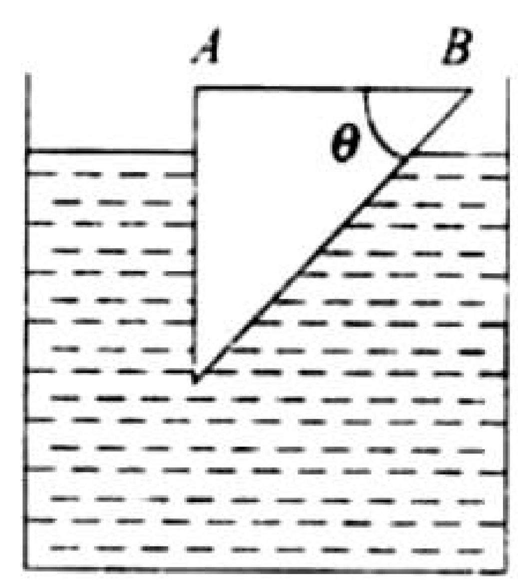
GEOMETRICAL OPTICS [JEE ADVANCED PREVIOUS YEAR SOLVED PAPERS]

JEE Advanced

Single Correct Answer Type

1. When a ray of light enters a glass slab from air.
   a. its wavelength decreases
   b. its wavelength increases
   c. its frequency increases
   d. neither its wavelength nor its frequency changes
   (IIT-JEE 1980)

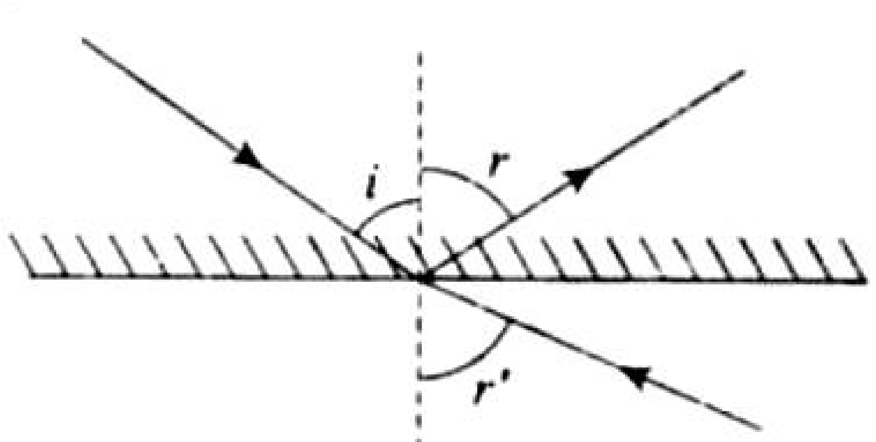
2. A glass prism of refractive index 1.5 is immersed in water (refractive index 4/3). A light beam normally on the face AB is totally reflected to reach on the face BC if



   a. $\sin\theta \geq \frac{8}{9}$
   b. $\frac{2}{3} < \sin\theta \geq < \frac{8}{9}$
   c. $\sin\theta \leq \frac{2}{3}$
   d. $\sin\theta \leq \frac{8}{9}$
   (IIT-JEE 1981)

3. A convex lens of focal length 40 cm is in contact with a concave lens of focal length 25 cm. The power of the combination is
   a. −1.5 dioptres b. −6.5 dioptres
   c. + 6.5 dioptres d. + 6.67 dioptres
   (IIT-JEE 1982)

