APPLICATION OF INTEGRAL

CHAPTER – 8

APPLICATION OF INTEGRAL

INTRODUCTION

The process of finding area of some plane region is called **Quadrature**. In this chapter we shall find the area bounded by some simple plane curves with the help of definite integral. For solving the problems on quadrature easily, if possible first draw the rough sketch of the required area.

CURVE TRACING

In chapter function, we have seen graphs of some simple elementary curves. Here we introduce some essential steps for curve tracing which will enable us to determine the required area.

(i) Symmetry

The curve f (x, y) = 0 is symmetrical

- about x-axis if all terms of y contain even powers.
- about y-axis if all terms of x contain even powers.
- about the origin if f(–x, –y) = f(x, y).

Example

$y^2 = 4ax$ is symmetrical about x-axis,

and $x^2 = 4ay$ is symmetrical about y-axis and the curve

$y = x^3$ is symmetrical about the origin

(ii) Origin

If the equation of the curve contains no constant term then it passes through the origin.

Example

$x^2 + y^2 + 2ax = 0$ passes through origin.

(iii) Points of intersection with the axes

If we get real values of x on putting y = 0 in the equation of the curve, then real values of x and y = 0 give those points where the curve cuts the x-axis. Similarly by putting x = 0, we can get the points of intersection of the curve and y-axis.

Example

the curve $x^2/a^2 + y^2/b^2 = 1$ intersects the axes at points

$(\pm a, 0)$ and $(0, \pm b)$.

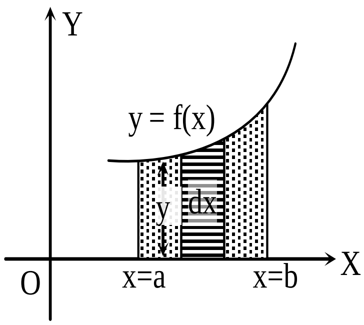
(iv) Region

Write the given equation as y = f(x) , and find minimum and maximum values of x which determine the region of the curve.

AREA BOUNDED BY CURVES

(i) The area bounded by a cartesian curve y = f(x), x-axis and ordinates x = a and x = b is given by,

$$\text{Area} = \int_a^b y\,dx = \int_a^b f(x)\,dx$$



Example

Find the area of the region bounded by $y^2 = 9x$, x = 2, x = 4 and the x-axis in the first quadrant.

Solution:

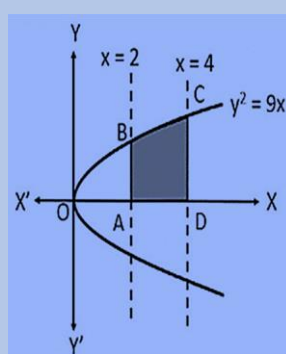
$= \int_2^4 y\,dx$

$= 3\int_2^4 \sqrt{x}\,dx$

$= 2\,[(2)^3 - (\sqrt{2})^3]$

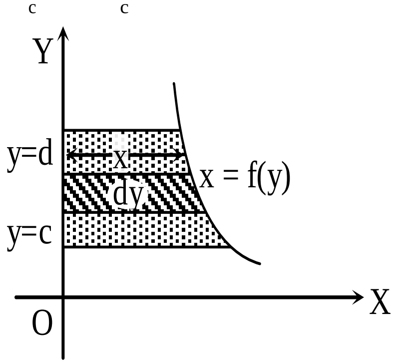
$= 2[8 - 2\sqrt{2}]$

$= 16 - 4\sqrt{2}$



(ii) The area bounded by a cartesian curve x = f(y), y-axis and abscissa y = c and y = d

$$\text{Area} = \int_c^d x\,dy = \int_c^d f(y)\,dy$$



Example

Find the area of the region A = {(x, y) ; $y^2/2 \le x \le y + 4$}.

Solution: Consider equations from the given inequalities,

$y^2 = 2x$ and $x - y = 4$

Here, $y^2 = 2x$ is equation of parabola open towards the +ve x-axis and having

and x – y = 4, is a straight line. focus $(\frac{1}{2}, 0)$

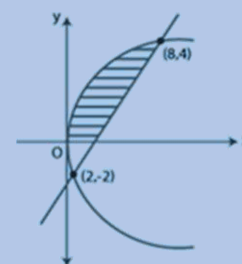
Solving above equations, we get

$y^2 = 2(y + 4)$

or $y^2 - 2y - 8 = 0$

or y = 4 or y = -2

Therefore, point of intersection of line and curve are (2, -2) and (8, 4).



Required Area $= \int_{-2}^{4} \left[(y+4) - \frac{y^2}{2}\right] dy$

$= \left[\frac{(y+4)^2}{2}\right] - \frac{1}{2}\left[\frac{y^3}{3}\right]$

= 1/2[64 – 4] – 1/6(64 + 8)

= 18 sq. units

(iii) If the equation of a curve is in parametric form, say x = f(t) , y = g(t), then the area $= \int_a^b y\,dx = \int_{t_1}^{t_2} g(t)f'(t)\,dt$.

Where t_1 and t_2 are the values of t respectively corresponding to the values of a & b of x.

SYMMETRICAL AREA

If the curve is symmetrical about a coordinate axis (or a line or origin), then we find the area of one symmetrical portion and multiply it by the number of symmetrical portions to get the required area.

POSITIVE AND NEGATIVE CURVE

Area is always taken as positive. If some part of the area lies in the positive side i.e. above x-axis and some part lies in the negative side i.e. below x-axis then the area of two parts should be calculated separately and then add their numerical values to get the desired area.

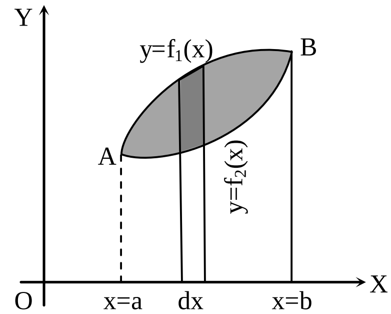
AREA BETWEEN TWO CURVES

I. When two curves intersect at two points and their common area lies between these points.

If $y = f_1(x)$ and $y = f_2(x)$ are two curves where $f_1(x) > f_2(x)$ which intersect at two points A (x = a) and B (x = b) and their common area lies between A & B, then their

Common area = $\int_a^b (y_1 - y_2)\,dx$ =

$$\int_a^b [f_1(x) - f_2(x)]\,dx$$



Example

Find the area of the region bounded by the parabolas $y = x^2$ and $x = y^2$.

Solution: When the graph of both the parabolas is sketched we see that the points of intersection of the curves are (0, 0) and (1, 1) as shown in the figure below.

So, we need to find the area enclosed between these points which would give us the area between two curves

Also, in the given region as we can see,

unit.

$y = x^2 = g(x)$ and

$x = y^2$

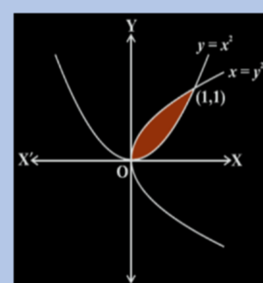
or, $y = \sqrt{x} = f(x)$.

As we can see in the given region,

The area enclosed will be given as,

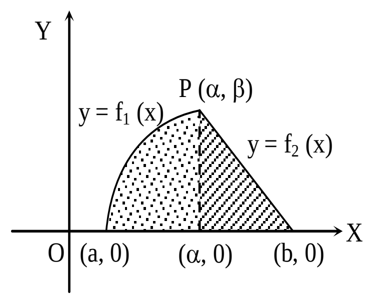
Area= $\int_0^1 \left(f(x) - g(x)\right)dx$

= $\int_0^1 (\sqrt{x} - x^2)\,dx$ $=\left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3}\right] = \frac{1}{3}\ sq$



II. When two curves intersect at a point and the area between them is bounded by x-axis. If $y = f_1(x)$ and $y = f_2(x)$ are two curves which intersect at P (α, β) and meet x-axis at A(a, 0), B(b, 0) respectively, then area between them and x-axis is given by

Area = $\int_a^\alpha f_1(x)\,dx + \int_\alpha^b f_2(x)\,dx$



Example

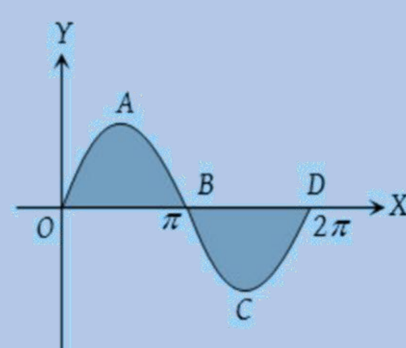
Area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$

Solution: We have $y = \sin x$

Required area = (Area of OAB) + (Area of BCD)

$= \int_0^\pi y\,dx + \int_\pi^{2\pi} (-y)dx$ [since ,area BCD is below x – axis]

$= \int_0^\pi \sin x\,dx - \int_\pi^{2\pi} \sin x dx = 4\ sq.unit$



QUESTIONS

MCQ

Q1. Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
(a) $2\pi ab$ (b) πab
(c) $4\pi ab$ (d) $\frac{\pi}{2}ab$

Q2. Find the area enclosed by $|x|+|y|=1$
(a) 2 (b) 4
(c) 8 (d) 1

Q3. Determine the area of the region enclosed by y = sin x , y = cos x , x = $\frac{\pi}{2}$ and the y – axis(Sq. unit)
(a) $2\sqrt{2}-1$
(b) $2\sqrt{2}-2$
(c) $\sqrt{2}-2$
(d) None of these

Q4. Find the area enclosed by the circle $x^2+y^2=a^2$.
(a) πa^2
(b) πb^2
(c) $\pi a\, b$
(d) $2\pi ab$

Q5. Find the area of the region bounded by the curve $y=x^2$ and the line $y=4$.
(a) $\frac{32}{3}$ (b) $\frac{16}{3}$
(c) $\frac{64}{3}$ (d) $\frac{8}{3}$

Q6. Find the area of the region bounded by the curve $y=x^2$, the x-axis, and the lines $x=1$ and $x=3$.
(a) $\frac{13}{2}$ (b) $\frac{26}{3}$
(c) 26 (d) 13

Q7. Find the area of the region bounded by the parabola $y^2=4x$, the x-axis, and the lines $x=1$ and $x=4$.
(a) 14 (b) 28
(c) $\frac{14}{3}$ (d) $\frac{28}{3}$

Q8. Find the area under the curve $y=\sqrt{6x+4}$ (above the x-axis) from x = 0 to x = 2
(a) $\frac{65}{2}$ (b) 56
(c) $\frac{56}{9}$ (d) $\frac{65}{9}$

Q9. Determine the area enclosed by curve y=x^3, and the lines $y=0, x=2$ and $x=4$.
(a) 25 (b) 50
(c) 80 (d) 60

Q10. Determine the area under the curve $y=\sqrt{a^2-x^2}$, included between the lines $x=0$ and $x=4$.
(a) $\frac{\pi a^2}{4}$ (b) $\frac{\pi a}{4}$
(c) $\frac{\pi a^2}{2}$ (d) $\frac{\pi a^2}{8}$

Q11. Using integration, find the area of the region bounded by the" lines $2y=5x+7$, the x-axis and the lines $x=2$ and $x=8$.
(a) 90 (b) 96
(c) 100 (d) 80

Q12. Find the area of the region bounded by the curve $y^2=4x$ and the lines $x=3$.
(a) $4\sqrt{3}$ (b) $16\sqrt{3}$
(c) $8\sqrt{3}$ (d) $4\sqrt{3}$

Q13. Evaluate the area bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ above the x-axis.
(a) 4π
(b) 2π
(c) 5π
(d) 3π

Q14. Using integration, find the area of the region bounded by the lines y=$1+|x+1|, x=-2, x=3$ and $y=0$.
(a) 27 (b) $\frac{27}{2}$
(c) 13 (d) $\frac{25}{2}$

Q15. Find the area bounded by the curve y= $(4-x^2)$, the y-axis and the lines $y=0, y=3$.
(a) 14 (b) $\frac{14}{3}$
(c) $\frac{14}{2}$ (d) 28

Q16. Using integration, find the area of region bounded by the line y-1 = x, the x-axis, and the ordinates x = -2 and x = 3.
(a) 8.5 (b) 12.5
(c) 10.5 (d) 11.5

Q17. Find the area of the region bounded by y $=4x^2, x=0, y=2$ and $y=4$ in the first quadrant.
(a) $\frac{1}{3}(8+2\sqrt{2})$ (b) $\frac{1}{3}(8-\sqrt{2})$
(c) $\frac{2}{3}(8-2\sqrt{2})$ (d) $\frac{1}{3}(8-2\sqrt{2})$

Q18. find the area of the region bounded by the parabolas $y^2=4x$ and $x^2=4y$
(a) $\frac{32}{3}$ (b) 8
(c) $\frac{16}{3}$ (d) $\frac{25}{3}$

Q19. Find by integration the area bounded by the curve $y^2=4ax$ and the lines $y=2a$ and $x=0$.
(a) $\frac{2a^2}{3}$ (b) $\frac{a^2}{3}$
(c) a^2 (d) $\frac{4a^2}{3}$

Q20. Find the area of bounded by the curve y=cos x, the " x-axis and the ordinates x = 0 and x = 2π.

