# GEOMETRY AND MENSURATION
# 11

## INTRODUCTION

The chapters on geometry and mensuration have their own share of questions in the CAT and other MBA entrance examinations. For doing well in questions based on this chapter, the student should familiarise himself/herself with the basic formulae and visualisations of the various shapes of solids and two-dimensional figures based on this chapter.

The following is a comprehensive collection of formulae based on two-dimensional and three-dimensional figures:

For the purpose of this chapter, we have divided the theory in two parts:

- Part I consists of geometry and mensuration of two-dimensional figures
- Part II consists of mensuration of three-dimensional figures

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## PART I: GEOMETRY

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## INTRODUCTION

Geometry and Mensuration are important areas in the CAT examination. In the Online CAT, the Quantitative Aptitude section has consisted of an average of 15–20% questions from these chapters. Besides, questions from these chapters appear prominently in all major aptitude based exams for MBAs, Bank POs, etc.

Hence, the student is advised to ensure that he/she studies this chapter completely and thoroughly. Skills to be developed while studying and practising this chapter will be based on the application of formula and visualisation of figures and solids.

The principal skill required for doing well in this chapter is the ability to apply the formulae and theorems.

The following is a comprehensive collection of formulae based on two-dimensional figures. The student is advised to remember the formulae in this chapter so that he/she is able to solve all the questions based on this chapter.

## THEORY

**Basic Conversions**

A. 1 m = 100 cm = 1000 mm

1 km = 1000 m

= 0.621 miles

1 inch = 2.54 cm

B. 1 m = 39.37 inches

1 mile = 1760 yd

= 6076 ft

1 nautical mile (knot)

= 6080 ft

C. 100 kg = 1 quintal

10 quintal = 1 tonne

= 1000 kg

1 kg = 2.2 pounds

(approx.)

D. 1 litre = 1000 cc

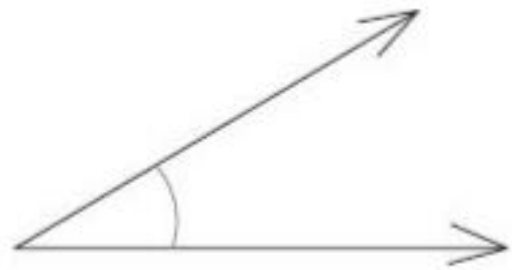
1 acre = 4046.86 sq m

1 hectare = 10000 sq m

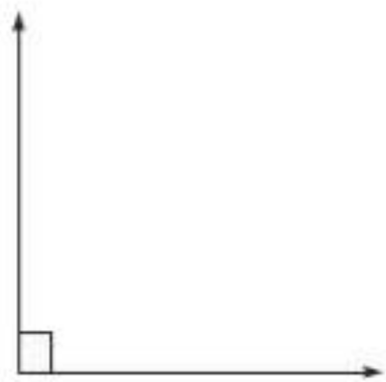
# TYPES OF ANGLES

## Basic Definitions

***Acute angle:*** An angle whose measure is less than 90 degrees. The following is an acute angle.



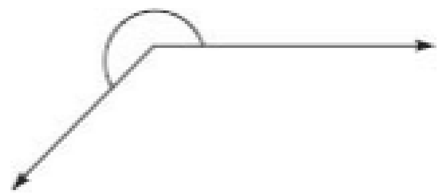
***Right angle:*** An angle whose measure is 90 degrees. The following is a right angle.



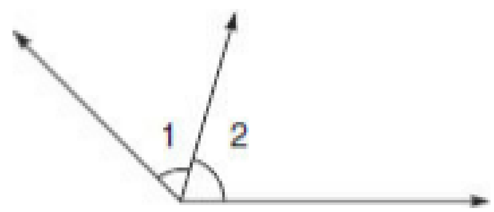
***Obtuse angle:*** An angle whose measure is bigger than 90 degrees but less than 180 degrees. Thus, it is between 90 degrees and 180 degrees. The following is an obtuse angle.

***Straight angle:*** It is an angle whose measure is 180 degrees.

***Reflex angle:*** An angle whose measure is more than 180 degrees but less than 360 degrees. The following is a reflex angle.

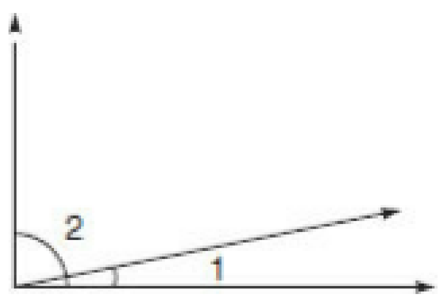


***Adjacent angles:*** Angles with a common vertex and one common side. In the figure below, ∠1 and ∠2 are adjacent angles.

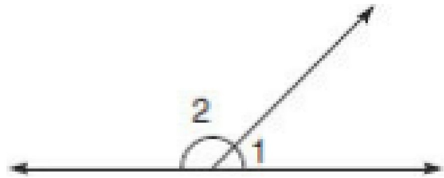



***Complementary angles:*** Two angles whose measures add to 90 degrees ∠1 and ∠2 are complementary angles because together they form a right angle.

However, one thing that you should note is that, even though in the figure given here, the two angles are shown as adjacent, they need not be so to be called complementary. As long as two angles add up to 90 degrees, they will be called complementary (even if they are not adjacent to each other).




***Supplementary angles:*** Two angles whose measures add up to 180 degrees. The following angles ∠1 and ∠2 are supplementary angles. However, supplementary angles do not need to be adjacent to be called supplementary (quite like complementary angles). The only condition for two angles to be called supplementary is if they are adding up to 180 degrees.

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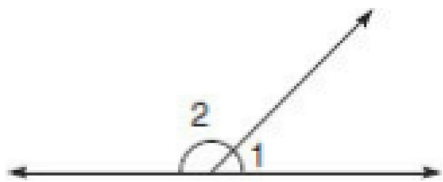
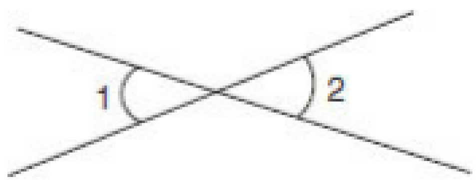
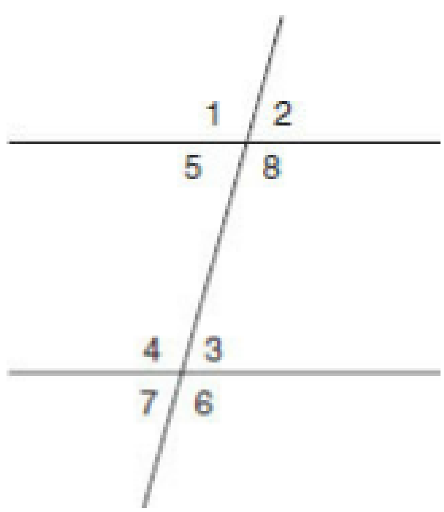



***Vertical angles:*** Angles that have a common vertex and whose sides are formed by the same lines. The following (∠1 and ∠2) are vertical angles.




***Angles formed when two parallel lines, are crossed by a transversal:*** When two parallel lines are crossed by a third line, (transversal), eight angles are formed. Take a look at the following figure:



Angles 3,4,5,8 are interior angles.

Angles 1,2,6,7 are exterior angles.

***Alternate interior angles:*** Pairs of interior angles on opposite sides of the transversal.

For instance, angle 3 and angle 5 are alternate interior angles. Angle 4 and angle 8 are also alternate interior angles. Both the angles in a pair of alternate interior angles are equal. Hence, in the figure we have: angle 3 = angle 5; also angle 4 = angle 8.

***Alternate exterior angles:*** Pairs of exterior angles on opposite sides of the transversal.

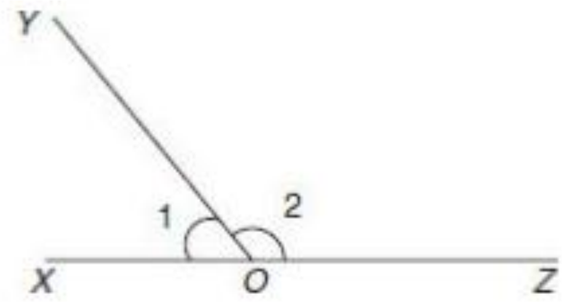
Angle 2 and angle 7 are alternate exterior angles. Angles 1 and 6 are also alternate exterior angles. Both the angles in a pair of alternate exterior angles are equal. Thus, in the figure angle 2 = angle 7 and angle 1 = angle 6.

***Co-interior angles:*** When two lines are cut by a third line (transversal), co-interior angles are between the pair of lines on the same side of the transversal. If the lines that are being cut by the transversal are parallel to each other, the co-interior angles are supplementary (add up to 180 degrees). In the given figure, angles 3 and 8 are co-interior angles. Also, angles 4 and 5 are co-interior angles, since, the lines being cut are parallel in this case, $\angle 3 + \angle 8 = 180$. Also, $\angle 4 + \angle 5 = 180$.

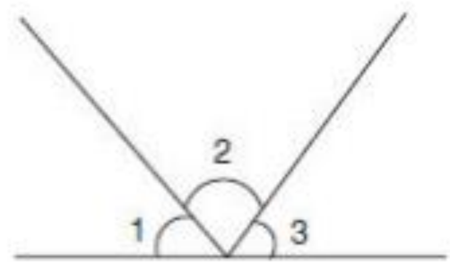
***Corresponding angles:*** Are pairs of angles that are in similar positions when two parallel lines are intersected by a transversal.

Angle 3 and angle 2 are corresponding angles. Similarly, the pairs of angles 1 and 4; 5 and 7; 6 and 8 are corresponding angles. Corresponding angles are equal. Thus, in the figure- $\angle 1 = \angle 4$; $\angle 5 = \angle 7$; $\angle 2 = \angle 3$ and $\angle 6 = \angle 8$.

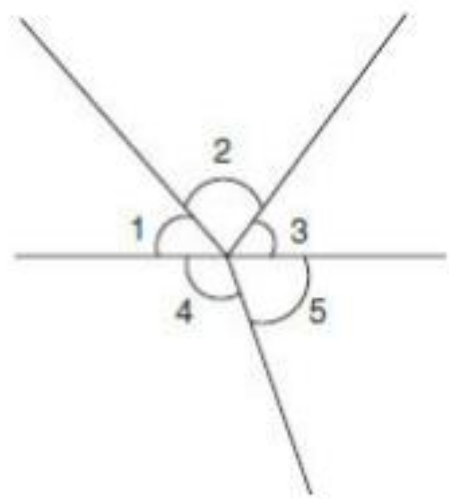
***Linear pair:*** $\angle XOY$ and $\angle YOZ$ are linear pair angles. One side must be common (e.g. *OY*) and these two angles must be supplementary.




***Angles on one side of a line:*** $\angle 1 + \angle 2 + \angle 3 = 180°$

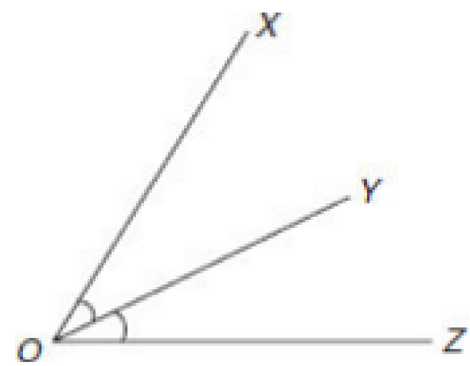



***Angles around the point:*** $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 360°$



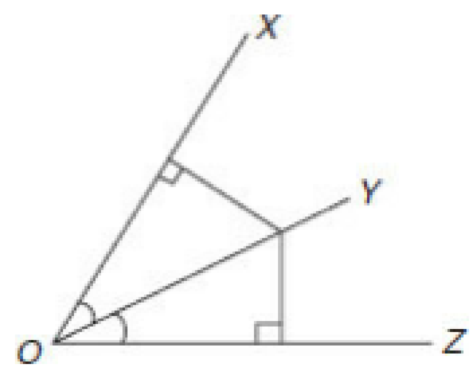

***Angle bisector:*** OY is the angle bisector for the $\angle$XOZ.

i.e. $\angle XOY = \angle ZOY = \frac{1}{2}\angle XOZ$




When a line segment divides an angle equally into two parts, then it is said to be the angle bisector (*OY*).

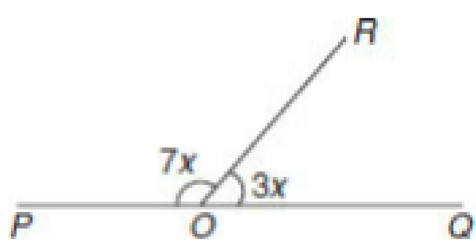
(Angle bisector is equidistant from the two sides of the angle.)




The distance between the lines *OX* and *OY* and the lines *OY* and *OZ* are equal to each other.

## PRACTICE EXERCISE

1. What is the value of $x$ in the given figure?



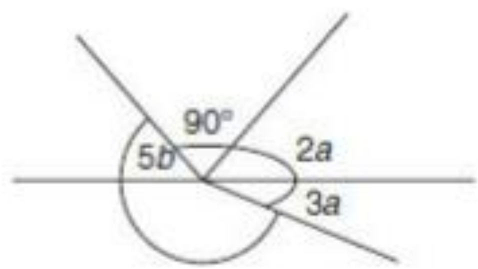
(a) 18°

(b) 20°

(c) 28°

(d) None of these

2. In the given figure, find the value of $(a + b)$




(a) 50°

(b) 54°

(c) 60°

(d) None of these

3. If $2a + 3$, $3a + 2$ are complementary, then find $a$,

(a) 17°

(b) 20°

(c) 23°

(d) 26°

4. If $5x + 17°$ and $x + 13°$ are supplementary, then find $x$,

(a) 20°

(b) 25°

(c) 30°

(d) None of these

5. An angle is exactly half of its complementary angle; find the angle.

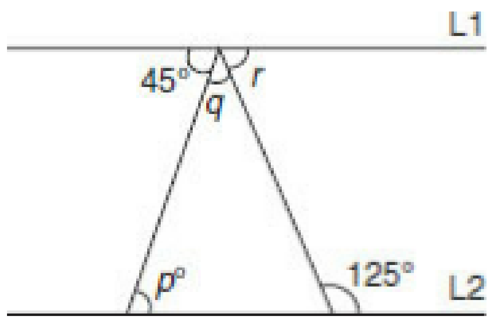
(a) 30°

(b) 40°

(c) 50°

(d) 60°

6. In the following figure, lines L1 and L2 are parallel to each other. Find the value of $q$.



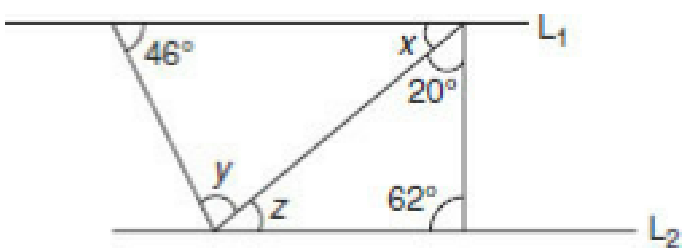

(a) 60°

(b) 80°

(c) 90°

(d) 85°

7. In the given figure if $L_1 || L_2$, then values of $x, y, z$ are:



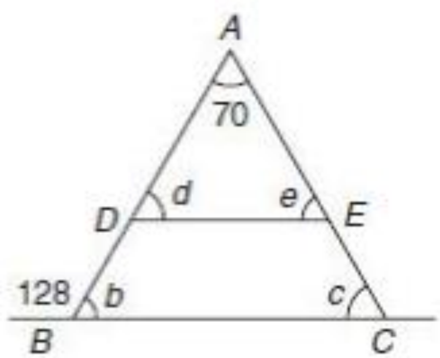
(a) 98°, 98°,36°

(b) 98°, 36°, 98°

(c) 36°,98°,36°

(d) None of these

8. In the given diagram if $BC||ED$ and $\angle BAC = 70°$, then find the value of $d$ and $c$.

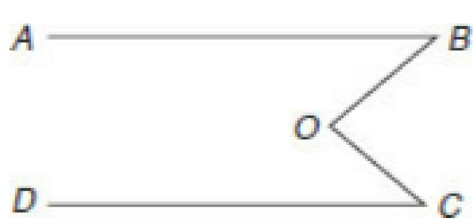



(a) 52°, 58°
(b) 58°, 52°

(c) 44°, 36°

(d) 36°, 44°

9. In the given diagram, if $AB||CD$ and $\angle ABO = 60°$ and $\angle BOC = 110°$, find $\angle OCD$.



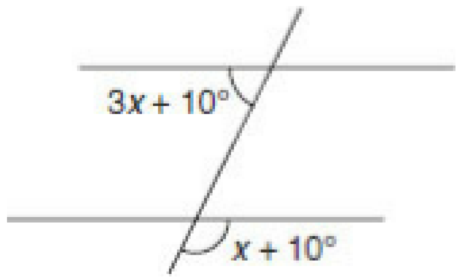
(a) 40°

(b) 50°

(c) 60°

(d) 70°

10. In the figure given, two parallel lines are intersected by a transversal. Then, find the value of $x$.




(a) 40°

(b) 50°

(c) 55°

(d) 65°

11. Maximum number of points of intersection of five lines on a plane is

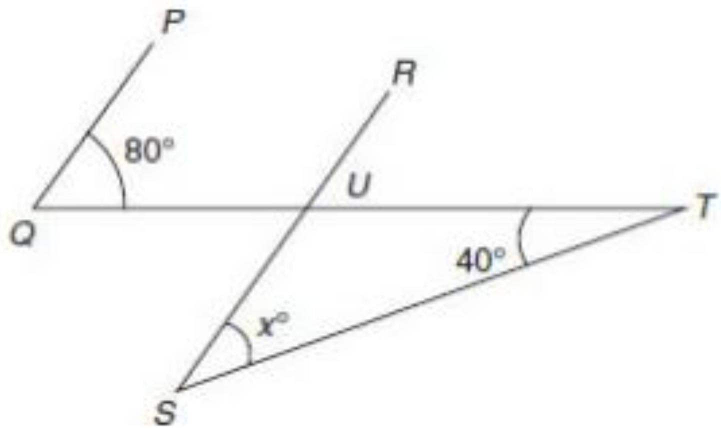
(a) 6

(b) 8

(c) 10

(d) 12

12. If $PQ || RS$, then find the value of $x$.



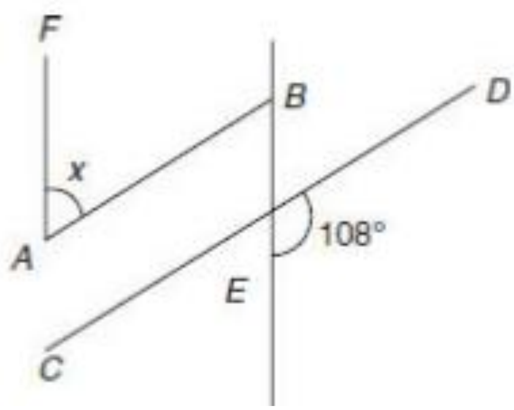

(a) 40°

(b) 60°

(c) 70°

(d) 80°

13. If $AB||CD$ and $AF||BE$, then the value of $x$ is



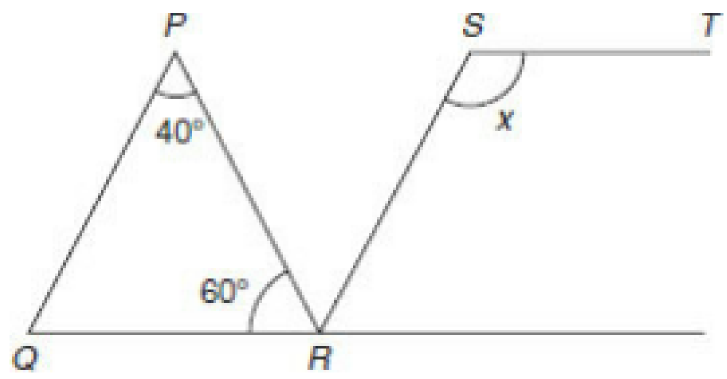

(a) 108°

(b) 72°

(c) 88°

(d) 82°

14. In the figure if $PQ||SR$ and $ST||QR$, then find $x$.



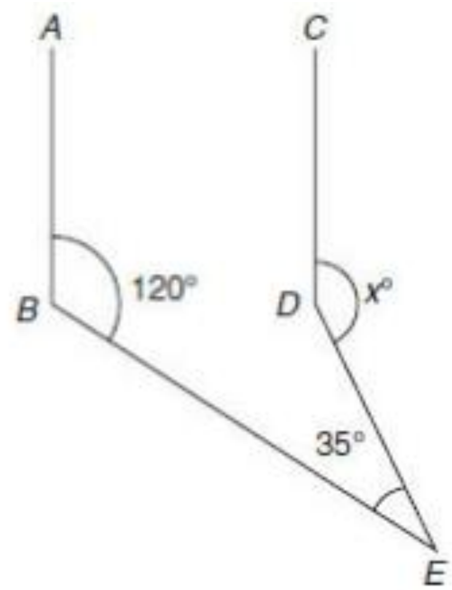

(a) 70°

(b) 80°

(c) 90°

(d) 100°

15. In the given figure, if $AB||CD$ then find the value of $x$.



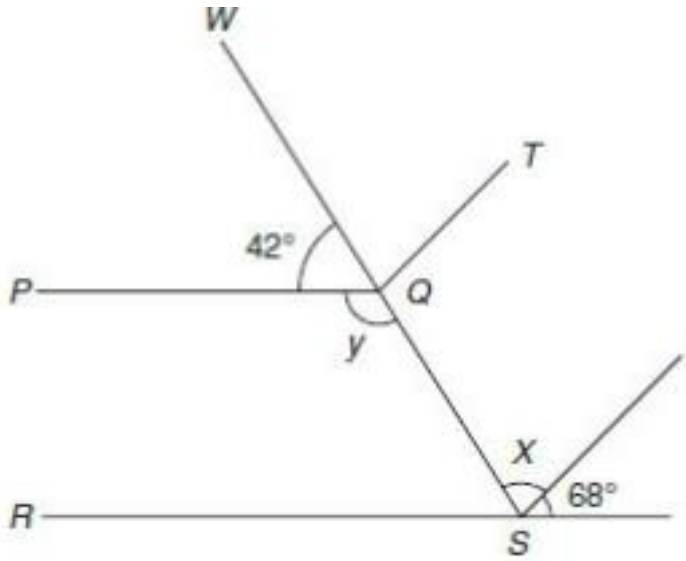

(a) 135°

(b) 145°

(c) 155°

(d) None of these

16. If $PQ||RS$ and $QT||SU$, then find the value of $x + y$.



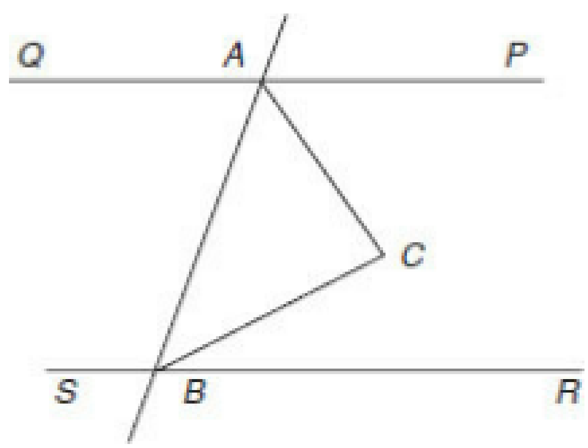

(a) 188°

(b) 202°

(c) 208°

(d) 212°

17. If $PQ||RS$ and $AC$ is angle bisector of $\angle PAB$, $BC$ is angle bisector of $\angle RBA$. Then find $\angle ACB$.



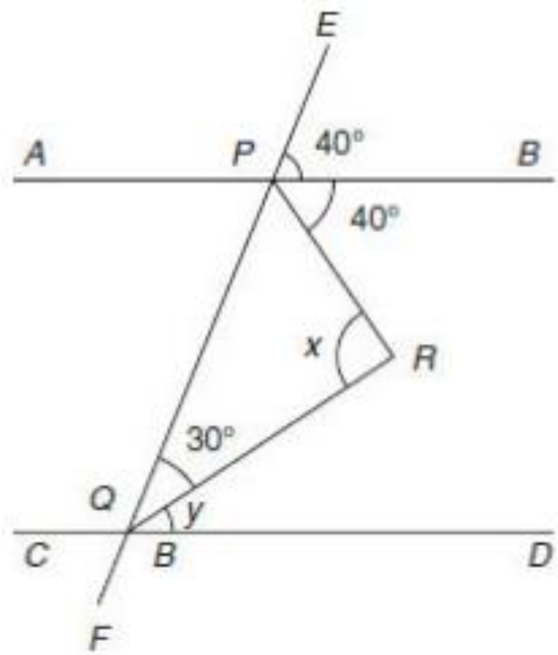
(a) 45°

(b) 75°

(c) 90°

(d) 110°

18. In the given figure, if $AB||CD$ then find $x + y$.



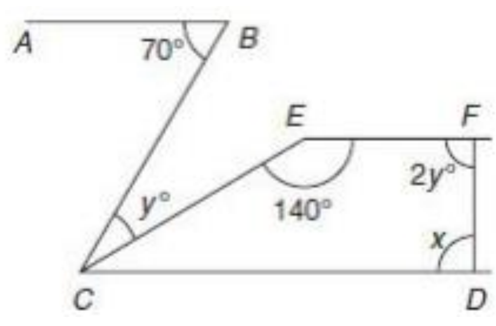

(a) 100°

(b) 110°

(c) 60°

(d) 125°

19. In the figure, if $CD||EF||AB$ then, find the value of $x$.



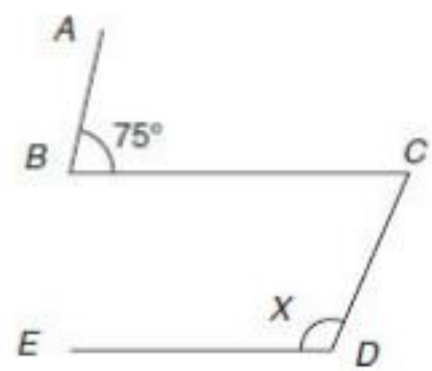
(a) 70°

(b) 90°

(c) 110°

(d) 120°

20. If in the given figure, $AB||CD$ and $BC||DE$, then find $x$.




(a) 95°

(b) 105°

(c) 115°

(d) 125°

## ANSWER KEY

1. (a)
2. (b)
3. (a)
4. (b)
5. (a)
6. (b)
7. (b)

8. (a)

9. (b)

10. (a)

11. (c)

12. (a)

13. (b)

14. (d)

15. (c)

16. (c)

17. (c)

18. (c)

19. (d)

20. (b)

**Solutions and Shortcuts**

1. $7x + 3x = 180°$

   $10x = 180°$ or $x = 18°$

   Option (a) is correct.

2. $90° + 2a + 3a + 5b = 360°$

   $5a + 5b = 270°$

   $a + b = 54°$

3. $2a + 3 + 3a + 2 = 90°$

   $5a + 5 = 90°$

   $$a = \frac{85°}{5} = 17°$$

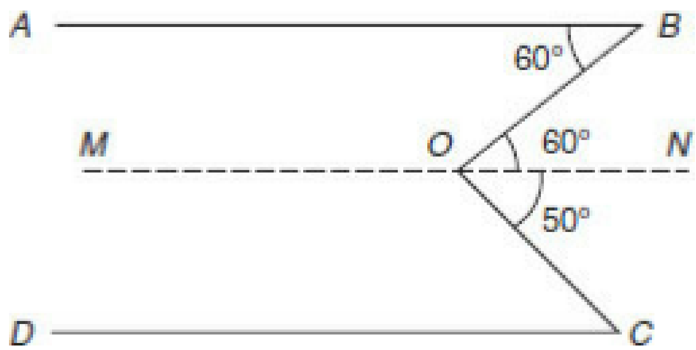
4. $5x + 17° + x + 13° = 180°$

$6x + 30° = 180°$

8. $\angle b = 180° - 128° = 52° = \angle d$ (Since they are corresponding angles)

$\angle c = 180° - (70° + 52°) = 58°$

9. Draw line $MON || AB || CD$




$\angle ABO = \angle BON$ [Alternate angles]

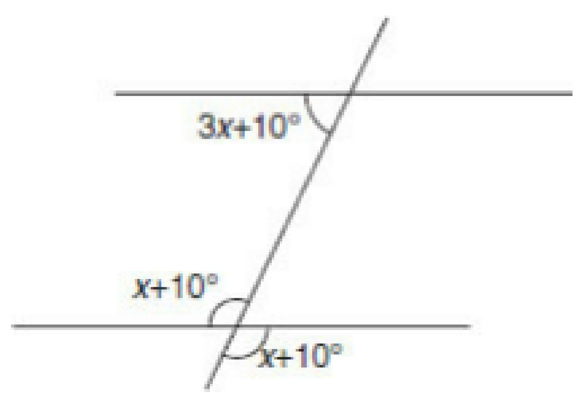
Hence, $\angle BON = 60°$

$\angle NOC = 110° - 60° = 50°$

Also, $\angle NOC = \angle OCD$ [Alternate angles]

$\angle OCD = 50°$

10. $3x + 10° + x + 10° = 180°$



$4x = 160°$

$x = 40°$

11. $^5C_2 = \frac{5!}{2! \times 3!} = 10$

Option (c) is correct.

12. $\angle PQU = \angle SUQ = 80°$ [Alternate angles]

$\angle SUT = 180° - 80° = 100°$

$\angle UST = x = 40°$

13. If $AB || CD$, then $\angle CEB + \angle ABE = 180°$

$\angle CEB = 108°$

$108° + \angle ABE = 180°$

$\angle ABE = 72°$

If $AB || BD$ then $\angle ABE = x$ [Alternate angles]

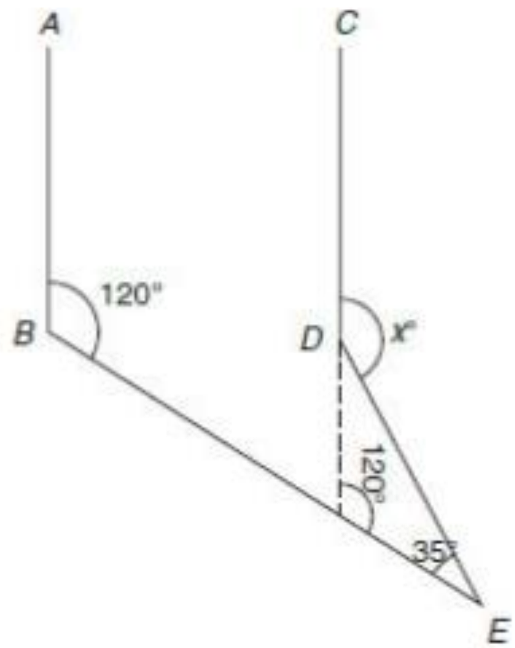
$x = 72°$

14. $\angle QPR = \angle SRP = 40°$ [Alternative angles]

$x = \angle SRQ = \angle SRP + \angle PRQ$ [Alternative angles]

$x = 60° + 40° = 100°$

15. Extend $CD$ to $x$




$\angle ABX = \angle CXE = 120°$

$x = 120° + 35°$ [$x$ is exterior angle of $\Delta DXE$)

$x = 155°$

16. $\angle y = 180° - 42 = 138°$

$\angle PQW = \angle RSQ = 42°$

$42° + x + 68° = 180°$

$x = 70°$

$x + y = 70° + 138° = 208°$

17. $\angle ACB = 180° - (\angle CAB + \angle CBA)$

$\angle PAB + \angle RBA = 180°$

$$\frac{\angle PAB}{2} + \frac{\angle RBA}{2} = 90°$$

$\angle CAB + \angle CBA = 90°$

$\angle ACB = 180° - (90°) = 90°$

18. $\angle RPQ = 180° - (40° + 40°) = 100°$

$x = 180° - (100° + 30°) = 50°$

$30° + y = 40°$

($\angle BPQ$ and $\angle DQP$ are corresponding angles).

$y = 10°$

$x + y = 50° + 10° = 60°$

19. $\angle ABC = \angle BCD = 70°$ [Alternate angles]

$\angle ECD = 180° - 140° = 40°$

[As angles $FEC$ and $ECD$ are co-interior angles]

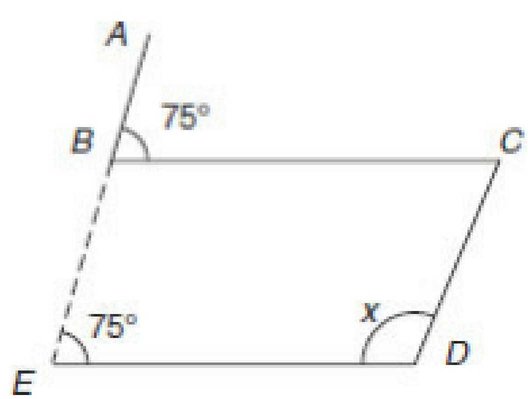
$40° + y = 70°$

$y = 30°$

$EF \| CD$ then $2y + x = 180°$

$x = 180° - 2y = 180° - 60° = 120°$

20. Extend $AB$ to $E$




$BC \| DE$, so $\angle ABC = \angle AFD = 75°$

$CD||BE$, hence $x + 75° = 180°$

$x = 105°$

## POLYGONS

Polygons are plane figures formed by a closed series of rectilinear (straight) segments. The following are examples of polygons:

triangle, rectangle, pentagon, hexagon, heptagon, octagon, nonagon (9-sided), decagon, undecagon or hendecagon (11-sided), dodecagon (12-sided), triskaidecagon or tridecagon (13-sided). Subsequent polygons are named as per the table below:

| *Number of sides* | *Name of the Polygon* |
|---|---|
| 14 | Tetradecagon, Terakaidecagon |
| 15 | Pentadecagon, Pentakaidecagon |
| 16 | Hexadecagon, Hexakaidecagon |
| 17 | Heptadecagon, Heptakaidecagon |
| 18 | Octadecagon, Octakaidecagon |
| 19 | Enneadecagon, Enneakaidecagon |
| 20 | Icosagon |
| 30 | Triacontagon |
| 40 | Tetracontagon |
| 50 | Pentacontagon |
| 60 | Hexacontagon |
| 70 | Heptacontagon |
| 80 | Ontacontagon |
| 90 | Enneacontagon |
| 100 | Hectogon, Hecatontagon |
| 1000 | Chiliagon |
| 10000 | Myriagon |

Polygons can broadly be divided into two types:

(a) *Regular polygons:* Polygons with all the sides and angles equal

(b) *Irregular polygons:* Polygons in which all the sides or angles are not of the same measure.

Polygons can also be divided as *concave* or *convex polygons*.

Convex polygons are the polygons in which all the diagonals lie inside the figure otherwise its a concave polygon

Polygons can also be divided on the basis of the number of sides they have.

| *Number of sides* | *Name of the polygon* | *Sum of all the angles* |
|---|---|---|
| 3 | Triangle | 180° |
| 4 | Quadrilateral | 360° |
| 5 | Pentagon | 540° |
| 6 | Hexagon | 720° |
| 7 | Heptagon | 900° |
| 8 | Octagon | 1080° |
| 9 | Nonagon | 1260° |
| 10 | Decagon | 1440° |

## Properties

1. Sum of all the angles of a polygon with $n$ sides = $(2n - 4)\pi/2$ or $(n - 2)\pi$

   Radians = $(n - 2)\ 180°$ degrees

2. Sum of all exterior angles = 360°◊

   i.e. in the figure below:

   $\theta_1 + \theta_2 + \ldots + \theta_6 = 360°$

   In general, $\theta_1 + \theta_2 + \ldots + \theta_n = 360°$

3. Number of sides = 360°/exterior angle.

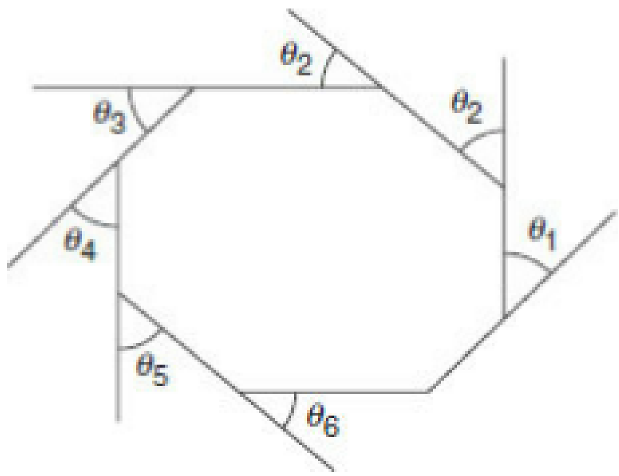
***Note:*** This property is true only for regular polygons.

4. Area = $(ns^2/4) \times \cot(180/n)$; where $s$ = length of side, $n$ = number of sides.

***Note:*** This property is true only for regular polygons.

5. Perimeter = $n \times s$

***Note:*** This property is true only for regular polygons.




## PRACTICE EXERCISE

1. Each interior angle of a regular polygon is 140°. Then the number of sides is

(a) 6

(b) 8

(c) 9

(d) 12

2. Each interior angle of a regular octagon is

(a) 90°
(b) 115°

(c) 125°

(d) 135°

3. The sum of the interior angles of a polygon is 1440°. The number of sides of the polygon is

(a) 8

(b) 10

(c) 12

(d) 14

4. Difference between interior and exterior angles of a polygon is 100°. Then the number of sides in the polygon is

(a) 8

(b) 9

(c) 10

(d) 11

5. If the ratio of interior and exterior angles of a regular polygon is 2:1, then find the number of sides of the polygon.

(a) 6

(b) 8

(c) 10

(d) 12

6. The ratio of the measure of an angle of a regular octagon to the measure of its exterior angle is:

(a) 2:1

(b) 1:3

(c) 3:1

(d) 1:1

7. Ratio between, the numbers of sides of two regular polygons is 2:3 and the ratio between their interior angles is 3:4. The number of sides of these polygons respectively is

(a) 4, 6

(b) 6, 9

(c) 8, 12

(d) None of these

8. Number of diagonals of a six-sided polygon is

(a) 6

(b) 9

(c) 12

(d) 15

9. Find the sum of all internal angles of a five-point star.

(a) 160°

(b) 180°

(c) 240°

(d) 300°

10. If the length of each side of a hexagon is 6 cm, then the area of the hexagon is

(a) 54 $cm^2$

(b) $54\sqrt{3}$ $cm^2$

(c) 68 $cm^2$

(d) None of these

---

## ANSWER KEY

---

1. (c)
2. (d)
3. (b)
4. (b)
5. (a)
6. (c)
7. (a)
8. (b)
9. (b)
10. (b)

**Solutions and Shortcuts**

1. Exterior angle of given polygon = 180° – 140° = 40°

   Number of sides = 360°/40° = 9. (Since the sum of all exterior angles of a polygon is 360°)

Option (c) is correct.

2. Total number of sides in octagon = 8

Each interior angle = $\frac{(8-2)\times180^\circ}{8} = \frac{6\times180^\circ}{8} = 135^\circ$

3. Let the number of sides be $x$.

Then according to the question

$(x-2) \times 180^\circ = 1440^\circ$

$x-2 = 8$

$x = 10$.

4. Let the internal angle be $x$ and external angle be $y$, according to the question

$x + y = 180^\circ$ (i)

$x - y = 100^\circ$ (ii)

$x = 140^\circ, y = 40^\circ$

Number of sides = $\frac{360^\circ}{40^\circ} = 9$

5. If interior angle be '$2x$' and exterior angle be $x$

Then $2x + x = 180^\circ$

$3x = 180^\circ$

$x = 60^\circ$

Number of sides = $360^\circ/60^\circ = 6$

6. Interior angle of a regular octagon = 135°

   Exterior angle of a regular octagon = 45°

   Required ratio = $\frac{135°}{45°} = 3:1$

7. We can solve this problem by checking the options.

   Option (a) 4, 6

   Interior angle of a four-sided polygon = 90°

   Interior angle of a six-sided polygon = 120°

   So the ratio of interior angles = 90°:120° = 3:4

   Hence, this option is correct.

8. Number of diagonals = $_6C_2 - 6$

   $$= \frac{6!}{2!4!} - 6 \Rightarrow 15 - 6 = 9$$

9. Sum of the angles of an $x$-pointed star = $(x - 4) \times \pi$

   So the required sum = $(5 - 4) \times \pi = 180°$

10. Required area = $6 \times \frac{\sqrt{3}}{4} \times 6^2$

    $$= 54\sqrt{3}\ \text{cm}^2$$

## TRIANGLES (Δ)

A triangle is a polygon having three sides. Sum of all the angles of a triangle = 180°.

***Types***

1. *Acute angle triangle:* Triangles with all three angles acute (less than 90°).

2. *Obtuse angle triangle:* Triangles with one of the angles obtuse (more than 90°).

***Note:*** We cannot have more than one obtuse angle in a triangle.

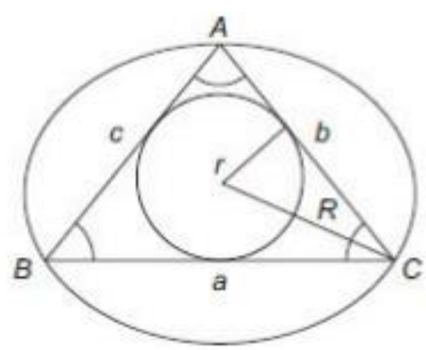
3. *Right angle triangle:* Triangle with one of the angles equal to 90°.
4. *Equilateral triangle:* Triangle with all sides equal. All the angles in such a triangle measure 60°.
5. *Isosceles triangle:* Triangle with two of its sides equal and consequently the angles opposite the equal sides are also equal.
6. *Scalene triangle:* Triangle with none of the sides equal to any other side.

## Properties (General)

- Sum of the length of any two sides of a triangle has to be always greater than the third side.
- Difference between the lengths of any two sides of a triangle has to be always lesser than the third side.
- Side opposite to the greatest angle will be the greatest and the side opposite to the smallest angle the smallest.
- The sine rule: $a/\sin A = b/\sin B = c/\sin C = 2R$

  (where $R$ = circum-radius.)
- The cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

  This is true for all sides and respective angles.



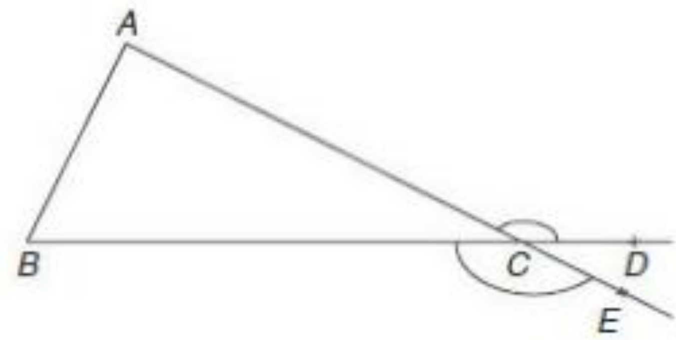
In case of a right triangle, the formula reduces to

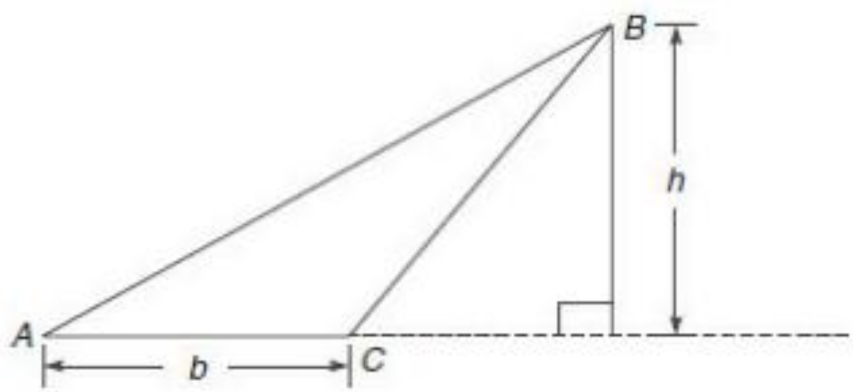
$a^2 = b^2 + c^2$

Since cos 90° = 0

- The exterior angle is equal to the sum of two interior angles not adjacent to it.

$$\angle ACD = \angle BCE = \angle A + \angle B$$




## Area

1. Area = 1/2 base × height or 1/2 $bh$

   Height = Perpendicular distance between the base and vertex opposite to it

2. Area = $\sqrt{s(s-a)(s-b)(s-c)}$ (Heron's formula)

   where $s = \frac{a+b+c}{2}$ ($a$, $b$ and $c$ being the length of the sides)

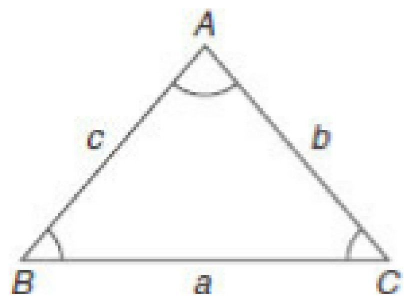
3. Area = $rs$ (where $r$ is **inradius**)

4. Area = 1/2 × product of two sides × sine of the included angle

$= 1/2\ ac \sin B$

$= 1/2\ ab \sin C$

$= 1/2\ bc \sin A$




4. Area = $abc/4R$

where $R$ = **circumradius**

***Congruency of Triangles*** Two triangles are congruent if all the sides of one are equal to the corresponding sides of another. It follows that all the angles of one are equal to the corresponding angles of another. The notation for congruency is ($\cong$).

## Conditions for Congruency

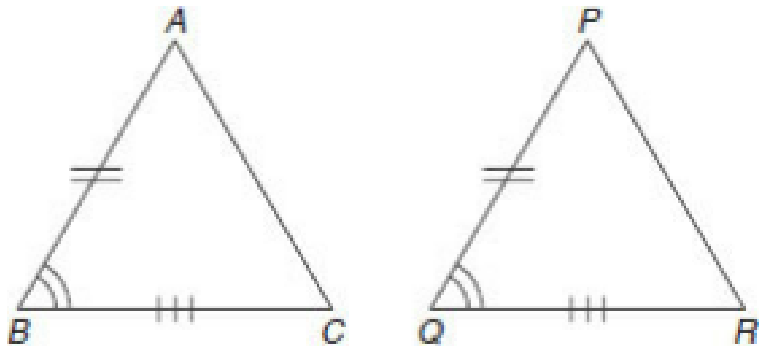
1. *SAS congruency*: If two sides and an included angle of one triangle are equal to two sides and an included angle of another, the two triangles are congruent. (See figure below.)

   Here, $AB = PQ$

   $BC = QR$

   and $\angle B = \angle Q$

   So, $\Delta ABC \cong \Delta PQR$



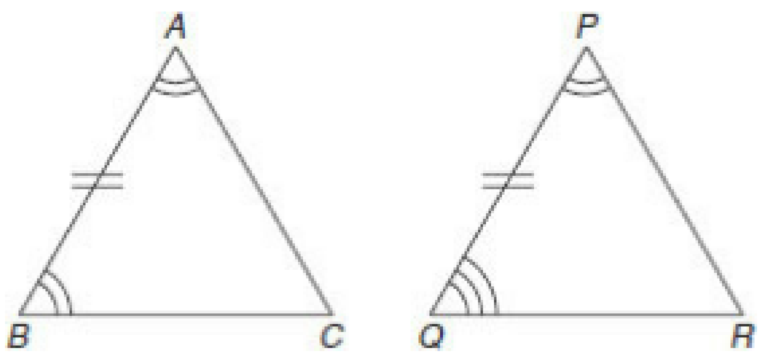

2. *ASA congruency*: If two angles and the included side of one triangle are equal to two angles and the included side of another, the triangles are congruent. (See figure below.)

Here, $\angle A = \angle P$

$\angle B = \angle Q$

and $AB = PQ$

So, $\Delta ABC \cong \Delta PQR$



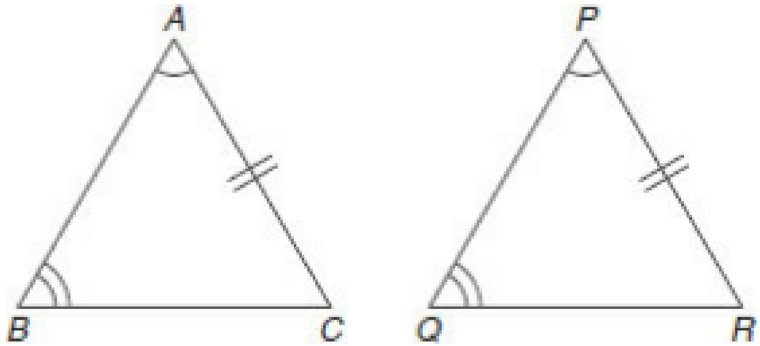

3. *AAS congruency*: If two angles and side opposite to one of the angles are equal to the corresponding angles and the side of another triangle, the triangles are congruent. In the figure below:

$\angle A = \angle P$

$\angle B = \angle Q$

and $AC = PR$

So, $\Delta ABC \cong \Delta PQR$



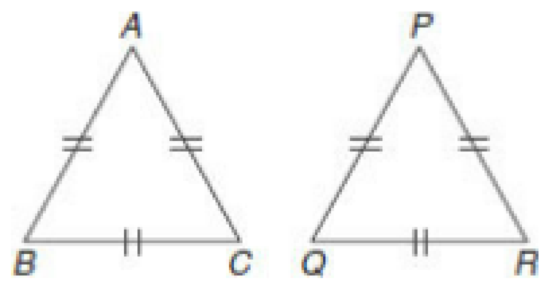

4. *SSS congruency*: If three sides of one triangle are equal to three sides of another triangle, the two triangles are congruent. In the figure below:

   $AB = PQ$

   $BC = QR$

   $AC = PR$

   $\therefore \Delta ABC \cong \Delta PQR$

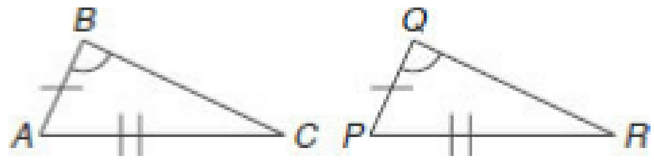



5. *SSA congruency*: If two sides and the angle opposite the greater side of one triangle are equal to the two sides and the angle opposite to the greater side of another triangle, then the triangles are congruent. The congruency does not hold if the equal angles lie opposite the shorter side. In the figure below, if

   $AB = PQ$

$AC = PR$

$\angle B = \angle Q$




then the triangles are congruent.

i.e. $\Delta ABC \cong \Delta PQR$.

***Similarity of triangles*** It is a special case where if either of the conditions of similarity of polygons holds, the other will hold automatically.

## Types of Similarity

1. *AAA similarity:* If in two triangles, corresponding angles are equal, that is, the two triangles are equiangular then the triangles are similar.
   *Corollary* (*AA similarity*)*:* If two angles of one triangle are respectively equal to two angles of another triangle then the two triangles are similar. The reason being, the third angle becomes equal automatically.

2. *SSS similarity:* If the corresponding sides of two triangles are proportional then they are similar.

   For $\Delta ABC$ to be similar to $\Delta PQR$, $AB/PQ = BC/QR = AC/PR$, must hold true.

3. *SAS similarity:* If in two triangles, one pair of corresponding sides are proportional and the included angles are equal then the two triangles are similar.

   $\Delta ABC \sim \Delta PQR$

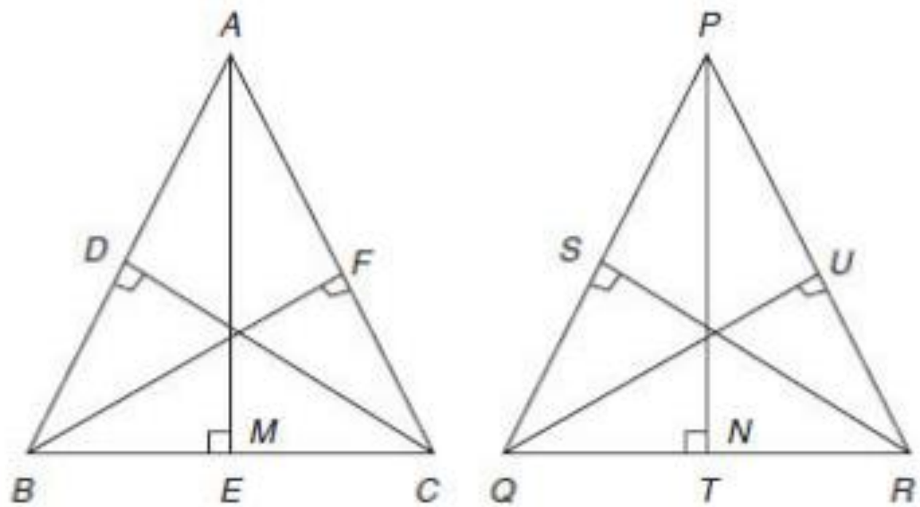
If $AB/BC = PQ/QR$ then $\angle B = \angle Q$

***Note:*** In similar triangles; the following identity holds:

Ratio of medians = Ratio of heights = Ratio of circumradii = Ratio of inradii = Ratio of angle bisectors

## Properties of similar triangles

If the two triangles are similar, then for the proportional/corresponding sides, we have the following results.




1. Ratio of sides = Ratio of heights (altitudes)

   = Ratio of medians

   = Ratio of angle bisectors

   = Ratio of inradii

   = Ratio of circumradii
2. Ratio of areas = Ratio of square of corresponding sides.

   i.e., if $\Delta ABC \sim \Delta PQR$, then

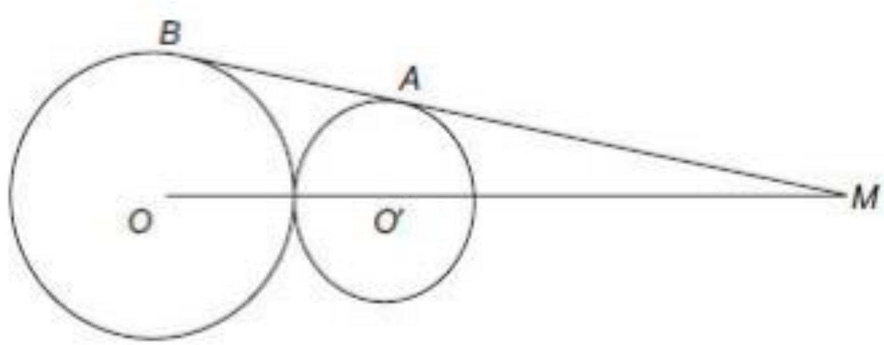
$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

While there are a lot of methods through which we see similarity of triangles, the one thing that all our Maths teachers forgot to tell us about similarity is the basic real life concept of similarity, i.e. **two things are similar if they look similar!!**

If you have been to a toy shop lately, you would have come across models of cars or bikes which are made so that they look like the original—but are made in a different size from the original. Thus, you might have seen a toy Maruti car which is built in a ratio of 1:25 of the original car. The result of this is that the toy car would look very much like the original car (of course, if it is built well!!). Thus, if you have ever seen a father and son looking exactly like each other, you have experienced similarity!!

You should use this principle to identify similar triangles. In a figure, two triangles would be similar simply if they look like one another.

Thus, in the figure below if you were to draw the radii OB and O′A, the two triangles MOB and MO′A will be similar to each other. Simply, because they look similar. Of course, the option of using the different rules of similarity of triangles still remains with you.

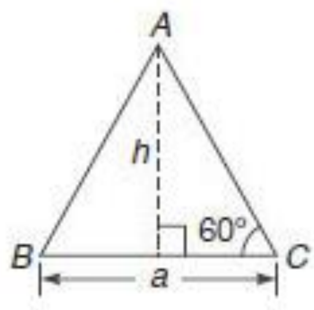


## Equilateral Triangles (of side *a*):

1. ($\because$ sin 60 = $\sqrt{3}/2 = h/\text{side}$)

   $$h = \frac{a\sqrt{3}}{2}$$

2. Area = 1/2 (base) × (height) = $\frac{1}{2} \times a \times \frac{a\sqrt{3}}{2} = \frac{\sqrt{3}}{4}a^2$



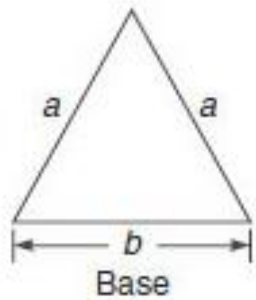

3. $R$ (circumradius) = $\frac{2h}{3} = \frac{a}{\sqrt{3}}$
4. $r$ (inradius) = $\frac{h}{3} = \frac{a}{2\sqrt{3}}$

## Properties

1. The in-center and circum-center lies at a point that divides the height in the ratio 2: 1.
2. The circum radius is always twice the inradius [$R = 2r$].
3. Among all the triangles that can be formed with a given perimeter, the equilateral triangle will have the maximum area.
4. An equilateral triangle in a circle will have the maximum area compared to other triangles inside the same circle.

## Isosceles Triangle

Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$




In an isosceles triangle, the angles opposite to the equal sides are equal.

## Right-Angled Triangle

***Pythagoras Theorem*** In the case of a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. In the figure below, for triangle *ABC*, $a^2 = b^2 + c^2$.

Area = 1/2 (product of perpendicular sides)

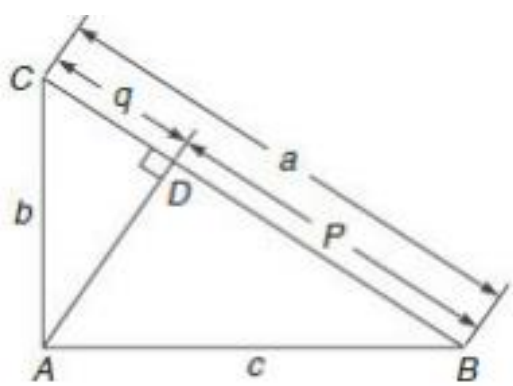
$R$(circum radius) = $\frac{\text{hypotenuse}}{2}$

Area = $rs$

(where $r$ = **inradius** and $s = (a + b + c)/2$ where $a$, $b$ and $c$ are sides of the triangle)

$\Rightarrow$ $1/2\ bc = r(a + b + c)/2$

$\Rightarrow$ $r = (bc)/(a + b + c)$




In the triangle *ABC*,

$\Delta ABC \sim \Delta DBA \sim \Delta DAC$

**Note:** A lot of questions are based on this figure.

Further, we find the following identities:

1. $\Delta ABC \sim \Delta DBA$

   $\therefore AB/BC = DB/BA$

   $\Rightarrow AB^2 = DB \times BC$

   $\Rightarrow c^2 = pa$

2. $\Delta ABC \sim \Delta DAC$

   $AC/BC = DC/AC$

   $\Rightarrow AC^2 = DC \times BC$

   $\Rightarrow b^2 = qa$

3. $\Delta DBA \sim \Delta DAC$

   $DA/DB = DC/DA$

   $DA^2 = DB \times DC$

   $\Rightarrow AD^2 = pq$

## Basic Pythagorean Triplets

→ 3, 4, 5 → 5, 12, 13 → 7, 24, 25 → 8, 15, 17 → 9, 40, 41 → 11, 60, 61 → 12, 35, 37 → 16, 63, 65 → 20, 21, 29 → 28, 45, 53 These triplets are very important since a lot of questions are based on them.

Any triplet formed by either multiplying or dividing one of the basic triplets by any positive real number will be another Pythagorean triplet.

Thus, since 3, 4, 5 form a triplet so also will 6, 8 and 10 as also 3.3, 4.4 and 5.5.

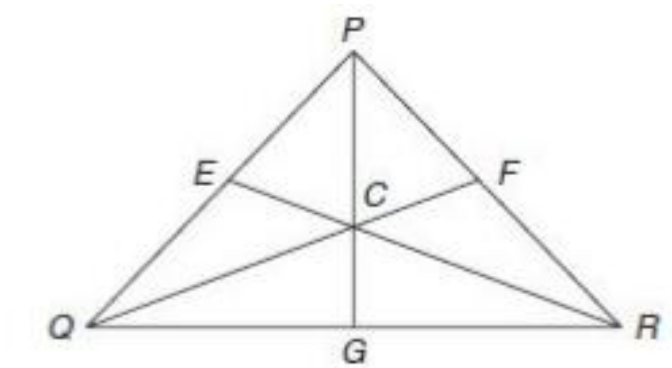
***Similarity of right triangles*** Two right triangles are similar if the hypotenuse and side of one is proportional to hypotenuse and side of another (RHS–similarity–Right angle hypotenuse side).

## Important Terms with Respect to a Triangle

***1. Median*** A line joining the mid-point of a side of a triangle to the opposite vertex is called a median. In the figure, the three medians are *PG*, *QF* and *RE* where *G*, *E* and *F* are mid-points of their respective sides.

- A median divides a triangle into two parts of equal area.
- The point where the three medians of a triangle meet is called the *centroid* of the triangle.
- The centroid of a triangle divides each median in the ratio 2: 1.

i.e. $PC : CG = 2 : 1 = QC : CF = RC : CE$




*Important formula with respect to a median*

→ 2 × (median)2 + 2 × (1/2 the third side)2

= Sum of the squares of other two sides

$\Rightarrow 2(PG)^2 + 2 \times \left(\frac{QR}{2}\right)^2$

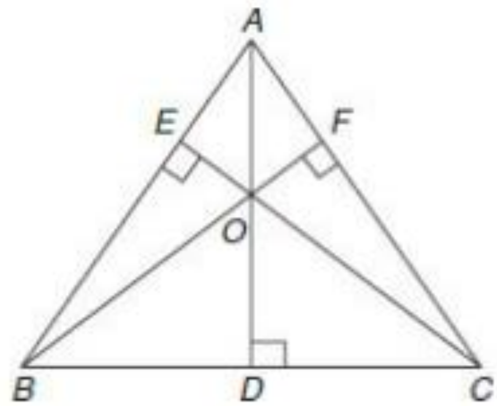
$= (PQ)^2 + (PR)^2$

***2. Altitude/Height*** A perpendicular drawn from any vertex to the opposite side is called the *altitude*. (In the figure, *AD*, *BF* and *CE* are the altitudes of the triangles).

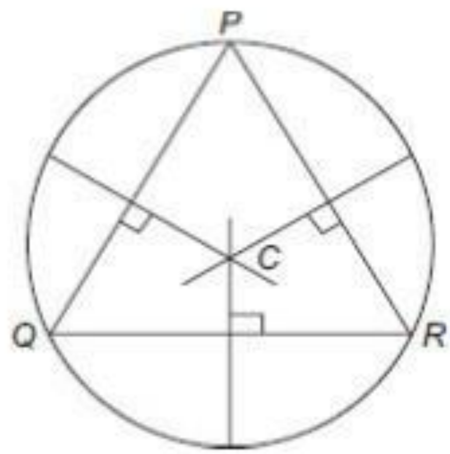
- All the altitudes of a triangle meet at a point called the *orthocenter* of the triangle.

- The angle made by any side at the orthocenter and the vertical angle make a supplementary pair (i.e. they both add up to 180°). In the figure below:

$$\angle A + \angle BOC = 180° = \angle C + \angle AOB$$




***3. Perpendicular Bisectors*** A line that is a perpendicular to a side and bisects it is the perpendicular bisector of the side.




- The point at which the perpendicular bisectors of the sides meet is called the *circumcenter* of the triangle
- The circumcenter is the center of the circle that circumscribes the triangle. There can be only one such circle.

- The point at which the perpendicular bisectors of the sides meet is called the *circumcenter* of the triangle
- The circumcenter is the center of the circle that circumscribes the triangle. There can be only one such circle.

• Angle formed by any side at the circumcenter is two times the vertical angle opposite to the side. This is the property of the circle whereby angles formed by an arc at the center are twice that of the angle formed by the same arc in the opposite arc. Here we can view this as:

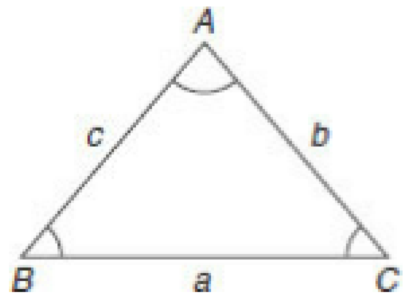
$\angle QCR = 2\ \angle QPR$ (when we consider arc $QR$ and its opposite arc $QPR$)

***4. In-center***

• The lines bisecting the interior angles of a triangle are the angle bisectors of that triangle.

• The angle bisectors meet at a point called the *in-center* of the triangle.

• The in-center is equidistant from all the sides of the triangle.




• From the in-center with a perpendicular drawn to any of the sides as the radius, a circle can be drawn touching all the three sides. This is called the *in-circle* of the triangle. The radius of the incircle is known as *in-radius*.

• The angle formed by any side at the in-center is always a right angle more than half the angle opposite to the side.

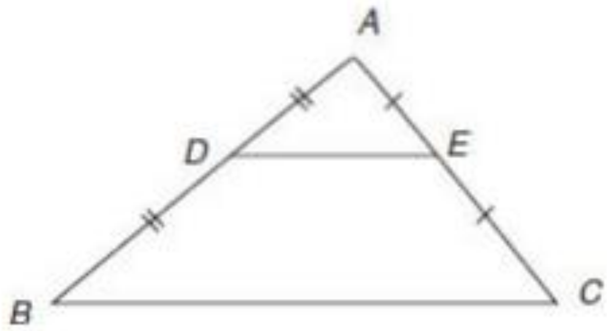
This can be illustrated as $\angle QXR = 90 + \frac{1}{2}\angle P$.

• If $QY$ and $RY$ are the angle bisectors of the exterior angles at $Q$ and $R$, then:

$$\angle QYR = 90 - \frac{1}{2}\angle P$$

## Mid-Point Theorem

The line segment joining the mid-points of two sides of a triangle is parallel to the third side and equal to half the third side.



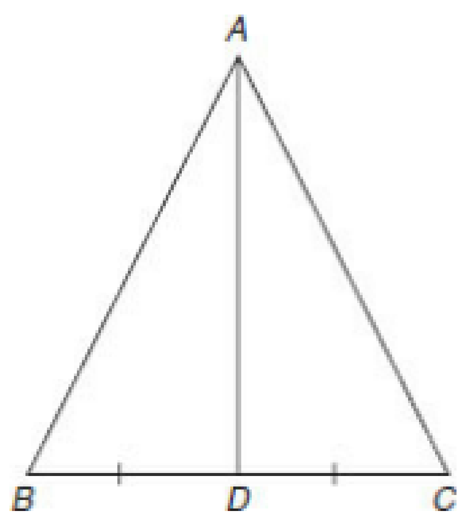

$AD = BD$ and $AE = CE$

$DE || BC$

## Apollonius' theorem

"The sum of the squares of any two sides of any triangle equals twice the square on half the third side plus twice the square of the median bisecting the third side".