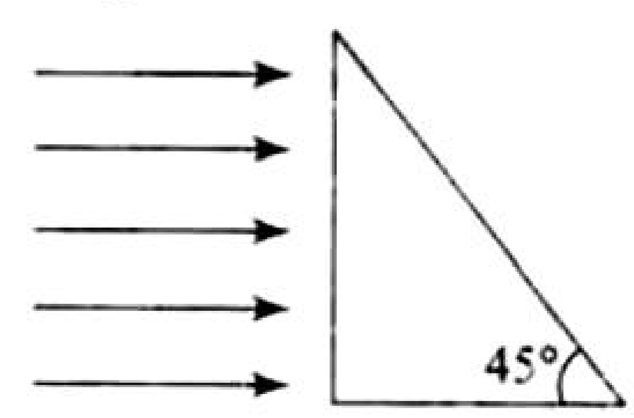
4. A ray of light from a denser medium strikes a rarer medium at an angle of incidence i (see figure). The reflected and refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r and r'. The critical angle is



   a. $\sin^{-1}(\tan r)$ b. $\sin^{-1}(\tan i)$
   c. $\sin^{-1}(\tan r')$ d. $\tan^{-1}(\sin i)$
   (IIT-JEE 1983)

5. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror. The size of the image is approximately equal to
   a. $b\left(\frac{u-f}{f}\right)^{1/2}$ b. $b\left(\frac{b}{u-f}\right)^{1/2}$
   c. $b\left(\frac{u-f}{f}\right)$ d. $b\left(\frac{f}{u-f}\right)^{2}$
   (IIT-JEE 1988)

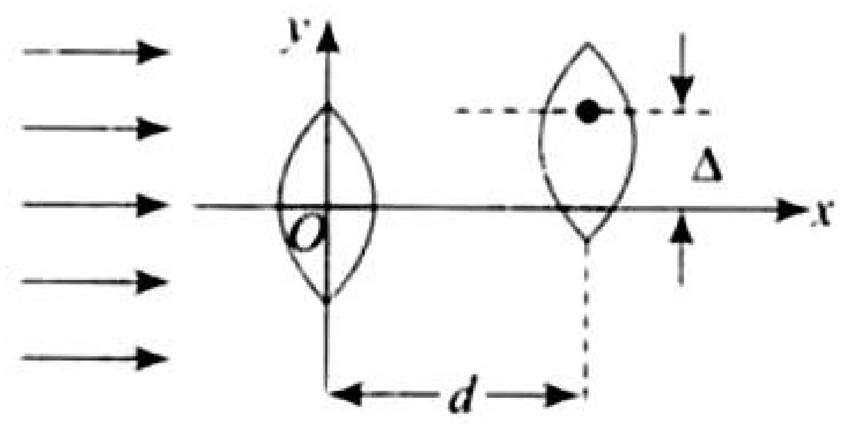
6. A beam of light consisting of red, green, and blue colors is incident on a right-angled prism. The refractive indices of the material of the prism for the above red, green, and blue wavelengths are 1.39, 1.44, and 1.47, respectively. The prism will



   a. separate part of the red color from the green and blue colors
   b. separate part of the blue color from the red and green colors
   c. separate all the three colors from one another
   d. not separate even partially any color from the other two colors **(IIT-JEE 1989)**

7. A thin prism P_1 with angle 4° and made from glass of refractive index 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of the prism P_2 is
   a. 5.33° b. 4°
   c. 3° d. 2.6° **(IIT-JEE 1990)**

8. Two thin convex lenses of focal length f_1 and f_2 are separated by a horizontal distance d (where $d < f_1, d < f_2$) and their centers are displaced by a vertical separation Δ as shown in figure.



Taking the origin of coordinates O at the center of the first lens, the x and y coordinates of the focal point of this lens system, for a parallel beam of rays coming from the left, are given by:

a. $x = \dfrac{f_1 f_2}{f_1 + f_2}, y = \Delta$

b. $x = \dfrac{f_1(f_2 + d)}{f_1 + f_2 - d}, y = \dfrac{\Delta}{f_1 + f_2}$

c. $x = \dfrac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}, y = \dfrac{\Delta(f_1 - d)}{f_1 + f_2 - d}$