(a) 5 (b) 4

(c) 3 (d) 6

Q21. Find the area of the triangle, the equations of whose sides are y=$2x+1, y=3x+1$ and $x=4$.

(a) 7 (b) 6

(c) 8 (d) 4

Q22. Find area of region $\{(x,y): x^2 \le y \le x\}$

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$

(c) $\frac{1}{6}$ (d) $\frac{1}{8}$

Q23. Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the ordinates $x=0$ and x = ae, where, $b^2=a^2(1-e^2)$ and $e<1$.

(a) $ab\left[e\sqrt{1-e^2}+\sin^{-1} e\right]$

(b) $2ab\left[e\sqrt{1-e^2}+\sin^{-1} e\right]$

(c) $ab\left[e\sqrt{1-e^2}-\sin^{-1} e\right]$

(d) $-ab\left[e\sqrt{1-e^2}+\sin^{-1} e\right]$

Q24. Find the area of the region bounded by x^2 =4y, y = 2, y = 4 and the y- axis in the first quadrant.

(a) $\frac{32+8\sqrt{2}}{3}$ (b) $\frac{32-8\sqrt{2}}{3}$

(c) $\frac{32-8\sqrt{2}}{6}$ (d) $\frac{32+8\sqrt{2}}{6}$

Q25. The area between x = y^2 and x = 4 is divided into two equal parts by the line **x** = a. Find the value of a

(a) $4^{\frac{2}{3}}$ (b) $4^{\frac{1}{3}}$

(c) $4^{\frac{5}{3}}$ (d) $4^{\frac{2}{5}}$

Q26. The area of the figure bounded by the curve $y=\log_e x$, the x-axis and the straight line $x=e$ is

(a) 5-e (b) 3+e

(c) 1 (d) None

Q27. The area of the region bounded by the curve y^2 = x, the y-axis and between y = 2 and y = 4 is

(a) 52/3 sq. units (b) 54/3 sq. units

(c) 56/3 sq. units (d) None

Q28. Area of the region bounded by the curve x = 2y + 3, the y-axis and between y = -1 and y = 1 is

(a) 6sq. units

(b) 4sq. units

(c) 8sq. units

(d) 12sq. units

Q29. Find half of the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(a) πab

(b) $\frac{\pi}{2}ab$

(c) $2\pi ab$

(d) $\frac{\pi}{4}ab$

Q30. Find 2 times the area enclosed by $|x|+|y|=1$

(a) 4 (b) 8

(c) 16 (d) 2

SUBJECTIVE QUESTIONS

Q1. The area between the curves y = 6 – x – x^2 and x-axis is –

Q2. The area bounded by the curve y = $4x^2$; x = 0, y = 1 and y = 4 in the first quadrant is-

Q3. The area bounded by the curve y^2 = x, straight line y = 4, and y-axis is-

Q4. The area between the curves y^2 = 4x and y = 2x is-

Q5. The parabolas y^2 = 4x and x^2 = 4y divide the square region bounded by the lines x = 4, y = 4 and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is

NUMERICAL TYPE QUESTIONS

Q1. Area bounded by the curves y = |x – 1|, y = 0 and |x| = 2 is______________.

Q2. The area of the region bounded by y^2 = x and x = 36 is divided in the ratio 1 : 7 by the line x = a, then a equals-

Q3. The area of the region {(x, y) : x^2 ≤ y ≤ |x|} is-

Q4. The area bounded by curves $3x^2$ + 5y = 32 and y = |x – 2| is-

Q5. The area bounded by the curves y = sin x, y = cos x and y-axis in first quadrant is-

TRUE AND FALSE

Q1. Area of the region bounded by the curve $y^2=4x$, y - axis and the line $y=3$ is $\frac{9}{4}$ sq. units.

Q2. Area of the region bounded by the curve y = cos x between x = 0 and x = π is 3 sq. units.

Q3. The area (in sq. units) of the region bounded by the curve $y=x^2$ and the line y = 16 is $\frac{128}{3}$ sq. units.

Q4. If we draw a rough sketch of curve y = $\sqrt{x-1}$ in the interval [1 ,5] ,then the area under the curve and between the lines x = 1 and x = 5 is $\frac{16}{3}$ sq. units.

Q5. If f(x) ≥ g(x) in [a , c] and f(x) ≤ g(x) in [c ,b] ,where a < c< b , then the area of the region bounded by the curves in [a, b] is A = $\int_a^c [f(x)-g(x)]\,dx + \int_c^b [g(x)-f(x)]dx$

ASSERTION AND REASONING

Q1. Assertion(A): The area bounded by $y^2 = 4x$ and y=x is 38 sq. units.

Reason(R): The area bounded by $y^2 = 4ax$ and y = mx is $\frac{8a^2}{3m^3}$ sq. units.

(a) both assertion and reason are correct and reason is correct explanation for assertion
(b) both assertion and reason are correct but reason is correct explanation for assertion
(c) assertion is correct but reason is false
(d) both assertion and reason are false

Q2. Assertion(A) : the area of the curve y=sin x **between** 0 and $\frac{\pi}{2}$ is 1 sq. unit

Reason(R): Area = $\int_0^{\frac{\pi}{2}} \sin x\, dx$

(a) both assertion and reason are correct and reason is correct explanation for assertion
(b) both assertion and reason are correct but reason is correct explanation for assertion
(c) assertion is correct but reason is false
(d) both assertion and reason are false

Q3. Assertion(A) : The area of the region bounded by the curve y = x^3, its tangent at (1, 1) and x-axis is $\frac{1}{12}$

Reason (R): Equation of tangent at A is y = 3x+2

(a) both assertion and reason are correct and reason is correct explanation for assertion
(b) both assertion and reason are correct but reason is correct explanation for assertion
(c) assertion is correct but reason is false
(d) both assertion and reason are false

Q4. Assertion (A): The area enclosed by y = $\sqrt{5-x^2}$ and $y = |x-1|$ is $\left(\frac{5\pi}{4} - \frac{1}{2}\right)$

Reason (B) : Area = $\int_{-1}^{2} \sqrt{5-x^2}dx + \int_{-1}^{2} |x-1|dx$

(a) both assertion and reason are correct and reason is correct explanation for assertion
(b) both assertion and reason are correct but reason is correct explanation for assertion
(c) assertion is correct but reason is false
(d) both assertion and reason are false

Q5. Assertion (A) : The area of the region bounded by the curve $y = \sqrt{16-x^2}$ and x – axis is 8π

Reason(R) : By using concept : $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$. Function $y = \sqrt{f(x)}$ is defined for f(x) ≥ 0 .Therefore y can not be negative.

(a) both assertion and reason are correct and reason is correct explanation for assertion
(b) both assertion and reason are correct but reason is correct explanation for assertion
(c) assertion is correct but reason is false
(d) both assertion and reason are false

HOMEWORK

MCQ

Q1. The area between the curve y = $\sin^2$ x, x-axis and the ordinates x = 0 and x = $\frac{\pi}{2}$ is-

(a) π (b) π/2
(c) π /4 (d) π/8

Q2. The area between the curve y = 4 + 3x – x^2 and x-axis is-

(a) 125/6 (b) 125/3
(c) 125/2 (d) None of these

Q3. The area bounded by the curve y^2 = 4x, y-axis and y = 3 is-

(a) 2 units (b) 9/4 units
(c) 7/3 units (d) 3 units

Q4. The area bounded by the curve x = a $\cos^3$ t, y = a $\sin^3$ t, is-

(a) $\frac{\pi a^2}{8}$ (b) $\frac{\pi a^2}{4}$
(c) $\frac{3\pi a^2}{8}$ (d) $\frac{2\pi a^2}{3}$

Q5. The area bounded by the curve y = sin x, x = 0 and x = 2π is-

(a) 4 units (b) 0 units
(c) 4π units (d) 2 units

Q6. The area between the curves y = tan x, y = cot x and x-axis in the interval [0, π/2] is-

(a) log 2 (b) log 3
(c) log $\sqrt{2}$ (d) None of these

Q7. The area between the curve y = $\cos^2$ x, x-axis and ordinates x = 0 and x = π in the interval (0, π) is-

(a) π (b) π/4
(c) π/2 (d) 2π

Q8. The area bounded by curve y = ex log x and y = $\frac{\log x}{e^x}$is-

(a) $\frac{e^2-5}{4}$ (b) $\frac{e^2+5}{4e}$
(c) $\frac{e}{4} - \frac{5}{4e}$ (d) None of these

Q9. If 0 ≤ x ≤ π; then the area bounded by the curve y = x and y = x + sin x is-

(a) 2 (b) 4
(c) 2π (d) 4π

Q10. The area bounded by curve y = |x – 1| and y = 1 is-
(a) 1 (b) 2
(c) 1/2 (d) None of these

SUBJECTIVE QUESTIONS

Q1. If area bounded by the curve $y = 8x^2 - x^5$ and ordinate x = 1, x = k is $\frac{16}{3}$ then k =

Q2. The area between the curves y = cos x and the line y = x + 1 in the second quadrant is-

Q3. The area between the curve y = sech x and x-axis is-

Q4. The area between the curves $y = \sqrt{x}$ and y = x is-

Q5. The area between the parabola $x^2 = 4y$ and line x = 4y – 2 is-

NUMERICAL TYPE QUESTIONS

Q1. The area bounded by the curve y = sin 2x, x- axis and the ordinate x = π/4 is__________.

Q2. Area under the curve y = sin 2x + cos 2x between x = 0 and x $= \frac{\pi}{4}$, is______.

Q3. The area bounded by the curve $y = 1 + 8/x^2$, x-axis, x = 2 and x = 4 is_______.

Q4. The area bounded by the curve $y = x \sin x^2$, x-axis and x = 0 and $x = \sqrt{\frac{\pi}{2}}$ is________.

Q5. The area (in square units) bounded by the curves $y = \sqrt{x}$, 2y – x + 3 = 0, x-axis, and lying in the first quadrant is ______..

TRUE AND FALSE

Q1. Let f(x) be a continuous function defined on [a, b] .Then ,the area bounded by the curve y = f(x) , the x -axis and the ordinates x = a and x = b is given by $\int_b^a f(x)dx$

Q2. If the curve y = f(x) lies below x – axis ,then the area bounded by the curve y = f(x) , the x -axis and the ordinates x = a and x = b is negative .So, the area is given by - $\int_a^b y dx$

Q3. The area bounded by the curve x = f(y) , the y -axis and the abscissae y = c and y = d is given by $\int_c^d f(y)dy$

Q4. The area of a loop bounded by the curve y = a sin x and x-axis is 2a.

Q5. The value of a for which the area of the region bounded by the curve y = sin 2x, the straight lines x = π/6, x = a and x-axis is equal to 1/2 is $\frac{\pi}{3}$.