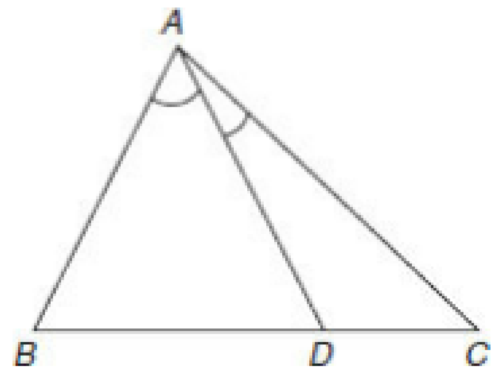
Specifically, in any triangle $ABC$, if $AD$ is a median, then




$BD = CD$

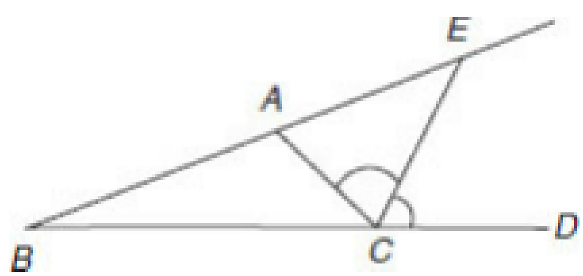
## Angle-bisector theorem

In a triangle, the angle bisector of an angle divides the opposite side, to the angle in the ratio of the remaining two sides, i.e. $\frac{BD}{CD} = \frac{AB}{AC}$ and $BD \times AC - CD \times AB = AD^2$.




## Exterior angle bisector theorem

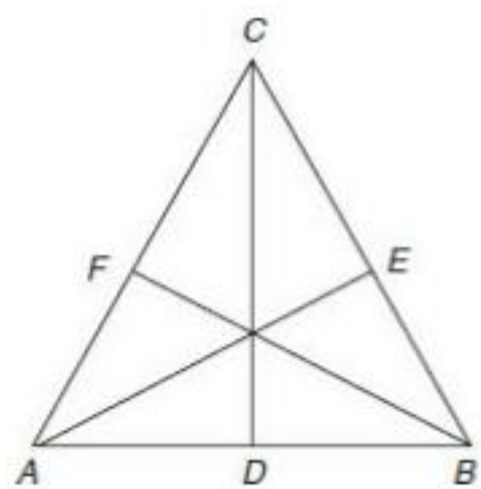
In a triangle, the angle bisector (represented by $CE$ in the figure) of any exterior angle of a triangle divides the side opposite to the external angle in the ratio of the remaining two sides i.e, $\frac{BE}{AE} = \frac{BC}{AC}$.

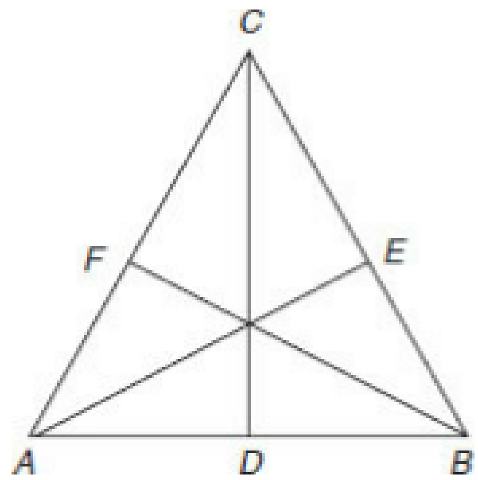



***Few important results:***

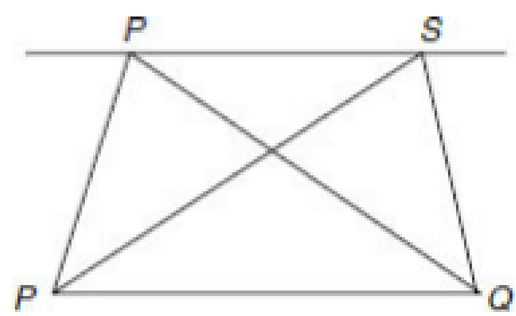
1. In a triangle $AE$, $CD$ and $BF$ are the medians then

   $3(AB^2 + BC^2 + AC^2) = 4(CD^2 + BF^2 + AE^2)$






2. If the two triangles have the same base and lie between the same parallel lines (as shown in figure), then the area of two triangles will be equal.




i.e. Area ($\Delta PQR$) = Area($\Delta PQS$)

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## PRACTICE EXERCISE

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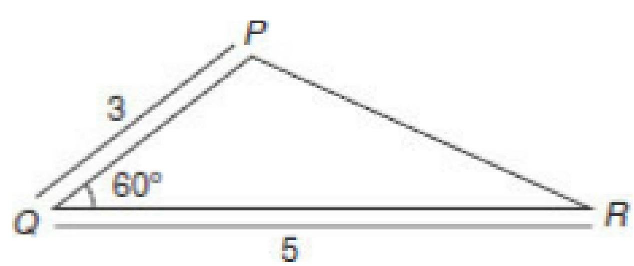
1. Find the area of $\Delta PQR$.



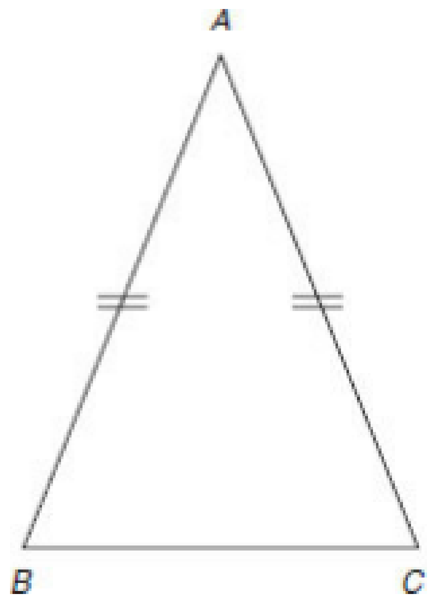
(a) $\frac{13\sqrt{3}}{4}$

(b) $\frac{15\sqrt{3}}{4}$

(c) $5\sqrt{3}$

(d) None of these

2. In $\Delta ABC$, $AB = AC = 5$ cm, $BC = 4$ cm, then find the area of $\Delta ABC$.




(a) $12\sqrt{3}$ cm2

(b) $2\sqrt{21}$ cm2

(c) $\sqrt{42}$ cm2

(d) None of these

3. If we draw a $\Delta\ ABC$ inside a circle ($A$, $B$, $C$ are on the circumference of a circle), then area of the $\Delta ABC$ is maximum when:

(a) $AB = BC \neq AC$

(b) $AB = BC = CA$

(c) $\angle BAC = 90°$

(d) $\Delta ABC$ is obtuse angle triangle

4. If height of an equilateral triangle is 10 cm, its area will be equal to:

(a) $100\sqrt{3}$ cm2

(b) $\frac{100}{3}\sqrt{3}$ cm2

(c) $\frac{100}{3}$ cm2

(d) $\frac{200\sqrt{3}}{3}$ cm2

5. Find the area of a triangle whose sides are 11, 60, 61.

(a) 210

(b) 330

(c) 315

(d) 275

6. If $AD$, $BE$, $CF$ are medians of a $\Delta ABC$ and $O$ is the centroid of $\Delta ABC$. If area of $\Delta AOF$ is 36 cm2, then find the area of $\Delta OFB$ + area of $\Delta OEC$.

(a) 36 cm2

(b) 54 cm2

(c) 72 cm2

(d) None of these

7. If three sides of a triangle are 5, 12, 13, then the circum-radius of the triangle is

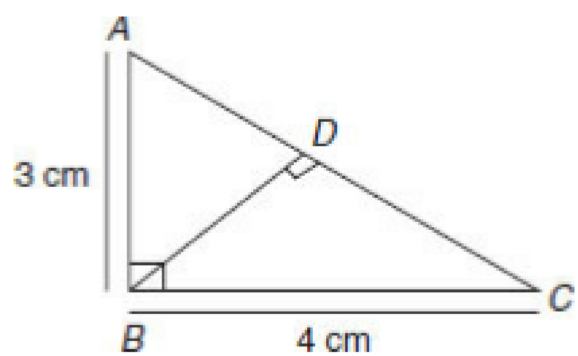
(a) 6cm

(b) 2.5cm

(c) 6.5cm

(d) None of these

8. $\Delta ABC$, $\angle B = 90°$, $BD \perp AC$ then find $BD$.



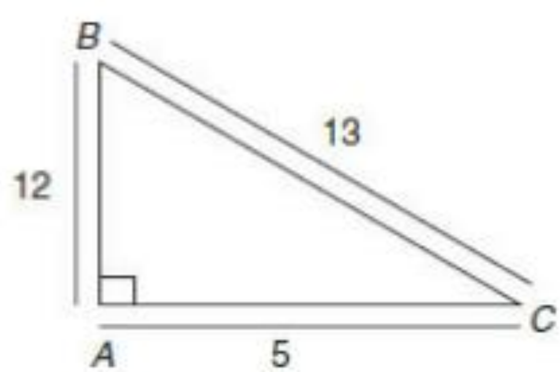

(a) 2.2 cm

(b) 2.4 cm

(c) 2.6 cm

(d) None of these

9. If $\angle A = 90°$, then the in-radius of $\Delta ABC$ is



(a) 2 cm

(b) 4 cm

(c) 6 cm

(d) 8 cm

10. In $\Delta ABC$ $AB = AC$, $\angle B = 80°$, $\angle BAD = 90°$, find $\angle ADE$.

11. $AD$ is the median of the triangle $ABC$ and $O$ is the centroid such that $AO$ = 12 cm. The length of $OD$ in cm is

(a) 4

(b) 5

(c) 6

(d) 8

12. If $\Delta ABC$ is an isosceles triangle with $\angle C = 90°$ and $AC$ = 7 cm, then $AB$ is

(a) 8.5 cm

(b) 8.2 cm

(c) $7\sqrt{2}$ cm

(d) 7.5 cm

13. In $\Delta$ $ABC$, $AB = AC$, $\angle BAC = 50°$, now $CB$ is extended to $D$, then the external angle at $\angle DBA$ is

(a) 90°

(b) 70°

(c) 115°

(d) 80°

14. The sides of a triangle are in the ratio 4:5:6. The triangle is

(a) Acute-angled

(b) Right-angled

(c) Obtuse-angled

(d) Either acute-angled or right angled

15. The sum of three altitudes of a triangle is

(a) Equal to the sum of three sides

(b) Less than the sum of sides

(c) $1/\sqrt{2}$ times of the sum of sides

(d) Half the sum of sides

16. Two medians *PS* and *RT* of Δ*PQR* intersect at *G* at right angles. If *PS* = 9 cm and *RT* = 6 cm, then the length of *RS* in cm is

(a) 10

(b) 6

(c) 5

(d) 3

17. Two triangles *ABC* and *PQR* are similar to each other in which *AB* = 5 cm, *PQ* = 4 cm. Then the ratio of the areas of triangles *ABC* and *PQR* is

(a) 4:5

(b) 25:16

(c) 64:125

(d) 4:7

18. In $\Delta ABC$, the internal bisectors of $\angle ACB$ and $\angle ABC$ meet at $X$ and $\angle BAC$ = 30°. The measure of $\angle BXC$ is

(a) 95°

(b) 105°

(c) 125°

(d) 130°

19. The area of an equilateral triangle is $900\sqrt{3}$ sq m. Its perimeter is

(a) 120 m

(b) 150 m

(c) 180 m

(d) 135 m

20. The sides of a triangle are 3 cm, 4 cm and 5 cm. The area (in $cm^2$) of the triangle formed by joining the mid points of this triangle is

(a) 6

(b) 3

(c) 3/2

(d) 3/4

## ANSWER KEY

1. (b)

2. (b)

3. (b)

4. (b)

5. (b)

6. (c)

7. (c)

8. (b)

9. (a)

10. 170o

11. (c)

12. (c)

13. (c)

14. (a)

15. (b)

17. (b)

18. (b)

19. (c)

20. (c)

**Solutions and Shortcuts**

1. $\text{Area} = \frac{1}{2} \times 3 \times 5 \times \sin 60^{o} = \frac{15}{2} \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{4}$

2. $\text{Area} = \frac{b}{4}\sqrt{4a^2 - b^2}$

$$= \frac{4}{4}\sqrt{4 \times 25 - 16} = \sqrt{84} = 2\sqrt{21} \text{ cm}^2$$

3. An equilateral triangle will have the maximum are compared to other triangles inside the same circle. So $AB = BC = CA$.

4. $h$ = 10 cm

$$h=\frac{a\sqrt{3}}{2}\Rightarrow a=\frac{10\times 2}{\sqrt{3}}=\frac{20}{\sqrt{3}}\text{ cm}$$

$$\text{Area}=\frac{1}{2}\times\frac{20}{\sqrt{3}}\times 10=\frac{100}{\sqrt{3}}\text{ cm}^2\text{ or }\frac{100\sqrt{3}}{3}\text{cm}^2$$

5. 11, 60, 61 form a Pythagoras triplet. Hence, the triangle is a right-angled triangle.

$$\text{Area}=\frac{1}{2}\times 11\times 60=330$$

6. '$O$' is the centroid of $\Delta ABC$

Then area of $\Delta AOF$ = area $\Delta OFB$ = area of $\Delta OEC$

Area ($\Delta OFB$)

+ Area($\Delta OEC$)

= 36 + 36 = 72 cm2

7. 5, 12, 13 form a Pythagoras triplet.

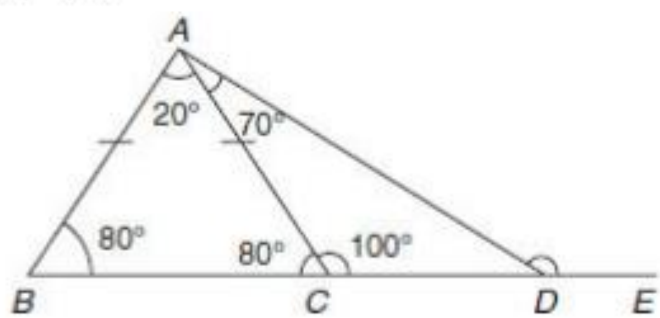
Circum-radius = 13/2 = 6.5 cm

8. $\frac{1}{BD^2}=\frac{1}{4^2}+\frac{1}{3^2}=\frac{25}{144}$

$BD=\left(\frac{144}{25}\right)^{\frac{1}{2}}=\frac{12}{5}=2.4$ cm

9. In-radius = $\frac{12\times 5}{12+5+13}=\frac{60}{30}=2$ cm

10. $\angle B = \angle ACB = 80°$



$\angle BAC = 180° - (80° - 80°) = 20°$

$\angle ADE = \angle CAD + \angle ACD = 70° + 100° = 170°$

11. $D$, is the mid-point of side $BC$.

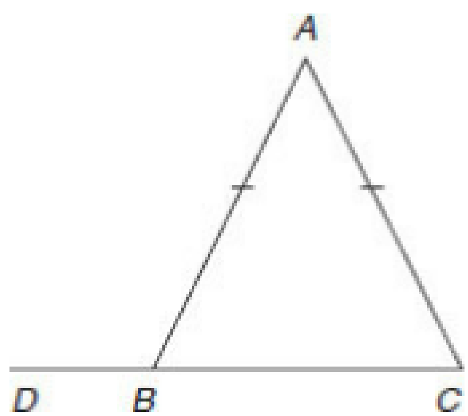
Centroid '$O$' divides $AD$ in the ratio 2:1

$\therefore OD = \dfrac{12}{2} = 6$ cm

12. $AC = BC = 7$ cm

$\therefore AB = \sqrt{AC^2 + BC^2} = \sqrt{7^2 + 7^2} = \sqrt{98} = 7\sqrt{2}$ cm

13.




$\angle ABC = \angle ACB$

$\angle BAC = 50°$

$\therefore \angle ABC + \angle ACB = 130°$

$\angle ABC = 65°$

$\therefore \angle ABD = 180° - 65° = 115°$

14. Let the sides of the triangle be $3x$, $4x$ and $6x$ units.

Clearly, $(4x)^2 + (5x)^2 > (6x)^2$

$\therefore$ the triangle will be acute-angled.

15. For a triangle $PQR$, let the altitudes be $AP$, $BR$ and $CQ$ respectively. Then:

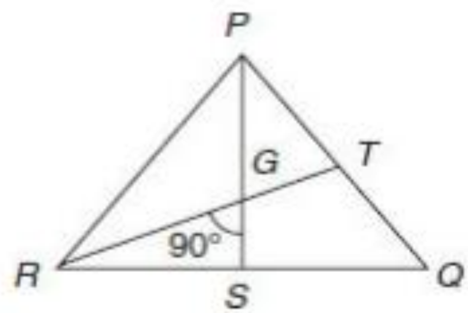
$AP < PR$

$BR < RQ$

$CQ < PQ$

$\therefore AP + BR + CQ < PQ + QR + PR$

16. $PS = 9$ cm




$\Rightarrow GS = \frac{1}{3} \times 9 = 3$ cm

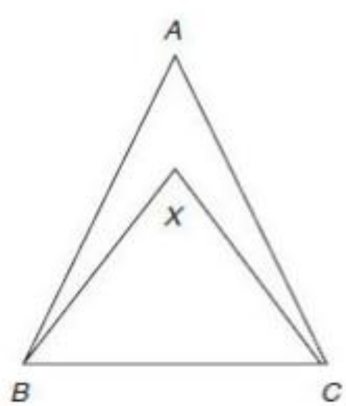
$RT = 6$ cm

$\Rightarrow RG = \frac{2}{3} \times 6 = 4$ cm

$\therefore RS = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$ cm

17. $\frac{\text{area of } \Delta ABC}{\text{area of } \Delta PQR} = \frac{AB^2}{PQ^2} = \frac{25}{16}$

18.



$\angle B + \angle C = 180° - 30° = 150°$

In $\Delta BXC$,

$$\frac{\angle B}{2} + \frac{\angle C}{2} + \angle BXC = 180°$$

$$\Rightarrow \angle BXC = 180° - \frac{1}{2}(\angle B + \angle C)$$

$$= 180° - \frac{150°}{2}$$

$= 180° - 75° = 105°$

19. Let the side of the equilateral triangle be $X$ cm. Area of equilateral triangle

$$= \frac{\sqrt{3}}{4} \times (X)^2$$

$$\Rightarrow \frac{\sqrt{3}}{4} \times (X)^2 = 900\sqrt{3}$$

$$\Rightarrow (X)^2 = \frac{900\sqrt{3} \times 4}{\sqrt{3}}$$

$\therefore X = \sqrt{4 \times 900} = 60$ meters

$\therefore$ Perimeter = $3 \times X = 3 \times 60 = 180$ meters

20. The area of the triangle formed by joining the midpoint of the triangle is 1/4th of the area of the original triangle.

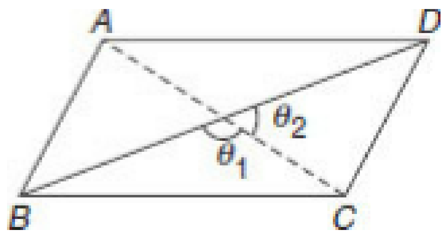
Area of the original triangle $= \frac{1}{2} \times 3 \times 4 = 6\text{ cm}^2$

$\therefore$ required area $= \frac{1}{4} \times 6 = \frac{3}{2}\text{ cm}^2$

## QUADRILATERALS

### Area

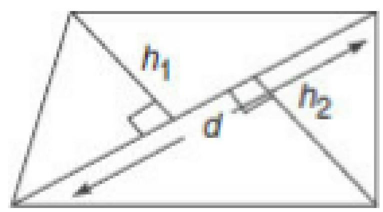
(a) Area = 1/2 (product of diagonals) × (sine of the angle between them)




If $\theta_1$ and $\theta_2$ are the two angles made between themselves by the two diagonals, we have by the property of intersecting lines → $\theta_1 + \theta_2 = 180°$.

Then, the area of the quadrilateral = $\frac{1}{2} d_1\, d_2 \sin \theta_1 = \frac{1}{2} d_1\, d_2 \sin \theta_2$.

(b) Area = 1/2 × diagonal × sum of the perpendiculars to it from opposite vertices = $\frac{d\,(h_1 + h_2)}{2}$.




(c) Area of a circumscribed quadrilateral

$$A = \sqrt{(S-a)(S-b)(S-c)(S-d)}$$

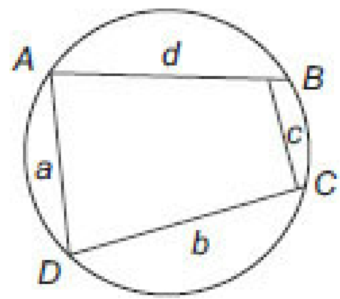
where $S = \frac{a+b+c+d}{2}$

(where $a$, $b$, $c$ and $d$ are the lengths of the sides)

## Properties

1. In a convex quadrilateral inscribed in a circle, the product of the diagonals is equal to the sum of the products of the opposite sides. For example, in the figure below:

$(a \times c) + (b \times d) = AC \times BD$




2. Sum of all the angles of a quadrilateral = 360°.

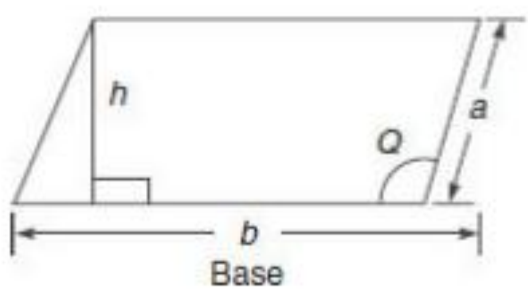
## TYPES OF QUADRILATERALS

### 1. Parallelogram (|| gm)

A parallelogram is a quadrilateral with opposite sides parallel (as shown in the figure).

(a) Area = Base ($b$) × height ($h$)

$= bh$




(b) Area = product of any two adjacent sides × sine of the included angle

$= ab \sin Q$

(c) Perimeter = $2(a + b)$

where, $a$ and $b$ are any two adjacent sides.

**Properties**

(a) Diagonals of a parallelogram bisect each other

(b) Bisectors of the angles of a parallelogram form a rectangle

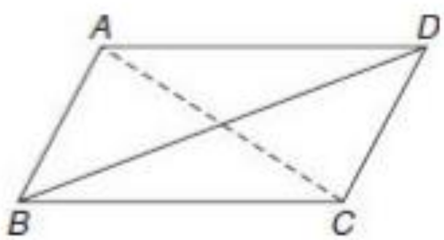
(c) A parallelogram inscribed in a circle is a rectangle

(d) A parallelogram circumscribed about a circle is a rhombus

(e) The opposite angles in a parallelogram are equal

(f) The sum of the squares of the diagonals is equal to the sum of the squares of the four sides. In the figure, $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$

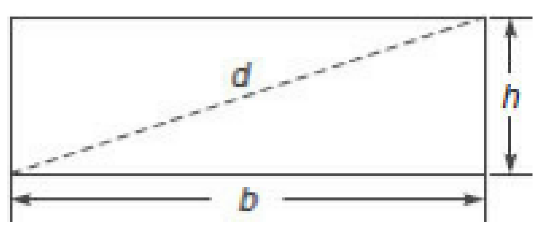
$= 2(AB^2 + BC^2)$




## 2. Rectangles

A rectangle is a parallelogram with all angles 90°.

(a) Area = Base × height = $b \times h$




***Note:*** Base and height are also referred to as the length and the breadth in a rectangle.

(b) Diagonal ($d$) = $\sqrt{b^2 + h^2}$ (by Pythagoras theorem)

**Properties of a Rectangle**

(a) Diagonals are equal and bisect each other

(b) Bisectors of the angles of a rectangle (a parallelogram) form another rectangle

(c) All rectangles are parallelograms but the reverse is not true

## 3. Rhombus

A parallelogram having all the sides equal is a rhombus.

(a) Area = 1/2 × product of diagonals × sine of the angle between them

= 1/2 × $d_1$ × $d_2$ sin 90° (Diagonals in a rhombus intersect at right angles)

= 1/2 × $d_1$ $d_2$ (since sin 90° = 1)

(b) Area = product of adjacent sides × sine of the angle between them

**Properties**

(a) Diagonals bisect each other at right angles

(b) All rhombuses are parallelograms but the reverse is not true

(c) A rhombus may or may not be a square but all squares are rhombuses

## 4. Square

A square is a rectangle with adjacent sides equal or a rhombus with each angle 90°.

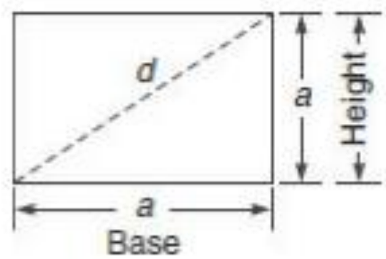
(a) Area = base × height = $a^2$

(b) Area = 1/2 (diagonal)$^2$ = 1/2 $d^2$ (square is a rhombus too)

(c) Perimeter = $4a$ ($a$ = side of the square)

(d) Diagonal = $a\sqrt{2}$

(e) Inradius = $\frac{a}{2}$




## Properties

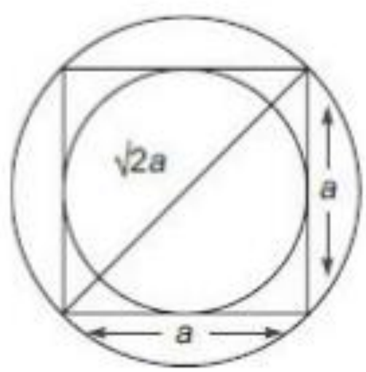
(a) Diagonals are equal and bisect each other at right angles

(b) Side is the diameter of the inscribed circle

(c) Diagonal is the diameter of the circumscribing circle

⇒ Diameter = $a\sqrt{2}$.

Circum-radius = $a/\sqrt{2}$




## 5. Trapezium

A trapezium is a quadrilateral with only two sides parallel to each other.

(a) Area = 1/2 × sum of parallel sides × height = 1/2 ($AB$ + $DC$) × $h$—for the figure below.

(b) Median = 1/2 × sum of the parallel sides (median is the line equidistant from the parallel sides)

For any line *EF* parallel to *AB*:

$$EF = \frac{\{[P \times (AB)] + [Q \times (DC)]\}}{AD}$$


## Properties

(a) If the non-parallel sides are equal then diagonals will be equal too.

---

## PRACTICE EXERCISE

---

1. Find the smallest angle of a quadrilateral if the measure of its interior angles is in the ratio of 1:2:3:4.

   (a) 18°

   (b) 36°

   (c) 54°

   (d) 72°

2. In a parallelogram *PQRS* if bisectors of *P* and *Q* meet at *X*, then the value of *PXQ* is

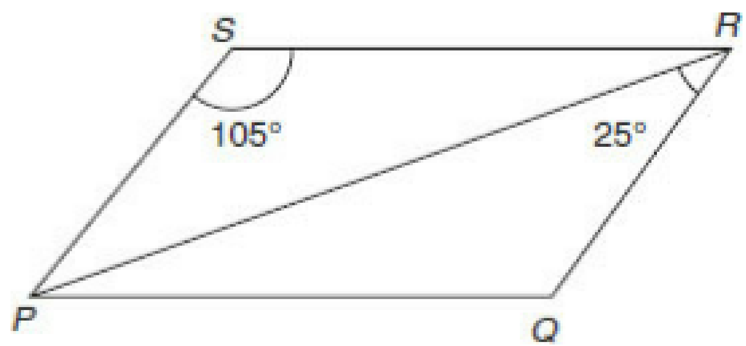
   (a) 45°

(b) 90°

(c) 75°

(d) 60°

3. In a parallelogram *PQRS*, if *S* = 105° and *PRQ* = 25° then find *QPR*.




(a) 40°

(b) 50°

(c) 60°

(d) 55°

4. If one diagonal of a rhombus is equal to its side, then the diagonals of the rhombus are in the ratio.

(a) $\sqrt{3}:1$

(b) 3 : 1

(c) 2 : 1

(d) None of these

5. A triangle and a parallelogram are constructed on the same base such that their areas are equal. If the altitude of the parallelogram is 100 m, then the altitude of the triangle is

(a) 50 m

(b) 100 m

(c) 200 m

(d) None of these

6. In a square *PQRS*, *A* is the midpoint of *PQ* and *B* is the midpoint of *QR*, if area of $\Delta AQB$ is 100 m2 then find the area of the square *PQRS*.

(a) 400 m2

(b) 250 m2

(c) 600 m2

(d) 800 m2

7. In the previous question, find the length of diagonal *PR*.

(a) 20 m

(b) 30 m

(c) 40 m

(d) $20\sqrt{2}$ m

8. If a triangle with area *x*, rectangle with area *y*, parallelogram with area *z* were all constructed on the same base and all have the same altitude, then which of the following options is true?

(a) $x = y = z$

(b) $x = y/2 = z$

(c) $2x = y = z$

(d) $2x = 2y = z$

9. □ $ABCD$ is a parallelogram; $AC$, $BD$ are the diagonals and intersect at point $O$. $X$ and $Y$ are the centroids of $\Delta ADC$ and $\Delta ABC$ respectively. If $BY$ = 6 cm, then find $OX$.

(a) 2 cm

(b) 3 cm

(c) 4 cm

(d) 6 cm

10. If area of a rectangle with sides $x$ and $y$ is $X$ and that of a parallelogram (which is strictly not a rectangle) with sides $x$ and $y$ is $Y$, then

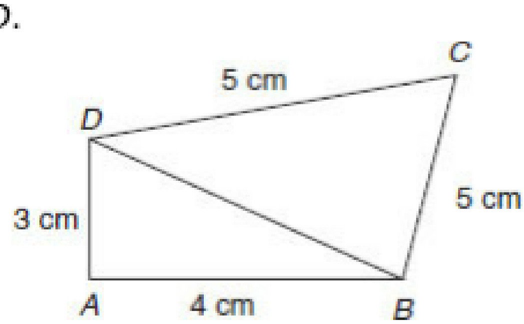
(a) $X = Y$

(b) $X \leq Y$

(c) $X < Y$

(d) $X > Y$

11. In □ $ABCD$, A = 90°, $BC = CD$ = 5 cm, $AD$ = 3 cm, $BA$ = 4 cm. Find the value ofÐ $\angle BCD$.



(a) 45°

(b) 60°

(c) 75°

(d) 85°

12. In the above question, what will be the area of ▫ ABCD?

(a) 16.83 cm2

(b) 15.36 cm2

(c) 14.72 cm2

(d) 13.76 cm2

13. ▫ *PQRS* is a parallelogram. '*O*' is a point within it and area of parallelogram *PQRS* is 50 cm2. Find the sum of areas of Δ*OPQ* and Δ*OSR* (in cm2).

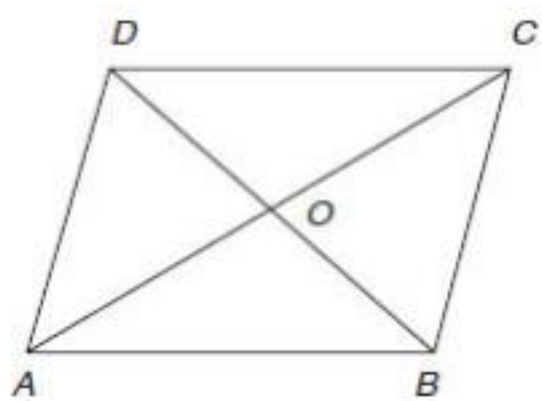
(a) 15

(b) 20

(c) 25

(d) 30

14. *ABCD* is a rhombus, such that *AB* = 5cm *AC* = 8 cm. Find the area of ▫ *ABCD*



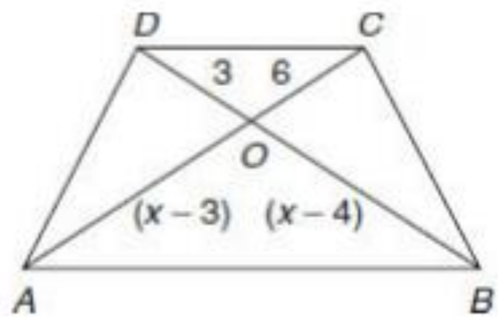
(a) 12 cm2

(b) 18 cm2

(c) 24 cm2

(d) 36 cm2

15. If *ABCD* is a trapezium, then find the value of *x*.




(a) 3

(b) 4

(c) 5

(d) 6

16. A square and a rhombus have the same base and the rhombus is inclined at 45° then what will be the ratio of area of the square to the area of the rhombus?

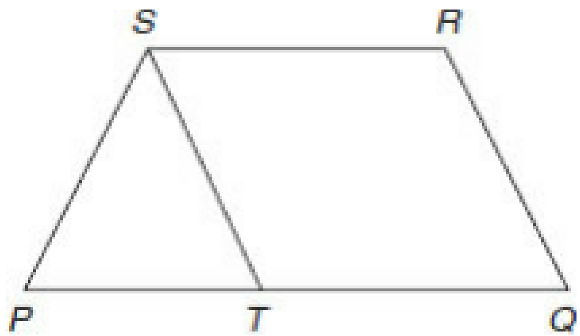
(a) 2:1

(b) $\sqrt{2}:1$

(c) $1:\sqrt{2}$

(d) $\sqrt{3}:1$

17. *PQRS* is a quadrilateral and $PQ \| RS$. *T* is the midpoint of *PQ*. $ST \| RQ$. If area of the triangle $\Delta PST$ is 50 $cm^2$ then area of ▫ *PQRS* is



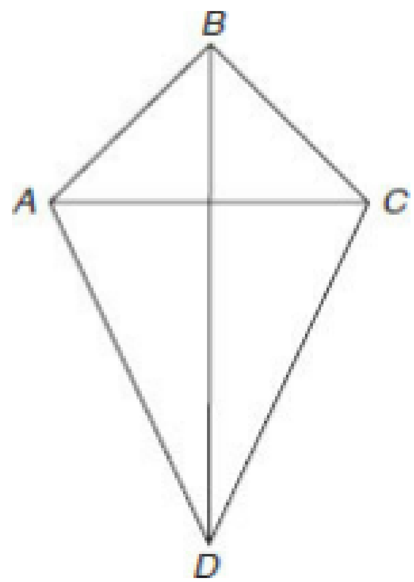

(a) 100 $cm^2$

(b) 125 $cm^2$

(c) 150 $cm^2$

(d) 175 $cm^2$

18. In ▫ *ABCD*, *AB* = *BC*, *AD* = *CD*. *BD* and *AC* are diagonals of ▫ *ABCD*, such that *BD* = 10 cm, *AC* = 5 cm. Find area of ▫ ABCD.




## ANSWER KEY

1. (b)

2. (b)

3. (b)

4. (a)

5. (c)

6. (d)

7. (c)

8. (c)

9. (b)

10. (d)

11. (b)

12. (a)

13. (c)

14. (c)

15. (c)

16. (b)

17. (c)

18. 25 $cm^2$

**Solutions and Shortcuts**

1. Let the angles be $x$, $2x$, $3x$, $4x$, respectively

   According to the question:

   $x + 2x + 3x + 4x = 360°$

   $10x = 360°$

   $X = 36°$

   Smallest angle = 36°

2. $P + Q = 180°$

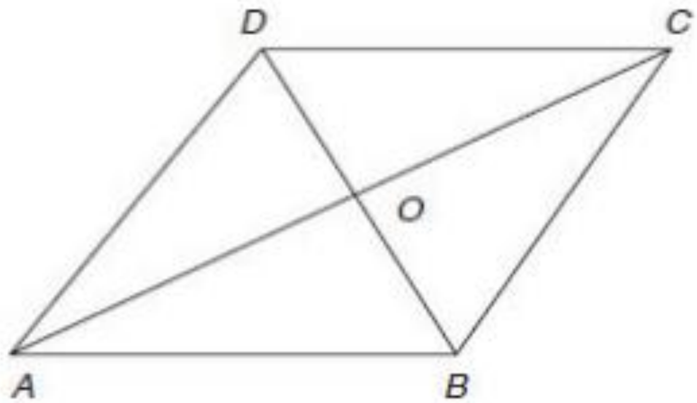
   $$\frac{\angle P}{2} + \frac{\angle Q}{2} = 90°$$

$$\angle PXQ = 180^o - \left[\frac{\angle P}{2} + \frac{\angle Q}{2}\right] = 180^o - 90^o = 90^o$$

3. $PQR = PSR = 105°$

   $RPQ = 180° - (105° + 25°) = 50°$

4. Let $AB = BD = DC = a, AC = b$




   In $\Delta COD$: $(CD)_2 = (OC)_2 + (OD)_2$

$$a^2 = \left(\frac{b}{2}\right)^2 + \left(\frac{a}{2}\right)^2$$

$$\frac{3a^2}{4} = \frac{b^2}{4}$$

$$b = a\sqrt{3}$$

$$\frac{b}{a} = \frac{\sqrt{3}}{1}$$

5. If '$b$' is the base and $h_1$, $h_2$ are altitudes of the triangle and parallelogram, respectively.

   Then, according to the question,

$$\frac{1}{2} \times b \times h_1 = b \times h_2$$

$h_1 = 2h_2$

$h_1 = 2 \times 100 = 200$ m

6. Area of $\Delta$AQB = $\frac{1}{2} \times AQ \times BQ = 100 = 100$

$$\frac{1}{2} \times \frac{PQ}{2} \times \frac{QR}{2} = 100$$
$$PQ \times QR = 2 \times 2 \times 2 \times 100 = 800 \text{ cm}^2$$

7. $PQ.QR$ = 800 cm2

$PQ = QR$ (□ $PQRS$ is a square)

$(PQ)2 = 800$

$PQ = 20\sqrt{2}$ cm

Length of the diagonal = PQ$\sqrt{2}$ = 20$\sqrt{2}$ × $\sqrt{2}$ = 40 m.

8. Area of triangle = $\frac{1}{2}$ × Area of Parallelogram

$x = z/2$

Area of parallelogram = Area of rectangle

$y = z$

$2x = y = z$

9. $\Delta ABC$ and $\Delta ADC$ are congruent to each other.

So $OD = OB$

$$\frac{OD}{3} = \frac{OB}{3}$$

$OX = OY$

$$OX = \frac{BY}{2} = \frac{6}{2} = 3 \text{ cm}$$

10. Area of rectangle $ABCD = X = xy$

Area of parallelogram $PQRS = Y = x.y \cos\theta$ (where $\theta$ is the angle between $x$ and $y$ and $\theta \neq 90°$)

As we know, $\cos q < 1$ (For $\theta \neq 90°$)

$Y < xy$

or

$Y < X$

11. $\Delta BAD$ is a right-angled triangle

$$BD = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ cm}$$

In $\Delta BCD$, all the sides are equal to each other, so $\Delta BCD$ is an equilateral triangle.

$\therefore BCD = 60°$

12. Area of $\square ABCD$ = Area of D$ABC$ + Area of D$BCD$

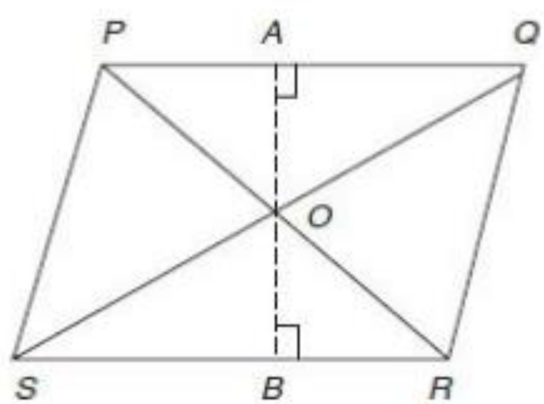
$$= \frac{1}{2} \times 3 \times 4 + \frac{\sqrt{3}}{4}(5)^2 \text{ cm}^2$$

$$= 6 + \frac{25\sqrt{3}}{4} \text{ cm}^2$$

$$= \left(6 + 6.25\sqrt{3}\right) \text{ cm}^2$$

= 16.83 cm2

13. Draw $OA \perp PQ$ and $OB \perp SR$.



If $OA = x$, $OB = y$ and $PQ = SR = a$, $QR = PS = b$

Then area of $\Delta OPQ = \frac{1}{2} \times x \times a = \frac{ax}{2}$

Area of $\Delta OSR = \frac{1}{2} \times y \times a = \frac{ay}{2}$

Area of $\Delta OPQ$ + Area of $\Delta OSR = \frac{ax}{2} + \frac{ay}{2}$

$= \frac{1}{2} a(x + y)$

$x + y$ = Altitude of parallelogram $PQRS$

Area of $PQRS = a(x + y)$

Area of $(\Delta OPQ + \Delta OSR) \frac{1}{2}$Area of □ $PQRS$

$= \frac{1}{2} \times 50 = 25 \text{ cm}^2$

14. $OC = \frac{AC}{2} = \frac{8}{2} = 4 \text{ cm}$

$\because \angle DOC = 90^\circ$

$\therefore OD^2 + OC^2 = CD^2$

$OD^2 + 4^2 = 5^2$

$OD^2 = 9$

OD = 3 cm

$BD = 2 \times' OD = 2 \times' 3 = 6$ cm

Area of $ABCD = \frac{1}{2} \times' 6 \times' 8 = 24$ cm2

15. $\frac{3}{x-4} = \frac{6}{x-3}$

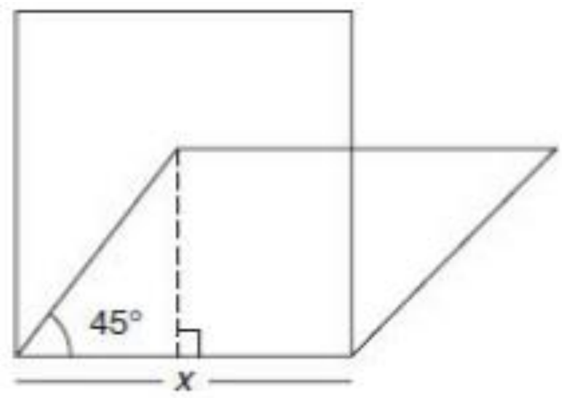
$3(x - 3) = 6(x - 4)$

$x - 3 = 2(x - 4)$

$x - 3 = 2x - 8$

$x = 5$

16. Let the length of base be '$x$' units. Area of square = x2




Area of rhombus = $x2$ ' × sin45°

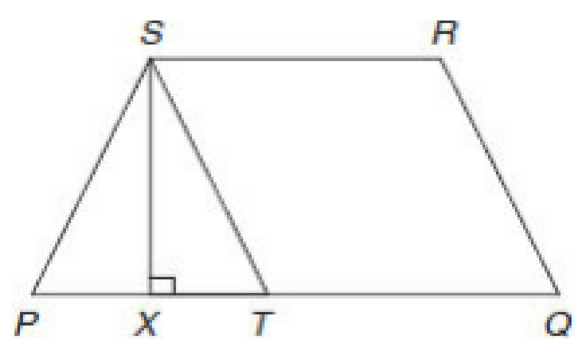
$$= x \times \frac{x}{\sqrt{2}} = \frac{x^2}{\sqrt{2}}$$

Required ratio $x^2 : \frac{x^2}{\sqrt{2}}$

$$= 1 : \frac{1}{\sqrt{2}}$$

$$= \sqrt{2} : 1$$

17. $T$ is the midpoint of $PQ$




$PT = TQ$

Draw $SX \perp PQ$, if $SX = h$ and $PT = TQ = a$

Area of $\Delta PST = \frac{1}{2} \times a \times h = \frac{ah}{2}$

Area of □ $PQRS$ = Area of $\Delta PST$ + Area of □ $STQR$

$= \frac{ah}{2} + ah$

$= \frac{3ah}{2}$

= 3[50] = 150 $cm^2$

18. □ $ABCD$ has a kite like structure, so it's diagonals intersect each other perpendicularly

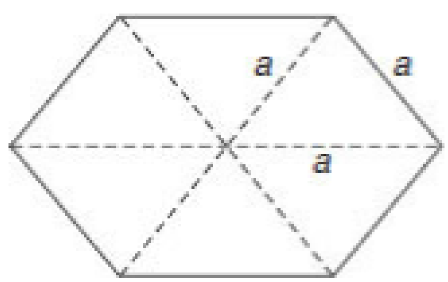
Area = $\frac{1}{2}$(product of diagonals)

$= \frac{1}{2} \times 10 \times 5 = 25 \text{ cm}^2$

## REGULAR HEXAGON

(a) Area = $[(3\sqrt{3})/2]\,(\text{side})^2$

$= \frac{3\sqrt{3} \times a^2}{2}$




(b) A regular hexagon is actually a combination of six equilateral triangles all of side '$a$'.

Hence, the area is also given by: 6 × area of an equilateral triangle having the same side as the side of the hexagon = $6 \times \frac{\sqrt{3}}{4} a^2$

(c) If you look at the figure closely, it will not be difficult to realise that circumradius ($R$) = $a$; i.e. the side of the hexagon is equal to the circum-radius of the same.

## CIRCLES

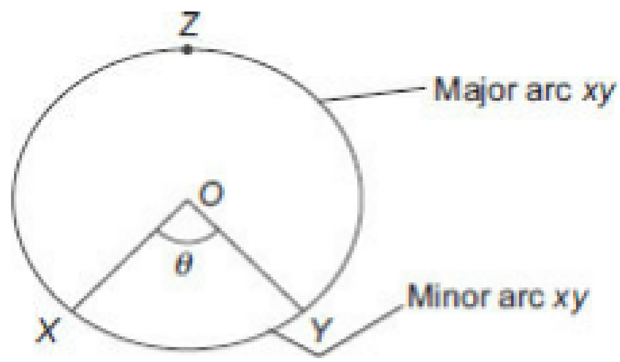
(a) Area = $\pi r2$

(b) Circumference = 2 $\pi r$ = ($r$ = radius)

(c) Area = 1/2 × circumference × $r$

**Arc:** It is a part of the circumference of the circle. The bigger one is called the *major arc* and the smaller one the *minor arc*.

(d) Length (Arc $XY$) = $\frac{\theta}{360} \times 2\pi r$




(e) **Sector of a circle** is a part of the area of a circle between two radii.

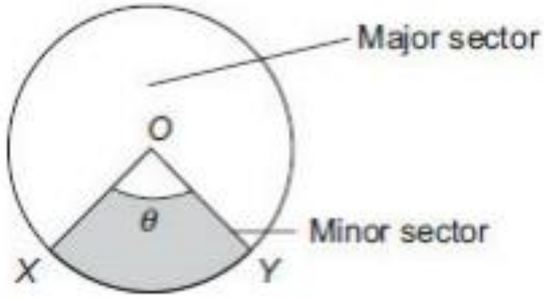
(f) Area of a sector = $\frac{\theta}{360} \times \pi r2$

(where θ is the angle between two radii)

= (1/2)$r$ × length (arc $xy$)

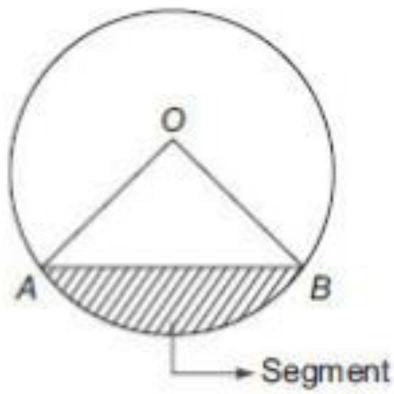
($\because \pi r\theta/180$ = length arc $xy$)

$$= \frac{1}{2} \times r \times \frac{\pi r\theta}{360}$$




(g) **Segment:** A sector minus the triangle formed by the two radii is called the segment of the circle.

(h) Area = Area of the sector – Area $\Delta OAB$ = $\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times r^2 \sin \theta$




(i) Perimeter of segment = length of the arc + length of segment $AB$

$$= \frac{\theta}{360} \times 2\pi r + 2r \sin\left(\frac{\theta}{2}\right)$$

$$= \frac{\pi r\theta}{180} + 2r \sin\left(\frac{\theta}{2}\right)$$

(j) **Congruency:** Two circles can be congruent if and only if they have equal radii.

## Properties

(a) The perpendicular from the center of a circle to a chord bisects the chord. The converse is also true.

(b) The perpendicular bisectors of two chords of a circle intersect at its center.

(c) There can be one and only one circle passing through three or more non-collinear points.

(d) If two circles intersect at two points then the line through the centers is the perpendicular bisector of the common chord.

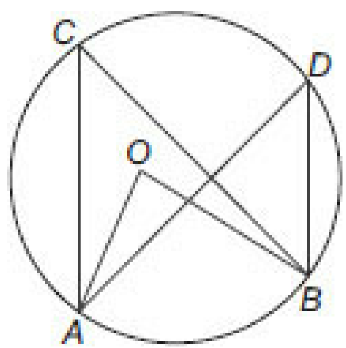
(e) If two chords of a circle are equal, then the center of the circle lies on the perpendicular bisector of the two chords.

(f) Equal chords of a circle or congruent circles are equidistant from the center.

(g) Equidistant chords from the center of a circle are equal to each other in terms of their length.

(h) The degree measure of an arc of a circle is twice the angle subtended by it at any point on the alternate segment of the circle. This can be clearly seen in the following figure:

With respect to the arc $AB$, $\angle AOB = 2\ \angle ACB$.

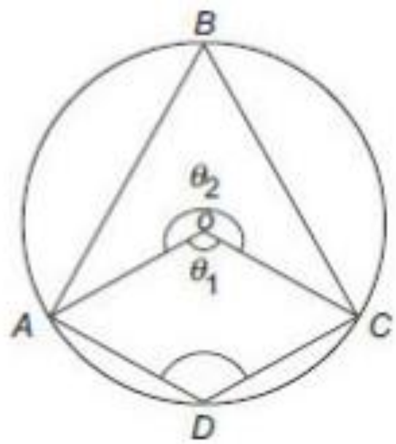


(i) Any two angles in the same segment are equal. Thus, $\angle ACB = \angle ADB$.

(j) The angle subtended by a semi-circle is a right angle. Conversely, the arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.

(k) Any angle subtended by a minor arc in the alternate segment is acute, and any angle subtended by a major arc in the alternate segment is obtuse.

In the figure below.




$\angle ABC$ is acute and

$\angle ADC$ = obtuse

Also $\theta_1 = 2\ \angle B$

And $\theta_2 = 2\ \angle D$

$\therefore \theta_1 + \theta_2 = 2(\angle B + \angle D)$

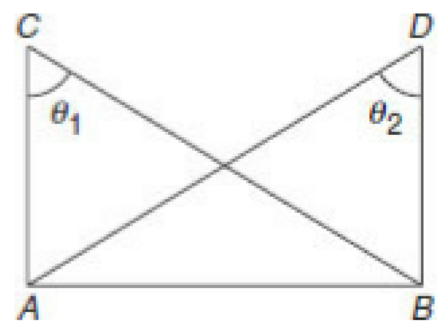
$= 360° = 2(\angle B + \angle D)$

or $\angle B + \angle D = 180°$
or sum of opposite angles of a cyclic quadrilateral is $180°$.

(l) If a line segment joining two points subtends equal angles at two other points lying on the same side of the line, the four points are concyclic. Thus, in the following figure:

If, $\theta_1 = \theta_2$

then *ABCD* are concyclic, that is, they lie on the same circle.




(m) Equal chords of a circle (or of congruent circles) subtend equal angles at the center (at the corresponding centers.) The converse is also true.

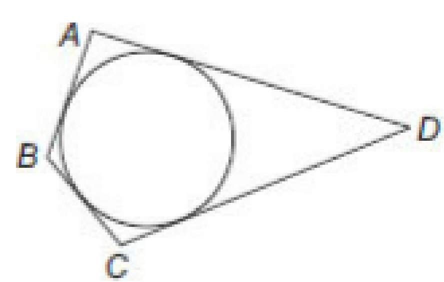
(n) If the sum of the opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

*Secant:* A line that intersects a circle at two points

*Tangent:* A line that touches a circle at exactly one point

(o) If a circle touches all the four sides of a quadrilateral then the sum of the two opposite sides is equal to the sum of other two.

$$AB + DC = AD + BC$$




(p) In two concentric circles, the chord of the larger circle that is tangent to the smaller circle is bisected at the point of contact.

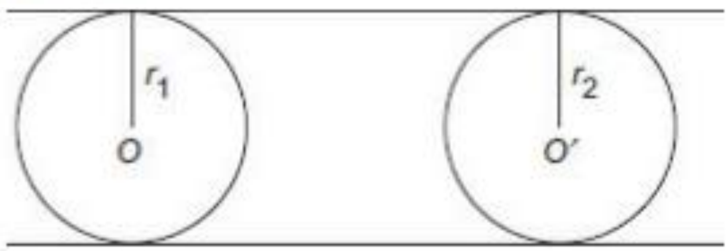
## Tangents

- Length of direct common tangents is

$$= \sqrt{(\text{Distance between their centres})^2 - (r_1 - r_2)^2}$$

where, $r_1$ and $r_2$ are the radii of the circles

$$= \sqrt{(OO')^2 - (r_1 - r_2)^2}$$



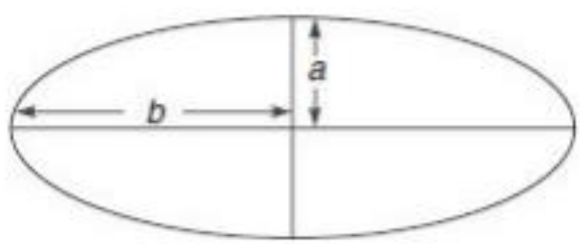

- Length of transverse common tangents is

$$= \sqrt{(\text{distance between their centres})^2 - (r_1 + r_2)^2}$$

$$= \sqrt{(OO')^2 - (r_1 + r_2)^2}$$

## ELLIPSE

- Perimeter = $\pi (a + b)$
- Area = $\pi ab$




## PRACTICE EXERCISE

1. Find the area of a circle of radius 5cm.

   (a) $25\pi$

   (b) $20\pi$

   (c) $22\pi$

(d) None of these

2. Find the circumference of the circle in the previous question:

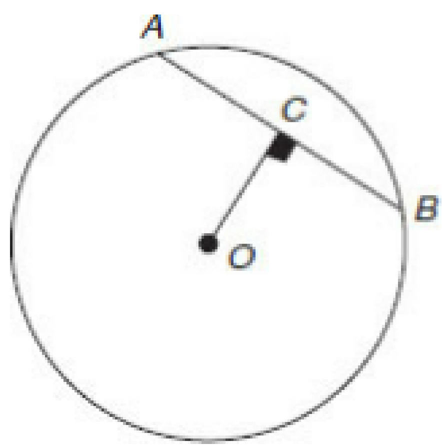
(a) $10\pi$

(b) $5\pi$

(c) $7\pi$

(d) None of these

3. If $O$ is the center of the circle and $OC \perp AB$ and $AC = x + 6$, $BC = 2x - 4$, then find $AB$.



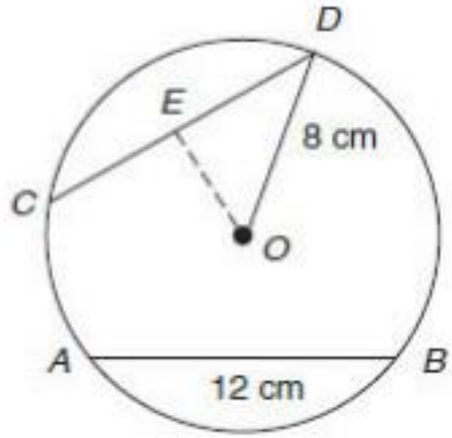

(a) 22

(b) 31

(c) 32

(d) 26

4. If $\overline{AB} = \overline{CD}$ and $AB$ = 12 cm and '$O$' is the center of the circle, $OD$ = 8 cm, $OE \perp CD$, then length of $OE$ is



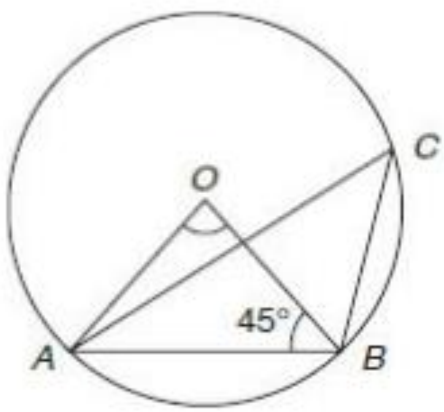

(a) 2 cm

(b) $2\sqrt{7}$ cm

(c) $2\sqrt{11}$ cm

(d) None of these

5. In the given figure, $O$ is the center of the circle. $ABO = 45°$. Find the value of $ACB$



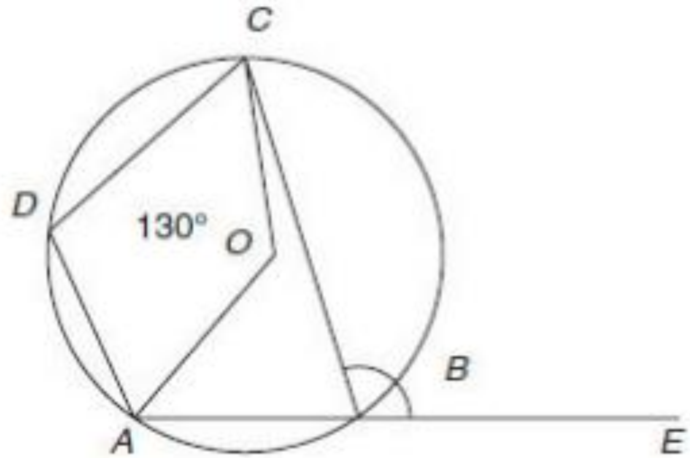

(a) 60°

(b) 75°

(c) 90°

(d) None of these

6. In the given figure, $AOC = 130°$, where $O$ is the center. Find $\angle CBE$



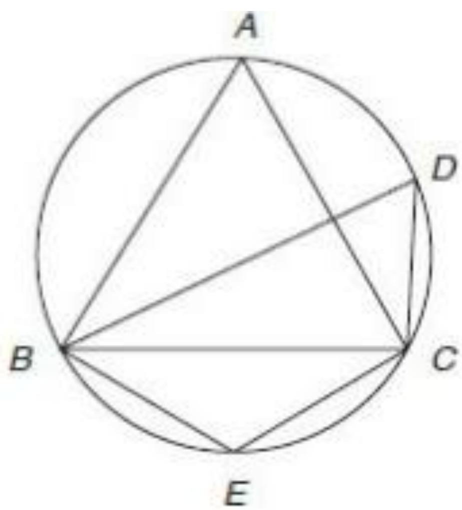

(a) 100°

(b) 70°

(c) 115°

(d) 130°

7. In the given figure, $\Delta ABC$ is an equilateral triangle. Find $\angle BEC$



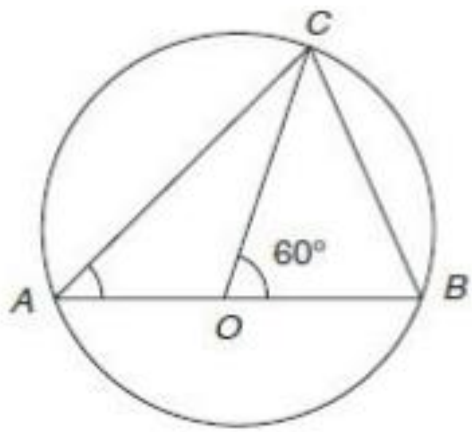

(a) 120°

(b) 60°

(c) 80°

(d) None of these

8. In the given figure, $COB = 60°$, $AB$ is the diameter of the circle. Find $\angle ACO$



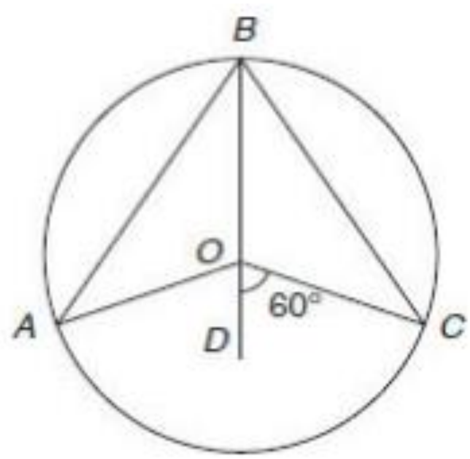

(a) 20°

(b) 30°

(c) 35°

(d) 40°

9. $O$ is the center of the circle; line segment $BOD$ is the angle bisector $\angle AOC$, and $\angle COD = 60°$. $\angle ABC$.



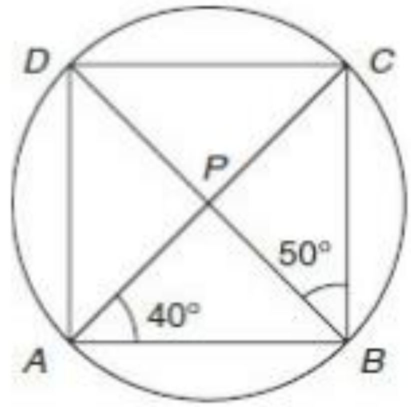

(a) 30°

(b) 40°

(c) 50°

(d) 60°

10. In the given figure, *ABCD* is a cyclic quadrilateral and the diagonals bisect each other at *P*. If $\angle CBD = 50°$ and $\angle CAB = 40°$, then find $\angle BCD$.




(a) 60°

(b) 75°

(c) 90°

(d) 105°

11. Two equal circles of radius 6 cm intersect each other such that each passes through the center of the other. The length of the common chord is:

(a) $2\sqrt{3}$ cm

(b) $6\sqrt{3}$ cm

(c) $2\sqrt{2}$ cm

(d) 8 cm

12. The length of the chord of a circle is 6 cm and perpendicular distance between center and the chord is 4 cm. Then the diameter of the circle is equal to:

(a) 12 cm

(b) 10 cm

(c) 16 cm

(d) 8 cm

13. The distance between two parallel chords of length 6 cm each in a circle of diameter 10 cm is

(a) 8 cm

(b) 7 cm

(c) 6 cm

(d) 5.5 cm

14. The length of the common chord of two intersecting circles is 24. If the diameters of the circles are 30 cm and 26 cm, then the distance between the centers of the circles (in cm) is

(a) 13

(b) 14

(c) 15

(d) 16

15. If two equal circles whose centers are $O$ and $O'$, intersect each other at the points $A$ and $B$. $OO'$ = 6 cm and $AB$ = 8 cm, then the radius of the circles is

(a) 5 cm

(b) 8 cm

(c) 12 cm

(d) 14 cm

16. Chords $BA$ and $DC$ of a circle intersect externally at $P$. If $AB$ = 7 cm, $CD$ = 5 cm and $PC$ = 1 cm, then the length of $PB$ is

(a) 11 cm

(b) 10 cm

(c) 9 cm

(d) 8 cm

17. Two circles touch each other internally. Their radii are 3 cm and 4 cm. The biggest chord of the greater circle which is outside the inner circle is of length

(a) $2\sqrt{3}$ cm

(b) $3\sqrt{2}$ cm

(c) $4\sqrt{3}$ cm

(d) $4\sqrt{2}$ cm

18. If the radii of two circles are 8 cm and 4 cm and the length of the transverse common tangent is 13 cm, then the distance between the two centers is

(a) $\sqrt{313}$ cm

(b) $\sqrt{125}$ cm

(c) $5\sqrt{2}$ cm

(d) $\sqrt{135}$ cm

19. The radii of two circles are 9 cm and 4 cm, the distance between their centers is 13 cm. Then the length of the transverse common tangent is

(a) 12 cm
(b) $12\sqrt{2}$ cm

(c) 5 cm

(d) 15 cm

20. The radii of two circles are 9cm and 4cm, the distance between their centers is 13cm. Then the length of the direct common tangent is

(a) 12 cm

(b) $12\sqrt{2}$ cm

(c) 5 cm

(d) 15 cm

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**ANSWER KEY**

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1. (a)
2. (a)
3. (c)
4. (b)
5. (d)
6. (c)
7. (a)
8. (b)
9. (d)
10. (c)
11. (b)

12. (b)
13. (a)
14. (b)
15. (a)
16. (b)
17. (c)
18. (a)
19. (c)
20. (a)

**Solutions and Shortcuts**

1. Area = $\pi r2 = \pi \times 52 = 25\pi$

2. Circumference = $2\pi$ ´ × 5 = $10\pi$

3. As $OC \perp AB$

   $AC = BC$

   $x + 6 = 2x - 4$

   $x = 10$

   $AB = x + 6 + 2x - 4 = 3x + 2 = 30 + 2 = 32$

4. If $\overline{AB} = \overline{CD}$

   Then $AB = CD = 12$ cm

   If $CD = 12$, then $CE = DE = 6$cm

   $OE = \sqrt{8^2 - 6^2} = \sqrt{28} = 2\sqrt{7}$ cm

5. $AO = BO$

   $ABO = BAO = 45°$

$AOB = 180° - (45° + 45°) = 90°$

$$\angle ACB = \frac{\angle AOB}{2} = \frac{90^\circ}{2} = 45^\circ$$

6. $\angle ABC = \frac{130^\circ}{2} = 65^\circ \quad \angle CBE = 180^\circ - 65^\circ = 115^\circ$

7. $BAC = 60°$

$$\angle BEC = 180^\circ - \angle BAC = 180^\circ - 60^\circ = 120^\circ$$

8. $\angle COB = 60^\circ$

$$\angle AOC = 180^\circ - 60^\circ = 120^\circ$$

$$\angle CAO = \frac{60^\circ}{2} = 30^\circ$$

$$\angle ACO = 180^\circ - (120^\circ + 30^\circ) = 180^\circ - 150^\circ = 30^\circ$$

9. $\angle COD = 60^\circ$

$$\angle AOC = 2 \times 60^\circ = 120^\circ$$

$$\angle ABC = \frac{120^\circ}{2} = 60^\circ$$

10. $\angle CDB = \angle CAB = 40^\circ$

In $\Delta BDC = \angle BCD + 50^\circ + 40^\circ = 180^\circ$

$$\angle BCD = 90^\circ$$

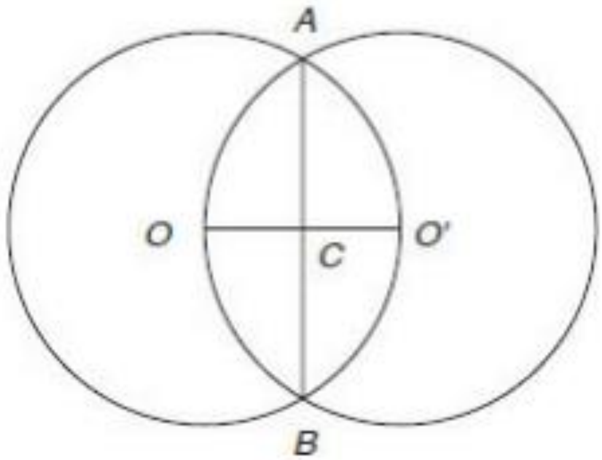
11. $OO' = 6$ cm

$OC = 3$ cm

$OA = 6$ cm

$\therefore AC = \sqrt{6^2 - 3^2} = \sqrt{36 - 9} = \sqrt{27} = 3\sqrt{3}$ cm

$\therefore AB = 6\sqrt{3}$ cm




12. $AC = CB = 3$ cm

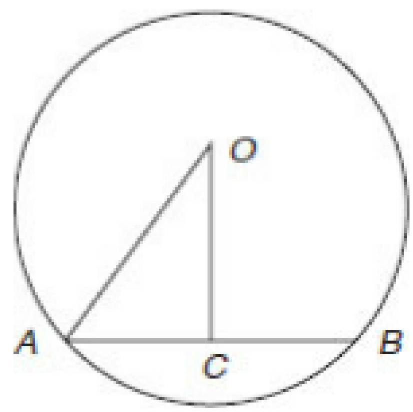
$OC = 4$ cm

$\therefore OA = \sqrt{OC^2 + CA^2}$

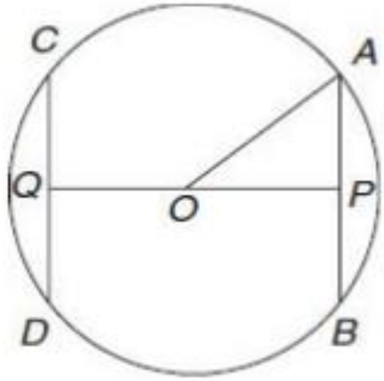
$= \sqrt{4^2 + 3^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$ cm

Diameter = 10 cm




13.