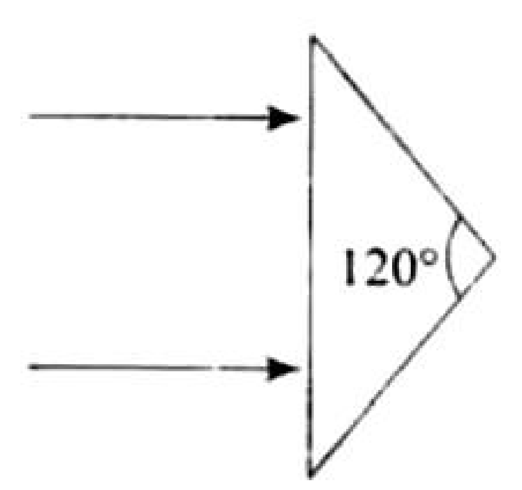
d. $x = \dfrac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}, y = \dfrac{\Delta(f_1 - d)}{f_1 + f_2 - d}$

(IIT-JEE 1993)

9. Spherical aberration in a thin lens can be reduced by

a. using a monochromatic light
b. using a doublet combination
c. using a circular annular mark over the lens
d. increasing the size of the lens **(IIT-JEE 1994)**

10. An isosceles prism of angle 120° has a refractive index 1.44. Two parallel monochromatic rays enter the prism parallel to each other in air as shown in the figure. The rays emerging from the opposite faces.



a. are parallel to each other
b. are diverging
c. make an angle of $2[\sin^{-1}(0.72)-30°]$ with each other
d. make an angle of $2\sin^{-1}(0.72)$ with each other

(IIT-JEE 1995)

11. A diminished image of an object is to be obtained on a screen 1.0 m from it. This can be achieved by appropriately placing

a. a concave mirror of suitable focal length
b. a convex mirror of suitable focal length
c. a convex lens of focal length less than 0.25 m
d. a concave lens of suitable focal length.

(IIT-JEE 1995)

12. The focal lengths of the objective and the eyepiece of a compound microscope are 2.0 cm and 3.0 cm, respectively. The distance between the objective and the eyepiece is 15.0 cm. The final image formed by the eyepiece is at infinity. The two lenses are thin. The distance, in cm, of the object and the image produced by the objective, measured from the objective lens, are respectively

a. 2.4 and 12.0 **b.** 2.4 and 15.0
c. 2.0 and 12.0 **d.** 2.0 and 3.0

(IIT-JEE 1995)

13. An eye specialist prescribes spectacles having combination of convex lens of focal length 40 cm in contact with a concave lens of focal length 25 cm. The power of this lens combination in diopters is

a. +1.5 **b.** −1.5 **c.** +6.67 **d.** −6.67

(IIT-JEE 1997)

14. A real image of a distant object is formed by a plano-convex lens on its principal axis. Spherical aberration

a. is absent
b. is smaller if the curved surface of the lens faces the object
c. is smaller if the plane surface of the lens faces the object
d. is the same whichever side of the lens faces the object **(IIT-JEE 1998)**

15. A concave mirror is placed on a horizontal table, with its axis directed vertically upward. Let O be the pole of the mirror and C its center of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be

a. real, and will remain at C
b. real, and located at a point between C and ∞
c. virtual, and located at a point between C and O
d. real, and located at a point between C and O

(IIT-JEE 1998)

16. A spherical surface of radius of curvature R separates air (refractive index 1.0) from glass (refractive index 1.5). The center of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O, and $PO = OQ$. The distance PO is equal to

a. $5R$ **b.** $3R$ **c.** $2R$ **d.** $1.5R$

(IIT-JEE 1998)

