CHAPTER 1 NUMBER SYSTEM

NUMBERS

The ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called *digits,* the group of which can represent any number.

1 **NUMBERS:** In Hindu- Arabic system, we have ten **digits**, namely 0, 1,2,3,4,5,6,7, 8, 9 Called zero, one two, three, four, five, six, seven, eight and nine respectively.

A number is denoted by a group of digits, called **numeral**. For denoting a numeral, we use the place-value chart, given below.

Example ***1:*** **Write each of the following numerals in words.**

	Ten-Crares	Crares	Ten-Lacs	Lacs	Ten-Thousands	Thousands	Hundreds	Ten	Units
(i)				6	3	8	5	4	9
(ii)			2	3	8	0	9	1	7
(iii)		8	5	4	1	6	0	0	8
(iv)	5	6	1	3	0	7	0	9	0

Solution: The given numerals in words are:

(i) Six lac thirty-eight thousand five hundred forty-nine.
(ii) Twenty-three lac eighty thousand nine hundred seventeen.
(iii) Eight crore fifty-four lac sixteen thousand eight.
(iv) Fifty-six crore thirteen lac seven thousand ninety.

2. **Face value and Place value (or Local Value) of a Digit In a Numeral**

(i) The face value of a digit in a numeral is its own value at whatever place it may be

Ex. In the numeral 6872, the face value of 2 is 2, the face value of 7 is 7, the face value of 8 is 8 and the face value of 6 is 6.

(ii) In a given numeral:

Place value of unit digit = (unit digit) x 1,
Place value of tens digit = (tens digit) x 10,
Place value of hundred's digit = (hundred's digit) x 100 and so on.

Ex. In the numeral 70984, we have
Place value of 4 = (4 x 1) = 4
Place value of 8 = (8 × 10) = 80,
Place value of 9 = (9 × 100) = 900,
Place value of 7 = (7 × 10000) = 70000.

NOTE: *Place value of 0 in a given numral is 0, at whatever place it may be.*

TYPES OF NUMBER

Natural Numbers

These are the numbers (1, 2, 3, etc.) that are used for counting.

- There are infinite natural numbers and the smallest natural number is one (1).

Even numbers

Natural numbers which are divisible by 2 are even numbers.
Thus 2, 4, 6, 8,... are all even numbers.

- Smallest even number is 2.
- There is no largest even number.

Odd numbers

Natural numbers which are not divisible by 2 are odd numbers.
Thus 1, 3, 5, 7, ... are all odd numbers.

- Smallest odd number is 1.
- There is no largest odd number.

Based on divisibility, there could be two types of natural numbers :

(a) **Prime numbers :** Natural numbers which have exactly two factors, i.e., 1 and the number itself are called prime numbers. The lowest prime number is 2.
2 is also the only even prime number.

(b) **Composite numbers :** Natural numbers which have atleast one divisor different from unity and itself are called composite numbers.

- Every composite number can be factorised into its prime factors.

Ex. $24 = 2 \times 2 \times 2 \times 3$. Hence, 24 is a composite number.
The smallest composite number is 4.

Whole Numbers

The natural numbers along with zero (0), form the system of whole numbers.
Thus 0, 1, 2, 3, ... are whole numbers.

- There is no largest whole number
- The smallest whole number is 0.

Integers

The number system consisting of natural numbers, their negative and zero is called integers.
Thus, ..., –3, –2, –1, 0, 1, 2, 3, ... are all integers.

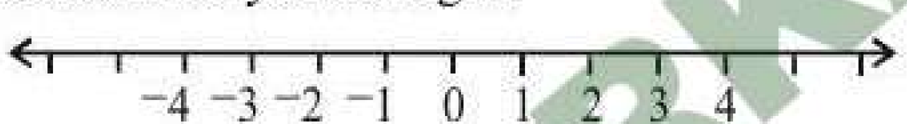
- The smallest and the largest integers cannot be determined.

Remember

✧ 1 is neither prime nor composite.
✧ 1 is an odd integer.
✧ 0 is neither positive nor negative.
✧ 0 is an even integer.
✧ 2 is prime & even both.
✧ All prime numbers (except 2) are odd.

The number line

The number line is a straight line between negative infinity on the left to positive infinity on the right.



Real Numbers

All numbers that can be represented on the number line are called real numbers.

- Real numbers = Rational numbers + Irrational numbers.

(a) Rational numbers

A number that can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number.
Here, p is called the numerator and q is called the denominator.
Every integer is a rational number.

- Zero (0) is also a rational number.
- The smallest and largest rational numbers cannot be determined.
- Every fraction (and decimal fraction) is a rational number.

If x and y are two rational numbers, then $\frac{x+y}{2}$ is also a rational number and its value lies between the given two rational numbers x and y.

An infinite number of rational numbers can be determined between any two rational numbers.

Example 2 : Find three rational numbers between 3 and 5.

Solution :

1st rational number $= \frac{3+5}{2} = \frac{8}{2} = 4$

2nd rational number (i.e., between 3 and 4)

$= \frac{3+4}{2} = \frac{7}{2}$

3rd rational number (i.e., between 4 and 5)

$= \frac{4+5}{2} = \frac{9}{2}$

(b) Irrational numbers

The numbers which are not rational or which cannot be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called an irrational number.

Ex. $\sqrt{2}, \sqrt{3}, \sqrt{5}, 2+\sqrt{3}, 3-\sqrt{5}, 3\sqrt{3}$ are irrational numbers.

NOTE :

(i) Every positive irrational number has a negative irrational number corresponding to it.

(ii) $\sqrt{2} + \sqrt{3} \neq \sqrt{5}$

$\sqrt{5} - \sqrt{3} \neq \sqrt{2}$

$\sqrt{3} \times \sqrt{2} = \sqrt{3 \times 2} = \sqrt{6}$

$\sqrt{6} \div \sqrt{2} = \sqrt{\frac{6}{2}} = \sqrt{3}$

(iii) Some times, product of two irrational numbers is a rational number.

Ex : $\sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2} = 2$

$(2+\sqrt{3}) \times (2-\sqrt{3}) = (2)^2 - (\sqrt{3})^2 = 4 - 3 = 1$

Both rational and irrational numbers can be represented on number line.
Every real number is either rational or irrational.

PRIME NUMBERS

We know that a number other than 1 is called a prime number if it is divisible by only 1 and itself.
So, all prime numbers less than 100 are:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Note that 2 is the smallest prime number. 2 is the only even prime number.
Smallest odd prime number is 3.

Twin Primes

A pair of prime numbers are said to be twin prime when they differ by 2.
For example: 3 and 5 are twin primes.

Co-primes or Relative primes

A pair of numbers are said to be co-primes or relative primes to each other if they do not have any common factor other than 1.
For example: 13 and 21 are co-primes.

To Test Whether a Given Number is Prime Number or Not

Let p be a given number and let n be the smallest counting number such that $n^2 \geq p$. Then, test whether p is divisible by any of the prime numbers less than or equal to n. It yes, then p is not prime. Otherwise, p is a prime number.

Example 3: Is 171 is a prime number ?

***Solution*:** Square root of 171 lies between 13 and 14, because $13^2 = 169$ and $14^2 = 196$. Therefore, the integer just greater than the square root of 171 is 14.
Now prime numbers less than 14 are 2, 3, 5, 7, 11 and 13.
Since 171 is divisible by 3, therefore 171 is not a prime number.

Example *4*: Is 167 is a prime number ?

***Solution*:**
Square root of 167 lies between 12 and 13, because $12^2 = 144$ and $13^2 = 169$. Therefore the integer just greater than the square root of 167 is 13.
Now prime numbers less than 13 are 2, 3, 5, 7 and 11.
Since 167 is not divisible by any of the prime numbers 2, 3, 5, 7 and 11; therefore 167 is a prime number.

Example *5*: Find the number of positive integers n in the range $12 < n < 40$ such that the product $(n-1)(n-2)(n-3)$... 3.2.1 is not divisible by *n*.

***Solution*:**
The product $(n-1)(n-2)(n-3)$...3.2.1 will not be divisible by *n* only when this product does not contain factors of *n*, i.e., *n* is a prime number. The prime numbers that satisfy the above conditions are 13, 17, 19, 23, 29, 31, and 37.
Hence there are 7 required prime numbers.

NUMBER OF FACTORS OF A COMPOSITE NUMBER

It is possible to find the number of factors of a composite number without lising all those factors.

Take 12 for instance, it can be expressed as $12 = 2^2 \times 3^1$.

The factors of 12 are $(2^0 \times 3^0)$, $(2^0 \times 3^1)$, $(2^1 \times 3^0)$, $(2^1 \times 3^1)$, $(2^2 \times 3^0)$ and $(2^2 \times 3^1)$.

Here the powers of 2 can be one of 0, 1, 2 and the powers of 3 can be one of 0, 1. So number of combinations of a power of 2 and a power of 3 is $3 \times 2 = 6$. All the combinations of power of 2 and a power of 3 are 0, 0; 0, 1; 1, 0; 1, 1; 2, 0; 2, 1. Each combination of the powers of 2 and 3 gives a distinctly different factor. Since there are 6 different combinations of the powers of 2 and 3, hence there are 6 distinctly different factors of 12.

Let *N* be a composite number such that $N = (x)^a (y)^b (z)^c$... where *x*, *y*, *z*... are different prime numbers. Then the number of divisors (or factors) of $N = (a+1)(b+1)(c+1)$...

Here factors and divisors means the same.

Example *6*: Find the total number of factors of 576.

***Solution*:**
The factorised form of $576 = 2^6 \times 3^2$
So the total number of factors $= (6+1)(2+1) = 21$

Example 7: Find the number of divisors of 21600.

***Solution*:**
$21600 = 2^5 \times 3^3 \times 5^2$
$\Rightarrow$ Number of divisors $= (5+1) \times (3+1) \times (2+1) = 72$

TESTS OF DIVISIBILITY

I. Divisibility by 2:
A number is divisible by 2 if its unit digit is any of 0, 2, 4, 6, 8.
Ex. 58694 is divisible by 2, while 86945 is not divisible by 2.

II. Divisible by 3:
A number is divisible by 3 only when the sum of its digits is divisible by 3.
Ex. (i) Sum of digits of the number 695421 = 27, which is divisible by 3.
$\therefore$ 695421 is divisible by 3.
(ii) Sum of digits of the number 948653 = 35, which is not divisible by 3.
$\therefore$ 948653 is not divisible by 3.

III. Divisible by 4:
A number is divisible by 4 if the number formed by its last two digits i.e. ten's and unit's digit of the given number is divisible by 4.
Ex. (i) 6879376 is divisible by 4, since 76 is divisible by 4.
(ii) 496138 is not divisible by 4, since 38 is not divisible by 4.

IV. Divisible by 5:
A number is divisible by 5 only when its unit digit is 0 or 5.
Ex. Each of the numbers 76895 and 68790 is divisible by 5.

V. Divisible by 6:
A number is divisible by 6 if it is simultaneously divisible by both 2 and 3.
Ex. 90 is divisible by 6 because it is divisible by both 2 and 3 simultaneously.

VI. Divisible by 7:

A number is divisible by 7 if and only if the difference of the number of its thousands and the remaining part of the given number is divisible by 7 respectively.

Ex. 473312 is divisible by 7, because the difference between 473 and 312 is 161, which is divisible by 7.

VII. Divisible by 8:

A number is divisible by 8 if the number formed by its last three digits i.e. hundred's, ten's and unit's digit of the given number is divisible by 8.

Ex. (i) In the number 16789352, the number formed by last 3 digits, namely 352 is divisible by 8.

$\therefore$ 16789352 is divisible by 8.

(ii) In the number 576484, the number formed by last 3 digits, namely 484 is not divisible by 8.

