

Probability

1. Experiment:

An operation which can produce some well-defined outcomes is called an experiment.

2. Random Experiment:

An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

Examples:

- i. Rolling an unbiased dice.
- ii. Tossing a fair coin.
- iii. Drawing a card from a pack of well-shuffled cards.
- iv. Picking up a ball of certain colour from a bag containing balls of different colours.

Details:

- i. When we throw a coin, then either a Head (H) or a Tail (T) appears.
- ii. A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.
- iii. A pack of cards has 52 cards.
It has 13 cards of each suit, name Spades, Clubs, Hearts and Diamonds.
Cards of spades and clubs are black cards.
Cards of hearts and diamonds are red cards.
There are 4 honours of each unit.
There are Kings, Queens and Jacks. These are all called face cards.

3. Sample Space:

When we perform an experiment, then the set S of all possible outcomes is called the sample space.

Examples:

1. In tossing a coin, $S = \{H, T\}$
2. If two coins are tossed, the $S = \{HH, HT, TH, TT\}$.
3. In rolling a dice, we have, $S = \{1, 2, 3, 4, 5, 6\}$.

4. Event:

Any subset of a sample space is called an event.

5. Probability of Occurrence of an Event:

Let S be the sample and let E be an event.

Then, $E \subseteq S$.

$$\therefore P(E) = \frac{n(E)}{n(S)}$$

6. Results on Probability:

- i. $P(S) = 1$
- ii. $0 \leq P(E) \leq 1$
- iii. $P(\emptyset) = 0$
- iv. For any events A and B we have : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- v. If \bar{A} denotes (not-A), then $P(\bar{A}) = 1 - P(A)$.

Exercise 15A

Question 1:

Total numbers of trials = 500

Numbers of heads = 285

Numbers of tails = 215

(i) Let E be the event of getting a head

$$\begin{aligned}\therefore P(\text{getting ahead}) &= P(E) = \frac{\text{numbers of heads coming up}}{\text{total number of trials}} \\ &= \frac{285}{500} = 0.57\end{aligned}$$

(ii) Let F be the event of getting a tail

$$\begin{aligned}\therefore P(\text{getting a tail}) &= P(F) = \frac{\text{numbers of tails coming up}}{\text{total number of trials}} \\ &= \frac{215}{500} \\ &= 0.43\end{aligned}$$

Question 2:

Total numbers of trials = 400

Numbers of times 2 head appears = 112

Number of times 1 head appears = 160

Number of times 0 head appears = 128

In a random toss of two coins, Let E_1, E_2, E_3 , be the events of
P(getting 2 heads)

$$= P(E_1) = \frac{\text{numbers of times 2 heads appear}}{\text{total number of trials}} = \frac{112}{400} = 0.28$$

$$\begin{aligned}P(\text{getting 1 head}) &= P(E_2) = \\ &= \frac{\text{numbers of times 1 head appears}}{\text{total number of trials}} = \frac{160}{400} = 0.4\end{aligned}$$

$$\begin{aligned}P(\text{getting 0 head}) &= P(E_3) = \\ &= \frac{\text{numbers of times 0 head appears}}{\text{total number of trials}} = \frac{128}{400} = 0.32\end{aligned}$$

Question 3:

Total number of trials=200

Number of times 3 heads appeared=39

Number of times 2 heads appeared = 58

Number of times 1 head appeared =67

Number of times 0 head appeared=36

The random toss of 3 coins , Let E₁, E₂, E₃ and E₄ be the events of getting 3 heads , 1 head and 0 head and 2 heads respectively . Then;

$$\begin{aligned}\text{(i) } P(\text{getting 3 heads}) &= P(E_1) = \\ &= \frac{\text{numbers of times 3 head appeared}}{\text{total number of trials}} \\ &= \frac{39}{200} \\ &= 0.195\end{aligned}$$

$$\begin{aligned}\text{(ii) } P(\text{getting 1 head}) &= P(E_2) = \\ &= \frac{\text{numbers of times 1 head appeared}}{\text{total number of trials}} \\ &= \frac{67}{200} \\ &= 0.335\end{aligned}$$

$$\begin{aligned}\text{(iii) } P(\text{getting 0head}) &= P(E_3) = \\ &= \frac{\text{numbers of times 0 head appeared}}{\text{total number of trials}} \\ &= \frac{36}{200} \\ &= 0.18\end{aligned}$$

$$\begin{aligned}\text{(iii) } P(\text{getting 2heads}) &= P(E_4) = \\ &= \frac{\text{numbers of times 2head appeared}}{\text{total number of trials}} \\ &= \frac{58}{200} \\ &= 0.29\end{aligned}$$