17. A concave lens of glass, refractive index 1.5, has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a

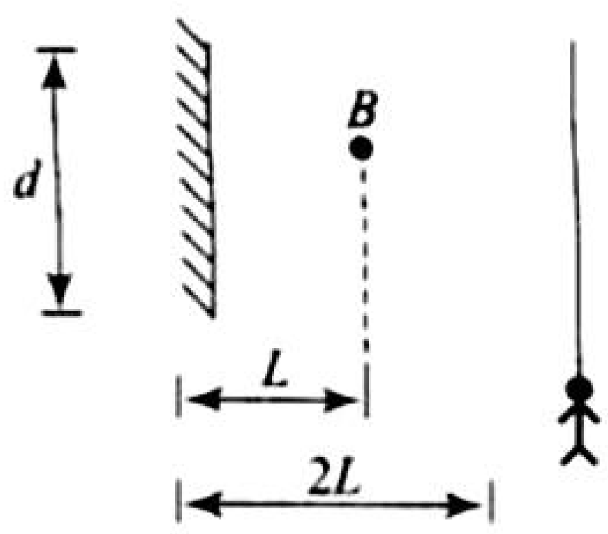
a. convergent lens of focal length $3.5R$
b. convergent lens of focal length $3.0R$
c. divergent lens of focal length $3.5R$
d. divergent lens of focal length $3.0R$

(IIT-JEE 1999)

18. A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 or L_2 having refractive indices μ_1 and μ_2 respectively ($\mu_2 > \mu_1 > 1$). The lens will diverge a parallel beam of light if it is filled with

a. air and placed in air
b. air and immersed in L_1
c. L_1 and immersed in L_2
d. L_2 and immersed in L_1. **(IIT-JEE 2000)**

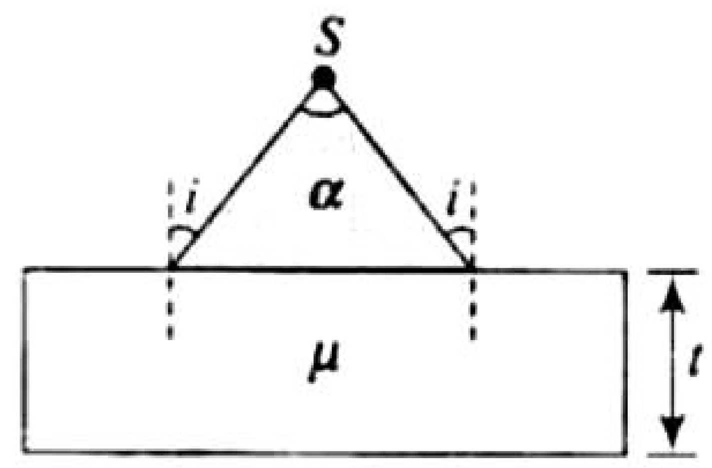
19. A point source of light B is placed at a distance L in front of the center of a mirror of width 'd' hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown in the figure. The greatest distance over which he can see the image of the light source in the mirror is



a. $d/2$ b. d c. $2d$ d. $3d$

(IIT-JEE 2000)

20. A diverging beam of light from a point source S having divergence angle α, falls symmetrically on a glass slab as shown. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and the refractive index n, then the divergence angle of the emergent beam is



a. zero

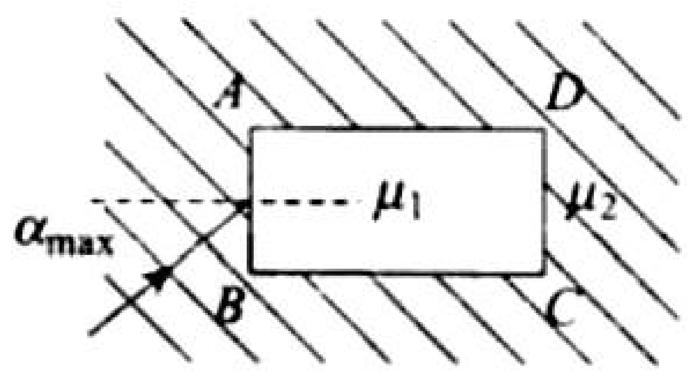
b. α

c. $\sin^{-1}\left(\frac{1}{n}\right)$

d. $2\sin^{-1}\left(\frac{1}{n}\right)$

(IIT-JEE 2000)