$\therefore$ 576484 is not divisible by 8.

VIII. Divisible by 9:

A number is divisible by 9 only when the sum of its digits is divisible by 9.

Ex. (i) Sum of digits of the number 246591 = 27, which is divisible by 9.

$\therefore$ 246591 is divisible by 9.

(ii) Sum of digits of the number 734519 = 29, which is not divisible by 9.

$\therefore$ 734519 is not divisible by 9.

IX. Divisible by 10:

A number is divisible by 10 only when its unit digit is 0.

Ex. (i) 7849320 is divisible by 10, since its unit digit is 0.

(ii) 678405 is not divisible by 10, since its unit digit is not 0.

X. Divisible by 11:

A number is divisible by 11 if the difference between the sum of its digits at odd places from right and the sum of its digits at even places also from right is either 0 or a number divisible by 11.

Ex. (i) Consider the number 29435417.

(Sum of its digits at odd places from right) – (Sum of its digits at even places from right)

$(7+4+3+9)-(1+5+4+2)=(23-12)=11$, which is divisible by 11.

$\therefore$ 29435417 is divisible by 11.

(ii) Consider the number 57463822.

(Sum of its digits at odd places) – (Sum of its digits at even places)

$= (2+8+6+7)-(2+3+4+5) = (23-14)$

$= 9$, which is neither 0 nor divisible by 11.

$\therefore$ 57463822 is not divisible by 11.

XI. Divisible by 12:

A number is divisible by 12, if it is simultaneously divisible by both 3 and 4.

Properties of Divisibility

(i) If a is divisible by b then ac is also divisible by b.

(ii) If a is divisible by b, and c is divisible by d then ac is divisible by bd.

(iii) If m and n both are divisible by d then $(m+n)$ and $(m-n)$ are both divisible by d.

(iv) Out of n consecutive whole numbers, one and only one is divisible by n.

For example, out of the five consecutive whole numbers 8, 9, 10, 11, 12 only one i.e., 10 is divisible by 5.

(v) The square of an odd integer when divided by 8 will always leave a remainder of 1.

(vi) The product of 3 consecutive natural numbers is divisible by 6.

(vii) The product of 3 consecutive natural numbers, the first of which is even, is divisible by 24.

(viii) Difference between any number and the number obtained by writing the digits in reverse order is divisible by 9.

(ix) Any number written in the form $(10^n - 1)$ is divisible by 3 and 9.

(x) Any six-digits, twelve-digits, eighteen-digits or any such number with number of digits equal to multiple of 6, is divisible by each of 7, 11 and 13 if all of its digits are the same.

For example 666666, 888888, 333333333333 are all divisible by 7, 11 and 13.

As 666666 can be written as $666 \times 1000 + 666$

$= 666(1000+1) = 666 \times (1001) = 666 \times (7 \times 11 \times 13)$

Hence, 666666 is divisible by all of 7, 11 and 13.

Example 8: Find the least value of * for which 7* 5462 is divisible by 9.

***Solution*:**

Let the required value be x. Then,

$(7+x+5+4+6+2) = (24+x)$ should be divisible by 9.

$\Rightarrow \quad x = 3$

Example 9: Find the least value of * for which 4832*18 is divisible by 11.

***Solution*:**

Let the digit in place of * be x.

(Sum of digits at odd places from right) – (Sum of digits at even places from right)

$= (8+x+3+4)-(1+2+8=(4+x),$

which should be divisible by 11.

$\therefore \quad x = 7.$

GENERAL OR EXPANDED FORM OF 2 AND 3 DIGITS NUMBERS

(i) In a two digits number AB, A is the digit of tenth place and B is the digit of unit place, therefore AB is written using place value in expanded form as

$AB = 10A + B$

Ex. $35 = 10 \times 3 + 5$

(ii) In a three digits number ABC, A is the digit of hundred place, B is the digit of tenth place and C is the digit of unit place, therefore ABC is written using place value in expanded form as

$ABC = 100A + 10B + C$

Ex. $247 = 100 \times 2 + 10 \times 4 + 7$

These expanded forms are used in forming equations related to 2 and 3 digits numbers.

Example 10: A two-digit number *pq* is added to the number formed by reversing its original digits. If their sum is divisible by 11, 9, and 2. Find the number *pq*.

Solution: Let the original number be pq. The value of the number $= 10p + q$.

The number formed by reversing the digits $= qp$. Value of this number $= 10q + p$.

Sum of the two numbers $= 11p + 11q = 11(p + q)$

Now, if the sum is divisible by 11, 9, 2, it means that $(p + q)$ must be divisible by both 9 and 2. Hence, $p + q = 18$. So, it means $p = q = 9$. The original number is 99.

Example 11: In a two digit prime number, if 18 is added, we get another prime number with reversed digits. How many such numbers are possible ?

Solution: Let a two-digit number be pq.

$\therefore \quad 10p + q + 18 = 10q + p$

$\Rightarrow \quad -9p + 9q = 18 \Rightarrow q - p = 2$

Satisfying this condition and also the condition of being a prime number (pq and qp both), there are 2 numbers 13 and 79.

COUNTING NUMBER OF ZEROS AT THE END OF A FACTORIAL

Sometimes we come across problems in which we have to count number of zeros at the end of factorial of any numbers.

Ex. Number of zeros at the end of 10!

$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Here basically we have to count number of fives, because multiplication of five by any even number will result in 0 at the end of final product. In 10! we have 2 fives thus total number of zeros are 2.

Shortcut :-

Counting number of zeros at the end of n! value will be

$$\frac{n}{5} + \frac{n}{5^2} + \frac{n}{5^3} + \frac{n}{5^4} + \ldots\ldots\ldots$$

The integral value of this number will be the total number of zeros.

Example 12 : Number of zeros at the end of 10!

Solution : $\frac{10}{5} + \frac{10}{5^2} + \ldots\ldots\ldots$ Integral value

$= 2 + 0$

So, number of zeros in $10! = 2$.

Note:- Here $\frac{10}{5^2}$ is less than 1 so will not count it.

Example 13 : Number of zeros at the end of 100!

Solution : $\frac{100}{5} + \frac{100}{5^2} + \frac{100}{5^3} + \ldots\ldots\ldots$

integral value will be

$= \quad 20 + 4 = 24$ zeros.

Example 14 : Number of zeros at the end of 126!

Solution :

$$\frac{126}{5} + \frac{126}{5^2} + \frac{126}{5^3} + \ldots\ldots\ldots$$

$\Rightarrow$ integral value will be

$= 25 + 5 + 1 = 31$ zeros.

Example 15 : Number of zeros at the end of 90!

Solution :

$$\frac{90}{5} + \frac{90}{5^2} + \frac{90}{5^3} + \ldots\ldots\ldots = 18 + 3 = 21 \text{ zeros}$$

POWER OF A NUMBER CONTAINED IN A FACTORIAL

Highest power of a prime number P in N!

$= \left[\frac{N}{P}\right] + \left[\frac{N}{P^2}\right] + \left[\frac{N}{P^3}\right] + \text{- - -} + \left[\frac{N}{P^r}\right]$, where [x] denotes the greatest integers less than or equal to x and is a natural number such that $P^r < n$.

Example 16 : Find highest power of 7^n in 50!

Solution :

The highest power 7 in 50!

$$= \left[\frac{50}{7}\right] + \left[\frac{50}{7^2}\right] = 7 + 1 = 8$$

Example 17 : Find highest power 15 in 100!

Solution :

Here given number 15 is not a prime number so first convert 15 as product of Primes $15 = 3 \times 5$ therefore we will find the highest power of 3 and 5 in 100!

Highest power of 3 in 100!

$$= \left[\frac{100}{3}\right] + \left[\frac{100}{3^2}\right] + \left[\frac{100}{3^3}\right] + \left[\frac{100}{3^4}\right]$$

$= 33 + 11 + 3 + 1 = 48$

Highest power of 5 in 100!

$$= \left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right] = 20 + 4 = 24$$

So 100! contains $(3)^{48} \times (5)^{24}$. Hence it contains 24 pairs of 3 and 5. Therefore, required power of 15 is 24, which is actually the power of the largest prime factor 5 of 15, because power of largest prime factor is away equal to or less than the other prime factor of any number.

TO FIND THE LAST DIGIT OR DIGIT AT THE UNIT'S PLACE OF a^n

(i) If the last digit or digit at the unit's place of a is 1, 5 or 6, whatever be the value of n, it will have the same digit at unit's place, i.e.,

$(\ldots..1)^n = (\ldots\ldots..1)$
$(\ldots..5)^n = (\ldots\ldots..5)$
$(\ldots..6)^n = (\ldots\ldots..6)$

(ii) If the last digit or digit at the units place of a is 2, 3, 5, 7 or 8, then the last digit of a^n depends upon the value of n and follows a repeating pattern in terms of 4 as given below :

n	last digit of $(....2)^n$	last digit of $(....3)^n$	last digit of $(....7)^n$	last digit of $(....8)^n$
4x+1	2	3	7	8
4x+2	4	9	9	4
4x+3	8	7	3	2
4x	6	1	1	6

(iii) If the last digit or digit at the unit's place of a is either 4 or 9, then the last digit of a^n depends upon the value of n and follows repeating pattern in terms of 2 as given below.

n	last digit of $(....4)^n$	last digit of $(....9)^n$
2x	6	1
2x + 1	4	9

Example *18* **: Find unit digit of 2^{323}.**

Solution **:** Here, 2, 4, 8, 6 will repeat after every four interval till 320 next digit will be 2, 4, $\boxed{8}$, so unit digit of 2^{323} will be 8.

Example *19* **: Find unit digit of 133^{133}.**

Solution **:**

Cycle of 3 is 3, 9, 7, 1 which repeats after every fourth interval will 133^{132}, so next unit digit will be 3.

Example *20* **: Find unit digit of $963^{63} \times 73^{73}$.**

Solution **:** Unit digit of $963^{63} = 7$

Unit digit of $73^{73} = 3$

So unit digit of $963^{63} \times 73^{73} = 7 \times 3 = 21$.

i.e. 1.

Example *21* **: Find unit digit of $17^{17} \times 27^{27} \times 37^{37}$.**

Solution **:** Unit digit of $17^{17} = 7$

Unit digit of $27^{27} = 3$

Unit digit of $37^{37} = 7$

So unit digit of $17^{17} \times 27^{27} \times 37^{37} = 7 \times 3 \times 7 = 147$

i.e., unit digit = 7

Example *22* **: Find unit digit of $18^{18} \times 28^{28} \times 288^{288}$.**

Solution **:**

Unit digit of 18^{18} is 4.

Unit digit of 28^{28} is 6.

Unit digit of 288^{288} is 6

So unit digit of $18^{18} \times 28^{28} \times 288^{288}$.

$= 4 \times 6 \times 6 = 144$ i.e., 4

Example *23* **: Find unit digit of $11^{11} + 12^{12} + 13^{13} + 14^{14} + 15^{15}$.**

Solution **:**

Unit digit of $11^{11} = 1$

Unit digit of $12^{12} = 6$

Unit digit of $13^{13} = 3$

Unit digit of $14^{14} = 6$

Unit digit of $15^{15} = 5$

So unit digit of given sum will be

$1 + 6 + 3 + 6 + 5 = 21$ i.e., 1

Example *24* **: Find unit digit of $21^{21} \times 22^{22} \times 23^{23} \times 24^{24} \times 25^{25}$.**

Solution **:**

25^{25} will give 5 in unit place, when multiplied by an even number i.e. 0, 2, 4, 6, 8. It will give zero at unit place. So, zero will be at the unit digit of given question.