Question 4:

Total number of trials = 300

In a random throw of a die let E_1 , E_2 , E_3 , and E_4 be the events of 3, 6, 5, and 1 respectively. Then;

$$\begin{aligned} \text{(i) } P(\text{getting 3}) &= P(E_1) = \frac{\text{numbers of times 3 appeared}}{\text{total number of trials}} \\ &= \frac{54}{300} \\ &= 0.18 \end{aligned}$$

$$\begin{aligned} \text{(ii) } P(\text{getting 6}) &= P(E_2) = \frac{\text{numbers of times 6 appeared}}{\text{total number of trials}} \\ &= \frac{33}{300} \\ &= 0.11 \end{aligned}$$

$$\begin{aligned} \text{(iii) } P(\text{getting 5}) &= P(E_3) = \frac{\text{numbers of times 5 appeared}}{\text{total number of trials}} \\ &= \frac{39}{300} \\ &= 0.13 \end{aligned}$$

$$\begin{aligned} \text{(iv) } P(\text{getting 1}) &= P(E_4) = \frac{\text{numbers of times 2 head appeared}}{\text{total number of trials}} \\ &= \frac{60}{300} \\ &= 0.2 \end{aligned}$$

Question 5:

The number of ladies = 200

Number of ladies who like coffee = 142

Number of ladies who do not like coffee = 58

Let E_1 = event that the selected lady likes coffee.

$$\therefore P(E_1) = \frac{\text{numbers of ladies who like coffee}}{\text{total number of trials}} = \frac{142}{200} = 0.71$$

Let (E_2) = event that the selected lady dislikes coffee. Then

$$\therefore P(E_2) = \frac{\text{numbers of ladies who dislike coffee}}{\text{total number of trials}} = \frac{58}{200} = 0.29$$

Question 6:

Number of tests in which he gets more than 60% marks = 2

Total numbers of tests = 6

\therefore Required probability

$$= \frac{\text{numbers of tests in which he gets more than 60\% marks}}{\text{total number of trials}}$$

$$= \frac{2}{6} = \frac{1}{3}$$

Question 7:

Total numbers of vehicles = 240

Numbers of two wheelers = 84

$$\therefore \text{Required probability} = \frac{\text{numbers of two wheelers}}{\text{total number of vehicles}}$$

$$= \frac{84}{240}$$

$$= 0.35$$

Question 8:

Total phone numbers = 200

Numbers of phone numbers with unit digit 5 = 24

$$\therefore \text{Required probability} = \frac{\text{numbers of phone numbers with units digits 5}}{\text{total number of numbers}}$$

$$= \frac{24}{200}$$

$$= 0.12$$

Numbers of phone numbers with units digit 8 = 16

\therefore Required probability

$$= \frac{\text{numbers of phone numbers with units digits 8}}{\text{total number of phone numbers}}$$

$$= \frac{16}{200}$$

$$= 0.08$$

Question 9:

Total number of students = 40

(i) Numbers of students having blood group O = 14

\therefore Required probability

$$= \frac{\text{numbers of students having blood group O}}{\text{total number of students}} = \frac{14}{40} = 0.35$$

(ii) Numbers of students having blood group AB = 6

\therefore Required probability

$$= \frac{\text{numbers of students having blood group AB}}{\text{total number of students}} = \frac{6}{40} = 0.15$$

Question 10:

Total numbers of students = 30

Numbers of students who lie in the interval 21-30 = 6

\therefore Required probability

$$= \frac{\text{numbers of students in the interval}}{\text{total number of students}} = \frac{6}{30} = 0.2$$

Question 11:

Total number of patients=360

(i) P (getting a patient of age 30 years or more but less than 40 years) = $\frac{60}{360} = \frac{1}{6}$

(ii) P (getting a patient of age 50 years or more but less than 70 years)

$$= \left(\frac{50 + 30}{360} \right) = \frac{80}{360} = \frac{2}{9}$$

(iii) P (getting a patient of age less than 10 years) = $\frac{0}{360} = 0$

(iv) P (getting a patient of age 10 years or more) = $\frac{360}{360} = 1$