21. A rectangular glass slab $ABCD$ of refractive index n_1 is immersed in water of refractive index $n_2 (n_1 < n_2)$. A ray of light is incident at the surface AB of the slab as shown. The maximum value of the angle of incidence α_{max} such that the ray comes out from the other surface CD is given by



a. $\sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\left(\frac{n_2}{n_1}\right)\right)\right]$

b. $\sin^{-1}\left[n_1\cos\left(\sin^{-1}\left(\frac{1}{n_2}\right)\right)\right]$

c. $\sin^{-1}\left(\frac{n_1}{n_2}\right)$

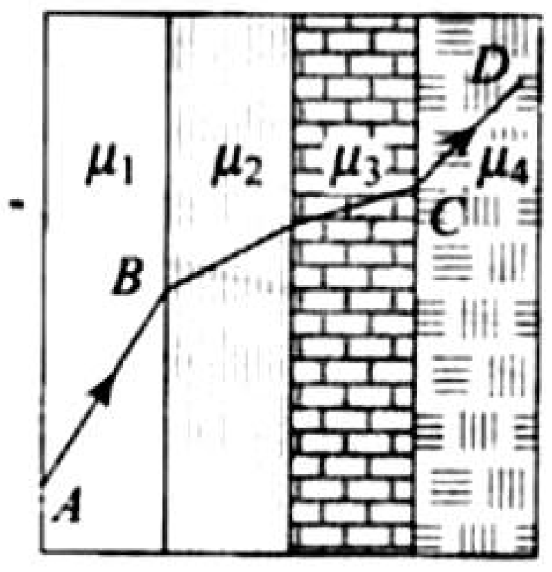
d. $\sin^{-1}\left(\frac{n_2}{n_1}\right)$ **(IIT-JEE 2000)**

22. In a compound microscope, the intermediate image is

a. virtual, erect, and magnified
b. real, erect, and magnified
c. real, inverted, and magnified
d. virtual, erect, and reduced

(IIT-JEE 2000)

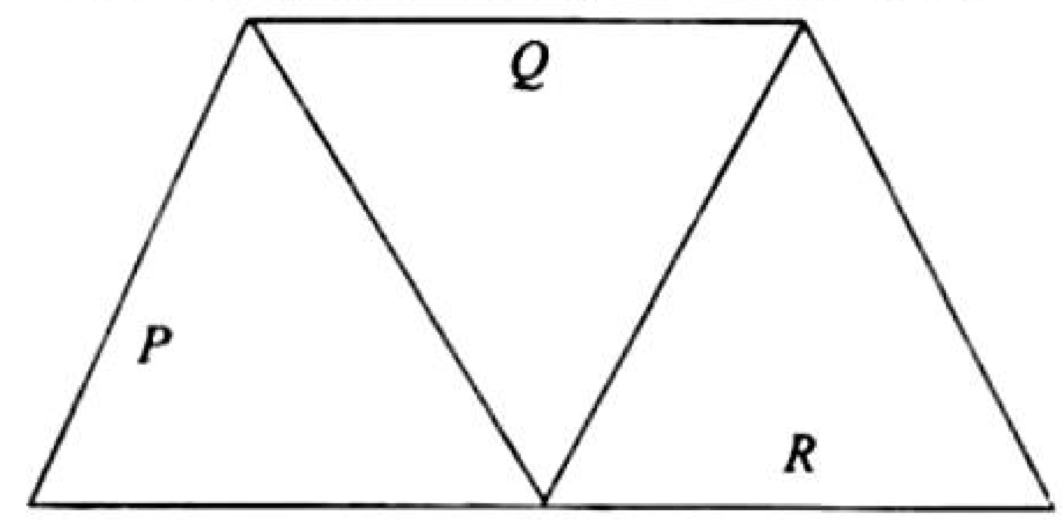
23. A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3, and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have