REMAINDER THEOREM

Remainder of expression $\frac{a \times b \times c}{n}$ [i.e. $a \times b \times c$ when divided by n] is equal to the remainder of expression $\frac{a_r \times b_r \times c_r}{n}$ [i.e. $a_r \times b_r \times c_r$ when divided by n] , where

a_r is remainder when a is divided by n.

b_r is remainder when b is divided by n. and

c_r is remainder when c is divided by n.

Example *25* **: Find the remainder of 15 × 17 × 19 when divided by 7.**

Solution **:**

Remainder of Expression $\frac{15 \times 17 \times 19}{7}$ will be equal to

$\frac{1 \times 3 \times 5}{7} = \frac{15}{7} = 2\frac{1}{7}$ i.e. 1

On dividing 15 by 7, we get 1 as remainder.

On dividing 17 by 7, we get 3 as remainder.

On dividing 19 by 7, we get 5 as remainder.

And combined remainder will be equal to remainder of $\frac{15}{7}$ i.e. 1.

Example *26* **: Find the remainder of expression $\frac{19 \times 20 \times 21}{9}$**

Solution **:**

Remainder of given expression $= \frac{1 \times 2 \times 3}{9} = \frac{6}{9}$ which is equal to 6.

POLYNOMIAL THEOREM

This is very useful theorem to find the remainder.

According to polynomial theorem.

$$(x + a)^n = x^n + {}^nc_1\, x^{n-1} \,.\, a^1 + {}^nc_2\, x^{n-2}\, a^2 + {}^nc_3\, x^{n-3}\, a^3 + \text{.........}\, {}^nc_{n-1}\, x^1 a^{n-1} + a^n \quad \ldots\text{(i)}$$

$$\therefore \quad \frac{(x+a)^n}{x}$$

$$= \frac{x^n + {}^nc_1 x^{n-1} a^1 + {}^nc_2 x^{n-2} a^2 + \cdots\cdots + {}^nc_{n-1} x^1 a^{n-1} + a^n}{x} \quad \ldots \text{(ii)}$$

remainder of expression (ii) will be equal to remainder of $\frac{a^n}{x}$ because rest of the terms contain x and are completely divisible by x.

Example 27 : Find the remainder of $\frac{9^{99}}{8}$.

Solution :

$$\frac{9^{99}}{8} = \frac{(8+1)^{99}}{8}$$

According to polynomial theorem remainder will be equal to remainder of the expression $\frac{1^{99}}{8} = \frac{1}{8}$, 1

Example 28: Find the remainder of $\frac{8^{99}}{7}$.

Solution :

$$\frac{8^{99}}{7} \Rightarrow \frac{(7+1)}{7} = \frac{1^{99}}{7} \text{ i.e. } 1$$

Example 29 : Find remainder of $\frac{11 \times 13 \times 17}{6}$.

Solution :

$$\frac{11 \times 13 \times 17}{6} = \frac{5 \times 1 \times 5}{6}$$

$$\frac{1}{6} \Rightarrow 1$$

Example 30 : Find remainder of $\frac{9^{100}}{7}$.

Solution :

$$\frac{9^{100}}{7} \Rightarrow \frac{(7+2)^{100}}{7}$$

$$= \frac{2^{100}}{7} = \frac{2^{99} \times 2}{7} = \frac{2^{3 \times 33} \times 2}{7} = \frac{8^{33} \times 2}{7}$$

$$= \frac{(7+1)^{33}}{7} \times 2 = \frac{1 \times 2}{7} = \frac{2}{7} \text{. i.e. } 2$$

Example 31 : Find remainder of $\frac{9^{50}}{7}$.

Solution :

$$\frac{9^{50}}{7} = \frac{(7+2)^{50}}{7} = \frac{2^{50}}{7} = \frac{\left(2^3\right)^{16} \times 2^2}{7} = \frac{8^{16} \times 4}{7}$$

$$\Rightarrow \frac{(7+1)^{16} \times 4}{7} = \frac{1 \times 4}{7} \text{ i.e., } 4$$

Example 32: Find remainder of $\frac{25^{100}}{7}$.

Solution :

$$\frac{25^{100}}{7} = \frac{(3 \times 7 + 4)^{50}}{7} = \frac{4^{50}}{7}$$

$$= \frac{2^{100}}{7} \Rightarrow \frac{\left(2^3\right)^{33} \times 2}{7} \Rightarrow \frac{(7+1)^{33}}{7} \times 2 \Rightarrow \frac{1 \times 2}{7}$$

$\Rightarrow$ Remainder is 2.

Example 33 : Find remainder of $\frac{3^{50}}{7}$.

Solution :

$$\frac{3^{50}}{7} = \frac{(3^2)^{25}}{7} \Rightarrow \frac{(7+2)^{25}}{7} = \frac{2^{25}}{7} = \frac{(2^3)^8 \times 2}{7}$$

$$= \frac{(7+1)^8 \times 2}{7} = \frac{1 \times 2}{7}$$

$\Rightarrow$ Remainder is 2.

Example 34 : Find remainder of $\frac{3^{250}}{7}$

Solution :

$$\frac{(3^2)^{125}}{7} = \frac{(7+2)^{125}}{7} = \frac{2^{125}}{7} = \frac{(2^3)^{41} \times 2^2}{7} = \frac{1 \times 4}{7}$$

$\Rightarrow$ Remainder is 4

DIVISION ALGORITHM

Dividend = (Divisor × Quotient) + Remainder

where, Dividend = The number which is being divided
Divisor = The number which performs the division process
Quotient = Greatest possible integer as a result of division
Remainder = Rest part of dividend which cannot be further divided by the divisor

Complete remainder

A complete remainder is the remainder obtained by a number by the method of successive division.

Complete remainder = [I divisor × II remainder] + I remainder

$$\text{C.R.} = d_1 r_2 + r_1$$

$$\text{C.R.} = d_1 d_2 r_3 + d_1 r_2 + r_1$$

- Two different numbers x and y when divided by a certain divisor D leave remainder r_1 and r_2 respectively. When the sum of them is divided by the same divisor, the remainder is r_3. Then,

$$\text{divisor } D = r_1 + r_2 - r_3$$

Example 35: A certain number when successively divided by 3 and 5 leaves remainder 1 and 2. What is the remainder if the same number be divided by 15?

Solution :

Let x be the dividend.

$x = 3y + 1$ ...(i)

now, y become dividend for 5.

$y = 5z + 2$ ...(ii)

putting y in (i)

$x = 3(5z + 2) + 1$

$= 15z + 6 + 1 = 15z + 7$

when x is divided by 15 gives remainder 7.

Alternate Method :

$d_1 = 3, d_2 = 5, r = 1$ and $r_2 = 2$

complete remainder $= d_1r_2 + r_1 = 3 \times 2 + 1 = 7$

Example 36: A certain number when divided by 899 leaves the remainder 63. Find the remainder when the same number is divided by 29.

Solution :

Number $= 899Q + 63$, where Q is quotient

$= 31 \times 29Q + (58 + 5) = 29(31Q + 2) + 5$

$\therefore$ Remainder = 5

Theorem 1: $(a^n + b^n)$ is divisible by $(a + b)$ when n is odd.

Theorem 2: $(a^n - b^n)$ is divisible by $(a + b)$ when n is even.

Theorem 3: $(a^n - b^n)$ is always divisible by $(a - b)$ when n is an integer.

Hence $(a^n - b^n)$ is divisible by both $(a + b)$ and $(a - b)$ when n is even and $(a^n - b^n)$ is divisible by only $(a - b)$ when n is odd.

Example 37: What is the remainder when $3^{444} + 4^{333}$ is divided by 5 ?

***Solution*:** The dividend is in the form $a^x + b^y$. We need to change it into the form $a^n + b^n$.

$3^{444} + 4^{333} = (3^4)^{111} + (4^3)^{111}$. Now $(3^4)^{111} + (4^3)^{111}$ will be divisible by $3^4 + 4^3 = 81 + 64 = 145$.

Since the number is divisible by 145, it will certainly be divisible by 5. Hence, the remainder is 0.

Example 38: What is the remainder when $(5555)^{2222} + (2222)^{5555}$ is divided by 7?

***Solution*:** The remainder when 5555 and 2222 are divided by 7 are 4 and 3 respectively. Hence, the problem reduces to finding the remainder when $(4)^{2222} + (3)^{5555}$ is divided by 7.

Now $(4)^{2222} + (3)^{5555} = (4^2)^{1111} + (3^5)^{1111} = (16)^{1111} + (243)^{1111}$.

Now $(16)^{1111} + (243)^{1111}$ is divisible by 16 + 243 or it is divisible by 259, which is a multiple of 7. Hence the remainder when $(5555)^{2222} + (2222)^{5555}$ is divided by 7 is zero.

PROGRESSIONS

Arithmetic Progressions (A.P.)

A sequence of numbers which are either continuously increased or continuously decreased by a common difference found by subtracting any term of the sequence from the next term.

The following sequences of numbers are arithmeticprogressions:

(i) 5, 8, 11, 14, ...

(ii) – 6, – 1, 4, 9, 14, ...

(iii) 10, 7, 4, 1, – 2, – 5, ...

(iv) $p, p + q, p + 2q, p + 3q, ...$

In the arithmetic progression (i); 5, 8, 11 and 14 are first term, second term, third term and fourth term respectively. Common difference of this A.P. is found out either by subtracting 5 from 8, 8 from 11 or 11 from 14. Thus common difference = 3. Similarly, common difference of arithmetic progression (ii), (iii) and (iv) are 5, – 3 and q respectively.

First term and common difference of an A.P. are denoted by a and d respectively. Hence

d of (i) A.P. = 3, d of (ii) A.P. = 5,

d of (iii) A.P. = – 3 and d of (iv) A.P. = q

nth TERM OF AN A.P.

To find an A.P. if first term and common difference are given, we add the common difference to first term to get the second term and add the common difference to second term to get the third term and so on.

The standard for m of an A.P. is

$$a, a + d, a + 2d, a + 3d, ...$$

Here 'a' is the first term and 'd' is the common difference. Also we see that coefficient of d is always less by one than the position of that term in the A.P. Thus n^{th} term of the A.P. is given by

$$T_n = a + (n - 1)d \quad ...(1)$$

This equation (1) is used as a formula to find any term of the A.P.

If l be the last term of a sequence containing n terms, then

$$l = T_n = a + (n - 1)d$$

To find any particular term of any A.P., generally we put the value of a, n and d in the formula (i) and then calculate the required term.

For example to find the 25^{th} term of the A.P. 6, 10, 14, 18, ... ; using the formula (i), we put the value of $a = 6$, $n = 25$ and $d = 4$ in formula and calculate as

$$T_{25} = 6 + (25 - 1) \times 4 = 6 + 24 \times 4 = 6 + 96 = 102$$

Example 39: In an A.P. if $a = -7.2$, $d = 3.6$, $a_n = 7.2$, then find the value of n.

***Solution*:** $a_n = a + (n - 1)d$

$\Rightarrow 7.2 = -7.2 + (n - 1)(3.6)$

$\Rightarrow 14.4 = (n - 1)(3.6)$

$\Rightarrow n - 1 = 4 \Rightarrow n = 5.$

Example 40: Which term of the A.P. 21, 42, 63, ... is 420 ?

Solution: $420 = a_n = a + (n - 1)d$

[Here $a = 21, d = 42 - 21 = 21$]

$= 21 + (n - 1)21$

$= 21n$

$\therefore \quad n = \dfrac{420}{21} = 20$

$\therefore$ required term is 20^{th} term.

Example 41: Is – 150 a term of the A.P. 11, 8, 5, 2, ... ?

