)



(B)



(C)



(D)

16. The graph of a network is shown in fig. P.1.1.16. The number of possible tree are

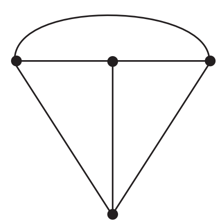


Fig. P.1.1.16

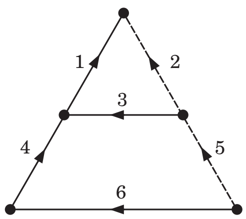
(A) 8 (B) 12

(C) 16 (D) 20

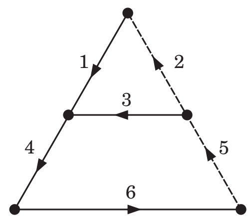
22. The fundamental cut-set matrix of a graph is

$$\mathbf{Q}_F = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

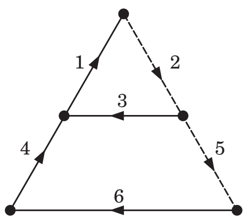
The oriented graph of the network is



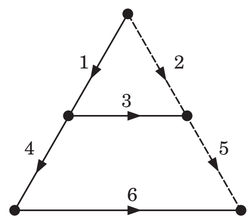
(A)



(B)



(C)



(D)

23. A graph is shown in fig. P.1.2.23 in which twigs are solid line and links are dotted line. For this chosen tree fundamental set matrix is given below.

$$\mathbf{B}_F = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

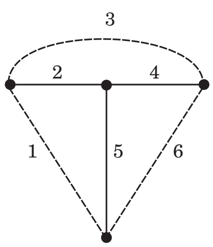
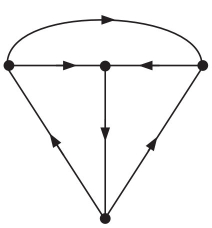
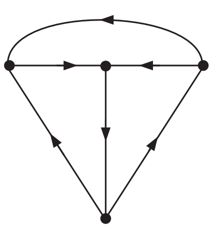


Fig. P. 1.2.23

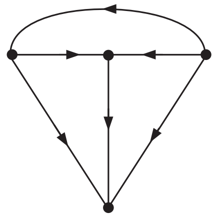
The oriented graph will be



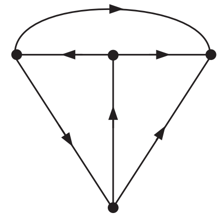
(A)



(B)



(C)



(D)

24. A graph is shown in fig. P.1.2.24 in which twigs are solid line and links are dotted line. For this tree fundamental loop matrix is given as below

$$\mathbf{B}_F = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

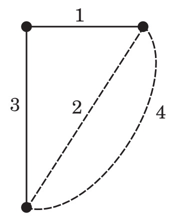
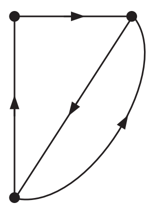
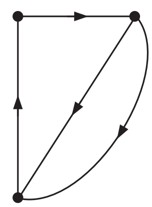


Fig. P.1.2.24

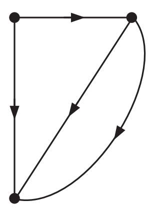
The oriented graph will be



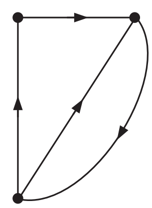
(A)



(B)



(C)



(D)

25. Consider the graph shown in fig. P.1.2.25 in which twigs are solid line and links are dotted line.

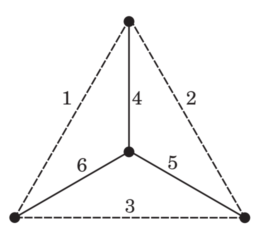
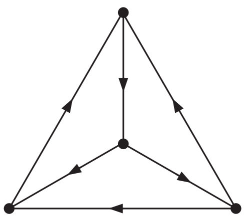


Fig. P. 1.2.25

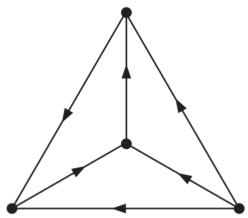
A fundamental loop matrix for this tree is given as below

$$\mathbf{B}_F = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix}$$

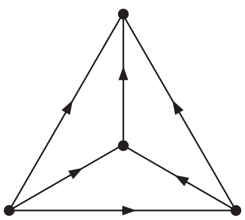
The oriented graph will be



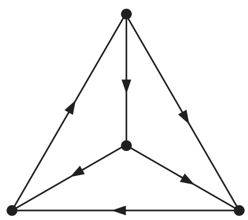
(A)



(B)



(C)



(D)

26. In the graph shown in fig. P.1.2.26 solid lines are twigs and dotted line are link. The fundamental loop matrix is

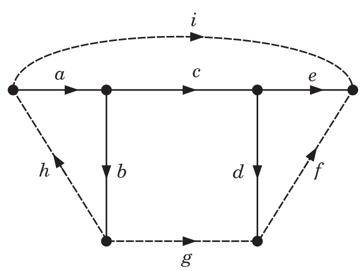


Fig. P.1.2.26

(A) $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

27. Branch current and loop current relation are expressed in matrix form as

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

where i_j represent branch current and I_k loop current. The number of independent node equation are