$AB = CD$

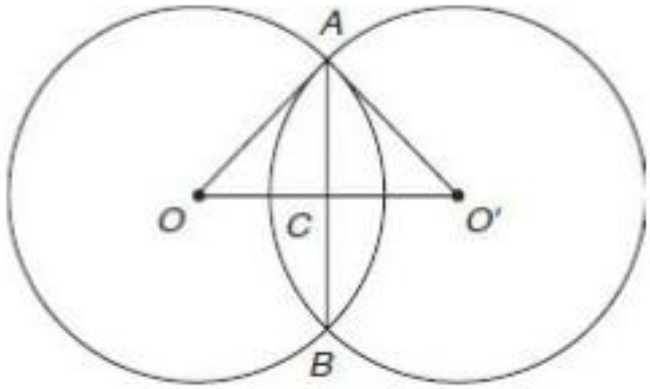
$OP = OQ$

From $\Delta OAP$

$OP = \sqrt{OA^2 - AP^2} = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ cm

$\therefore QP = 2 \times OP = 8$ cm

14. $AC = CB = 12$ cm

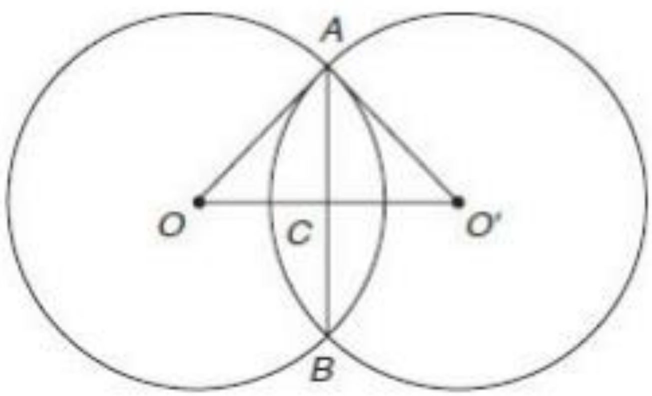



$OC = \sqrt{15^2 - 12^2} = \sqrt{225 - 144} = \sqrt{81} = 9$ cm

$O'C = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5$ cm

$\therefore OO' = 9 + 5 = 14$ cm

15.



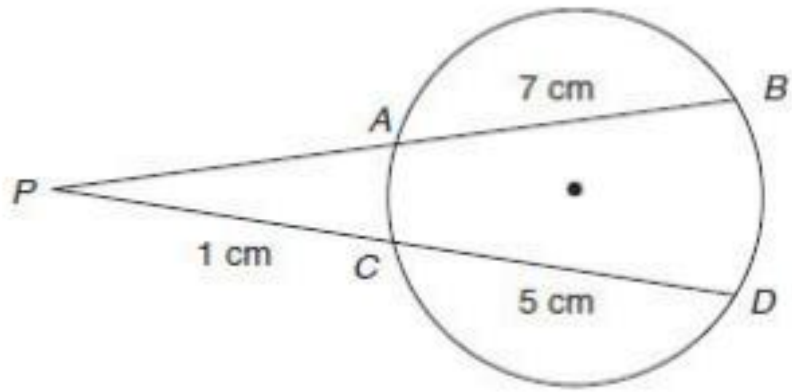
$AB = 8$ cm

$AC = BC = 4$ cm

$OC = CO' = 3$ cm

$\therefore OA = \sqrt{OC^2 + CA^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$ cm

16.




$AB = 7$ cm, $CD = 5$ cm

$PC = 1$ cm, $PA = x$ cm

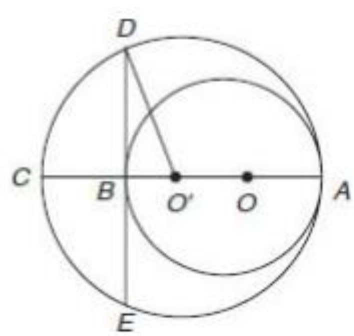
$PA \times PB = PC \times PD$

$\Rightarrow x(x + 7) = 6 \times 5$

By solving, we get $x = 3$ cm

$PB = 3 + 7 = 10$ cm

17.



$O'A = 4$ cm

$AB = 6$ cm

$O'B = AB - O'A = 6 - 4 = 2$ cm

$BD = \sqrt{4^2 - 2^2} = 2\sqrt{3}$ cm

$DE = 4\sqrt{3}$ cm

18. Let the distance between the centers be $x$ cm.

$\Rightarrow 13 = \sqrt{x^2 - (8+4)^2}$

$\Rightarrow 169 = x^2 - 144$

$\Rightarrow x^2 = 169 + 144 = 313$

$\Rightarrow x = \sqrt{313}$ cm

19. Transverse common tangent

$= \sqrt{(\text{Distance between centres})^2 - (r_1 + r_2)^2}$

$= \sqrt{13^2 - 12^2} = \sqrt{25} = 5$ cm

20. Direct common tangent

$= \sqrt{(13)^2 - (9-4)^2} = \sqrt{169 - 25} = \sqrt{144} = 12$ cm

**STAR**

Sum of angles of a star = $(2n - 8) \times \pi/2 = (n - 4)\pi$

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## PART II: MENSURATION

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The following formulae hold true in the area of mensuration:

***1. Cuboid***

A cuboid is a three-dimensional box. It is defined by the virtue of its length $l$, breadth $b$ and height $h$. It can be visualised as a room which has its length, breadth and height different from each other.

(a) Total surface area of a cuboid = $2(lb + bh + lh)$

(b) Volume of the cuboid = $lbh$

***2. Cube of side s***

A cube is a cuboid which has all its edges equal i.e. length = breadth = height = $s$.

(a) Total surface area of a cube = $6s2$.

(b) Volume of the cube = $s3$.

***3. Prism***

A prism is a solid which can have any polygon at both its ends. Its dimensions are defined by the dimensions of the polygon at its ends and its height.

(a) Lateral surface area of a right prism = perimeter of base × height

(b) Volume of a right prism = area of base × height

(c) Whole surface of a right prism = lateral surface of the prism + the area of the two plane ends

***4. Cylinder***

A cylinder is a solid which has both its ends in the form of a circle. Its dimensions are defined in the form of the radius of the base ($r$) and the height $h$. A gas cylinder is a close approximation of a cylinder.

(a) Curved surface of a right cylinder = $2\pi rh$, where $r$ is the radius of the base and $h$ the height.

(b) Whole surface of a right circular cylinder = $2\pi rh + 2\pi r2$

(c) Volume of a right circular cylinder = $\pi r2h$

***5. Pyramid***

A pyramid is a solid which can have any polygon as its base and its edges converge to a single apex. Its dimensions are defined by the dimensions of the polygon at its base and the length of its lateral edges which lead to the apex. The Egyptian pyramids are examples of pyramids.

(a) Slant surface of a pyramid = 1/2 × perimeter of the base × slant height

(b) Whole surface of a pyramid = slant surface + area of the base

(c) Volume of a pyramid = $\frac{\text{area of the base}}{3}$ × height

***6. Cone***

A cone is a solid which has a circle at its base and a slanting lateral surface that converges at the apex. Its dimensions are defined by the radius of the base ($r$), the height ($h$) and the slant height ($l$). A structure similar to a cone is used in ice-cream cones.

(a) Curved surface of a cone = $\pi rl$ where $l$ is the slant height

(b) Whole surface of a cone = $\pi rl + \pi r2$

(c) Volume of a cone = $\frac{\pi r^2 h}{3}$

***7. Sphere***

A sphere is a solid in the form of a ball with radius $r$.

(a) Surface area of a sphere = $4\pi r2$

(b) Volume of a sphere = $\frac{4}{3}\pi r3$

***8. Frustum of a pyramid***

When a pyramid is cut, the left-over part is called the frustum of the pyramid.

(a) Slant surface of the frustum of a pyramid = 1/2 * sum of perimeters of end * slant height

(b) Volume of the frustum of a pyramid = $\frac{k}{3}[E_1 + (E_1 E_2)^{1/2} + E_2]$, where $k$ is the height of the frustum and $E_1$, $E_2$ the areas of the ends

***9. Frustum of a cone***

When a cone is cut, the left-over part is called the frustum of the cone.

(a) Slant surface of the frustum of a cone = $\pi(r_1 + r_2)l$ where $l$, is the slant height

(b) Volume of the frustum of a cone = $\frac{\pi}{3} k(r_1^2 + r_1 r_2 + r_2^2)$

Where 'k' is the height of the frustum.

## WORKED-OUT PROBLEMS

**Problem 11.1** A right triangle with hypotenuse 10 inches and other two sides of variable length is rotated about its longest side thus giving rise to a solid. Find the maximum possible volume of such a solid.

(a) $(250/3)\pi \text{in}^3$

(b) $(160/3)\pi \text{in}^3$

(c) $325/3\pi \text{in}^3$

(d) None of these

**Solution** Most of the questions like this that are asked in the CAT will not have figures accompanying them. Drawing a figure takes time, so it is always better to strengthen our imagination. The beginners can start off by trying to imagine the figure first and trying to solve the problem. They can draw the figure only when they do not arrive at the right answer and then find out where exactly they went wrong. The key is to spend as much time with the problem as possible trying to understand it fully and analysing the different aspects of the same without investing too much time on it.

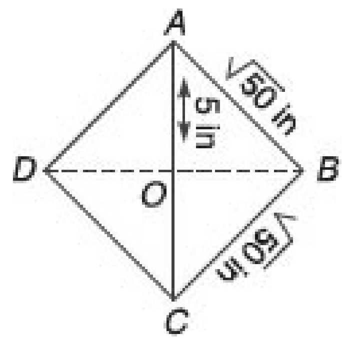
Let's now look into this problem. the key here lies in how quickly you are able to visualise the figure and are able to see that

(i) the triangle has to be an isosceles triangle,

(ii) the solid, thus, formed is actually a combination of two cones,

(iii) the radius of the base has to be the altitude of the triangle to the hypotenuse.

After you have visualised, this comes the calculation aspect of the problem. this is one aspect where you can score over others.




In this question, the figure will be somewhat like this (as shown alongside) with triangles *ABC* and *ADC* representing the cones and *AC* being the hypotenuse around which the triangle *ABC* revolves. Now that the area has to be maximum with *AC* as the hypotenuse, we must realise that *ADB* has to be an isosceles triangle, which automatically makes *BCD* an isosceles triangle too. the next step is to calculate the radius of the base, which is essentially the height of the triangle *ABC*. to find that, we have to first find *AB*. We know

$$AC2 = AB2 + BC2$$

For triangle to be one with the greatest possible area, *AB* must be equal to *BC*, that is, $AB = BC = \sqrt{50}$, since $AO = 1/2AC = 5$ inches.

Now take the right angle triangle *ABO*, *BO* being the altitude of triangle *AOC*.

By Pythagoras theorem, $AB^2 = AO^2 + BO^2$, so $BO^2 = 25$ inches.

The next step is to find the volume of the cone $ABD$ and multiply it with two to get the volume of the whole solid.

Volume of the cone $ADB = \frac{\pi}{3} \times BO^2 \times AO = \frac{125\pi}{3}$.

Therefore volume of the solid $ABCD = 2 \times \frac{125\neq}{3} = \frac{250\neq}{3}$.

**Problem 11.2** A right circular cylinder is to be made out of a metal sheet such that the sum of its height and radius does not exceed 9 cm can have a maximum volume of

(a) 54 p $cm^2$

(b) 108 p $cm^2$

(c) 81 p $cm^2$

(d) None of these

**Solution** Solving this question requires the knowledge of ratio and proportion also. To solve this question, one must know that for $a^2b^3c^4$ to have the maximum value when $(a + b + c)$ is constant, $a$, $b$ and $c$ must be in the ratio 1: 2: 3.

Now let's look at this problem.

Volume of a cylinder = $\pi r^2 h$.

If you analyse this formula closely, you will find that $r$ and $h$ are the only variable term. So for volume of the cylinder to be maximum, $r^2h$ has to be maximum under the condition that $r + h = 9$. By the information given above, this is possible only when $r: h = 2: 1$, that is, $r = 6$, $h = 3$. So,

Volume of the cylinder = $\pi r^2 h$

$= \pi \times 6^2 \times 3$

= **108π**

**Problem 11.3** There are five concentric squares. If the area of the circle inside the smallest square is 77 square units and the distance between the corresponding corners of consecutive squares is 1.5 units, find the difference in the areas of the outermost and innermost squares.

**Solution** Here again the ability to visualise the diagram would be the key. Once you gain expertise in this aspect, you will be able to see that the diameter of the circle is equal to the side of the innermost square, that is

$\pi r2 = 77$

or $r = 3.5\sqrt{2}$

or $2r = 7\sqrt{2}$

Then the diagonal of the square is 14 sq units

which means the diagonal of the fifth square would be 14 + 12 units = 26

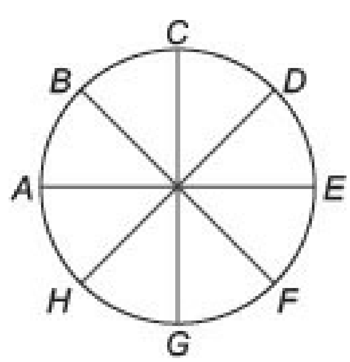
Which means the side of the fifth square would be $13\sqrt{2}$.

therefore, the area of the fifth square = 338 sq units.

Area of the first square = 98 sq units.

Hence, the difference would be 240 sq. units.

**Problem 11.4** A spherical pear of radius 4 cm is to be divided into eight equal parts by cutting it in halves along the same axis. Find the surface area of each of the final piece.




(a) $20\,\pi$

(b) 25 $\pi$

(c) 24 $\pi$

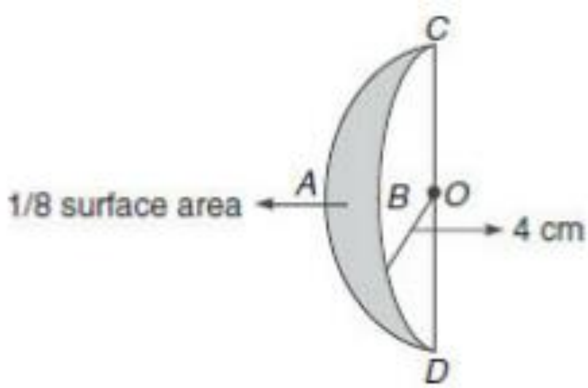
(d) 19 $\pi$

**Solution** The pear after being cut will have eight parts each of same volume and surface area. The figure will be somewhat like the above figure if seen from the top before cutting. After cutting, it looks something like the figure given below.

Now the surface area of each piece = Area *ACBD* + 2 (Area *CODB*).

The darkened surface is nothing but the arc *AB* from side glance which means its surface area is one eighth the area of the sphere, that is, $1/8 \times 4\pi r^2 = (1/2)\pi r^2$.

Now *CODB* can be seen as a semicircle with radius 4 cm.




Therefore, 2 (Area *CODB*) = $2[(1/2)]\pi r^2 = \pi r^2$

$\Rightarrow$ surface area of each piece = $(1/2)\pi r^2 + \pi r^2$

$= (3/2)\pi r^2$

$= 24\pi$

**Problem 11.5** A solid metal sphere is melted and smaller spheres of equal radii are formed. Ten percent of the volume of the sphere is lost in the process. The smaller spheres have a radius, that is $1/9^{th}$ the larger sphere. If ten litres of paint were needed to paint the larger sphere, how many litres are needed to paint all the smaller spheres?

(a) 90

(b) 81

(c) 900

(d) 810

**Solution** questions like this require, along with your knowledge of formulae, your ability to form equations. Stepwise, it will be something like this;

*Step 1:* Assume values

*Step 2:* Find out the volume left

*Step 3:* Find out the number of small spheres possible

*Step 4:* Find out the total surface area of each small spheres as a ratio of the original sphere

*Step 5:* Multiply it by 10

**Step 1:** Let radius of the larger sphere be $R$ and that of smaller ones be $r$.

Then, volume = $\frac{4}{3}\pi r3$ and $\frac{4}{3}\pi r3 = \frac{4}{3}\pi(R/9)3$, respectively for the larger and smaller spheres.

**Step 2:** Volume lost due to melting = $\frac{4}{3}\pi r3 \times \frac{10}{100} = \frac{10}{100}$

$$\text{Volume left} = \frac{4}{3}\pi R^3 \times \frac{90}{100} = \frac{4\pi R^3 \times 0.9}{3}$$

**Step 3:** Number of small spheres possible = volume left/volume of the smaller sphere

$$= \frac{\frac{4}{3}\pi R^3 \times 0.9}{\frac{4}{3}\pi \times (R/9)^3} = 9^3 \times 0.9$$

**Step 4:** Surface area of larger sphere = $4\pi r2$

Surface area of smaller sphere = $4\pi r2 = 4p\,(R/9)2 = \frac{4\pi R^2}{81}$

Surface area of all smaller spheres = number of small spheres × surface area of smaller sphere

$= (9^3 \times .9) \times (4\pi R^2)/81$

$= 8.1 \times (4\pi R^2)$

Therefore, ratio of the surface area is $\frac{[8.1 \times (4\pi R^2)]}{4\pi R^2} = 8.1$

**Step 5:**

8.1 × number of litres = 8.1 × 10 = 81

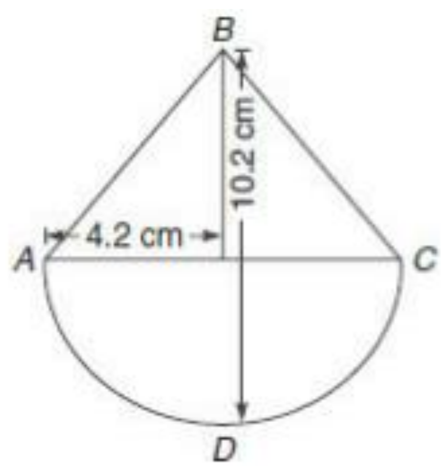
**Problem 11.6** A solid wooden toy in the shape of a right circular cone is mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.

(a) 104 $cm^3$

(b) 162 $cm^3$

(c) 427 $cm^3$

(d) 266 $cm^2$




**Solution** Volume of the cone is given by $-1/3 \times \pi r^2 h$

Here, $r = 4.2$ cm; $h = 10.2 - r = 6$ cm

Therefore, the volume of the cone = $1/3\ \pi \times (4.2)^2 \times 6$ cm

= 110.88 $cm^3$

Volume of the hemisphere = $\frac{1}{2} \times \frac{4}{3} \pi r^3$ = 155.23

Total volume = 110.88 + 155.232 = 266.112

**Problem 11.7** A vessel is in the form of an inverted cone. Its depth is 8 cm and the diameter of its top, which is open, is 10 cm. It is filled with water up to the brim. When bullets, each of which is a sphere of radius 0.5 cm, are dropped into the vessel 1/4 of the water flows out. Find the number of bullets dropped in the vessel.

(a) 50

(b) 100

(c) 150

(d) 200

**Solution** In these types of questions, it is just your calculation skills that are being tested. You just need to take care that while trying to be fast you do not end up making mistakes like taking the diameter to be the radius and so forth. The best way to avoid such mistakes is to proceed systematically. For example, in this problem we can proceed, thus:

Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{200}{3}\pi$ $cm^3$

Volume of all the lead shots = Volume of water that spilled out = $\frac{50}{3}\pi$ $cm^3$

Volume of each lead shot = $\frac{4}{3}\pi r^3 = \frac{\pi}{6}$ $cm^3$

Number of lead shots = (volume of water that spilled out)/(volume of each lead shot)

$$= \frac{\frac{50}{3}\pi}{\frac{\pi}{6}} = \frac{50}{3} \times 6 = 100$$

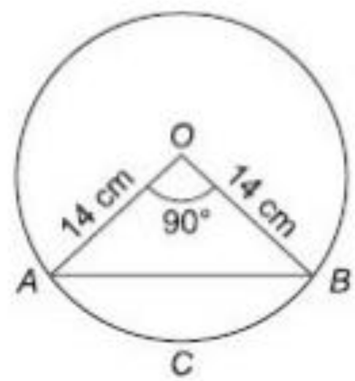
**Problem 11.8** *AB* is a chord of a circle of radius 14 cm. The chord subtends a right angle at the center of the circle. Find the area of the minor segment.

(a) 98 sq cm

(b) 56 sq cm

(c) 112 sq cm

(d) None of these




**Solution** Area of the sector *ACBO* = $\frac{90\pi \times 14^2}{360}$

= 154 sq cm

Area of the triangle *AOB* = $\frac{14 \times 14}{2}$

= 98 sq cm

Area of the segment *ACB* = Area sector *ACBO* – Area of the triangle *AOB* = 56 sq cm

**Problem 11.9** A sphere of diameter 12.6 cm is melted and cast into a right circular cone of height 25.2 cm. Find the diameter of the base of the cone.

(a) 158.76 cm

(b) 79.38 cm

(c) 39.64 cm

(d) None of these

**Solution** In questions like this, do not go for complete calculations. As far as possible, try to cancel out values in the resulting equations.

volume of the sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6.3)^3$

Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{\pi}{3} r^2(25.2)$

Now, volume of the cone = volume of the sphere

Therefore, $r$ (radius of the cone) = 6.3 cm

Hence the diameter = 12.6 cm

**Problem 11.10** A chord $AB$ of a circle of radius 5.25 cm makes an angle of 60° at the center of the circle. Find the area of the major and minor segments. (Take $\pi$ = 3.14)

(a) 168 $cm^2$

(b) 42 $cm^2$

(c) 84 $cm^2$

(d) None of these

**Solution** The moment you finish reading this question, it should occur to you that this has to be an equilateral triangle. Once you realise this, the question is reduced to just calculations.

Area of the minor sector = $\frac{60}{360} \times \pi \times 5.25^2$

= 14.4375 $cm^2$

Area of the triangle = $\frac{\sqrt{3}}{4} \times 5.25^2$ = 11.93 $cm^2$

Area of the minor segment = Area of the minor sector – Area of the triangle = 2.5 $cm^2$

Area of the major segment = area of the circle – Area of the minor segment.

= 86.625 $cm^2$ – 2.5 $cm^2$ = 84.125 $cm^2$

**Problem 11.11** A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

(a) 1 : 2

(b) 2 : 1

(c) 3: 1

(d) None of these

**Solution** Questions of this type should be solved without the use of pen and paper. A good authority over formulae will make things easier.

Volume of the cone = $\frac{\pi r^2 h}{3}$ = volume of a hemisphere = $\frac{2}{3}\pi r^3$πr3,

Height of a hemisphere = radius of its base

So the question is effectively asking us to find out $h/r$.

By the formula above we can easily see that $h/r = 2/1$.

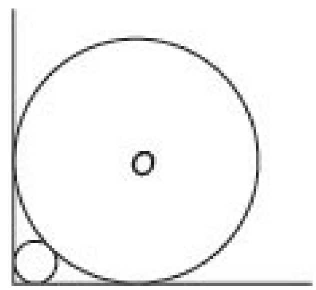
## TRAINING GROUND FOR BLOCK IV

### HOW TO THINK IN PROBLEMS ON BLOCK IV?

In the Back to School section of this block, it has already been mentioned that there is very little use of complex and obscure formulae and results while solving questions on this block.

The following is a list of questions (with solutions) of what has been asked in previous years' CAT questions from this chapter. Hopefully you will realise through this exercise, what I am talking about when I say this. For each of the questions given below, try to solve on your own first, before looking at the solution provided.

1. A circle with radius 2 is placed against a right angle. As shown in the figure below, another smaller circle is placed in the gap between the circle and the right angle. What is the radius of the smaller circle?



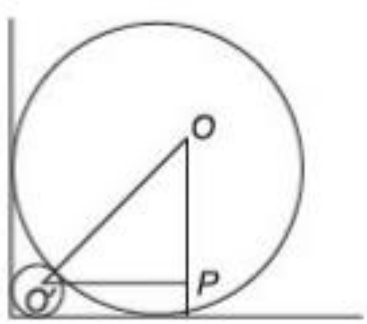

(a) $3 - 2\sqrt{2}$

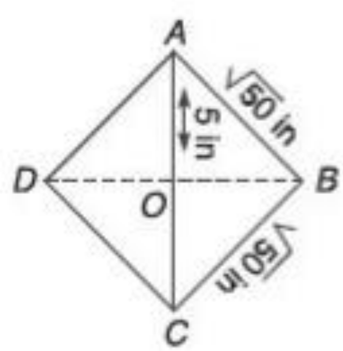
(b) $4 - 2\sqrt{2}$

(c) $7 - 4\sqrt{2}$

(d) $6 - 4\sqrt{2}$

**Solution:** The solution of the above question is based on the following construction.



In the right triangle OO'P,

OP = (2 – r), O'P = (2 – r) and OO' = 2 + r

Where, $r$ is the radius of the smaller circle.

Using pythagoras theorem:

$(2 + r)^2 = (2 - r)^2 + (2 - r)^2$

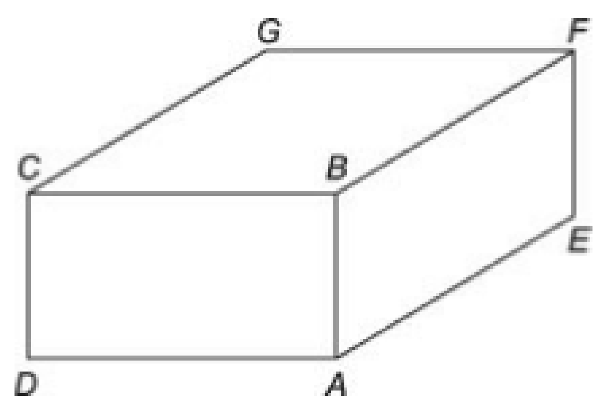
Solving, we get $r = 6 \pm 4\sqrt{2}$

$6 + 4\sqrt{2}$ cannot be correct since the value of $r$ should be less than 2.

***Note:*** The key to solving this question is in the visualisation of the construction. If you try to use complex formulae while solving, your mind unnecessarily gets cluttered. The key to your thinking in this question is:

1. Realise that you only have to use length-measuring formulae. Hence, put all the angle-measurement formulae into the back-seat.
2. A quick mental search of the length-measuring formulae available for this situation will narrow down your mind to the pythagoras theorem.
3. The key then becomes the construction of a triangle (right-angled of course) where the only unknown is $r$.

2. *ABCDEFGH* is a cube. If the length of the diagonals *DF*, *AG* and *CE* are equal to the sides of a triangle, then the circum-radius of that triangle would be



(a) Equal to the side of the cube

(b) $\sqrt{3}$ times the side of the cube

(c) $1/\sqrt{3}$ times the side of the cube

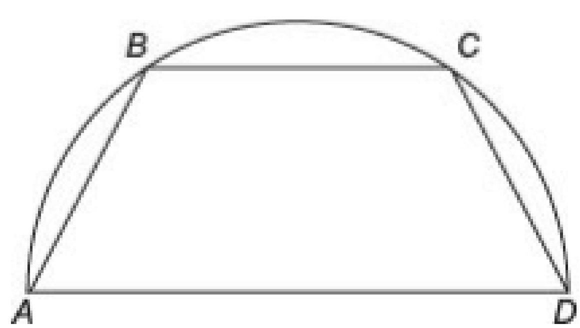
(d) Indeterminate

**Solution:** If we assume the side of the cube to be $a$, the triangle will be an equilateral triangle with side $a\sqrt{3}$. (we get this using pythagoras theorem). Also, we know that the circum-radius of an equilateral triangle is $1/\sqrt{3}$ times the side of the triangle.

Hence, in this case the circum-radius would be $a$—equal to the side of the cube.

(Again the only formula used in this question would be the pythagoras theorem.)

3. On a semicircle (diameter $AD$), chord $BC$ is parallel to the diameter $AD$. Also, $AB = CD = 2$, while $AD = 8$, what is the length of $BC$?



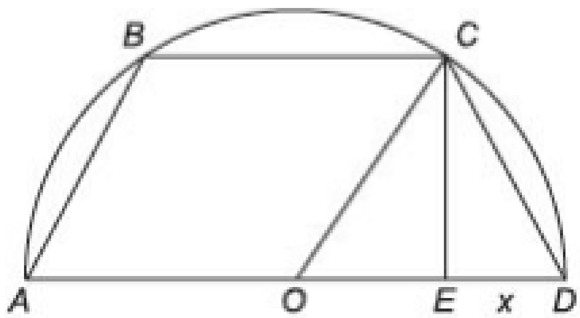

(a) 7.5

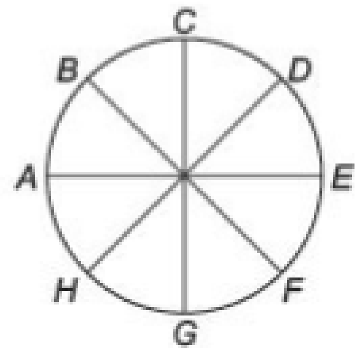
(b) 7

(c) 7.75

(d) None of these

**Solution:** Think only of length-measuring formulae (pythagoras theorem is obvious in this case).




If we can find the value of $x$, we will get the answer for $BC$ as $AD - 2x$. Hence, we need to focus our energies in finding the value of $x$.

The construction above gives us two right-angled triangles ($OEC$ and $DEC$).

In $\Delta OCE$, $OC = 4$ (radius) and $OE = (4 - x)$, Then: $(CE)2 = 8x - x_2$. (Using pythagoras Theorem)

Then in triangle $CED$:

$(8x - x_2) + x_2 = 22$

Hence, $x = 0.5$

Thus, $BC = 8 - 2 \times 0.5 = 7$

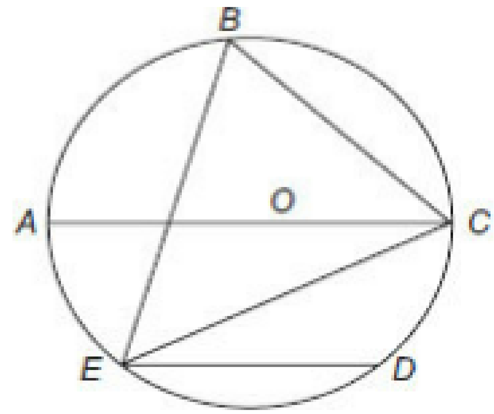
4. In the given circle, $AC$ is the diameter of the circle. $ED$ is parallel to $AC$. $\angle CBE = 65°$, find $\angle DEC$.

(a) 35°

(b) 55°

(c) 45°

(d) 25°




**Solution:** Obviously, this question has to be solved using only angle measuring tools. Further from the figure, it is obvious that we have to use angle measurement tools related to arcs of circles.

Reacting to the 65° information in the question above, you will get $\angle EOC$ = 130° (since, the angle at the center of the circle is twice the angle at any point of the circle).

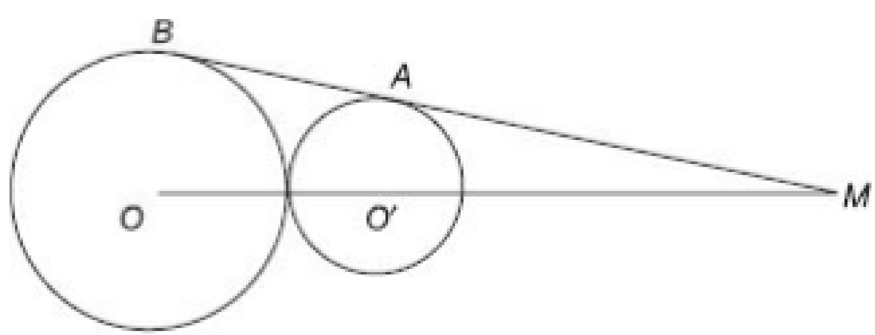
Hence, $\angle AOE$ = 180 – 130 = 50°.

This will be the same as $\angle COD$ since the minor arc *AE* = minor arc *CD*.

Also, $\angle$DEC = 1/2 × $\angle$COD

Hence, $\angle$DEC = 25°

***Directions for Questions 5 to 7:*** In the figure below, *X* and *Y* are circles with centers *O* and *O′* respectively. *MAB* is a common tangent. The radii of *X* and *Y* are in the ratio 4: 3 and *OM* = 28 cm.



5. What is the ratio of the length of $OO'$ to that of $O'M$?

(a) 1 : 4

(b) 1 : 3

(c) 3 : 8

(d) 3 : 4

6. What is the radius of circle $X$?

(a) 2 cm

(b) 3 cm

(c) 4 cm

(d) 5 cm

7. The length of $AM$ is

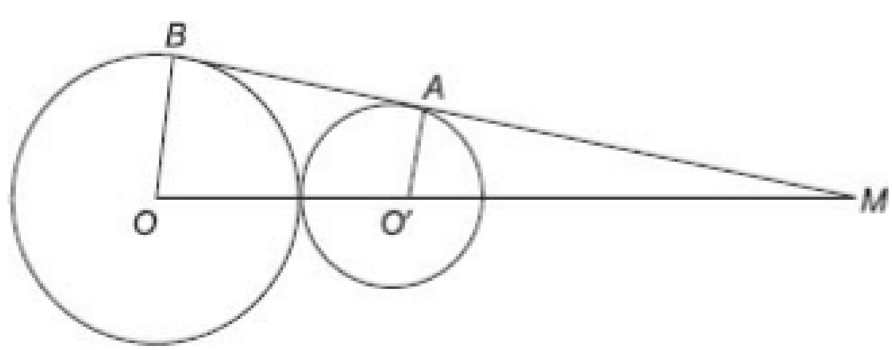
(a) $8\sqrt{3}$ cm

(b) $10\sqrt{3}$ cm

(c) $12\sqrt{3}$ cm

(d) $14\sqrt{3}$ cm

**Solution:** Construct $OB$ and $O'A$ as shown below.



In this construction, it is evident that the two right-angled triangles formed are similar to each other, i.e $\Delta OBM$ is similar to $\Delta O'AM$.

Hence, $OM$: $O'M$ = 4: 3 (since $OB$: $O'A$ = 4: 3)

Also, $OM$ = 28 cm, ∴$O'M$ = 21 cm → $OO'$ = 7 cm. Hence, the radius of circle $X$ is 4 cm (Answer to $Q$. 6).

5. Also: $OO'$= 7 and $O'M$ = 21. Hence, required ratio = 1: 3

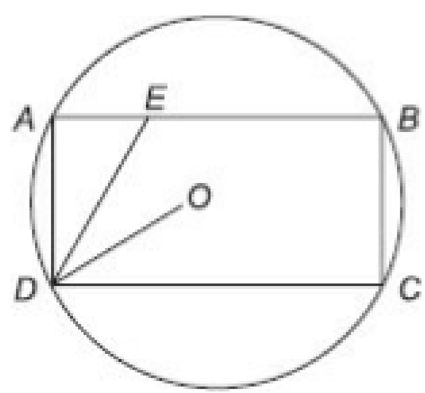
7. $AM$ can be found easily using pythagoras theorem.

$AM^2 = 21^2 - 3^2 = 432$

∴ AM = $\sqrt{432} = 12\sqrt{3}$

(**Note**: Only similarity of triangles and pythagoras theorem was used here.)

8. In the figure, $ABCD$ is a rectangle inscribed inside a circle with center $O$. Side $AB$> Side $BC$. The ratio of the area of the circle to the area of the rectangle is p:$\sqrt{3}$. Also, $\angle ODC = \angle ADE$. Find the ratio $AE$: $AD$.




(a) 1 :

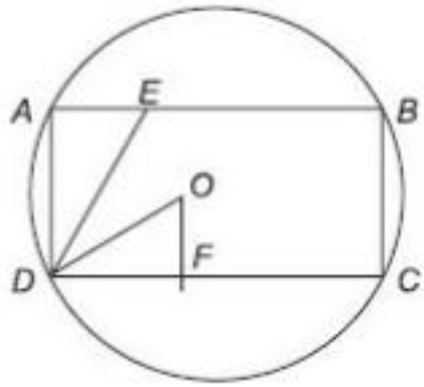
(b) 1 :

(c) 2 : 1

(d) 1 : 2

**Solution:** In my experience, questions involving ratios of length typically involve the use of similar triangles. This question is no different.

Make the following construction:




$\Delta OFD$ is similar to $\Delta AED$. Hence, the required ratio $AE: AD = OF/FD$

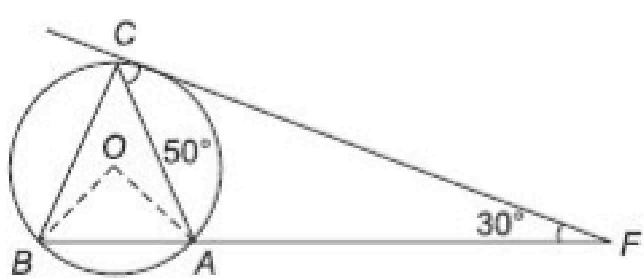
But, $OF$ = ½ side $BC$ while $FD$ = ½ side $CD$.

Hence, we need the ratio of the side of the rectangle $BC: DC$ (This will give the required answer.)

From this point, you can get to the answer through a little bit of unconventional thinking.

The ratio of the area of the circle to that of the rectangle is given as p:$\sqrt{3}$. Hence, it is obvious that one of the sides has to have a component in it. Hence, options (b) and (d) can be rejected. Also, the required ratio has to be less than 1, hence, option (a) is correct.

9. Find $\angle BOA$.




(a) 100°

(b) 150°

(c) 80°

(d) Indeterminate

**Solution:** Obviously this question has to be solved using angle measurement tools.

In order to measure $\angle BOA$, you could either try to use theorems related to the angle subtended by arcs of a circle or solve using the isosceles $\angle BOA$.

With this thought in mind, start reacting to the information in the question.

$\angle CAF = 100°$. Hence $\angle BAC = 80°$

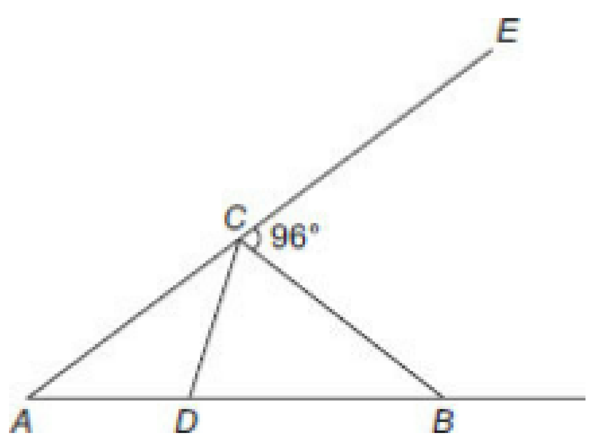
Also, $\angle OCA = (90 - ACF) = 90 - 50 = 40° = \angle OAC$ (Since the triangle $OCA$ is isosceles)

Hence, $\angle OAB = 40°$

In isosceles $\angle OAB$, $\angle OBA$ will also be 40°.

Hence, $\angle BOA = 180 - 40 - 40 = 100°$

10. In the figure $AD = CD = BC$ and $\angle BCE = 96°$. How much is $\angle DBC$?



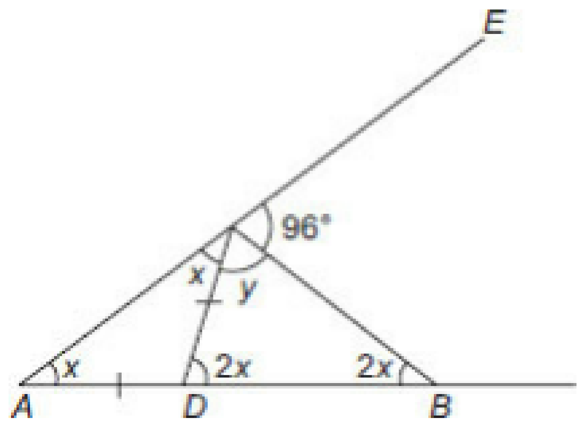

(a) 32°

(b) 84°

(c) 64°

(d) Indeterminate

**Solution:** Bring out your angle measuring formulae and start reacting to the information.




From the figure above, it is clear that

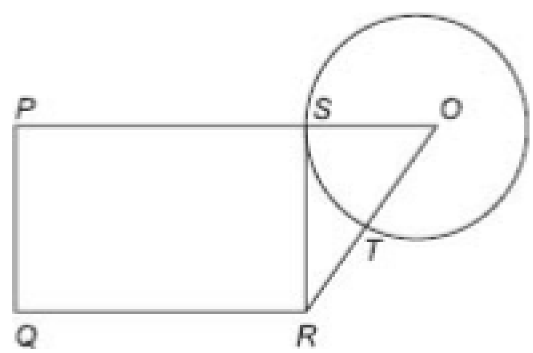
$x + y = 180 - 96 = 84°$

Also $4x + y = 180°$

Solving, we get $x = 32°$

Hence, $\angle DBC = 2x = 64°$.

11. *PQRS* is a square. *SR* is a tangent (at point *S*) to the circle with center *O* and *TR* = *OS*. Then the ratio of area of the circle to the area of the square is



(a) $p/3$

(b) 11/7

(c) $3/p$

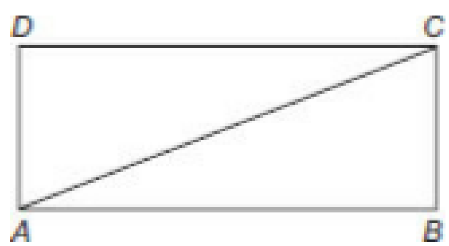
(d) 7/11

**Solution:** Looking at the options, we can easily eliminate Option (b) and (d), because in the ratio of the area of the circle to the area of the triangle we cannot eliminate $p$ and hence, the answer should contain $p$.

Further the question is asking for the ratio $\frac{\text{Area of the circle}}{\text{Area of the square}}$ so, $p$ should be in the numerator.

Hence, (a) is the correct answer.

12. In the adjoining figure, $AC + AB = 5AD$ and $AC - AD = 8$. Then the area of the rectangle $ABCD$ is




(a) 36

(b) 50

(c) 60

(d) Cannot be answered

**Solution:** Think only of length-measuring formulae (pythagoras theorem is obvious in this case).

There is no need of forming equations if you have the knowledge of some basic triplets like 3, 4, 5; 5, 12, 13, etc.

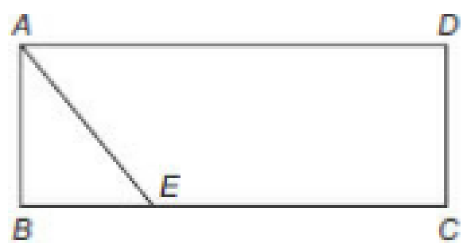
Now looking at the equations given in the question and considering $\Delta CDA$ where $AD$ is the height and $AC$ is the hypotenuse, we will easily get,

$AC - AD = 13 - 5 = 8$ and

$AC + AB = 13 + 12 = 25$, i.e. $5AD$

Hence, area of rectangle is length × breadth, i.e. $5 \times 12 = 60$.

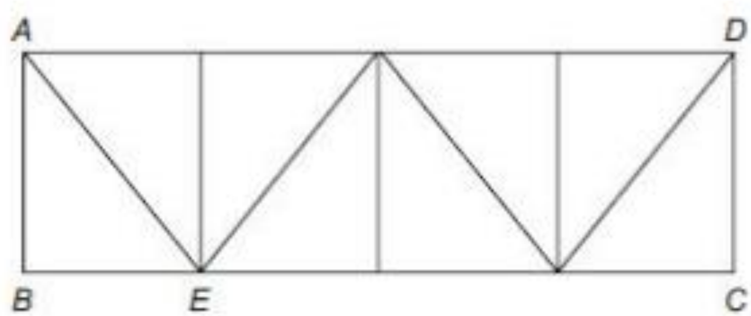
13. In the figure given below, $ABCD$ is a rectangle. The area of the isosceles right triangle $ABE$ = 7 cm2, $EC = 3(BE)$. The area of $ABCD$ (in cm2) is




**Solution:** The key to solve this question is in the visualisation of the construction and the equations.

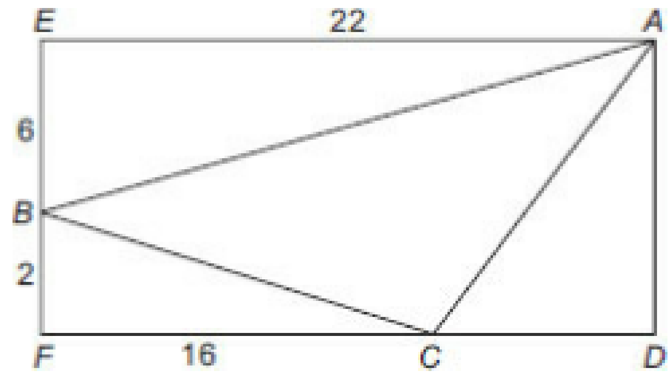
It is given that $EC = 3\ (BE)$; from this we can conclude that the whole side $BC$ can be divided in four equal parts of measurement $BE$.

Now look at this construction




Each part is of equal area as 7 cm2. Hence, $7 \times 8 = 56$ cm2.

14. In the given figure, *EADF* is a rectangle and *ABC* is a triangle whose vertices lie on the sides of *EADF*. *AE* = 22, *BE* = 6 *CF* = 16 and *BF* = 2. Find the length of the line joining the mid–points of the side *AB* and *BC*.




(a) 4

(b) 5

(c) 3.5

(d) None of these

**Solution:** Think only of length-measuring formulae and triplets.

*EA* = 22 and *FC* = 16, So, *CD* = 6

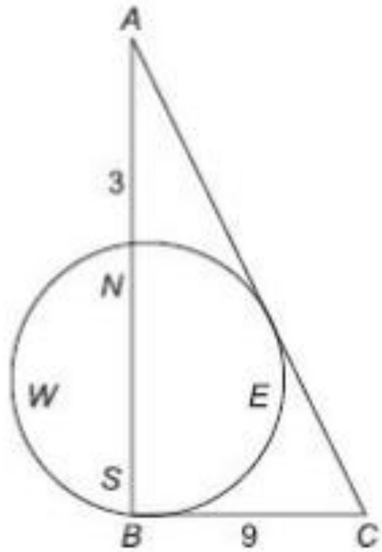
*EF* = 8 So, *AD* is also 8. Now using the triplet 6, 8, 10 based on basic triplet 3, 4, 5, we will get that *AC* = 10.

The line joining the mid-points of the sides *AB* and *BC* will be exactly half the side *AC* (using similar triangles).

Hence, 5 is the correct answer.

15. A certain city has a circular wall around it and this wall has four gates pointing North, South, East and West. A house stands outside the city, three km north of the North gate and it can just be seen from a point nine km east of south gate. What is the diameter of the wall that surrounds the city?

**Solution:** Make this construction

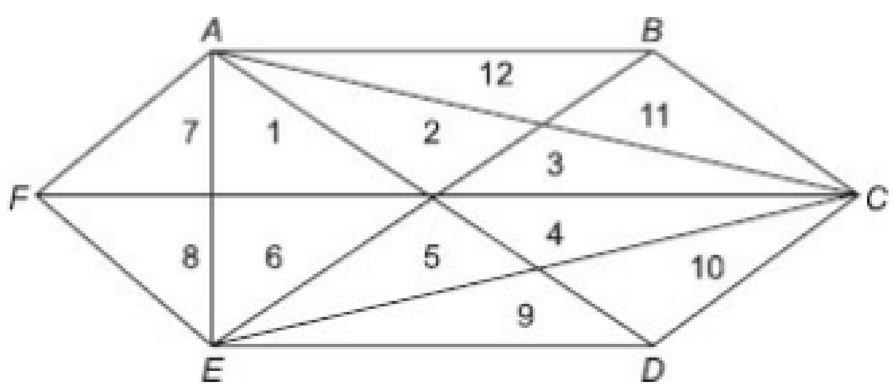



Given $AN = 3$, $BC = 9$ and $\angle B$ is 90°. Now according to conventional method, we have to use tangent theorem to get to the answer, which will be very long.

Instead if we were to use the pythagorean triplets again, we would easily see the (9, 12, 15) triplet which is based on the basic triplet 3, 4, 5. Here $BC = 9$. Hence, $AC = 15$ and $AB = 12$. Therefore, the diameter will be 12 – 3 = 9 km.

16. Let *ABCDEF* be a regular hexagon: What is the ratio of the area of the triangle *ACE* to that of the hexagon *ABCDEF*?

**Solution:** Make the following construction:



Now we have to find the ratio $\frac{\text{Area of } ACE}{\text{Area of } ABCDEF}$.

In order to do so, we use the property of a regular hexagon (that it is a combination of six equilateral triangles).

We can easily see that we have divided all six equilateral triangles into two equal parts of the same area.

If we number all the equal areas as 1, 2, 3 ---- 12 as shown in the above construction, we will get the answer as

$$\frac{\text{Sum of area of triangles } 1+2+3+4+5+6}{\text{Sum of area of triangle } 1+2+3\ldots+12}$$

Hence, ½ is the ratio.

17. Euclid has a triangle in mind. Its longest side has length 20 and another of its side has length 10. Its area is 80. What is the exact length of its third side?

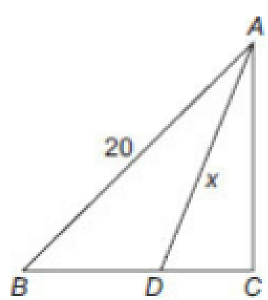
(a) $\sqrt{260}$

(b) $\sqrt{250}$

(c) $\sqrt{240}$

(d) $\sqrt{270}$

**Solution:** The solution of the above question is based on the following construction, where $AB = 20$ and $BD = 10$.



The question is asking for the exact length *AD*, of triangle *ABD*.

Think only of length-measuring formulae (pythagoras theorem is obvious in this case).

If we extend the side *BD* upto a point C, the length *AC* will give the altitude or height of the $\Delta ABD$. Then we will get:

$1/2\ b \times h \Rightarrow 80 \Rightarrow 1/2 \times 10 \times h = 80 \Rightarrow h = 16$, i.e. $AC = 16$.

And now as $\Delta ABC$ is a right-angled triangle, we can easily get the length of *DC* as 2, based on the triplet 12, 16, 20.

Now, if $AC = 16$, $DC = 2$, we can easily get the exact length of *AD* using pythagoras theorem, i.e. $AC = \sqrt{2^2 + 16^2} = \sqrt{260}$.

Hence, (a) is the correct answer.

## GEOMETRY

### LEVEL OF DIFFICULTY (I)

1. A vertical stick 20 m long casts a shadow 10 m long on the ground. At the same time, a tower casts the shadow 50 m long on the ground. Find the height of the tower.

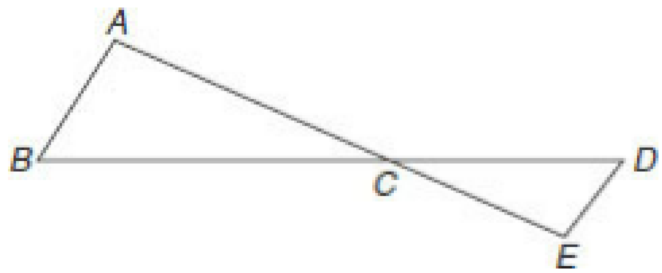
   (a) 100 m

   (b) 120 m

   (c) 25 m

   (d) 200 m

2. In the figure, $\Delta ABC$ is similar to $\Delta EDC$.




If we have $AB = 4$ cm,

$ED = 3$ cm, $CE = 4.2$ and

$CD = 4.8$ cm, find the value of $CA$ and $CB$.

(a) 6 cm, 6.4 cm

(b) 4.8 cm, 6.4 cm

(c) 5.4 cm, 7.2 cm

(d) 5.6 cm, 6.4 cm

3. The area of similar triangles, $ABC$ and $DEF$ are 144 $cm^2$ and 81 $cm^2$ respectively. If the longest side of larger $\Delta ABC$ be 36 cm, then the longest side of smaller $\Delta DEF$ is

(a) 20 cm

(b) 26 cm

(c) 27 cm

(d) 30 cm

4. Two isosceles triangles have equal angles and their areas are in the ratio 16 : 25. Find the ratio of their corresponding heights.

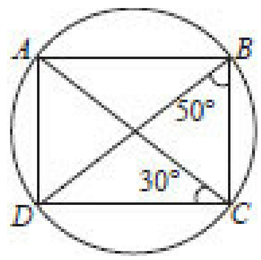
(a) 4/5

(b) 5/4

(c) 3/2

(d) 5/7

5. *ABCD* is a quadrilateral inscribed in a acircle.




$\angle BDC$ is thrice of $\angle BCA$. Find the value of $\angle BAC$.

6. Two poles of height 6 m and 11 m respectively stand vertically upright on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

(a) 12 m

(b) 14 m

(c) 13 m

(d) 11 m

7. The radius of a circle is 9 cm and length of one of its chords is 14 cm. Find the distance of the chord from the center.

(a) 5.66 cm

(b) 6.3 cm

(c) 4 cm

(d) 7 cm

8. The chord $AB$ of a circle is the perpendicular bisector of the chord $CD$ of the same circle. The two chords intersect at point $E$. If $CD = BE = 4$ cm, then what is the diameter of the circle?

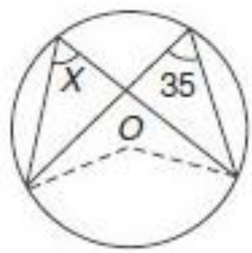
(a) 4 cm

(b) 5 cm

(c) 6 cm

(d) 8 cm

9. If $O$ is the center of circle, find $\angle x$.



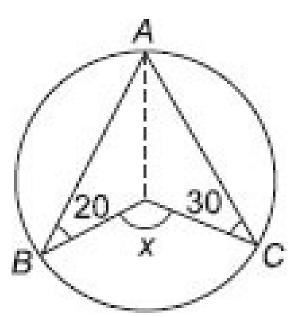

(a) 35°

(b) 30°

(c) 39°

(d) 40°

10. Find the value of $\angle x$ in the given figure.



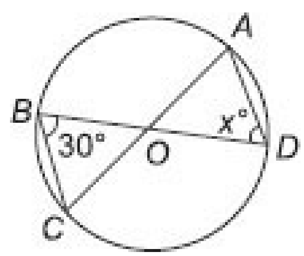
(a) 120°

(b) 130°

(c) 110°

(d) 100°

11. Find the value of $x$ in the figure, if it is given that $AC$ and $BD$ are diameters of the circle.



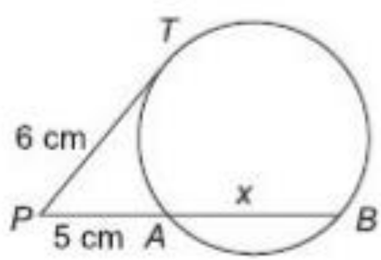

(a) 60°

(b) 45°

(c) 15°

(d) 30°

12. Find the value of $x$ in the given figure.



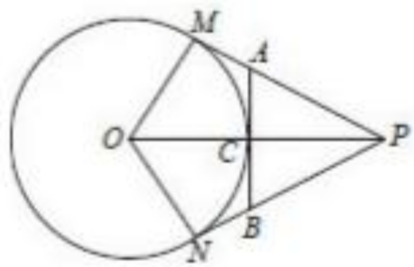

(a) 2.2 cm

(b) 1.6 cm

(c) 3 cm

(d) 2.6 cm

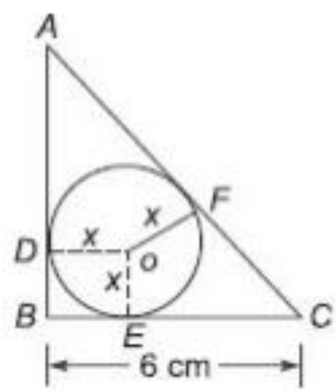
***Directions for Questions 13 and 14:*** *P* is a point outside a circle with center *O* and radius 6 units, such that *OP* = 10 units. Tangents are drawn from *P* to the circle touching it at *M* and *N* (see the figure given below). The line segment *OP* cuts the circle at point *C* and the tangent drawn to the circle at *C* meets *PM* and *PN* at points *A* and *B*, respectively.




13. What is the length of *AP* (in cm)?

14. What is the length (in cm) of in-radius of triangle *APB*?

15. *ABC* is a right angled triangle with *BC* = 6 cm and *AB* = 8 cm. A circle with center *O* and radius *x* has been inscribed in Δ*ABC*. What is the value of *x*?



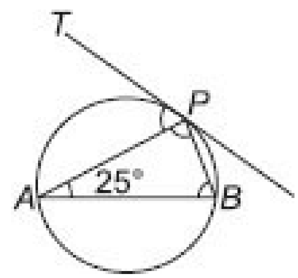

(a) 2.4 cm

(b) 2 cm

(c) 3.6 cm

(d) 4 cm

16. In the given figure, $AB$ is the diameter of the circle and $\angle PAB = 25°$. Find $\angle TPA$.




(a) 50°

(b) 65°

(c) 70°

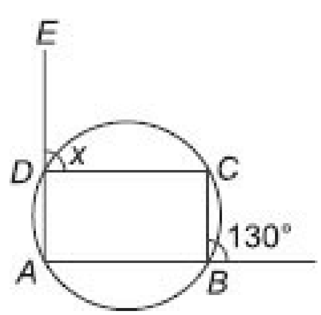
(d) 45°

***Directions for Questions 17 and 18:*** There are two concentric circles of radii 7 cm and 8 cm. $PQ$, a diameter of the larger circle, cuts the smaller circle at $S$ and $T$. A tangent drawn from $Q$ touches the inner circle at $R$.

17. What is the length of $QR$ (in cm)?

18. What is the length of $PR$ (in cm)?

19. In the following figure $A$, $B$, $C$ and $D$ are the concyclic points. Find the value of $x$.



(a) 130°

(b) 50°

(c) 60°

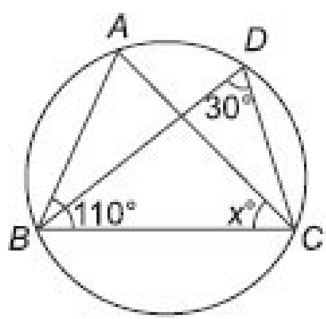
(d) 30°

***Directions for Questions 20 and 21:*** In $\Delta ABC$, $AB$ = 16 cm and $AC$ = 9 cm. $AD$ is the perpendicular drawn from the vertex $A$ to the side $BC$ and the circum-radius of the triangle is 9 cm.

20. Find the length (in cm) of $AD$.

21. What is the length (in cm) of $BC$?

22. In the following figure, find the value of $x$




(a) 40°

(b) 25°

(c) 30°

(d) 45°

23. If $L_1 \parallel L_2$ in the figure below, what is the value of $x$?

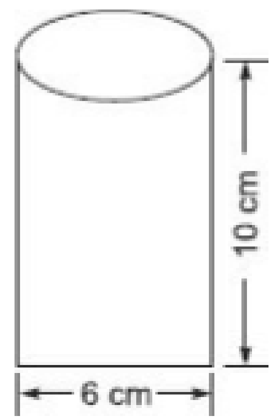
(a) 80°

(b) 100°

(c) 40°

(d) Cannot be determined

24. Find the perimeter of the given figure.



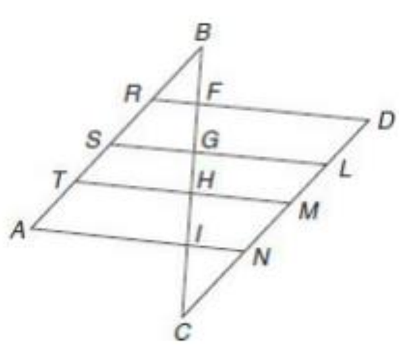

(a) (32 + 3π) cm

(b) (36 + 6π) cm

(c) (46 + 3π) cm

(d) (26 + 6π) cm

25. In the figure, *AB* is parallel to *CD* and *RD* || *SL* || *TM* || *AN*, and *BR*: *RS*: *ST*: *TA* = 3: 5: 2: 7. If it is known that *CN* = 1.333 *BR*. Find the ratio of *BF*: *FG*: *GH*: *HI*: *IC*.



(a) 3: 7: 2: 5: 4

(b) 3: 5: 2: 7: 4

(c) 4: 7: 2: 5: 3

(d) 4: 5: 2: 7: 3

26. In a regular polygon, the number of diagonals is '*k*' times the number of sides. If the interior angle of the polygon is *x*, then the value of *k* is

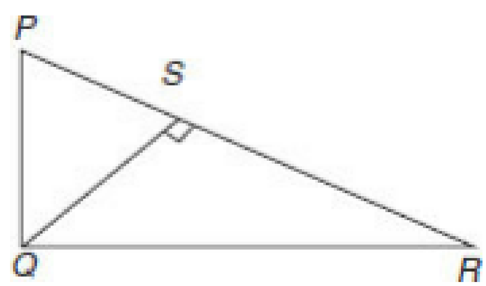
(a) $\frac{3x-\pi}{2(\pi-x)}$

(b) $\frac{3x+\pi}{2(\pi-x)}$

(c) $\frac{3x-\pi}{2(\pi+x)}$

(d) None of these

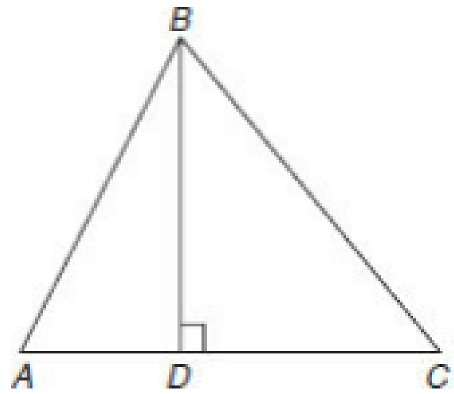
***Directions for Questions 27 and 28:*** $\Delta PQR$ is a right-angled triangle and $\angle Q = 90°$, $PQ = 15$ cm, $QR = 20$ cm and $QS \perp PR$, then answer the following questions.




27. Find the length of *SR* (in cm).

28. Find the length of *SQ* (in cm).

29. In the given diagram $\Delta ABC$ is a right-angled triangle, $\angle ABC = 90°$, $BD \perp AC$. If $AB:AC = 3:5$ and area of $\Delta ABD$ is 90 $cm^2$, then the area of $\Delta ABC$ (in $cm^2$) is




30. If the inradius of an equilateral triangle is $\sqrt{3}$ cm, then its area is

(a) $7\sqrt{3}$ cm2

(b) $9\sqrt{3}$ cm2

(c) $10\sqrt{3}$ cm2

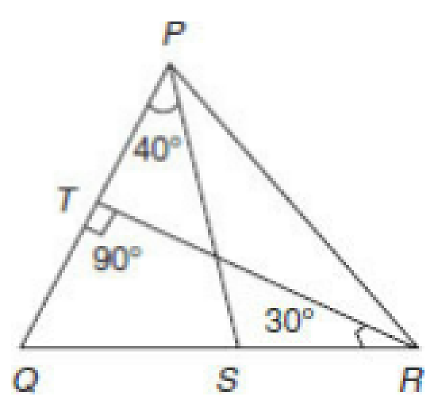
(d) $12\sqrt{3}$ cm2

31. If $\Delta ABC$ is a right-angled triangle such that $\angle B = 90°$, $(AB + BC) - AC = 20$ cm and perimeter of $\Delta ABC = 60$ cm, then find the area of the triangle (in cm2).

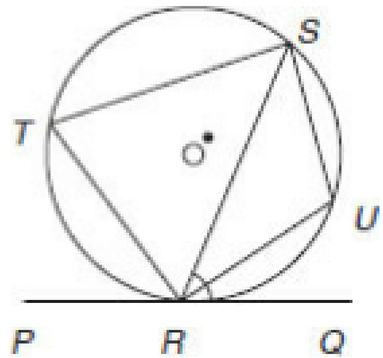
32. Find the sum of the squares of the medians of a triangle whose sides are 6 cm, 7 cm, and 8 cm.

33. In the diagram given below, $RT \perp PQ$ $S$ is a point on QR such that

$\angle QPS = 40°$, $\angle TRQ = 30°$, then find $\angle PSQ$.



34. In the given diagram, $PQ$ touches the circle at $R$. $T, S, U$ are the points on the circle. If $\angle SRQ = 60°$ then find $\angle SUR$.