***Solution*:** Here $a = 11, d = -3$

$$-150 = a_n = a + (n-1)d$$
$$= 11 + (n-1)(-3)$$
$$= 11 - 3n + 3$$
$$= 14 - 3n$$
$$3n = 14 + 150$$
$$n = \frac{164}{3} = 54\frac{2}{3},$$

which is not possible because n is +ve integer.

$\therefore$ – 150 is not a term of the given A.P.

Sum of First n Terms of an A.P.

Sum of first n terms means sum of terms from first term to n^{th} term. Consider an A.P. whose first term and common difference are 'a' and 'd' respectively. Sum of first n terms S_n of this A.P. is given by

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad ...(1)$$

If last term of an A.P. containing n terms be l, then n^{th} term $= l = a + (n-1)d$.

$$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + \{a + (n-1)d\}]$$

$$\Rightarrow \quad S_n = \frac{n}{2}(a + l) \quad ...(2)$$

Arithmetic Mean of n Numbers

Arithmetic mean of n numbers $a_1, a_2, a_3, a_4, ..., a_n$

$$= \frac{a_1 + a_2 + a_3 + a_4 + ... + a_n}{n}$$

If A be the arithmetic mean between any two given numbers a and b; then a, A, b will be in A.P.

$$\therefore \quad b - A = A - a \Rightarrow A = \frac{a+b}{2}$$

Geometric Progression (G.P.)

A sequence of numbers whose each term (except first term) is found out by multiplying the just previous term by the same number. The number by which we multiply to any term to get its next term is called common ratio of the G.P.

For example, 5, 10, 20, 40, ... is a G.P. whose first term is 5, second term is 10, third term is 20 and so on. Its common ratio is 2, because to get any term (except first term) we multiply its just previous term by 2.

Common ratio is also found out by dividing any term (except first term) by its just previous term,

Thus

$$\text{common ratio} = \frac{10}{5} = \frac{20}{10} = \frac{40}{20} = ... = 2$$

First term of a G.P. is denoted by 'a' and its common ratio is denoted by r.

$\therefore \quad a = 5, r = 2$

Standard form of a G.P. is

$$a, ar, ar^2, ar^3, ar^4, ...$$

n^{th} term of a G.P.,

$$a_n = ar^{n-1}$$

Sum of first *n* terms of a G.P.,

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{if} |r| > 1$$

and $$S_n = \frac{a(1 - r^n)}{1 - r}, \text{if} |r| < 1$$

If $|r| < 1$, then sum of infinite terms of the G.P.,

$$S_\infty = \frac{a}{1 - r}$$

If $|r|$? 1, then sum of infinite terms cannot exist.

Example 42: Which term of the G.P. 2, 1, $\frac{1}{2}, \frac{1}{4}, \frac{1}{128}$ ... is ?

Solution: Let the nth term be $\frac{1}{128}$. Then,

$$a_n = \frac{1}{128}$$

$$\Rightarrow \quad ar^{n-1} = \frac{1}{128}$$

$$\Rightarrow 2\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^7 \Rightarrow \left(\frac{1}{2}\right)^{n-2} = \left(\frac{1}{2}\right)^7$$

$\therefore \quad n - 2 = 7$

$\Rightarrow \quad n = 9.$

Example 43: The third term of a G.P. is 4. Find the product of its first five terms.

Solution: Let a be the first term and r the common ratio. Then,

$$a_3 = 4 \Rightarrow ar^2 = 4$$

Product of first five terms $= a_1a_2a_3a_4a_5 = a(ar)(ar^2)(ar^3)(ar^4)$
$= a^5r^{10} = (ar^2)^5 = (4)^5 = 1024.$

Example 44: Find the sum of the series

2 + 6 + 18 + ... + 4374.

Solution: Required sum $= \frac{a(r^n - 1)}{r - 1} = \frac{(ar^{n-1})r - a}{r - 1}$

$$= \frac{4374 \times 3 - 2}{3 - 1} = 6560.$$

[Here $a = 2, r = 3, ar^{n-1} = 4374$]

Geometric Mean of n Numbers

Geometric mean of n positive numbers $a_1, a_2, a_3, a_4, ..., a_n$ $= (a_1 . a_2 . a_3 . a_4 ... a_n)^{1/n}$.

Let G be the geometric mean (G.M.) between any two given numbers a and b; then a, G, b are in G.P.

$\therefore \quad \frac{b}{G} = \frac{G}{a}$

$\Rightarrow \quad G = \sqrt{ab}$

Harmonic Progression (H.P.)

Harmonic progression is defined as a sequence, reciprocal of whose terms in order are in A.P.

Thus, if $a, b, c, d, \ldots$ are in H.P., then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}, \ldots$ are in A.P.

The standard form of a H.P. is

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots$$

Remember that a, b, c are in H.P. $\Leftrightarrow b = \frac{2ac}{a+c}$

General Term of a H.P.

General term (n^{th} term) of a H.P. is given by $T_n = \frac{1}{a+(n-1)d}$

There is no formula and procedure for finding the sum of any number of terms in H.P.

Questions based on H.P. are generally solved by inverting the terms (i.e., converting H.P. into A.P.) and use of formula and properties of the A.P.

Harmonic Mean of n Numbers

Harmonic mean of n numbers (or quantities) $a_1, a_2, a_3, a_4, \ldots, a_n$

$$= \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \frac{1}{a_4} + \ldots + \frac{1}{a_n}}$$

To Find a Harmonic Mean Between Two Given Numbers

Let H be the harmonic mean between two given numbers a and b;

then a, H, b are in H.P. or $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A.P.

$$\therefore \quad \frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H} \Rightarrow \frac{2}{H} = \frac{1}{a} + \frac{1}{b}$$

$$\Rightarrow \quad H = \frac{2ab}{a+b}.$$

Relation Between Arithmetic Mean (A.M.), Geometric Mean (G.M.) and Harmonic Mean (H.M.)

If A, G, H are the arithmetic, geometric, and harmonic means between a and b, then we have

$$A = \frac{a+b}{2} \quad \ldots(1)$$

$$G = \sqrt{ab} \quad \ldots(2)$$

$$H = \frac{2ab}{a+b} \quad \ldots(3)$$

Therefore, $A \times H = \frac{(a+b)}{2} \times \frac{2ab}{(a+b)} = ab = G^2$. Hence G is the geometric mean between A and H. From these results we see that

$$A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2}$$

$= \left[\frac{\sqrt{a}-\sqrt{b}}{\sqrt{2}}\right]^2$, which is positive if a and b are positive. Therefore, the arithmetic mean of any two positive numbers is greater than their geometric mean.

Also $G^2 = AH$

Hence G is the intermediate in value between A and H, therefore $A > G > H$.

Example 45: Find two numbers whose A.M. is 34 and G.M. is 16.

Solution: Let two numbers be a and b.

$$\text{A.M.} = 34 = \frac{a+b}{2} \Rightarrow a+b = 68 \quad \ldots(1)$$

$$\text{G.M.} = 16 = \sqrt{ab} \Rightarrow ab = 256$$

$$a - b = \sqrt{(a+b)^2 - 4ab} = \sqrt{4624 - 4 \times 256} = \sqrt{3600}$$

$$\therefore \quad a - b = 60 \quad \ldots(2)$$

By (1) and (2)

$a = 64, \ b = 4$

$\therefore$ Required numbers are 64 and 4.

Some Important Formulae

(i) **(a)** Sum of first n natural numbers

$$= 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

(b) Sum of first n odd natural numbers

$$= 1 + 3 + 5 + \ldots + (2n-1) = n^2$$

(c) Sum of first n even natural numbers

$$= 2 + 4 + 6 + \ldots + 2n = n(n+1)$$

(d) Sum of odd numbers $\le n$

$$= \begin{cases} \left(\frac{n+1}{2}\right)^2, & \text{if } n \text{ is odd} \\ \left(\frac{n}{2}\right)^2, & \text{if } n \text{ is even} \end{cases}$$

(e) Sum of even numbers $\le n$

$$= \begin{cases} \frac{n}{2}\left(\frac{n}{2}+1\right), & \text{if } n \text{ is odd} \\ \left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right), & \text{if } n \text{ is even} \end{cases}$$

(ii) Sum of squares of first n natural numbers

$$= 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(iii) Sum of cubes of first n natural numbers

$$= 1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

CLOCK

Introduction

A clock has two hands : Hour hand and Minute hand.
The minute hand (M.H.) is also called the long hand and the hour hand (H.H.) is also called the short hand.
The clock has 12 hours numbered from 1 to 12.
Also, the clock is divided into 60 equal minute divisions.
Therefore, each hour number is separated by five minute divisions. Therefore,

- One minute division $= \frac{360}{60} = 6°$ apart. ie. In one minute, the minute hand moves 6°.
- One hour division = 6° × 5 = 30° apart. ie., in one hour, the hour hand moves 30° apart.

  Also, in one minute, the hour hand moves $= \frac{30°}{60°} = \frac{1°}{2}$ apart.
- Since, in one minute, minute hand moves 6° and hour hand moves $\frac{1°}{2}$, therefore, in one minute, the minute hand gains $5\frac{1°}{2}$ more than hour hand.
- In one hour, the minute hand gains $5\frac{1°}{2} \times 60 = 330°$ over the hour hand. i.e., the minute hand gains 55 minutes divisions over the hour hand.

Relative Position of the Hands

The position of the M.H. relative to the H.H. is said to be the same, whenever the M.H. is separated from the H.H. by the same number of minute divisions and is on same side (clockwise or anticlockwise) of the H.H.

Any relative position of the hands of a clock is repeated 11 times in every 12 hours.

(a) When both hands are 15 minute spaces apart, they are at right angle.

(b) When they are 30 minute spaces apart, they point in opposite directions.

(c) The hands are in the same straight line when they are coincident or opposite to each other.

- In every hour, both the hands coincide once.
- In a day, the hands are coinciding 22 times.
- In every 12 hours, the hands of clock coincide 11 times.
- In every 12 hours, the hands of clock are in opposite direction 11 times.
- In every 12 hours, the hands of clock are at right angles 22 times.
- In every hour, the two hands are at right angles 2 times.
- In every hour, the two hands are in opposite direction once.
- In a day, the two hands are at right angles 44 times.
- If both the hands coincide, then they will again coincide after $65\frac{5}{11}$ minutes. i.e. in correct clock, both hand coincide at an interval of $65\frac{5}{11}$ minutes.
- If the two hands coincide in time less than $65\frac{5}{11}$ minutes, then clock is too fast and if the two hands coincides in time more than $65\frac{5}{11}$ minutes, then the clock is too slow.

Another one shortcut formula for clocks

$$\text{Angle} = \left|30H - \left(\frac{11}{2}\right)M\right|;$$

H ⇒ hour
M ⇒ minutes.

Example 46: At what time between 4 and 5 will the hands of a watch
(i) coincide, and
(ii) point in opposite directions.

Solution : (i) At 4 O'clock, the hands are 20 minutes apart. Clearly the minute hand must gain 20 minutes before two hands can be coincident.
But the minute-hand gains 55 minutes in 60 minutes.
Let minute hand will gain x minute in 20 minutes.

So, $\frac{55}{20} = \frac{60}{x}$

$\Rightarrow \quad x = \frac{20 \times 60}{55} = \frac{240}{11} = 21\frac{9}{11}$ min.

∴ The hands will be together at $21\frac{9}{11}$ min past 4.

(ii) Hands will be opposite to each other when there is a space of 30 minutes between them. This will happen when the minute hand gains (20 + 30) = 50 minutes.

Now, the minute hand gains 50 min in $\frac{50 \times 60}{55}$ or $54\frac{6}{11}$ min.

∴ The hands are opposite to each other at $54\frac{6}{11}$ min past 4.

Example 47: What is the angle between the hour hand and minute hand when it was 5 : 05 pm.