(A) 4 (B) 5

(C) 6 (D) 7

28. If the number of branch in a network is b, the number of nodes is n and the number of dependent loop is l, then the number of independent node equations will be

(A) $n + l - 1$ (B) $b - 1$

(C) $b - n + 1$ (D) $n - 1$

Statement for Q.29–30:

Branch current and loop current relation are expressed in matrix form as

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

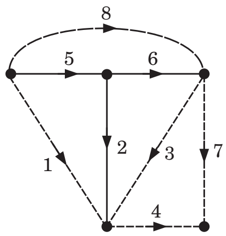
where i_j represent branch current and I_k loop current.

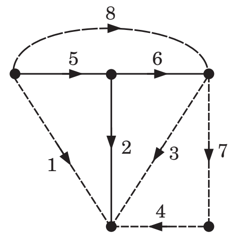
29. The rank of incidence matrix is

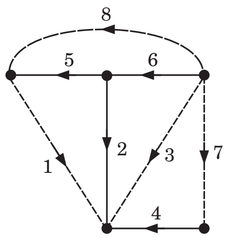
(A) 4 (B) 5

(C) 6 (D) 8

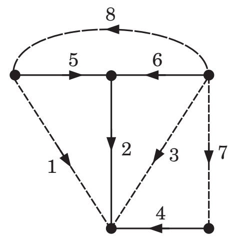
30. The directed graph will be







(C)



(D)

31. A network has 8 nodes and 5 independent loops. The number of branches in the network is

(A) 11 (B) 12

(C) 8 (D) 6

32. A branch has 6 node and 9 branch. The independent loops are

(A) 3 (B) 4

(C) 5 (D) 6

Statement for Q.33–34:

For a network branch voltage and node voltage relation are expressed in matrix form as follows:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

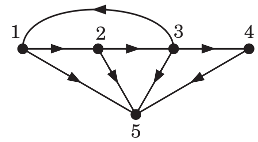
where v_i is the branch voltage and V_k is the node voltage with respect to datum node.

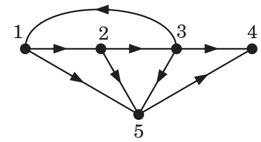
33. The independent mesh equation for this network are

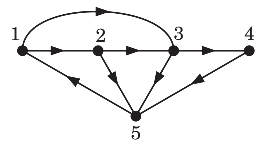
(A) 4 (B) 5

(C) 6 (D 7

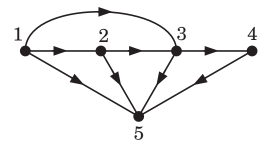
33. The oriented graph for this network is







(C)



(D)

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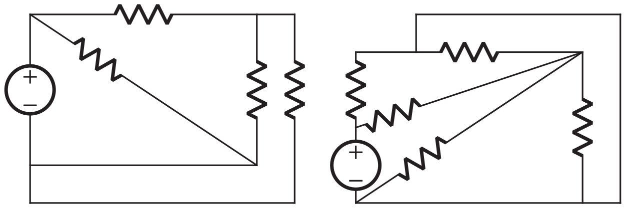
SOLUTIONS

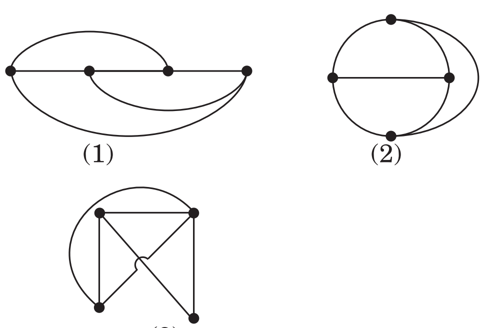
1. (A) The circuit 1 and 2 are redrawn as below. 3 and 4 can not be redrawn on a plane without crossing other branch.



(1) (2)

Fig. S1.2.1

2. (B) Other three circuits can be drawn on plane without crossing



(1) (2)

(3)

Fig. S1.2.1

3. (C) $l = b - (n - 1) = 4$.

4. (B) There are 4 node and 6 branches.

$t = n - 1 = 3, \quad l = b - n + 1 = 3$

5. (C) From fig. it can be seen that $a\ f\ h\ g$ is a tree of given graph

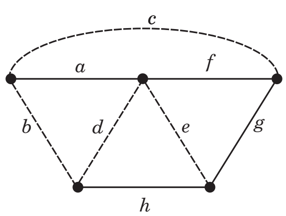


Fig. S 1.2.5

6. (B) From fig. it can be seen that $a\ d\ f$ is a tree.