ASSERTION AND REASONING

Q1. **Assertion(A):** The area between the curve $x = 2y - y^2$ and y-axis is $\frac{4}{3}$ sq. units
Reason(R): Required area = $\int_0^2 x\,dy$
(a) both assertion and reason are correct and reason is correct explanation for assertion
(b) both assertion and reason are correct but reason is correct explanation for assertion
(c) assertion is correct but reason is false
(d) both assertion and reason are false

Q2. **Assertion(A) :** The area between the curve $y = \sin^3 x$, x-axis, and the ordinates x = 0 to x = π/2 is $\frac{1}{6}$
Reason(R): Using sin 3x = 3 sin x + $4\sin^3 x$
(a) both assertion and reason are correct and reason is correct explanation for assertion
(b) both assertion and reason are correct but reason is correct explanation for assertion
(c) assertion is correct but reason is false
(d) both assertion and reason are false

Q3. **Assertion(A) :** The area of the figure bounded by the parabola $y = x^2 + 1$ and the straight line x + y = 3 is $\frac{9}{2}$ sq. units.
Reason (R): , $y^2 = 4ax$ is symmetrical about x-axis,
(a) both assertion and reason are correct and reason is correct explanation for assertion
(b) both assertion and reason are correct but reason is correct explanation for assertion
(c) assertion is correct but reason is false
(d) both assertion and reason are false

Q4. **Assertion (A):** Let f(x) be a non-negative continuous function such that the area bounded by the curve y = f(x), x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is $1 - \frac{\pi}{4} + \sqrt{2}$
Reason (B) : Using this property: $\int_{\frac{\pi}{4}}^{\beta} f(x)dx = \beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta$
(a) both assertion and reason are correct and reason is correct explanation for assertion
(b) both assertion and reason are correct but reason is correct explanation for assertion
(c) assertion is correct but reason is false
(d) both assertion and reason are false

Q5. **Assertion (A) :** The area bounded by the curve y = 1 – |x| and x-axis is 1 sq. unit

Reason(R) : $y = \begin{cases} 1 - x \,; x \geq 0 \\ 1 + x \,; x < 0 \end{cases}$

(a) both assertion and reason are correct and reason is correct explanation for assertion

(b) both assertion and reason are correct but reason is correct explanation for assertion

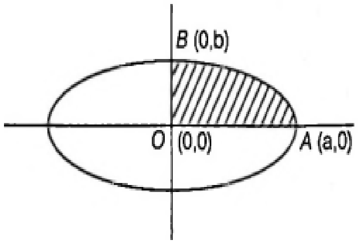
(c) assertion is correct but reason is false

(d) both assertion and reason are false

SOLUTIONS

MCQ

S1. **(b)**



$A = 4(\text{ area } OABO)$

$= 4\int_0^a y dx$, where $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore \quad \frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$

$\Rightarrow \quad y^2 = \frac{b^2}{a^2}(a^2 - x^2)$

$\Rightarrow \quad y = \pm\frac{b}{a}\sqrt{a^2 - x^2}$

In the first quadrant,

$y\& = \frac{b}{a}\sqrt{a^2 - x^2}$

$A\& = 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}dx = 4\frac{b}{a}\int_0^a \sqrt{a^2 - x^2}dx$

$= 4\frac{b}{a}\left\{\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

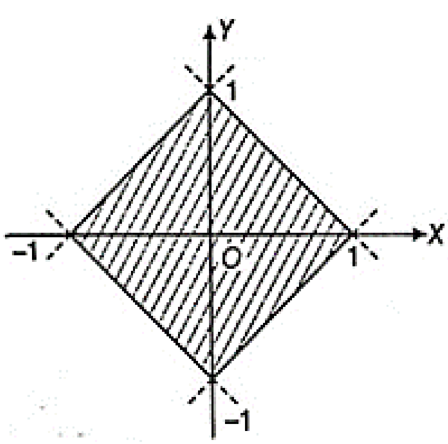
$= 4\frac{b}{a}\left[\left\{0 + \frac{a^2}{2}\sin^{-1}\frac{a}{a}\right\} - \{0 + 0\}\right] = \frac{4b}{a} \cdot$

$\frac{a^2}{2}\sin^{-1}(1)$

$= 2ab\left(\frac{\pi}{2}\right)$

$A\& = \pi ab$ sq units

S2. **(a)**



Given, $|x| + |y| = 1$

Case 1. x+y=1 then (x,y)=(1,0)and (x, y)=(0,1)

Case 2. -x – y = 1 then (x, y) = (0 , -1) and (x, y) = (-1, 0)

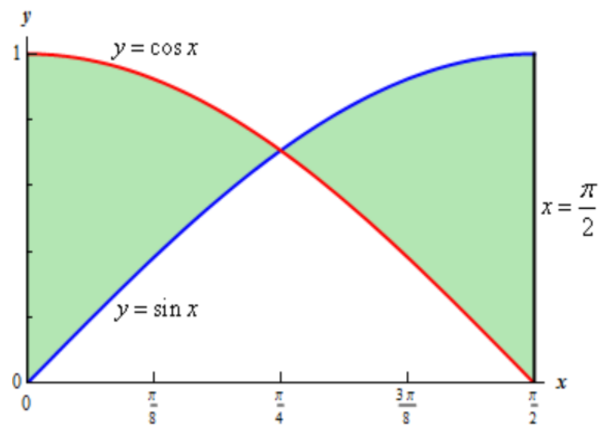
Case3. – x +y = 1 then (x, y) = (0 ,1) and (x, y) = (-1, 0)

Case 4. X – y = 1 then (x, y) = (0, -1) and (x, y) = (1, 0)

The above graph form a square of side $\sqrt{2}$ unit.

So, Area = $(\sqrt{2})^2$ = 2 sq. unit

S3. **(b)** First let's get a graph of region



So, we have another situation where we will need to do two integrals to get the area. The intersection point will be there sin x =cos x in the interval.

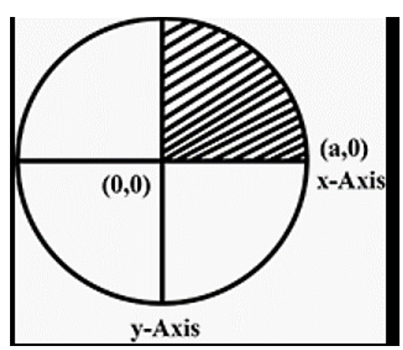
Then area = $\int_0^{\frac{\pi}{4}} \cos x - \sin x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x - \cos x \, dx$

Area = $\sin x + \cos x|_0^{\frac{\pi}{4}} + (-\cos x - sinx)|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

Area = $\sqrt{2} - 1 + \sqrt{2} - 1$

= $2\sqrt{2}$ – 2 Sq. unit

S4. **(a)**



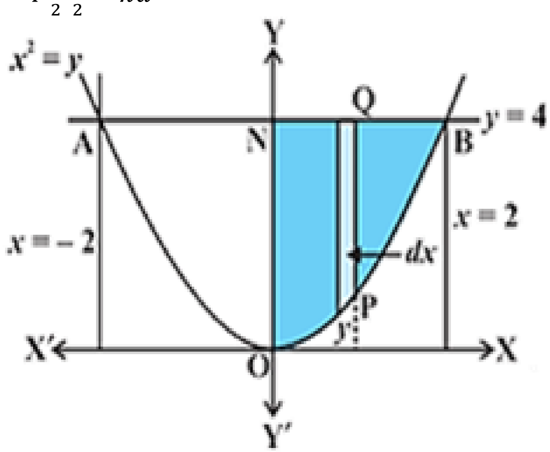
considering horizontal strips, the whole area of the region enclosed by circle

$= 4\int_0^a xdy = 4\int_0^a \sqrt{a^2 - y^2}dy$

$= 4\left[\frac{y}{2}\sqrt{a^2 - y^2} + \frac{a^2}{2}\sin^{-1}\frac{y}{a}\right]_0^a$

$= 4\left[\left(\frac{a}{2} \times 0 + \frac{a^2}{2}\sin^{-1} 1\right) - 0\right]$

$= 4\frac{a^2}{2}\frac{\pi}{2} = \pi a^2$



S5. (a)

the area of the region AOBA

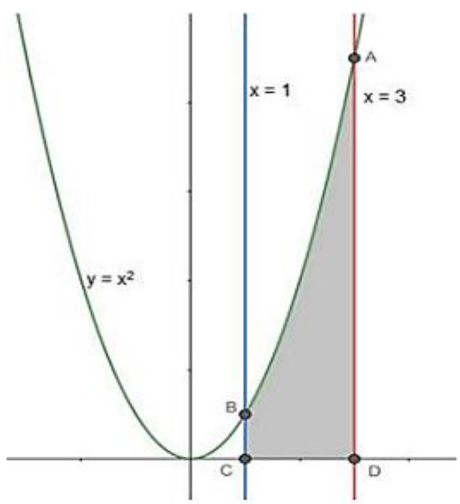
$= \int_{-2}^{2} ydx$

[y = (y-coordinate of Q) - (y-coordinate of P) = 4-x^2]

$= 2\int_0^2 (4 - x^2)dx$

$= 2\left[4x - \frac{x^3}{3}\right]_0^2 = 2\left[4 \times 2 - \frac{8}{3}\right] = \frac{32}{3}$ sq. unit

S6. (b)



Area of the required region = Area of ABCD.

Area of $ABCD = \int_1^3 x^2dx$

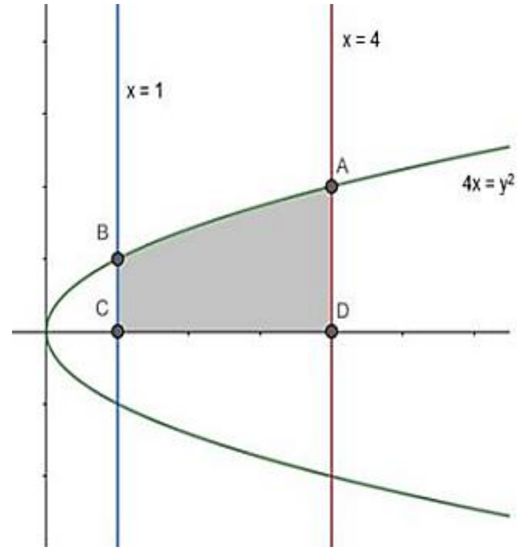
$= \left[\frac{x^3}{3}\right]_1^3 = \left(\frac{3^3}{3} - \frac{1^3}{3}\right)$

[Using the formula $\int x^n dx = \frac{x^{n+1}}{n+1}$]

$= \left(\frac{27}{3} - \frac{1}{3}\right) = \frac{26}{3}$

The area of the required region $= \frac{26}{3}$ sq. units

S7. (d)



Area of $ABCD = \int_1^4 ydx = \int_1^4 \sqrt{4x}dx$

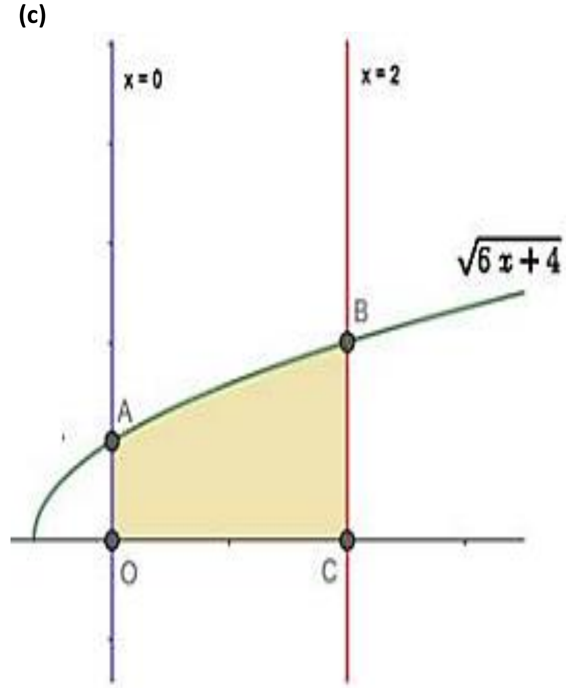
$= 2\int_1^4 \sqrt{x}dx = 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4 = 2\left[\frac{2x^{\frac{3}{2}}}{3}\right]_1^4$

[Using the formula $\int x^n dx = \frac{x^{n+1}}{n+1}$]

$= 2\left(\frac{2(4)^{\frac{3}{2}}}{3} - \frac{2(1)^{\frac{3}{2}}}{3}\right) = 4\left(\frac{8}{3} - \frac{1}{3}\right) = 4\left(\frac{7}{3}\right)$

$= \frac{28}{3}$

S8. (c)



Area of BCD $= \int_0^2 y\text{dx} = \int_0^2 \sqrt{6x + 4}\text{dx}$

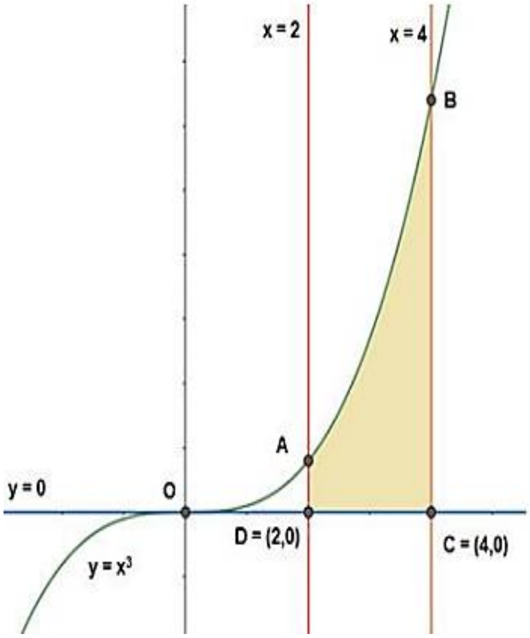
$= \int_0^2 \sqrt{6x + 4}dx = \left[\frac{(6x+4)^{\frac{3}{2}}}{\frac{3}{2}(6)}\right]_0^2 = \frac{1}{9}\left[(6x + 4)^{\frac{3}{2}}\right]_0^2$

$= \frac{1}{9}\left(((6 \times 2) + 4)^{\frac{3}{2}} - ((6 \times 0) + 4)^{\frac{3}{2}}\right)$

$= \frac{1}{9}(64 - 8) = \frac{1}{9}(56)$

$= \frac{56}{9}$

S9. (d)



Area of the required region $=$ Area of $ABCD$.