Solution : 5.05 pm means hour hand was on 5 and minute hand was on 1, i.e., there will be 20 minutes gap.

$\therefore$ Angle $= 20 \times 6° = 120°$ $[\because 1 \text{ minute} = 6°]$

Incorrect clock

If a clock indicates 6 : 10, when the correct time is 6 : 00, it is said to be 10 minute too fast and if it indicates 5 : 50 when the correct time is 6 : 00, it is said to be 10 minute too slow.

- Also, if both hands coincide at an interval x minutes and $x < 65\frac{5}{11}$, then total time gained $= \left(\frac{65\frac{5}{11} - x}{x}\right)$ minutes and clock is said to be 'fast'.
- If both hands coincide at an interval x minutes and $x > 65\frac{5}{11}$, then total time lost $= \left(\frac{x - 65\frac{5}{11}}{x}\right)$ minutes and clock is said to be 'slow'.

Example 48: My watch, which gains uniformly, is 2 min slow at noon on Sunday, and is 4 minutes 48 seconds fast at 2 pm on the following Sunday. When was it correct.

Solution : From Sunday noon to the following Sunday at 2 pm = 7 days 2 hours = 170 hours.

The watch gains $\left(2 + 4\frac{48}{60}\right) = 6\frac{4}{5}$ minutes in 170 hours.

$\therefore$ The watch gains 2 minutes in $\frac{2}{6\frac{4}{5}} \times 170 = 50$ hours

Now, 50 hours = 2 days 2 hours

2 days 2 hours from Sunday noon = 2 pm on Tuesday.

Example 49: The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose ?

Solution : In a correct clock, the minute hand gains 55 min. spaces over the hour hand in 60 minutes.

To be together again, the minute hand must gain 60 minutes over the hour hand.

55 min. are gained in $\left(\frac{60}{55} \times 60\right)$ min. $= 65\frac{5}{11}$ min.

But, they are together after 65 min.

$\therefore$ Gain in 65 min. $= \left(65\frac{5}{11} - 65\right) = \frac{5}{11}$ min.

Gain in 24 hours $= \left(\frac{5}{11} \times \frac{60 \times 24}{65}\right)$ min. $= 10\frac{10}{143}$ min.

$\therefore$ The clock gains $10\frac{10}{143}$ minutes in 24 hours.

Example 50: A man who went out between 5 or 6 and returned between 6 and 7 found that the hands of the watch had exactly changed place. When did he go out ?

Solution : Between 5 and 6 to 6 and 7, hands will change place after crossing each other one time. ie., they together will make 1 + 1 = 2 complete revolutions.

H.H. will move through $2 \times \frac{60}{13}$ or $\frac{120}{13}$ minute divisions.

Between 5 and 6 $\rightarrow \frac{120}{13}$ minute divisions.

At 5, minute hand is 25 minute divisions behind the hour-hand.

Hence it will have to gain $25 + \frac{120}{13}$ minute divisions on the hour-hand $= \frac{445}{13}$ minute divisions on the hour hand.

The minute hand gains $\frac{445}{13}$ minute divisions in $\frac{445}{13} \times \frac{12}{11}$ minutes $= \frac{5340}{143} = 37\frac{49}{143}$ minutes

$\therefore$ The required time of departure is $37\frac{49}{143}$ minutes past 5.

EXERCISE

1. Minimum difference between x and y such that 1x 71y 61 is exactly divisible by 11 is
(a) 2 (b) 3 (c) 1 (d) 0

2. Two different numbers when divided by the same divisor, left remainder 11 and 21 respectively, and when their sum was divided by the same divisor, remainder was 4. What is the divisor?
(a) 36 (b) 28
(c) 14 (d) 9

3. A number when divided by a divisor, left remainder 23. When twice of the number was divided by the same divisor, remainder was 11. Find the divisor.
(a) 12 (b) 34
(c) 35 (d) data inadequate

4. A number when divided by 5 leaves a remainder 3. What is the remainder when the square of the same number is divided by 5?
(a) 9 (b) 3
(c) 0 (d) 4

5. Find the unit digit in the product $(2467)^{153} \times (341)^{72}$.
(a) 6 (b) 7
(c) 8 (d) 9

6. Which digits should come in place of * and $ if the number 62684*$ is divisible by both 8 and 5?
(a) 4, 0 (b) 0, 4
(c) 2, 0 (d) 4, 4

7. There is one number which is formed by writing one digit 6 times (e.g. 111111, 444444 etc.). Such a number is always divisible by:
(a) 7 and 11 (b) 11 and 13
(c) 7, 11 and 13 (d) None of these

8. If the product of first sixty positive consecutive integers be divisible by 8^n, where n is an integer, then the largest possible value of n is
(a) 18 (b) 19
(c) 17 (d) 16

9. The digit in the unit's place of the number represented by $(7^{95} - 3^{58})$ is:
(a) 0 (b) 4
(c) 6 (d) 7

10. In the product of first forty positive consecutive integers be divisible by 5^n, where n is an integer, then the largest possible value of n is
(a) 8 (b) 9
(c) 10 (d) 7

11. A number A4571203B is divisible by 18. Find the value of A and B.
(a) 8, 4 (b) 6, 8
(c) 4, 6 (d) 6, 6

12. Let x and y be positive integers such that x is prime and y is composite. Then
(a) y – x cannot be an even integer
(b) xy cannot be an even integer.
(c) (x + y)/x cannot be an even integer
(d) None of the above statements is true.

13. The sum of $5^2 + 6^2 + 7^2 + ... + 15^2$ is
(a) 1110 (b) 1120
(c) 1310 (d) 1210

14. If x 959 y is divisible by 44 and y > 5, then what are values of the digit x and y ?
(a) x = 7, y = 6 (b) x = 4, y = 8
(c) x = 6, y = 7 (d) None of these

15. The unit's digit in the product $(3127)^{173}$ is :
(a) 1 (b) 3
(c) 7 (d) 9

16. The unit's digit in the product $(7^{71} \times 6^{59} \times 3^{65})$ is :
(a) 1 (b) 2
(c) 4 (d) 6

17. Let n(> 1) be a composite integer such that $\sqrt{n}$ is not an integer. Consider the following statements
I : *n* has a perfect integer-valued divisor which is greater than 1 and less than $\sqrt{n}$.
II : *n* has a perfect integer-valued divisor which is greater than $\sqrt{n}$ but less than *n*
Then,
(a) Both I and II are false (b) I is true but II is false
(c) I is false but II is true (d) Both I and II are true

18. If a, a + 2 and a + 4 are prime numbers, then the number of possible solutions for a is
(a) one (b) two
(c) three (d) more than three

19. If the numerator and the denominator of a proper fraction are increased by the same quantity, then the resulting fraction is :
(a) always greater than the original fraction
(b) always less than the original fraction
(c) always equal to the original fraction
(d) none of these

20. If x = – 0.5, then which of the following has the smallest value ?
(a) $2^{\frac{1}{x}}$ (b) $\frac{1}{x}$
(c) $\frac{1}{x^2}$ (d) 2^x

21. What is the number of terms in the series 117, 120, 123, 126,............., 333 ?
(a) 72 (b) 73
(c) 76 (d) 79

22. At what approximate time between 4 and 5 am will the hands of a clock be at right angle?
(a) 4 : 40 am (b) 4 : 38 am
(c) 4 : 35 am (d) 4 : 39 am

23. What will be the acute angle between hands of a clock at 2 : 30 ?
(a) 105° (b) 115°
(c) 95° (d) 135°

24. At what time between 9'o clock and 10'o clock will the hands of a clock point in the opposite directions ?
(a) $16\frac{4}{11}$ minutes past 9 (b) $16\frac{4}{11}$ minutes past 8
(c) $55\frac{5}{61}$ minutes past 7 (d) $55\frac{5}{61}$ minutes to 8

25. A clock gains 15 minutes per day. It is set right at 12 noon. What time will it show at 4.00 am, the next day ?
(a) 4:10 am (b) 4:45 am
(c) 4:20 am (d) 5:00 am

26. At what time between 3 and 4 o'clock, the hands of a clock coincide ?
(a) $16\frac{4}{11}$ minutes past 3 (b) $15\frac{5}{61}$ minutes past 3
(c) $15\frac{5}{60}$ minutes to 2 (d) $16\frac{4}{11}$ minutes to 4

27. Consider the following assumption and two statements:
Assumption: A number, 'ABCDE' is divisible by 11.
Statement I: E – D + C – B + A is divisible by 11.
Statement II: E – D + C – B + A = 0
Which one of the following is correct ?
(a) Only statement I can be drawn from the assumption
(b) Only statement II can be drawn from the assumption
(c) Both the statements can be drawn from the assumption
(d) Neither of the statements can be drawn from the assumption

28. If k is any even positive integer, then $(k^2 + 2k)$ is
(a) divisible by 24
(b) divisible by 8 but may not be divisible by 24
(c) divisible by 4 but may not be divisible by 8
(d) divisible by 2 but may not be divisible by 4

29. If n is a positive integer, then what is the digit in the unit place of $3^{2n+1} + 2^{2n+1}$?
(a) 0 (b) 3
(c) 5 (d) 7

30. If the 14th term of an arithmetic series is 6 and 6th term is 14, then what is the 95th term?
(a) – 75 (b) 75
(c) 80 (d) – 80

31. For a positive integer n, define d (n) = The number of positive divisors of n. What is the value of d [d {d (12)}]?
(a) 1 (b) 2
(c) 4 (d) None of these

32. Consider the following statements:
If p is a prime such that p + 2 is also a prime, then
I. p (p + 2) + 1 is a perfect square.
II. 12 is a divisor of p + (p + 2), if p > 3.
Which of the above statements is/are correct ?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

33. If three sides of a right angled triangle are integers in their lowest form, then one of its sides is always divisible by
(a) 6 (b) 5
(c) 7 (d) None of these

34. What is the number of prime factors of 30030?
(a) 4 (b) 5
(c) 6 (d) None of these

35. Which one of the following is a prime number ?
(a) 161 (b) 171
(c) 173 (d) 221

36. Consider the following statements:
I. The product of any three consecutive integers is divisible by 6.
II. Any integer can be expressed in one of the three forms 3k, 3k + 1, 3k + 2, where k is an integer.
Which of the above statements is/are correct ?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

37. Consider the following statements:
I. If n is a prime number greater than 5, then $n^4 – 1$ is divisible by 2400.
II. Every square number is of the form 5n, (5n – 1) or (5n + 1), where n is a whole number.
Which of the above statements is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

38. What is the harmonic mean of 10, 20, 25, 40 and 50?
(a) 25 (b) 30
(c) 26.1 (d) 21.3

39. Consider the following statements:
(I) There is a finite number of rational numbers between any two rational numbers.
(II) There is an infinite number of rational numbers between any two rational numbers.
(III) There is a finite number of irrational numbers between any two rational numbers.
Which of the above statements is/are correct?
(a) Only I (b) Only II
(c) Only III (d) Both I and II

40. If m and n are natural number, then $\sqrt[m]{n}$ is
(a) always irrational
(b) irrational unless n is the mth power of an integer
(c) irrational unless m is the nth power of an integer
(d) irrational unless m and n are coprime

41. Every prime number of the form 3k + 1 can be represented in the form 6m + 1 (where, k and m are integers), when
(a) k is odd
(b) k is even
(c) k can be both odd and even
(d) No such form is possible