a. $\mu_1 = \mu_2$ b. $\mu_2 = \mu_3$

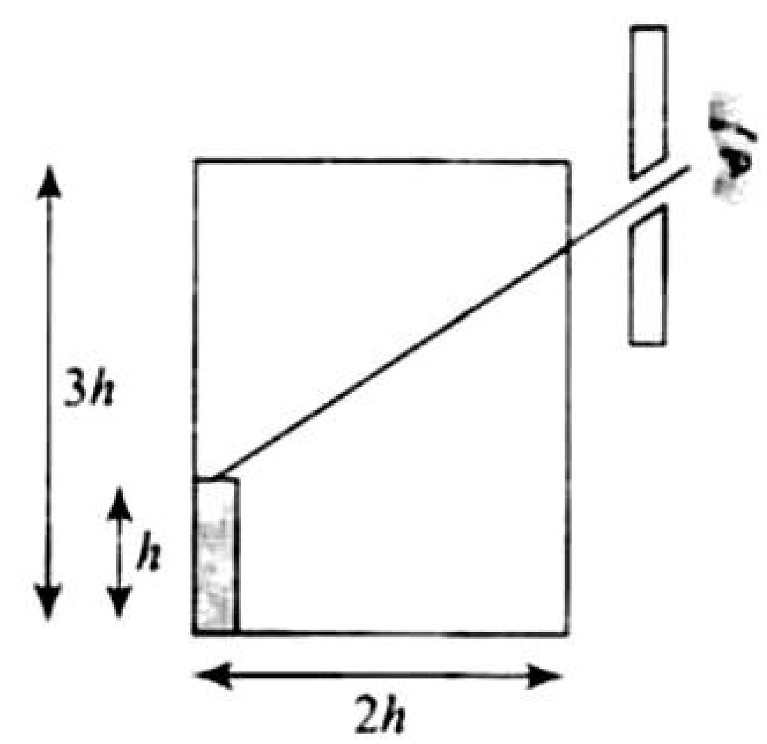
c. $\mu_3 = \mu_4$ d. $\mu_4 = \mu_1$ **(IIT-JEE 2001)**

24. A given ray of light suffers minimum deviation in an equilateral prism P. Additional prism Q and R of identical shape and of the same material as P are now added as shown in the figure. The ray will now suffer



a. greater deviation
b. no deviation
c. same deviation as before
d. total internal reflection **(IIT-JEE 2001)**

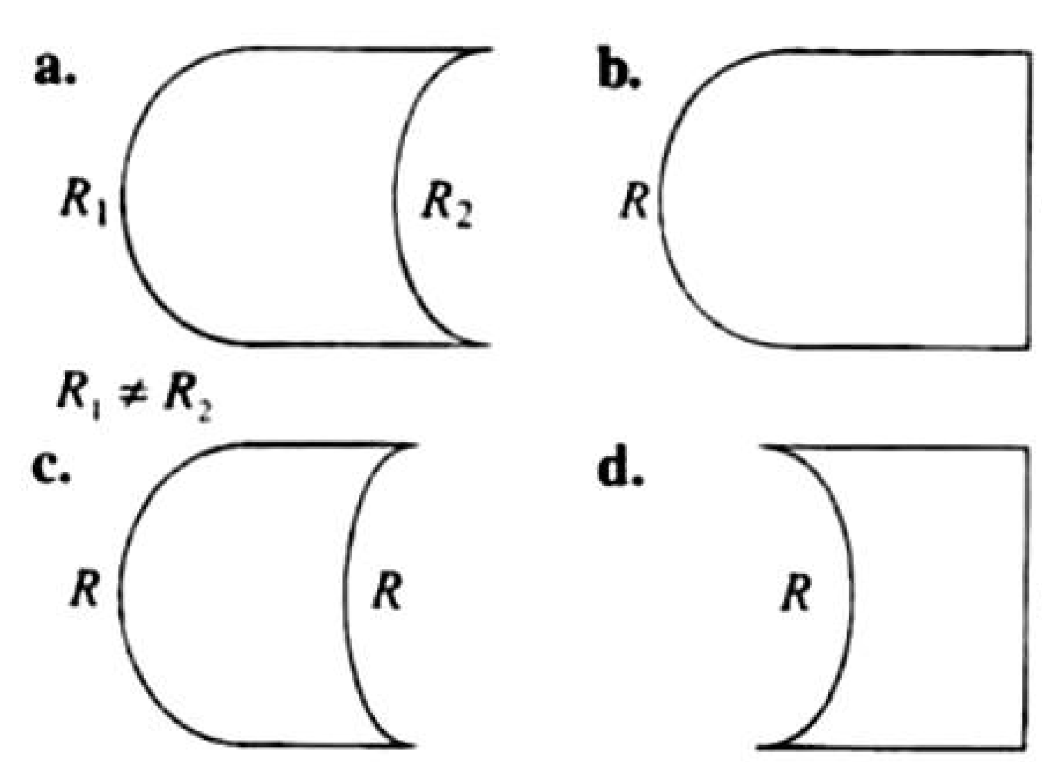
25. An observer can see through a pin-hole the top end of a thin of height h, placed as shown in the figure. The beaker height is $3h$ and its radius h. When the beaker is filled with a liquid up to a height $2h$, he can see the lower end of the rod. Then the refractive index of the liquid is