35. In a right-angled triangle $\Delta PQR$, $\angle Q = 90°$, if $A$ and $B$ are points on the sides $PQ$ and $QR$ respectively then:

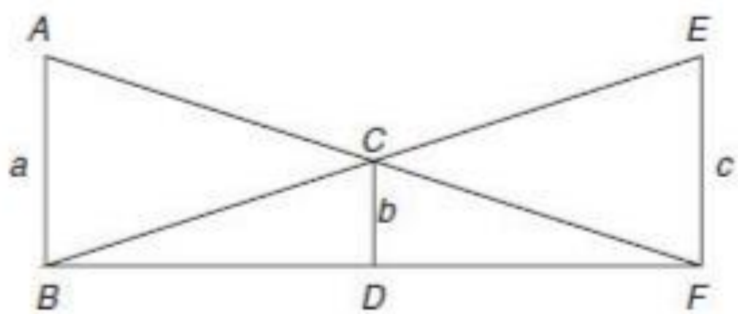
(a) $AR_2 + PB_2 = 2(PR_2 + AB_2)$

(b) $AR_2 + PB_2 = PR_2 + AB_2$

(c) $AR_2 + RB_2 = 0.5(PR_2 + AB_2)$

(d) None of these

36. In the given diagram, if $AB||CD||EF$ then which of the given options is true?



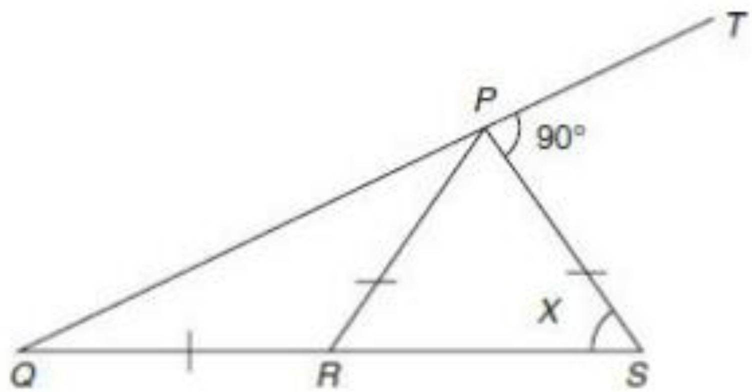

(a) $\frac{1}{a} - \frac{1}{c} = \frac{1}{b}$

(b) $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$

(c) $\frac{1}{b}-\frac{1}{a}=\frac{1}{c}$

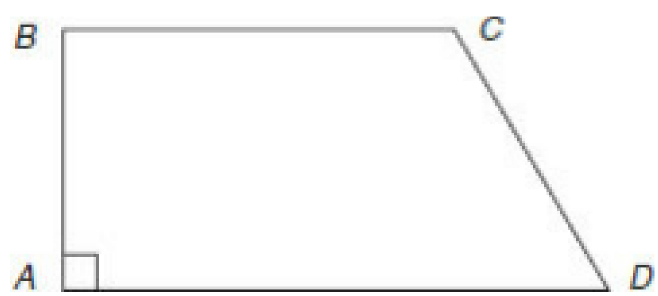
(d) $\frac{1}{c}-\frac{1}{b}=\frac{1}{a}$

37. In the given figure, $PR = QR = PS$, $\angle PSR = x$, $\angle TPS = 90°$ then find $x$.




38. If the sides of a triangle measure 72, 75 and 21. What is the measure of its in-radius?

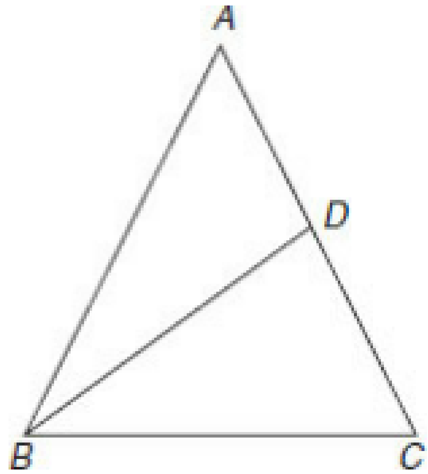
39. $ABCD$ has area equals to 28. $BC$ is parallel to $AD$. $BA$ is perpendicular to $AD$. If $BC$ is 6 and $AD$ is 8, then what is $AB$?




40. In the previous question, find the length of $CD$..

41. Two tangents are drawn to a circle from an exterior point $A$; they touch the circle at points $B$ and $C$, respectively. A third tangent intersects segment $AB$ in $P$ and $AC$ in $R$, and touches the circle at $Q$. If $AB = 20$, then find the perimeter of triangle $APR$.

***Directions for Questions 42 and 43:*** In the diagram given below, if $D$ is the mid-point of side $AC$ and $DB = AD = DC$, then answer the following questions:




42. $\Delta ABC$ is a

    (a) Right-angled triangle

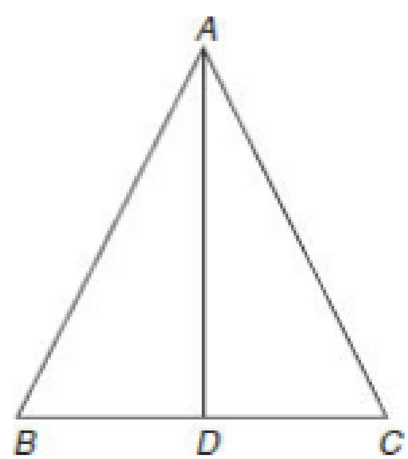
    (b) Equilateral triangle

    (c) Acute-angled triangle

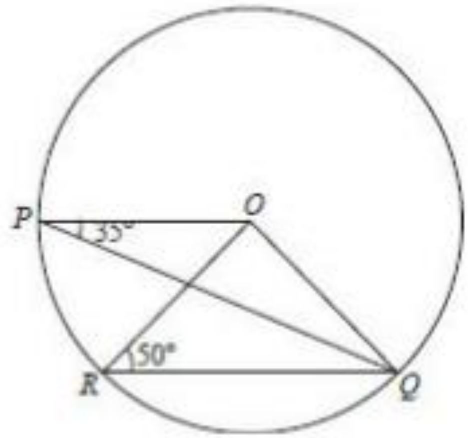
    (d) Obtuse-angled triangle

43. If $\Delta ABC$ is an isosceles triangle then find $\angle BDC$.

44. In the diagram given below, if $AB = AC$ and $\angle ADC = 2\angle ABD$, $\angle DAC = 30°$ then find $\angle BAD$.



45. In the figure given below, '$O$' is the center of the circle. If $PQ$ and $QR$ are chords of the circle, find the measure of $\angle POR$.




(a) 15°

(b) 30°

(c) 45°

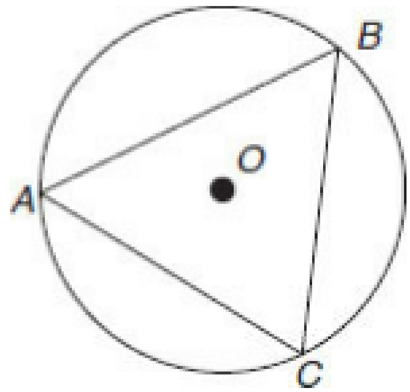
(d) 60°

46. If the interior angle of a regular polygon is 120°, find the number of diagonals of the polygon.

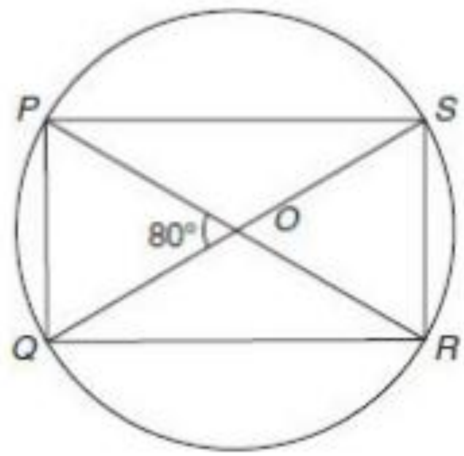
47. The internal angle of a regular polygon exceeds the internal angle of another regular polygon by 18°. If the second polygon has half the number of sides as the first, then find the number of sides in the first polygon.

48. The sum of the interior angles of a regular polygon is 40 times the exterior angle. Find the number of sides of the polygon.

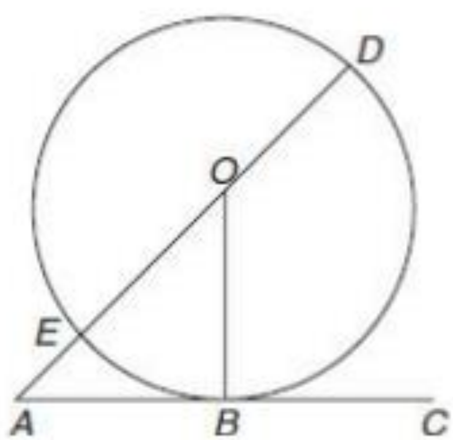
49. In the given diagram, $O$ is the center of the circle and $\angle AOC$ = 140°. If $AB$ = $BC$ then find $\angle BCA$.




50. In the given diagram, if '$O$' is the center of the circle, chord $PR$ and $SQ$ intersect each other at $O$, and $\angle POQ = 80°$, then find $\angle QSR$.

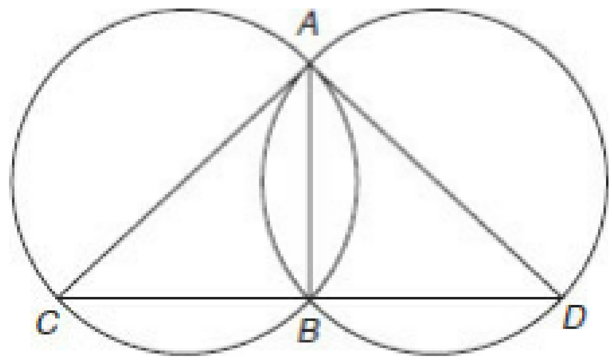



51. $O$ is the center and $AC$ is the tangent of the circle at $B$. In the diagram given below, if $\angle OBE = 70°$, find $\angle BOE$.




52. In the previous question, find $\angle BAO$.

***Directions for Questions 53 and 54:*** Two circles having equal radius intersect each other at $A$ and $B$ as shown in the diagram below. The diameters $AC$ and $AD$ intersect at $A$. If $C, B, D$ are collinear and $\angle ACB = 30°$, then answer the following questions:




53. Find $\angle CAD$.

54. If $BC = 2$ cm, then $AD$ (in cm) is

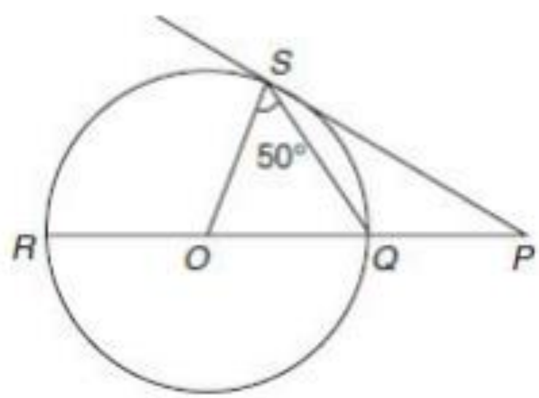
(a) 4

(b) $4/\sqrt{2}$

(c) 3

(d) None of these

55. In the diagram $O$ is the center of the circle. $PS$ is the tangent of the circle at $S$ and $\angle OSQ = 50°$. Find $\angle SPR$.



56. Lines joining midpoint of a quadrilateral form a

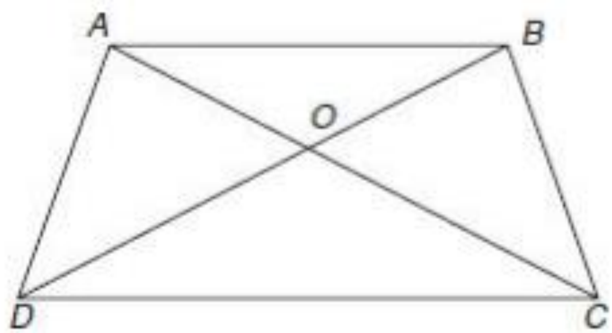
(a) Square

(b) Parallelogram

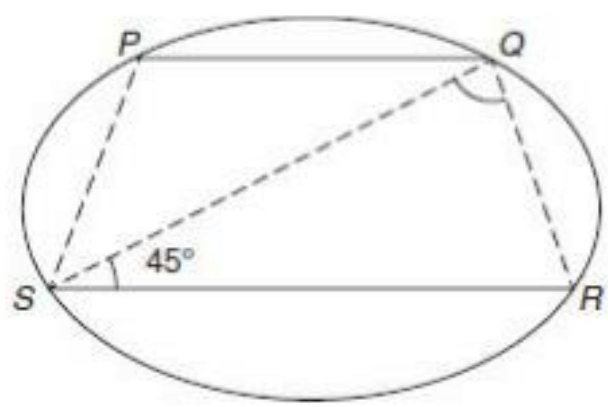
(c) Rectangle

(d) None of these

57. *In the given diagram,* ▫ *ABCD* is a trapezium. If *AB* = 4 cm, *CD* = 6 cm and *OB* = 5 cm, then BD = ____cm.




***Directions for Questions 58 to 59:*** In the given diagram, *PQRS* is a trapezium and SR = $4\sqrt{6}$ cm and ∠*SQR* = 60° ∠*QSR* = 45°. Answer the following questions:




58. Length of *QR* is ___cm.

59. *PS* + *QR* = ___cm.

60. $ABCD$ is a rectangle with $AD = 10$. $P$ is a point on $BC$ such that $\angle APD = 90°$. If $DP = 8$, then the length of $BP$ is.

61. $ABCD$ is a quadrilateral. The diagonals of $ABCD$ intersect at the point $P$. The area of the triangles $APD$ and $BPC$ are 27 and 12, respectively. If the areas of the triangles $APB$ and $CPD$ are equal then the area of triangle $APB$ is

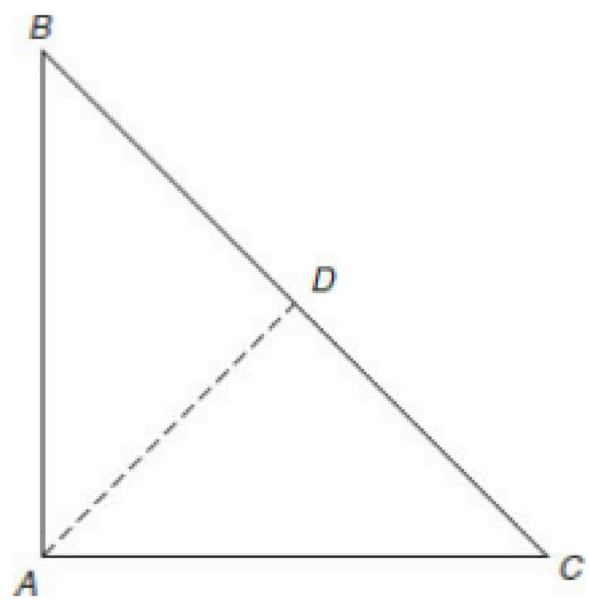
(a) 12

(b) 15

(c) 16

(d) 18

62. In a right angle triangle $BAC$ given below, $AD$ is the altitude of the hypotenuse BC. The figure is followed by three possible inferences.




1. Triangle $ABD$ and triangle $CAD$ are similar

2. Triangle $ADB$ and triangle $CDA$ are congruent

3. Triangle *ADB* and triangle *CAB* are similar

Mark the correct option.

(a) 1 and 2 are correct

(b) 1 and 3 are correct

(c) Only 3 is correct

(d) All three are correct

63. The area of an isosceles triangle is 12 $cm^2$. If one of the equal sides is 5 cm long, mark the option which can give the length of the base.

(a) 4 cm

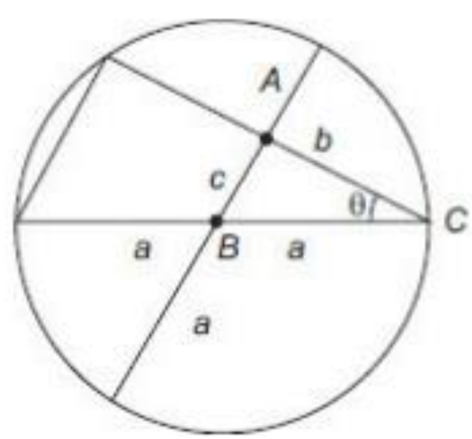
(b) 6 cm

(c) 10 cm

(d) 9cm

64. An arc *AB* of a circle subtends an angle '*x*' radian at the center *O* of the circle. If the area of the sector *AOB* is equal to the square of the length of the arc *AB*, then find *x*.

65. What is the value of $c^2$ in the given figure, where the radius of the circle is '*a*' units?



(a) $c^2 = a^2 + b^2 - 2\,ab\cos\theta$

(b) $c^2 = a^2 + b^2 - 2ab\sin\theta$

(c) $c^2 = a^2 - b^2 + 2ab\cos\theta$

(d) None of these

66. In a circle, the height of an arc is 21 cm and the diameter is 84 cm. Find the chord of 'half of the arc'.

67. The perimeter of a right-angled triangle measures 234 m and the hypotenuse measures 97 m. Then the other two sides of the triangle are measured as

(a) 100 m and 37 m

(b) 72 m and 65 m

(c) 80 m and 57 m

(d) None of these

68. A 25 feet long ladder is placed against the wall with its base 7 feet from the wall. The base of the ladder is drawn out so that the top comes down by half the distance that the base is drawn out. This distance is in the range

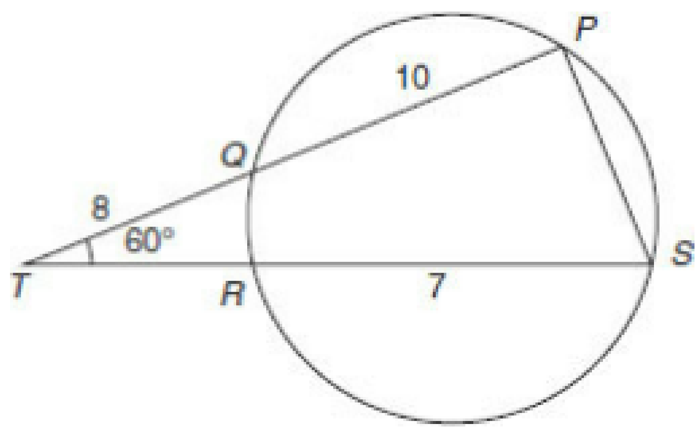
(a) (2, 7)

(b) (5, 8)

(c) (9, 10)

(d) None of these

69. In KyaKya Island, there is a circular park. There are four points of entry into the park, namely – $P$, $Q$, $R$ and $S$. The king of the island His Excellency Mr. Honolulu got three paths constructed which connected the points $PQ$, $RS$, and $PS$. The length of the path $PQ$ is 10 units, and the length of the path $RS$ is 7 units. Later, the municipal corporation extended the paths $PQ$ and $RS$ past $Q$ and $R$ respectively, and they meet at a point $T$ on the main road outside the park. The path from $Q$ to $T$ measures 8 units, and it was found that the $\angle PTS$ is $60°$. Find the area (in square units) enclosed by the paths $PT$, $TS$, and $PS$.




(a) $36\sqrt{3}$

(b) $54\sqrt{3}$

(c) $72\sqrt{3}$

(d) $90\sqrt{3}$

70. There are two circles $C_1$, and $C_2$ of radii 3 and 8 units respectively. The common internal tangent $T$ touches the circles at points $P$ and $Q$ respectively. The line joining the centers of the circles intersects $T$ at $X$. The distance of $X$ from the center of the smaller circle is 5 units. What is the length of the line segment $PQ$?

(a) $\leq 13$

(b) $>13$ and $\leq 14$

(c) $> 14$ and $< 15$

(d) $>15$ and $\leq 16$

71. In quadrilateral $PQRS$, $PQ = 5$ units, $QR = 17$ units, $RS = 5$ units, and $PS = 9$ units. The length of the diagonal $QS$ can be

(a) $> 10$ and $< 12$

(b) $> 12$ and $< 14$

(c) $> 14$ and $< 16$

(d) $>16$ and $<18$

72. In an equilateral triangle $ABC$, whose length of each side is 3 cm, $D$ is a point on $BC$ such that $BD = CD/2$. What is the length of $AD$?

(a) $\sqrt{5}$ cm

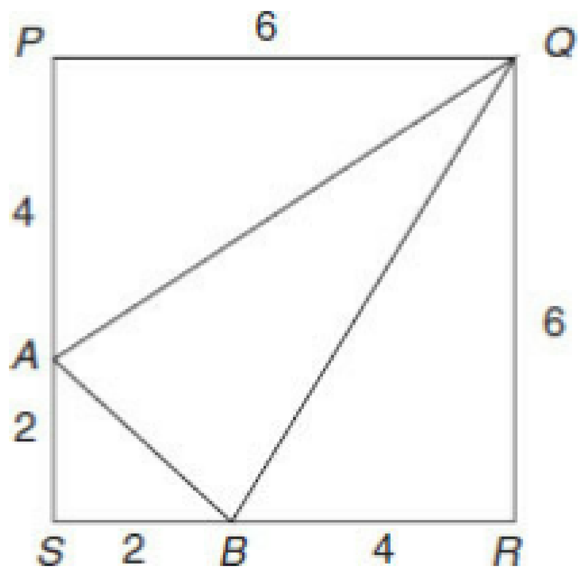
(b) $\sqrt{6}$ cm

(c) $\sqrt{7}$ cm

(d) $\sqrt{8}$ cm

73. Eight points lie on the circumference of a circle. Find the difference between the number of triangles and the number of quadrilaterals that can be formed by connecting these points.

74. In a square $PQRS$, $A$ and $B$ are two points on $PS$ and $SR$ such that $PA = 2AS$, and $RB = 2BS$. If $PQ = 6$, the area of the triangle $ABQ$ is




75. A pole has to be erected on the boundary of a circular park of diameter 13 meters in such a way that the difference of its distances from two diametrically opposite fixed gates *A* and *B* on the boundary is 7 meters. The shortest distance of the pole from one of the gates is ____.

## LEVEL OF DIFFICULTY (II)

1. The angles of a convex nonagon are in an arithmetic progression. Then, which of the following can never be the value of any of its angles?

   (a) 900

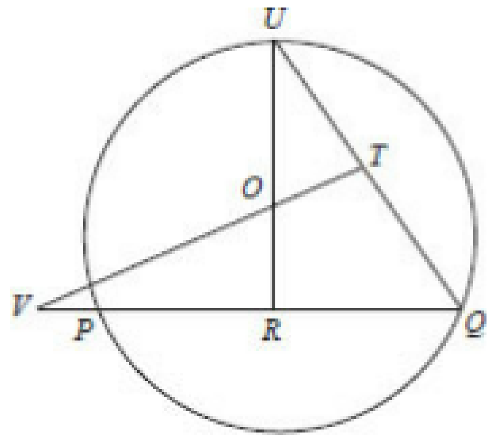
   (b) 1100

   (c) 1200

   (d) None of these

***Directions for Questions 2 to 5:*** A circle with center at *O* is shown in the figure given below. *PQ* is a chord of the circle having length

16 cm. $OR$ = 6 cm and is perpendicular to the chord $PQ$. $RO$ when extended intersects the circumference of the circle at $U$. T is the mid-point of $QU$. TO, when extended, intersects $QP$ extended at the point V.




2. What is the length of chord $QU$ (in cm)?

3. What is the length of $VT$ (in cm)?

4. What is the length of $VP$ (in cm)?

5. What is the area of triangle $VOQ$ (in $cm^2$)?

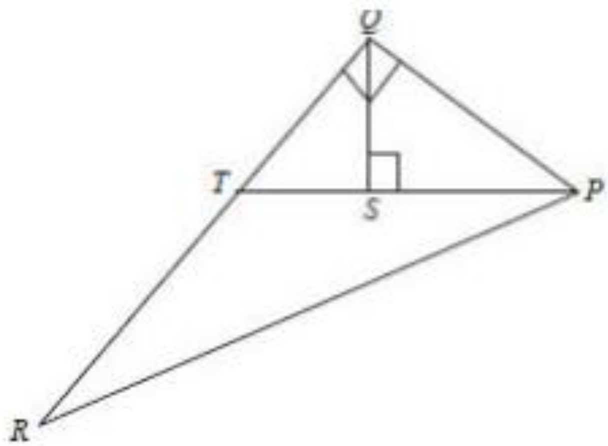
6. The area of a regular polygon of side '$a$' cm is '$25a$' square cm. If the length of the radius of the in-circle is an integer then, how many such polygons will be there?

7. Each exterior angle of an $n$-sided regular polygon is an integer. If $n$ is an odd number, then how many values are possible for $n$?

8. Each exterior angle of an $n$-sided regular polygon is an integer. If $n$ is an odd number, then find the maximum possible value of $n$.

9. Each exterior angle of an $n$-sided regular polygon is an integer. If $n$ is an odd number, then find the maximum possible value of interior angle of $n$.

10. In the right-angled $\Delta PQR$ (see figure), $PR$ = 25 units and $PQ$ = 7 units. It is given that $QT$: $TR$ = 1: 3. Find the approximate length of $QS$.

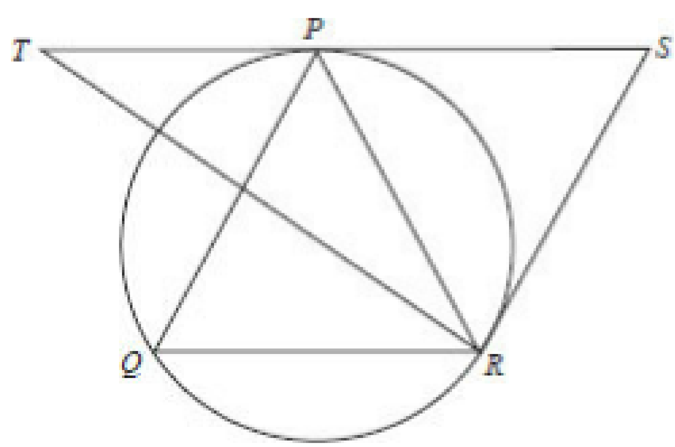



***Directions for Questions 11 and 12:*** A circle is drawn with the center at $O$ and radius as $OA$, where $OA$ is one of the sides of a parallelogram $ABCO$. The circle cuts $CO$ and $BA$ at $Q$ and $P$ respectively. If the area of the circle is $36\pi$ and $BP$ = 2 cm, $PA$ = 8 cm, then answer the following questions:

11. What is the area of the triangle $POA$ (in cm2)?

12. What is the length of $PQ$ (in cm)?

***Directions for Questions 13 to 15:*** In the figure $\Delta PQR$ is an equilateral triangle. $ST$ is a tangent to the circle at $P$. If $\angle PRS = 40°$ and $\angle PSR = 4 \times \angle RTS$ and $\angle TPQ = 60°$



**Answer the following questions:**

13. What is the measure of the $\angle PRQ$?

14. What is the measure of the $\angle PSR$?

15. What is the measure of the $\angle TRQ$?

***Directions for Questions 16 and 17:*** In $\Delta ABC$, $AD \perp BC$, $BE \perp AC$ and $CF \perp AB$. $AD$, $BE$ and $CF$ intersect each other at $O$.

If $AO$ = 18 cm, $BO$ = 12 cm and $AD$ = 20 cm, then answer the following questions:

16. What is the length of $BE$?

17. What is the length of $AB$?

***Directions for Questions 18 to 20:*** In an obtuse-angled triangle $ABC$, $AB = a$, $BC = b$, $AC = c$, where $a$, $b$ and $c$ are integers.

It is given that $a \times b = 9$. Based on this information, answer the following questions:

18. How many such triangles are possible?

19. Find the maximum possible value of $c$.

20. Find the maximum area of the $ABC$.

***Directions for Questions 21 and 22:*** Sum of two sides of an isosceles triangle is 10 cm and the length of each side is a positive integer.

21. How many such triangles are possible, such that they are strictly isosceles?

22. Maximum possible perimeter of the triangle = $x$ cm. What is the value of $x$?

***Directions for Questions 23 and 24:***

23. Sides of a triangle are 12, 20 and $x$. For what value of $x$, is the area of the triangle maximum?

24. What is the value of maximum possible area of the triangle?

25. All of the following are true except:

(a) The points of intersection of direct common tan gents and indirect common tan-gents of two circles divide the line segment joining the two centers respectively externally and internally in the ratio of their radii.

(b) In a cyclic quadrilateral $ABCD$, if the diagonal $CA$ bisects the angle $C$, then diagonal $BD$ is parallel to the tangent at $A$ to the circle through $A$, $B$, $C$, $D$.

(c) If $TA$, $TB$ are tangent segments to a circle $C(O, r)$ from an external point $T$ and $OT$ intersects the circle in $P$, then $AP$ bisects the angle $TAB$.

(d) If in a right triangle $ABC$, $BD$ is the perpendicular on the hypotenuse $AC$, then

(i) $AC \mid AD = AB2$ and

(ii) $AC \cdot AD = BC2$

***Directions for Questions 26 and 27:*** Two cows are tethered at the mid-points of two adjacent sides of a square field. Each of them is tied with a rope in such a way that grazing area of each cow is a semicircular region of diameter equal to the side of square. If side of the square is 10 m, then answer the following questions:

26. What is the area of the grazing field that is grazed by both the cows (in m2)?

(a) $12.5(\pi + 2)$

(b) $12.5(-2+\pi)$

(c) $25(2\pi+1)$

(d) $25(2\pi-1)$

27. For the given situation, the area of the non-grazed regions (in m2) is

(a) 75 + 12.5π

(b) 75 – 12.5π

(c) 75 + 25π

(d) 75 – 13π

***Directions for Questions 28 and 29:*** In the rectangle *PQRS*, *M* and *N* are two points on *SR* and *PQ* respectively such that *MN* bisects *SQ* perpendicularly at *O*. If *SR* = 4 cm, *RQ* = 3 cm.

28. What will be the value of *MO*/*SO*?

(a) 0.8

(b) 0.75

(c) 0.65

(d) None of these

29. Find the area of quadrilateral *SONP*.

(a) 3.65 cm2

(b) 4.66 cm2

(c) 2.66 cm2

(d) None of these

30. A rectangle *PQRS* is inscribed in a semicircle of center *O* and diameter *MN*. *M*, *S*, *R*, *N* are collinear. If *RN* = 2 cm and *QR* = 4 cm, then what is the area of the semicircle not overlapped by the rectangle *PQRS*?

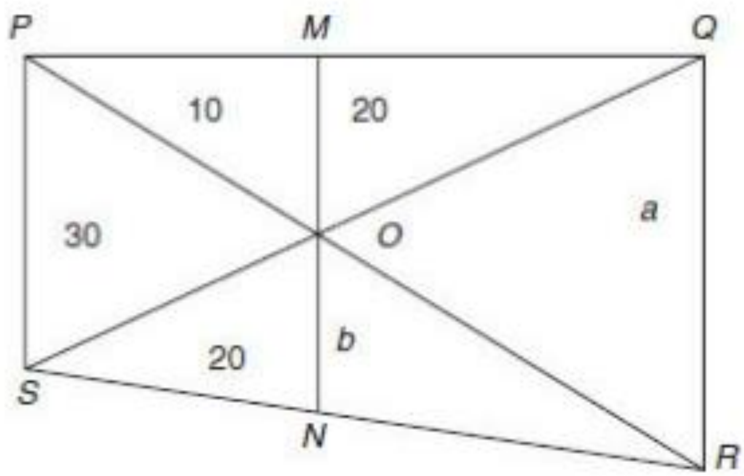
(a) (12.5π + 12) cm2

(b) (12.5π − 12) cm2

(c) (12.5π + 24) cm2

(d) (12.5π − 24) cm2

31. In the diagram given below, quadrilateral *PQRS* is divided into six smaller triangles. The number inside the triangle mentions its area. If *SN*/*NR* = 1.25, then find the value of *a* + *b*.



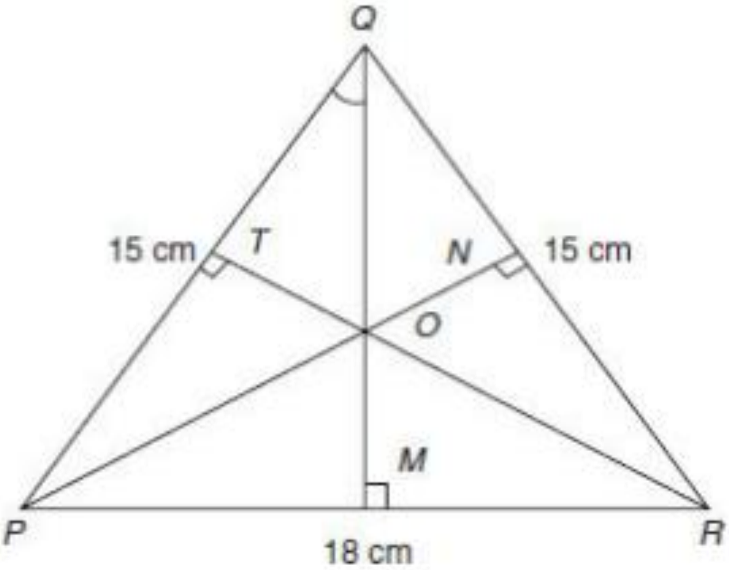

(a) 42

(b) 52

(c) 54

(d) 60

***Directions for Questions 32 and 33:*** In the figure given below: *QM*, *PN*, *RT* are the altitudes of an isosceles triangle and *PQ* = *QR* = 15 cm and *PR* = 18 cm. Answer the following questions:




32. Find the ratio of $QT/QO$.

(a) 0.8

(b) 0.6

(c) 0.7

(d) 0.9

33. The value of $OT$ is

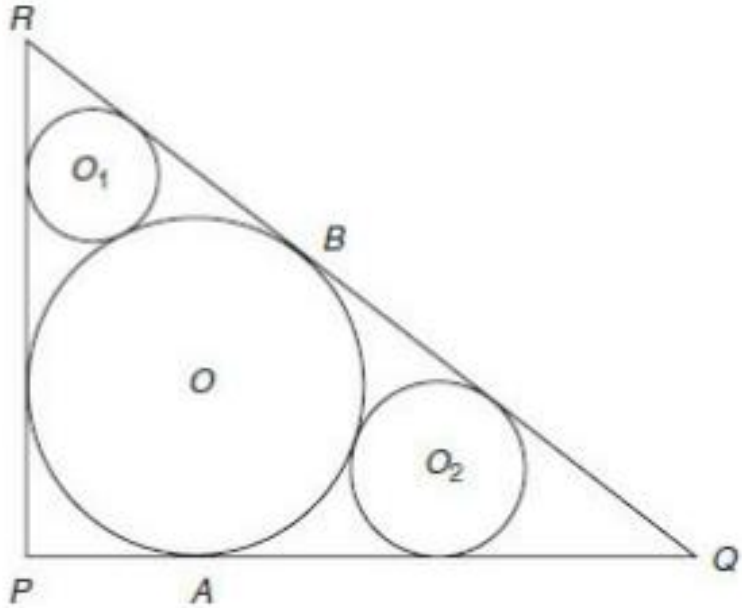
(a) 3.15 cm

(b) 3.35 cm

(c) 3.05 cm

(d) 3.55 cm

34. In the figure given below ΔPQR is a right angled triangle with $\angle P = 90°$. The center of the incircle of the given triangle is $O$. Circles with centers $O_1$ and $O_2$ touch the circle and two sides as shown in the figure. If the radius of the incircle of $\Delta PQR$ is 1 cm and $BR : BQ = 2:3$, then find the value of $r_1 : r_2$ (where $r_1$ is the radius of circle with center $O_1$ and $r_2$ is the radius of circle with center $O_2$).



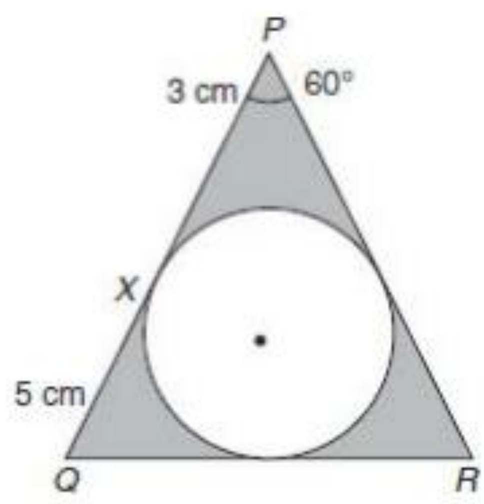

(a) $(33-11\sqrt{5}-6\sqrt{10}+10\sqrt{2})/18$

(b) $(33+11\sqrt{5}-6\sqrt{10}-10\sqrt{2})/18$

(c) $(33-11\sqrt{5}+6\sqrt{10}-10\sqrt{2})/18$

(d) $(33+11\sqrt{5}-6\sqrt{10}-10\sqrt{2})/18$

***Directions for Questions 35 and 36:*** A circle is inscribed in a triangle $PQR$. It touches side $PQ$ at point $X$. If $\angle P = 60°$, $PX = 3$ cm, $QX = 5$ cm, then answer the following questions:




35. Find the radius of the circle.

(a) $3\sqrt{\frac{5}{7}}$

(b) $\frac{5}{7}\sqrt{3}$

(c) $3\sqrt{\frac{5}{17}}$

(d) None of these

36. Find the area of the shaded portion.

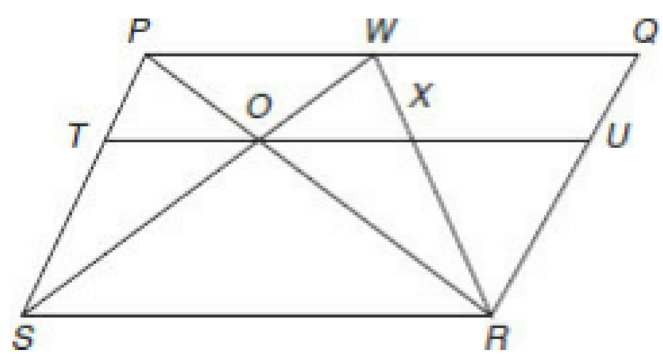
(a) $10\sqrt{3}-3\pi$ cm2

(b) $10\sqrt{3}-13\pi$ cm2

(c) $10\sqrt{3}-2\pi$ cm2

(d) None of these

***Directions for Questions 37 and 38:*** In quadrilateral *PQRS*, *PQ*||*SR* and *PS*||*QR*. *TU*||*SR* and *TU* passes through the point of intersection of *PR* and *WS*. If the ratio of the area of Δ*POW* to Δ*SOR* is 9:25, then answer the following questions:




37. Find the ratio of area of quadrilateral *WQUX* and Δ*XUR*.

(a) 38:25

(b) 39:25

(c) 41:25

(d) None of these

38. If $WQ = 2$ cm, then find $TX \times XU$.

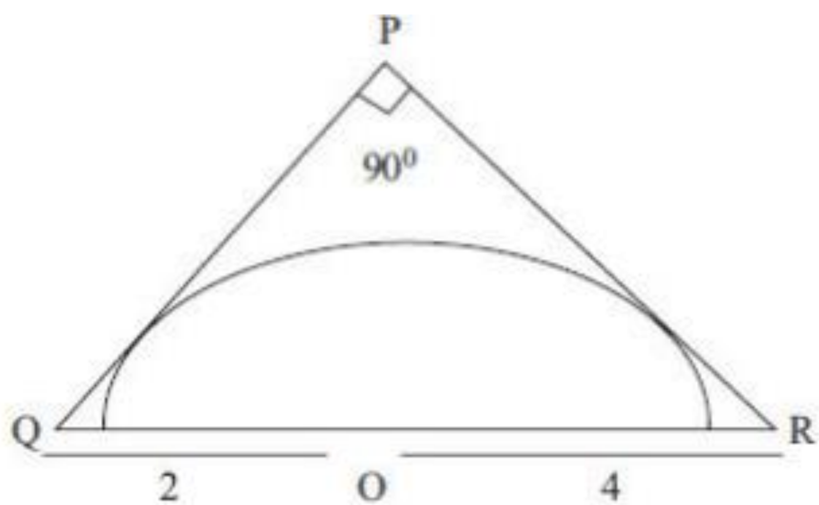
(a) 25/4 $cm^2$

(b) 75/16 $cm^2$

(c) 50/33 $cm^2$

(d) 100/67 $cm^2$

39. In the given figure, $PQ$ and $PR$ are tangents of a semicircle. Centre '$O$' of this semicircle lies on $QR$. If $QO = 2$ cm, $OR = 4$ cm, and $\angle P = 90°$, then radius of the semicircle is




(a) $\sqrt{5}/4$ cm

(b) $\frac{4}{\sqrt{5}}$ cm

(c) $\sqrt{5}/3$ cm

(d) $\sqrt{5}/6$ cm

40. The radius of a circle with center $O$ is $\sqrt{50}$cm. $A$ and $C$ are two points on the circle, and $B$ is a point inside the circle. The length of $AB$ is 6 cm, and the length of $BC$ is 2 cm. The angle $ABC$ is a right angle. Find the square of the distance $OB$.

(a) 26

(b) 25

(c) 24

(d) 23

41. Triangle $ABC$ is a right-angled triangle. $D$ and $E$ are mid points of $AB$ and $BC$ respectively. Read the following statements.

I. $AE = 19$

II. $CD = 22$

III. Angle $B$ is a right angle

Which of the following statements would be sufficient to determine the length of $AC$?

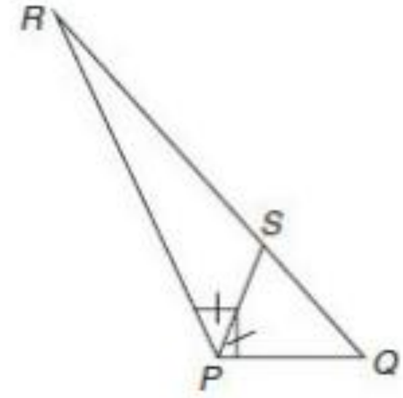
(a) Statements I and II

(b) Statements I and III

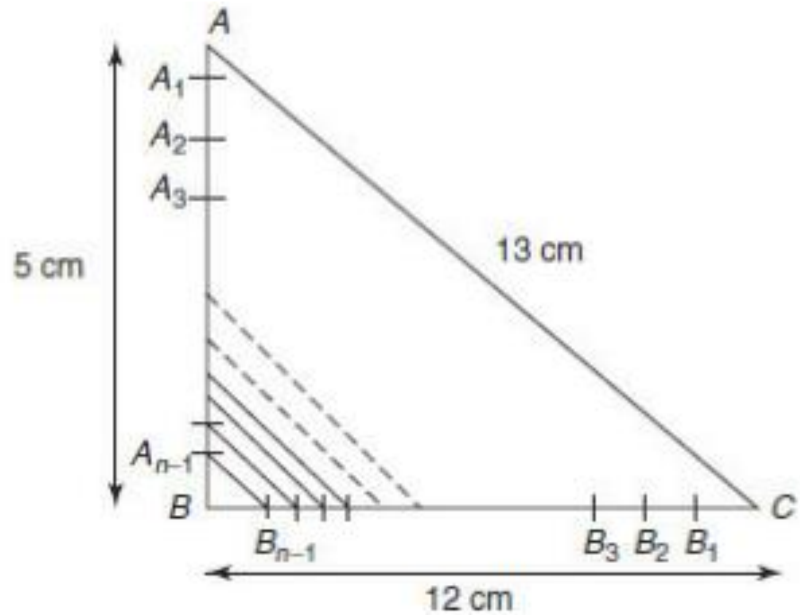
(c) Statements II and III

(d) All three statements

42. In $\Delta\, RPQ$, $\angle P = 120°$ angle bisector of $\angle P$ meets $RQ$ at $S$ and $PQ = 9$ cm, $PS = 6$ cm. Then the value of $PR$ (in cm) is




***Directions for Questions 43 and 44:*** In the diagram given below in $\Delta ABC$ in which $AB$ = 5 cm, $BC$ = 12 cm, $AC$ = 13 cm. Sides $AB$ and $BC$ are divided in $n$-equal parts by $n$ – 1 equally spaced points as shown in the diagram. $A_1$ joined to $B_1$, $A_2$ joined to $B_2$ and so on, then answer the following questions:




43. Find the value of '$n$' for which $A_{n-1}B_{n-1} + A_{n-2}B_{n-2} + A_{n-3}B_{n-3} + \ldots + AC$ = 130 cm.

44. What will be the value of $n$ for which:

Area of $\Delta ABC$ + Area of $\Delta A_1BB_1$ + Area of $\Delta A_2BB_2$ + Area of $\Delta A_3BB_3$ + … + Area of $\Delta A_{n-1}BB_{(n-1)}$ = 66 $cm^2$

***Directions for Questions 45 and 46:*** Two circles of radii 3 cm and 6 cm intersect each other in such a way that their common chord is of maximum possible length.

Then answer the following questions:

45. What is the area of the triangle formed by joining the points of intersection of the two circles to the center of the bigger circle?

(a) $7\sqrt{3}$ cm2

(b) $9\sqrt{3}$ cm2

(c) $10\sqrt{3}$ cm2

(d) $12\sqrt{3}$ cm2

46. What is the area of the region that is common to the two circles (in cm2)?

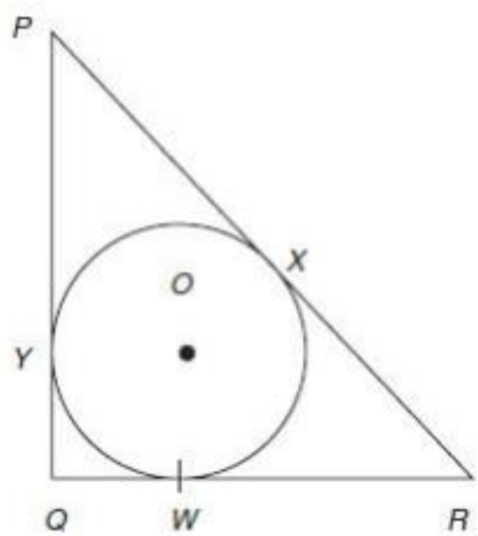
(a) $\left[\frac{21}{2}\pi + 9\sqrt{3}\right]$

(b) $\left[\frac{21}{2}\pi - 9\sqrt{3}\right]$

(c) $\left[\frac{11}{2} + 9\sqrt{3}\right]$

(d) $\left[\frac{11}{2} - 9\sqrt{3}\right]$

***Directions for Questions 47 and 48:*** In the given diagram, *PQR* is an isosceles right-angled triangle with the center *O* touching the side *QR* at *W*, *PR* at *X* and *PQ* at *Y*. If *PR* = $3\sqrt{2}$, then answer the following questions:



47. The ratio $PX$: $QW$: $PY$ is:

(a) $1:(\sqrt{2}-1):1$

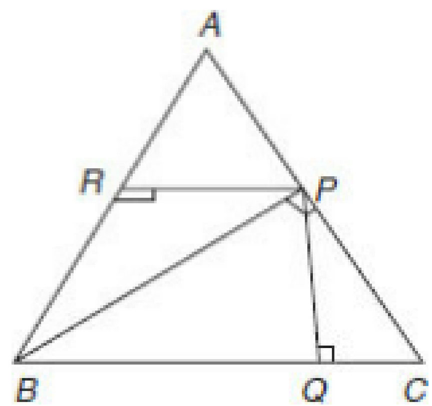
(b) $1:\left(1-\frac{1}{\sqrt{2}}\right):1$

(c) $1:\left(\sqrt{2}-\frac{1}{2}\right):1$

(d) None of these

48. The area of quadrilateral $PYOX$ is:

(a) $9(\sqrt{2}+1)$ cm2




(a) 4:1

(b) 3:1

(c) 2:1

(d) None of these

50. In the previous question, if length of side of equilateral triangle is 4 cm, then area of $\Delta RPQ$ is

(a) $\frac{3\sqrt{3}}{2}$

(b) $\frac{3\sqrt{3}}{4}$

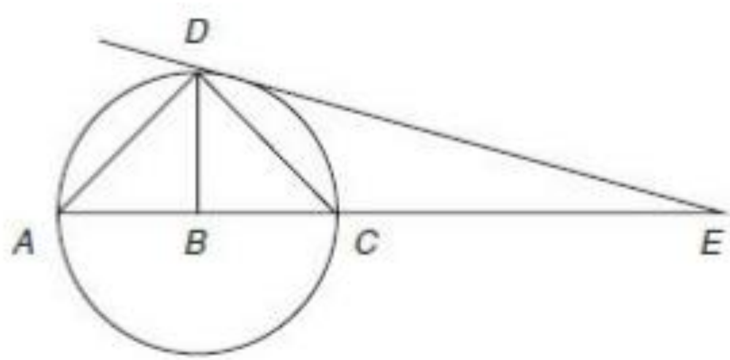
(c) $\frac{\sqrt{3}}{2}$

(d) None of these

51. In an isosceles triangle $PQR$ $PQ = PR$; $A$ and $B$ are points on $PR$ and $PQ$ respectively such that $AB||QR$. $C$ and $D$ are the points on $QR$ such that $AC||PD$. If $PQ = 10$ cm, $PA: AR = 2:3$ and $\angle APD = \angle BAC$, then find the length of $DC$ (in cm)

52. In a $\Delta PQR$, $X$, $Y$, $Z$ are points on sides $PQ$, $QR$, $PR$ such that $PX: XQ = 1:1$, $PZ: ZR = 1:2$, and $QY: YR = 2:3$. What is the ratio of the area of quadrilateral $XYRZ$ to that of $\Delta PXZ$?

***Directions for Questions 53 and 54:*** In the diagram given below, $ED$ is a tangent to the circle and line $AE$ intersects the circle at point $C$. $B$ is a point on $AC$ such that $DB$ is angle bisector of $\angle ADC$. If $\angle ADB = 30°$ and $\angle EDC: \angle ECD = 2: 5$, then answer the following questions:




53. Find $\angle DEC$ (in degrees).

54. If $AB: AC = 1:3$ and $DC = 6$ cm, then find $AD$ (in cm).

55. In a regular polygon, the number of sides is '$p$' times the number of diagonals. If the interior angle of the polygon is $x$, then $x$ is

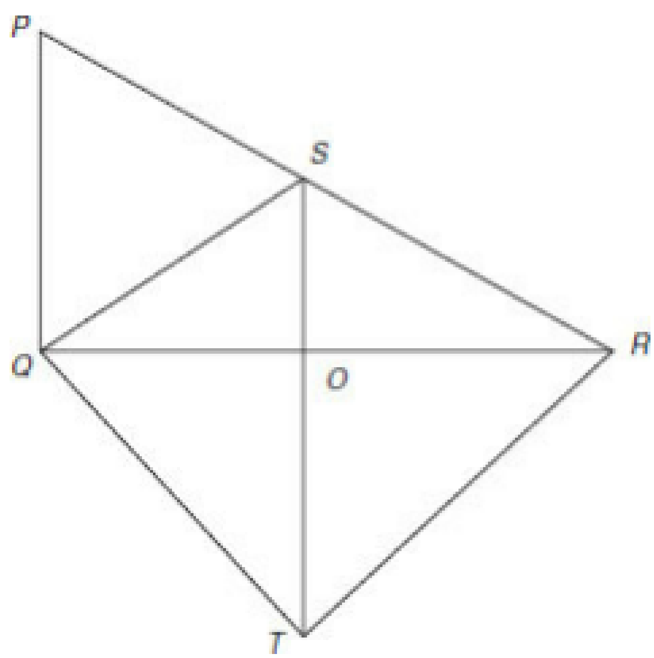
(a) $\frac{2\pi(P+1)}{3P+2}$

(b) $\frac{\pi(2+P)}{3P+2}$

(c) $\frac{3\pi(P+2)}{3P+2}$

(d) None of these

***Directions for Questions 56 and 57:*** In the diagram given below, $\Delta PQR$, and $\Delta QTR$ are right-angled triangles with $\angle PQR = \angle QTR = 90°$, $PR = 25$ cm, $PQ = 15$ cm, $QS = 12$ cm, $RT = 16$ cm. Then answer the following questions:




56. Find the ratio of $RO:OQ$.

(a) 3: 4

(b) 5: 4

(c) 16: 9

(d) 25: 16

57. If $ST = x$ cm, then find $x$.

(a) 18.20

(b) 18.60

(c) 19.20

(d) 19.60

58. In a triangle $ABC$, $AB = 3$, $BC = 4$ and $CA = 5$. Point $D$ is the midpoint of $AB$, point $E$ is on the segment $AC$ and point $F$ is on the segment $BC$. If $AE = 1.5$ and $BF = 0.5$, then find $\angle DEF$.

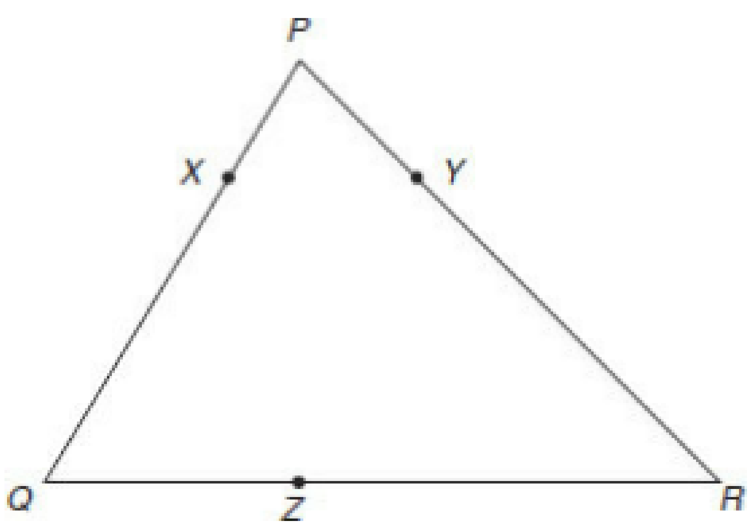
(a) 30°

(b) 60°

(c) 45°

(d) 75°

59. In a $\Delta PQR$, three points $X, Y, Z$ lie on $PQ, QR, PR$ respectively and if $PX : QZ = 1:2$, $QX : QZ = 3:2$ and $PY : YR = 1:3$, then which of the following options is true?



(a) Area of $\Delta XYZ$ is maximum when $QZ : QR = 1:2$

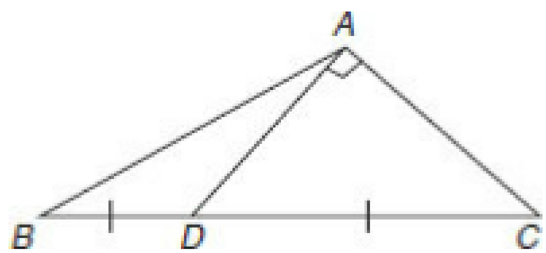
(b) Area of $\Delta XYZ$ is minimum when $QZ : QR = 2:3$

(c) Area of $\Delta XYZ$ is maximum when $QZ : QR = 2:3$

(d) Area of $\Delta XYZ$ will be same for any value of $QZ : QR$.

60. *ABCD* is a square with sides of length 10 units. *OCD* is an isosceles triangle with base *CD*. *OC* cuts *AB* at point *Q* and *OD* cuts *AB* at point *P*. The area of trapezoid *PQCD* is 80 square units. Find the altitude from *O* of the triangle *OPQ*.

61. If *D* is the midpoint of side *BC* of a triangle *ABC* and *AD* is perpendicular to *AC* then




(a) $3AC^2 = BC^2 - AB^2$

(b) $3BC^2 = AC^2 - 3AB^2$

(c) $5AB^2 = BC^2 + AC^2$

(d) None of these

62. In a triangle *ABC*, the length of side *BC* is 295. If the length of side *AB* is a perfect square, then the length of side *AC* is a power of 2, and the length of side *AC* is twice the length of side *AB*. Determine the perimeter of the triangle.

63. There is a triangular building ($ABC$) located in the heart of Aurangabad, the city of Aurangzeb. The length of the one wall in the East ($BC$) direction is 397 feet. If the length of South wall ($AB$) is a perfect cube, the length of the Southwest wall ($AC$) is a power of three, and the length of wall in Southwest ($AC$) is thrice the length of side $AB$, determine the perimeter of this triangular building.

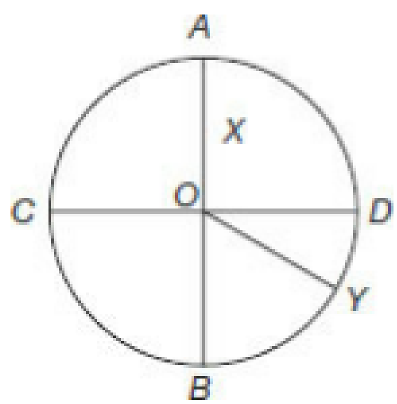
(a) 3609 feet

(b) 3813 feet

(c) 3773 feet

(d) 3313 feet

64. In a circular field, $AOB$ and $COD$ are two mutually perpendicular diameters having length of 4 meters. $X$ is the mid-point of $OA$. $Y$ is a point on the circumference such that $\angle YOD = 30°$. Which of the following correctly gives the relation among the three alternate paths from $X$ to $Y$?




(a) $XOBY$: $XODY$: $XADY$: 5.15: 4.50: 5.06

(b) $XADY$: $XODY$: $XOBY$: 6.25: 5.34: 4.24

(c) $XODY$: $XOBY$: $XADY$: 4.04: 5.35: 5.25

(d) $XADY : XOBY : XODY : 5.19 : 5.09 : 4.04$

65. $ABCD$ is a parallelogram with $\angle ABC = 60°$. If the longer diagonal is of length 7 cm and the area of the parallelogram $ABCD$ is $15\frac{\sqrt{3}}{2}$ cm2, then find the perimeter of the parallelogram (in cm).

66. The center of a circle inside a triangle is at a distance of 625 cm, from each of the vertices of the triangle. If the diameter of the circle is 350 cm and the circle is touching only two sides of the triangle, find the area of the triangle.

(a) 240000

(b) 387072

(c) 480000

(d) 506447

67. Two poles, of height 2 meters and 3 meters, are 5 meters apart. The height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is

(a) 1.2 meters

(b) 1.0 meters

(c) 5.0 meters

(d) 3.0 meters

69. In $\Delta ABC$, $\frac{\angle A}{\angle B} = 1 - \frac{\angle C}{\angle B}$, then which of the following statement is true?

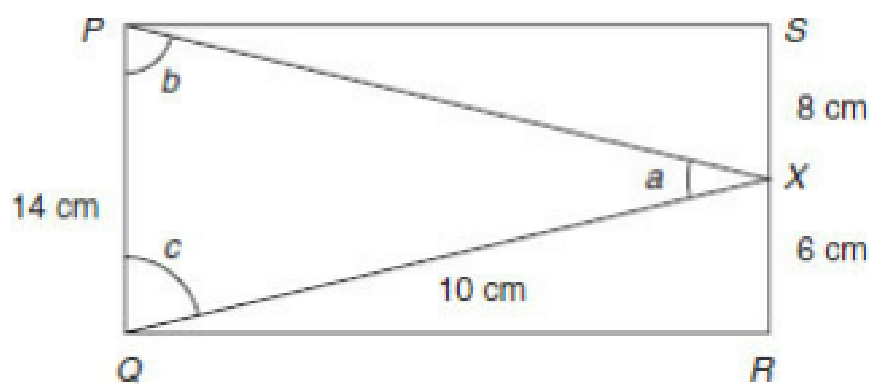
(a) $\Delta ABC$ is always an acute-angled triangle

(b) $\angle > 90°$

(c) $AB^2 + BC^2 = AC^2$

(d) None of these

70. A city has a park which is shaped as a right-angled triangle. The length of the longest side of this park is 80 m. The Mayor of the city wants to construct three paths from the corner point opposite to the longest side such that these three paths divide the longest side into four equal segments. Determine the sum of the squares of the lengths of the three paths (in $m^2$).




71. In a rectangle $PQRS$, $PQ$ = 14 cm, $X$ is a point on $SR$ such that $SX$: $XR$ = 4:3 and $QX$ = 10 cm. If $\angle PXQ = a$, $\angle XPQ = b$, $\angle XQP = c$, then which of the following is correct?

(a) $a > b > c$

(b) $b > c > a$

(c) $a > c > b$

(d) $c > b > a$

## MENSURATION

## LEVEL OF DIFFICULTY (I)

1. In a right-angled triangle, find the hypotenuse if base and perpendicular are respectively 36015 cm and 48020 cm.
   (a) 69125 cm
   (b) 60025 cm
   (c) 391025 cm
   (d) 60125 cm
2. The perimeter of an equilateral triangle is $72\sqrt{3}$ m. Find its height.
   (a) 63 meters
   (b) 24 meters
   (c) 18 meters
   (d) 36 meters
3. The inner circumference of a circular track is 440 cm. The track is 14 cm wide. Find the diameter of the outer circle of the track.
   (a) 84 cm
   (b) 168 cm
   (c) 336 cm
   (d) 77 cm
4. A race track is in the form of a ring whose inner and outer circumferences are 352 meter and 396 meter respectively. Find the width of the track.
   (a) 7 meters
   (b) 14 meters

4. A race track is in the form of a ring whose inner and outer circumferences are 352 meter and 396 meter respectively. Find the width of the track.

(a) 7 meters

(b) 14 meters

(c) 14p meters

(d) 7p meters

5. The outer circumference of a circular track is 220 meter. The track is 7 meter wide everywhere. Calculate the cost of levelling the track at the rate of 50 paisa per square meter.

(a) ₹1556.5

(b) ₹3113

(c) ₹593

(d) ₹693

6. Find the area of a quadrant of a circle whose circumference is 44 cm.

(a) 77 $cm^2$

(b) 38.5 $cm^2$

(c) 19.25 $cm^2$

(d) $19.25\pi$ $cm^2$

7. A pit 7.5 meters long, 6 meters wide and 1.5 meters deep is dug in a field. Find the volume of soil removed in cubic meters.

(a) 135 $meters^3$

(b) 101.25 $meters^3$

(c) 50.625 $meters^3$

(d) 67.5 $meters^3$

8. Find the length of the longest pole that can be placed in an indoor stadium 24 meters long, 18 meters wide and 16 meters high.

(a) 30 meters

(b) 25 meters

(c) 34 meters

(d) $\sqrt{580}$ meters

9. The length, breadth and height of a room are in the ratio of 3: 2: 1. If its volume be 1296 $m^3$, find its breadth

(a) 18 meters

(b) 6 meters

(c) 16 meters

(d) 12 meters

10. The volume of a cube is 216 $cm^3$. Part of this cube is then melted to form a cylinder of length 8 cm. Find the volume of the cylinder.

(a) 342 $cm^3$

(b) 216 $cm^3$

(c) 36 $cm^3$

(d) Data inadequate

11. The whole surface of a rectangular block is 8788 square cm. If length, breadth and height are in the ratio of 4: 3: 2, find length.

(a) 26 cm

(b) 52 cm

(c) 104 cm

(d) 13 cm

12. Three metal cubes with edges 6 cm, 8 cm and 10 cm respectively are melted together and formed into a single cube. Find the side of the resulting cube.

(a) 11 cm

(b) 12 cm

(c) 13 cm

(d) 24 cm

13. Find curved and total surface area of a conical flask of radius 6 cm and height 8 cm.

(a) 60π, 96π

(b) 20π, 96π

(c) 60π, 48π

(d) 30π, 48π

14. The volume of a right circular cone is 100π cm3 and its height is 12 cm. Find its curved surface area.

(a) 130 π $cm^2$

(b) 65 π $cm^2$

(c) 204 π $cm^2$

(d) 65 $cm^2$

15. The diameters of two cones are equal. If their slant height be in the ratio 5 : 7, find the ratio of their curved surface areas.

(a) 25 : 7

(b) 25 : 49

(c) 5: 49

(d) 5: 7

16. The curved surface area of a cone is 2376 square cm and its slant height is 18 cm. Find the diameter.

(a) 6 cm

(b) 18 cm

(c) 84 cm

(d) 12 cm

17. The ratio of radii of a cylinder to a cone is 1: 2. If their heights are equal, find the ratio of their volumes.

(a) 1 : 3

(b) 2 : 3

(c) 3 : 4

(d) 3 : 1

18. A silver wire when bent in the form of a square encloses an area of 484 cm2. Now if the same wire is bent to form a circle, the area enclosed by it would be

(a) 308 cm2

(b) 196 cm2

(c) 616 cm2

(d) 88 cm2

19. The circumference of a circle exceeds its diameter by 16.8 cm. Find the circumference of the circle.

(a) 12.32 cm

(b) 49.28 cm

(c) 58.64 cm

(d) 24.64 cm

20. A bicycle wheel makes 5000 revolutions in moving 11 km. What is the radius of the wheel?

(a) 70 cm

(b) 135 cm

(c) 17.5 cm

(d) 35 cm

21. The volume of a right circular cone is 100π $cm^3$ and its height is 12 cm. Find its slant height.

(a) 13 cm

(b) 16 cm

(c) 9 cm

(d) 26 cm

22. The short and the long hands of a clock are 4 cm and 6 cm long respectively. What will be sum of distances travelled by their tips in 4 days? (Take π = 3.14)

(a) 954.56 cm

(b) 3818.24 cm

(c) 2909.12 cm

(d) 2703.56 cm

23. The surface areas of two spheres are in the ratio of 1: 4. find the ratio of their volumes.

(a) 1 : 2

(b) 1 : 8

(c) 1 : 4

(d) 2 : 1

24. The outer and inner diameters of a spherical shell are 10 cm and 9 cm respectively. Find the volume of the metal contained in the shell. (Use π = 22/7)

(a) 6956 $cm^3$

(b) 141.95 $cm^3$

(c) 283.9 $cm^3$

(d) 478.3 $cm^3$

25. The radii of two spheres are in the ratio of 1 : 2. Find the ratio of their surface areas.

(a) 1 : 3

(b) 2 : 3

(c) 1 : 4

(d) 3 : 4

26. A sphere of radius $r$ has the same volume as that of a cone with a circular base of radius $r$. Find the height of cone.

(a) $2r$

(b) $r/3$

(c) $4r$

(d) $(2/3)r$

27. Find the number of bricks, each measuring 25 cm × 12.5 cm × 7.5 cm, required to construct a wall 12 meters long, 5 meters high and 0.25 meters thick, while the sand and cement mixture occupies 5% of the total volume of wall.

(a) 6080

(b) 3040

(c) 1520

(d) 12160

28. A road that is 7 m wide surrounds a circular path whose circumference is 352 m. What will be the area of the road?

(a) 2618 meters$^2$

(b) 654.5 meters$^2$

(c) 1309 meters$^2$

(d) 5236 meters$^2$

29. In a shower, 10 cm of rain falls. What will be the volume of water that falls on 1 hectare area of ground?

(a) 500 meters$^3$

(b) 650 meters$^3$

(c) 1000 meters$^3$

(d) 750 meters$^3$

30. Seven equal cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.

(a) 750 cm$^2$

(b) 1500 cm$^2$

(c) 2250 cm$^2$

(d) 700 cm$^2$

31. In a swimming pool measuring 90 meters by 40 meters, 150 men take a dip. If the average displacement of water by a man is 8 cubic meters, what will be the rise in the water level?

(a) 30 cm

(b) 50 cm

(c) 20 cm

(d) 33.33 cm

32. How many meters of cloth 5 meters wide will be required to make a conical tent, the radius of whose base is 7 meters and height is 24 meters?

(a) 55 meters

(b) 330 meters

(c) 220 meters

(d) 110 meters

33. Two cones have their heights in the ratio 1: 2 and the diameters of their bases are in the ratio 2: 1. What will be the ratio of their volumes?

(a) 4 : 1

(b) 2 : 1

(c) 3 : 2

(d) 1 : 1

34. A conical tent is to accommodate ten persons. Each person must have 6 meters$_2$ space to sit and 30 meters$_3$ of air to breathe. What will be the height of the cone?

(a) 37.5 meters

(b) 150 meters

(c) 75 meters

(d) None of these

35. A closed wooden box measures externally 10 cm long, 8 cm broad and 6 cm high. Thickness of wood is 0.5 cm. Find the volume of wood used.

(a) 230 cubic cm

(b) 165 cubic cm

(c) 330 cubic cm

(d) 300 cubic cm.

36. A cuboid of dimension 24 cm × 9 cm × 8 cm is melted and smaller cubes of side 3 cm are formed. Find how many such cubes can be formed.

(a) 27

(b) 64

(c) 54

(d) 32

37. Three cubes each of volume of 216 $meters^3$ are joined end to end. Find the surface area of the resulting figure.

(a) 504 $meters^2$

(b) 216 $meters^2$

(c) 432 $meters^2$

(d) 480 $meters^2$

38. A hollow spherical shell is made of a metal of density 4.9 $g/cm^3$. If its internal and external radii are 10 cm and 12 cm respectively, find the weight of the shell. (Take $\pi = \frac{22}{7}$)

(a) 5016 g

(b) 1416.8 g

(c) 14948.26 g

(d) 5667.1 g

39. The largest cone is formed at the base of a cube of side measuring 7 cm. Find the ratio of volume of cone to cube.

(a) 20 : 21

(b) 22 : 21

(c) 21: 22

(d) 11: 42

40. A spherical cannon ball, 28 cm in diameter, is melted and cast into a right circular conical mould the base of which is 35 cm in diameter. Find the height of the cone correct up to two places of decimals.