42. If k is a positive integer, then every square integer is of the form
(a) only 4k (b) 4k or 4k + 3
(c) 4k + 1 or 4k + 3 (d) 4k or 4k + 1
43. If b is the largest square divisor of c and a^2 divides c, then which one of the following is correct (where a, b and c are integers) ?
(a) b divides a (b) a does not divide b
(c) a divides b (d) a and b are coprime
44. Consider the following statements:
I. 7710312401 is divisible by 11.
II. 173 is a prime number.
Which of the statements given above is/are correct ?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II
45. Consider the following statements:
I. To obtain prime numbers less than 121, we have to reject all the multiples of 2, 3, 5 and 7.
II. Every composite number less than 121 is divisible by a prime number less than 11.
Which of the statements given above is/are correct ?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II
46. Consider the following statements:
I. No integer of the form 4k + 3, where k is an integer, can be expressed as the sum of two squares.
II. Square of an odd integer can expressed in the form $8k + 1$, where k is an integer.
Which of the above statements is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II
47. $7^{10} - 5^{10}$ is divisible by
(a) 10 (b) 7
(c) 5 (d) 11
48. If $N^2 - 33$, $N^2 - 31$ and $N^2 - 29$ are prime numbers, then what is the number of possible values of N, where N is an integer ?
(a) 1 (b) 2
(c) 6 (d) None of these
49. Consider all those two-digits positive integers less than 50, which when divided by 4 yield unity as remainder. What is their sum ?
(a) 310 (b) 314
(c) 218 (d) 323
50. What is the remainder when 4^{1012} is divided by 7?
(a) 1 (b) 2
(c) 3 (d) 4
51. What is the remainder when $(17^{23} + 23^{23} + 29^{23})$ is divided by 23 ?
(a) 0 (b) 1
(c) 2 (d) 3
52. p,q and r are prime numbers such that p < q < r < 13. In how many cases would $(p + q + r)$ also be a prime number ?
(a) 1 (b) 2
(c) 3 (d) None of these
53. What is the number of divisors of 360 ?
(a) 12 (b) 18
(c) 24 (d) None do these
54. A student was asked to multiply a number by 25. He instead multiplied the number by 52 and got the answer 324 more than the correct answer. The number to be multiplied was **(CDS)**
(a) 12 (b) 15
(c) 25 (d) 32
55. The difference between the squares of two consecutive odd integers is always divisible by **(CDS)**
(a) 3 (b) 7
(c) 8 (d) 16
56. What is the maximum value of m if the number $N = 35 \times 45 \times 55 \times 60 \times 124 \times 75$ is divisible by 5^m? **(CDS)**
(a) 4 (b) 5
(c) 6 (d) 7
57. A person goes to a market between 4 p.m. and 5 p.m. When he comes back, he finds that the hour hand and minute hand have interchanged their positions. For how much time (approximately) was he out of his house? **(CDS)**
(a) 55.25 minutes (b) 55.30 minutes
(c) 55.34 minutes (d) 55.38 minutes
58. When a ball bounces, it rises to $\frac{2}{3}$ of the height from which it fell. If the ball is dropped from a height of 36 m, how high will it rise at the third bounce? **(CDS)**
(a) $10\frac{1}{3}$ m (b) $10\frac{2}{3}$m
(c) $12\frac{1}{3}$ m (d) $12\frac{2}{3}$ m
59. A light was seen regularly at an interval of 13 seconds. It was seen for the first time at 1 hour 54 minutes 50 seconds (a.m.) and the last time at 3 hours 17 minutes 49 seconds (a.m.). How many times was the light seen? **(CDS)**
(a) 375 (b) 378
(c) 383 (d) 384
60. Consider the following statements for the sequence of numbers given below : **(CDS)**
11, 111, 1111, 11111, ...
1. Each number can be expressed in the form $(4m + 3)$, where m is a natural number.
2. Some numbers are squares.
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2
61. Consider the following statements : **(CDS)**
1. There exists only one prime number p such that $(17p + 1)$ is a square.
2. If x is the product of 10 consecutive prime numbers starting from 2, then $(x + 1)$ is also a prime number.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

62. Out of 532 saving accounts held in a post office, 218 accounts have deposits over ₹ 10,000 each. Further, in 302 accounts, the first or sole depositors are men, of which the deposits exceed ₹ 10,000 in 102 accounts. In how many accounts the first or sole depositors are women and the deposits are up to ₹ 10,000 only? **(CDS)**

(a) 116

(b) 114

(c) 100

(d) Cannot be determined from the given data

63. What is the remainder obtained when 1421 × 1423 × 1425 is divided by 12? **(CDS)**

(a) 1 (b) 2

(c) 3 (d) 4

64. What is the remainder when 4^{96} is divided by 6? **(CDS)**

(a) 4 (b) 3

(c) 2 (d) 1

65. Let a two-digit number be k times the sum of its digits. If the number formed by interchanging the digits is m times the sum of the digits, then the value of m is **(CDS)**

(a) $9-k$ (b) $10-k$

(c) $11-k$ (d) $k-1$

66. Let S be a set of first fourteen natural numbers. The possible number of pairs (a, b), where a, b, S and a, b such that ab leaves remainder 1 when divided buy 15, is **(CDS)**

(a) 3 (b) 5

(c) 6 (d) None of the above

67. A clock strickes once at 1 o'clock, twice at 2 o'clock and thrice at 3 o'clock and so on. If it takes 8 seconds to strike at 5 o'clock, the time taken by it to strike at 10 o'clock is **(CDS)**

(a) 14 seconds (b) 16 seconds

(c) 18 seconds (d) None of the above

68. What is the maximum value of m, if the number $N = 90 \times 42 \times 324 \times 55$ is divisible by 3^m? **(CDS)**

(a) 8 (b) 7

(c) 6 (d) 5

69. If a and b are negative real numbers and c is a positive real number, then which of the following is/are correct? **(CDS)**

1. $a-b<a-c$

2. If $a<b$ then $\frac{a}{c}<\frac{b}{c}$.

3. $\frac{1}{b}<\frac{1}{c}$

Select the Correct answer using the code given below.

(a) 1 (b) 2 only

(c) 3 only (d) 2 and 3

HINTS & SOLUTIONS

1. (a) As 1x 71y 61 is exactly divisible by 11.
$(1 + 7 + y + 1) - (x + 1 + 6) = 0$ or multiple of 11 for minimum difference
$9 + y - 7 - x = 0$
$\Rightarrow x - y = 2$

2. (b) Divisor = [Sum of remainders] – [Remainder when sum is divided]
$= 11 + 21 - 4 = 28$

3. (c) Let the original number be N, divisor be 'd' and quotients be 'k'
Then $N = kd + 23$ ....(i)
$2N = 2(kd + 23)$
$= 2kd + 46$
Now, Rem $[2N/d] = 11$
Rem $[(2kd + 46)/d] = 11$
Rem $[46/d] = 11$
$d = 35$
divisor = 35

4. (d) Let the number be $5q + 3$, where q is quotient
Now, $(5q+3)^2 = 25q^2 + 30q + 9$
$= 25q^2 + 30q + 5 + 4$
$= 5[5q^2 + 6q + 1] + 4$
Hence, remainder is 4

5. (b) Clearly, unit's digit in the given product = unit's digit in $7^{153} \times 1^{72}$.
Now, 7^4 gives unit digit 1.
$\therefore$ 7^{153} gives unit digit $(1 \times 7) = 7$. Also 1^{72} gives unit digit 1.
Hence, unit's digit in the product $= (7 \times 1) = 7$.

6. (a) Since the given number is divisible by 5, so 0 or 5 must come in place of \$. But, a number ending with 5 is never divisible by 8. So, 0 will replace \$.
Now, the number formed by the last three digits is 4*0, which becomes divisible by 8, if * is replaced by 4.
Hence, digits in place of * and \$ are 4 and 0 respectively.

7. (c) Since 111111 is divisible by each one of 7, 11 and 13, so each one of given type of numbers is divisible by each one of 7, 11, and 13. as we may write, 222222 $= 2 \times 111111$, $333333 = 3 \times 111111$, etc.

8. (a) Product of first sixty consecutive integers = 60!
$8 = 2 \times 2 \times 2 = 2^3$
highest power of 2 is 60!
$= \left[\frac{60}{2}\right] + \left[\frac{60}{2^2}\right] + \left[\frac{60}{2^3}\right] + \left[\frac{60}{2^4}\right] + \left[\frac{60}{2^5}\right]$
$= 30 + 15 + 7 + 3 + 1 = 56$
highest power of 80 or $(2^3) = \left[\frac{56}{3}\right] = 18$

9. (b) Unit digit in 7^4 is 1. So, unit digit in 7^{92} is 1.
$\therefore$ Unit digit in 7^{95} is 3.
Unit digit in 3^4 is 1.
$\therefore$ Unit digit in 3^{56} is 1.
$\therefore$ Unit digit in 3^{58} is 9.
$\therefore$ Unit digit in $(7^{95} - 3^{58}) = (13 - 9) = 4$.

10. (b) Product of first forty positive integers
$1 \times 2 \times 3 \times ---- \times 40 = 40!$
Highset power of 5 $= \left[\frac{40}{5}\right] + \left[\frac{40}{5^2}\right] = 8 + 1 = 9$
largest possible value of n is 9

11. (b) The number is divisible by 18 i.e., it has to be divisible by 2 and 9.
$\therefore$ B may be 0, 2, 4, 6, 8.
$A + 4 + 5 + 7 + 1 + 2 + 0 + 3 + B = A + B + 22$.
A + B could be 5, 14 (as the sum can't exceed 18, since A and B are each less than 10).
So, A and B can take the values of 6, 8.

12. (d) x is prime say 7
y is not prime but composite no. say 8, 9, 21
(a) $9 - 7 = 2$ (b) $7 \times 8 = 56$
(c) $\frac{21+7}{7} = 4$
By hit and trial all the 3 options can be proved wrong

13. (d) Sum of square of first 'n' natural number
$= \frac{n(n+1)(2n+1)}{6}$
$S = 5^2 + 6^2 + ... + 15^2$
$= (1^2 + 2^2 + ... + 15)^2 - (1^2 + 2^2 + ... + 4^2)$
$S = \frac{15 \times 16 \times 31}{6} - \frac{4 \times 5 \times 9}{6}$
$= 1210$

14. (a) Here $44 = 11 \times 4$
$\therefore$ the number must be divisible by 4 and 11 respectively.
Test of 4 says that 9y must be divisible by 4 and since $y > 5$, so $y = 6$
Again , x 9596 is divisible by 11, so $x + 5 + 6 = 9 + 9$
$\Rightarrow$ $x = 7$
Thus $x = 7$, $y = 6$

15. (c) Unit digit in $(3127)^{173}$ = Unit digit in $(7)^{173}$. Now, 7^4 gives unit digit 1.
$\therefore$ $(7)^{173} = (7^4)^{43} \times 7^1$. Thus, $(7)^{173}$ gives unit digit 7.

16. (c) Unit digit in 7^4 is 1.
Unit digit in 7^{68} is 1.
$\therefore$ Unit digit in $7^{71} = 1 \times 7^3 = 3$

Again, every power of 6 will give unit digit 6.

$\therefore$ Unit digit in 6^{59} is 6.

Unit digit in 3^4 is 1.

$\therefore$ Unit digit in 3^{64} is 1. Unit digit in 3^{65} is 3.

$\therefore$ Unit digit in $(7^{71} \times 6^{59} \times 3^{65})$

= Unit digit in $(3 \times 6 \times 3) = 4$.

17. (d) Let n = 6

Therefore $\sqrt{n} = \sqrt{6} \approx 2.4$

Now, the divisor of 6 are 1, 2, 3

If we take 2 as divisor then $\sqrt{n} > 2 > 1$.

Statement I is true.

If we take 3 as divisor then $6 > 3 > 2.4$, i.e. $n > \sqrt{n}$

Therefore statement II is true

18. (a) a, a + 2, a + 4 are prime numbers.

Put value of 'a' starting from 3, we will have 3, 5 and 7 as the only set of prime numbers satisfying the given relationships.