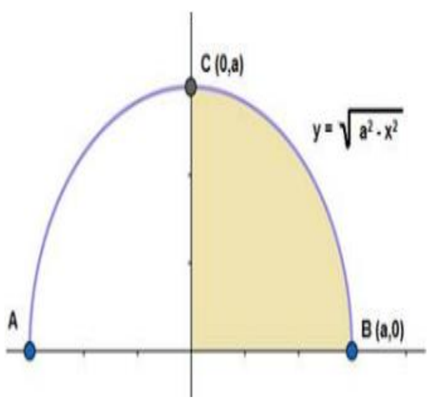
Area of ABCD $= \int_2^4 y\mathrm{dx} = \int_2^4 \mathrm{x}^3\mathrm{dx}$

$= \int_2^4 x^3 dx = \left[\frac{x^4}{4}\right]_2^4 = \frac{1}{4}[(x)^4]_2^4$

$= \frac{1}{4}(4^4 - 2^4) = \frac{1}{4}(256 - 16) = \frac{1}{4}(240)$

$= 60$ sq. units

S10. (a)



Area of OBC $= \int_0^a \mathrm{ydx} = \int_0^a \sqrt{a^2 - x^2}\mathrm{dx}$

$= \int_0^a \sqrt{a^2 - x^2}\mathrm{dx} = \left[\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$

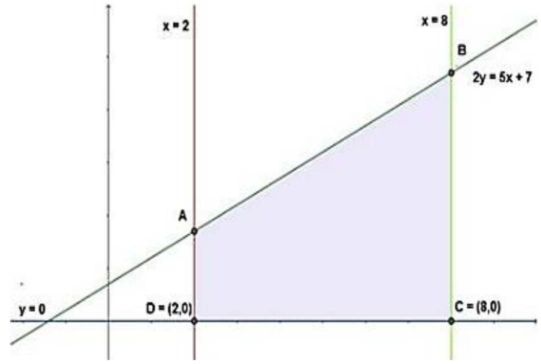
[Using the formula, $\int \sqrt{a^2 - x^2}\mathrm{dx} = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$]

$= \left[\frac{a\sqrt{a^2-a^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{a}{a}\right)\right] - \left[\frac{0\sqrt{a^2-0^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{0}{a}\right)\right]$

& $= \frac{a^2}{2}\left(\frac{\pi}{2}\right) - (0 + 0) = \frac{\pi a^2}{4}$

The Area of the required region $= \frac{\pi a^2}{4}$ sq. units

S11. (b)



Area of $ABCD = \int_2^8 ydx = \int_2^8 \frac{5x+7}{2}dx$

$= \frac{1}{2}\int_2^8 (5x + 7)dx = \frac{1}{2}\left[5\left(\frac{x^2}{2}\right) + 7x\right]_2^8$

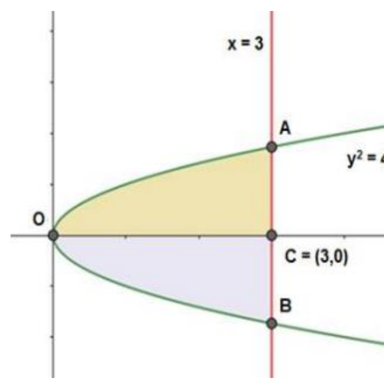
$= \frac{1}{2}\left\{\left[5\left(\frac{8^2}{2}\right) + 7(8)\right] - \left[5\left(\frac{2^2}{2}\right) + 7(2)\right]\right\}$

[Using the formula $\int x^n dx = \frac{x^{n+1}}{n+1}$ and $\int cdx = cx$]

$= \frac{1}{2}\left\{\left[5\left(\frac{64}{2}\right) + 56\right] - [10 + 14]\right\} = \frac{1}{2}[(5 \times 32) + 56 - 24] = \frac{1}{2}(160 + 32)$

& $= \frac{1}{2}(192) = 96$

S12. (c)

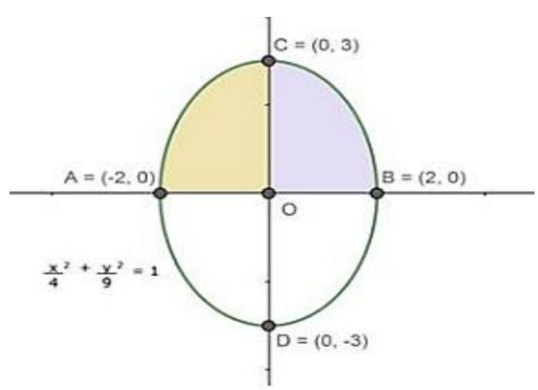


Area of OAB $= 2\int_0^3 ydx = 2\int_0^3 \sqrt{4x}dx$

$= 4\int_0^3 \sqrt{x}dx = 4\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^3 = 4\left[\frac{2x^{\frac{3}{2}}}{3}\right]_0^3$

$= 4\left(\frac{2(3)^{\frac{3}{2}}}{3} - \frac{2(0)^{\frac{3}{2}}}{3}\right) = \frac{8}{3}(3\sqrt{3}) = 8\sqrt{3}$

S13. (d)



Area of ABC $= 2\int_0^2 y dx = 2\int_0^2 \frac{3}{2}\sqrt{4-x^2}dx$

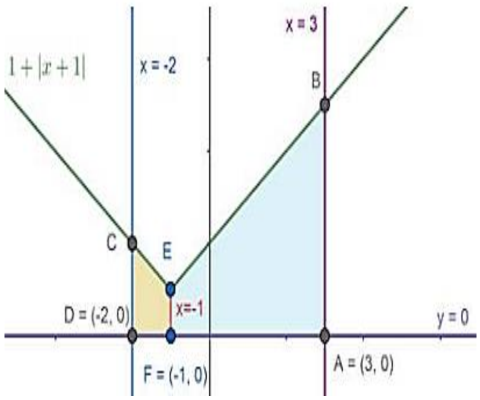
$= 3\int_0^2 \sqrt{(2)^2-x^2}dx = 3\left[\frac{x\sqrt{4-x^2}}{2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$

[Using the formula, $\int \sqrt{a^2-x^2}dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$]

$= 3\left[\frac{2\sqrt{4-2^2}}{2}+\frac{4}{2}\sin^{-1}\left(\frac{2}{2}\right)\right] - 3\left[\frac{0\sqrt{a^2-0^2}}{2}+\frac{4}{2}\sin^{-1}\left(\frac{0}{2}\right)\right]$

& $= 3\times 2\left(\frac{\pi}{2}\right) - 3(0+0) = 3\pi$

S14. (b)



Area of ABCD $= \int_{-2}^{-1} ydx + \int_{-1}^{3} ydx =$

$\int_{-2}^{-1} (-x)dx + \int_{-1}^{3} (x+2)dx$

& $= -\int_{-2}^{-1} (x)dx + \int_{-1}^{3} (x+2)dx =$

$= -\left[\frac{x^2}{2}\right]_{-2}^{-1} + \left[\left(\frac{x^2}{2}\right)+2x\right]_{-1}^{3}$

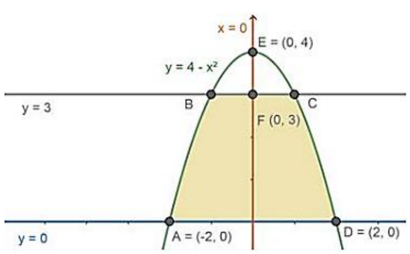
[Using the formula $\int x^n dx = \frac{x^{n+1}}{n+1}$ and $\int c dx = cx$]

$= -\left[\frac{(-1)^2}{2} - \frac{(-2)^2}{2}\right] + \left\{\left[\left(\frac{3^2}{2}\right)+2(3)\right] - \left[\left(\frac{(-1)^2}{2}\right)+2(-1)\right]\right\}$

$= -\left[\frac{1-4}{2}\right] + \left\{\left[\frac{9+12}{2}\right] - \left[\frac{1-4}{2}\right]\right\} = \frac{3}{2} + \left(\frac{21}{2}+\frac{3}{2}\right) = \frac{27}{2}$

The Area of the required region = $\frac{27}{2}$ sq. units.

S15. (b)



Area of $ABCD = \int_0^3 xdy = \int_0^3 \sqrt{4-y}dy$

$= \left[\frac{-(4-y)^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^3 = -\frac{2}{3}\left[(4-y)^{\frac{3}{2}}\right]_0^3$

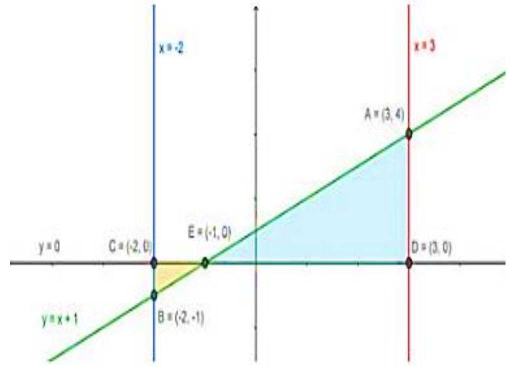
[Using the formula $\int x^n dx = \frac{x^{n+1}}{n+1}$]

$= -\frac{2}{3}\left[(4-3)^{\frac{3}{2}} - (4-0)^{\frac{3}{2}}\right] = -\frac{2}{3}\left[1-(2^2)^{\frac{3}{2}}\right] = -\frac{2}{3}(1-8)$

& $= -\frac{2}{3}(-7) = \frac{14}{3}$

The Area of the required region $= \frac{14}{3}$ sq. units

S16. (a)



Area of ABCD $= \int_{-2}^{-1} -ydx + \int_{-1}^{3} ydx =$

$\int_{-1}^{3} (x+1)dx - \int_{-2}^{-1} (x+1)dx$

$= \int_{-1}^{3} (x+1)dx - \int_{-2}^{-1} (x+1)dx$

$= \left[\left(\frac{x^2}{2}\right)+x\right]_{-1}^{3} - \left[\left(\frac{x^2}{2}\right)+x\right]_{-2}^{-1}$

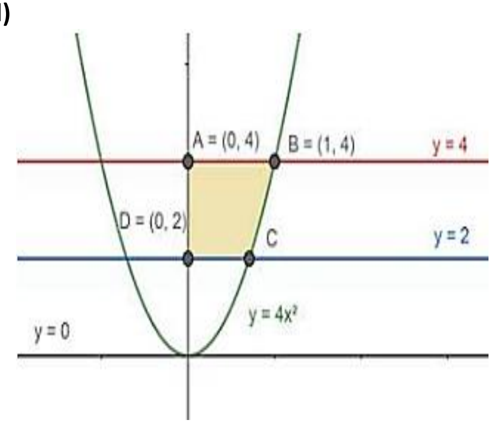
$= \left\{\left[\left(\frac{3^2}{2}\right)+(3)\right] - \left[\left(\frac{(-1)^2}{2}\right)+(-1)\right]\right\} - \left\{\left[\left(\frac{(-1)^2}{2}\right)+(-1)\right]\right.$

$\left. - \left[\left(\frac{(-2)^2}{2}\right)+(-2)\right]\right\}$

$\left\{\left[\frac{9+6}{2}\right] - \left[\frac{1-2}{2}\right]\right\} - \left\{\left[\frac{1-2}{2}\right] - \left[\frac{4-4}{2}\right]\right\} = \left(\frac{15+1}{2}\right) - \left(-\frac{1}{2}\right)$

$= \frac{17}{2} = 8.5$

S17. (d)



Area of the required region $=$ Area of $ABCD$.