(a) 8.96 cm

(b) 35.84 cm

(c) 5.97 cm

(d) 17.92 cm

41. Find the area of the circle circumscribed about a square each side of which is 10 cm.

(a) 314.28 cm2

(b) 157.14 cm2

(c) 150.38 cm2

(d) 78.57 $cm^2$

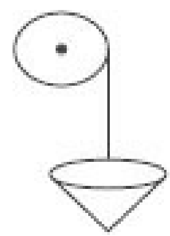
42. Find the radius of the circle inscribed in a triangle whose sides are 8 cm, 15 cm and 17 cm.

(a) 4 cm

(b) 5 cm

(c) 3 cm

(d) $2\sqrt{2}$ cm



43. In the given diagram, a rope is wound around the outside of a circular drum whose diameter is 70 cm and a bucket is tied to the other end of the rope. Find the number of revolutions made by the drum if the bucket is raised by 11 m.

(a) 10

(b) 2.5

(c) 5

(d) 5.5

44. A cube whose edge is 20 cm long has circles on each of its faces painted black. What is the total area of the unpainted surface of the cube if the circles are of the largest area possible?

(a) 85.71 $cm^2$

(b) 257.14 $cm^2$

(c) 514.28 $cm^2$

(d) 331.33 $cm^2$

45. The areas of three adjacent faces of a cuboid are $x, y, z$. If the volume is $V$, then $V^2$ will be equal to

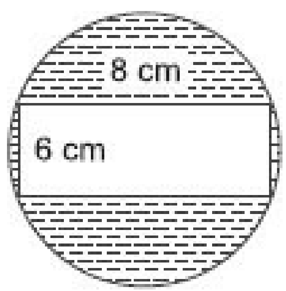
(a) $xy/z$

(b) $yz/x^2$

(c) $x^2y^2/z^2$

(d) $xyz$

46. In the adjacent figure, find the area of the shaded region. (Use π = 22/7)



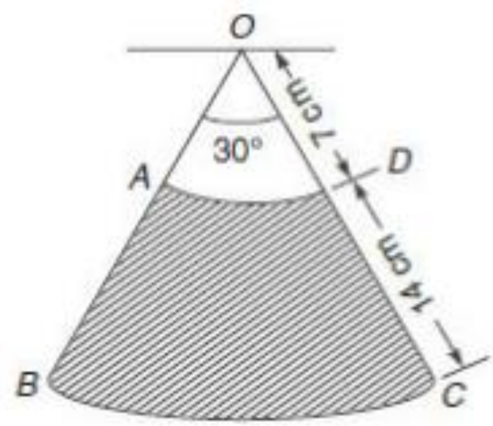

(a) 15.28 $cm^2$

(b) 61.14 $cm^2$

(c) 30.57 $cm^2$

(d) 40.76 $cm^2$

47. The diagram represents the area swept by the wiper of a car. With the dimensions given in the figure, calculate the shaded area swept by the wiper.



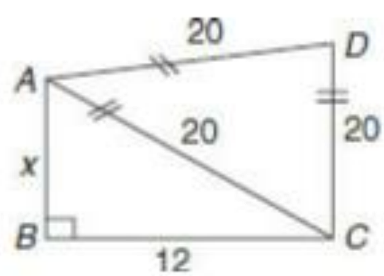

(a) 102.67 cm

(b) 205.34 cm

(c) 51.33 cm

(d) 208.16 cm

48. Find the area of the quadrilateral *ABCD*. (Given, $\sqrt{3}$ = 1.73)



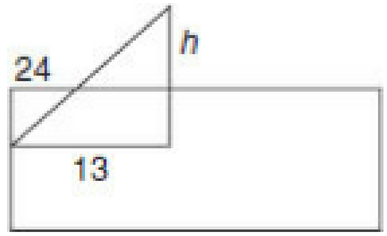

(a) 452 sq units

(b) 269 sq units

(c) 134.5 sq units

(d) 1445 g cm

49. The base of a pyramid is a rectangle of sides 18 m × 26 m and its slant height to the shorter side of the base is 24 m. Find its volume.




(a) $156\sqrt{407}$

(b) $78\sqrt{407}$

(c) $312\sqrt{407}$

(d) $234\sqrt{407}$

50. A wire is looped in the form of a circle of radius 28 cm. It is bent again into a square form. What will be the length of the diagonal of the largest square possible thus?

(a) 44 cm

(b) $44\sqrt{2}$

(c) $176/2\sqrt{2}$

(d) $88\sqrt{2}$

51. If $x$ units are added to the length of the radius of a circle, what is the number of units by which the area of the circle is increased?

52. A man walks diagonally across a square lot. Approximately, what is the percentage reduction in the total distance that he walked vis-à-vis the distance he would have walked had he walked along the edges? (To the closest 1%)

53. The radius of circle is so increased that its circumference is increased by 5%. The area of the circle then increases by

53. The radius of circle is so increased that its circumference is increased by 5%. The area of the circle then increases by

(a) 12.5%

(b) 10.25%

(c) 10.5%

(d) 11.25%

54. The biggest possible cube is taken out of a right solid cylinder of radius 15 cm and height 20 cm respectively. What will be the volume of the cube?

55. How many cuboids of different dimensions can be assembled with 100 identical cubes?

56. What is the least number of square tiles required to pave the floor of a room 1517 cm long and 902 cm broad?

57. A wire, if bent into *a* square, encloses an area of 484 cm2. This wire is cut into two pieces with the bigger piece having *a* length three-fourth of the original wire's length. Now, if *a* circle and *a* square are formed with the bigger and the smaller piece respectively, what would be the area enclosed by the two pieces?

58. A spiral staircase is made up of 13 successive semicircles, with center alternately at *A* and *B*, starting with center at *A*. The radii of semicircles, thus developed, are 0.5 cm, 1.0 cm, 1.5 cm, and 2.0 cm and so on. Find the total length of the spiral Use $\pi = \frac{22}{7}$

59. A cylinder, a hemisphere and a cone stand on the same base and have the same heights. The ratio of the areas of their curved surface is

(a) 2: 2: 1

(b) $\sqrt{2}:\sqrt{2}:1$

(c) $2:\sqrt{2}:1$

(d) None of these

60. The radius of a spherical balloon, of radii 30 cm, increases at the rate of 2 cm per second. Then by how much does its curved surface area increases?

61. A rectangular piece of paper is 22 cm long and 10 cm wide. A cylinder is formed by rolling the paper along its length. Find the volume of the cylinder.

62. Consider the volumes of the following objects and arrange them in **decreasing** order of their volumes:

1. A parallelepiped of length 5 cm, breadth 3 cm and height 4 cm

2. A cube of each side 4 cm

3. A cylinder of radius 3 cm and length 3 cm

4. A sphere of radius 3 cm

(a) 4, 3, 2, 1

(b) 4, 2, 3, 1

(c) 4, 3, 1, 2

(d) None of these

63. A hemispherical bowl is filled with hot water to the brim. The contents of the bowl are transferred into a cylindrical vessel whose radius is 50% more than its height. If diameter of the bowl is the same as that of the vessel and the volume of the hot water in the cylindrical vessel is $x$% of the volume of the cylindrical vessel then $x$ =?

64. In a circular field, there is a rectangular tank of length 130 m and breadth 110 m. If the area of the land portion of the field is 20350 $m^2$, then find the radius of the field.

65. A tank internally measuring 150 cm × 120 cm × 100 cm has 1281600 $cm^3$ water in it. Porous bricks are placed in the water until the tank is full up to its brim. Each brick absorbs one tenth of its volume of water. How many bricks, of 20 cm × 6 cm × 4 cm, can be put in the tank without spilling over the water?

66. A spherical metal ball of radius 10 cm is molten and made into 1000 smaller spheres of equal sizes. In this process the surface area of the metal is increased by $n$ %. Then $n$ =?

67. Suresh, who runs a bakery, uses a conical-shaped equipment to write decorative labels (e.g., Happy Birthday etc.) using cream. The height of this equipment is 7 cm and the diameter of the base is 5 mm. A full charge of the equipment will write 330 words on an average. How many words can be written using two fifth of a litre of cream?

68. Your friend's cap is in the shape of a right circular cone of base radius 14 cm and height 26.5 cm. Find the approximate area of the sheet required to make seven such caps.

69. In an engineering college there is a rectangular garden of dimensions 34 m by 21 m. Two mutually perpendicular walking corridors of 4 m width have been made in the central part and flowers have been grown in the rest of the garden. Find the area under the flowers.

70. A right circular cone is enveloping a right circular cylinder that rests on the base of the cone.

If the radius and the height of the cone is 4 cm and 10 cm respectively, and the radius of the cylinder is '$r$' cm, the largest possible curved surface area of the cylinder is '$a\pi r(b-r)$' then $a \times b =$?

## LEVEL OF DIFFICULTY (II)

1. The perimeter of a sector of a circle of radius 5.7 m is 27.2 m. Find the area of the sector.

   (a) 90.06 $cm^2$

   (b) 135.09 $cm^2$

   (c) 45 $cm^2$

   (d) None of these

2. The dimensions of a field are 20 m by 9 m. A pit 10 m long, 4.5 m wide and 3 m deep is dug in one corner of the field and the earth removed has been evenly spread over the remaining area of the field. What will be the rise in the height of field as a result of this operation?

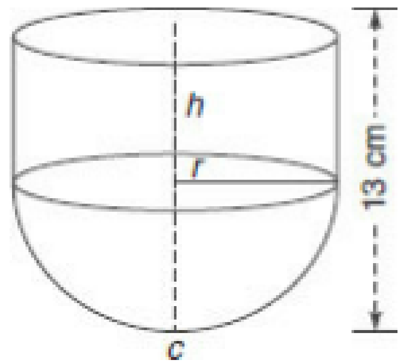
   (a) 1 m

   (b) 2 m

   (c) 3 m

   (d) 4 m

3. A vessel is in the form of a hollow cylinder mounted on a hemispherical bowl. The diameter of the sphere is 14 cm and the total height of the vessel is 13 cm. Find the capacity of the vessel.(Take $\pi = 22/7$)




(a) 321.33 cm

(b) 1642.67 cm3

(c) 1232 cm3

(d) 1632.33 cm3

4. The sides of a triangle are 21, 20 and 13 cm. Find the area of the larger triangle into which the given triangle is divided by the perpendicular upon the longest side from the opposite vertex.

(a) 72 cm2

(b) 96 cm2

(c) 168 cm2

(d) 144 cm2

5. A circular tent is cylindrical to a height of 3 meters and conical above it. If its diameter is 105 m and the slant height of the conical portion is 53 m, calculate the length of the canvas 5 m wide to make the required tent.

(a) 3894 m

(b) 973.5 m

(c) 1947 m

(d) 1800 m

6. A steel sphere of radius 4 cm is drawn into a wire of diameter 4 mm. Find the length of wire.

(a) 10,665 mm

(b) 42,660 mm

(c) 21,333 mm

(d) 14,220 mm

7. A cylinder and a cone having equal diameter of their bases are placed in the Qutab Minar one on the other, with the cylinder placed in the bottom. If their curved surface area are in the ratio of 8: 5, find the ratio of their heights. Assume the height of the cylinder to be equal to the radius of Qutab Minar. (Assume Qutab Minar to be having same radius throughout.)

(a) 1 : 4

(b) 3 : 4

(c) 4 : 3

(d) 2 : 3

8. If the curved surface area of a cone is thrice that of another cone and slant height of the second cone is thrice that of the first, find the ratio of the area of their base.

(a) 81 : 1

(b) 9 : 1

(c) 3 : 1

(d) 27 : 1

9. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, find the uniform thickness of the cylinder.

(a) 2 cm

(b) 3 cm

(c) 1 cm

(d) 3.5 cm

10. A hollow sphere of external and internal diameters 6 cm and 4 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone.

(a) 4.75 cm

(b) 9.5 cm

(c) 19 cm

(d) 38 cm

11. Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes.

(a) 7 : 9

(b) 49 : 81

(c) 9 : 7

(d) 27 : 23

12. If $V$ be the volume of a cuboid of dimension $x, y, z$ and $A$ is its surface, then $A/V$ will be equal to

(a) $x^2y^2z^2$

(b) $1/2\ (1/xy + 1/xz + 1/yz)$

(c) $2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

(d) $1/xyz$

13. The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute-hand between 9 AM and 9: 35 AM

(a) 183.3 $cm^2$

(b) 366.6 $cm^2$

(c) 244.4 $cm^2$

(d) 188.39 $cm^2$

14. Two circles touch internally. The sum of their areas is $116\pi$ $cm^2$ and distance between their centers is 6 cm. Find the radii of the circles.

(a) 10 cm, 4 cm

(b) 11 cm, 4 cm

(c) 9 cm, 5 cm

(d) 10 cm, 5 cm

15. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface

area of the toy if the height of conical part is 12 cm.

(a) 1440 $cm^2$

(b) 385 $cm^2$

(c) 1580 $cm^2$

(d) 770 $cm^2$

16. A solid wooden toy is in the form of a cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of wood used in the toy.

(a) 343.72 $cm^3$

(b) 266.11 $cm^3$

(c) 532.22 $cm^3$

(d) 133.55 $cm^3$

17. A cylindrical container whose diameter is 12 cm and height is 15 cm, is filled with ice cream. The whole ice-cream is distributed to ten children in equal cones having hemispherical tops. If the height of the conical portion is twice the diameter of its base, find the diameter of the ice-cream cone.

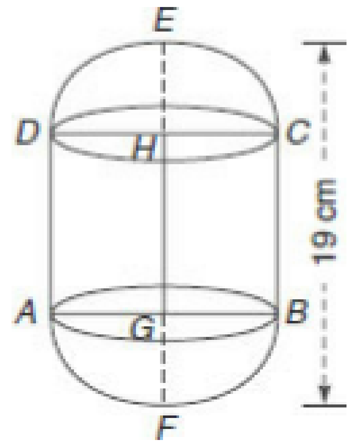
(a) 6 cm

(b) 12 cm

(c) 3 cm

(d) 18 cm

18. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the total surface area of the solid.(Use = 22/7)




(a) 398.75 cm2

(b) 418 cm2

(c) 444 cm2

(d) 412 cm2

19. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. What is the ratio of their volumes?

(a) 2: 1: 3

(b) 2.5: 1: 3

(c) 1: 2: 3

(d) 1.5: 2: 3

20. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The cost of painting 1 cm2 of the surface is ₹0.05. Find the total cost of painting the vessel all over. (Take π = 22/7)

(a) ₹97.65

(b) ₹86.4

(c) ₹184

(d) ₹96.28

21. A solid is in the form of a right circular cylinder with a hemisphere at one end and a cone at the other end. Their common diameter is 3.5 cm and the heights of conical and cylindrical portions are respectively 6 cm and 10 cm. Find the volume of the solid.

(Use π = 3.14)

(a) 117 cm2

(b) 234 cm2

(c) 58.5 cm2

(d) None of these

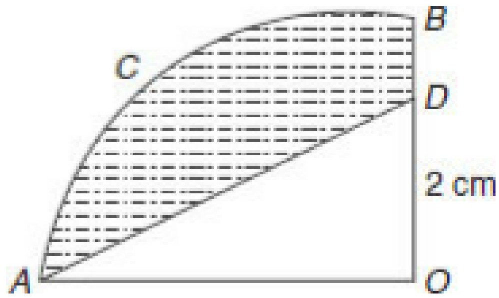
22. In the adjoining figure, *AOBCA* represents a quadrant of a circle of radius 3.5 cm with center *O*. Calculate the area of the shaded portion. (Use π = 22/7)

(a) 35 cm2

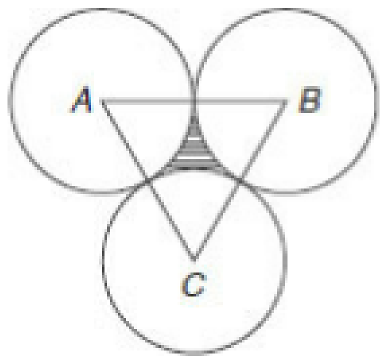
(b) 7.875 cm2

(c) 9.625 cm2

(d) 6.125 cm2




23. Find the area of the shaded region if the radius of each of the circles is 1 cm.




(a) $2 - \frac{\pi}{3}$

(b) $\sqrt{3}$ – p

(c) $\sqrt{3} - \frac{\pi}{2}$

(d) $\sqrt{3} - \pi/4$

24. A right elliptical cylinder full of petrol has its widest elliptical side 2.4 m and the shortest 1.6 m. Its height is 7 m. Find the time required to empty half the tank through a hose of diameter 4 cm if the rate of flow of petrol is 120 m/min

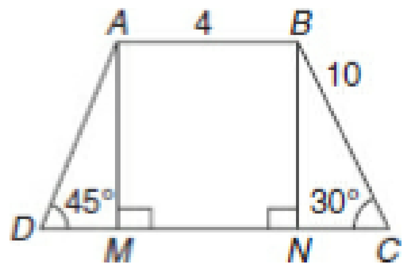
(a) 60 min

(b) 90 min

(c) 75 min

(d) 70 min

25. Find the area of the trapezium *ABCD*.



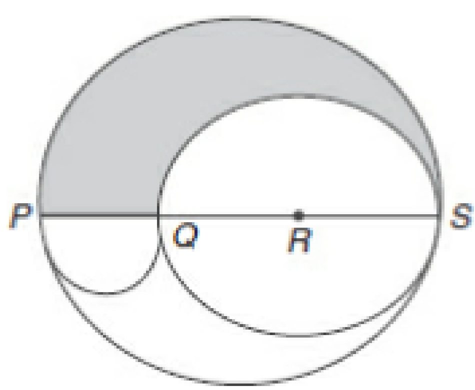

(a) $5/2(13 + 2\sqrt{3})$

(b) $\frac{5\sqrt{3}(13+5\sqrt{3})}{2}$

(c) $13(13 + 2\sqrt{3})$

(d) None of these

26. *PQRS* is the diameter of a circle of radius 6 cm. The lengths *PQ*, *QR* and *RS* are equal. Semi-circles are drawn with *PQ* and *QS* as diameters as shown in the figure alongside. Find the ratio of the area of the shaded region to that of the un-shaded region.



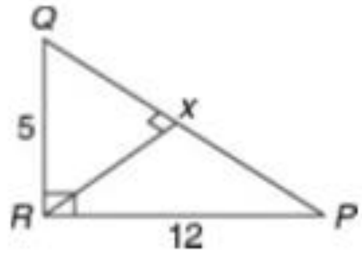

(a) 1 : 2

(b) 25 : 121

(c) 5: 18

(d) 5: 13

27. In the right-angled triangle *PQR*, find *Rx*.




(a) 13/60

(b) 13/45

(c) 60/13

(d) 23/29

28. The radius of a right circular cylinder is increased by 50%. Find the percentage increase in volume

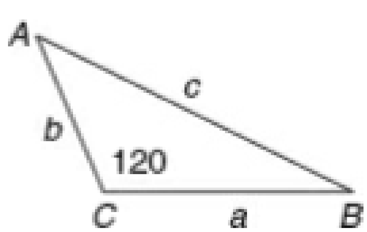
(a) 120%

(b) 75%

(c) 150%

(d) 125%

29. Two persons start walking on a road that diverges at an angle of 120°. If they walk at the rate of 3 km/h and 2 km/h repectively, find the distance between them after four hours.



(a) $4\sqrt{19}$ km

(b) 5 km

(c) 60/13

(d) 23/29

28. The radius of a right circular cylinder is increased by 50%. Find the percentage increase in volume

(a) 120%

(b) 75%

(c) 150%

(d) 125%

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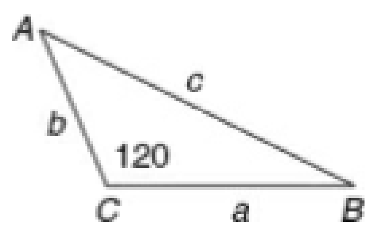



(a) $4\sqrt{19}$ km

(b) 5 km

(c) 7 km

(d) $8\sqrt{19}$ km

30. Water flows out at the rate of 10 m/min from a cylindrical pipe of diameter 5 mm. Find the time taken to fill a conical tank whose diameter at the surface is 40 cm and depth 24 cm.

(a) 50 min

(b) 102.4 min

(c) 51.2 min

(d) 25.6 min

31. The section of a solid right circular cone by a plane containing vertex and perpendicular to base is an equilateral triangle of side 12 cm. Find the volume of the cone.

(a) 72 cc

(b) 144 cc

(c) $72\sqrt{2}$p cc

(d) $72\sqrt{3}$ p cc

32. Iron weighs eight times the weight of oak. Find the diameter of an iron ball whose weight is equal to that of a ball of oak 18 cm in diameter.

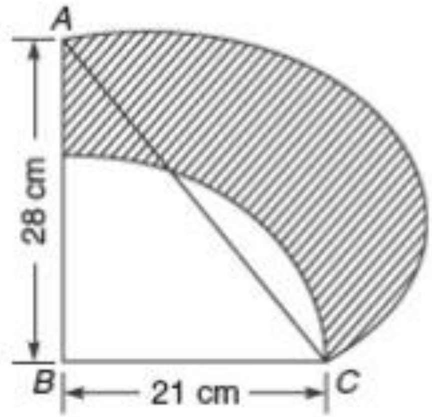
(a) 4.5 cm

(b) 9 cm

(c) 12 cm

(d) 15 cm

33. In the figure, $ABC$ is a right-angled triangle with $\angle B = 90°$, $BC = 21$ cm and $AB = 28$ cm. With $AC$ as diameter of a semicircle and with $BC$ as radius, a quarter circle is drawn. Find the area of the shaded portion correct to two decimal places.



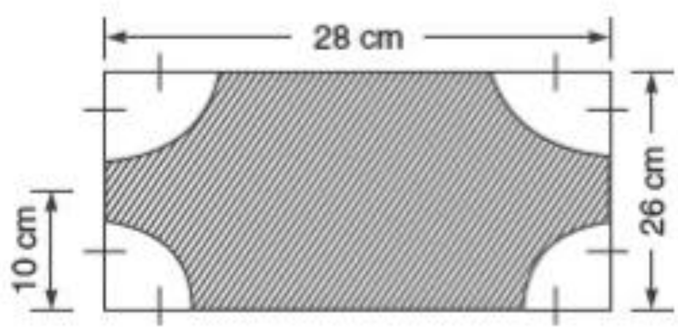

(a) 428.75 $cm^2$

(b) 857.50 $cm^2$

(c) 214.37 $cm^2$

(d) 371.56 $cm^2$

34. Find the perimeter and area of the shaded portion of the adjoining diagram:



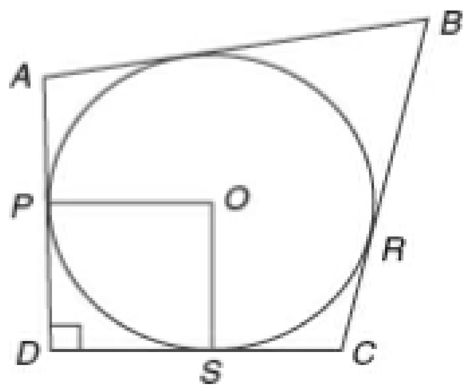

(a) 90.8 cm, 414 $cm^2$

(b) 181.6 cm, 423.7 $cm^2$

(c) 90.8 cm, 827.4 cm2

(d) 181.6 cm, 827.4 cm2

35. In the adjoining figure, a circle is inscribed in the quadrilateral *ABCD*. Given that *BC* = 38 cm, *AB* = 27 cm and *DC* = 25 cm, and that *AD* is perpendicular to *DC*. Find the maximum limit of the radius and the area of the circle.




(a) 10 cm; 226 cm2

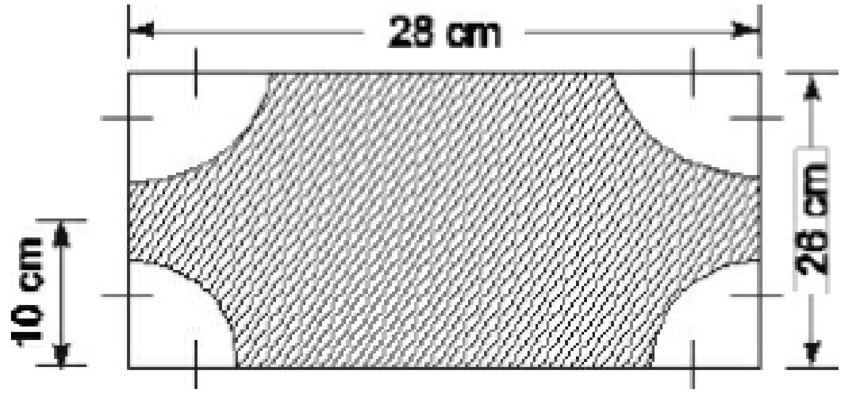
(b) 14 cm; 616 cm2

(c) 14 cm; 216 cm2

(d) 28 cm; 616 cm2

36. From a piece of cardboard, in the shape of a trapezium *ABCD* and *AB* || *DC* and ∠*BCD* = 90°, a quarter circle (*BFEC*) with *C* as its center is removed. Given *AB* = *BC* = 3.5 cm and *DE* = 2 cm, calculate the area of the remaining piece of the cardboard.

(Take π = 22/7)



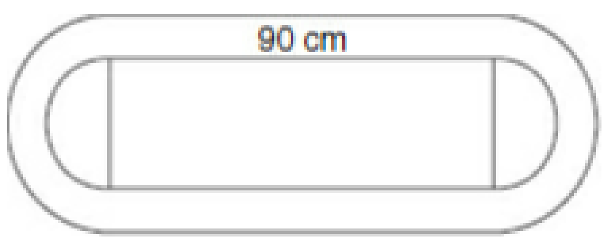

(a) 3.325 cm2

(b) 3.125 cm2

(c) 6.075 cm2

(d) 12.25 cm2

37. The inside perimeter of a practice running track with semi-circular ends and straight parallel sides is 312 m. The length of the straight portion of the track is 90 m. If the track has a uniform width of 2 m throughout, find its area.




(a) 5166 m2

(b) 5802.57 m2

(c) 636.57 m2

(d) 1273.14 m2

38. Find the area of the triangle inscribed in a circle circumscribed by a square made by joining the mid-points of the adjacent sides of a square of side $a$.

(a) $3a^2/16$

(b) $\frac{3\sqrt{3}\,a^2}{16}$

(c) $3/4\, a^2(\pi - 1/2)$

(d) $\frac{3\sqrt{3}\,a^2}{32}$

39. Two goats are tethered to the diagonally opposite vertices of a square field formed by joining the mid-points of the adjacent sides of another square field of side $20\sqrt{2}$ meters. The inner square field is fenced on all sides and the goats are allowed to graze only inside the inner field. If their grazing ropes are of a length of $10\sqrt{2}$ meters each, find the total area grazed by the two goats together.

(a) $100\pi$ m$^2$

(b) $50(\sqrt{2} - 1)\pi$ m$^2$

(c) $100\pi(3 - 2\sqrt{2})$ m$^2$

(d) $200\pi(2 - \sqrt{2})$ m$^2$

40. The area of the circle circumscribing three circles of unit radius touching each other is

(a) $(\pi/3)(2 + \sqrt{3})^2$

(b) $6\pi(2 + \sqrt{3})^2$

(c) $3\pi (2 + \sqrt{3})2$

(d) $\left(\frac{\pi}{6}\right)(2 + \sqrt{3})2$

41. Find the ratio of the diameter of the circles inscribed in and circumscribing an equilateral triangle to its height.

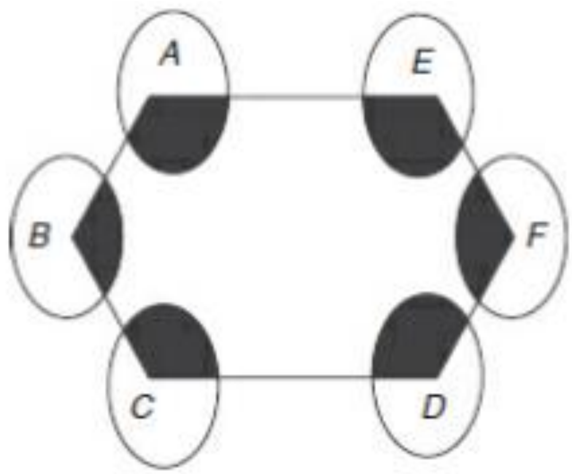
(a) 1: 2: 1

(b) 2:4: 3

(c) 1: 3: 4

(d) 3: 2: 1

42. Find the sum of the areas of the shaded sectors given that *ABCDFE* is any hexagon and all the circles are of same radius *r* with different vertices of the hexagon as their centers as shown in the figure.




(a) $\pi r2$

(b) $2\pi r2$

(c) $5\pi r2/4$

(d) $3\pi r2/2$

43. Circles are drawn with four vertices as the center and radius equal to the side of a square. If the square is formed by joining the mid-points of another square of side $2\sqrt{6}$, find the area common to all the four circles.

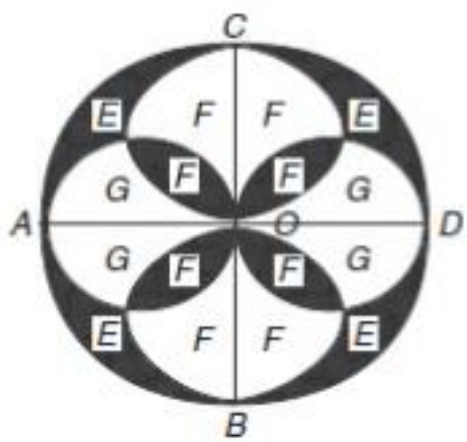
(a) $[(\sqrt{3} - 1)/2)]\ 6\pi$

(b) $4\pi - 3\sqrt{3}$

(c) $1/2\ (\pi - 3\sqrt{3})$

(d) $4\pi - 12(\sqrt{3} - 1)$

44. *ABDC* is a circle and circles are drawn with *AO*, *CO*, *DO* and *OB* as diameters. Areas *E* and *F* are shaded. *E*/*F* is equal to




(a) 1

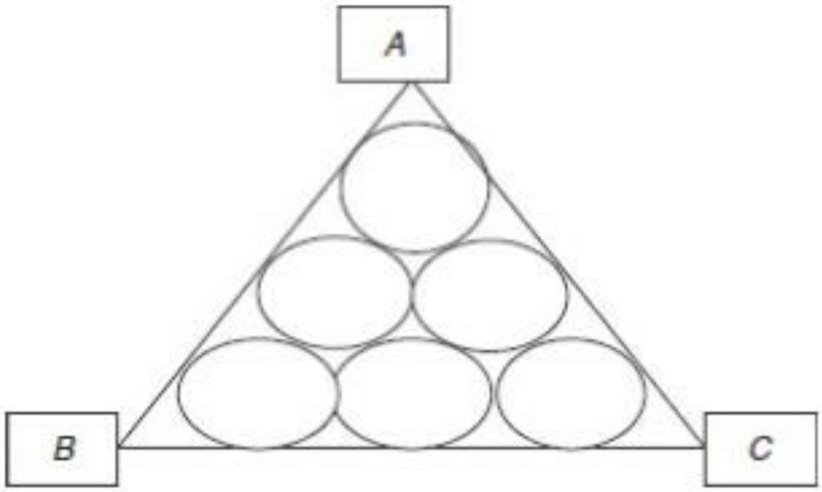
(b) 1/2

(c) 1/ 2

(d) $\pi/4$

45. The diagram shows six equal circles inscribed in an equilateral triangle *ABC*. The circles touch externally among themselves and also touch the

sides of the triangle. If the radius of each circle is $R$, then area of the triangle is




(a) $(6 + \pi\sqrt{3})\, R1$

(b) $9\, R2$

(c) $R2\, (12 + 7\sqrt{3})$

(d) $R2\, (9 + 6\sqrt{3})$

46. A boy Mithilesh was playing with a square cardboard of side 2 meters. While playing, he accidentally sliced off the corners of the cardboard in such a manner that a figure having all its sides equal was generated. The area of this eight-sided figure is

(a) $\frac{4\sqrt{2}}{\sqrt{2}+1}$

(b) $\frac{4}{\sqrt{2}+1}$

(c) $\frac{2\sqrt{2}}{\sqrt{2}+1}$

(d) $\frac{8}{\sqrt{2}+1}$

47. In a painting competition, students were asked to draw alternate squares and circles, circumscribing each other. The first student drew $A_1$ a square whose side is '$a$' meters. The second student drew circle $C_1$ circumscribing the square $A_1$ such that all its vertices are on $C_1$. Subsequent students drew square $A_2$ circumscribing $C_1$, circle $C_2$ circumscribing $A_2$ and $A_3$ circumscribing $C_2$, and so on. If $D_N$ is the area between the square $A_N$ and the circle $C_N$, where $N$ is a natural number, then the ratio of the sum of all $D_N$ to $D_1$ for $N = 12$ is

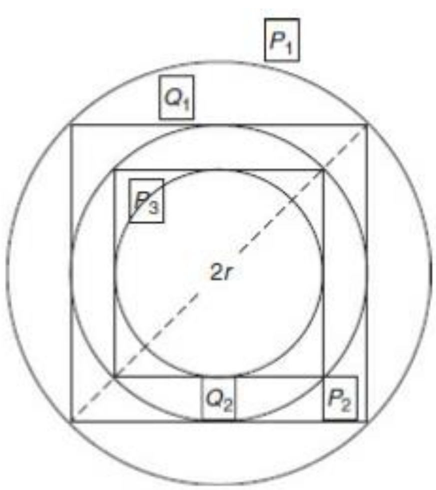
(a) 1

(b) $\frac{\pi}{2} - 1$

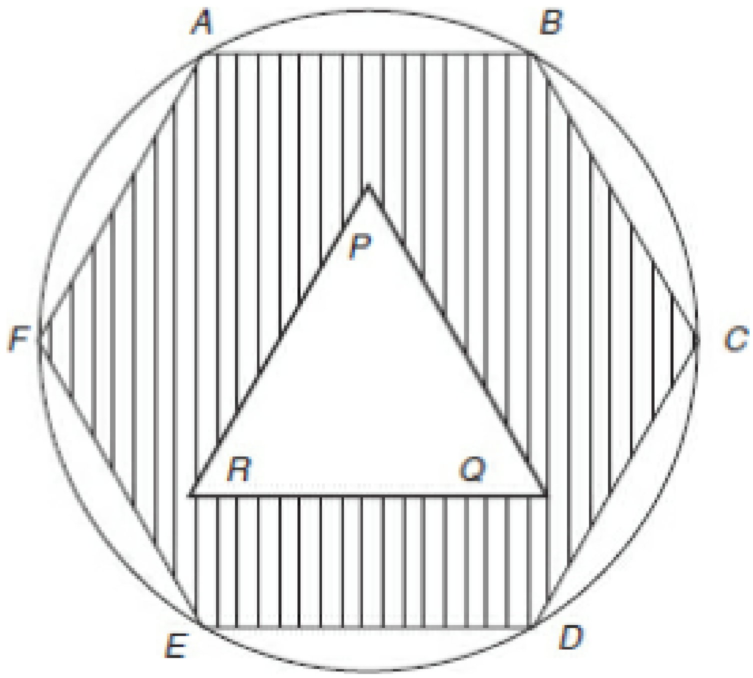
(c) $2^{12} - 1$

(d) $2^{11} - 1$

48. Let $P_1$ be the circle of radius $r$. A square $Q_1$ is inscribed in $P_1$ such that all the vertices of the square $Q_1$ lie on the circumference of $P_1$. Another circle $P_2$ is inscribed in $Q_1$. Another square $Q_2$ is inscribed in the circle $P_2$. Circle $P_3$ is inscribed in the square $Q_2$ and so on. $S_N$ is the area between $Q_N$ and $P_{N+1}$ where $N$ represents the set of natural numbers. If the ratio of sum of all such $S_N$ to that of the area of the square $Q_1$ is $\frac{a - \pi}{b}$ then $a + b$ =?



49. In the figure, *ABCDEF* is a regular hexagon and *PQR* is an equilateral triangle of side '*a*'. The area of the shaded portion is $23\sqrt{3}$ cm2 and *CD*: *PQ*:: 2 : 1. If the area of the circle circumscribing the hexagon is $X\pi$ cm2, then $X$ =?




50. Let $S_1, S_2$, ....be the squares such that for each $n \geq 1$, the length of the diagonal of $S_n$ is equal to the length of the side of $S_{n+1}$. If the length of the side of $S_3$ is 4 cm, what is the area of the square $S_{11}$ ?

51. At the center of a city's municipal park there is a large circular pool. A fish is released in the water at the edge of the pool. The fish swims north for 30 meters before it hits the edge of the pool. It then turns east and swims for 40 meters before it hits the edge of the pool. If the area of the pool is $X\pi$ m2 then $X$ is

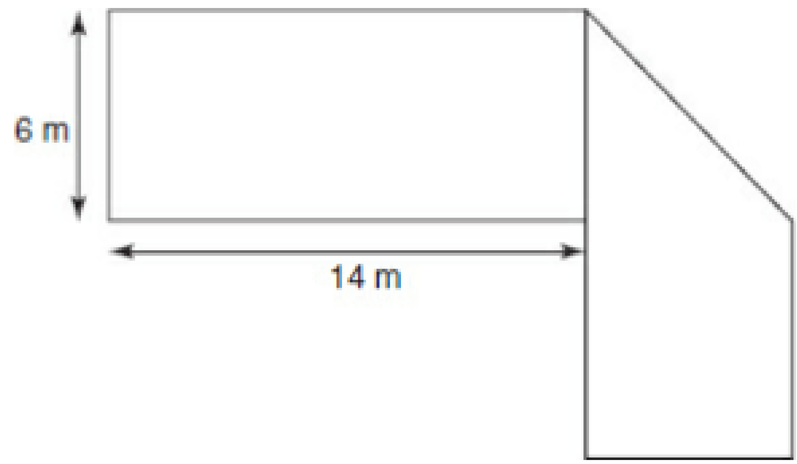
(a) 625

(b) 125

(c) 250

(d) 500

52. The figure below has been obtained by folding a rectangle. The total area of the figure (as visible) is 144 square meters. Had the rectangle not been folded, the current overlapping part would have been a square. What would have been the total area of the original unfolded rectangle (in m2)?




53. A solid metal cylinder of 10 cm height and 14 cm diameter is melted and re-cast into two cones in the proportion of 3: 4 (volume), keeping the height 10 cm. What would be the percentage change in the flat surface area before and after?

(a) 9%

(b) 16%

(c) 25%

(d) 50%

54. A circular road is constructed outside a square field. The perimeter of the square field is 200 ft. If the width of the road is $7\sqrt{2}$ ft. and cost of construction is ₹100 per sq. ft, find the lowest possible cost to construct 50% of the total road.

(a) ₹70,400

(b) ₹125,400

(c) ₹140,800

(d) ₹235,400

55. Diameter of the base of a water-filled inverted right circular cone is 26 cm. A cylindrical pipe, 5 mm inradius, is attached to the surface of the cone at a point. The perpendicular distance between the point and the base (the top) is 15 cm. The distance from the edge of the base to the point is 17 cm, along the surface. If water flows at the rate of 10 meters per minute through the pipe, how much time would elapse before water stops coming out of the pipe?

(a) ≥ 5.2 minutes

(b) ≥ 4.5 minutes but < 4.8 minutes

(c) ≥ 4.8 minutes but < 5 minutes

(d) ≥ 5 minutes but < 5.2 minutes

56. Consider a rectangle *ABCD* of area 90 units. The points *P* and *Q* trisect *AB*, and *R* bisects *CD*. The diagonal *AC* intersects the line segments *PR* and *QR* at *M* and *N* respectively. What is the area of the quadrilateral *PQNM*?

(a) > 9.5 and ≤ 10

(b) > 10 and ≤ 10.5

(c) > 10.5 and ≤ 11

(d) > 11 and ≤ 11.5

57. The central park of the city is 40 meters long and 30 meters wide. The Mayor wants to construct two roads of equal width in the park such that the roads intersect each other at right angles and the diagonals of the

park are also the diagonals of the small rectangle formed at the intersection of the two roads. Further, the mayor wants that the area of the two roads to be equal to the remaining area of the park. What should be the width of the roads?

(a) 10 meters

(b) 12.5 meters

(c) 14 meters

(d) 15 meters

58. A rectangular swimming pool is 48 m long and 20 m wide. The shallow edge of the pool is 1 m deep. For every 2.6 m that one walks up the inclined base of the swimming pool, one gains an elevation of 1 m. What is the volume of water (in cubic meters), in the swimming pool? Assume that the pool is filled up to the brim.

(a) 528

(b) 960

(c) 6790

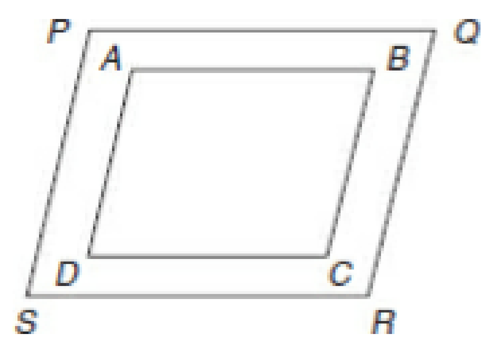
(d) 10560

59. A thread is wound on a cylinder such that it makes exactly twenty-four complete turns around the cylinder. The two ends of thread touch the top and bottom of cylinder. If cylinder has a radius of 15 cm and its curved surface area is 2880 $cm^2$ then find the length of the string

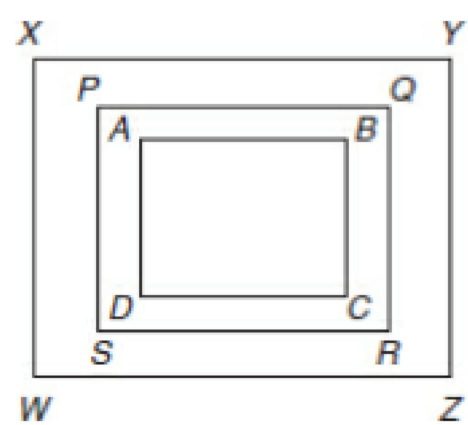
60. If the length of the minute-hand and the hour-hand of a clock are 4.2 cm and 2.1 cm and the minute hand covers an area of 110.88 cm2, then find the area covered by hour hand during the same period.

[take if $\pi = 22/7$]

61. Hiru has a rhombus-shaped farm *ABCD*. This farm is surrounded, by a path of width 2 m, as shown in the diagram. If $\angle ADC = 30°$, $AD = 10$ m. Then find the area of the path.




62. A series of infinite concentric squares are drawn as shown below. Starting with the first square *ABCD*, subsequent squares drawn are *PQRS*, *XYZW* and so on as shown in the diagram. If areas of the squares *ABCD*, *PQRS*, *XYZW*, .... are 1, 3/2, 7/4, 15/8... and so on, then find the area of the diagram when the infinite number square is drawn.



63. Three regular hexagons are drawn such that their diagonals cut each other at the same point and area $A_1:A_2:A_3$ = 1:2:3. Then the ratio of the length of the sides of the regular hexagons (from the smallest to the largest) is:

(a) $1:\sqrt{2}:\sqrt{6}$

(b) $1:\sqrt{2}:\sqrt{3}$

(c) $1:\sqrt{3}:2\sqrt{2}$

(d) $1:\sqrt{3}:\sqrt{6}$


## ANSWER KEY

### *GEOMETRY*

***Level of Difficulty (I)***

1. (a)
2. (d)
3. (c)
4. (a)
5. 75°
6. (c)
7. (a)

8. (b)

9. (a)

10. (d)

11. (d)

12. (a)

13. 5

14. 1.5

15. (b)

16. (b)

17. $\sqrt{15}$cm

18. $\sqrt{211}$cm

19. (b)

20. 8

21. 17.97

22. (a)

23. (d)

24. (d)

25. (b)

26. (a)

27. 16 cm

28. 12 cm

29. 250 $cm^2$

30. (b)

31. 300

32. 111.75

33. 80

34. 120

35. (b)

36. (c)

37. 60°

38. 9

39. 4 cm

40. $\sqrt{20}$cm

41. 40

42. (a)

43. 90°

44. 50°

45. 30°

46. 9

47. 20

48. 10

49. 55°

50. 50°

51. 40°

52. 50°

53. 120°

54. (d)

55. 10°

56. (b)

57. 12.5cm

58. 8cm

59. 16cm

60. 3.6

61. (d)

62. (b)

63. b

64. 0.5

65. (a)

66. 42 cm

67. (b)

68. (d)

69. (c)

70. (c)

71. (b)

72. (c)

73. 14

74. 10

75. 5 m

***Level of Difficulty (II)***

1 (a)

2. $\sqrt{320}$cm

3. $8\sqrt{5}$ cm

4. 4

5. 60 $cm^2$

6. 4

7. 5

8. 45°

9. 172°

10. 4.56 cm

11. $8\sqrt{5}$ $cm^2$

12. $2\sqrt{6}$cm

13. 60°

14. 80°

15. 20°

16. 15 cm

17. $\sqrt{540}$cm

18. 1

19. 5

20. $\frac{3}{4} \times \sqrt{11}$

21. 13

22. 19

23. $\sqrt{544}$ cm

24. 120

25. (d)

26. (b)

27. (b)

28. (b)

29. (a)

30. (d)

31. (b)

32. (a)

33. (a)

34. (c)

35. (d)

36. (a)

37. (b)

38. (b)

39. (b)

40. (a)

41. (d)

42. 18

43. 19

44. 5

45. (b)

46. (b)

47. (a)

48. (d)

49. (b)

50. (b)

51. 4cm

52. 19:5

53. 40°

54. 3

55. (b)

56. 16:9

57. (c)

58. (c)

59. (d)

60. 15

61. (a)

62. 1063

63. (d)

64. (d)

65. 16
66. (b)
67. (a)
68. (d)
69. (c)
70. 5600
71. (c)

## *MENSURATION*

***Level of Difficulty (I)***

1. (b)
2. (d)
3. (b)
4. (a)
5. (d)
6. (b)
7. (d)
8. (c)
9. (d)
10. (d)
11. (b)
12. (b)
13. (a)
14. (b)
15. (d)
16. (c)
17. (c)

18. (c)
19. (d)
20. (d)
21. (a)
22. (b)
23. (b)
24. (b)
25. (c)
26. (c)
27. (a)
28. (a)
29. (c)
30. (a)
31. (d)
32. (d)
33. (b)
34. (d)
35. (b)
36. (b)
37. (a)
38. (c)
39. (d)
40. (b)
41. (b)
42. (c)
43. (c)
44. (c)
45. (d)

46. (c)

47. (a)

48. (b)

49. (a)

50. (b)

51. $\pi x(2r + x)$

52. 29%

53. (b)

54. 8000

55. 8

56. 814

57. 376.75 $cm^2$

58. 143 cm

59. (b)

60. 480 $\pi$

61. 385 $cm^3$

62. (a)

63. 100

64. 105 m

65. 1200

66. 900

67. 288000

68. 9240 $cm^2$

69. 510 sq. m.

70. 20

***Level of Difficulty (II)***

1. (d)

2. (a)

3. (b)

4. (b)

5. (c)

6. (c)

7. (b)

8. (a)

9. (c)

10. (d)

11. (a)

12. (c)

13. (a)

14. (a)

15. (d)

16. (b)

17. (a)

18. (b)

19. (c)

20. (d)

21. (d)

22. (d)

23. (c)

24. (d)

25. (d)

26. (d)

27. (c)

28. (d)

29. (a)
30. (c)
31. (d)
32. (b)
33. (a)
34. (a)
35. (b)
36. (c)
37. (c)
38. (d)
39. (a)
40. (a)
41. (b)
42. (b)
43. (d)
44. (a)
45. (c)
46. (d)
47. (c)
48. 6
49. 16
50. 4096 $cm^2$
51. (a)
52. 162 $m^2$
53. (d)
54. (b)
55. (d)

56. (d)

57. (a)

58. (d)

59. 408cm

60. 2.31

61. 112m2

62. 2

63. (d)

**Solutions and Shortcuts**

**GEOMETRY**

***Level of Difficulty (I)***

1. When the length of stick = 20 m, then length of shadow = 10 m, i.e. in this case, length = 2 × shadow. With the same angle of inclination of the sun, the length of tower that casts a shadow of 50 m ⇒ 2 × 50 m = 100 m.

   Therefore, height of tower = 100 m

2. D$ABC$ ~ $\Delta EDC$

   Then $\frac{AC}{EC} = \frac{BC}{DC} = \frac{AB}{ED}$

   Then $\frac{AC}{4.2} = \frac{4}{3} = AC$ = 5.6 cm and

   $\frac{BC}{4.8} = \frac{4}{3} = BC$ = 6.4 cm

3. For similar triangles ⇒ (Ratio of sides)2 = Ratio of areas

   Then as per question = $\left(\frac{36}{x}\right)^2 = \frac{144}{81}$

   {Let the longest side of $\Delta DEF = x$}

   ⇒ $\frac{36}{x} = \frac{12}{9}$ ⇒ $x$ = 27 cm

4. (Ratio of corresponding sides)$^2$ = Ratio of area of similar triangles

∴ Ratio of corresponding sides in this question

= $\sqrt{\frac{16}{25}} = \frac{4}{5}$. This will also be their height ratio.

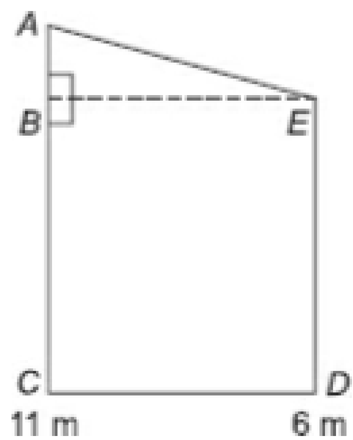
5. Let $\angle BCA = x,\ \angle BDC = 3x$

$x + 3x + 30° + 50° = 180°$ or $x = 25°$

$\angle BDC = 3x = 75°$

$\angle BAC = \angle BDC = 75°$

6.




$BC = ED = 6$ m

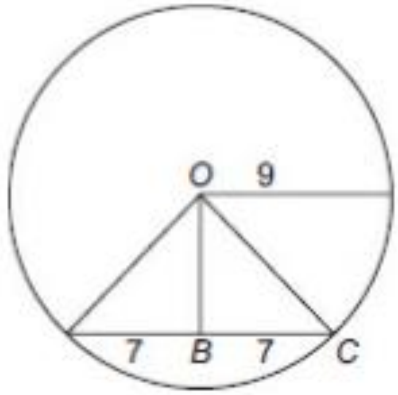
So $AB = AC - BC = 11 - 6 = 5$ m

$CD = BE = 12$ m

Then by Pythagoras theorem:

$AE^2 = AB^2 + BE^2 \Rightarrow AE = 13$ m

7.




In the $\Delta OBC$; $BC$ = 7 cm and $OC$ = 9 cm, then using Pythagoras theorem

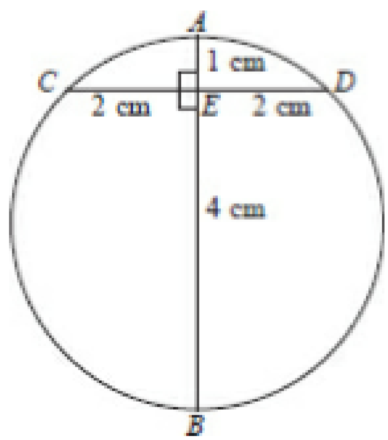
$OB2 = OC2 - BC2$

$OB = \sqrt{32}$ = 5.66 cm (approx.)

8. $AE \times BE = CE \times DE$

$AE \times 4 = 2 \times 2$

$AE$ = 1 cm

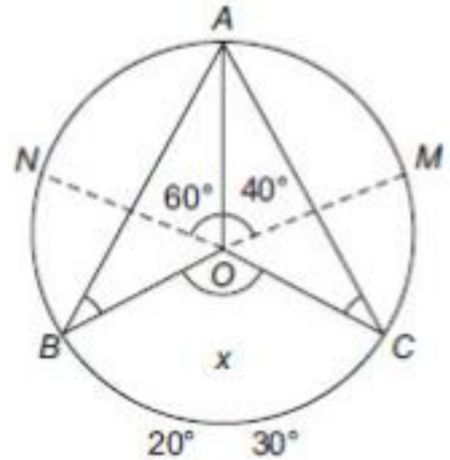



Chord $AB$ of the circle is the perpendicular bisector of the chord $CD$. Hence, $AB$ is the diameter of the circle.

$AB = AE + EB = 1 + 4 = 5$ cm. Hence, Option (b) is correct.

9. $\angle x = 35°$; because angles subtended by an arc, anywhere on the circumference are equal.

10.




$\angle AOM = 2\angle ABM$ and

$\angle AON = 2\angle ACN$

because angle subtended by an arc at the center of the circle is twice the angle subtended by it on the circumference on the same segment.

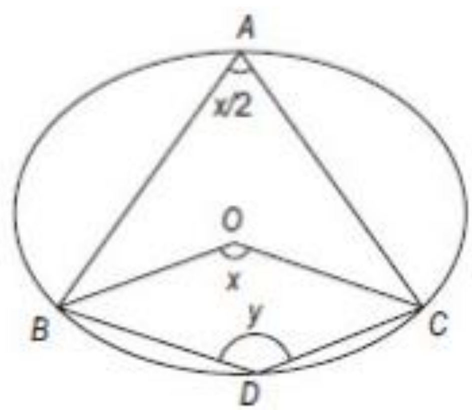
$\angle AON = 60°$ and $\angle AOM = 40°$

$\angle X = \angle AON + \angle AOM$

($\because$ vertically opposite angles).

$\angle X = 100°$

Alternately, you could also solve this using the following process:




In the given figure, join the points $BD$ and $CD$. Then, in the cyclic quadrilateral $ABDC$, the sum of angles $x/2$ and $y$ would be 180°. Hence, $y$ =

180 – $x/2$. Also, the sum of the angles $OBD + OCD = 180 - 20 - 30 = 130°$. Therefore, $x + y = 230$ (as the sum of the angles of the quadrilateral $OBDC$ is 360). Solving, the two equations, we get $x = 100$.

11. The triangle $BOC$ is an isosceles triangle with sides $OB$ and $OC$ both being equal as they are the radii of the circle. Hence, the angle $OBC$ = angle $OCB$ = 30°. Hence, the third angle of the triangle $BOC$ viz: angle $BOC$ would be equal to 120°. Also, $BOC = AOD = 120°$. Hence, in the isosceles triangle $DOA$, angle $ODA$ = angle $DAO = x = 30°$.

12. By the rule of tangents, we know:

    $6_2 = (5 + x)5 \Rightarrow 36 = 25 + 5x \Rightarrow 11 = 5x \Rightarrow x = 2.2$ cm

***Solutions for Questions 13 and 14*:**

13. $MP = \sqrt{OP^2 - OM^2} = \sqrt{10^2 - 6^2} = 8$cm

    Triangle $OMP$ and triangle $ACP$ are similar.

    $OP/MP = AP/CP$

    $AP = CP \times \frac{OP}{MP} = 4 \times \frac{10}{8} = 5$cm

14. Similarly, $AC$ = 3 cm, $AB$ = 6 cm

    In-radius of triangle $ABP$ =

    $$\frac{\frac{1}{2} \times 4 \times 6}{\frac{1}{2}(6+5+5)} = \frac{24}{16} = 1.5\text{cm}$$

15. Use the formula: In-radius = Area/ Semi-perimeter = 24/12 = 2 cm

16. $\angle APB = 90°$ (angle in a semicircle = 90°)

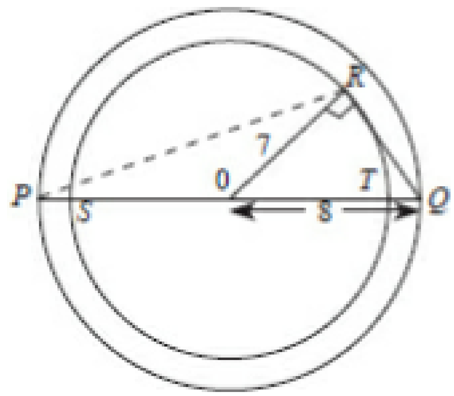
    $\angle PBA = 180 - (90 + 25) = 65°$

$\angle TPA = \angle PBA$ (the angle that a chord makes with the tangent, is subtended by the chord on the circumference in the alternate segment)

$= 65°$

(**Note:** This is also called as the Alternate Segment Theorem.)

17.




$RQ$ is a tangent of inner circle.

$RQ = \sqrt{8^2 - 7^2} = \sqrt{15}$ cm

18. In triangle $PQR$, apply Apollonius theorem:

$PR^2 + RQ^2 = 2(OR^2 + OQ^2)$

$PR^2 = 2(7^2 + 8^2) - 15 = \sqrt{211}$

19. $\angle ABC = 180° - 130° = 50°$

($\therefore$ sum of angles on a line = 180°)

$\angle ADC = 180° - \angle ABC = 130°$

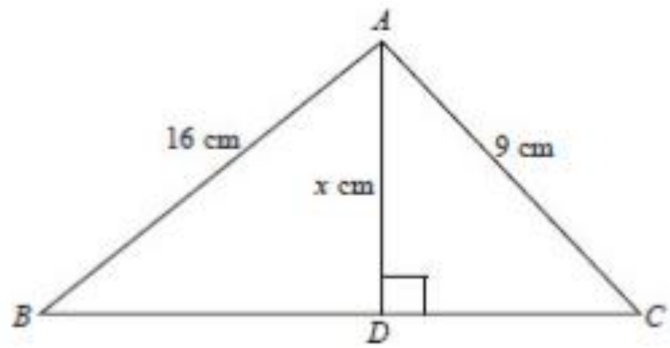
($\because$ opposite angles of a cyclic quadrilateral are supplementary)

$\angle x = 180° - 130° = 50°$

($\therefore$ sum of angles on a line = 180°)

***Solutions for Questions 20 and 21:***

20.




Area of the triangle = $\frac{1}{2} \times BC \times x = \frac{16 \times 9 \times BC}{4 \times 9}$

$x = 8$ cm

21. $BD = \sqrt{16^2 - 8^2} = 13.85$ cmcm

$DC = \sqrt{9^2 - 8^2} = 4.12$

$BC = BD + DC = 13.85 + 4.12 = 17.97$ cm

22. $\angle BAC = 30°$ ($\because$ angles subtended by an arc anywhere on the circumference in the same segment are equal)

In $\Delta BAC$; $\angle x = 180° - (110° + 30°) = 40°$

($\because$ sum of angles of a triangle = 180°)

23. As $L_4$ and $L_3$ are not parallel lines, so there cannot be any relation between 80° and $x°$.

Hence, the answer cannot be determined.

24. Perimeter of the figure = 10 + 10 + 6 + 6π = 26 + 6π

25. Since the lines *AB* and *CD* are parallel to each other, and the lines *RD* and *AN* are parallel, it means that the triangles *RBF* and *NCI* are similar to each other. Since the ratio of *CN*: *BR* = 1.333, if we take *BR* as 3, we will get *CN* as 4. This means that the ratio of *BF*: *CI* will also be 3:4. Also, the ratio of *BR*: *RS*: *ST*: *TA* = *BF*: *FG*: *GH*: *HI* = 3:5:2:7 (given). Hence, the correct answer is 3:5:2:7:4.

26. Interior angle of *n*-sided regular polygon =

$$\frac{n-2}{n}\times\pi = x$$

Hence, the number of sides, $n = \frac{2\pi}{\pi - x}$

The number of diagonals in an *n*-sided polygon = $n_{C_2} - n = k \times n$ (given)

$\frac{n(n-1)}{2} - n = kn$ or $k = \frac{n-3}{2}$

Hence, $k = \frac{\frac{2\pi}{\pi - x} - 3}{2} = \frac{3x-\pi}{2(\pi - x)}$. Option (a) is correct.

27. $PR = \sqrt{PQ^2 + QR^2}$ (Using Pythagoras Theorem)

$= \sqrt{15^2 + 20^2} = \sqrt{625} = 25 \text{ cm}$

$$SR = \frac{QR^2}{PR} = \frac{20^2}{25} = 16 \text{ cm}$$

(Property of a right-angled triangle)

28. $\frac{1}{SQ^2} = \frac{1}{PQ^2} + \frac{1}{QR^2}$ (Property of *a* right-angled triangle)

$$\frac{1}{SQ^2} = \frac{1}{15^2} + \frac{1}{20^2}$$

$SQ_2 = \frac{20^2 \times 15^2}{20^2 + 15^2} \rightarrow SQ = \frac{20 \times 15}{25} = 12\,\text{cm}$

29. $\Delta ABD \sim \Delta ACB$

$$\frac{\text{area of } \Delta\ ABC}{\text{area of } \Delta\ ADB} = \frac{AC^2}{AB^2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

(Using the property that in similar figures, area ratios are squared off the side ratios)

Area of $\Delta ABC = 90 \times \frac{25}{9} = 250$

30. Inradius of an equilateral triangle = $\frac{side}{2\sqrt{3}}$ (formula for in-radius)

Side = $2\sqrt{3} \times \sqrt{3} = 6\,\text{cm}$

Required area = $\frac{\sqrt{3}}{4} \times 6^2 = 9\sqrt{3}\,\text{cm}^2$(Using the formula that the area of an equilateral triangle of side $a$ is given by the formula: $\frac{\sqrt{3}}{4} \times a^2$.

31. In-radius of $\Delta ABC = \frac{(AB + BC) - AC}{2} = \frac{20}{2} = 10$ (Formula for inradius)

Semi-perimeter = $s = \frac{60}{2}$

$$r = \frac{\text{Area}}{s}$$

$\text{Area} = r \times s$(formula for area of any triangle using the semi-perimeter and inradius)

$= 10 \times 30 = 300\ \text{cm}^2$

32. 3 × (Sum of squares of the sides of a triangle) = 4 × (Sum of squares of the medians of the triangle)

$$\frac{3}{4}\left(6^2 + 7^2 + 8^2\right) = \text{Sum of square of medians}$$

$= \frac{3}{4}[149] = 111.75 \text{ cm}^2$

33. In $\Delta QTR$: $\angle TOR = 180° - (90° + 30°) = 180° - 120° = 60°$ (angles of a triangle add up to 180)

In $\Delta QPS : \angle PSQ = 180^o - (60^o + 40^o) = 80^o$

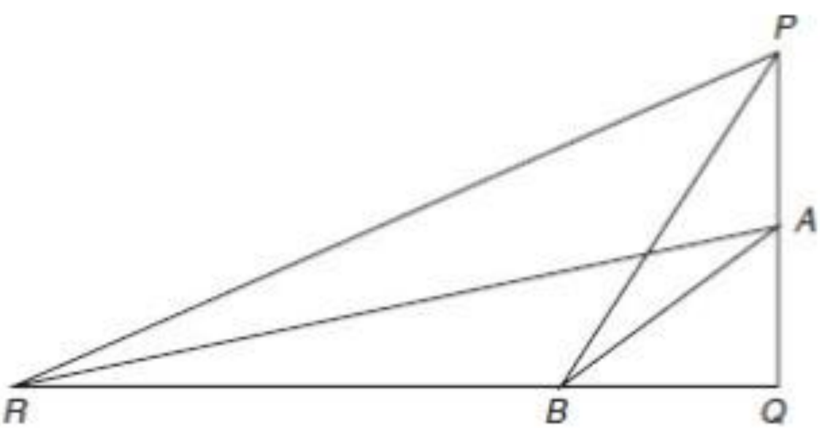
34. According to the Alternate Segment Theorem:

$\angle SRQ = = \angle STR = 60°$

□$TSUR$ is a cyclic quadrilateral. Therefore $\angle STR + \angle SUR = 180°$

$\angle SUR = 180° - 60° = 120°$

35. $AR^2 = AQ^2 + RQ^2$ (Using Pythagoras Theorem)




$PB^2 = PQ^2 + BQ^2$ (Pythagoras Theorem)

$AR^2 + PB^2 = (PQ^2 + RQ^2) + (AQ^2 + BQ^2)$

$= PR^2 + AB^2$

36. $AB \parallel CD$ therefore $\Delta ABF$ and $\Delta CDF$ are similar to each other.

$\frac{DF}{BF} = \frac{b}{a}$ (1)

Similarly $\Delta BCD$ and $\Delta BEF$ are similar each other:

$\frac{BD}{BF}=\frac{b}{c}$ (2)

By adding (1) and (2), we get

$$\frac{BD+DF}{BF}=\frac{b}{a}+\frac{b}{c}$$

$$\frac{BF}{BF}=\frac{b}{a}+\frac{b}{c}=1$$

$$\frac{1}{b}=\frac{1}{a}+\frac{1}{c}$$

$$\frac{1}{b}-\frac{1}{a}=\frac{1}{c}$$

37. $PR = PS$

$\angle PRS = \angle PSR$ (angles opposite equal sides are equal in an isosceles triangle)

$\angle PRS = x$

Let $\angle RPQ = y$

$PR = QR$

$\angle RPQ = \angle PQR = y$(angles opposite equal sides are equal in an isosceles triangle)

$\angle PRQ = 180° - 2y = 180° - x$

$x = 2y \; or \; y = \frac{x}{2}$

$\angle RPS = 180° - 2x$

$\angle QPR + \angle RPS + 90° = 180°$

$$\frac{x}{2}+180^o-2x+90^o=180^o$$

$$90^o=\frac{3x}{2}$$

$$x=60^o$$

38. 72, 21, 75 form a Pythagorean triplet. The triangle is a right-angled triangle.

The measure of in-radius of a right angle triangle =

$$\frac{\text{Sum of legs of the right angled triangle-hypotenuse}}{2}$$

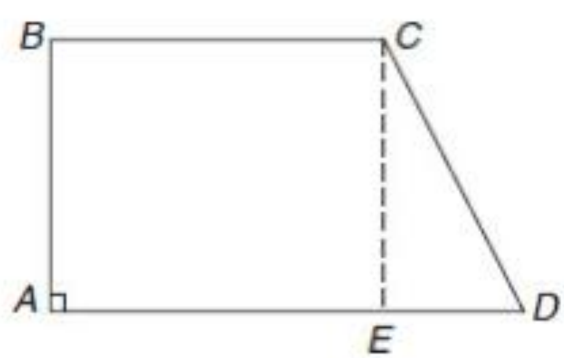
$$\frac{72+21-75}{2}=\frac{18}{2}=9$$

39. Area of a trapezium $=\frac{1}{2}\times$ (sum of parallel sides) $\times$(sum of parallel sides) × height

$$28=\frac{1}{2}(6+8)AB$$

$AB$ = 4 cm

40.




Draw $CE \perp AD$.