19. (a) Let us take a proper fraction, such as $\frac{1}{2}$.

Now, the new fraction = $\frac{1+2}{2+2} = \frac{3}{4}$

Thus, $\frac{3}{4} > \frac{1}{2}$

20. (b) Putting the value of x = – 0.5 in all the options.

(a) $2^{1/-0.5} = 2^{-2} = \frac{1}{4}$ (b) $\frac{1}{-0.5} = -2$

(c) $\frac{1}{(-0.5)^2} = 4$ (d) $2^{-0.5} = \frac{1}{\sqrt{2}}$

So, clearly (b) is smallest.

21. (b) 117, 120, 123, 126, ..., 333

Given series is an A.P series with first term, a = 117, last term ℓ = 333 and common difference, d = 3

last term, $\ell = a + (n-1)d$

where, n = number of terms.

$117 + (n-1)3 = 333$

$(n-1)3 = 216 \quad n = 73$

22. (b) Here H × 30 = 4 × 30 = 120°.

(Since initially the hour hand is at 4. $\therefore$ H = 4).

Required angle A = 90° and since, H × 30 > A° so, there will be two timings.

Required time $T = \frac{2}{11}(H \times 30 \pm A)$ minutes past H.

$\therefore$ One timing $= \frac{2}{11}(4 \times 30 + 90)$ minutes past 4

$= 38\frac{2}{11}$ minutes past 4.

Or 4 : 38 approx.

23. (a) At 2'O Clock, Minute Hand will be 10 × 6 = 60° behind the Hour Hand.

In 30 minutes, Minute Hand will gain $\left(5\frac{1}{2}\right)^\circ \times 30$

$= 150 + 15 = 165°$

$\therefore$ Angle between Hour Hand and Minute Hand

$= 165 - 60 = 105°$

24. (a) At 9'o clock, the Minute Hand is ahead of Hour Hand by 45 minutes. The hands will be opposite to each other when there is a space of 30 minutes between them.

This will happen when the Minute Hand gains 15 minutes space over Hour Hand.

Time taken by Minutes Hand to gain 15 minutes

$= 15 \times \left(1 + \frac{1}{11}\right) = 15 + \frac{15}{11} = 15 + 1\frac{4}{11} = 16\frac{4}{11}$ minutes.

Hence the Hands are opposite to each other at $16\frac{4}{11}$ minutes past 9.

25. (a) The clock gains 15 min in 24 hours.

Therefore, in 16 hours, it will gain 10 minutes.

Hence, the time shown by the clock will be 4.10 am.

26. (a) Since, in one hour, two hands of a clock coincide only once, so, there will be value.

Required time $T = \frac{2}{11}(H \times 30 + A°)$ minutes past H.

Here H = initial position of hour hand = 3 (Since 3 o'clock)

A° = required angle = 0° (Since it coincides)

$T = \frac{2}{11}(3 \times 30 + 0)$ minutes past 3

$= 16\frac{4}{11}$ minutes past 3

27. (c) We know that, if the difference of the sum of odd digits and sum of even digits is either 0 or mlultiple of 11, then the number is divisible by 11.

Given number is $ABCDE$.

Here, $A + C + E - (B + D) = 0$ or divisible by 11

Hence, both statements are true.

28. (b) If k is any even positive integer, then $(k^2 + 2k)$ is divisible by 8 but may not be divisible by 24.

Let $k = 2$

$k^2 + k.2 = 4 + 4 = 8$

So it is divisible by 8 but not be divisible by 24, in this case.

29. (c) $3^{2n+1} + 2^{2n+1} = 3 \times (3)^{2n} + 2 \times (2)^{2n}$

$= 3 \times (9)^n + 2 \times (4)^n$

n	Unit digit of $(9)^n$	Unit digit of $(4)^n$
1	9	4
2	1	6
3	9	4
4	1	6
5	9	4
⋮	⋮	⋮

Thus, when n is odd, then unit digit of $(9)^n = 9$ and $(4)^n = 4$
and when n is even, then unit digit of $(9)^n = 1$ and $(4)^n = 6$
Hence, when n is odd positive integer, then
$3 \times$ (unit digit of 9) + $2 \times$ (unit digit of 4)
$= 3 \times 9 + 2 \times 4 = 35$
Hence, unit digit of $(3)^{2n+1} + (2)^{2n+1} = 5$
Also, when n is even positive integer, then
$3 \times$ (unit digit of 9) + $2 \times$ (unit digit of 4)
$= 3 \times 1 + 2 \times 6 = 15$
Hence, unit digit of $(3)^{2n+1} + (2)^{2n+1} = 5$

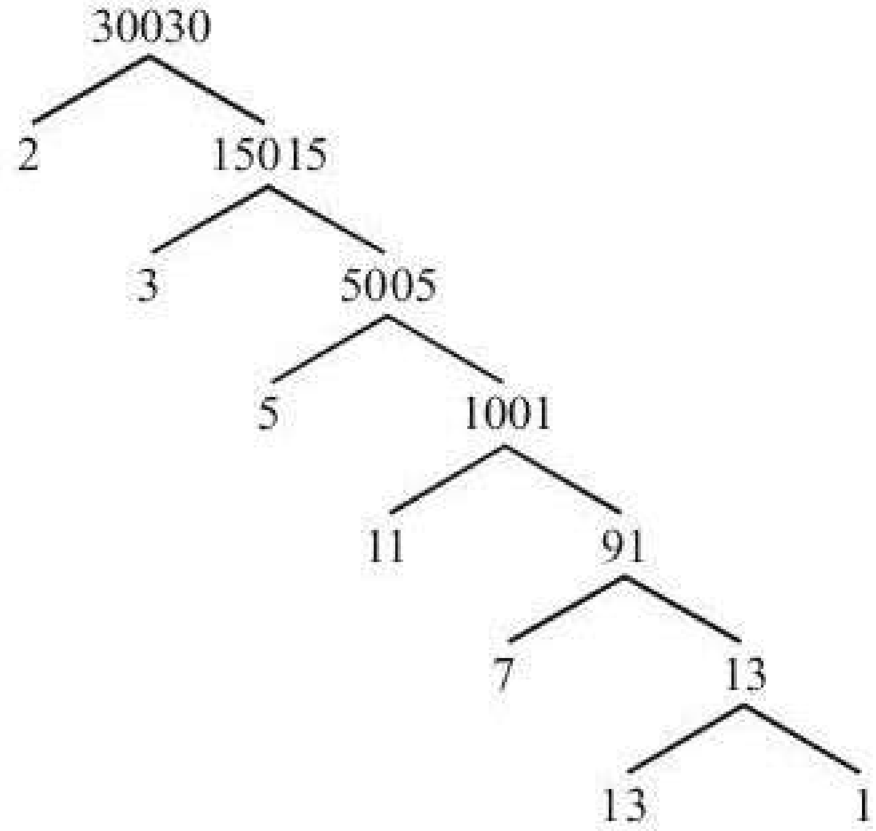
30. (a) $\because \quad T_{14} = 6$
$\Rightarrow \quad a + 13d = 6$
and $T_6 = 14 \Rightarrow a + 5d = 14$
On solving equations (i) and (ii), we get
$a = 19, d = -1$
$\therefore \quad T_{95} = a + 94d = 19 - 94 = -75$

31. (d) $d\,[d\,\{d\,(12)\}] = d\,[d\,(6)]$
($\because$ positive integer divisor of 12 = 1, 2, 3, 4, 6, 12)
$= d\,(4)$ ($\because$ positive integer divisor of 6 = 1, 2, 3, 6)
$= 3$ ($\because$ positive integer divisor of 4 = 1, 2, 4)

32. (c) On taking $p = 11$,
$p + 2 = 13$ (prime number)
I. $11 \times 13 + 1 = 144$ (a square number)
II. $11 + 13 = 24$ (12 is a divisor of 24)
Hence, both statements I and II are correct.

33. (b) Let the lowest sides of a right triangle be 3, 4, 5.
By Pythagoras theorem, $(3)^2 + (4)^2 = (5)^2$
Hence, one of its sides is always divisible by 5.

34. (c)



So, prime factors of 30030 are
2, 3, 5, 11, 7, and 13
So, number of prime factors of 30030 is 6.

35. (c) 161, 171, 221, are divisible by 7, 3 and 13 respectively.
But 173 is not divisible by any others numbers except 1 and 173.

36. (c) I. The product of any three consecutive integers is divisible by 3! i.e., 6.
II. Here, $3k = \{..., -6, -3, 0, 3, 6, ...\}$
$3k + 1 = \{..., -5, -2, 1, 4, 7, ...\}$
and $3k + 2 = \{..., -4, -1, 2, 5, 8, ...\}$
$\therefore \{3k, 3k+1, 3k+2\}$
$= \{..., -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, ...\}$
Hence, it is true.

37. (c) I. Given, n is a prime number greater than 5.
Now, $n^4 - 1 = (n^2 - 1)(n^2 + 1)$
$= (n - 1)(n + 1)(n^2 + 1)$
Put $n = 7$ (prime number greater than 5)
$n^4 - 1 = (7 - 1)(7 + 1)(49 + 1)$
$= 6 \cdot 8 \cdot 50 = 2400$
So, statements I is true.
II. Given, $n \in W$ (whole number)
i.e., $n = 0, 1, 2, 3, 4, 5, ...$
For $n = 0$,
$5n, (5n - 1), (5n + 1) = 0, -1, 1 = (0)^2, -1, (1)^2$
For $n = 1$,
$5n, (5n - 1), (5n + 1) = 5, 4, 6 = 5, (2)^2, 6$
For $n = 2$,
$5n, (5n - 1), (5n + 1) = 10, 9, 11$
$= 10, (3)^2, 11$
For $n = 3$,
$5n, (5n - 1), (5n + 1) = 15, 14, 16 = 15, 14, (4)^2$
For $n = 4$,
$5n, (5n - 1), (5n + 1) = 20, 19, 21$
For $n = 5$,
$5n, (5n - 1), (5n + 1) = 25, 24, 26 = (5)^2, 24, 26$
... so on.
So, statements II is true.

38. (d) Let the number are $a_1 = 10, a_2 = 20, a_3 = 25, a_4 = 40$ and $a_5 = 50$.

$$\therefore \text{Harmonic mean} = \frac{\text{Number of observations}}{\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\frac{1}{a_4}+\frac{1}{a_5}}$$

$$= \frac{5}{\frac{1}{10}+\frac{1}{20}+\frac{1}{25}+\frac{1}{40}+\frac{1}{50}}$$

$$= \frac{5}{\frac{20+10+8+5+4}{200}}$$

$$= \frac{5 \times 200}{47} = \frac{1000}{7} = 21.27 \approx 21.3$$

39. (b) We know that, between any two rational numbers, there are an infinite number of rational and irrational numbers.
Hence, statement II is correct.