a. $\frac{5}{2}$ b. $\sqrt{\frac{5}{2}}$ c. $\sqrt{\frac{3}{2}}$ d. $\frac{3}{2}$

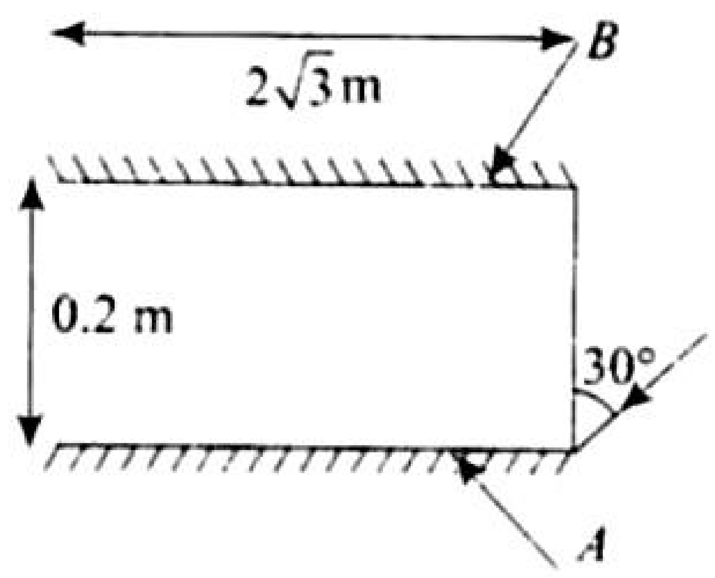
(IIT-JEE 2002)

26. Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams.



(IIT-JEE 2002)

27. Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle of 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflection (including the first one) before it emerges out is



a. 28 b. 30 c. 32 d. 34

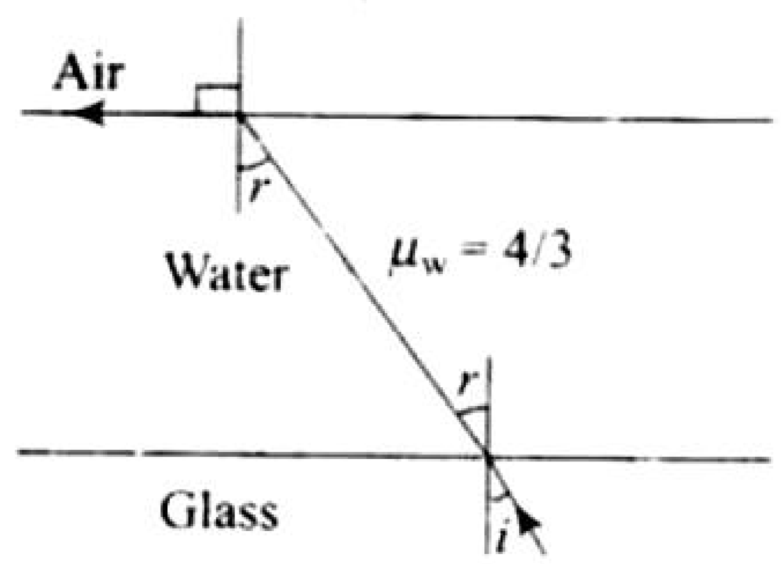
(IIT-JEE 2002)