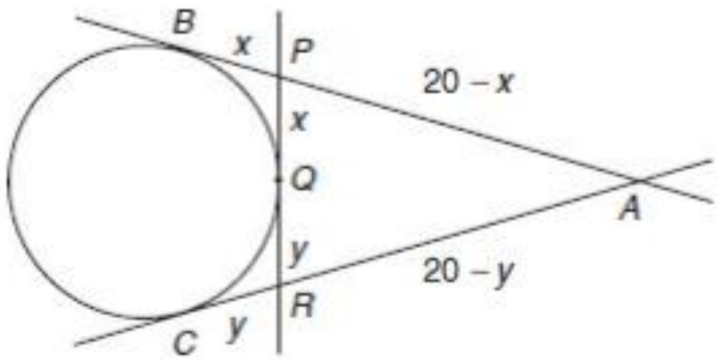
$DE$ = 8- 6 = 2 cm

$CE = AB = 4$ cm

$CD = \sqrt{(4)^2 + (2)^2} = \sqrt{20}$ cm

(Using Pythagoras theorem)

41. If $BP = x$, $CR = y$ then $PQ = BP = x$, $RC = QR = y$ (The two tangents to a circle from an external point are equal in length)




Perimeter of $\Delta APR = AP + PR + AR$

$= 20 - x + x + y + 20 - y$

$= 40$ units

42. $BD$ is the median of the triangle $ABC$ and $AD = DC = BD$. therefore $\Delta ABC$ is a right-angled triangle. Option (a) is correct.

43. $\Delta ABC$ is a right-angled isosceles triangle.

$\therefore AB = BC$ and $\angle A = \angle C = 45°$

$\therefore DB = DC$

$\therefore \angle DBC = \angle DCB$

$\angle DBC = 45°$

$\angle BDC = 180° - (45° + 45°) = 90°$

44. $\angle ADC = \angle ABD + \angle BAD$ (Using the property that the exterior angle is equal to opposite interior angles on the triangle $ABD$)

$2\angle ABD = \angle ABD + \angle BAD$

Hence, $\angle BAD = \angle ABD$

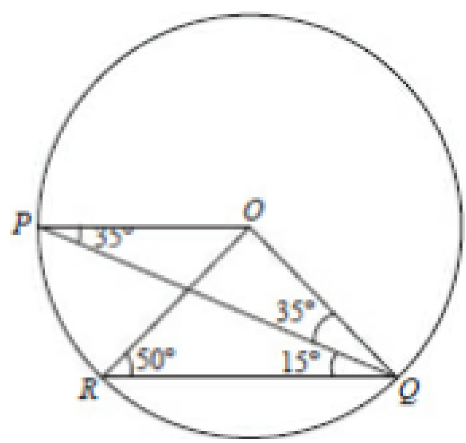
Let $\angle ABD = \angle BAD = \angle ACB = x$ (**Note:** $\angle ACB$ and $\angle ABC$ are equal as the triangle $ABC$ is an isosceles triangle)

Then in $\Delta ABC$ $30^\circ + x + x + x = 180^\circ$

$3x = 150^\circ$

$x = 50^\circ = \angle BAD = \angle BAD$

45.




In $\Delta POQ$:

$OP = OQ$

$\angle OQP = \angle OPQ = 35^\circ$

In $\Delta ORQ$:

$\angle OQR = \angle ORQ = 50^\circ$

$\angle PQR = \angle OQR - \angle OQP = 50° - 35° = 15°$

Hence, $\angle POR = 2\angle PQR = 2 \times 15° = 30°$ (Since, POR is an angle subtended by the minor arc PR at the center, while the angle $PQR$ is an angle subtended by the same arc at a point on the circumference of a circle).

46. $\frac{(2n-4)90^{\circ}}{n} = 120^{\circ}$ (Formula for interior angle of a regular polygon)

$$\frac{n-2}{n} = \frac{120^{o}}{180^{o}} = \frac{2}{3}$$

$3n - 6 = 2n$

$n = 6$

Number of diagonals = ${}_6C_2$ -6 = 9 (Number of diagonals of any $n$ sided polygon is given by the formula ${}_nC_2$-$n$)

47. Let the number of sides of the polygons be $2n$ and $n$ respectively.

As per the question:

$18 = \frac{2n-2}{2n}\times 180° - \frac{n-2}{n}\times 180°$ (formula for interior angle of a regular polygon applied to both the polygons)

$$18 = \left[\frac{n-1}{n} - \frac{n-2}{n}\right]180°$$

$$18 = \frac{1}{n}\times 180°$$

$\Rightarrow n = 10$ or $2n = 20$

48. Let the number of sides in the polygon be $n$ as per the question:

$$(n-2)180° = 40\left(180° - \left[\frac{n-2}{n}\right]180°\right)$$

$$18 = \left[\frac{n-1}{n} - \frac{n-2}{n}\right]180^\circ$$

$$18 = \frac{1}{n} \times 180^\circ$$

$\Rightarrow n = 10$ or $2n = 20$

48. Let the number of sides in the polygon be $n$ as per the question:

$$(n-2)180^\circ = 40\left(180^\circ - \left[\frac{n-2}{n}\right]180^\circ\right)$$

$$(n-2)180^\circ = 40\left\{\frac{n-n+2}{n}\right\}180^\circ$$

$$\frac{2}{n} = \frac{(n-2)}{40}$$

$n = 10$

49. $\angle ABC = \frac{\angle AOC}{2} = \frac{140^\circ}{2} = 70^\circ$ (Using the logic that the angle subtended by an arc at the center of a circle is twice the angle subtended by the same arc at any point on the circle).

$AB = BC$
$\angle BAC = \angle BCA$
$2\angle BCA + \angle ABC = 180^\circ$
$2\angle BCA = 180^\circ - \angle ABC = 180^\circ - 70^\circ = 110^\circ$

$$\angle BCA = \frac{110^\circ}{2} = 55^\circ$$

50. $\angle QOR = 180^\circ - \angle POQ = 180^\circ - 80^\circ = 100^\circ$

$\angle QSR = \frac{\angle QOR}{2} = 50^\circ$ (Using the logic that the angle subtended by an arc at the center of a circle is twice the angle subtended by the same arc at any point on the circle)

51. $OE = OB$ = radius of the circle

$\angle OEB = \angle OBE$(angles opposite equal sides of an isosceles triangle are equal)

In $\Delta OEB : \angle OEB + \angle OBE + \angle BOE = 180^\circ$

$$2\angle OEB + \angle BOE = 180^\circ$$
$$\angle BOE = 180^\circ - 2\angle OEB = 180^\circ - 2\times 70^\circ$$
$$= 40^\circ$$

52. In $\Delta OBA \Rightarrow \angle BAO + 90^\circ + \angle AOB = 180^\circ$

$$\angle BAO = 180^\circ - (90^\circ + \angle AOB)$$

$= 180^\circ - (90^\circ + 40^\circ)$ (Since we have already found the angle $AOB$ as $40°$ in the previous question) = $50°$

53. In $\Delta ACD : AC = AD$

$\angle ACD = \angle ADC = 30^\circ$ (Angles opposite equal sides on an isosceles triangle are equal)

$$\angle CAD = 180^o - (\angle ACD + \angle ADC)$$

$$= 180^\circ - (30^\circ + 30^\circ) = 120^\circ$$

54. In $\Delta ABC$ & $\Delta ABD$

$$\angle ACB = \angle ADB$$
$$\angle ABC = \angle ABD = 90^\circ$$

$AC = AD$

$$\Delta ABC \cong \Delta ABD$$
$$\therefore BC = BD$$
$$\therefore CD = 4 \text{ cm}$$

In $\Delta CAD = \dfrac{CD}{\sin 120^\circ} = \dfrac{AD}{\sin 30^\circ}$ (Using the sine rule and values of sin 30 = 1/2 and value of sin 120 = $\dfrac{\sqrt{3}}{2}$)

$$AD = \frac{4}{\sqrt{3}} \times 2 \times \frac{1}{2} = \frac{4}{\sqrt{3}} \text{ cm}$$

55. $OS = OQ$

$\angle OSQ = \angle OQS = 50^\circ$

$\angle SOQ = 180^\circ - 100^\circ = 80^\circ$ (Angles of a triangle add up to 180)

In $\Delta OSP$:

$80^\circ + 90^\circ + \angle SPR = 180^\circ$ (Angles of a triangle add up to 180)

$\angle SPR = 180^\circ - (80^\circ + 90^\circ) = 10^\circ$

56. Lines joining midpoint of a quadrilateral form a parallelogram.

57. Diagonals of trapezium intersect each other proportionally in the ratio of length of parallel sides. Therefore,

$$\frac{AB}{CD} = \frac{OB}{OD}$$

$$OD = \frac{OB \times CD}{AB} = \frac{5 \times 6}{4} = 7.5 \text{ cm}$$

Length of diagonal $BD = 5 + 7.5 = 12.5$ cm

58. Using the sine rule we get: $\frac{QR}{\sin 45^\circ} = \frac{SR}{\sin 60^\circ}$

$$QR = 4\sqrt{6} \times \frac{1}{\sqrt{2}} \times \frac{2}{\sqrt{3}}$$

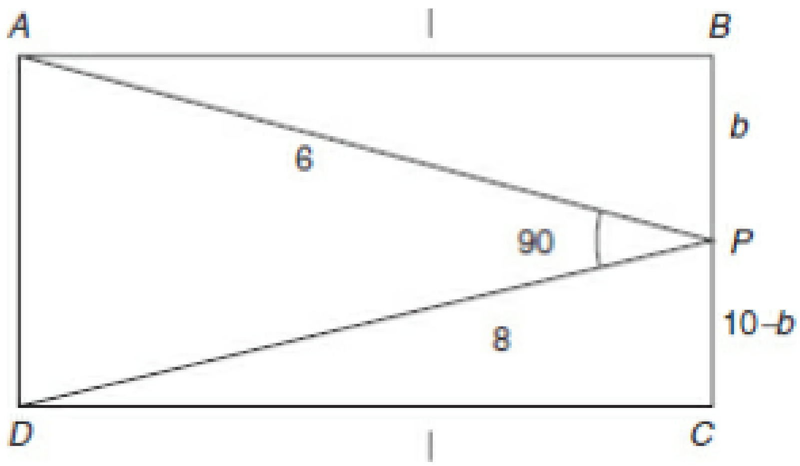
$QR = 8$ cm

59. If a trapezium is inscribed in a circle then it is an isosceles trapezium with equal oblique sides.

$PS = QR = 8$ cm

$PS + QR = 8 + 8 = 16$ cm

60.




In $\Delta APD$: $AP = \sqrt{10^2 - 8^2} = \sqrt{36} = 6$

Let $AB = l$, $BP = b$

Now using the Pythagoras theorem on the triangles *ABP* and *PCD* we get two equations between *l* and *b* as follows:

In $\Delta ABP : l^2 + b^2 = 6^2$ (1)

In $\Delta PCD : l^2 + (10 - b)^2 = 8^2$(2)

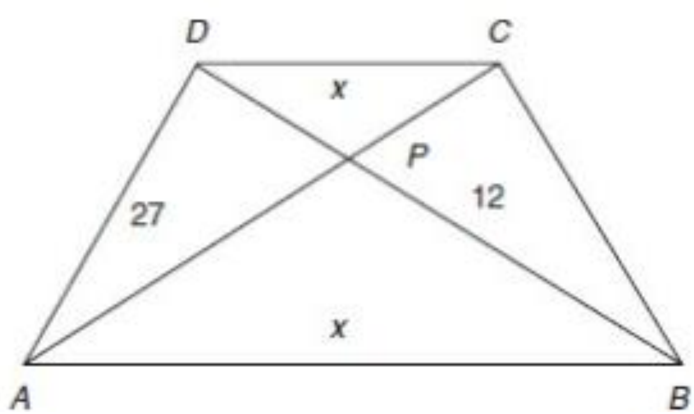
From equation (2) – (1), we get

$-b^2 + (10 - b)^2 = -6^2 + 8^2$

$100 - 20b = 28$

$b = 3.6$

61.



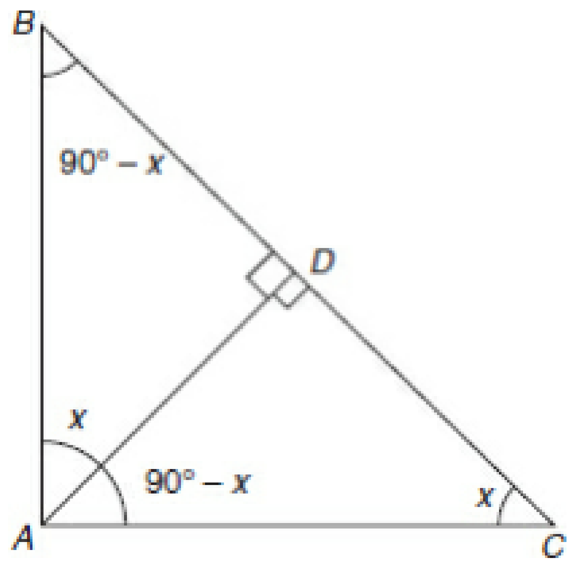
Let the area of $\Delta ABP$ and $\Delta DPC$ is $x$.

$27 \times 12 = x \times x$

$x^2 = 27 \times 12$

$x = 18$

62.




All the corresponding angles of$\Delta ABD$ and in $\Delta CAD$ are equal, so $\Delta ABD$ and $\Delta CAD$ are similar to each other.

So, the first statement is correct.

$\Delta ADB \sim \Delta CDA$.But it is not necessary that both triangles are congruent.

So, the second statement is incorrect.

In $\Delta ADB$ and $\Delta CAB$:

$\angle BAC = \angle ADB$

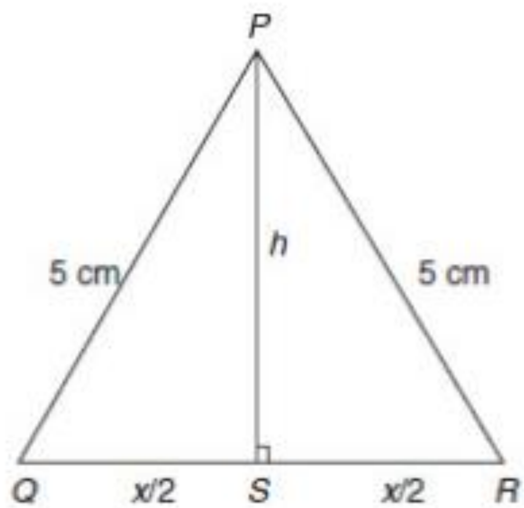
$\angle ABC = \angle ABD$

$\angle BCA = \angle BAD$

So $\Delta ADB \sim \Delta CAB$

So, third statement is also correct. Hence Option (b) is correct.

63.




Area of $\Delta PQR = \frac{1}{2} \times x \times h = 12 \text{ cm}^2$ = 12 cm2

$xh = 24cm^2$

Also in the right angle $\Delta PQS$,

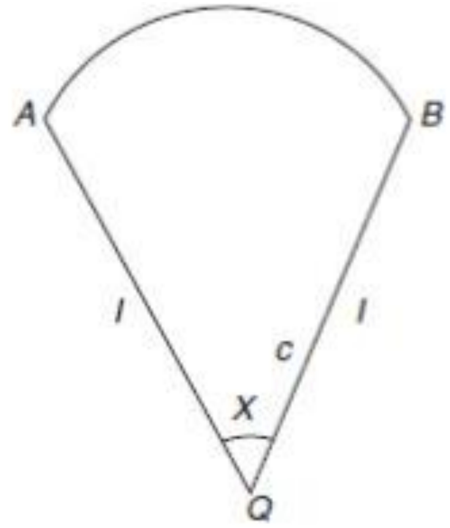
$(5)^2 = \left(\frac{x}{2}\right)^2 + h^2$

$x^2 + 4h^2 = 100$

$x^2 + 4\left(\frac{24}{x}\right)^2 = 100$

The above equation is satisfied only for $x = 6$. So Option (b) is correct.

64. Let the radius of the circle be '$l$'.




Length of $arc = lx$

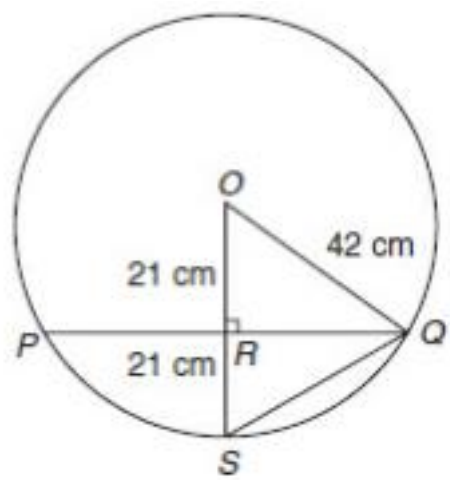
Area of the sector = $\frac{x}{2\pi} \times \pi l^2 = \frac{xl^2}{2}$

According to the question: $\frac{xl^2}{2} = (lx)^2$ or $x = 0.5$. or $x = 0.5$.

65. By applying cosine formula in $\Delta ABC$, we get:

$$cos\theta = \frac{a^2 + b^2 - c^2}{2ab} \text{ or } a^2 + b^2 - 2abcos\theta = c^2$$

66. Here $PQ$ is the chord. Height of chord = $RS$ = 21 cm




Here we need to find the length of chord ($QS$) of half arc $QS$.

$RO = 42 - 21 = 21$ cm.

In $\Delta ORQ$ and $\Delta SRQ$ : $OR = RS$, $QR = QR$, $\angle ORQ = \angle SRQ = 90^\circ$

So both the triangles are congruent.

So $OQ = SQ = 42$ cm.

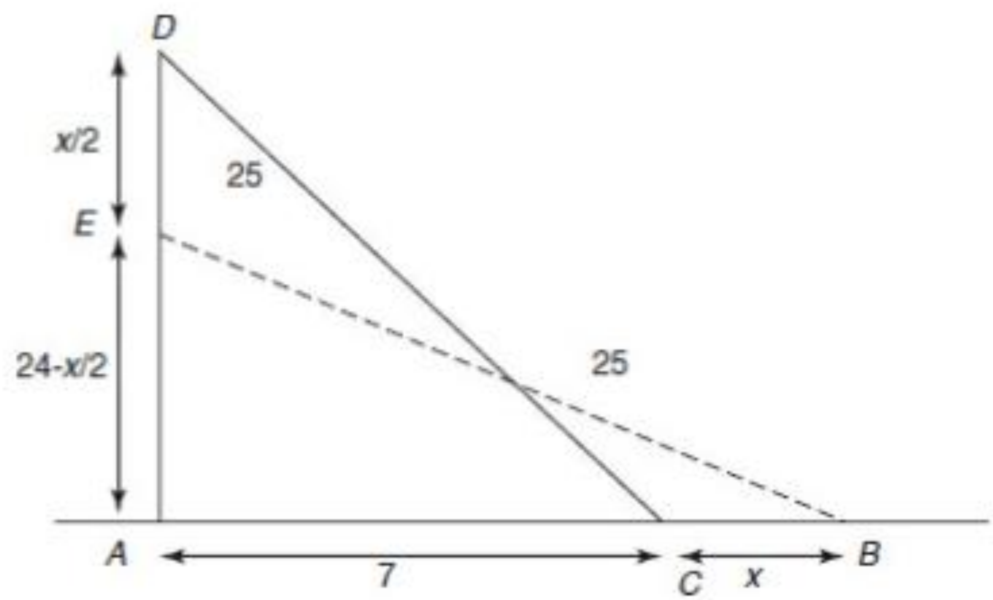
67. Let the other sides of the right angle triangle be $x$ and $y$ respectively.

Then according to the question: $\sqrt{x^2 + y^2} = 97, x + y = 234 - 97 = 137$

Now by checking the options we can see that only Option (b) satisfies both the equations.

So Option (b) is correct.

68. Let the base of the ladder is drawn out by $x$ feet.




In $\Delta EAB$: $\left(24 - \frac{x}{2}\right)^2 + (7 + x)^2 = 25^2$ (Using the Pythagoras theorem)

By solving the above quadratic equation, we get $x = 0, 8$. So, Option (d) is correct.

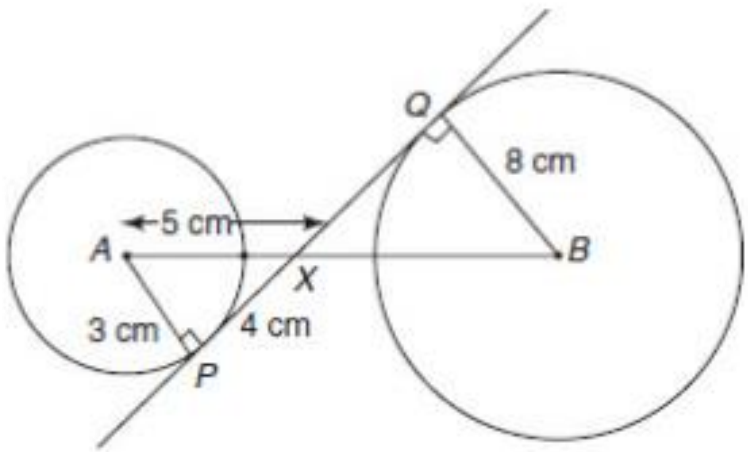
69. $TQ \times TP = TR \times TS$

$8 \times 18 = TR\,(TR + 7)$

$TR = 9$ units

Area of the $\Delta PTS = \frac{1}{2} \times 16 \times 18 \times \sin 60^0 = 72\sqrt{3}$ sq. units

70.




Let $A$ and $B$ are centers of circles. $PX = \sqrt{5^2 - 3^2} = 4$cm

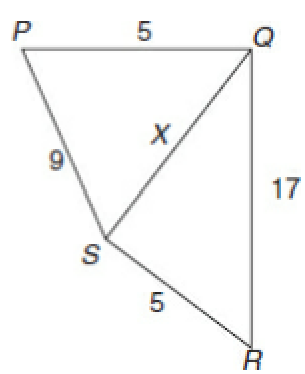
$\Delta APX$ and $\Delta BQX$ are similar to each other as their three angles are equal.

$$\frac{AP}{BQ} = \frac{PX}{QX}$$

$QX = 8 \times \frac{4}{3} = 10.66$cm

$PQ = PX + XQ = 4 + 10.66 = 14.66$cm

71. In a triangle the sum of any two sides > third side.

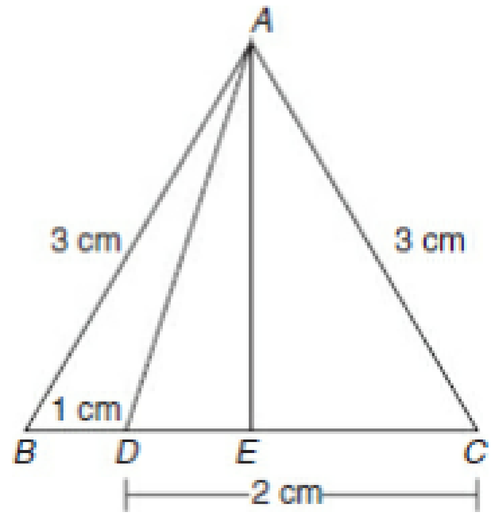


In $\Delta PQS$ $5 + 9 > x$ ; $x < 14$

In $\Delta RQS$ $x + 5 > 17$ ; $x > 12$

$14 > x > 12$

72.




Draw $AE \perp BC$. Since $ABC$ is an equilateral triangle so $AE$ will bisect $BC$.

$DE = DC - EC = 2 - 3/2 = ½$ cm

$AE = \sqrt{3^2 - 1.5^2} = \frac{3}{2}\sqrt{3}$ cmcm

$AD = \sqrt{\left(\frac{3}{2}\sqrt{3}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{7}$ cm

73. Number of quadrilaterals that can be formed = ${}_8C_4$ = 70.

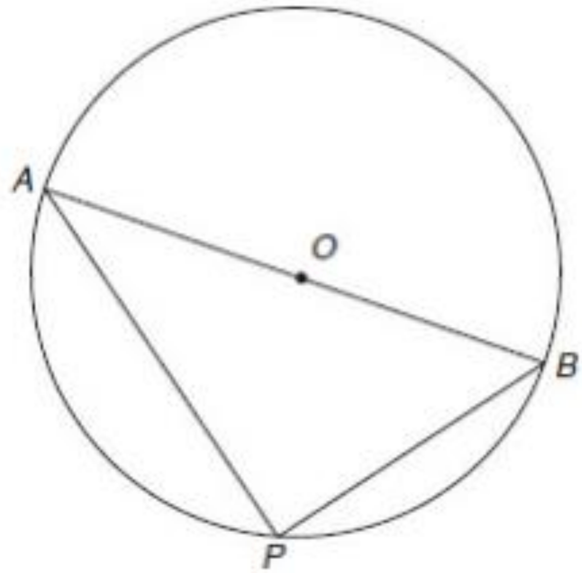
Number of triangles that can be formed = ${}_8C_3$ = 56.

Required difference = 70 – 56 = 14

74. According to the figure, the area of the triangle $QAB$ = area of square $PQRS$ – area of $\Delta$ $ABS$- area of $\Delta PAQ$ - area of $\Delta QBR$

= 36 – 2 – 12 – 12 = 10

75.




Let '$p$' be the position of pole and $A$ and $B$ are the gates referred to in the question. We are given that $AP - BP = 7$;

$AB$ is the diameter. Therefore $\Delta APB$ is a right-angled triangle:

$$AB^2 = AP^2 + BP^2$$
$$13^2 = (7 + BP)^2 + BP^2$$

By solving we get $BP = 5$ m and $AP = 12$ m.

Required shortest distance = 5 m.

***Level of Difficulty (II)***

1. Let the angles of the convex nonagon be $x - 4y, x - 3y, x - 2y, x - y, x, x + y, x + 2y, x + 3y, x + 4y$. (assuming 9 numbers in an arithmetic progression)
   Sum of all the angles of the nonagon = $(9 - 2) \times 180° = 1260°$.

   Sum of all the angles of the nonagon = $9x$

   According to the question: $9x = 1260°$ or $x = 140°$.

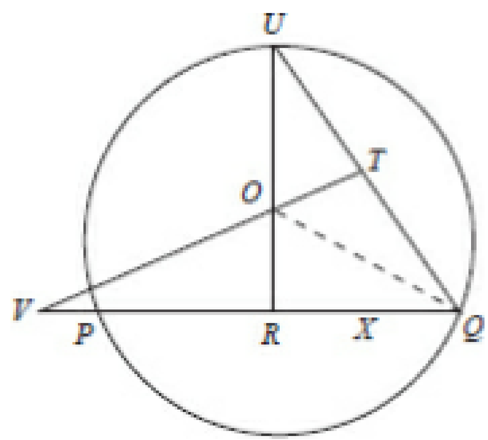
The smallest and the largest angles are $x - 4y$ and $x + 4y$. As this is a convex polygon, so $x + 4y < 180°$

$140° + 4y < 180°$

$y < 10°$.

The smallest angle must be greater than $140° - 40° = 100°$. Hence, option (a) is correct.

***Solutions for Questions 2 to 5:***




2. Connect points $Q$ and $O$.

   $OR = 6$ cm, $QR = PQ/2 = 16/2 = 8$ cm

   $OQ = \sqrt{6^2 + 8^2} = 10$ cm (and this is the radius of the circle)

   $UR = UO + OR = 10 + 6 = 16$ cm

   $\Delta URQ$ is a right-angle triangle. $UQ =$

   $\sqrt{UR^2 + RQ^2} = \sqrt{16^2 + 8^2} = \sqrt{320}$ cm

3. As $OT \perp UQ$, triangles $URQ$ and $VTQ$ are similar to each other. Hence,

   $\frac{UR}{VT} = \frac{RQ}{TQ} \rightarrow VT = UR \times \frac{TQ}{RQ} = \frac{16 \times 80^{0.5}}{8} = 8\sqrt{5}$ cm

4. $VP = VQ - PQ = 20 - 16 = 4$ cm

5. Area of triangle *VOQ* =

$$\frac{1}{2} \times VQ \times OR = \frac{1}{2} \times 20 \times 6 = 60 \text{ cm}^2$$

6. Area of the polygon = semi-perimeter of the polygon × in-radius

$225a = \frac{na}{2} \times r$(Let the polygon has '$n$' sides).

$n \times r = 50$

As $n$ and $r$ both are positive integers, possible values of $(n, r)$ are (50, 1), (25, 2), (10, 5), (5, 10). Hence, four such polygons are possible.

7. Every exterior angle of an $n$-sided polygon is $360°/n$.

$360°/n$ must be an integer and $n$ must be an odd number and $n > 2$.

$360 = 2^3 \times 3^2 \times 5$

Number of odd factors of 360 = (2 + 1) (1 + 1) = 6.

out of these six odd factors, one factor is 1, so total five values are possible for $n$.

8. Every exterior angle of an $n$-sided polygon is $360°/n$. $360°/n$ must be an integer and $n$ must be an odd number and $n > 2$.

$360 = 2^3 \times 3^2 \times 5$

The largest odd factor of $360 = 3^2 \times 5 = 45$. Hence, maximum possible value of $n = 45$.

9. Every exterior angle of an $n$-sided polygon is $360°/n$. $360°/n$ must be an integer and $n$ must be an odd number and $n > 2$.

$360 = 2^3 \times 3^2 \times 5$

Largest odd factor of 360 = $3^2 \times 5 = 45$. Hence, maximum possible value of $n$ = 45.

The sum of every pair of interior and exterior angles in a polygon is always equal to 180. Hence, the maximum possible value of the interior angle =

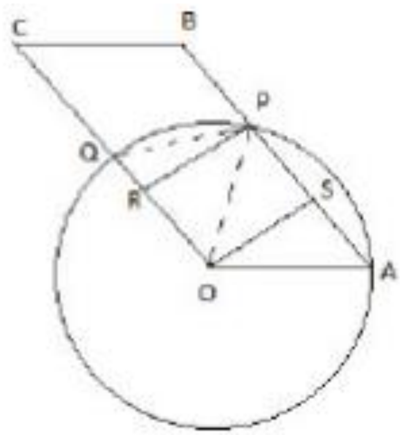
$$180^\circ - \frac{360^\circ}{45} = 180^\circ - 8^\circ = 172^\circ.$$

10. $QR = \sqrt{25^2 - 7^2} = 24$ cm

$QT = \frac{1}{1+3} \times 24 = 6$ cm

$$\frac{1}{QS^2} = \frac{1}{QT^2} + \frac{1}{PQ^2} = \frac{1}{6^2} + \frac{1}{7^2} = \frac{85}{42^2}.\ QS \approx \frac{42}{9.21} \approx 4.56 \text{ cm}$$

11.




Radius of the circle = $\left(\frac{36\pi}{\pi}\right)^{\frac{1}{2}}$ cm.

Join $OP$, triangle $OPA$ is an isosceles triangle since $OP = OA = 6$ (radii). Also, $AP = 8$ (given). So the area of the triangle is =

$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{10 \times 4 \times 4 \times 2}\ 8\sqrt{5}$ cm2.

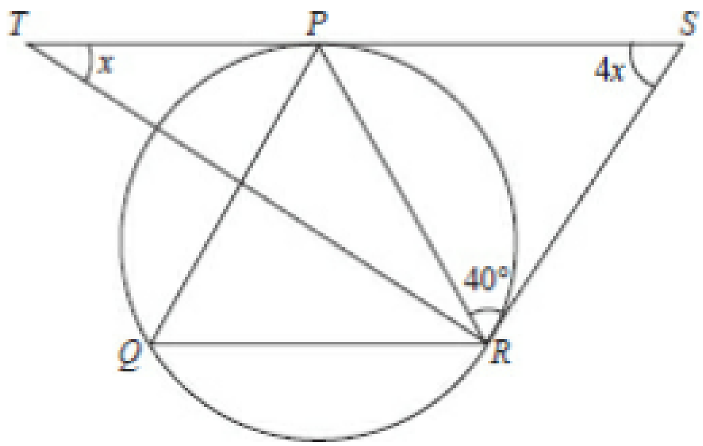
12. Construct *OS* and *PR*, such that *OS* is perpendicular of *AB* and *PR* is perpendicular to *OC*. Since, the area of the triangle *POA* is as found in the previous question, we can use for triangle *POA*, area =

$\frac{1}{2} \times OS \times AP = 8\sqrt{5} \rightarrow OS = 2\sqrt{5}$cm = *PR*. Also, *RO* = *PS* = $\sqrt{6^2 - \left(2\sqrt{5}\right)^2}$ = 4 cm

Side *QR* of the triangle *PQR*, *QR* = *OQ* – *OR* = 6 – 4 = 2 cm

*PQ* = $\sqrt{PR^2 + QR^2} = \sqrt{2\sqrt{5}^2 + 2^2} = 2\sqrt{6}c$

***Solutions for Questions 13 to 15:***




$\angle TPQ = \angle PRQ = 60^\circ = \angle SPR$

ST || QR

$\angle PTR = \angle TRQ = x$

$\angle PSR = 4x$

In $\Delta PSR$:

$\angle SPR + 40° + 4x = 180°$

Or $60° + 40° + 4x = 180°$ or $x = 20°$

$\angle TRQ = \angle PTR = 20°$

$\angle PSR = 80°$

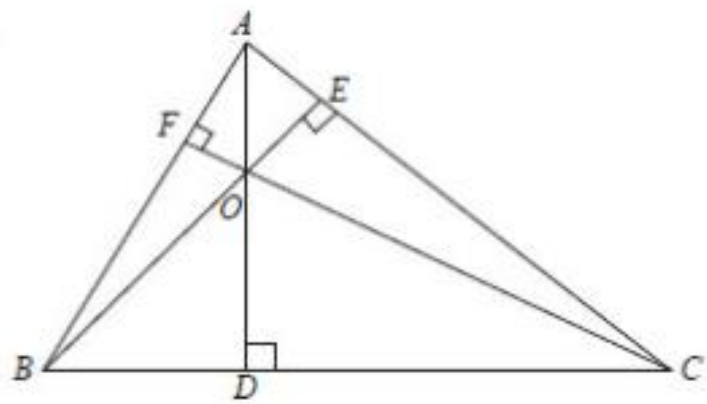
13. $\angle PRQ = 60°$

14. $\angle PSR = 80°$

15. $\angle TRQ = 20°$

***Solutions for Questions 16 and 17*:**

16.




$\Delta BOD$ and $\Delta AOE$ are similar.

$$\frac{BO}{AO} = \frac{OD}{OE}$$

$BO \times OE = AO \times OD$

$BO \times (BE - BO) = AO \times (AD - AO)$

$12 \times (BE - 12) = 18 \times (20 - 18)$

$BE = 15$ cm

17. $AB = \sqrt{AE^2 + BE^2}$. From the solution to the previous question, we know that $BE = 15$. So, we need to focus on finding $AE$. $OD = AD - AO = 20 - 18 =$ 2 cm.

$\Delta AOE$ and $\Delta BOD$ are similar to each other.

$$\frac{OE}{OD} = \frac{AO}{BO}$$

$$OE = \frac{AO \times DO}{BO} = \frac{18 \times 2}{12} = 3\text{cm}$$

$$AE = \sqrt{18^2 - 3^2} = 3\sqrt{35}$$

Hence, $AB = \sqrt{AE^2 + BE^2} = \sqrt{315+225} = \sqrt{540}$ cm

***Solutions for Questions 18 to 20:***

$a \times b = 9$

$(a, b) = (3, 3)$ or $(1, 9)$ or $(9, 1)$

For all triangles, the sum of two sides is always greater than the third side. Hence, $a + b > c$.

Also, for an obtuse-angled triangle $a2 + b2 < c2$ (obtuse-angle triangle property)

Possible values of $c$ are 1, 2, 3, 4 and 5, if $a$ and $b$ are 3 each. Also, if $a$ and $b$ are 1, 9 or 9, 1 the value of $c < 10$. For these combinations of values, the value of: $a2 + b2 < c2$ is possible only for $(a, b, c) = (3, 3, 5)$.

18. One

19. The maximum possible value of $c = 5$.

20. Area of $\Delta ABC = \frac{5}{2} \times \sqrt{3^2 - \left(\frac{5}{2}\right)^2} = \frac{5}{4} \times \sqrt{11}$.

***Solutions for Questions 21 to 22:***

21. **Case 1:** Two equal sides could add up to 10.

Let the sides are $a$, $a$ and $c$.

$a + a = 10$ or $a = 5$

Possible combinations: (5, 5, 1), (5, 5, 2), (5, 5, 3), (5, 5, 4), (5, 5, 6), (5, 5, 7), (5, 5, 8), (5, 5, 9)

There are eight possible triangles. (**Note:** we exclude the triangle (5, 5, 5) because it would become an equilateral triangle.)

**Case 2:** $a + c = 10$

Possible values of $(a, c)$ are (4, 6), (6, 4), (7, 3), (8, 2), (9, 1)

There are five possible triangles.

Hence, total 8 + 5 = 13 triangles are possible.

22. Maximum possible perimeter = 5 + 5 + 9 = 19 cm.

$x = 19$

***Solutions for Questions 23 and 24:***

23. Let the angle between 12 and 20 be '*i*'.

$$\frac{1}{2} \times 12 \times 20 \times \sin A = \text{Area}$$

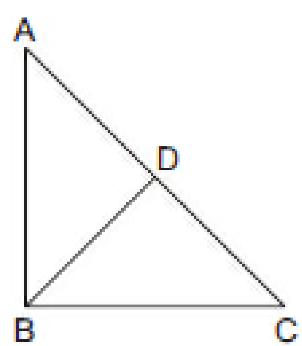
To maximise the area value of sin $A$ must be 1.

Maximum possible area = $\frac{1}{2} \times 12 \times 20 = 120$square units.

In this case: $x = \sqrt{12^2 + 20^2} = \sqrt{544}$cm.

24. 120 square cm

25.



$AC . AD = AB^2$

$AC . AD = BC^2$

$\Delta ABC$ ª $\Delta ADB$

$\therefore \frac{AC}{AB} = \frac{AB}{AD}$

(Corresponding sides of similar triangle are proportional)

$AC . AD = AB^2$ (1)

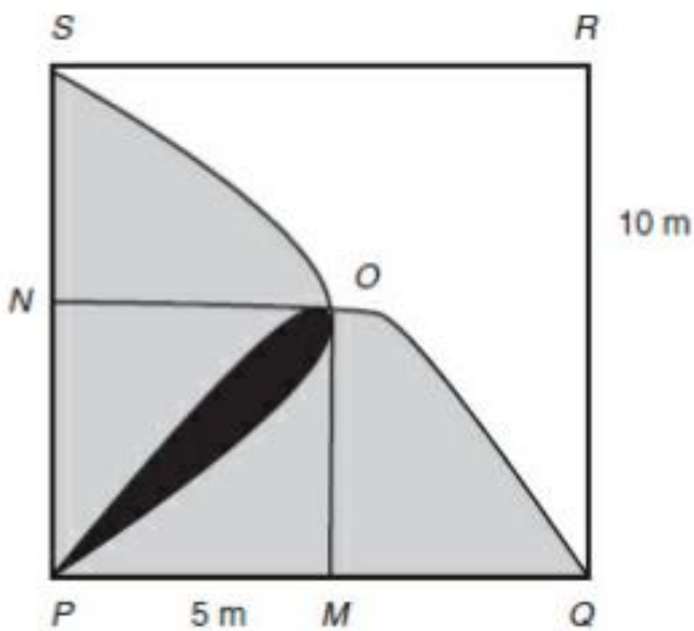
Also, $\Delta ABC$ *N* ΔBDC $\Rightarrow \frac{AC}{BC} = \frac{BC}{CD}$

$AC . CD = BC^2$ (2)

Both conditions of Option (d) are found; therefore

(d) is the answer.

26.




Let *PQRS* be the square grazing field and *M*, *N* are the points at which cows are tethered.

Area grazed by both cows = [Area of quadrant *POM* + Area of quadrant *NOP*] – [Area of square *PMON*]

$$= \frac{\pi}{4}(5)^2 \times 2 - 5 \times 5$$

$$= \frac{50}{4} \times \pi - 25$$

$$= \frac{25\pi}{2} - 25$$

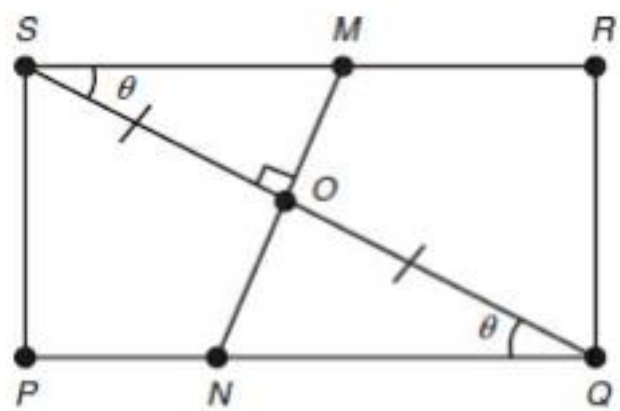
27. Area of grazed region = Area of *OMPN* + Area of *OMQ* + Area of *OSN*

$$= 5^2 + \frac{\pi}{4}(5)^2 \times 2$$

$$= 25 + \frac{25\pi}{2} = \frac{25}{2}[\pi + 2] = 12.5(\pi + 2)$$

Area of non-grazed region = 10 × 10 – 12.5π – 25 = 75 – 12.5π m2

28.




$SQ = \sqrt{3^2 + 4^2} = \sqrt{25} = 5\text{cm}$

$SO = OQ = 2.5\text{cm}$

If $\angle RSQ$ = □

$$\tan\theta = \frac{MO}{SO}(1)$$

$\tan\theta = \frac{RQ}{SR}(2)$

From equation (1) and (2)

$$\frac{MO}{SO} = \frac{RQ}{SR} = \frac{3}{4}$$

$$\frac{MO}{SO} = 0.75$$

29. $OQ = 2.5\text{cm}$

$$\tan\theta = \frac{ON}{OQ}$$

$ON = OQ\tan\theta$

Area of $\Delta QNO = \frac{1}{2}\times OQ\times ON = \frac{1}{2}\times OQ\times OQ\tan\theta$

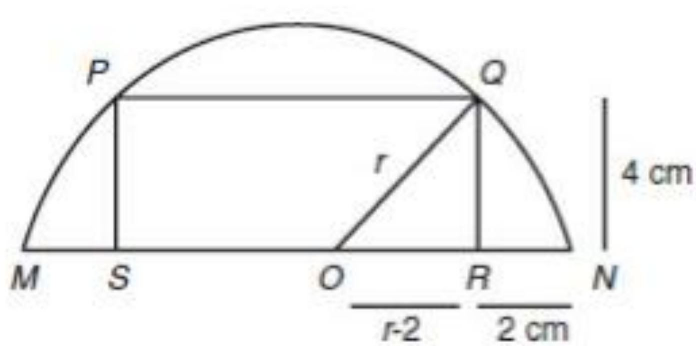
$$= \frac{1}{2}(OQ)^2\tan\theta$$

$$= \frac{1}{2}\times\left(\frac{5}{2}\right)^2\times\frac{3}{4} = \frac{75}{32}$$

Area of $SONP = \frac{1}{2}\times 4\times 3 - \frac{75}{32}$

$$= 6 - \frac{75}{32} = \frac{192-75}{32} = \frac{117}{32}\text{cm}^2 = 3.65\text{cm}^2$$

30. Let '$r$' be the radius of the semicircle.




In $\Delta ORQ$: $r^2 = (r-2)^2 + 4^2$

$r^2 = r^2 + 4 - 4r + 16$

$4r = 20$

$r = 5$ cm

Area of rectangle PQRS = $2(r - 2) \times 4$

$= 2(5 - 2) \times 4 = 24$ cm2

Required area = Area of the semi-circle − Area of rectangle $PQRS$ = $\frac{\pi}{2}(5)^2 - 24$

$= (12.5\pi - 24)$ cm2

31. $\frac{\text{Area of } \Delta SNO}{\text{Area of } \Delta RNO} = \frac{20}{b} = \frac{SN}{NR} = 1.25$

$b = \frac{20}{1.25} = 16$

Area of $\Delta POS$ × Area of $\Delta QOR$ = Area of $\Delta POQ$ × Area of $\Delta SOR${Property of the diagonals of a quadrilateral]

$30 \times a = 30 \times 36$

$a = 36$

$a + b = 36 + 16 = 52$

32. In $\Delta PMQ : QM = \sqrt{(PQ)^2 - (PM)^2}$

$QM = \sqrt{(15)^2 - 9^2} = 12$ cm

In $\Delta QTO$ and $\Delta QMP$:

$\angle QTO = \angle QMP = 90°$

$\angle PQM = \angle TQO$[They are angles between the same lines]

Hence, $\Delta QTO \sim \Delta QMP$

$$\frac{QT}{QO}=\frac{QM}{QP}=\frac{12}{15}=\frac{4}{5}=0.8$$

33. $TR^2 = QR^2 - QT^2 = PR^2 - PT^2$

$$15^2 - QT^2 = 182 - (15 - QT)^2$$

$$15^2 - QT^2 = 18^2 - 15^2 - QT^2 + 30QT$$

$$30QT = 450 - 324$$

$$QT = \frac{126}{30} = 4.2 \text{ cm}$$

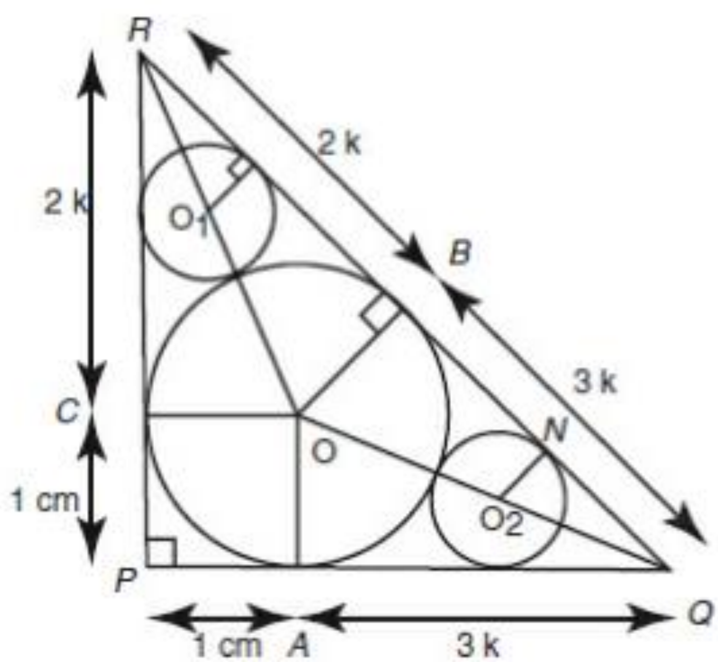
$\Delta QTO$ and $\Delta QMP$ are similar to each other, then:

$$\frac{QT}{OT}=\frac{QM}{PM}$$

$$OT = QT \times \frac{PM}{QM} = 4.2 \times \frac{9}{12}$$

$$= 3.15 \text{ cm}$$

34.



Let $RB = RC = 2k$

$BQ = AQ = 3k$

In $\Delta PQR$, $(2k + 1)^2 + (3k + 1)^2 = (2k + 3k)^2$

By solving, we get $k = 1$

So $RB = RC = 2$ cm

$BQ = AQ = 3$ cm

$PQ = 1 + 3 = 4$ cm, $PR = 1 + 2 = 3$ cm, $RQ = 2 + 3 = 5$ cm

$$OQ = \sqrt{3^2 + 1^2} = \sqrt{10} \text{ cm}$$

$$OR = \sqrt{2^2 + 1^2} = \sqrt{5} \text{ cm}$$

In the solution figure mark point $M$, where the perpendicular from $O_1$ cuts $RB$.

$O_1M \perp RB$, $O_2N \perp BQ$

In $\Delta OBR$ *and* $O_1MR$:

$$\frac{O_1M}{OB} = \frac{O_1R}{OR}$$

$$\frac{r_1}{1} = \frac{OR - (r_1 + 1)}{OR}$$

$$r_1 = \frac{\sqrt{5} - (r_1 + 1)}{\sqrt{5}}$$

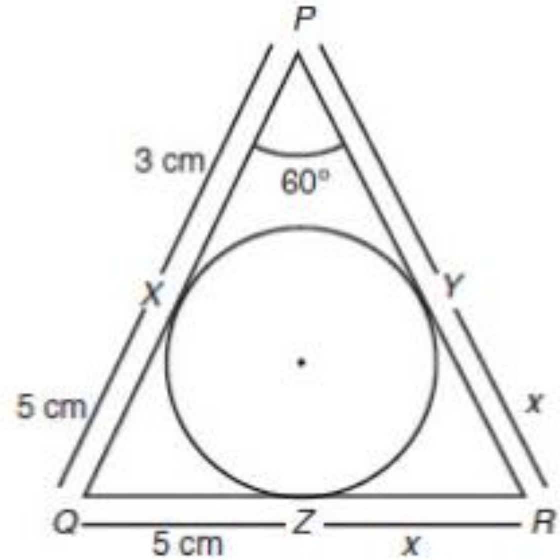
By solving, we get $r_1 = \dfrac{\sqrt{5} - 1}{\sqrt{5} + 1}$

Similarly $r_2 = \dfrac{\sqrt{10} - 1}{\sqrt{10} + 1}$

$$\frac{r_1}{r_2} = \frac{\sqrt{5} - 1}{\sqrt{5} + 1} \times \frac{\sqrt{10} + 1}{\sqrt{10} - 1} = \frac{(\sqrt{5} - 1)^2}{4} \times \frac{(\sqrt{10} + 1)^2}{9}$$

$= (33 - 11\sqrt{5} + 6\sqrt{10} - 10\sqrt{2}) / 18$

35.




Let $ZR = x$ cm = $YR$

In $\Delta PQR$:

Using the Cosine rule, we get:

$$\cos 60^o = \frac{(3+5)^2 + (3+x)^2 - (5+x)^2}{2(3+5)(3+x)}$$

$8(3 + x) = 64 + 9 + x2 + 6x - 25 - x2 - 10x$

$24 + 8x = 64 - 16 - 4x$

$12x = 24$

$x = 2$ cm

Sides of the $\Delta PQR$: $PQ = 8$ cm

$RQ = 7$ cm

$PR = 5$ cm

Semi-perimeters = $\frac{8+7+5}{2} = 10\text{cm}$

Area of $\Delta PQR = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{10(2)(3)(5)}$

$= 10\sqrt{3}$ cm$^2$

Radius of in-circle = $\frac{10\sqrt{3}}{10} = \sqrt{3}$ cm (Since area of a triangle is given by $s \times r$)

36. Area of the shaded portion =

$10\sqrt{3} - \pi\left(\sqrt{3}\right)^2 = 10\sqrt{3} - 3\pi$ cm$^2$ cm2

37. $\Delta POW$ and $\Delta ROS$ are similar to each other,

$$\frac{PO}{OR} = \frac{OW}{OS} = \frac{PW}{SR} = \sqrt{\frac{\text{area of}\left(\Delta POW\right)}{\text{area of}\left(\Delta SOR\right)}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$PQ \parallel TU \parallel SR$.

According to the basic proportionality theorem:

$$\frac{PT}{TS} = \frac{PO}{OR} = \frac{WO}{OS} = \frac{WX}{XR} = \frac{QU}{UR} = \frac{3}{5}$$

$\Delta WRQ$ and $\Delta XRU$ are similar and

$$\frac{XU}{WQ} = \frac{RX}{RW}$$

$$\frac{WX}{XR} = \frac{3}{5} \Rightarrow \frac{WX}{XR} + 1 = \frac{3}{5} + 1$$

$$\frac{WX + XR}{XR} = \frac{8}{5}$$

$$\frac{WR}{XR} = \frac{8}{5}$$

$$\frac{XR}{WR} = \frac{5}{8}$$

$$\frac{\text{Area of}\left(\Delta XRU\right)}{\text{Area of}\left(\Delta WRQ\right)} = \frac{5^2}{8^2} = \frac{25}{64}$$

$$\frac{\text{area of } \square WQUX}{\text{area of } \Delta XRU} = \frac{64-25}{25} = \frac{39}{25}$$

38. Let $SR = 5a$, then $PW = 3a$

$WQ = 5a - 3a = 2a$

But $WQ = 2$ cm

$2a = 2$ cm

$a = 1$ cm

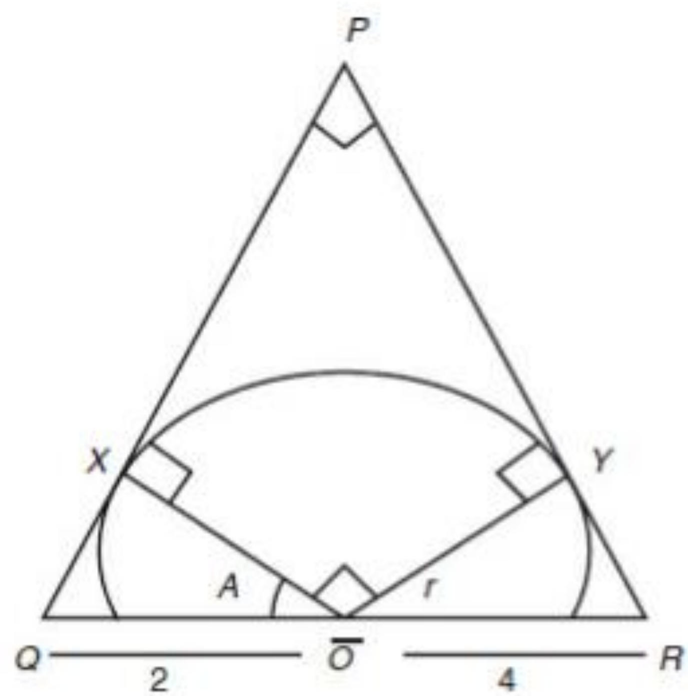
$$\frac{WQ}{XU} = \frac{WR}{XR} = \frac{3+5}{5} = \frac{8}{5}$$

$$XU = 2 \times \frac{5}{8} = \frac{5}{4} \text{cm}$$

$$TX = 5 - \frac{5}{4} = \frac{15}{4} \text{cm}$$

$$TX \times XU = \frac{15}{4} \times \frac{5}{4} = \frac{75}{16} \text{ cm}^2$$

39.



Let $PQ$ and $PR$ touches the semicircle at $X$ and $Y$ respectively.

$OX \perp PQ$, $OY \perp PR$

□$PXOY$ is a square, $\angle XOY = 90°$

Let the radius of the semicircle be r.

Let $\angle XOQ = A$, then in $\Delta OXQ$:

$\cos A = \frac{r}{2}$ (1)

Similarly in $\Delta OYR$:

$\cos(90^\circ - A) = \frac{r}{4}$ ($\angle YOR = 180° - A - 90° - A$)

$\sin A = \frac{r}{4}$ (2)

Squaring and adding equation (1) and (2), we get

$$\cos^2 A + \sin^2 A = \left(\frac{r}{2}\right)^2 + \left(\frac{r}{4}\right)^2$$

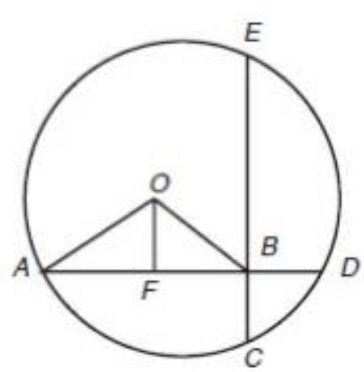
$$1 = \frac{r^2}{4} + \frac{r^2}{16}$$

$$\frac{5r^2}{16} = 1$$

$$r^2 = 16/5$$

$$r = \frac{4}{\sqrt{5}} \text{ cm}$$

40.



In this figure, our objective is to find the value of *OB*.

Obviously, the value of *OB* depends on the values of *OF* and *FB*. Hence, we would need to think of a way in which we can work out the values of *OF* and *FB* respectively.

If we were to take the values of *BD* as *x* and *BE* as *y* respectively, then using the intersecting chord theorem (for chords intersecting at right angles), we get:

$AB \times BD = BE \times BC$

$\rightarrow 6x = 2y \rightarrow y = 3x$

Now, $OF = \frac{3x-2}{2}$ and $AF = \frac{6+x}{2}$. Solving for *x*, using Pythagoras theorem, we get *x* as 4.

Then, in right angled triangle *OFA*, we have $OA^2 = OF^2 + AF^2$

$$OF^2 = 50 - \frac{(6+x)^2}{4} = 41 - 3x - \frac{x^2}{4}$$

Hence, $OF = 5$ and $BF = AB - AF = 1$.

Then, in the right-angled triangle *OFB*, we get $OB^2 = OF^2 + FB^2 = 25 + 1 = 26$.

Hence, option (a) is correct.

41. First-things-first, while solving this. If we do not include Statement III, we do not know which angle is a right angle and hence cannot uniquely calculate the value of the side *AC*.

Also, Statement III alone does not give us anything. Hence, we can reject options (a). Option (b) and

(c) gives us similar set of information – i.e. the value of one median and the fact that *B* is the right angle in the triangle. This is also clearly not sufficient to answer the question.

For option (d): We have two medians of a right-angled triangle, and we know *B* is the right angle. Hence, we can find AC.

Hence, Option (d) is the correct answer.

42. Area of $\Delta RPQ = \frac{1}{2} \times PR \times PQ \times \sin 120°$

$$\text{Area of } \Delta PSR = \frac{1}{2} \times PS \times PR \times \sin 60°$$

$$\text{Area of } \Delta PSQ = \frac{1}{2} \times PS \times PQ \times \sin 60°$$

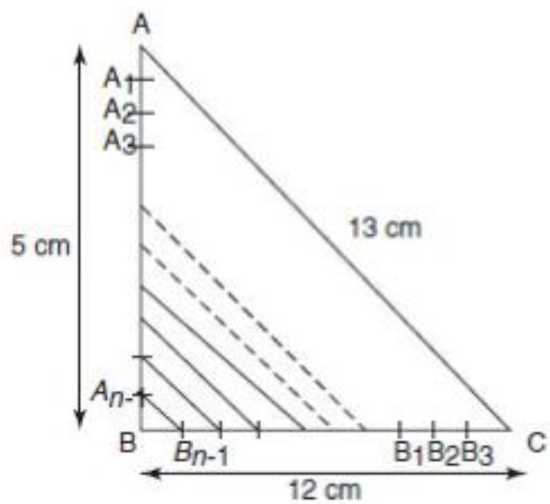
$$\frac{1}{2} \times PQ \times PR \times \sin 120° = \frac{1}{2} \times PS \times PR \times \sin 60° + \frac{1}{2} \times PS \times PQ \times \sin 60°$$

$$\frac{9\sqrt{3}}{2} \times PR = 6 \times PR \times \frac{\sqrt{3}}{2} + 6 \times 9 \times \frac{\sqrt{3}}{2}$$

$$9PR = 6PR + 54$$

$$PR = \frac{54}{3} = 18 \text{ cm}$$

43.



$AB = 5\text{cm}$

$$A_{n-1}B = \frac{5}{n}$$

$$A_{n-2}B = \frac{10}{n},\ A_{n-3}B = \frac{15}{n},\ \ldots\ldots$$

Similarly $BB_{n-1} = \frac{12}{n}, BB_{n-2} = \frac{24}{n}, BB_{n-3} = \frac{36}{n}, \ldots$

$$A_{n-1}B_{n-1} = \sqrt{\left(\frac{5}{n}\right)^2 + \left(\frac{12}{n}\right)^2} = \frac{13}{n}$$

$$A_{n-2}B_{n-2} = \sqrt{\left(\frac{10}{n}\right)^2 + \left(\frac{24}{n}\right)^2} = \frac{26}{n}$$

And so on…

$A_{n-1}B_{n-1} + A_{n-2}B_{n-2} + A_{n-3}B_{n-3} + \ldots +$

$$AC = \frac{13}{n} + \frac{26}{n} + \frac{39}{n} + \ldots n \text{ terms} =$$

$$\frac{13}{n}[1+2+3+\ldots+n] = 130$$

$$\frac{13}{n} \times \frac{n}{2}(n+1) = 130$$

$(n + 1) = 20$

$n = 19$

44. *Area of* $\Delta A_{n-1} BB_{n-1}$ + *Area of* $\Delta A_{n-2} BB(n-2)$ + *Area of* $\Delta A_{n-3}BB(n-3) + \ldots +$

*Area of* $\Delta ABC = 66\ \text{cm}2$

*Area of* $\Delta A_{n-1} BB_{n-1}$ + *Area of* $\Delta A_{n-2} BB(n-2)$ + *Area of* $\Delta A_{n-3}BB(n-3) + \ldots +$

*Area of* $\Delta ABC = 66\ \text{cm}2$

$$\Rightarrow \frac{1}{2} \times \frac{5}{n} \times \frac{12}{n} + \frac{1}{2} \times \frac{10}{n} \times \frac{24}{n} + \frac{1}{2} \times \frac{15}{n} \times \frac{36}{n} + \ldots + \frac{1}{2} \times 5 \times 12 = 66$$

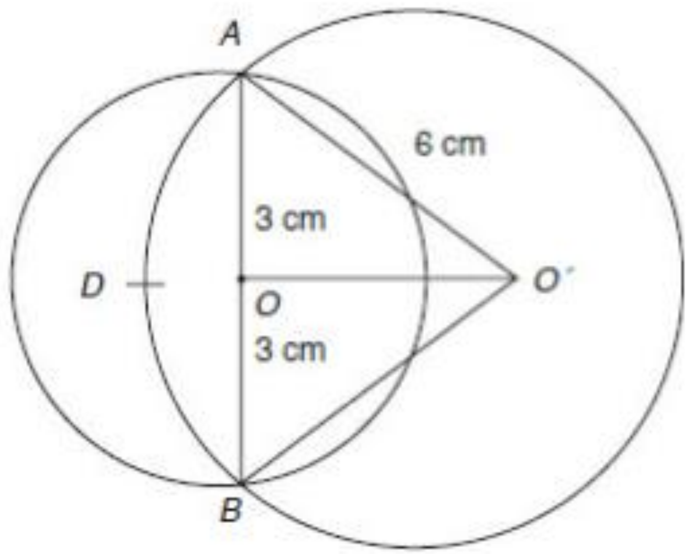
$$\Rightarrow \frac{60}{2n^2}\left[1^2+2^2+3^2+\ldots+n^2\right]=66$$

$$\Rightarrow \frac{30}{n^2}\times\frac{n(n+1)(2n+1)}{6}=66$$

$$\Rightarrow \frac{5}{n}(n+1)(2n+1)=66$$

$$\Rightarrow n = 5$$

45.




Let the center of the smaller and the bigger circle be $O$, $O'$ respectively (as shown in the diagram above). And $A$, $B$ are the points of intersection of the circles.

The common chord will be of maximum length, if it is the diameter of the smaller circle. (**Note:** You cannot make the common chord longer than the diameter of the smaller circle.)

$AB = 3 \times 2 = 6$ cm

$O'A = O'B = 6$cm

This means that the $\Delta ABO'$ is an equilateral triangle having side 6 cm.

So area of $\Delta ABO' = \frac{\sqrt{3}}{4}(6)^2 = 9\sqrt{3}\ \text{cm}^2$.

46. Area of intersection of the two circles =

Area of smaller circle/2 + Area of segment $ABD$

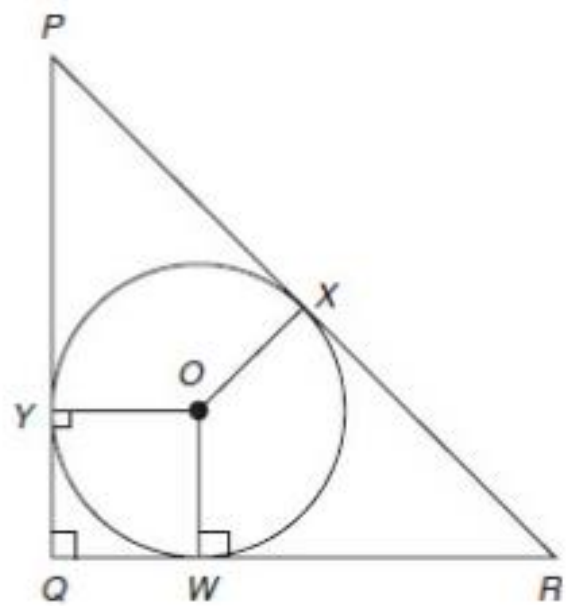
$$= \frac{\pi(3)^2}{2} + (\text{area of segment } ADBO' - \text{area of } \Delta ABO')$$

$$= \frac{\pi}{2}(3)^2 + \left(\pi(6)^2 \times \frac{60^\circ}{360^\circ} - 9\sqrt{3}\right)$$

$$= \frac{9\pi}{2} + 6\pi - 9\sqrt{3}$$

$$= \left[\frac{21}{2}\pi - 9\sqrt{3}\right]\text{cm}^2$$

47.




$PR = 3\sqrt{2}$cm

$PQ = QR = 3$ cm

In-circle of an isosceles right angle triangle touches the hypotenuse at its mid-point.

So, $PX = XR = 1.5\sqrt{2}$cm

and $WR = XR = \sqrt{2}$ cm

$QW = QR - WR = (3 - 1.5\sqrt{2})$cm

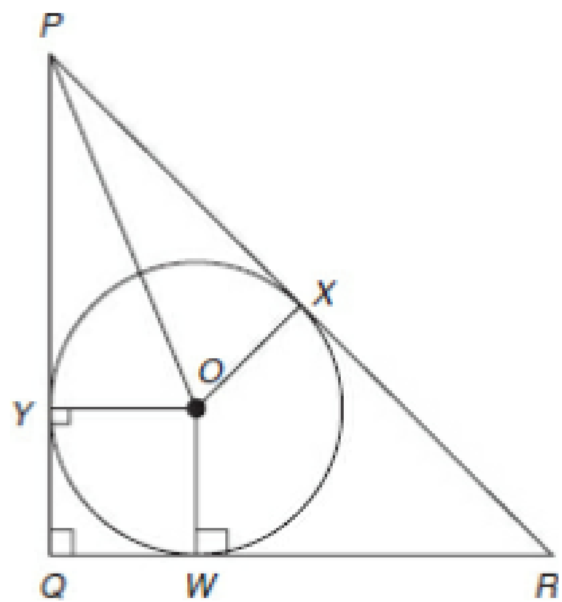
So the required ratio is:

$1.5\sqrt{2} : 3 - 1.5\sqrt{2} : 1.5\sqrt{2}$

$1 : (\sqrt{2} - 1) : 1$

48. $\angle XOY = 180° - \angle YPX$

$= 180° - 45° = 135°$




Radius of in-circle = $OY = QW = \left[3 - \frac{3}{2}\sqrt{2}\right]$cm

Area of *PYOX* = *area of* $\Delta PYO$ + *area of PXO* = 2 (*area of* $\Delta PYO$)

$$= \frac{1}{2} \times \left[3 - \frac{3}{2}\sqrt{2}\right] \times \frac{3}{2}\sqrt{2} \times 2$$

$$= \frac{9}{2}\sqrt{2} - \frac{9}{4} \times 2 = \frac{9}{2}\sqrt{2} - \frac{9}{2} = \frac{9}{2}\left(\sqrt{2} - 1\right) \text{cm}^2$$

49. $BP \perp AC$, $\Delta ABC$ is an equilateral triangle so $AP = PC$ and if side of $\Delta ABC$ is a. Then $BP = \frac{a\sqrt{3}}{2}$ (Altitude of an equilateral triangle). In order to find the area of $\Delta PQC$ we would need to find the base $QC$ and the height $PQ$. Also, in order to find the area of the $\Delta PRB$, we would need to find the base $BR$ and the height PR of the triangle.