$ABCD = \int_2^4 xdy = \frac{1}{2}\int_2^4 \sqrt{y}dy$

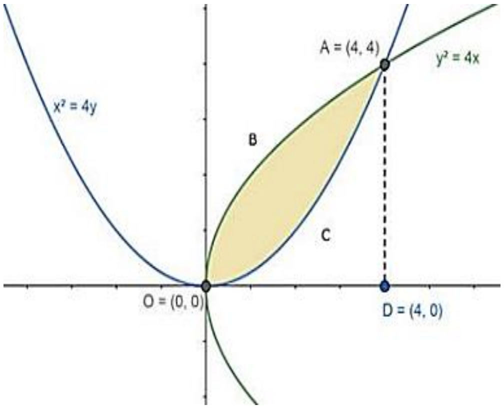
$= \frac{1}{2}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4 = \frac{1}{2}\times\frac{2}{3}\left[y^{\frac{3}{2}}\right]_2^4$

[Using the formula $\int x^n dx = \frac{x^{n+1}}{n+1}$]

$= \frac{1}{3}\left(4^{\frac{3}{2}} - 2^{\frac{3}{2}}\right) = \frac{1}{3}(8-2\sqrt{2})$

The Area of the required region $= \frac{(8-2\sqrt{2})}{3}$ sq. units

S18. (c)



the first parabola, $y^2 = 4x$ ... (1)
the second parabola, $x^2 = 4y \cdots (2)$
Now to find the point of intersection of (1) and (2), substitute $y = \frac{x^2}{4}$ in (1)

$\left(\frac{x^2}{4}\right)^2 = 4x$

$\Rightarrow x^4 = 64x$

$\Rightarrow x(x^3 - 64) = 0$

$\Rightarrow x = 0$ (or) $x = 4$

So the two points, A and B where (1) and (2) meet are $A = (4,4)$ and $O = (0,0)$

Area of OBACO $= \int_0^4 2\sqrt{x}dx - \frac{1}{4}\int_0^4 x^2$

$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$

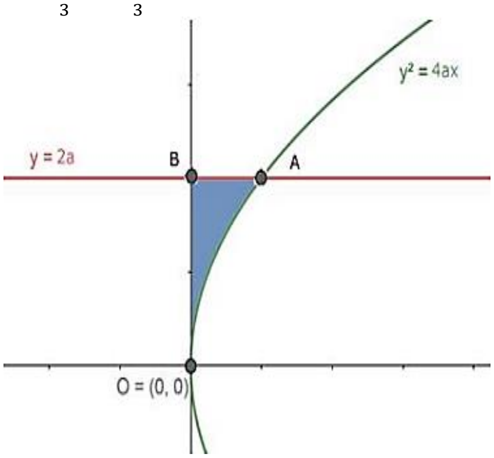
[Using the formula, $\int x^n dx = \frac{x^{n+1}}{n+1}$]

$= \frac{4}{3}\left[4^{\frac{3}{2}} - 0^{\frac{3}{2}}\right] - \frac{1}{12}[4^3 - 0^3]$

$= \frac{4}{3}(8) - \frac{1}{12}(64)$

$= \frac{32-16}{3} = \frac{16}{3}$

S19. (a)



Area of the required region = Area of OBA.

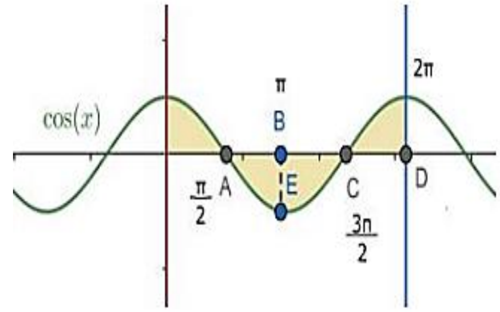
Area of OBA $= \int_0^{2a} x\mathrm{d}y = \int_0^{2a} \frac{y^2}{4a}\mathrm{d}y$

$= \frac{1}{4a}\left[\frac{y^3}{3}\right]_0^{2a}$

$= \frac{1}{12a}[(2a)^3 - 0^3]$

$= \frac{8a^3}{12a} = \frac{2a^2}{3}$

S20. (b)



For $0 < x < \frac{\pi}{2}, y = \cos x$

For $\frac{\pi}{2} < x < \frac{3\pi}{2}, y = -\cos x$

For $\frac{3\pi}{2} < x < 2\pi, y = \cos x$

Area required = Area under of OA + Area of ABC + Area under CD

Area required $= \int_0^{\frac{\pi}{2}} \cos x dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$ (-cosx) dx+ $\int_{\frac{3\pi}{2}}^{2\pi}$ cos x dx

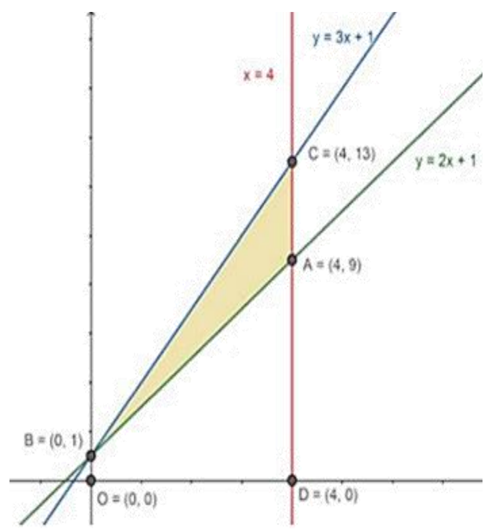
$= \int_0^{\frac{\pi}{2}} \cos x dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x dx$

$= (\sin x)_0^{\frac{\pi}{2}} - (\sin x)_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + (\sin x)_{\frac{3\pi}{2}}^{2\pi}$

$= \left[\sin\frac{\pi}{2} - \sin 0\right] - \left[\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right] + \left[\sin 2\pi - \sin\frac{3\pi}{2}\right]$

$= [1 - 0] - [-1 - 1] + [0 - (-1)] = 1 + 2 + 1 = 4$

S21. (c)



Area of ABC$= \int_0^4 (3x+1)dx - \int_0^4 (2x+1)dx$

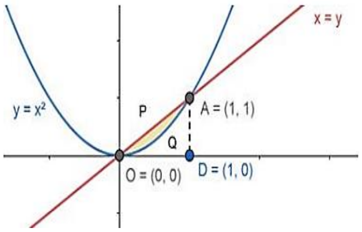
$= \left[\frac{3x^2}{2} + x\right]_0^4 - \left[\frac{2x^2}{2} + x\right]_0^4$

$= \left\{\left[\frac{3(4)^2}{2} - \frac{3(0)^2}{2}\right] + [4 - 0]\right\} - \{[4^2 - 0] + [4 - 0]\}$

$= \left\{\left[\frac{3(16)}{2}\right] + 4\right\} - \{[16] + [4]\}$

$= 24 + 4 - 20 = 8$

S22. **(c)**



Area of PAQO $= \int_0^1 y\text{dx} - \int_0^1 y\text{dx}$

$= \int_0^1 \text{xdx} - \int_0^1 \text{x}^2\text{dx}$

$\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$

$\left[\text{Using the formula } \int x^n dx = \frac{x^{n+1}}{n+1}\right]$

$= \frac{1}{2}(1^2 - 0^2) - \frac{1}{3}(1^3 - 0^3) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

S23. **(a)** the area of the region

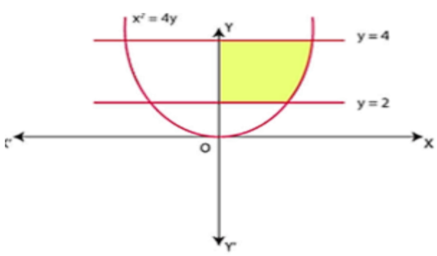
$= 2\int_0^{ae} ydx = 2\frac{b}{a}\int_0^{ae} \sqrt{a^2 - x^2}dx$

$= \frac{2b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{ae}$

$= \frac{2b}{2a}\left[ae\sqrt{a^2 - a^2e^2} + a^2\sin^{-1} e\right]$

$= ab\left[e\sqrt{1 - e^2} + \sin^{-1} e\right]$

S24. **(b)**

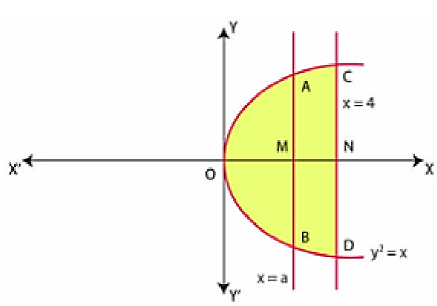


Equation of curve (parabola) is $x^2 = 4y$.
Required region is shaded, that is area bounded by curve $x^2 = 4y$, and Horizontal lines $y = 2, y = 4$ and y-axis in first quadrant.

$= \left|\int_2^4 xd\right| = \left|\int_2^4 2\sqrt{y}dy\right| = \left|2\int_2^4 y^{\frac{1}{2}}dy\right|$

$= 2\left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)\Big| = \frac{4}{3}\left(4^{\frac{3}{2}} - 2^{\frac{3}{2}}\right) = \left(\frac{32 - 8\sqrt{2}}{3}\right)$ sq. units

S25. **(a)**



Area OAMB = Area AMBDNC

$\Rightarrow 2\left|\int_0^a ydx\right| = 2\left|\int_a^4 ydx\right|$

$\Rightarrow 2\left|\int_0^a \sqrt{x}\, dx\right| = 2\left|\int_a^4 \sqrt{x}dx\right|$

$\Rightarrow \frac{4}{3}\left[x^{\frac{3}{2}}\right]_0^a = \frac{4}{3}\left[x^{\frac{3}{2}}\right]_a^4$

$\Rightarrow a^{\frac{3}{2}} = 8 - a^{\frac{3}{2}}$

$\Rightarrow$ a = $4^{\frac{2}{3}}$

S26. **(c)** At, $x = 1, y = \log_e(1) = 0$

At, $x = e, y = \log_e(e) = 1$

Therefore, $A = \int_1^e \log_e x\, dx$

Using integration by parts,

$A = [x\log_e x - x]_1^e$

Now, apply the limits, we get

$A = [e - e - 0 + 1]$

$A = 1$

S27. **(c)** Given, $y^2 = x$

Hence, the required area, $A = \int_2^4 y^2 dy$

$A = [y^3/3]_2^4$

$A = (4^3/3) - (2^3/3)$

$A = (64/3) - (8/3)$

$A = 56/3$ sq. units.

S28. **(a)** Required Area $= \int_{-1}^1 (2y + 3)$ dy

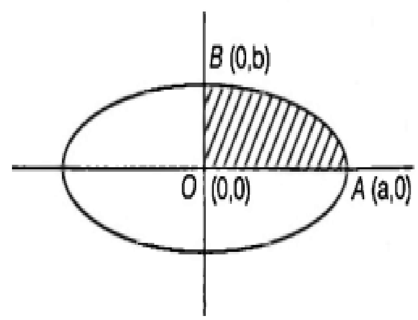
$A = [(2y^2/2) + 3y]_{-1}{}^1$

Now, apply the limits, we get

$A = 1 + 3 - 1 + 3$

$A = 6$sq. units.

S29. **(b)**



$A = 4($ area $OABO)$

$= 4\int_0^a ydx$, where $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore \quad \frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$

$\Rightarrow \quad y^2 = \frac{b^2}{a^2}(a^2 - x^2)$

$\Rightarrow \quad y = \pm\frac{b}{a}\sqrt{a^2 - x^2}$

In the first quadrant,

$y\& = \frac{b}{a}\sqrt{a^2 - x^2}$

$A\& = 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}dx = 4\frac{b}{a}\int_0^a \sqrt{a^2 - x^2}dx$

$= 4\frac{b}{a}\left\{\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

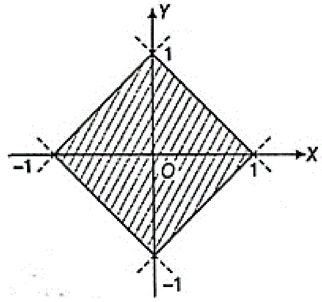
$= 4\frac{b}{a}\left[\left\{0 + \frac{a^2}{2}\sin^{-1}\frac{a}{a}\right\} - \{0 + 0\}\right] = \frac{4b}{a}\cdot$

$\frac{a^2}{2}\sin^{-1}(1)$

$= 2ab\left(\frac{\pi}{2}\right)$

$\Rightarrow \frac{A}{2} = \frac{\pi}{2}ab$ sq units

S30. **(a)**



Given, $|x| + |y| = 1$

Case 1. X + y = 1 then (x, y) = (1, 0) and (x, y) = (0,1)

Case 2. -x – y = 1 then (x, y) = (0 , -1) and (x, y) = (-1, 0)

Case3. – x +y = 1 then (x, y) = (0 ,1) and (x, y) = (-1, 0)

Case 4. X – y = 1 then (x, y) = (0, -1) and (x, y) = (1, 0)

The above graph form a square of side $\sqrt{2}$ unit.