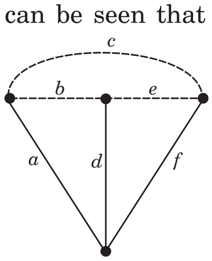
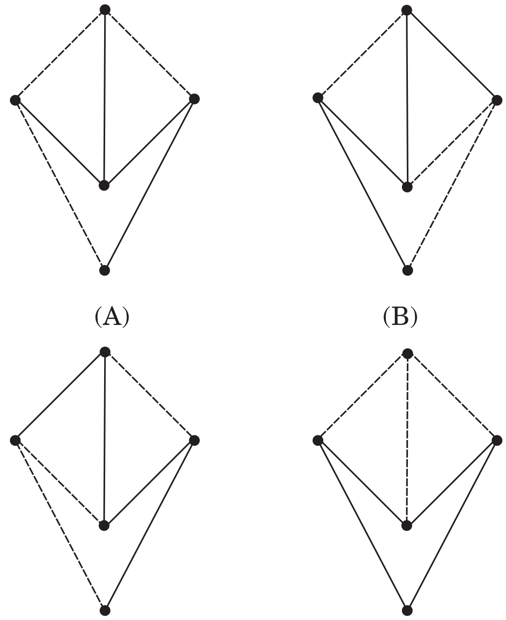


Fig. S. 1.2.6

7. (D) D is not a tree

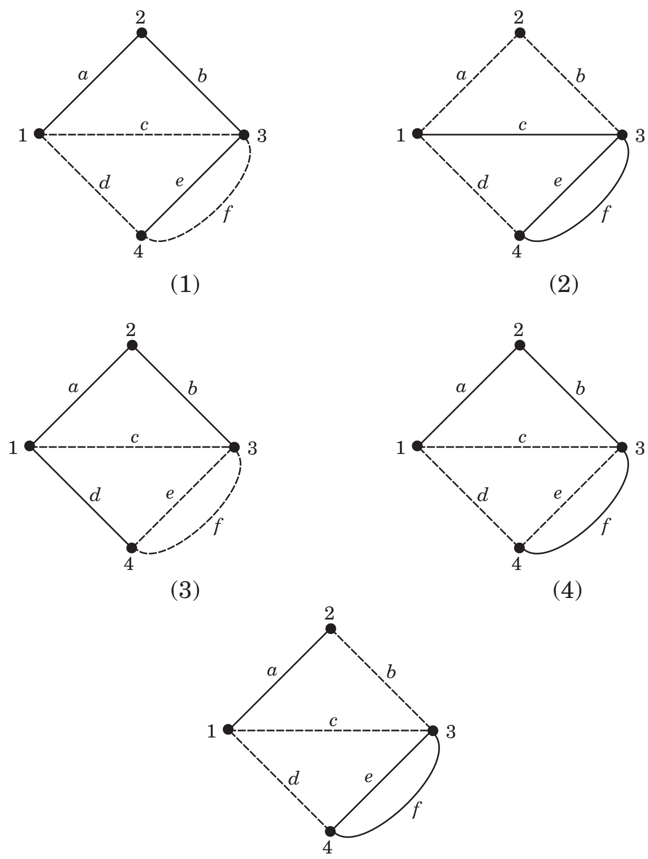


(A) (B)

(C) (D)

Fig. S .1.2.7

8. (D) it is obvious from the following figure that 1, 3, and 4 are tree



(1) (2)

(3) (4)

(5)

Fig. S. 1.2.8

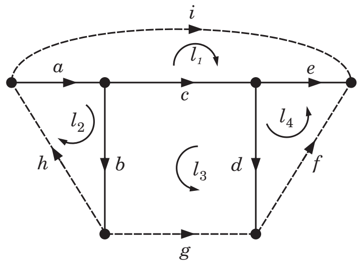


Fig. S 1.2.26

This in similar to matrix in (A). Only place of rows has been changed.

27. (A) Number of branch =8

Number of link =4

Number of twigs $=8-4=4$

Number of twigs =number of independent node equation.

28. (D) The number of independent node equation are $n-1$.

29. (A) Number of branch $b=8$

Number of link $l=4$

Number of twigs $t=b-l=4$

rank of matrix $=n-1=t=4$

30. (B) We know the branch current and loop current are related as

$[i_b]=[B^T][I_L]$

So fundamental loop matrix is

$$\mathbf{B}_f=\begin{bmatrix} 0 & -1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 \end{bmatrix}$$

f-loop 1 include branch (2, 4, 6, 7) and direction of branch–2 is opposite to other (B only).

31. (B) Independent loops =link

$l=b-(n-1)$

$\Rightarrow \quad 5=b-7,\ b=12$

32. (B) Independent loop =link

$l=b-(n-1)=4$

33. (A) There are 8 branches and $4+1=5$ node

Number of link $=8-5+1=4$

So independent mesh equation =Number of link.

34. (D) We know that $[v_b]=A_r^T\,[V_n]$

So reduced incidence matrix is

$$\mathbf{A}_r=\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

At node-1, three branch leaves so the only option is (D).

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