In $\Delta PQC$:

$$\sin 60^\circ = \frac{PQ}{PC} = \frac{\sqrt{3}}{2} \Rightarrow \frac{\sqrt{3}}{2} = \frac{PQ}{\frac{a}{2}} \Rightarrow PQ = \frac{\sqrt{3}}{4}a$$

$$\cos 60^\circ = \frac{QC}{PC} = \frac{1}{2} \Rightarrow QC = \frac{a}{4}$$

$$\text{Area of } \Delta PQC = \frac{1}{2} \times \frac{a}{4} \times \frac{a\sqrt{3}}{4} = \frac{a^2\sqrt{3}}{32}\text{cm}^2$$

In $\Delta ARP$

$$\sin \angle RAP = \frac{RP}{AP} = \frac{RP}{\frac{a}{2}} \Rightarrow \sin 60^\circ = \frac{2RP}{a}$$

$$RP = \frac{a\sqrt{3}}{4}\text{cm}$$

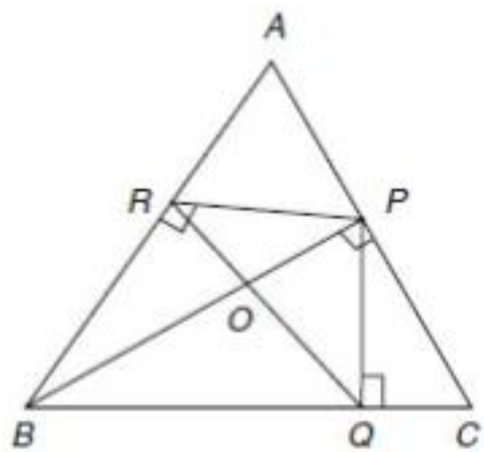
$\angle PBR = 30°$

$$BR = PB\cos 30^\circ = \frac{a\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = a \times \frac{3}{4}$$

$$\text{Area of } \Delta PRB = \frac{1}{2} \times RB \times PR = \frac{1}{2} \times \frac{3a}{4} \times \frac{a\sqrt{3}}{4}$$

$$= \frac{3a^2\sqrt{3}}{32}$$

50.




We have already seen $AR = QC = a/4$

$\Rightarrow RQ \parallel AC$

$\Rightarrow \Delta BRQ \sim$ D$BAC$

$$\frac{RQ}{AC} = \frac{BO}{BP} = \frac{BR}{AB} = \frac{AB - AR}{AB} = \frac{\frac{3a}{4}}{a} = \frac{3}{4}$$

$$\frac{RQ}{AC} = \frac{3}{4}$$

$$RQ = \frac{3a}{4}\text{cm}$$

$$BO = \frac{3}{4} \times \frac{a\sqrt{3}}{2} = \frac{3a\sqrt{3}}{8}$$

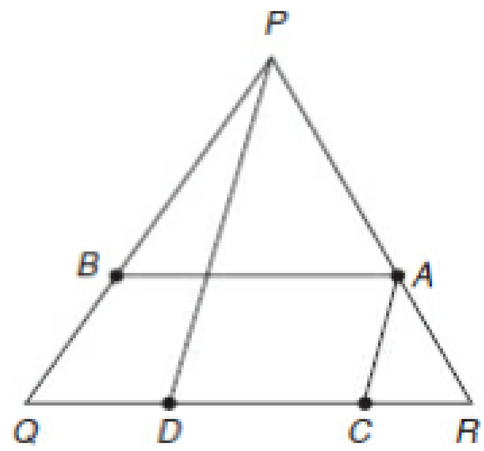
$$OP = BP - BO = \frac{a\sqrt{3}}{2} - \frac{3a\sqrt{3}}{8}$$

$$= = \frac{a\sqrt{3}}{8}\text{cm}$$

Area of $\Delta PRQ = \frac{1}{2} \times \frac{3a}{4} \times \frac{a\sqrt{3}}{8}$

$$= \frac{1}{2} \times \frac{3 \times 4}{4} \times \frac{4\sqrt{3}}{8} = \frac{3\sqrt{3}}{4}\text{cm}^2$$

51.




$PQ = PR = 10$cm.

$PA : AR = 2:3$

$\Rightarrow PA = 10 \times \frac{2}{2+3} = 4$cm, AR = $10 \times \frac{3}{2+3}$ = 6 cm

In $\Delta PRD$ & $\Delta ACR$: $AC \parallel PD$

$\Rightarrow \Delta ACR \sim \Delta PRD$

$\frac{PA}{AR} = \frac{DC}{CR} = \frac{2}{3}(1)$

$AB \parallel DC$ and $AC \parallel PD$

$\Rightarrow \angle PDC = \angle BAC$

$\angle APD = \angle BAC$ (given)

$\Rightarrow \angle$PDC = $\angle APD$ or $PR = RD = 10$cm.

$\frac{DC}{CR} = \frac{2}{3}$(From equation 1)

$DC = 10 \times \frac{2}{2+3} = 4$cm

52.

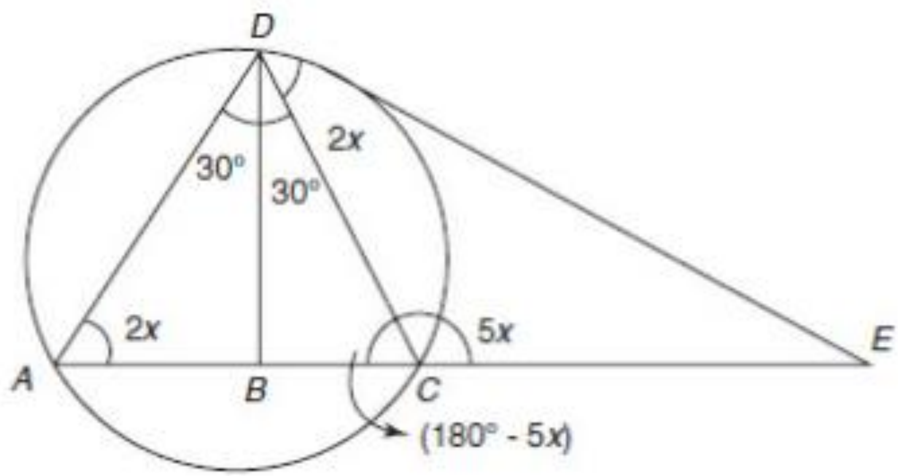
$$\frac{\text{Area of } \Delta QXY}{\text{Area of } \Delta PQR} = \frac{\frac{1}{2} QX.QY.sinQ}{\frac{1}{2} PQ.QR.sinQ}$$

$$= \frac{QX}{PQ} \times \frac{QY}{QR} = \frac{1}{2} \times \frac{2}{5} = \frac{2}{10}$$

$$\frac{\text{Area of } \square XYRZ}{\text{Area of} \Delta PQR} = \frac{1 - \left(\frac{1}{6} + \frac{2}{10}\right)}{1} = \frac{19}{30}$$

$$\frac{\text{Area of } \square XYRZ}{\text{Area of} \Delta PXZ} = \frac{\frac{19}{30}}{\frac{1}{6}} = \frac{19}{5}$$

53.




Let $\angle EDC = 2x$ and $\angle ECD = 5x$

According to alternate segment theorem:

$\angle EDC = \angle DAC$

$\angle DAC = 2x$

$\angle BCD = 180° - 5x$

$\angle ADB = \angle CDB = 30°$

In $\Delta ADC$: $2x + (30° + 30°) + 180° - 5x = 180°$

$x = \dfrac{60^\circ}{3} = 20^\circ$

$\angle DEC = 180° - (40° + 100°) = 40°$

54. According to the internal angle bisector theorem:

$$\frac{AB}{BC} = \frac{AD}{DC}$$
$$\frac{AB}{AC} = \frac{1}{3} \Rightarrow \frac{AB}{BC} = \frac{1}{2}$$
$$\Rightarrow \frac{AD}{DC} = \frac{1}{2}$$
$$AD = \frac{DC}{2} = \frac{6}{2} = 3\,\text{cm}$$

55. In an $n$-sided regular polygon number of diagonals = ${}_nC_2 - n$

$$= \frac{n(n-3)}{2}$$
$$p\left[\frac{n(n-3)}{2}\right] = n$$
$$p(n - 3) = 2 \Rightarrow \frac{2}{n-3} = p$$

Internal angle $x = \dfrac{(n-2)\pi}{n}$. On transformation, we get $n = \dfrac{2\pi}{\pi - x}$.

Hence, $P$ =

$$\frac{2}{n-3} = \frac{2}{\left[\dfrac{2\pi}{\pi - x}\right] - 3} = \frac{2(\pi - x)}{2\pi - 3\pi + 3x} = \frac{2(\pi - x)}{3x - \pi}$$

$$p = \frac{2(\pi - x)}{3x - \pi}$$

$3xp - \pi p = 2\pi - 2x$

$x(3p + 2) = 2\pi + \pi p$

$x = \frac{\pi(2+P)}{(3P+2)}$

**Alternative method:**

Let the polygon is a square then $n = 4$, $p = 2$ and $x = 90°$

Only Option (b) satisfies these conditions. Hence, it is the correct option.

56. $\Delta PQR$ is a right-angle triangle.

$PQ = 15\text{cm}, PR = 25\text{cm}$

$QR = \sqrt{PR^2 - PQ^2} = \sqrt{25^2 - 15^2} = \sqrt{400} = 20\,\text{cm}$

Since $PQ \times QR = 20 \times 15 = 300$ and $QS \times PR = 12 \times 25 = 300$, therefore $QS \perp PR$ (**Note:** This is a property of right-angled triangles that you should know).

$SR = \sqrt{(20)^2 - 12^2} = 16\,\text{cm}$

In $\Delta QTR$: $QT = \sqrt{QR^2 - RT^2} = \sqrt{20^2 - 16^2} = 12\,\text{cm}$

$\Delta QTR$ and $\Delta QSR$ are congruent to each other therefore $\angle SOQ = \angle TOQ = 90°$

It is obvious that $ST \perp QR$.

Hence, $\Delta ROS \sim RQP$

$\frac{RO}{RQ} = \frac{RS}{RP} = \frac{16}{25}$

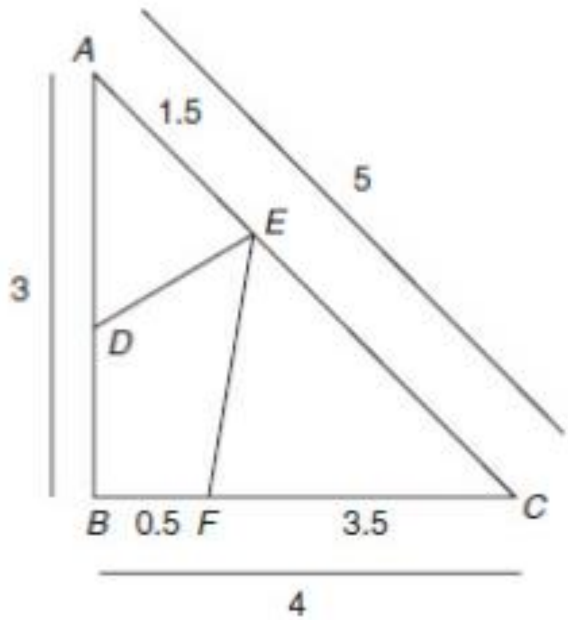
$\Rightarrow \frac{RO}{OQ} = \frac{16}{9}$

57. Area of quadrilateral $SQTR$ = Area of $\Delta SQR$ + Area of $\Delta QTR$. We know that the area of a quadrilateral is given by product of diagonals ÷ 2. Hence, we get:

$\frac{ST \times 20}{2} = \frac{1}{2} \times 12 \times 16 + \frac{1}{2} \times 12 \times 16$ (Using the lengths of $SQ$, $QR$, $QT$ and $TR$ which we know from the previous question and the given information)

Hence, $ST$ = 19.20 cm

58.




$\Delta ABC$ is right angle triangle and $\angle B = 90°$

Let $\angle BAC = A$

$\Rightarrow \angle BCA = 90° - A$

In $\Delta ADE$: $AD = AE = 1.5$

$$\Rightarrow \angle ADE = \angle AED = \frac{180° - A}{2} = 90° - \frac{A}{2}$$

In $\Delta EFC$: $EC = FC = 3.5$ cm

$$\angle CEF = \angle CFE = \frac{180° - (90° - A)}{2} = 45° + \frac{A}{2}$$

$\angle AED + \angle DEF + \angle FEC = 180°$

$90^\circ - \frac{A}{2} + \angle DEF + 45^\circ + \frac{A}{2} = 180^\circ$

$\angle DEF = 45°$

59. $\frac{PX}{QZ} = \frac{1}{2}$ and $\frac{QX}{QZ} = \frac{3}{2}$

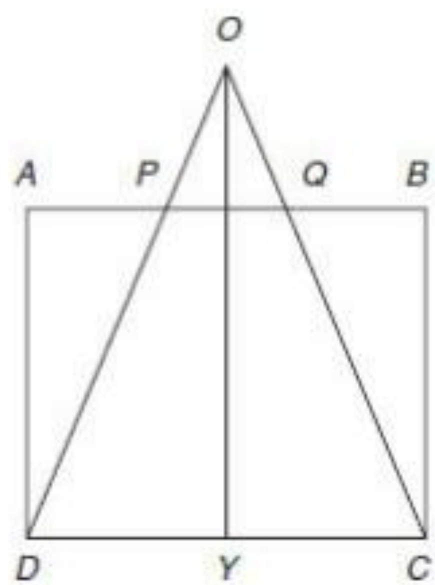
$\therefore PX : XQ = 1 : 3$

$\Rightarrow \frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{3}$

$\therefore XY \parallel QR$

If $XY \parallel QR$, then for any point $Z$ on the line $QR$, $\Delta XYZ$ will have the same area as the base and the height of the triangle would be constant irrespective of where $Z$ is placed. So area of $\Delta XYZ$ will be same for any value of $QZ : QR$.

60.



Let the altitude from $O$ of $\Delta OPQ$ is $OX$ and $OY \perp DC$.

Area of $\square PQCD = \frac{1}{2}(PQ + DC)BC = 80$ sq units

$$\Rightarrow \frac{1}{2}(PQ + 10)10 = 80$$

$PQ = 6$ units

$\Delta OPQ$ and $\Delta ODC$ are similar triangles.

$$\frac{OX}{OY} = \frac{PQ}{DC} = \frac{6}{10}$$

$$\frac{OX}{OX + 10} = \frac{6}{10}$$

$10\,OX = 6\,OX + 60$

$OX = \frac{60}{4} = 15$ units

61. In $\Delta\, CAD : AD^2 + CA^2 = CD^2$

Using Apollonius theorem we get : $AB^2 + AC^2 = 2(AD^2 + CD^2)$

$AB^2 + AC^2 = 2AD^2 + BC^2/2$

$AB^2 + AC^2 = 2(CD^2 - AC^2) + BC^2/2$

$AB^2 + AC^2 = 2(BC^2/4 - AC^2) + BC^2/2$

$3AC^2 = BC^2 - AB^2$

**Theory Note: Apollonius' theorem** I relating the length of a median of a triangle to the lengths of its side is a theorem: t states that the sum of the squares of any two sides of any triangle equals twice thesquare on half the third side, together with twice the square on the median bisecting the third side" Specifically, in any triangle ABC, if AD then, is a median.

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

62. Let $AC = 2^a$, where $a \in N$

$AB = AC/2 = \frac{2^a}{2} = 2^{a-1}$

According to the question $AB$ is a perfect square. So a − 1 should be even or 'a' should be odd.

$AB + AC > BC$

$3AB > 295$

$AB > 98.33$

$2^{a-1} > 98.33$ (1)

$AC - AB < BC$

$2^a - 2^{a-1} < 295$ or $2^{a-1} < 295$(2)

Only a = 9 satisfies equation (1) and (2).

$\therefore AB = 2^8 = 256, AC = 2^9 = 512$

∴Perimeter of the triangle = 256 + 512 + 295 = 1063

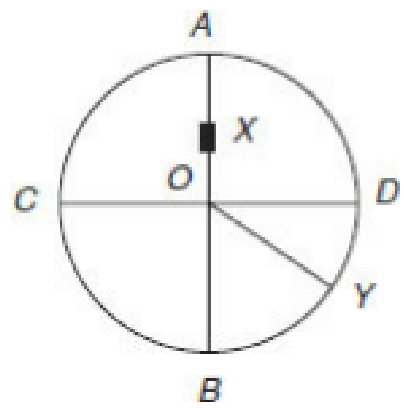
63. Let $AB = a^3$, $AC = 3^n$ according to the question : $3^n = 3.a^3$ or $a^3 = 3^{n-1}$

Perimeter $P = 397 + 3^n + 3^{n-1}$

$P - 397 = 3^{n-1}[3 + 1] = 4.3^{n-1}$

Thus $P$- 397 should be a multiple of both 3 and 4, only Option (d) $p$ = 3313 feet satisfies this condition, so option (d), is correct.

64.



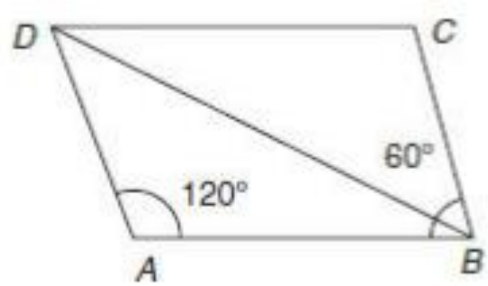

$XADY = 1 + 4\pi \times \frac{120^\circ}{360^\circ} = 1 + \frac{4\pi}{3} 5.19$

$XOBY = 1 + 2 + 4\pi \times \frac{60^\circ}{360^\circ} = 3 + \frac{4\pi}{6} 5.09$

$XODY = 1 + 2 + 4\pi \times \frac{30^\circ}{360^\circ} = 4.04$

$XADY : XOBY : XODY :: 5.19 : 5.09 : 4.04$

65.




Let $AB = a$ and $AD = b$

Area of parallelogram = $ab \sin 60^\circ = ab\frac{\sqrt{3}}{2} = 15\frac{\sqrt{3}}{2}$

$ab = 15$ .....(1)

By applying cosine rule in $\Delta ABD$:

$\cos 120^\circ = \frac{a^2 + b^2 - 49}{2ab} = -\frac{1}{2}$ or $a2 + b2 = 34$

$$a^2 + \frac{225}{a^2} = 34$$

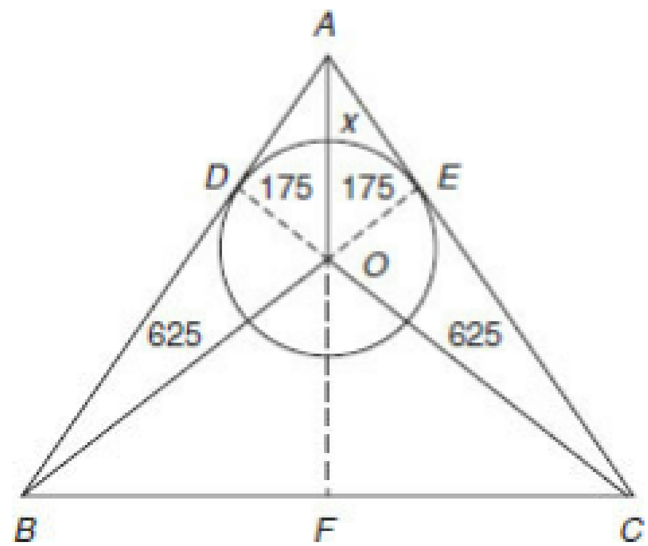
By solving the above equation we get $a^2 = 9$ or $25$

$a = 3$ or $5$

$ab = 15$ so $b = 5$ or $3$

Perimeter = $2(a + b) = 2(5 + 3)$ or $2(3 + 5) = 16$ cm

66. Let $ABC$ is the triangle and the circle touches $AB$, $AC$ at $D$, $E$ respectively as shown in the diagram.




$OD \perp AB$ and $OE \perp AC$

$OA = OB = OC = 625$ cm (Given)

In $\Delta ODB$, $BD^2 + OD^2 = OB^2$ (Using Pythagoras theorem)

$BD^2 + 175^2 = 625^2$

$\Rightarrow BD = 600$ cm.

Similarly, $AD = AE = EC = 600$ cm

Hence, $\Delta ABC$ is an isosceles triangle and $AB = AC = 1200$ cm

So, $AF \perp BC$

In $\Delta AEO$ and $\Delta AFC$ :

$\angle OAE = \angle CAF$

$\angle AEO = \angle AFC = 90^0$

So, $\Delta AEO \sim \Delta AFC$

$$\frac{AE}{AF} = \frac{OE}{CF} = \frac{OA}{AC}$$

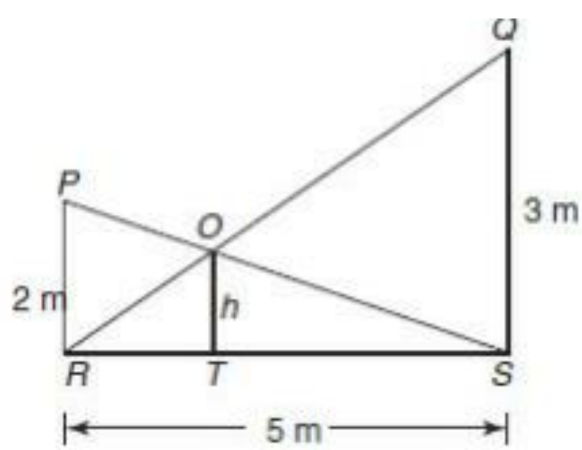
$$\frac{600}{AF} = \frac{175}{CF} = \frac{625}{1200}$$

So, $AF = 1200 \times \frac{600}{625}$ cm and $CF = 1200 \times \frac{175}{625} = 336$ cm

$CB = 672$ cm

Area of $\Delta ABC = \frac{1}{2} \times 672 \times 1152 = 387072$ $cm^2$

67.




$\Delta ROT$ and $\Delta RQS$ are similar to each other:

$OT/QS = RT/RS$

$RT/RS = h/3$ .....(1)

$\Delta SOT$ and $\Delta SPR$ are similar to each other:

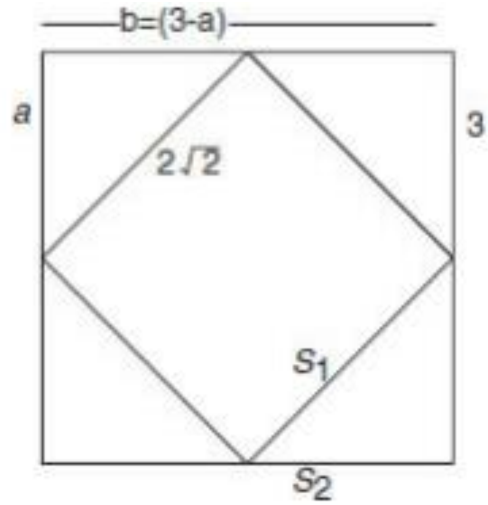
$ST/SR = h/2$ ......(2)

Adding (1) and (2), we get:

$(RT + ST)/RS = RS/RS = 1 = h/2 + h/3$

$h = 1.2$ m

68. If we visualise a figure for this situation, you would be able to see something as follows:




Solving through Pythagoras theorem, we will get $a = 0.18$ and $b = 3 - 0.18 = 2.82$.

Hence, the value of $b/a = 2.82/0.18 = 15.666$.

69. $\frac{\angle A}{\angle B} = 1 - \frac{\angle C}{\angle B}$

$\angle A = \angle B - \angle C$

$\angle A + \angle C = \angle B$

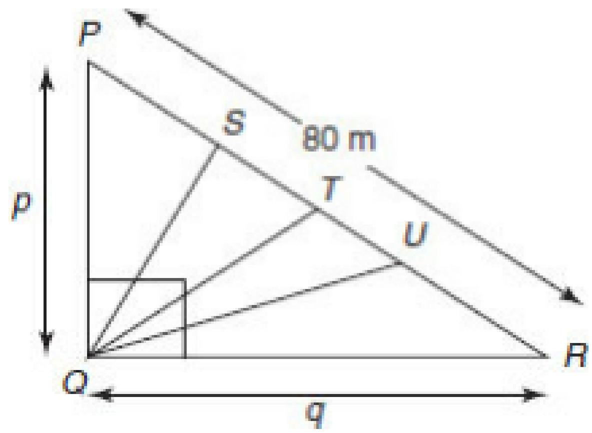
In $\Delta ABC$: $\angle A + \angle B + C = 180°$

$2\angle B = 180°$

$\angle B = 90°$

$\therefore AB^2 + BC^2 = AC^2$

70.




$PR = 80$ m, $PS = ST = TU = UR = 20$m

$T$ is the midpoint of $PR$(hypotenuse) so $PT = TR = QT = 40$m

Applying Apollonius theorem in $\Delta PQT$,

$p^2 + 40^2 = 2(QS^2 + 20^2)$ .....(1)

Again applying Apollonius theorem in $\Delta QRT$,

$q^2 + 40^2 = 2(QU^2 + 20^2)$ .....(2)

By adding equation 1 and 2 we get:

$p^2 + q^2 + 2$
$40^2 = 2\,(QS^2 + QU^2) + 40^2$

$QS^2 + QU^2 = (p^2 + q^2 + 40^2)/2 = (80^2 + 40^2)/2 = 4000$ $m^2$

$QS^2 + QU^2 + QT^2 = 4000$ $m^2 + 40^2$ $m^2 = 5600$ $m^2$

71. $PQ = SR = 14$cm

$SX : XR = 4:3$

$SX = 8$cm, $XR = 6$cm

$QR = \sqrt{10^2 - 6^2} = 8\text{cm}$

$PS = 8$cm

$PX = \sqrt{8^2 + 8^2} = 8\sqrt{2}$ cm

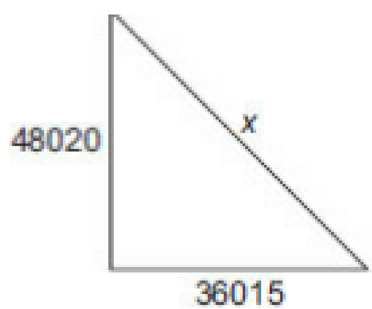
In $\Delta PXQ$: $PQ = 14$cm, $PX = 8\sqrt{2}$ cm, $QX = 10$cm

$\therefore a > c > b$

## MENSURATION

### Level of Difficulty (I)

1.



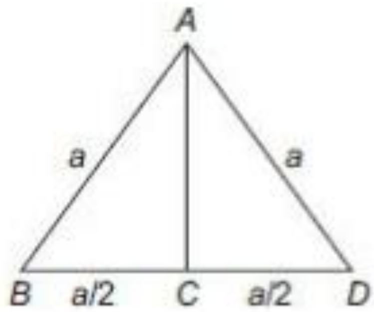

Let hypotenuse = $x$ cm

Then, by Pythagoras theorem:

x2 = (48020)2 + (36015)2

$x \Rightarrow 60025$ cm

2.




Let one side of the D be = $a$

Perimeter of equilateral triangle = $3a$

$\therefore 3a = 72\sqrt{3} \rightarrow a = 24\sqrt{3}$ m

Height = $AC$; by Pythagoras theorem

$AC2 = a2 - \left(\frac{a}{2}\right)^2$

$AC = 36$ m

3. Let inner radius = $r$; then $2\pi r = 440 \therefore |r = 70$

Radius of outer circle = 70 + 14 = 84 cm

$\therefore$ Outer diameter = 2 × radius = 2 × 84 = 168

4. Let inner radius = $r$ and outer radius = $R$

Width = $R - r = \frac{396}{2\pi} - \frac{352}{2\pi}$

$\Rightarrow (R - r) = \frac{44}{2\pi} = 7$ meters

5. Let outer radius = $R$; then inner radius = $r = R - 7$

$2\pi r = 220 \Rightarrow R = 35$ m;

$r = 35 - 7 = 28$ m

Area of track = $\pi r2 - \pi r2 \Rightarrow \pi(R2 - r2) = 1386$ m2

Cost of levelling it = $1386 \times \frac{1}{2}$ = ₹693

6. Circumference of circle = $2\pi r = 44$

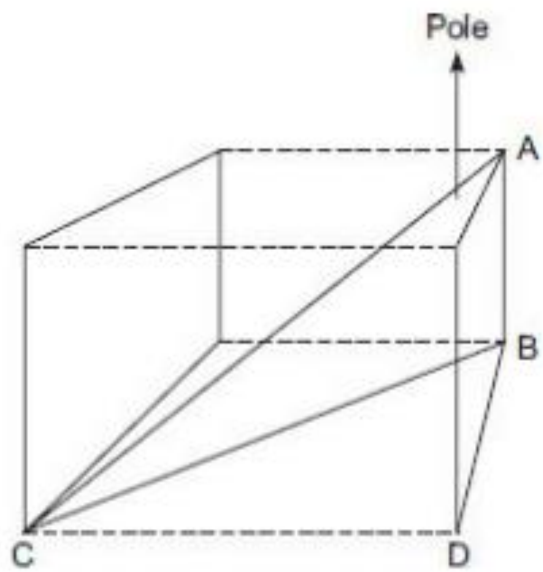
   $= r = 7$ cm

   Area of a quadrant = $\frac{\pi r^2}{4} = 38.5$ cm2

7. Volume of soil removed = $l \times b \times h$

   $= 7.5 \times 6 \times 1.5 = 67.5$ m3

8. The longest pole can be placed diagonally (three-dimensional).




   $BC = \sqrt{18^2 + 24^2} = 30$

   $AC = \sqrt{30^2 + 16^2} = 34$ m

9. Let the common ratio be = $x$

   Then; length = $3x$, breadth = $2x$ and height = $x$

   Then; as per question $3x \,.\, 2x \,.\, x = 1296 \Rightarrow 6x3 = 1296$

⇒ $x = 6$ m

Breadth = $2x = 12$ m

10. Data is inadequate as it is not mentioned that what part of the cube is melted to form cylinder.

11. Let the common ratio be = $x$

Then, length = $4x$, breadth = $3x$ and height = $2x$

As per question;

$2(4x \cdot 3x + 3x \cdot 2x + 2x \cdot 4x) = 8788$

$2(12x^2 + 6x^2 + 8x^2) = 8788 \Rightarrow 52x^2 = 8788$

⇒ $x = 13$

Length = $4x = 52$ cm

12. The total volume will remain the same, let the side of the resulting cube be = $a$. Then,

$6^3 + 8^3 + 10^3 = a^3 \Rightarrow a = \sqrt[3]{1728} = 12$ cm

13. Slant length = $l = \sqrt{6^2 + 8^2} = 10$ cm

Then curved surface area = $\pi rl = \pi \times 6 \times 10 \Rightarrow 60\pi$

And total surface area = $\pi rl + \pi r^2 \Rightarrow \pi((6 \times 10) + 6^2) = 96\pi$ cm$^2$

14. Volume of a cone = $\frac{\pi r^2 h}{3}$

Then, 100p = $\frac{\pi r^2 \cdot 12}{3} \Rightarrow r = 5$ cm

Curved surface area = $\pi rl$

$l = \sqrt{h^2 + r^2} \Rightarrow \sqrt{12^2 + 5^2} = 13$

Then, $\pi rl = \pi \times 13 \times 5 = 65\pi$ cm2

15. Let the radius of the two cones be = $x$ cm

Let slant height of first cone = 5 cm and

Slant height of second cone = 7 cm

Then, ratio of covered surface area = $\frac{\pi \times 5}{\pi \times 7}$ = 5: 7

16. Radius = $\frac{\pi rl}{\pi l} = \frac{2376}{3.14 \times 18}$ = 42 cm

Diameter = 2 × Radius = 2 × 42 = 84 cm

17. Let the radius of cylinder = 1($r$)

Then the radius of cone be = 2($R$)

Then, as per question = $\frac{\pi r^2 h}{\frac{\pi R^2 h}{3}} \Rightarrow \frac{3\pi r^2 h}{\pi R^2 h}$

$\Rightarrow \frac{3r^2}{R^2} \Rightarrow$ 3: 4

18. The perimeter will remain the same in any case.

Let one side of a square be = $a$ cm

Then, $a$2 = 484 ⇒ $a$ = 22 cm perimeter = 4$a$ = 88 cm

Let the radius of the circle be = $r$ cm

Then, $2\pi r = 88 \Rightarrow r = 14$ cm

Then, area = $\pi r$2 = 616 cm2

19. Let the radius of the circle be = $r$

Then, $2\pi r - 2r = 16.8 \Rightarrow r = 3.92$ cm

Then, $2\pi r = 24.64$ cm

20. Let the radius of the wheel be = $r$

Then $5000 \times 2\pi r = 1100000$ cm $\Rightarrow r = 35$ cm

21. Let the slant height be = $l$

Let radius = $r$

Then $v = \frac{\pi r^2 h}{3} \Rightarrow r = \sqrt{\frac{3v}{\pi h}} \Rightarrow \sqrt{\frac{3\times 100\pi}{\pi \times 12}} = 5$ cm

$l = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2} = 13$ cm

22. In four days the short-hand will cover the circumference $4\times2 = 8$ times, while the long-hand will cover its circumference $4\times24 = 96$ times.

Then, the total distance they will cover would be:

$(2 \times \pi \times 4)8 + (2 \times \pi \times 6)96 = 3818.24$ cm

23. Let the radius of the smaller sphere = $r$

Then, the radius of the bigger sphere = $R$

Let the surface area of the smaller sphere = 1

Then, the surface area of the bigger sphere = 4

Then, as per question

$\Rightarrow \frac{4\pi r^2}{4\pi R^2} = \frac{1}{4} \Rightarrow \frac{r}{R} = \frac{1}{2} \Rightarrow R = 2r$

Ratio of their volumes

$= \frac{4\pi r^3}{3} \times \frac{3}{4\pi(2r)^3} \Rightarrow 1:8$

24. Inner radius ($r$) = $\frac{9}{2}$ = 4.5 cm

Outer radius ($R$) = $\frac{10}{2}$ = 5 cm

Volume of metal contained in the shell = $\frac{4\pi R^3 - 4\pi r^3}{3}$

$\Rightarrow \frac{4\pi}{3}(R3 - r3)$

$\Rightarrow$ 141.9 cm3

25. Let smaller radius ($r$) = 1

Then bigger radius ($R$) = 2

Then, as per question

$\Rightarrow \frac{4\pi r^2}{4\pi R^2} = \left(\frac{r}{R}\right)^2 \Rightarrow \left(\frac{1}{2}\right)^2 = 1:4$

26. As per question $\Rightarrow \frac{4\pi r^3}{3} = \frac{\pi r^2 h}{3} \Rightarrow h = 4r$

27. Volume of wall = 1200 × 500 × 25 = 15000000 cm3

Volume of cement = 5% of 15000000 = 750000 cm3

Remaining volume = 15000000 – 750000

= 14250000 cm3

Volume of a brick = 25 × 12.5 × 7.5 = 2343.75 cm3

Number of bricks used = $\frac{14250000}{2343.75}$ = 6080

28. Let the inner radius = $r$

Then, $2\pi r$ = 352 m. Then $r$ = 56

Then, outer radius = $r$ + 7 = 63 = $R$

Now, $\pi r^2 - \pi r^2$ = area of road

$\Rightarrow \pi(R^2 - r^2) = 2618\ m^2$

29. 1 hectare = 10000 $m^2$

Height = 10 cm = $\frac{1}{10}$ m

Volume = $10000 \times \frac{1}{10} = 1000\ m^3$

30. Total surface area of seven cubes $\Rightarrow 7 \times 6a^2 = 1050$

But on joining end to end, twelve sides will be covered.

So, their area = $12 \times a^2 \Rightarrow 12 \times 25 = 300$

So, the surface area of the resulting figure = 1050 – 300 = 750

31. Let the rise in height be = $h$

Then, as per the question, the volume of water should be equal in both the cases.

Now, $90 \times 40 \times h = 150 \times 8$

$h = \frac{150 \times 8}{90 \times 40} = \frac{1}{3}\ m = \frac{100}{3}$ cm

= 33.33 cm

32. Slant height ($l$) = $\sqrt{7^2 + 24^2}$ = 25 m

Area of cloth required = curved surface area of cone = $\pi rl = \frac{22}{7} \times 7 \times 25$ = 550 $m^2$

Amount of cloth required = $\frac{550}{5}$ = 110 m

33. If the ratio of their diameters = 2 : 1, then the ratio of their radii will also be = 2 : 1

Let the radii of the broader cone = 2 and height be = 1

Then the radii of the smaller cone = 1 and height be = 2

Ratio of volumes = $\frac{\pi 2^2 \cdot 1}{3} \div \frac{\pi 1^2 \cdot 2}{3}$

$\frac{4\pi}{3}(12^3 - 10^3)$

34. Area of base = 6 × 10 = 60 m2

Volume of tent = 30 × 10 = 300 m3

Let the radius be = $r$, height = $h$, slant height = $l$

$\pi r2 = 60 \Rightarrow r = \sqrt{\frac{60}{\pi}}$

$\frac{\pi r^2 h}{3}$

35. Volume of wood used = External volume – Internal volume

⇒ (10 × 8 × 6) – (10 – 1) × (8 – 1) × (6 – 1)

⇒ 480 – (9 × 7 × 5) = 165 cubic cm.

36. Total volume in both the objects will be equal. Let the number of smaller cubes = $x$

$x \,.\, 33 = 24 \times 9 \times 8 \Rightarrow x = \frac{24 \times 72}{27} = 64$

37. Let one side of the cube = $a$

Then, $a3 = 216 \Rightarrow a = 6$ m

Area of the resultant figure

= Area of all three cubes – Area of covered figure

$\Rightarrow 216 \times 3 - (4 \times a^2) \Rightarrow 648 - 144 \Rightarrow 504\ m^2$

38. Volume of metal used = $\frac{4\pi R^3}{3} - \frac{4\pi r^3}{3}$

$= \frac{4\pi}{3}(12^3 - 10^3)$

$= 3050.66\ cm^3$

Weight = volume × density ⇒ 4.9 × 3050.66

⇒ 14948.26

39. Volume of cube = $7^3 = 343\ cm^3$

Radius of cone = $\frac{7}{2} = 3.5$ cm

Height of cone = 7 cm

Ratio of volumes = $\frac{\frac{\pi r^2 h}{3}}{343} = \frac{22\times3.5\times3.5\times7}{7\times3\times343}$

⇒ 11:42

40. The volume in both the cases will be equal. Let the height of cone be = $h$

$4 \times \frac{22}{7} \times (14)^3 \times \frac{1}{3} = \frac{22}{7} \times \left(\frac{35}{2}\right)^2 \times \frac{h}{3}$

$\Rightarrow 4(14)^3 = h\left(\frac{35}{2}\right)^2 = h$

$= \frac{4\times14\times14\times14\times2\times2}{35\times35}$

$= h = 35.84$ cm

41. Diameter of circle = diagonal of square

$= \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$

$\therefore$ Radius = $\frac{10\sqrt{2}}{2}$

Area of circle = $\pi r^2 \Rightarrow 50\pi = 157.14$ cm$^3$

42. Area of triangle = $rS$; where $r$ = in-radius

$S = \frac{15+8+17}{2} = 20$ cm

$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$\Rightarrow \Delta\sqrt{20(20-15)(20-8)(20-17)}$

$\Delta = \sqrt{20\times5\times12\times3} = 60$ cm$^2$

$r = \frac{\Delta}{S} = \frac{60}{20} = 3$ cm

43. Circumference of the circular face of the cylinder = $2\pi r$

$\Rightarrow 2 \times \frac{22}{7} \times \frac{35}{100} = 2.2$ m

Number of revolutions required to lift the bucket by 11 m = $\frac{11}{2.2} = 5$

44. Surface area of the cube = $6a^2 = 6 \times (20)^2$

= 2400

Area of six circles of radius 10 cm = $6\pi r^2$

= $6 \times \pi \times 100$

= 1885.71

Remaining area = 2400 – 1885.71 = 514.28

45. $x\,.\,y\,.\,z = lb \times bh \times lh = (lbh)^2$

(v) Volume of a cuboid = $lbh$

So, $V^2 = (lbh)^2 = xyz$

46. Diameter of the circle = diagonal of rectangle

$= \sqrt{8^2 + 6^2} = 10$ cm

Radius $= \frac{10}{2} = 5$ cm

Area of shaded portion $= \pi r^2 - lb$

$= (22/7) \times 5^2 - 8 \times 6$

$= 30.57$ cm$^2$

47. Larger radius ($R$) = 14 + 7 = 21 cm

Smaller radius ($r$) = 7 cm

Area of shaded portion $\pi r^2 \frac{\theta}{360} - \frac{\pi r^2 \theta}{360}$

$\Rightarrow \frac{\pi\theta}{360}(21^2 - 7^2) \Rightarrow 102.67$ cm

48. Area of quadrilateral = Area of right angled triangle + Area of equilateral triangle $x = \sqrt{20^2 - 12^2} = 16$

Area of quadrilateral $= \left(\frac{1}{2} \times 16 \times 12\right) + \frac{\sqrt{3}}{4} \times 20 \times 20$

$= 269$ units$^2$

49. Height $= \sqrt{24^2 - 13^2} = \sqrt{407}$

Volume $= \frac{\text{Area of base} \times \text{height}}{3} \Rightarrow \frac{18 \times 26 \times \sqrt{407}}{3}$

$\Rightarrow 156\sqrt{407}$

50. The perimeter will remain the same in both cases. Circumference of circle $= 2\pi r = 2 \times \frac{22}{7} \times 28 = 176$ cm

Perimeter of square = 176

Greatest side possible = $\frac{176}{4}$ = 44 cm

Length of diagonal = $\sqrt{44^2 + 44^2}$

= $\frac{88}{2} \cdot \sqrt{2}$ = $44\sqrt{2}$

51. Let the initial radius of the circle = $r$

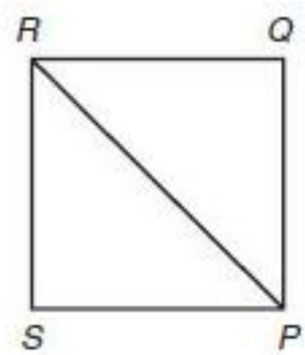
Area = $\pi r2$

New radius = $(r + x)$

New area = $\pi(r + x)2$

So the area increased by $\pi(r + x)2 - r2 = \pi x\,(2r + x)$

52. Let the side of square *PQRS* be $x$ meters.




$PR = \text{x}\sqrt{2}$ meters or 1.414$x$ meters

$PQ + QR = x + x = 2x$ m

Total saving = 2x -1.41$x$ =0.586 $x$ meters

% saving = $\frac{0.586x}{2x} \times 100 = 29.3\% \approx 29\%$

53. Two circles will be similar to each other. When you increase the length measures of a figure by a certain percentage, the effect is the multiplication of the length measure by a certain value. (Thus a 5% increase is the

same as a multiplication of the original length by 1.05). In such a case the area measures, for a similar figure, get multiplied twice (by the same multiplier). Thus, the new area = original area ×1.05×1.05 → 10.25% increase.

This can also be understood as below:

Circumference of a circle= $2\pi r$, where $r$ is the radius.

Circumference a radius

As the circumference increases by 5%, the radius also increases by 5%

∴ New radius = $1.05r$

∴ As area a(radius)2

∴ New area = $(1.05)^2$ × old area= 1.1025 ×old area

∴ Percentage increase in area = 10.25%

54. Side of the biggest possible cube in given cylinder = 20 cm

Volume of cube = $(20)^3$ = 8000

55. Only possible values are: 1 × 1 × 100; 1 × 2 × 50; 1 × 4 × 25; 1 × 5 × 20; 1 × 10 × 10; 2 × 2 × 25; 2 × 5 × 10 and 4 × 5 ×5.

Hence, there are total eight possibilities:

56. Length of largest tile = H.C.F. of 1517 cm and 902 cm = 41 cm.

Area = (41 × 41)$cm^2$

Required number of tiles = $\frac{1517\times 902}{41\times 41} = 814$

57. Side of square = $\sqrt{484} = 22$ cm

So length of the wire = $4 \times 22 = 88$ cm

Longer part = $88 \times \frac{3}{4} = 66$ cm. So the radius of the circle will be $\frac{66}{2\pi}$.

Shorter part = 88 $\frac{1}{4}$= 22 cm. So the length of the side of the square will be $= \frac{11}{2}$.

Area of circle formed by longer part =

$$\pi\left(\frac{66}{2\pi}\right)^2 = \frac{693}{2}\text{cm}^2(\text{Using}\,\pi = \frac{22}{7}).$$

Area of square = $\left(\frac{11}{2}\right)^2 = \frac{121}{4}\text{cm}^2$

Therefore, the total area of both the pieces = $\left(\frac{693}{2}+\frac{121}{4}\right)\text{cm}^2 = 376.75\ \text{cm}^2$

58. There are thirteen successive semicircles with radii 0.5 cm, 1.0 cm, 1.5 cm, and so on.

Total length of the spiral = $\pi \times 0.5 + \pi \times 1.0 + ... + \pi \times 6.5 = \pi\,(0.5 + 1.0 + ... + 6.5) = 143$ cm.

59. Let the three solids have base radius of *r* units. Height of hemisphere = *r* = height of cylinder = height of cone. The curved surface areas of the three solids are:

The curved surface area of the cylinder = $2\pi r \times r = 2\pi r^2$
The curved surface area of the hemisphere: $2\pi r^2$

The curved surface area of the cone = $\pi\, r\sqrt{r^2 + r^2} = \sqrt{2}\pi r^2$

So the required ratio = $2\pi r^2 : 2\pi r^2 : \sqrt{2}\pi r^2$ = $\sqrt{2} : \sqrt{2} : 1$.

60. The curved surface area of sphere, $S = 4\pi r^2$

The rate of change of the radius per unit time can be represented mathematically using:

$dr/dt$ = + 2 cm per second and $r$ = 30 cm

The rate of change of the surface area ($s$) will then be given by: $ds/dt$ = $d(4\pi r^2)/dt = 4\pi \times 2r \times dr/dt = 8\pi \times 30 \times 2 = 480\pi$.

61. Circumference of the base of the cylinder = 22

Let the radius of base of cylinder $r$.

$2\pi r = 22$ or $r = \frac{11}{\pi}$

Height of the cylinder = 10 cm

Volume of the cylinder = $\pi\left(\frac{11}{\pi}\right)^2 10 = 385 \text{ cm}^3$

62. Volume of parallelepiped = $3\times4\times5 = 60$ cm$^3$

Volume of cube = $4\times4\times4 = 64$ cm$^3$

Volume of the cylinder = $\pi(3)^2 3 = 27\pi$ cm$^3$

Volume of sphere = $\frac{4}{3}\pi(3)^3 = 36\pi \text{ cm}^3$

Therefore 4 > 3 > 2> 1

63. If radius of the hemispherical bowl is '$r$' then its volume would be $\frac{2}{3}\pi r^3$

Radius of cylinder = $r$ and height = 2/3 $r$

Volume of cylinder = $\pi r^2.\frac{2}{3}r = \frac{2}{3}\pi r^3$

It means that the volume of the hot water in the cylindrical vessel is 100% of the cylindrical vessel, therefore $x$= 100

64. Let the radius of the field be '$r$' meters. According to the question

$\pi r^2 - (130\times110) = 20350$

$\pi r^2 = 20350 + 14300 = 34650$

$r^2 = 34650 \times \frac{7}{22} = 11025$ or $r$ = 105 m

65. Let $n$ bricks can be put in the tank without spilling over the water. According to the question, the volume of the tank should be totally occupied by the available water and the bricks. Also, since the question tells us that each brick absorbs 10% of its' own volume of water, the additional volume added to the current water by each brick added to the tank would only by 90% of the brick's own volume. Let the number of bricks required by $n$. Then:

$$150 \times 120 \times 100 = n \times 20 \times 6 \times 4\left(1 - \frac{10}{100}\right) + 1281600$$

$$150 \times 120 \times 100 - 1281600 = n \times 20 \times 6 \times 4 \times 0.9$$

$$n = \frac{518400}{20 \times 6 \times 4 \times 0.9} = 1200$$

66. Let radius of smaller sphere be '$r$ cm'.

$$\frac{4}{3} \times \pi \times (10)^3 = 1000 \times \frac{4}{3} \times \pi \times (r)^3$$

$r = 1$ cm

Surface area of the larger sphere = $4\pi(10)^2 = 400\pi$

Total surface area of 1000 smaller spheres = 1000 $4\pi(1)^2$= 4000π

Increase in the surface area = 4000π – 400π = 3600π

Hence, surface area of the metal is increased by 900%. Therefore, $n$ = 900.

67. Volume of the conical writing equipment = $\frac{1}{3}\pi\left(2.5 \times 10^{-1}\right)^2 \times 7 = \frac{11}{24}$ cm$^3$

11/24 cm3 cream can be used to write 330 words.

Number of words that can be written with 1 cm3 cream = $330 \times \frac{24}{11} = 720$

Since 1 litre is 1000 cm3, 2/5 litres = $\frac{2}{5} \times 1000 = 400$ cm3.

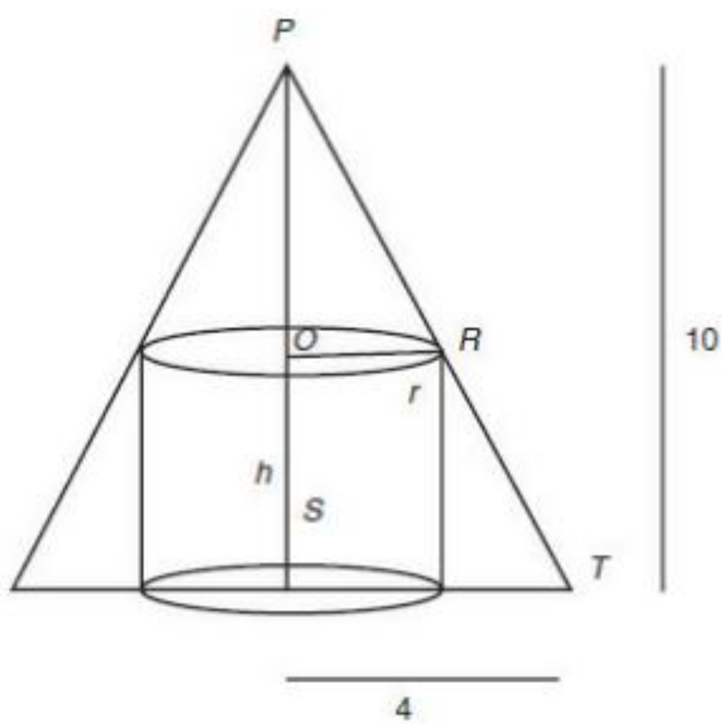
Therefore, number of words that can be written with $\frac{2}{5}$ litre or 400 cm3 = 400 × 720 = 288000.

68. The surface area of the sheet required to make the cap will be equal to the lateral surface area of the cap, which is given by the formula $\pi \times r \times l$ Since the base radius is 14cm and height is 26.5cm, the value of '*l*' can be calculated using $r^2 + h^2 = l^2 \rightarrow l = 29.97 \approx 30$.

Thus, seven caps would require 7 × *p* × 14 × 30 = 9240 cm2.

69. The area of the garden is $34 \times 21 = 714$ sq. m. Out of this the area covered by the paths = $4 \times 34 + 4 \times 21 - 4 \times 4 = 204$ sq. m. The remaining area being covered by flowers will be equal to: 714 – 204 = 510 sq. m.

70. Surface area of the cylinder will be largest when the cylinder touches as shown in the diagram given below:



If $ST$ and $PS$ are the radius and height of the right circular cone respectively and $h$ is the height of the cylinder.

$\Delta POR$ and $\Delta PST$ are similar to each other.

$$\frac{PO}{PS} = \frac{OR}{ST}$$

$$\frac{10-h}{10} = \frac{r}{4}$$

$$h = \frac{20-5r}{2}$$

*Curved surface area of the cylinder* $= 2\pi rh$

$$2\pi r\left(\frac{20-5r}{2}\right)$$

$$5\pi r(4-r)$$

By comparison we get $a = 5, b = 4$

$a \times b = 5 \times 4 = 20$

***Level of Difficulty (II)***

1. Let the angle subtended by the sector at the center be = q.

   Then,

   $$5.7 + 5.7 + (2\pi) \times 5.7 \times \frac{\theta}{360} = 27.2$$

   $$11.4 + \frac{11.4 \times 3.14 \times \theta}{360} = 27.2$$

   $$\Rightarrow \quad \frac{\theta}{360} = 0.44$$

   $$\pi r^2 \frac{\theta}{360} \Rightarrow (22/7) \times (5.7)^2 \times 0.44$$

   $$\Rightarrow \quad \frac{\theta}{360} = 0.44$$

   $$\pi r^2 \frac{\theta}{360} \Rightarrow (22/7) \times (5.7)^2 \times 0.44$$

= 44.92 (approx.)

2. Volume of mud dug out = 10 × 4.5 × 3 = 135 $m^3$

Let the remaining ground rise by = $h$ m

Then, {(20 × 9) – (10 × 4.5)}$h$ = 135

135$h$ = 135 ⇒ $h$ = 1 m

3. Height of the cylinder = 13 – 7 = 6 cm

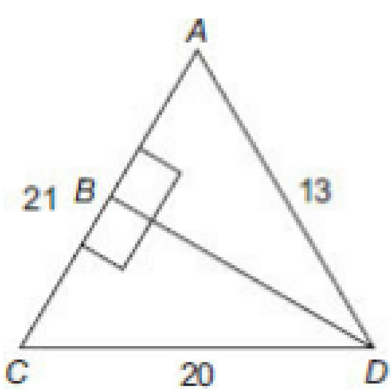
Radius of the cylinder and the hemisphere = 7 cm

Volume of the vessel = volume of cylinder + volume of hemisphere

$$\Rightarrow \pi r^2 h + \frac{4\pi r^3}{3\times 2} \Rightarrow 3.14 \times (7)^2 \times 6 + \frac{4\times 3.14\times (7)^3}{3\times 2}$$

⇒ 1642.6 $cm^3$

4.




Let the original triangle be = $ACD$

Longest side = $AC$ = 21cm

In the right angled $\Delta ABD$, by Pythagorean triplets, we get $AB$ = 5 and $BD$ = 12.

Then, $BC$ = 21 – 5 = 16

By Pythagoras theorem,

$BD^2 = CD^2 - BC^2 \Rightarrow BD = 12$ cm

Thus, our assumption is correct.

Area of the larger $\Delta BDC = \frac{1}{2} \times 16 \times 12 \Rightarrow 96$ cm$^2$

5. Radius = 52.5 m

   Area of the entire canvas, used for the tent

   = Surface area of cylinder + Surface area of cone

   = $2\pi rh + \pi rl$

   $= -2 \times \frac{22}{7} \times 52.5 \times 3 + \frac{22}{7} \times 52.5 \times 53$

   This surface area has to be equal to 5w ×.

   Thus, we have $5w = 2\times\frac{22}{7}\times52.5\times3+\frac{22}{7}\times52.5\times53$

   $\rightarrow w = 1947$

6. The volume in both the cases will be the same.

   Therefore = $\frac{4\pi r^3}{3} = \pi r^2 h$

   $\frac{4\times3.14\times(4\times10)^3}{3} = 3.14 \times 2^2 \times h$

   $\Rightarrow h = \frac{64000}{3} = 21333.33$ mm

7. As the cylinder and cone have equal diameters. So they have equal area.

   Let cone's height be $h_2$ and as per question, cylinder's height be $h_1$.

   $$\frac{2\pi rh_1}{\pi r\sqrt{h_2^2 + r^2}} = \frac{8}{5}$$

On solving, we get the desired ratio as 4: 3

8. Let the slant height of first cone = $L$

Then the slant height of second cone = $3L$

Let the radius of first cone = $r_1$

And let the radius of second cone = $r_2$

Then, $\pi r_1 L = 3 \times \pi r_2 \times 3L$

$\Rightarrow \pi r_1 L = 9\pi r_2 L \Rightarrow r_1 = 9r_2$

Ratio of area of the base

$$\frac{\pi r_1^2}{\pi r_2^2} \Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{9}{1}\right)^2 \Rightarrow 81:1$$

9. Let the internal radius of the cylinder = $r$

Then, the volume of sphere = volume of cylinder

$$\Rightarrow \frac{4\pi \cdot 6^3}{3} = \pi h(5_2 - r_2)$$

$$\Rightarrow \frac{864\pi}{3} = 32\pi(25 - r_2)$$

$\Rightarrow r_2 = 16 \Rightarrow r = 4$ cm

So thickness of the cylinder = 5 – 4 = 1 cm

10. The volume in both the cases will be the same.

Let the height of the cone = $h$

Then, external radius = 6 cm

Internal radius = 4 cm

$$\Rightarrow \frac{4\pi(6^3 - 4^3)}{3} = \frac{\pi \cdot 4^2 \cdot h}{3}$$

$\Rightarrow h = \frac{6^3 - 4^3}{4} \Rightarrow h = \frac{216 - 64}{4} = 38$ cm

11. Let the side of the cube be = $a$ units

Total surface area of three cubes = $3 \times 6a^2$

$= 18a^2$

Total surface area of cuboid = $18a^2 - 4a^2 = 14a^2$

Ratio = $\frac{14a^2}{18a^2} = 7 : 9$

12. $A = 2(xy + yz + zx)$

$V = xyz$

$$A/V = \frac{2(xy + yz + zx)}{xyz} = \frac{2}{z} + \frac{2}{x} + \frac{2}{y}$$

$$\Rightarrow 2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

13. The entire dial of the clock = $360^0$

In 35 minutes, the hand would traverse $210^0$ on the dial.

Hence, the required area = $\frac{210}{360} \times \pi \times 10 \times 10 = 183.33$ cm$^2$.

14. Let the radius of the bigger circle = $R$

Let the radius of the smaller circle = $r$

Then as per question; $R - r = 6$

Solving through options; only option (a) satisfies this condition.

15. Radius of cylinder, hemisphere and cone = 5 cm

Height of cylinder = 13 cm

Height of cone = 12 cm

Surface area of toy = $2\pi rh + \frac{4\pi r^2}{2}$ + p$rL$

$L = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2} = 13$

Then ⇒ (2 × 3.14 × 5 × 13) + (2 × 3.14 × 25) + (3.14 × 5 × 13) ⇒ 770 cm2

16. Height of cone = 10.2 – 4.2 = 6 cm

Volume of wood = $\frac{\pi r^2 h}{3} + \frac{4\pi r^3}{3\times 2}$

⇒ $\frac{3.14\times(4.2)^2\times 6}{3} + \frac{4\times 3.14\times(4.2)^3}{3\times 2}$

⇒ 266 cm3

17. Volume of cylindrical container= $\pi(6)^2 15$

Volume of one cone= volume of the cone + volume of hemispherical top

= $\frac{1}{3}\pi r^2 4r + \frac{2}{3}\pi r^3 = 2\pi r^3$

(Where '$r$' is the radius of the cone).

According to the question:

$10\times 2\pi r^3 = \pi(6)^2 15 or r^3 = 27 \Rightarrow 2r = 2(27)\frac{1}{3} = 6$ cm

Thus, option (a) is true.

18. Radius of cylinder and hemispheres = $\frac{7}{2}$ = 3.5 cm

Height of cylinder = 19 – (3.5 × 2) = 12 cm

Total surface area of solid = $2\pi rh + 4\pi r$2

⇒ 2 × 3.14 × 3.5 × 12 + 4 × 3.14 × (3.5)2

⇒ 418 cm2

19. As they stand on the same base so their radius is also same.

Then; volume of cone = $\frac{\pi r^2 h}{3}$

Volume of hemisphere = $\frac{2\pi r^3}{3}$

Volume of cylinder = $\pi r^2 h$

Ratio = $\frac{\pi r^2 h}{3}$

$\Rightarrow \frac{h}{3} : \frac{2r}{3} : h$

$\Rightarrow h : 2r : 3h$

Radius of a hemisphere = its height

So $h : 2h : 3h \Rightarrow 1 : 2 : 3$

20. Total cost of painting = Total surface to be painted × 0.05 = {External Surface Area + Internal Surface area + Area of ring} ×0.05

$= \{2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)\} \times 0.05 =$ ₹96.28

21. Radius = $\frac{3.5}{2} = 1.75$ cm

Volume of solid = $\pi r^2 h + \frac{\pi r^2 h}{3} + \frac{2\pi r^3}{3}$

$\Rightarrow \pi r^2 \left(h + \frac{h}{3} + \frac{r}{3}\right)$

$\Rightarrow 3.14 \times (1.75)^2 \times \left(10 + \frac{6}{3} + \frac{1.75}{3}\right)$

$\Rightarrow 121$ cm$^3$

22. Area of shaded portion = area of quadrant – area of triangle

$\Rightarrow \frac{\pi r^2}{4} - \frac{1}{2} \times 3.5 \times 2 = \frac{3.14 \times (3.5)^2}{4} - 3.5$

⇒ 6.125 cm2

23. *ABC* is an equilateral triangle with sides = 2 cm

Area of shaded portion = Area of equilateral triangle –Area of three quadrants

⇒ i.e. $\frac{\sqrt{3}}{4} \times 2^2 - 3\left(3.14 \times 1 \times \frac{60}{360}\right)$a2 – 3$\left(\pi r^2 \frac{\theta}{360}\right)$; θ = 60° (Since Δ*ABC* is an equilateral triangle)

⇒ $\frac{\sqrt{3}}{4} \times 2^2 - 3\left(3.14 \times 1 \times \frac{60}{360}\right)$

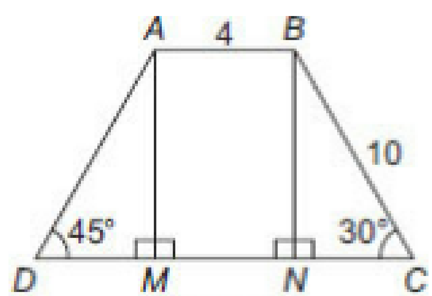
⇒ $\sqrt{3} - \frac{3.14}{2} = \sqrt{3} - \frac{\pi}{2}$

24. Volume of elliptical cylinder = $\pi\left(\frac{2.4}{2}\right)\left(\frac{1.6}{2}\right).7 = 21.12 \text{ m}^3$

Amount of water emptied per minute = $\pi\left(\frac{2}{100}\right)^2 120 \text{ m}^3$

Time required to empty half the tank = $\frac{\frac{21.12}{2}}{\pi\left(\frac{2}{100}\right)^2 120} = 70 \text{ mir}$

25.



Triangles

*AB* and *DC* are the parallel sides

Height = *AM* = *BN*

*AB* = *MN* = 4

DBNC and $\Delta AMD$ are right-angled triangles

In $\Delta BNC \Rightarrow \sin 30 = \frac{BN}{10} \Rightarrow BN = 5$

Using Pythagoras theorem, $NC = \sqrt{10^2 - 5^2} = 5\sqrt{3}$

In $\Delta ADM$; $AM = 5$; $\tan 45 = \frac{AM}{DM} = 1 = \frac{5}{DM}$

$\Rightarrow DM = 5$

Area of trapezium $\Rightarrow \frac{1}{2}(4 + 4 + 5\sqrt{3} + 5) \times 5 = \frac{5(13+5\sqrt{3})}{2}$ (Sum of parallel sides) × height

$\Rightarrow \frac{1}{2}(4 + 4 + 5\sqrt{3} + 5) \times 5 = \frac{5(13+5\sqrt{3})}{2}$

26. $PQ = QR = RS = \frac{12}{3} = 4$ cm

Area of unshaded region $\Rightarrow \frac{\pi 6^2}{2} + \frac{\pi 4^2}{2}$

$\Rightarrow 18\pi + 8\pi \Rightarrow 26\pi$

Area of shaded region $\Rightarrow \frac{\pi 6^2}{2} - \frac{\pi 4^2}{2}$

$\Rightarrow 18\pi - 8\pi = 10\pi$

Ratio $= \frac{10\pi}{26\pi} \Rightarrow \frac{5}{13} \Rightarrow 5 : 13$

27. $QP = \sqrt{5^2 + 12^2} = 13$

Area of the triangle $= \frac{1}{2} \times b \times h = 30$

$\Rightarrow$ As $Rx$ is $a \perp$ drawn to the hypotenuse

So $Rx = \frac{2 \times \text{Area}}{\text{Hypotenuse}} = \frac{60}{13}$

28. A fifty-percent increase in the radius without increasing the height will mean a multiplication of the radius by 1.5. This would mean that the volume would get multiplied by 1.5 × 1.5 (since the volume formula is $\pi r^2 h$).

29. Distance after four hours = $AB = C$

$a = 3 \times 4 = 12$; $b = 2 \times 4 = 8$

and $\frac{a+b+c}{2} \Rightarrow \frac{12+8+C}{2} \Rightarrow \left(10+\frac{C}{2}\right)$

Area = $\sqrt{S(S-a)(S-b)(S-c)}$

Area = $\frac{1}{2}ab \sin 120°$

Area ⇒ $48 \times \frac{\sqrt{3}}{2} = 24\sqrt{3}$

As per question:

$24\sqrt{3}$

On solving, we get $c = 4\sqrt{19}$ km

30. Volume of the cone =

$$\frac{\pi r^2 h}{3} = \frac{22 \times 20 \times 20 \times 24}{3 \times 7} = 10057.14 \text{ cm}^3$$

Diameter of the pipe = 5 mm = 0.5 cm.

Volume of water flowing out of the pipe per minute (in $cm^3$) =

$1000 \times 0.25 \times 0.25 \times \pi = 196.42$ $cm^3$

Hence, the time taken to fill the tank = 10057.14 ÷ 196.42 = 51.2 minutes.

31. One side of the equilateral triangle = diameter of cone.

Therefore radius of cone = $\frac{12}{2}$ = 6

Height of cone = height of equilateral triangle

Height of cone = $6\sqrt{3}$

Volume of cone = $\frac{\pi r^2 h}{3}$

$\Rightarrow \frac{\pi \times 6^2 \times 6\sqrt{3}}{3} = 72\sqrt{3}\pi \text{ cm}^3$

32. Let the radius of the iron ball = $r_1$;

Let the radius of the oak ball = $r_0$;

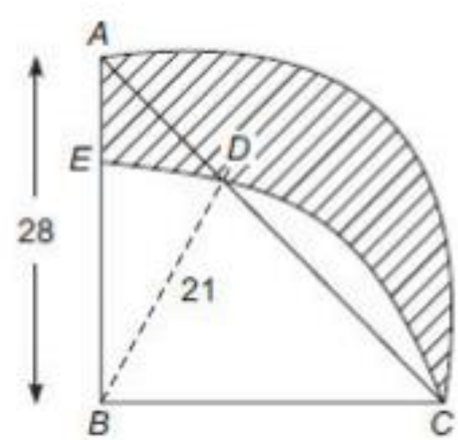
Since, the weight of iron is eight times the weight of oak the volume of the oak ball would need to be eight times the volume of the iron ball for the same weight of the two balls.