So, Area = $(\sqrt{2})^2$ = 2 sq. unit

Therefore, Required area = 2×2 = 4 sq. unit

SUBJECTIVE QUESTIONS

S1. Given, $y = -x^2 + x + 6$

Solving the above equation we get, $(-x+3),(x+2)$

Area= $\int_{-2}^{3} 6 + x - x^2 dx$

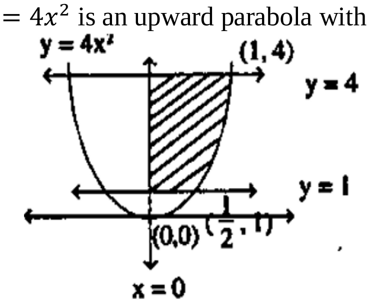
$= \left[6x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{3}$

$= \left[18 + \frac{9}{2} - \frac{27}{3}\right] - \left[-12 + \frac{4}{2} + \frac{8}{3}\right]$

$= \left[30 + \frac{5}{2} - \frac{35}{3}\right]$

$= \frac{125}{6}$ sq. units

S2. $y = 4x^2$ is an upward parabola with vertex (0,0)



x = 0 is line with points (0 ,1) ,(0 ,0) etc…

y = 1 is line with points (0 ,1) ,(1,1) etc …

y = 4 is line with points (0, 4) ,(1,4) etc…

parabola meets at points (0 ,0) with line x = 0

$\left(\frac{1}{2}, 1\right)$ with line y = 1 , (1,4) with line y = 4

Here we need to do along 'y' axis as we don't know equation of the line with respect to 'x'

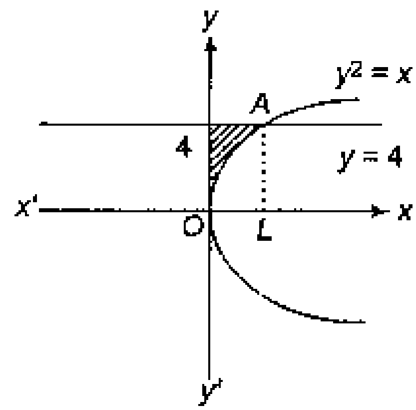
Required area (shaded one) = Area under parabola from y = 1 to y=4

$= \int_1^4 |x| dy = \int_1^4 \sqrt{\frac{y}{4}} dy = \frac{1}{2}\int_1^4 \sqrt{y}\, dy$

$= \frac{1}{2}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4 = \frac{1}{2} \times \frac{2}{3}\left[y^{\frac{3}{2}}\right]_1^4 = \frac{1}{3}\left[4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right]$

$= \frac{1}{3}[8 - 1] = \frac{7}{3} sq. units$

S3.



Line y = 4 meets the parabola $y^2 = x$ at A

$\therefore$ 16 = x and A is (16, 4)

$\therefore$ Required area

$= \int_0^4 x\, dy$

$= \int_0^4 y^2\, dy$

$= [y^3/3]_0^4$

$= \frac{64}{3}$ sq units

S4. The area lying between the curve, $y^2 = 4x$ and y = 2x is represented by the shaded area OBAO as shaded in the diagram.

The points of intersection of these curves are O(0, 0) and A(1, 2).

We draw AC perpendicular to x-axis such that the coordinates of C are (1, 0).

∴ Area OBAO = Area (ΔOCA)-Area (OCABO)

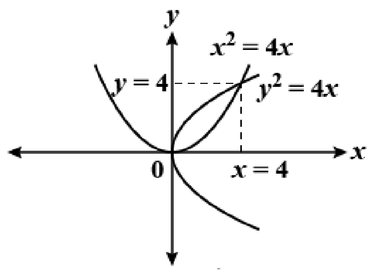
$= \int_0^1 2x dx - \int_0^1 2\sqrt{x} dx$

$= 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$

$= \left|1 - \frac{4}{3}\right|$

$= \left|-\frac{1}{3}\right| = \frac{1}{3}$ sq. units

S5.



Area of region bounded by $y^2 = 4ax$ and $y^2 = 4by$ is $\frac{16ab}{3}$ sq. units.

Now $S_1 = S_3 = \int_0^4 \frac{x^2}{4}dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4 = \frac{1}{12} \times 64 = \frac{16}{3}$ sq. unit

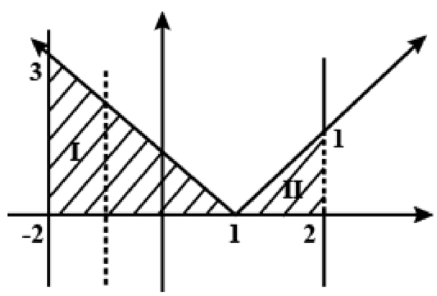
$\therefore S_2 + S_3 = \int_0^4 \sqrt{4x}dx = 2\left[\frac{x^{3/2}}{3/2}\right]_0^4 = \frac{4}{3} \times 8 = \frac{32}{3}$ sq. unit

$\Rightarrow S_2 = \frac{16}{3}$ sq. unit

$\therefore S_1 : S_2 : S_3 = \frac{16}{3} : \frac{16}{3} : \frac{16}{3} = 1 : 1 : 1$

NUMERICAL TYPE QUESTIONS

S1.

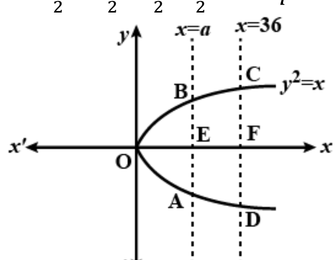


$y = |x - 1|, y = 0$ and $|x| = 2^{nd}$ triangle

Area of 1st triangle + Area of 2nd triangle

$= \frac{3\times3}{2} + \frac{1}{2} = \frac{9}{2} + \frac{1}{2} = 5\ sq.units$

S2.



Since the line x = a divides the region CBOAD in the ration 1: 7

Therefore , ar(ABCD)= 7 ar (OAB)

$\Rightarrow 2 \times \int_a^{36} y dx = 2 \times \left(7\int_0^a y\, dx\right)$

$\Rightarrow 2 \times \int_a^{36} \sqrt{x} dx = 2 \times \left(7\int_0^a \sqrt{x}\, dx\right)$ [since $y^2 = x$]

$\Rightarrow \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_a^{36} = 7\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$

$\Rightarrow (36)^{\frac{3}{2}} - a^{\frac{3}{2}} = 7 \times a^{\frac{3}{2}}$

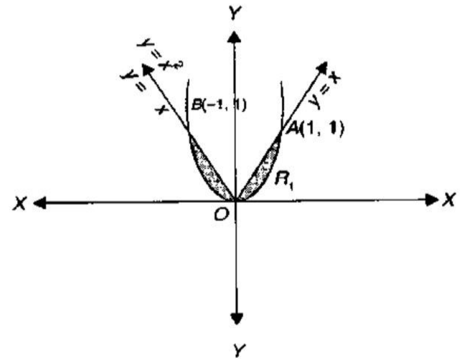
$\Rightarrow 216 = 8 \times a^{\frac{3}{2}}$

$\Rightarrow 27 = a^{\frac{3}{2}}$

$\Rightarrow a = 9$

S3. The required area is bounded between two curves $y = x^2$ and $y = |x|$. Both of these curves are symmetric about y-axis and shaded region in the fig. shows the region in the fig. shows the region whose area is required.

Therefore, the required area



Now, to find the point of intersection of the curves $y = |x|$ and $y = x^2$, we solve them simultaneously.

Clearly, the region R_1 is in the first quadrant, where x > 0,

$\therefore\ |x| = x$ or $y = x$ ...(i)

and $y = x^2$ ...(ii)

Solving these two equations, we get

$x = x^2$

or either $x = 0$ or $x = 1$

The limits are, when x = 0, y = 0 and when x =1, y = 1. So, the point of intersection of the curves are O(0, 0) and A (1, 1).

Now ,required area = 2 × Area of line region R_1

$= 2\int_0^1[(\text{y of the line } y = x) -$

$(\text{y of the parabola } y = x^2)]\, dx$

S4. Here the first curve can be written in the following form

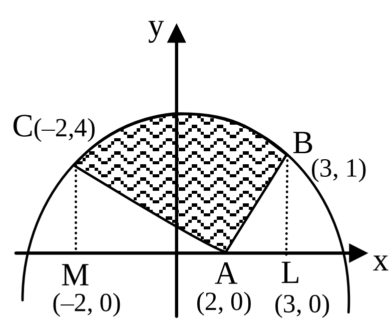
$x^2 = -\frac{5}{3}\left(y - \frac{32}{5}\right)$

which is a parabola whose vertex lies on the y-axis.

Again second curve is given by

$y = \begin{cases} x-2, x \geq 2 \\ -(x-2), x < 2 \end{cases}$

which consists of two perpendicular lines AB and AC as shown in the fig.



These lines meet the parabola at B(3,1) and C(–2,4)
Hence the reqd. area A is given by

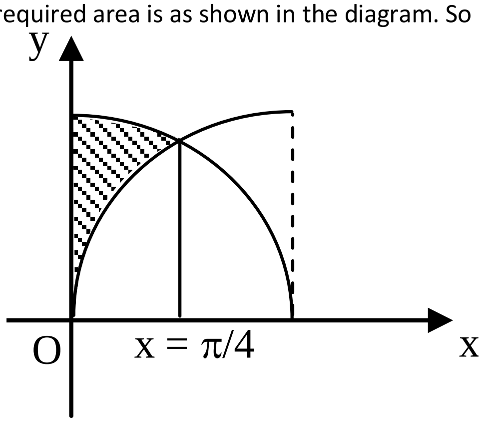
$$A = \int_{-2}^{3} y\,dx - \Delta\,ABL - \Delta ACM$$

$$\int_{-2}^{3}\frac{1}{5}(32-3x^2)\,dx - \frac{1}{2}\,.\,1.1 - \frac{1}{2}\,(4\,.\,4)$$

$$= \frac{1}{5}\left[32x - x^3\right]_{-2}^{3} - \frac{17}{2}$$

$$= \frac{1}{5}\,[69+56] - \frac{17}{2} = \frac{33}{2}$$

S5. In first quadrant sin x and cos x meet at x = $\pi/4$. The required area is as shown in the diagram. So



$$\text{Required area} = \int_{0}^{\pi/4}(\cos x - \sin x)\,dx$$

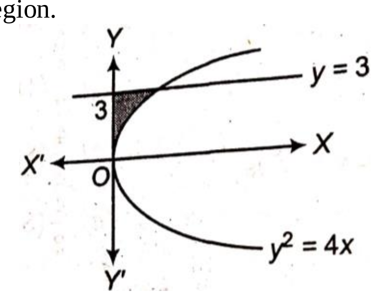
$= [\sin x + \cos x]_0^{\pi/4}$

$= (1/\sqrt{2} + 1/\sqrt{2}) - (0+1)$

$= \sqrt{2} - 1$

TRUE AND FALSE

S1. **(True)** The area bounded by the curve $y^2 = 4x$, y – axis and y = 3 is represented in the figure by shaded region.



Required area = $\int_0^3 |x|dy$

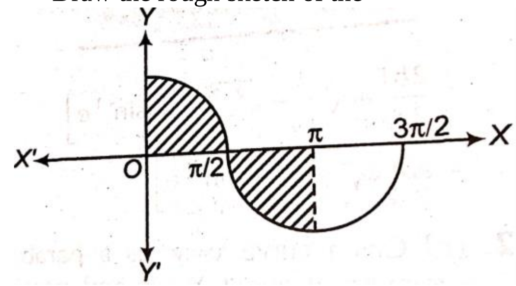
$= \int_0^3 \frac{y^2}{4} = \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$

$= \frac{1}{12}(3^3 - 0)$

$= \frac{9}{4}$ sq. units

S2. **(False)** We have , y = cos x and lines x = 0 , x = π

Draw the rough sketch of the



Required area = $\int_0^{\frac{\pi}{2}} \cos x\,dx + \left|\int_{\frac{\pi}{2}}^{\pi} \cos x dx\right|$

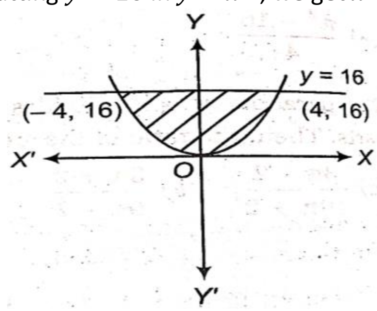
$= [\sin x]_0^{\frac{\pi}{2}} + \left|[\sin x]_{\frac{\pi}{2}}^{\pi}\right|$

$= \left[\sin\frac{\pi}{2} - \sin 0\right] + \left|\left[\sin\pi - \sin\frac{\pi}{2}\right]\right|$

$= [\,1-0] + |0-1| = 1+1 = 2$ sq. units.