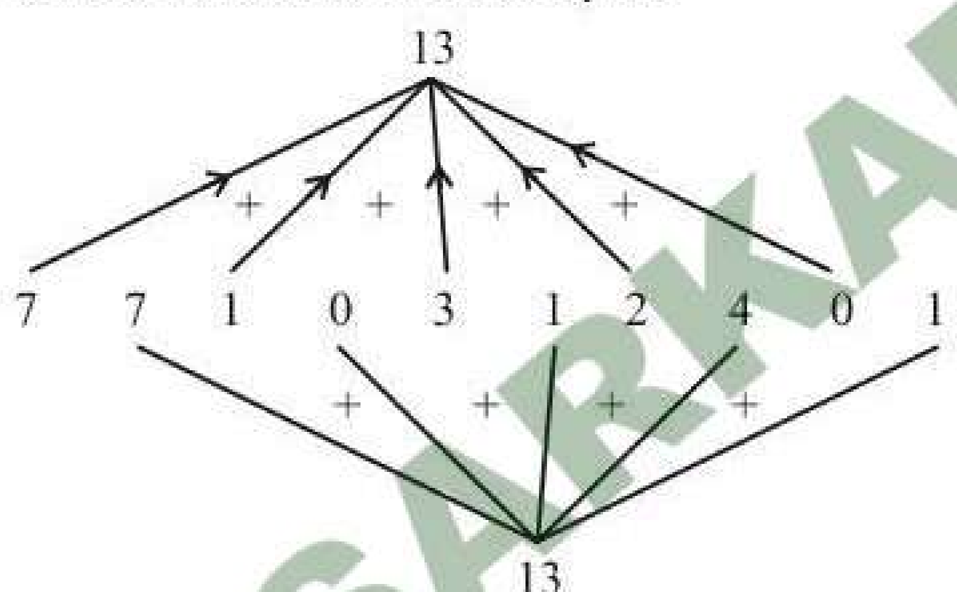
40. (b) If m and n are natural numbers, then $\sqrt[m]{n}$ is irrational unless n is mth power of an integer.

41. (b) Every prime number of the form $3k + 1$ can be represented in the form $6m + 1$ only, when k is even.

42. (d) For square integer 25, $4k + 1$ mean $4 \times 6 + 1$
and for 36
$4k$ mean 4×9
Now, if k is a positive integer, then every square integer is of the form $4k$ or $4k + 1$

43. (c) Here b is the largest square divisor of c and a^2 divides c then, it is sure that a divides b.

44. (c) **Statement I:**
Divisibility rule of 11: The difference between the sum of even places and the sum of odd places is 0 or that number is divisible by 11.



Difference = Sum of odd place – Sum of even place = 13 – 13 = 0, it is divisible by 11.
Statement II:
173 is square of approximately 13.
So, below 13 prime number are -
2, 3, 5, 7, 11
Now, 173 is not divisible by 2, 3, 5, 7 and 11.
So, it is a prime number.
So, both the statements I and II are correct.

45. (c) Both the statements given are correct. As 121 is the square of 11. So, to obtain prime numbers less than 121, we reject all the multiples of prime numbers less than 11 *i.e.*, 2, 3, 5 and 7. Similarly, every composite number less than 121 is divisible by a prime number less than 11 *i.e.*, 2, 3, 5 or 7.

46. (a) I. $f(k) = 4k + 3$
For $k = 1, f(1) = 4 \times 1 + 3 = 7$
For $k = 2, f(2) = 4 \times 2 + 3 = 11$
For $k = 3, f(3) = 4 \times 3 + 3 = 15$
Values of $f(k)$ for $k = 1, 2, ...$ cannot be expressed as sum of squares, since
$1^2 + 2^2 = 5$, $1^2 + 3^2 = 10$, $2^2 + 3^2 = 13$
II. $f(k) = 8k + 1$
For $k = 1, f(1) = (8 \times 1) + 1 = 9 = (3)^2$
For $k = 2, f(2) = (8 \times 2) + 1 = 17$
For $k = 3, f(3) = (8 \times 3) + 1 = 25 = (5)^2$
For $k = 4, f(4) = (8 \times 4) + 1 = 33$
For $k = 5, f(5) = (8 \times 5) + 1 = 41$
$f(k) = 8k + 1$ is square of an odd integer only for some values of k. So, only Statement I is correct.

47. (d) $a^n - b^n$ is divisible by (a+b) & (a–b) if n is even
$\therefore 7^{10} - 5^{10}$ is divisible by (7+5) i.e. 12.

48. (a) By Hook and Crook $N = 6$
$N^2 - 33 = 6^2 - 33 = 36 - 33 = 3$, which is prime.
$N^2 - 31 = 6^2 - 31 = 36 - 31 = 5$, which is prime.
$N^2 - 29 = 6^2 - 29 = 36 - 29 = 7$, which is prime.
So, $N = 6$ only. possible value
Hence, only one value is the possible value.

49. (a) Let the two-digits numbers less than 50 which when divided by 4 yield unity as remainder be 13, 17, ..., 49. Here, first term, a = 13, common difference, $d = 4$ and $n = 10$.

$\therefore$ Required sum $= \frac{n}{2}[a + a_n]$

$$= \frac{10}{2}[13 + 49]$$

$$= \frac{10 \times 62}{2} = 310$$

50. (d) $= \frac{4^{1012}}{7} = \frac{4^{3 \times 337 + 1}}{7} = \frac{4^{3 \times 337} \times 4}{7}$

$$= \frac{(64)^{337} \times 4}{7} = \frac{(9 \times 7 + 1)^{337} \times 4}{7}$$

We know that $= \frac{(ax+1)^n}{a}$ then its Remainder is 1
$= 1 \times 4 = 4$

51. (a) $\frac{17^{23} + 23^{23} + 29^{23}}{23}$

$$= \frac{17^{23} + (0)^{23} + (29)^{23}}{23}$$

$$= \frac{17^{23} + (29)^{23}}{23} = \frac{(17)^{23} + (6)^{23}}{23}$$

$\therefore \frac{a^n + b^n}{n}$ is divisible by a + b if n is add. So $17^{22} + 6^{23}$ is divisible by (17+6) = 23. So remainder = 0
So remainder is always zero.

52. (b) The prime numbers less than 13 are 2, 3, 5, 7, 11.
Also, using the condition, $p < q < r < 13$ and $p + q + r$ is a prime number
Hence, only two possible pairs exist i.e. (3, 5, 11) and (5, 7, 11).

53. (c) $\because \quad 360 = 2^3 \times 3^2 \times 5$
$\therefore$ Number of divisors = (3 + 1) (2 + 1) (1 + 1)
= 4 × 3 × 2 = 24

54. (a) Let *a* number be x
multiply by 25 we get another number y.
According to question
$x \times 25 = y$ ....(i)
Again $x \times 52 = 324 + y$ ....(ii)
Now subtract equation (i) from equation (ii)
$\Rightarrow x \times 52 - x \times 25 = 324 + y - y$
$\Rightarrow 27x = 324$
$$x = \frac{324}{27} = 12$$

55. (c) Let two consecutive odd numbers = $(2x + 1)$ and $(2x + 3)$
According to question
$= (2x + 3)^2 - (2x + 1)^2$
$= 4x^2 + 12x + 9 - 4x^2 - 1 - 4x$
$= 8x + 8 = 8(x + 1)$
So, it is divisible by 8.

56. (c) N = 35 × 45 × 55 × 60 × 124 × 75
$= 7\times5\times9\times5\times11\times5\times12\times5\times124\times5\times5\times3$
$= 5^6 \times 7 \times 9 \times 11 \times 12 \times 124 \times 3$
$m = 6$, The given number has maximum factor of 5 is 6.

57. (d) Let us assume that he was out of house for 't' min.
So angle formed by min. hand = $6 \times t$
Angle formed by hour hand = $0.5 \times t$
Now, $0.5 \times t + 6 \times t = 360$
$\Rightarrow 6.5\, t = 360$
$$t = \frac{360}{6.5} = 55.38 \text{ min}$$

58. (b) When a ball is dropped from a height = 36 m
1st bounce back = $\frac{2}{3} \times 36 = 24$ m
2nd bounce back = $\frac{2}{3} \times 24 = 16$ m
3rd bounce back $\frac{2}{3} \times 16 = \frac{32}{3} = 10\frac{2}{3}$ m.

59. (d) 1st time seen
= 1 hour 54 min 50 sec
= 3600 sec + 54 × 60 sec + 50 sec
= 6890 sec
2nd time seen = 3 hour + 17 min + 49 sec
= 11869 sec.
Interval between light seen = 11869 – 6890
= 4979
Number of times light was seen
$$= \frac{4979}{13} + 1 = 384$$

60. (a) 11, 111, 1111, 11111, ...
1 Each number is divided by 4 then their remainder are always 3.
So, it can be written as = $4m + 3$
So statement is true.
2 No, this type of numbers are not squares. So it is not true.
Only 1 is true.

61. (c) **Statement 1**
Yes, there exists only one prime number p such that $(17p + 1)$ is a square.
Let $p = 19$
$\Rightarrow (17 \times 19 + 1) = 324$ which is square of 18, so it is true.
Statement 2
Product of first 10 consecutive number
x (2, 3, 5, 7, 11, 13, 17, 19, 23, 29) = 6469693230
$\therefore x + 1 = 6469693231$
Divisibility by 7: The difference of the numbers upto thousands place and remaning part of the number if it is divisible by 7 then the number is divisible by 7
6469693 231
Thousand part Remaining part
Difference = 6469693 – 231
= 6469462
Again difference = 6469 – 462
= 6007
Again difference 7 – 6 = 1 which is not divisible by 7.
Therefore, number 6469693231 is not divisible by 7
So it is a prime number
So both statements are true.

62. (b) Number of account up to ₹ 10,000
= 532 – 218
= 314 accounts.
Rest of accounts of men deposits
= 302 – 102 = 200 accounts
Number of accounts of women deposits = 314 – 200
= 114

63. (c) $\frac{1421 \times 1423 \times 1425}{12}$
When we divide 1421, 1423 and 1425 then 5,7 at 9 are the remainders respectively.
$$= \frac{5 \times 7 \times 9}{12} = \frac{315}{12} = 3$$

64. (a) $\frac{4^{96}}{6}$

When 4^1 is divided by 6 then remainder = 4
4^2 is divided by 6 then remainder = 4
4^3 is divided by 6 then remainder = 4
4^4 is divided by 6 then remainder = 4
....
....
4^{96} divided by 6 then remainder = 4

65. (c) Let two digit number = 10y + x
According to question,
1st condition, $10y + x = k(x + y)$ ...(1)
2nd condition, $10x + y = m(x + y)$ ...(2)
Adding (1) and (2) we get
$11x + 11y = (k + m) \times (x + y)$
$11(x + y) = (k + m)(x + y)$
$k + m = 11$
$m = 11 - k$
∴ Option (c) is correct

66. (d) Here S be a set of 14 Natural Numbers i.e. {1, 2, 3, 4, 5. ---- 14}
Possible no. of pairs.
{a, b} is {(2, 8), (7, 13)}
⇒ Pairs = 2
⇒ Option (d) is correct.

67. (b) Time taken to reach 5'O Clock = 8 seconds

Time taken to reach 10'O Clock = $\frac{8}{5}$ seconds

Time taken to reach 10'O Clock = $\frac{8}{5} \times 10$ = 16 seconds

∴ Option (b) is correct.

68. (b) Here $N = 90 \times 42 \times 324 \times 55$
Now $90 = 3 \times 3 \times 10 = 3^2 \times 10$
$42 = 14 \times 3 = 14 \times 3^1$
$324 = 3 \times 3 \times 3 \times 3 \times 4 = 3^4 \times 4$
$55 = 11 \times 5$
$N = 3^2 \times 3^1 \times 3^4 \times 10 \times 14 \times 4 \times 11 \times 5$
$N = 3^7 \times 10 \times 14 \times 4 \times 11 \times 5$
Maximum value of m = 7
∴ Option (b) is correct.

69. (b) Since a, b are negative numbers.
$a < 0$ and $b < 0$
C is a positive real number
⇒ $c > 0$
(1) $a - b < a - c$
⇒ $-b < -c$
⇒ $b > c$

It is not true as $b < c$

(2) if $a < b \Rightarrow \frac{a}{c} < \frac{b}{c}$ this is true.

(3) $\frac{1}{b} < \frac{1}{c}$

Since $c > b \Rightarrow \frac{1}{c} < \frac{1}{b}$ this is not true.

∴ option (b) is correct.