Thus, we have:

$\frac{4\pi r_0^3}{3} = \frac{8 \times 4\pi r_1^3}{3} \rightarrow r_0 = 2r_1$

Hence, the diameter of the iron ball is half the diameter of the oak ball.

33.



Area of shaded portion = area of $ADC$ – area of sector $DC$ + area of $\Delta ADB$ – sector $BED$

$\Rightarrow$ Area of $ADC = \pi \times (17.5)^2 \times \frac{1}{2} = 481 \text{ cm}^2$

$$\frac{\angle DBC}{\angle ABC} = \frac{21}{28} \Rightarrow \angle DBC = 67.5 \text{ and } \angle DBA = 22.5$$

$\Rightarrow$ Area of sector $DC = \left(\pi \times 21^2 \times \frac{67.5}{360}\right)$

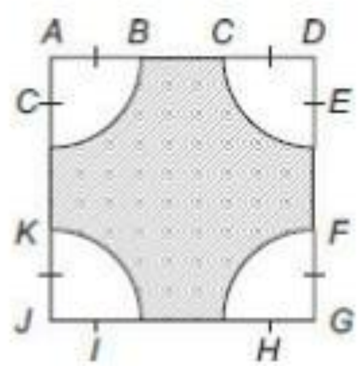
$-\left(\frac{1}{2} \times 21^2 \times \sin 67.5\right) = 56 \text{ cm}^2$

$\Rightarrow$ Area of $ADE = \left(\frac{1}{2} \times 28 \times 21\right)$

$-\left(204 + \frac{1}{2} \times 21^2 \times \sin 22.5\right) = 5.6 \text{ cm}^2$

Thus, area of shaded portion = $480 - 56 + 5.6 = 429 \text{ cm}^2$

34.




$KJ$ = radius of semicircles = 10 cm

Four quadrants of equal radius = 1 circle of that radius

Area of shaded portion $\Rightarrow$ area of rectangle – area of circle

$\Rightarrow (28 \times 26) - (3.14 \times 10^2) \Rightarrow 414 \text{ cm}^2$

$BC = 28 - (10 + 10) = 8$ and $EF = 26 - (10 + 10) = 6$

Perimeter of shaded portion = 28 cm + $2\pi r$

Answer ⇒ 414 $cm^2$ = area and

perimeter = 90.8 cm

35. Go through the option

Only Option (b) is correct as it's area matches with the radius.

36. Area of remaining cardboard = area of trapezium

– area of quadrant

⇒ Area of trapezium = $\frac{1}{2}$(sum of parallel sides)

× height

$= \frac{1}{2} \times (AB + DC) \times BC$

$\Rightarrow \frac{1}{2} \times (3.5 + 5.5) \times 3.5$

= 4.5 × 3.5 = 15.75 $cm^2$

Area of quadrant = $\frac{\pi r^2}{4} \Rightarrow \frac{3.14 \times 3.5 \times 3.5}{4} = 9.625$

⇒ Area of remaining cardboard = 15.7 – 9.6 = 6.075 $cm^2$

37. Circumference of the two semicircles = 312 – (90 + 90) = 132

Two semicircles = one circle with equal radius

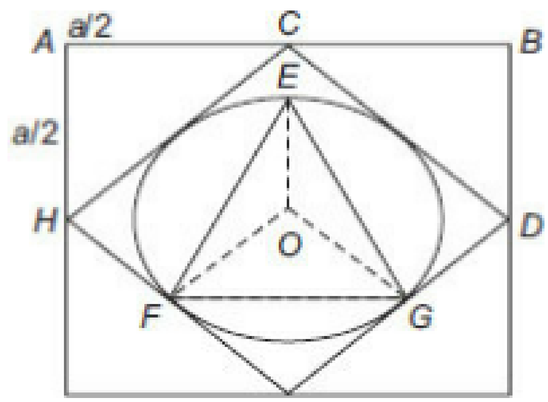
So $2\pi r = 132 \Rightarrow 2r = \frac{132}{3.14}$ ⇒ 42 m diameter

Area of track = Area within external border – Area within internal border

$\Rightarrow \pi(23^2 - 21^2) + 90 \times 46 - 90 \times 42$

$\Rightarrow 88\pi + 360 \Rightarrow 636.57$ $m^2$

38.




$AB$ = side of the outermost square = $a$

$AC = CB = a/2$

$HC = \sqrt{\frac{a^2}{4}+\frac{a^2}{4}} = \frac{a}{\sqrt{2}}$

Diameter of circle = $\frac{a}{\sqrt{2}}$; radius = $\frac{a}{2\sqrt{2}}$

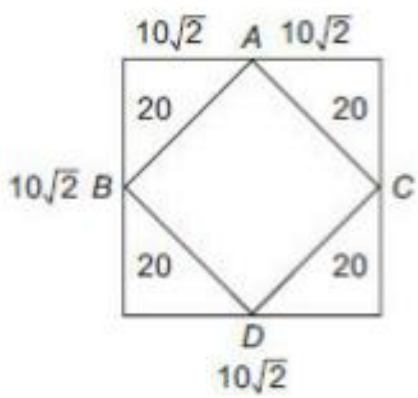
$O$ is the center of the circle. Then $\angle EOF = 120°$

Then area of $\Delta EOF = \frac{1}{2} EO \,.\, OF \,.\, \sin 120°$

$\Rightarrow \frac{1}{2} \times \frac{a^2}{8} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}a^2}{32}$

Then, area of $\Delta EFG = \frac{3\sqrt{3}a^2}{32}$

39.

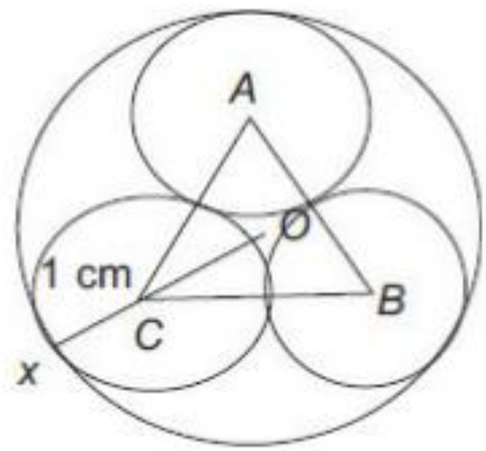


The length of rope of goat = $10\sqrt{2}$m.

Then the two goats will graze an area = area of a semicircle with radius $10\sqrt{2}$m.

So total area grazed = $\frac{\pi r^2}{2} \Rightarrow 100\pi\ m^2$

40.




Let $A$, $B$, $C$ be centers of circles having radius 1 cm and $O$ is the center of the circle circumscribing these three circles.

$AC = AB = BC = 2$ cm

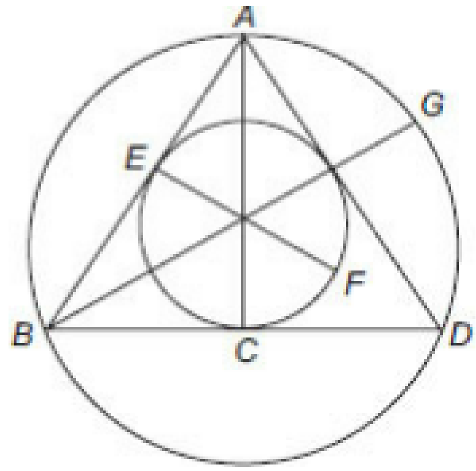
By using the formula of the circum-radius, we can calculate $OC$.

$OC = \dfrac{2\times2\times2}{4\times\frac{\sqrt{3}}{4}(2)^2} = 2/\sqrt{3}$

$OX = OC + CX = \dfrac{2}{\sqrt{3}} + 1$ cm

Required area= $\pi\left(\dfrac{2}{\sqrt{3}}+1\right)^2 = \dfrac{\pi}{3}\left(2+\sqrt{3}\right)^2$ cm$^2$.

41.




Let side of equilateral triangle = $a$

Then height = $\frac{a\sqrt{3}}{2}$

Area = $\frac{\sqrt{3}}{4}a^2$ ; $S = \frac{a+a+a}{2} = \frac{3a}{2}$

Diameter of inner circle = $\frac{2\times \text{Area}}{S}$

$= \frac{\sqrt{3}}{2}a^2 \times \frac{2}{3a} = \frac{a}{\sqrt{3}}$

Diameter of outer circle = $\frac{a^3}{2\times \text{Area}} = a^3 \times \frac{2}{\sqrt{3}a^2}$

$\Rightarrow \frac{2a}{\sqrt{3}}$

Ratio = $\frac{a}{\sqrt{3}} : \frac{2a}{\sqrt{3}} : \frac{a\sqrt{3}}{2}$ ⇒ Ratio = 2 : 4 : 3

42. Sum of interior angles of a hexagon = 720°

six sectors with same radius $r$ = two full circles of same radius

So area of shaded region ⇒ $2\pi r_2$

44. $AO = CO = DO = OB$ = radius of bigger circle = $r$(let)

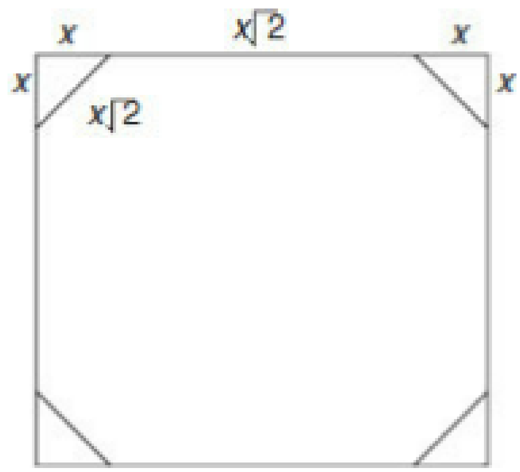
Then area of $(G + F) = \frac{\pi r^2}{2}$

Area of $2(G + F) = \pi r^2$. Also area of $2G + F + E = \pi r^2$

i.e. $2G + F + F = 2G + F + E \Rightarrow F = E$

So the ratio of areas $E$ and $F$ = 1: 1

46. If Mithilesh cuts the cardboard as shown in the diagram below:



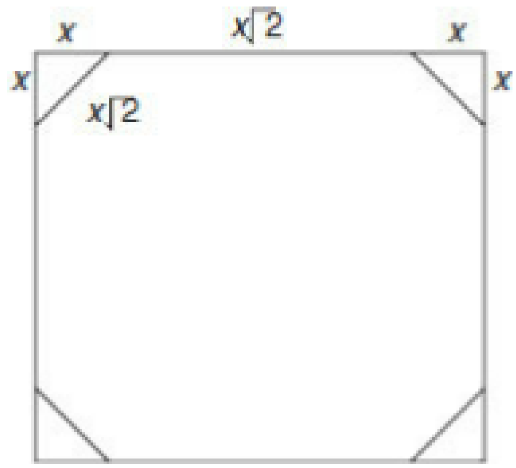

Length of the side of the square = $x\sqrt{2} + x + x = 2x + x\sqrt{2} = 2$

$$x = \frac{2}{2+\sqrt{2}}$$

Reduced area = area of square (PQSR) – (sum of areas of four right-angle triangles)

$$= 4 - \left(4 \times \frac{1}{2} \times \frac{4}{\left(2+\sqrt{2}\right)^2}\right) = \frac{8}{\sqrt{2}+1} \text{ Sq. units}$$ Sq. units

47.




Let the radius of the circle be $x$, then according to the question:

$x^2 + x^2 = a^2$ or $x = \frac{a}{\sqrt{2}}$

$$D_1 = \frac{\pi a^2}{2} - a^2$$

For $A_2$ and $C_2$:

$C_2 = \pi a^2$ and $A_2 = 2a_2$

$$D_2 = \pi a^2 - 2a^2 = 2\left(\frac{\pi}{2}a^2\right) = 2D_1$$

Similarly $D_3 = 4D_1$ and so on. Hence, $DN = 2^{N-1}D_1$

Required ratio = $(D_1 + D_2 + D_3 + D_4 + \ldots DN)/D_1 = (1 + 2 + 4 + \ldots + 2^{(N-1)})D_1/D_1 = (2^N - 1)/(2 - 1) = (2^{12} - 1)$ (for $N = 12$).

48. $S_1 = Q_1 - P_2 = \frac{1}{2}(2r)^2 - \pi\left(\frac{r}{\sqrt{2}}\right)^2 = 2r^2 - \frac{\pi r^2}{2} = r^2\left(\frac{4-\pi}{2}\right)$

$S_2 = Q_2 - P_3 = \frac{1}{2}(\sqrt{2}r)^2 - \frac{\pi r^2}{4} = r^2\left(\frac{4-\pi}{4}\right)$

Required sum, $Sn = S_1 + S_2 + S_3 \ldots$

i.e. Sum of infinite GP having common ratio ½

$$S_n = \left(\frac{r^2\frac{(4-\pi)}{2}}{1-\frac{1}{2}}\right) = 2r^2\left(\frac{4-\pi}{2}\right) = r^2(4-\pi)$$

Required ratio $= \frac{S_n}{Q_1} = \left(\frac{r^2(4-\pi)}{2r^2}\right) = \frac{4-\pi}{2}$

By comparing we get $a = 4$, $b = 2$, therefore $a + b = 4 + 2 = 6$

49. Area of hexagon = area of six equilateral triangles having their side equal to the side of the hexagon $= 6\times\frac{\sqrt{3}}{4}\times(2a)^2 = 6\sqrt{3}a^2$

Area of $PQR = \frac{\sqrt{3}}{4}\times a^2 = \frac{\sqrt{3}}{4}a^2$

Difference $= \sqrt{3}a^2\left(6-\frac{1}{4}\right) = \frac{23}{4}\sqrt{3}a^2 = 23\sqrt{3} \Rightarrow a^2 = \frac{4\times 23\sqrt{3}}{23\sqrt{3}} \Rightarrow a^2 = 4$

Now, we know for such circles circumscribed around a regular hexagon, the radius of the circle is equal to the side of the hexagon. Hence, the radius of the circle is $2a$.

Area of circle $= \pi r^2 = \pi(2a)^2 = 4\pi a^2 = 4\pi \times 4 = 16\pi$ cm$^2$ $\because a^2 = 4$.

Therefore, $X = 16$.

50. Side of the square $S_3$ = diagonal of the square $S_2$ = 4 cm

Side of a square is $\frac{1}{\sqrt{2}}$ times of its diagonal. So side of square $S_2 = \frac{4}{\sqrt{2}}$ cm = $2\sqrt{2}$ cm

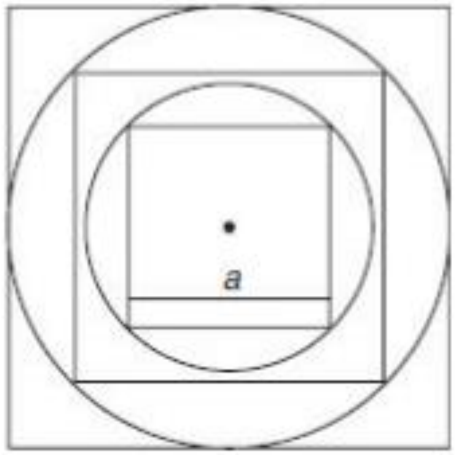
Similarly side of square $S_1 = \frac{2\sqrt{2}}{\sqrt{2}} = 2$ cm. This means that the sides of the consecutive squares form a geometric progression, with common ratio = $\sqrt{2}$ cm.

So side of $S_n = 2\left(\sqrt{2}\right)^{n-1} = 2^{\frac{n+1}{2}}$ cm

Side of $S_{11} = 2^{\frac{11+1}{2}} = 2^6 = 64$ cm cm

Area of the square = $64^2$ = 4096 $cm^2$

51.

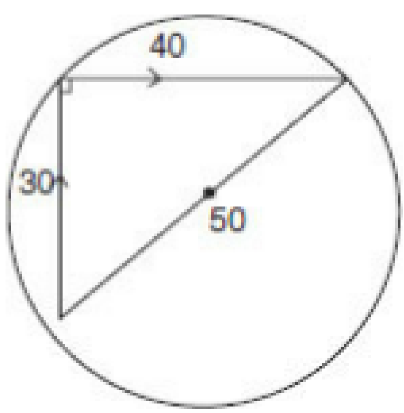



Radius of the circle = $\frac{1}{2}\left(\sqrt{(30^2+40^2)}\right)$ = 25 m

Area of the pool = $\pi\,(25)^2 = 625\,\pi m^2 = X\pi$

Therefore, $X$ = 625

52.



It can be seen from the diagram that actual area = folded area + area of triangular portion ($ABC$).

Area of triangular portion ($ABC$) = $\frac{1}{2} \times 6 \times 6 = 18$ m$^2$

So, there is an increase of 18 m$^2$. Total area of the unfolded rectangle = 144 m$^2$ + 18 m$^2$ = 162 m$^2$.

53. Volume of cylinder= $\pi r^2 h = \pi \times 7^2 \times 10 = 1540$ cm$^3$

Flat surface area of cylinder = $2\pi \times 7^2 = 308$ cm$^2$

*Cone 1*: Volume= $(3/7) \times 1540 = 660$ cm$^3$

Volume of cone = $(1/3)\pi r^2 h = 660$

$\Rightarrow (1/3)\pi r^2\ 10 = 660 \Rightarrow \pi r^2 = 66 \times 3 = 198$ cm$^2$

Flat surface area= $\pi r^2 = 198$ cm$^2$

*Cone 2*: Volume= $(4/7) \times 1540 = 880$ cm$^3$

Volume of cone = $(1/3)\pi r^2 h = 880$

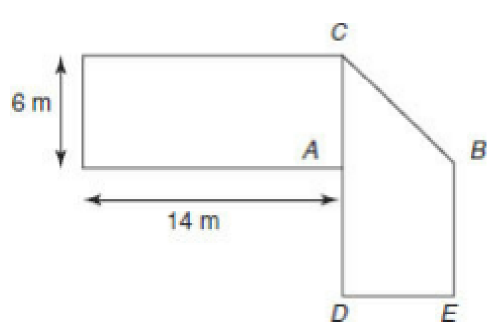
$\Rightarrow (1/3)\pi r^2\ 10 = 880 \Rightarrow \pi r^2 = 88 \times 3 = 264$ cm$^2$

Final flat surface area = 198 + 264 cm$^2$ = 462 cm$^2$

Increase in flat surface area = 462 − 308 = 154 cm$^2$

Percentage increase = (154/308) × 100 = 50%

54. The following image explains the construction.



Radius of inner circle= (1/2) × $50\sqrt{2}$ ft. = $25\sqrt{2}$ ft.

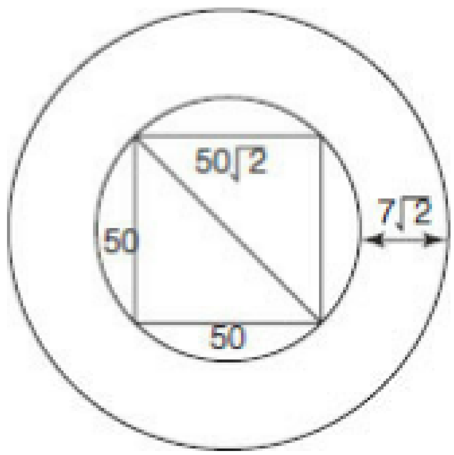
Radius of outer circle = $32\sqrt{2}$ ft.

Area of the path = π × [($32\sqrt{2}$ ft.)2 – ($25\sqrt{2}$ ft.)2] = 2508 ft2

Total cost = 2508 × 100 = ₹250800

50% of this = ₹125400

55. The following figure would exemplify the situation, with the pipe attached at a height of *h* from the apex (bottom) of the cone.




In the above figure, the cones with height *h* and the cone with height *h* + 15 are similar to each other. Hence, using similarity, we will get:

$$\frac{10}{26} = \frac{h}{h+15}$$

$$\rightarrow 10h + 150 = 26h \rightarrow 16h = 150 \rightarrow h = 9.375 \text{ cm}$$

Based on this information, we can then calculate the volume of water that would flow out from the pipe as: total volume of the cone with height (*h* + 15) – volume of cone of height *h*.

$$= \frac{1}{3}\pi[(13^2 \times (15 + 9.375) - (5^2 \times 9.375)]$$

Calculating this, we get the volume of water that overflows = 1295 π cm3.

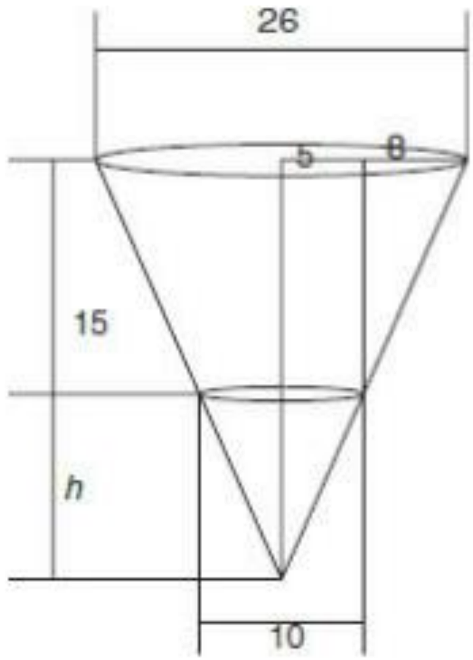
Further, the rate at which the water flows out of the hole per minute is given by:

$\pi \times (0.5^2 \times 1000)$ cm3

Hence, the required time can be given as,

$$\frac{1295\pi}{\pi \times \left(0.5^2 \times 1000\right)} = 5.18 \text{ minutes}$$

56. The given situation can be visualised based on the following figure:




The area of the quadrilateral *PQNM* = area of *PQFE* – area triangle *MVE* + area triangle *VFN*.

So our focus to find the area of the required quadrilateral should shift to the area of the three individual components on the right-hand side of the above equation.

Finding the area of quadrilateral *PQFE*:

Being a parallelogram, the required area would be given by:

$\frac{1}{2} \times$ sum of parallel sides $\times$
perpendicular distance between the parallel sides.

In the figure, let side $AB = 3x$ and side $BC = y$. Then, for the quadrilateral $PQFE$, the perpendicular distance between the parallel side will be $y/2$.

Further, the sum of parallel sides would be equal to

$$PQ + EF = x + \frac{x}{2} = \frac{3x}{2}$$

Area of $PQFE$ = $\frac{1}{2} \times \frac{3x}{2} \times \frac{y}{2}$

We know that the area of the rectangle is $3x \times y = 90$.

Hence, area of $PQFE = 90 \div 8 = 11.25$.

Finding the area of the triangle $MEV$:

$$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times EV \times \text{height}$$

Using similarity between $APM$ and $VEM$, we can see that since $AP = x$ and $EV = x/4$, the ratios of the lengths of $APM$ and $VEM$ will be 4:1.

Thus, if the height of $APM = 4h$, the height of $VEM = h$ and also $4h + h = y/2 \rightarrow h$ = height of $VEM$ with base $VE = y/10$

Hence, area triangle MEV = $\frac{1}{2} \times EV \times \text{height} = \frac{xy}{80}$

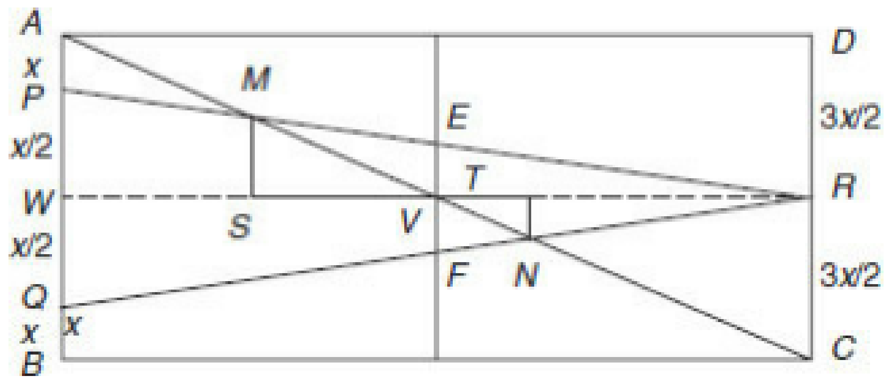
= 0.375

Similarly, the area of $\Delta VFN \approx 0.27$.

Thus, the required area = $11.25 - 0.375 + 0.27 \approx 11.145$.

57. Think of this question as follows:

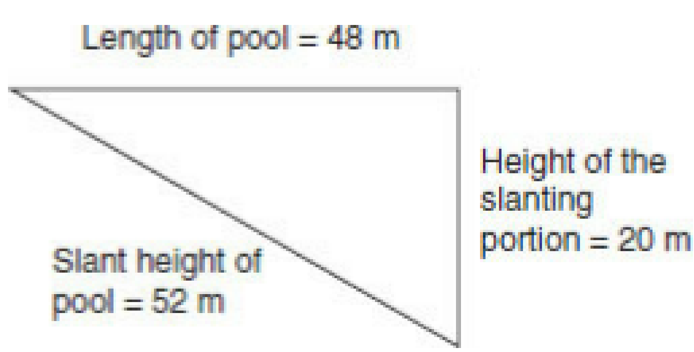
The entire park is 1200 square meters. Out of this, the area of the road formed has to be 600 meters (based on the condition that 'the Mayor wants that the area of the two roads to be equal to the remaining area of the park'). Also, the width of all the roads should be equal (since, the diagonals of the parks have to be diagonals of the small rectangle formed at the intersection of the two roads - it means that the rectangle formed at the intersection of the two roads should be a square). The figure would look as follows:



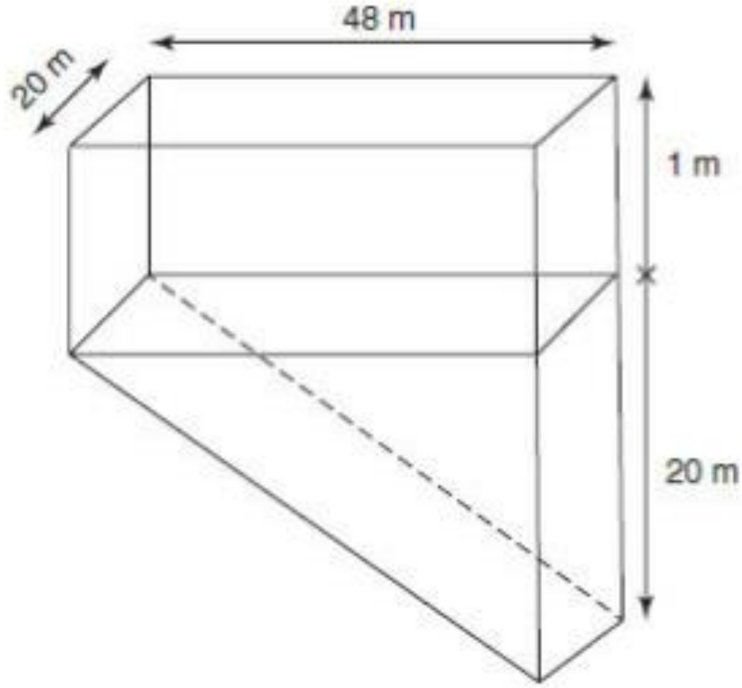

Using the options, we can see that if the road width is 10 meters, the areas covered by the roads will be 10 × 40 + 10 × 30 − 10 × 10 = 600, which will mean that exactly half the area of the park will be covered by roads.

Hence, option (a) is correct.

58. For every 2.6m that one walks along the slant part of the pool, there is a height of 1 m that is gained. Also, since the length of the pool is 48 m, we get the following dimensions of the pool.



The pool would look as given in the figure below:



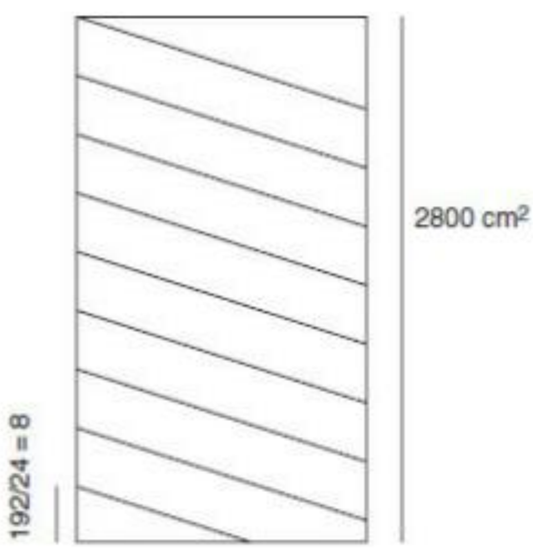

The volume of water in the pool = volume of the upper part + volume of the standard triangular vessel

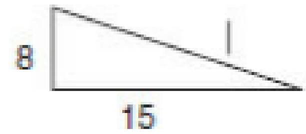
$$= \left(\frac{1}{2} \times 48 \times 20\right) \times 20 + (48 \times 20 \times 1)$$

$$\Rightarrow 48 \times 20 \times 11 \Rightarrow 10560 \ m^3$$

Hence, Option (d) is the correct answer.

59.






$$h = \frac{2\pi rh}{2\pi r} = \frac{2880}{15} = 192 \text{ cm}$$

$$l = \sqrt{8^2 + 15^2} = 17 \text{ cm}$$

Therefore, length of one complete turn = 17 cm

Hence, total length of the thread = 17 × 24 = 408 cm

60. Let θ be the angle made by minute-hand to cover an area of 110.88 $cm^2$.

$$\Rightarrow \frac{22}{7} \times (4.2)^2 \times \frac{\theta}{360°} = 110.88$$

$$\theta = \frac{110.88 \times 360° \times 7}{22 \times (4.2)^2}$$

$$\theta = 720°$$

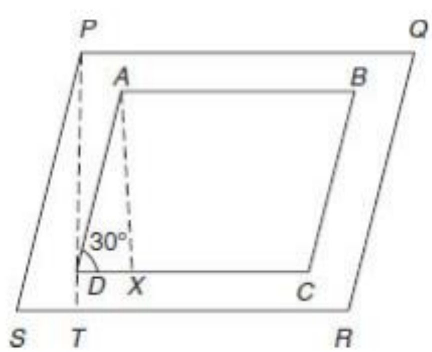
As we know that speed of hour-hand is $\frac{1}{12}$ of the speed of minute-hand, therefore, angle covered by hour-hand during this period is $\frac{720°}{12} = 60°$.

Area covered by hour-hand = $\pi(2.1)^2 \times \frac{60°}{360°}$.

$$= \; = \frac{22}{7} \times 2.1 \times 2.1 \times \frac{1}{6}$$

= 2.31 $cm^2$

61.



Draw $AX \perp DC$

$AX = 10 \sin 30° = 5$ m

If $PT \perp SR$

$PT = 5 + 2 + 2 = 9$ m

$PT = 5 + 2 + 2 = 9$ m

$$PS = \frac{PT}{\sin 30°} = \frac{9}{1/2} = 18 \text{ m}$$

Area of the path = area of $PQRS$ – area of $ABCD$ (Using area of rhombus = base × height)

= 18 × 9 – (10 × 5)

= 162 – 50 = 112 m2

62. Area of $\square ABCD = 1$

Area of portion between $ABCD$ and $PQRS = \frac{3}{2} - 1 = \frac{1}{2}$

Area of the next portion (between $PQRS$ and $XYZW$) $= \frac{7}{4} - \frac{3}{2} = \frac{1}{4}$

Area of the next portion $= \frac{15}{8} - \frac{7}{4} = \frac{1}{8}$

So the required area is the sum of the infinite geometric progression represented by: $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

$$= \frac{1}{1 - \frac{1}{2}} = 2 = 2$$

63. If side of the smallest hexagon is '$a$' and the side of the second largest and largest hexagons are $b$ and $c$ respectively. Then according to the question:

$$\frac{3\sqrt{3}}{2} a^2 = A_1$$

$$\frac{3\sqrt{3}}{2}b^2 = A_1 + A_2 = 3A_1$$

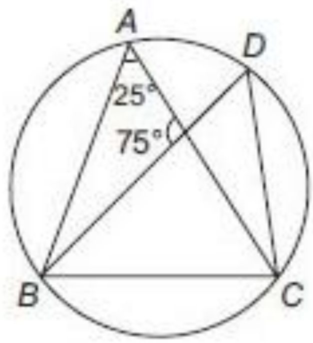
$$\frac{3\sqrt{3}}{2}c^2 = A_1 + A_2 + A_3 = 6A_1$$

$$a^2 : b^2 : c^2 = 1:3:6$$

$$a:b:c = 1:\sqrt{3}:\sqrt{6}$$

**EXTRA PRACTICE EXERCISE ON GEOMETRY AND MENSURATION**

1. In the figure given what is the measure of $\angle ACD$?



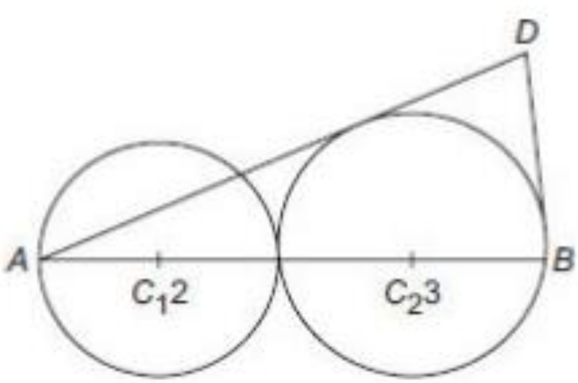

(a) 75°

(b) 80°

(c) 90°

(d) 105°

2. Two circles $C_1$ and $C_2$ of radius 2 and 3 respectively touch each other as shown in the figure. If $AD$ and $BD$ are tangents then the length of $BD$ is



(a) $3\sqrt{6}$

(b) $5\sqrt{6}$

(c) $(3\sqrt{10})/2$

(d) 6

3. If the sides of a triangle measure 13, 14, 15 cm respectively, what is the height of the triangle for the base side 14?

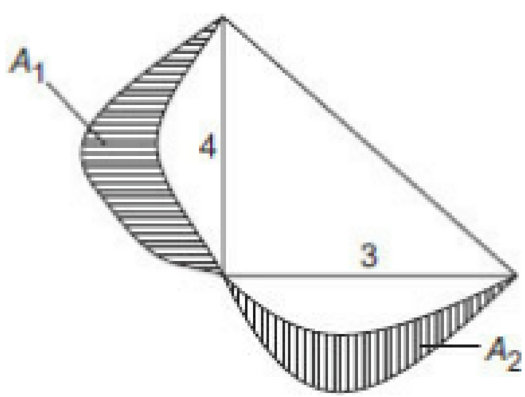
(a) 10

(b) 12

(c) 14

(d) 13

4. A right-angled triangle is drawn on a plane such that sides adjacent to right angle are 3 cm and 4 cm. Now three semi-circles are drawn taking all three sides of the triangle as diameters respectively (as shown in the figure). What is the area of the shaded regions $A_1 + A_2$?




(a) $3\pi$

(b) $4\pi$

(c) $5\pi$

(d) None of these

5. A lateral side of an isosceles triangle is 15 cm and the altitude is 8 cm. What is the radius of the circumscribed circle?

(a) 9.625

(b) 9.375

(c) 9.5

(d) 9.125

6. Let $a$, $b$, $c$ be the length of the sides of triangle $ABC$. Given $(a + b + c)(b + c - a) = abc$. Then the value of a will lie in between

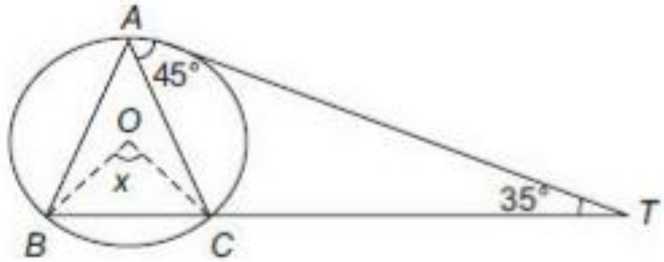
(a) –1 and 1

(b) 0 and 4

(c) 0 and 1

(d) 0 and 2

7. In the figure given below (not drawn to scale). $A$, $B$ and $C$ are three points on a circle with center $O$. The chord $BC$ is extended to point $T$ such that $AT$ becomes a tangent to the circle at point $A$. If $\angle CTA = 35°$ and $\angle CAT = 45°$. calculate $x°$ ($\angle BOC$).



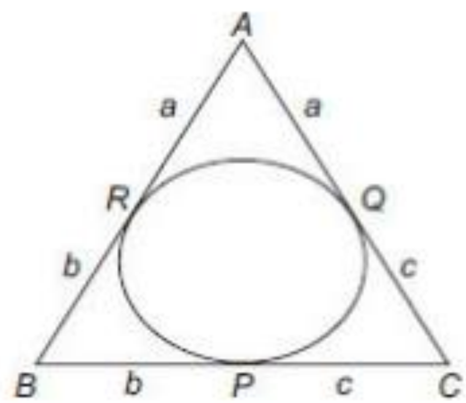

(a) 100°

(b) 90°

(c) 110°

(d) 65°

8. In the given figure.




$AB = 20$

$BC = 15$

$CA = 19$

Calculate $a, b, c$.

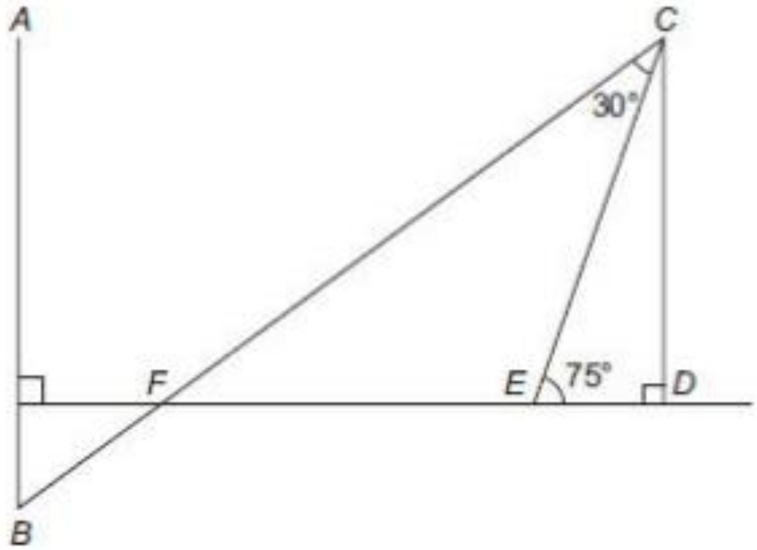
(a) $a = 12, b = 8, c = 7$

(b) $a = 8, b = 12, c = 7$

(c) $a = 9, b = 10, c = 15$

(d) $a = 10, b = 15, c = 9$

9. In the figure given below, $AB$ is perpendicular to $ED$. $\angle CED = 75°$ and $\angle ECF = 30°$. What is the measure of $\angle ABC$?



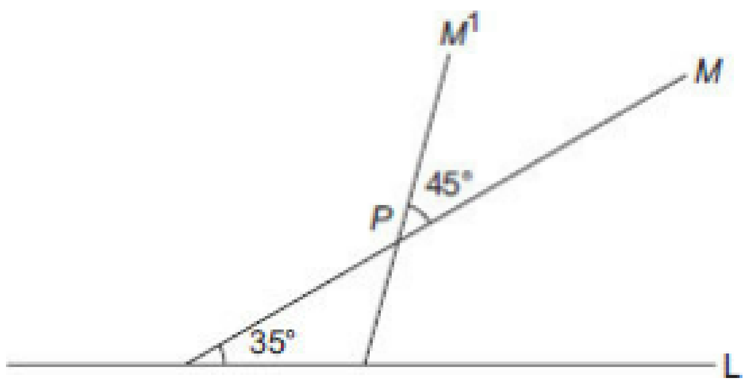

(a) 60°

(b) 45°

(c) 55°

(d) 30°

10. The angle between lines *L* and *M* measures 35° degrees. If line *M*. is rotated 45° degrees counter clockwise about point *P* to line *M*1 what is the angle in degrees between lines *L* and *M*1?



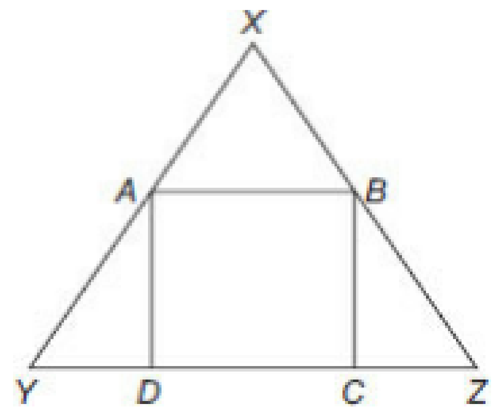

(a) 90°

(b) 80°

(c) 75°

(d) 60°

11. In the figure given below, $XYZ$ is a right-angled triangle in which $\angle Y$ = 45° and $\angle X$ = 90°. $ABCD$ is a square inscribed in it whose area is 64 cm2. What is the area of triangle $XYZ$?




(a) 100

(b) 64

(c) 144

(d) 81

12. The area of circle circumscribed about a regular hexagon is 144π. What is the area of hexagon?

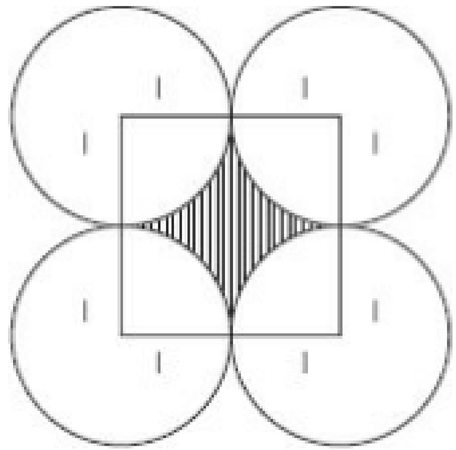
(a) $300\sqrt{3}$

(b) $216\sqrt{3}$

(c) 256

(d) 225

13. Find the area of the shaded portion.



(a) $4 - \pi$

(b) $6 - \pi$

(c) $5 - \pi$

(d) $\pi$

14. The numerical value of the product of the three sides (which are integers when measured in cm) of a right-angled triangle having a perimeter of 56 cm is 4200. Find the length of the hypotenuse.

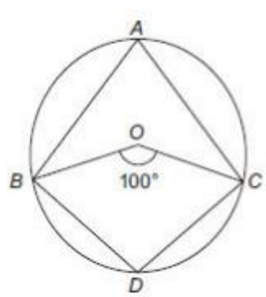
(a) 24

(b) 25

(c) 15

(d) 30

15. In the figure, *ABDC* is a cyclic quadrilateral with *O* as center of the circle. Find $\angle BDC$.



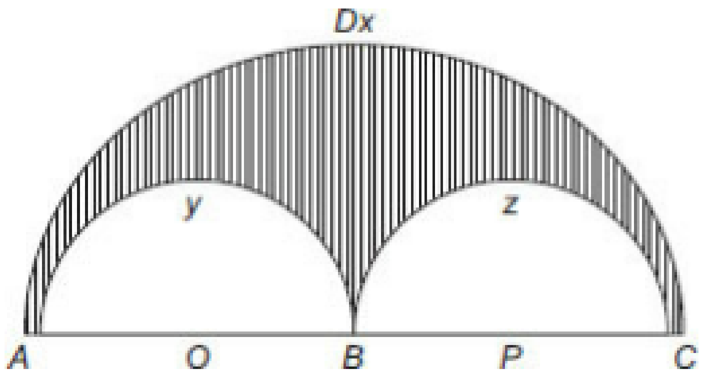
(a) 105°

(b) 120°

(c) 130°

(d) 95°

16. $B$, $O$, and $P$ are centers of semicircles $AXC$, $AYB$ and $BZC$ respectively and $AC$ = 12 cm. Find the area of the shaded region.



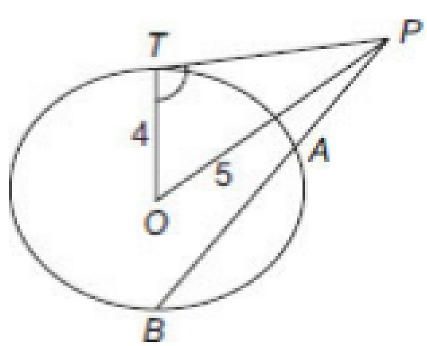

(a) 9π

(b) 18π

(c) 20π

(d) 25π

17. $O$ is the center of the circle. $OP$ = 5 and $OT$ = 4, and $AB$ = 8. The line $PT$ is a tangent to the circle. Find $PB$.



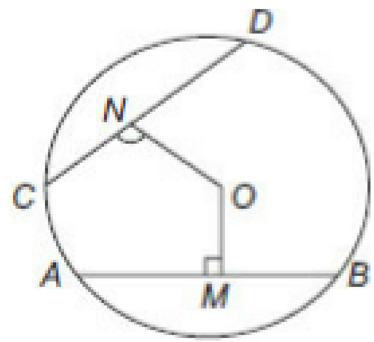
(a) 9 cm

(b) 10 cm

(c) 7 cm

(d) 8 cm

18. In the figure given below, $AB = 16$, $CD = 12$ and $OM = 6$. Calculate $ON$.



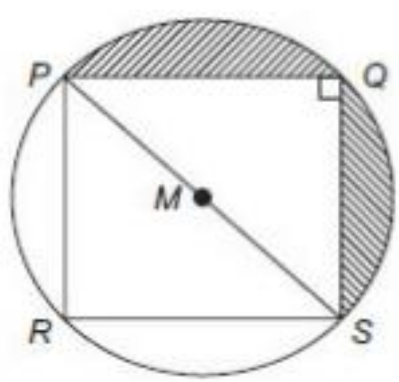

(a) 8

(b) 10

(c) 12

(d) 14

19. In the figure, $M$ is the center of the circle. $L(QS) = 10\sqrt{2}$, $L(PR) = L(RS)$ and $PR$ is parallel to $QS$. Find the area of the shaded region.



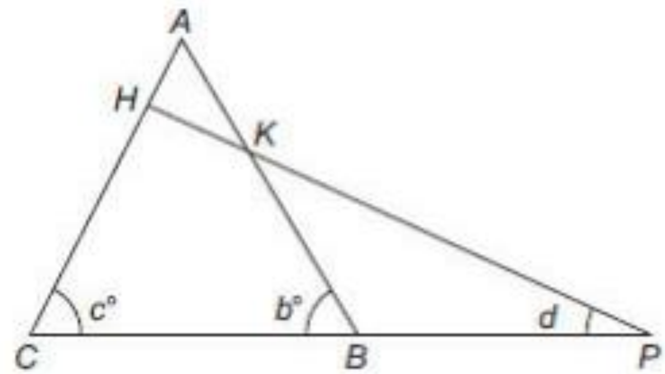
(a) $90\pi - 90$

(b) $50\pi - 100$

(c) $150\pi - 150$

(d) $125\pi - 125$

20. In the given figure *PBC* and *PKH* are straight lines. If $AH = AK$, $b = 70°$, $c = 40°$, the value of $d$ is



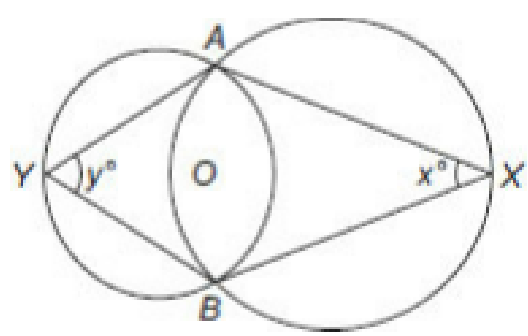

(a) 20°

(b) 25°

(c) 15°

(d) 35°

21. In the given figure, circle *AXB* passes through '*O*', the center of circle *AYB*. *AX* and *BX* and *AY* and *BY* are tangents to the circles *AYB* and *AXB* respectively. The value of $y°$ is



(a) $180° - x°$,

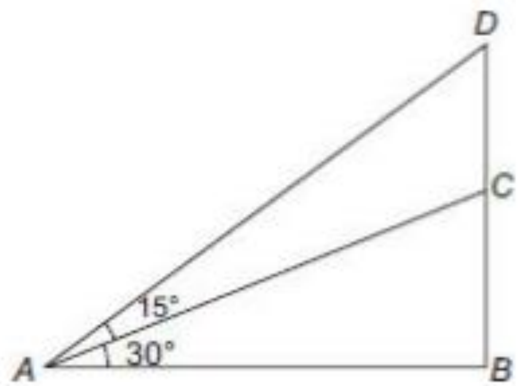
(b) $180° - 2x°$

(c) $\frac{1}{2}(90° - x°)$

(d) $90° - (x°/2)$

22. In the figure, $AB = x$

Calculate the area of triangle $ADC$ ($\angle B = 90°$).




(a) $\frac{1}{2}x2 \sin 30°$

(b) $\frac{1}{2}x2 \cos 30°$

(c) $\frac{1}{2}x2 \tan 30°$

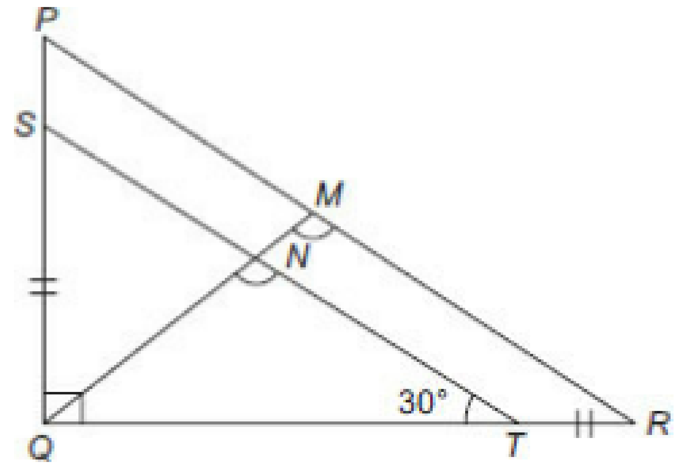
(d) $\frac{1}{2}x2(\tan 45° - \tan 30°)$

23. In the given figure. $SQ = TR = a$, $QT = b$, $QM \perp PR$, $ST$ is parallel to $PR$.

$m \angle STQ = 30°$

$m \angle SQT = 90°$

Find $QM$.



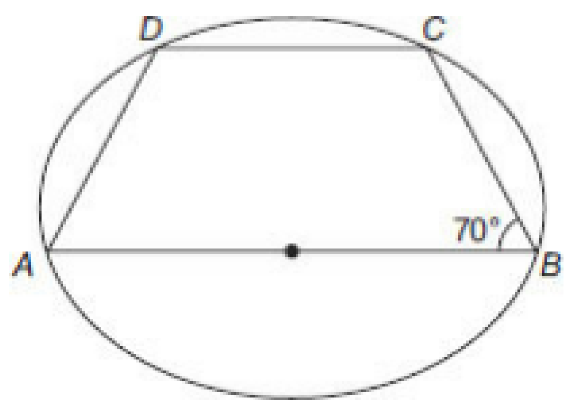

(a) $(a + b)/2$

(b) $2(a + b)$

(c) $(2a + b)/2$

(d) $(a + 2b)/2$

24. In the figure given, $AB$ is a diameter of the circle and $C$ and $D$ are on the circumference such that $\angle CAD = 40°$. Find the measure of the $\angle ACD$




(a) 40°

(b) 50°

(c) 60°

(d) None of these

25. Six solid hemispherical balls have to be arranged one upon the other vertically. Find the minimum total surface area of the cylinder in which the hemispherical balls can be arranged, if the radius of each hemispherical ball is 7 cm.

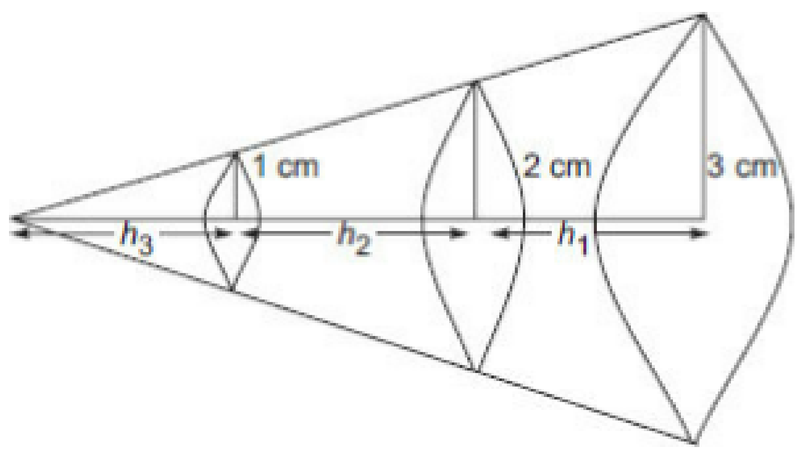
(a) 2056

(b) 2156

(c) 1232

(d) None of these

***Directions for Questions 26 and 27:*** In the following figure, there is a cone which is being cut and extracted in three segments having heights $h_1$, $h_2$ and $h_3$ and the radius of their bases 1 cm, 2 cm and 3 cm respectively, then




26. The ratio of the volumes of the smallest segment to that of the largest segment is

(a) 1 : 27

(b) 27 : 1

(c) 1: 19

(d) None of these

27. The ratio of the curved surface area of the second largest segment to that of the full cone is:

(a) 1 : 3

(b) 4 : 9

(c) Cannot be determined

(d) None of these

28. On a semicircle with diameter *AD*, chord *BC* is parallel to the diameter. Further each of the chords *AB* and *CD* has length 2 cm while *AD* has length 8 cm. Find the length of *BC*.

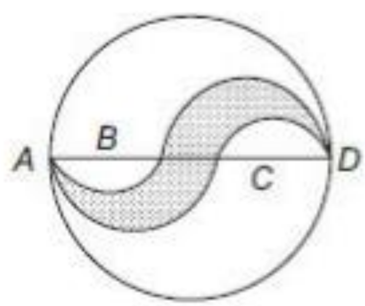
(a) 7.5 cm

(b) 7 cm

(c) 7.75 cm

(d) Cannot be determined

29. In the given figure, *B* and *C* are points on the diameter *AD* of the circle such that *AB* = *BC* = *CD*. Then find the ratio of area of the shaded portion to that of the whole circle.



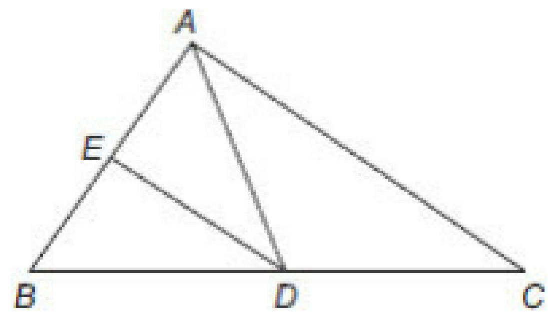

(a) 1 : 3

(b) 2 : 3

(c) 1: 2

(d) None of these

30. In the given figure, $ABC$ is a triangle in which $AD$ and $DE$ are medians to $BC$ and $AB$ respectively, then the ratio of the area of $\Delta BED$ to that of $\Delta ABC$ is




(a) 1 : 4

(b) 1 : 16

(c) Data inadequate

(d) None of these

31. Two identical circles intersect so that their centers, and the points at which they intersect, form a square of side 1 cm. The area in square cm of the portion that is common to the two circles is

(a) $\pi/4$

(b) $\pi/2 - 1$

(c) $\pi/5$

(d) $\sqrt{2} - 1$

32. If the height of a cone is trebled and its base diameter is doubled, then the ratio of the volume of the resultant cone to that of the original cone is

(a) 9 : 1

(b) 9 : 2

(c) 12 : 1

(d) 6 : 1

33. If two cylinders of equal volume have their heights in the ratio 2: 3, then the ratio of their radii is

(a) $\sqrt{3} : \sqrt{2}$

(b) 2 : 3

(c) $\sqrt{5} : \sqrt{3}$

(d) $\sqrt{6} : \sqrt{3}$

34. Through three given non-collinear points, how many circles can pass?

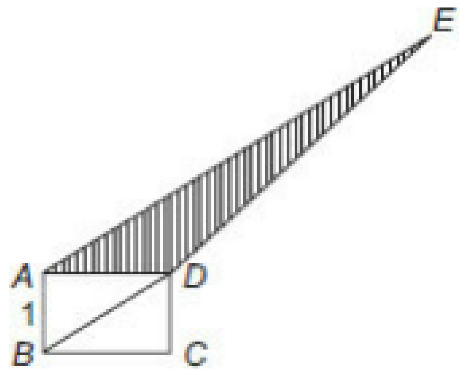
(a) 2

(b) 3

(c) Both 1 and 2

(d) None of these

35. The area of the rectangle *ABCD* is 2 and *BD* = *DE*. Find the area of the shaded region.




(a) $\sqrt{5}$

(b) $2\sqrt{5}$

(c) $\sqrt{(5/2)}$

(d) 1

36. In a right-angled triangle, the square of the hypotenuse is equal to twice the product of the other two sides. The acute angles of the triangle are

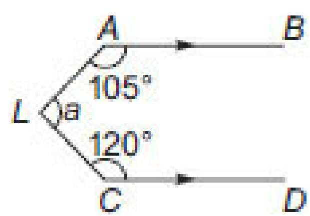
(a) 30° and 30°

(b) 30° and 60°

(c) 15° and 75°

(d) 45° and 45°

37. Find $\angle ALC$ if $AB \parallel CD$.




(a) 75

(b) 135

(c) 110

(d) 145

38. If the sides of a triangle are in the ratio of ½ : 1/3: ¼ , the perimeter is 52 cm, then the length of the smallest side is

(a) 12 cm

(b) 11 cm

(c) 8 cm

(d) None of these

39. The number of distinct triangles with integral valued sides and perimeter as 14 is

(a) 2

(b) 3

(c) 4

(d) 5

40. A polygon has 65 diagonals. Then, what is the number of sides of the same polygon?

(a) 11

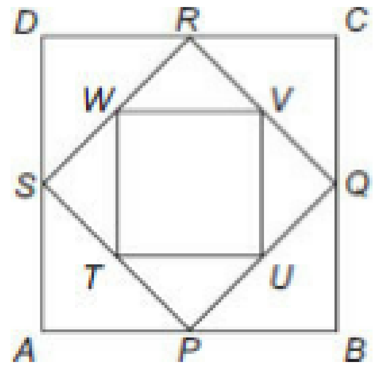
(b) 12

(c) 14

(d) None of these

41. *PQRS* is a square drawn inside square *ABCD* of side $2x$ units by joining the mid-points of the sides *AB*, *BC*, *CD*, and *DA*. The square *TUVW* is drawn inside *PQRS*, where *T*, *U*, *V*, *W* are the mid-points of *SP*, *PQ*, *QR* and *RS*. If

the process is repeated an infinite number of times the sum of the areas of all the squares will be equal to




(a) 8x2

(b) 6x2

(c) 16x2

(d) 6x2/2

42. Suppose the same thing is done with an equilateral triangle of side *x*, wherein the mid-points of the sides are connected to each other to form a second triangle and the mid-points of the sides of the second triangle are connected to form a third triangle and so on an infinite number of times —then the sum of the areas of all such equilateral triangles would be

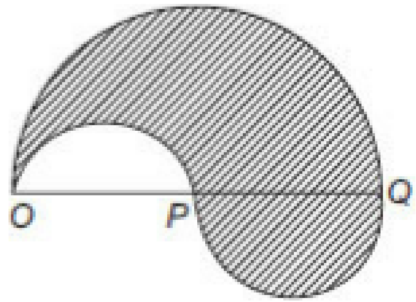
(a) 3x2

(b) 6x2

(c) 12x2

(d) None of these

43. If in the figure given below *OP* = *PQ* = 28 cm and *OQ*, *PQ* and *OP* are all joined by semicircles, then the perimeter of the figure (shaded area) is equal to




(a) 352 cm

(b) 264 cm

(c) 176 cm

(d) 88 cm

44. For the question above, what is the shaded area?

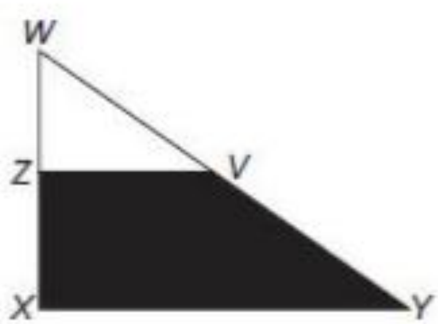
(a) 1352 sq. cm

(b) 1264 sq. cm

(c) 1232 sq. cm

(d) 1188 sq. cm

45. What is the area of the shaded portion? It is given that $ZV || XY$, $WZ = ZX$, $ZV = 2a$ and $ZX = 2b$.



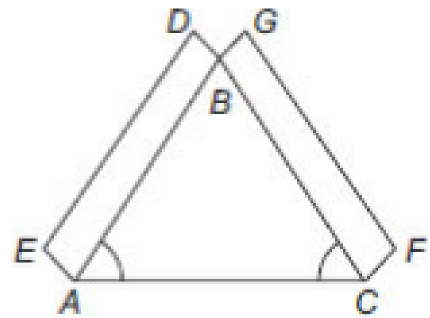

(a) $\frac{4ab}{2}$

(b) $\frac{8ab}{3}$

(c) $6ab$

(d) $3ab$

46. In the given figure, there is an isosceles triangle $ABC$ with angle $A$ = angle $C$ = 50°. $ABDE$ and $BCFG$ are two rectangles drawn on the sides $AB$ and $BC$ respectively, such that $BD = BG = AE = CF$.




Find the value of the angle $DBG$.

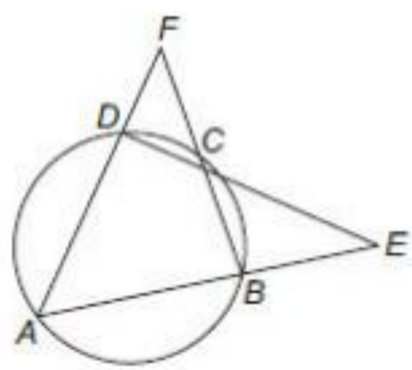
(a) 80°

(b) 120°

(c) 100°

(d) 140°

47. In the figure, $ABE$, $DCE$, $BCF$ and $ADF$ are straight lines. If $E$ = 50°, $F$ = 56°, find $\angle A$.



(a) 47°

(b) 37°

(c) 40°

(d) 42°

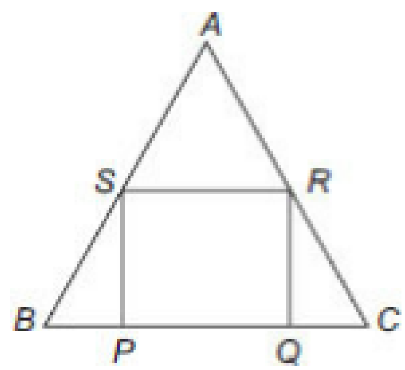
48. *ABC* is an equilateral triangle. *PQRS* is a square inscribed in it. Therefore.

(a) $AR^2 = RC^2$

(b) $2AR^2 = RC^2$

(c) $3AR^2 = 4RC^2$

(d) $4AR^2 = 3RC^2$




49. Consider the five points comprising the vertices of a square and the intersection point of its diagonals. How many triangles can be formed using these points?

(a) 4

(b) 6

(c) 8

(d) 10

50. In a triangle $ABC$, the internal bisector of the angle $A$ meets $BC$ at $D$. If $AB = 4$, $AC = 3$ and $\angle A = 60°$. Then, the length of $AD$ is

(a) $2\sqrt{3}$

(b) $(12\sqrt{3})/7$

(c) $(15\sqrt{3})/8$

(d) $(6\sqrt{3})/7$

***Directions for Questions 51 and 52:*** Answer the questions based on the following information.

A rectangle $PRSU$, is divided into two smaller rectangles $PQTU$ and $QRST$ by the line $QT$. $PQ = 40$ cm. $QR = 20$ cm, and $RS = 40$cm. Points $A, B, F$ are within rectangle $PQTU$, and points $C, D, E$ are within the rectangle $QRST$. The closest pair of points among the pairs $(A, C), (A, D), (A, E), (F, C), (F, D), (F, E), (B, C), (B, D), (B, E)$ are $40\sqrt{3}$ cm apart.

51. Which of the following statements is necessarily true?

(a) The closest pair of points among the six given points cannot be $(F, C)$.

(b) Distance between $A$ and $B$ is greater than that between $F$ and $C$.

(c) The closest pair of points among the six given points is $(C, D)$, $(D, E)$ or $(C, E)$.

(d) None of these.

52. $AB > AF > BF$; $CD > DE > CE$; and $BF = 24\sqrt{5}$ cm. Which is the closest pair of points among all the six given points?

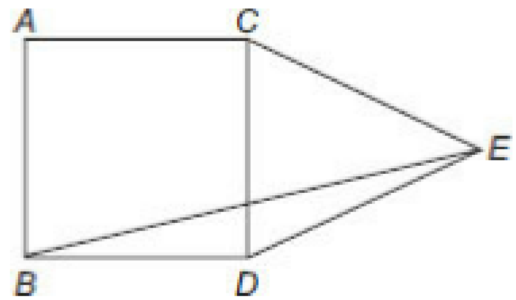
(a) $B, F$

(b) $C, D$

(c) $A, B$

(d) None of these

53. If $ABCD$ is a square and $CDE$ is an equilateral triangle, what is the measure of $\angle DEB$?



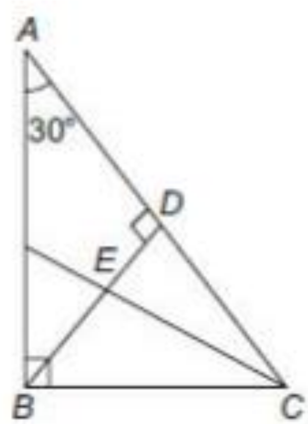

(a) 15°

(b) 30°

(c) 20°

(d) 45°

54. $AB \perp BC$, $BD \perp AC$ and $CE$ bisects $\angle C$, $\angle A = 30°$. Then, what is $\angle CED$?




(a) 30°

(b) 60°

(c) 45°

(d) 65°

55. Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then, the ratio of the shorter side to the longer side is

(a) 1/2

(b) 2/3

(c) 1/4

(d) 3/4

## ANSWER KEY

1. (b)
2. (c)
3. (b)
4. (d)
5. (b)
6. (b)
7. (c)
8. (a)
9. (b)
10. (b)
11. (c)
12. (b)
13. (a)

14. (b)
15. (c)
16. (a)
17. (a)
18. (a)
19. (b)
20. (c)
21. (d)
22. (d)
23. (a)
24. (d)
25. (b)
26. (c)
27. (a)
28. (b)
29. (a)
30. (a)
31. (b)
32. (c)
33. (a)
34. (d)
35. (d)
36. (d)
37. (b)
38. (a)
39. (c)
40. (d)

41. (a)
42. (d)
43. (c)
44. (c)
45. (c)
46. (c)
47. (b)
48. (d)
49. (c)
50. (b)
51. (d)
52. (d)
53. (a)
54. (b)
55. (d)