S3. **(False)** Area of region = $2\int_0^{16}\sqrt{y}dy$

Putting $y = 16$ in $y = x^2$, we get $x = \pm 4$

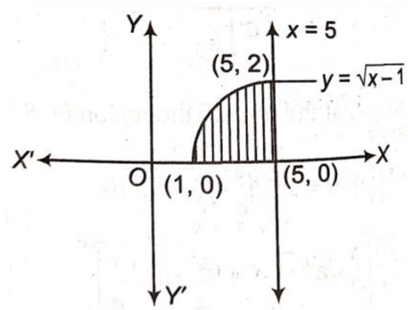


$= 2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{16} = \frac{2}{3}\times 2\left[16^{\frac{3}{2}} - 0\right]$

$= \frac{4\times 64}{3} = \frac{256}{3}$ sq. units.

S4. **(True)** We have , Equation of the curve $y = \sqrt{x-1}$ On squaring both sides , we get

Now , sketch the graph of given curve



Therefore , area of the shaded region = $\int_1^5 \sqrt{x-1}dx$

$= \left[\frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^5 = \frac{2}{3}\left[(x-1)^{\frac{3}{2}}\right]_1^5$

$= \frac{2}{3}\left[(4)^{\frac{3}{2}} - 0\right]$

$= \frac{2}{3}\times 8 = \frac{16}{3}$ sq. units.

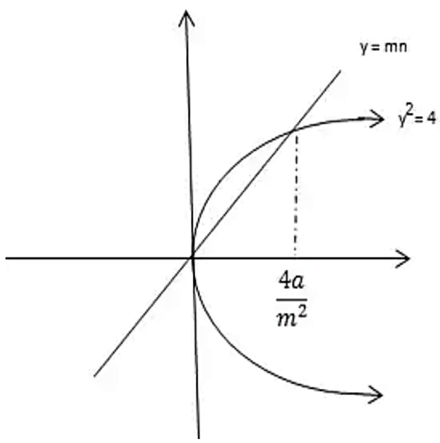
S5. **(True)** If f(x) $\geq$ g(x) in [a , c] and f(x)$\leq$ g(x) in [a, b] where a < b< c ,then the area of the regions bounded by curves can be written as

Total area = Area of the region ACBDA + Area of the region BPRQB

A = $\int_a^c [f(x) - g(x)]\, dx + \int_c^b [g(x) - f(x)] dx$

ASSERTION AND REASONING

S1. **(a)**



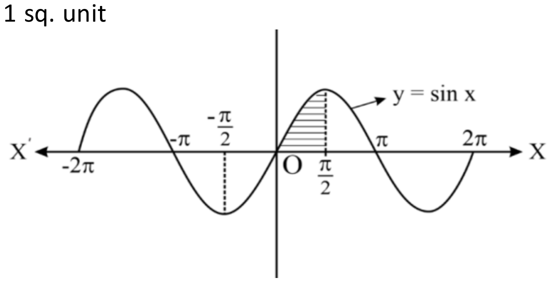
$\Rightarrow \int_0^{\frac{4a}{m^2}} (\sqrt{4x} - x) dx$

$= \left[\frac{2}{3} \times 2\sqrt{a} \times x^{\frac{3}{2}} - \frac{x^2}{2}\right]_0^{\frac{4a}{m^2}}$

$= \frac{8a^2}{3m^3}$

So, A , R are true and R explains A.

S2. **(a)** **the area of the curve** y=sin x **between** 0 and $\frac{\pi}{2}$ is 1 sq. unit



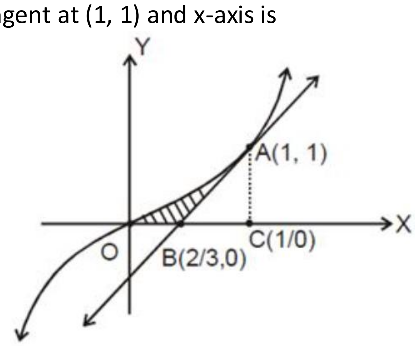
Area = $\int_0^{\frac{\pi}{2}} y\, dx = \int_0^{\frac{\pi}{2}} \sin x dx$

$\Rightarrow [-\cos x]_0^{\frac{\pi}{2}} = \left[-\cos\frac{\pi}{2} + \cos 0\right]$

$\Rightarrow [0 + 1] = 1$

So, A , R are true and R explains A.

S3. **(d)** The area of the region bounded by the curve y = x^3, its tangent at (1, 1) and x-axis is

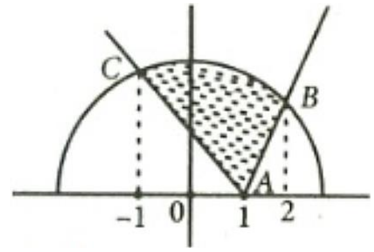


Equation of tangent at A is y = 3x -2

Area of shaded region = $\int_0^1 x^3 dx - \int_{\frac{2}{3}}^1 (3x - 2) dx = \frac{1}{4} - \frac{15}{18} + \frac{2}{3} = \frac{1}{12}$

Assertion is correct but reason is false

S4. **(d)**



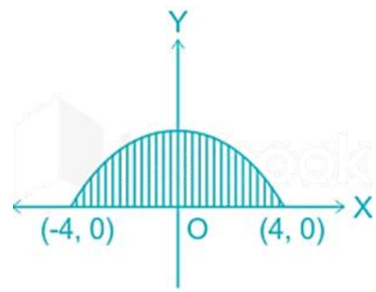
Requires area ABCA = $\int_{-1}^2 \sqrt{5 - x^2} dx - \int_{-1}^2 |x - 1| dx$

= $\int_{-1}^2 \sqrt{(\sqrt{5})^2 - x^2} dx - \int_{-1}^1 (1 - x) dx - \int_1^2 (x - 1) dx$

= $\left(\frac{5\pi}{4} - \frac{1}{2}\right)$ sq. units

Assertion is correct but reason is false

S5.(a)



Given : $y = \sqrt{16 - x^2}$ and x – axis

At x – axis , y will be zero

$y = \sqrt{16 - x^2}$

$\Rightarrow 0 = \sqrt{16 - x^2}$

$\Rightarrow 16 - x^2 = 0$

$\Rightarrow x = \pm 4$

So, the intersection points are (4 ,0) and (-4 ,0)Since the curve is $y = \sqrt{16 - x^2}$

So, $y \geq 0$ [always]

So, we will take the circular part which is above the x-axis

Area of the curve , A = $\int_{-4}^4 \sqrt{16 - x^2}$

Using $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$\Rightarrow \int_{-4}^4 \sqrt{16 - x^2} = \left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_{-4}^4$

$\Rightarrow \left[\frac{4}{2}\sqrt{16 - 16} + 8\sin^{-1}\frac{4}{4} + \frac{4}{2}\sqrt{16 - 16} - 8\sin^{-1}\frac{-4}{4}\right]$

$\Rightarrow [8\sin^{-1} 1 - 8\sin^{-1}(-1)]$

$\Rightarrow 16\sin^{-1}(1)$

$\Rightarrow 16 \times \frac{\pi}{2} = 8\pi$ sq. units

So, A , R are true and R explains A.

HOMEWORK

MCQ

S1. **(c)** Required area = $\int_0^{\pi/2} \sin^2 x\,dx$

$= \int_0^{\pi/2} \frac{1-\cos 2x}{2} dx$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2} = \frac{\pi}{4}$

S2. **(a)** Putting y = 0, we get,

$x^2 - 3x - 4 = 0$

$\Rightarrow (x-4)(x+1) = 0$

$\Rightarrow x = -1$ or $x = 4$

$\therefore$ required area = $\int_{-1}^{4} (4 + 3x - x^2)\,dx$

$= \left(4x + \frac{3x^2}{2} - \frac{x^3}{3}\right)_{-1}^{4} = \frac{125}{6}$

S3. **(b)** Area = $\int_0^3 x\,dy = \int_0^3 \frac{y^2}{4} dy$

$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3 = \frac{1}{12}(27 - 0)$

= 9/4 units

S4. **(c)** Given curve $\left(\frac{x}{a}\right)^{1/3} = \cos t$, $\left(\frac{y}{a}\right)^{1/3} = \sin t$

Squaring and adding $x^{2/3} + y^{2/3} = a^{2/3}$

Clearly it is symmetric with respect to both the axis, so whole area is

$= 4\int_0^a y\,dx$

$= 4\int_{\pi/2}^{0} a\sin^3 t\; 3a\cos^2 t\,(-\sin t)\,dt$

By given equation at x = 0; t = $\frac{\pi}{2}$ at x=a; t = 0

$= 12a^2 \int_0^{\pi/2} \sin^4 t\cos^2 t\,dt$

$= 12a^2 . \frac{3.1.1}{6.4.2} . \frac{\pi}{2} = \frac{3\pi a^2}{8}$

S5. **(a)** f(x) = y = sin x

when $x \in [0, \pi]$, $\sin x \geq 0$

and when $x \in [\pi, 2\pi]$, $\sin x \leq 0$

$\therefore$ required area = $\int_0^{\pi} y\,dx + \int_{\pi}^{2\pi} (-y)\,dx$

$= \int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} (-\sin x)\,dx$

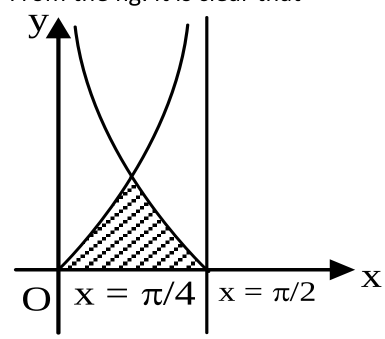
$= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi}$

$= (-\cos\pi + \cos 0) + (\cos 2\pi - \cos\pi)$

= (1 + 1) + (1 + 1)

= 4 units

S6. **(a)** From the fig. it is clear that



$= \int_0^{\pi/4} \tan x\,dx - \int_{\pi/4}^{\pi/2} \cot x\,dx$

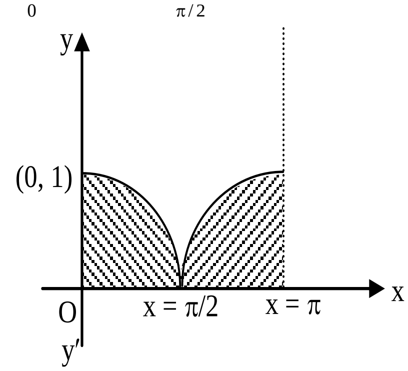
$= [\log \sec x]_0^{\pi/4} + [\log \sin x]_{\pi/4}^{\pi/2}$

$= \log\sqrt{2} - \log\frac{1}{\sqrt{2}}$

= log 2

S7. **(c)** Required area = $\int_0^{\pi} \cos^2 x\,dx$

$= \int_0^{\pi/2} \cos^2 x\,dx + \int_{\pi/2}^{\pi} \cos^2 x\,dx$



$= \frac{1}{2} \times \frac{\pi}{2} + \frac{1}{2}\int_{\pi/2}^{\pi} (1 + \cos 2x)\,dx$

$= \frac{\pi}{4} + \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_{\pi/2}^{\pi}$

$= \frac{\pi}{4} + \frac{1}{2}\left[\left(\pi - \frac{\pi}{2}\right)\right]$

$= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

S8. **(c)** Solving the equation of curves

$e^x \log x = \frac{\log x}{e^x}$

$\Rightarrow \log x\left(e^x - \frac{1}{e^x}\right) = 0$

$\Rightarrow x = 1, 1/e$

$\therefore$ required area $= \int_{\frac{1}{e}}^{1}\left(\frac{\log x}{e^x} - e^x \log x\right) dx$

$= \left[\frac{1}{e}\frac{(\log x)^2}{2} - e\left((\log x).\frac{x^2}{2} - \frac{x^2}{4}\right)\right]_{1/e}^{1}$

$= \frac{1}{2e}\,[0-(-1)^2] - e\left[0 - \frac{1}{4} - \left(-\frac{1}{2e^2} - \frac{1}{4e^2}\right)\right]$

$= -\frac{1}{2e} - \frac{1}{2e} + \frac{e}{4} - \frac{1}{4e} = \frac{e}{4} - \frac{5}{4e}$

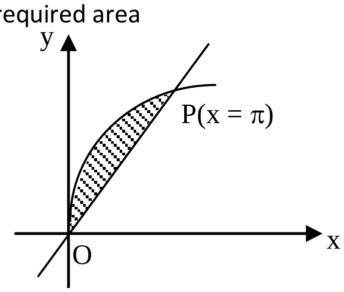
S9. **(a)** For the points of intersection of the given curves

x = x + sin x

$\Rightarrow$ sin x = 0

$\Rightarrow$ x = 0, π

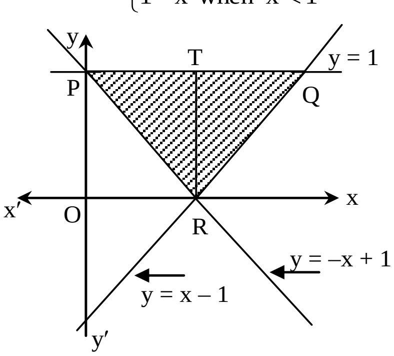
$\therefore$ required area



$= \int_0^{\pi} [(x + \sin x) - x]\,dx$

$= \int_0^{\pi} \sin x\,dx = -[\cos x]_0^{\pi} = 2$

S10. **(a)** $y = |x-1| = \begin{cases} x-1 & \text{when } x \geq 1 \\ 1-x & \text{when } x < 1 \end{cases}$



Point of intersection of y = x – 1, y = 1 is (2, 1)

Point of intersection of y = 1 – x, y = 1 is (0, 1)

Required area = Area of Δ PQR

$= \frac{1}{2}$ (PQ) . (RT)

$= \frac{1}{2}$. 2.1 = 1

SUBJECTIVE QUESTIONS

S1. $\int_1^k (8x^2 - x^5)\,dx = \frac{16}{3}$

$\Rightarrow \left[\frac{8x^3}{3} - \frac{x^6}{6}\right]_1^k = \frac{16}{3}$

$\Rightarrow \frac{8}{3}(k^3 - 1) - \left(\frac{k^6 - 1}{6}\right) = \frac{16}{3}$

$\Rightarrow 16k^3 - k^6 - 15 = 32$

$\Rightarrow k^6 - 16k^3 + 47 = 0$

$\Rightarrow k^3 = 8 \pm \sqrt{17}$

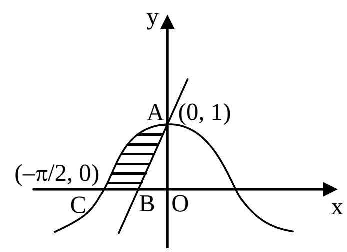
$\Rightarrow k = (8 \pm \sqrt{17})^{1/3}$

S2. Let the line y = x + 1, meets x-axis at the point A (0, 1). Also suppose that the curve y = cos x meets x-axis and y-axis respectively at the points C and A. From the adjoint figure it is obvious that

Required area = area of ABC

= area of OAC – area of OAB

$= \int_{-\pi/2}^{0} \cos x\,dx - \frac{1}{2} \times OB \times OA$

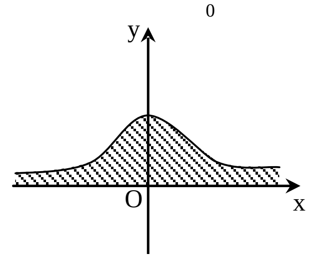


$= [\sin x]_{-\pi/2}^{0} - \frac{1}{2} \times 1 \times 1$

= 1 – (1/2) = (1/2).

S3. Given curve is symmetrical about y-axis as shown in the diagram.

Reqd. area $= 2\int_0^{\infty} \text{sech}\,x\,dx$



$$= 2\int_0^{\infty} \frac{2}{e^x + e^{-x}} dx = 4\int_0^{\infty} \frac{e^x}{e^{2x}+1} dx$$

$$= 4\left[\tan^{-1}(e^x)\right]_0^{\infty} = 4\left[\frac{\pi}{2} - \frac{\pi}{4}\right] = \pi$$

S4. The points of intersection of curves are x = 0 and x = 1.

$\therefore$ required area = $\int_0^1 (\sqrt{x} - x)\,dx$

$$= \left[\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

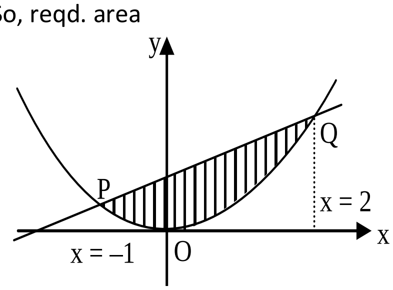
S5. Solving the equation of the given curves for x, we get

$x^2 = x + 2$

$\Rightarrow (x - 2)(x + 1) = 0$

$\Rightarrow x = -1, 2$

So, reqd. area



$$= \int_{-1}^{2}\left[\frac{x+2}{4} - \frac{x^2}{4}\right]dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}\left[(2 + 4 - 8/3) - (1/2 - 2 + 1/3)\right] = 9/8$$

NUMERICAL TYPE QUESTIONS

S1. **(1)**

$$\text{Area} = \int_{\pi/4}^{0} (\sin 2x + \cos 2x)dx$$

$$= \left[\frac{-\cos 2x}{2} + \frac{\sin 2x}{2}\right]_0^{\pi/4}$$

$$= \left(\frac{-\cos\frac{\pi}{2} + \sin\frac{\pi}{2}}{2}\right) - \left(\frac{-\cos 0 + \sin 0}{2}\right)$$

$$= -1\frac{1}{2} - \left(\frac{-1}{2}\right)$$

$$= 1$$

S2. **(1)**

$y = \sin 2x + \cos 2x$

$= \sqrt{2}\sin\left(2x + \frac{\pi}{4}\right)$

Area under the curve between

$x = 0, x = \frac{\pi}{4}$ is,

$$\int_0^{\pi/4} \sqrt{2}\sin\left(2x + \frac{\pi}{4}\right)dx$$

$$\sqrt{2}\left[-\frac{\cos\left(2x + \frac{\pi}{4}\right)}{2}\right]_0^{\frac{\pi}{4}}$$

$$\sqrt{2}\left[-\frac{\left(\frac{-1}{\sqrt{2}}\right)}{2} - \left(\frac{-1/\sqrt{2}}{2}\right)\right]$$

$$\sqrt{2} \times \left[\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right]$$

1 sq. unit.

S3. **(4)**

$y = 1 + \frac{8}{x^2}$

from x = 2 and x = 4

So, $\int_2^4 y\,dx$

$= \int_2^4 (1 + 8/x^2)dx$

$= |x - 8/x|_2^4$

$= [4 - \frac{8}{4}] - [2 - \frac{8}{2}]$

$= 4 - 2 - 2 + 4$

$= 4$ sq units.

S4. **(0.5)** $A = \frac{1}{2}\int_0^{\sqrt{\frac{\pi}{2}}} 2x \sin x^2\, dx$

$X^2 = t$

$2xdx = dt$

$$A = \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin t\,dt$$

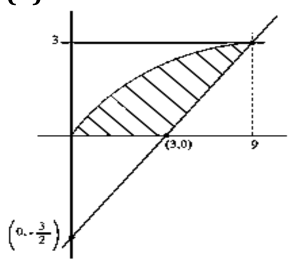
$$= \frac{1}{2} \times -\left[\cos t\right]_0^{\pi/2}$$

$$= \frac{-1}{2}[0 - 1]$$

$$= \frac{1}{2}$$

$= 0.5$

S5. **(9)**



Given, $y = \sqrt{x},\ 2y - x + 3 = 0$

from above we have,

$2\sqrt{x} - x + 3 = 0$

$2\sqrt{x} = x - 3$

$4x = x^2 - 6x + 9$

$x^2 - 10x + 9 = 0$

$x = 9,\ x = 1$

now,

$$\int_0^3 [(2y+3) - y^2]dy$$

$$= \left[y^2 + 3y - \frac{y^3}{3}\right]_0^3 = 9 + 9 - 9 = 9$$

TRUE AND FALSE

S1. **(False)** Let f(x) be a continuous function defined on [a, b] .Then ,the area bounded by the curve y = f(x) , the x -axis and the ordinates x = a and x = b is given by $\int_a^b f(x)dx$

S2. **(False)** If the curve y = f(x) lies below x – axis ,then the area bounded by the curve y = f(x) , the x -axis and the ordinates x = a and x = b is negative .So, the area is given by $|\int_a^b ydx|$

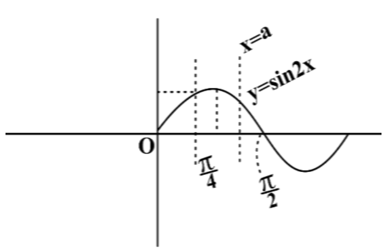
S3. **(True)** The area bounded by the curve x = f(y) , the y - axis and the abscissae y = c and y = d is given by $\int_c^d f(y)dy$

S4. **(True)** $y = a\sin x$

$$\int_0^\pi a\sin dx = -a\cos x\Big|_0^\pi$$

$$= -a(-1-1) = 2a$$

S5.(True)



$$\int_{\pi/6}^{a} \sin 2x = \frac{1}{2}$$

$$\left[\frac{-\cos 2x}{2}\right]_{\pi/6}^{a} = \frac{1}{2}$$

$$-\cos 2a - \cos\frac{2\pi}{6} = 1$$

$$-\cos 2a + \frac{1}{2} = 1$$

$$\cos 2a = \frac{-1}{2}$$

$$2a = \frac{2\pi}{3}$$

$$a = \frac{\pi}{3}$$

ASSERTION AND REASONING

S1. **(a)**

$x = 2y - y^2 \qquad y axis : -x = 0$

$2y - y^2 = 0$

$y(2-y) = 0$

$y = 0, y = 2$

$A = \int_0^2 (2y - y^2)dy$

$A = \left[y^2 - \frac{y^3}{3}\right]_0^2$

$A = \left(2^2 - \frac{2^3}{3}\right)$

$A = \frac{12-8}{3}$

$A = \frac{4}{3}$ sq. units

Both assertion and reason are correct and reason is correct explanation for assertion

S2. (d)

$4\sin^3 x = 3\sin x - \sin 3x$

$$\sin^3 x = \frac{3\sin - \sin 3x}{4}$$

$$= \frac{3}{4}\sin x - \frac{1}{4}\sin 3x$$

$$\int_0^{\pi/2} ydx$$

$$\int_0^{\pi/2} \sin^3 xdx$$

$$= \int_0^{\pi/y} \frac{3}{4}\sin x - \frac{1}{4}\sin 3xdx$$

$$= \left[\frac{-3}{4}\cos x - \frac{1}{4}\frac{\cos 3x}{3}\right]_0^{\pi/2}$$

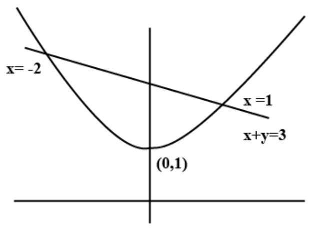
$$= \left[\left(\frac{-3}{4}\cos\frac{\pi}{2}\right) - \frac{1}{4}0 + \frac{3}{4}(1) + \frac{1}{4\times 3}\right]$$

$$= \frac{3}{4} + \frac{1}{12} = \frac{9+1}{12}$$

$$= \frac{10}{12} = \frac{5}{6}$$

Both assertion and reason are false

S3. (b)



The two meet at $3 - x = x^2 + 1 \Rightarrow x^2 + x - 2 = 0 \Rightarrow (x+2)(x-1) = 0 \Rightarrow x = -2, 1$

$\therefore$ Required area $= \int_{-2}^{1} [(3-x) - (x^2+1)]\, dx$

$= \int_{-2}^{1} (2 - x - x^2)\, dx = \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{1}$

$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - \frac{4}{2} + \frac{8}{3}\right) = \frac{9}{2}$

Reason (R) : $y^2 = 4ax$ is symmetrical about x-axis

Both assertion and reason are correct but reason is correct explanation for assertion

S4. (a)

Given, $\int_{\pi/4}^{\beta} f(x)\, dx = \beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta$

On differentiating with respect to β on both sides, we get

$f(\beta) = \sin\beta + \beta\cos\beta - \frac{\pi}{4}\sin\beta + \sqrt{2}$ (by Leibnitz rule)

Put $\beta = \frac{\pi}{2}$

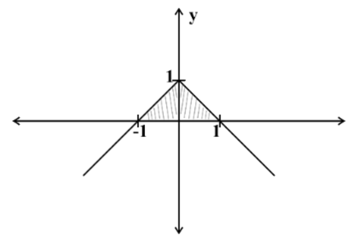
Then, $f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + \frac{\pi}{2}\cos\frac{\pi}{2} - \frac{\pi}{4}\sin\frac{\pi}{2} + \sqrt{2}$

$= 1 + 0 - \frac{\pi}{4} + \sqrt{2}$

$= 1 - \frac{\pi}{4} + \sqrt{2}$

Both assertion and reason are correct and reason is correct explanation for assertion

S5. (a)



$\Rightarrow y = \begin{cases} 1 - x\,; x \geq 0 \\ 1 + x\,; x < 0 \end{cases}$

$\Rightarrow A = Area\ of\ two\ triangles = 2 \times \left(\frac{1}{2} \times 1 \times 1\right) = 1\ sq.unit$

Both assertion and reason are correct and reason is correct explanation for assertion