28. The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image.

a. 1.25 cm b. 2.5 cm c. 1.05 cm d. 2 cm

(IIT-JEE 2003)

29. A ray of light is incident at the glass–water interface at an angle i. It merges finally parallel to the surface of water. Then, the value of μ_g would be



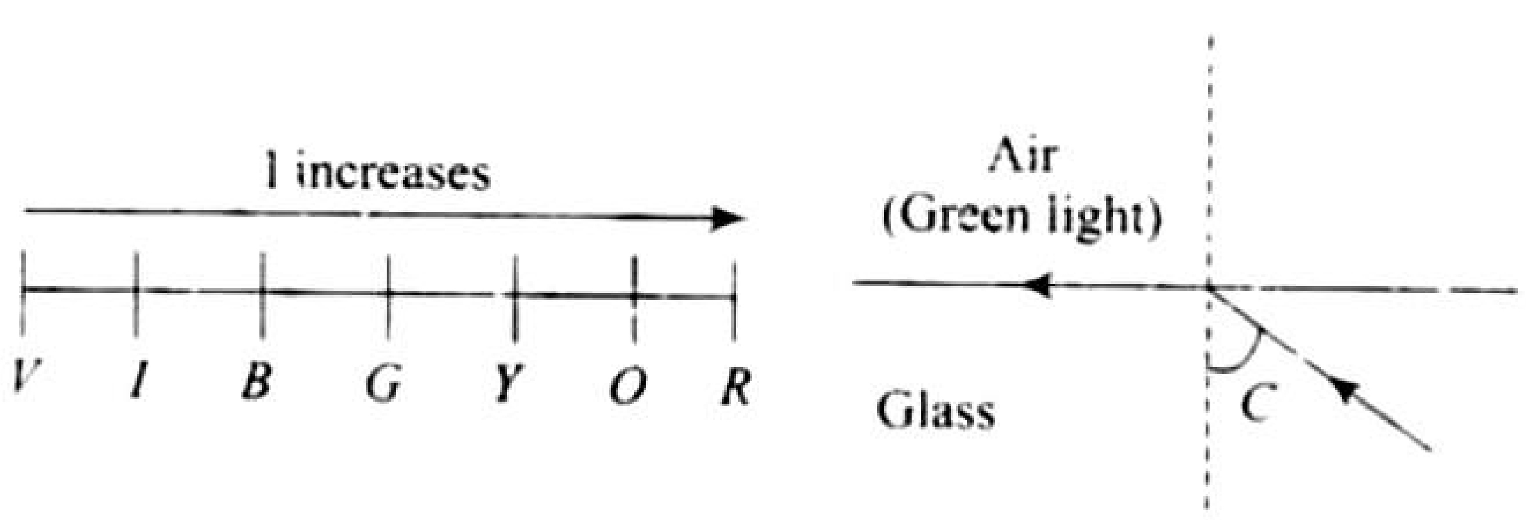
a. (4/3) sin i

b. 1/sin i

c. 4/3

d. 1 **(IIT-JEE 2003)**

30. A beam of white light is incident on glass–air interface from glass to air such that green light just suffers total internal reflection. The colors of the light which will come out to air are



a. Violet, Indigo, Blue

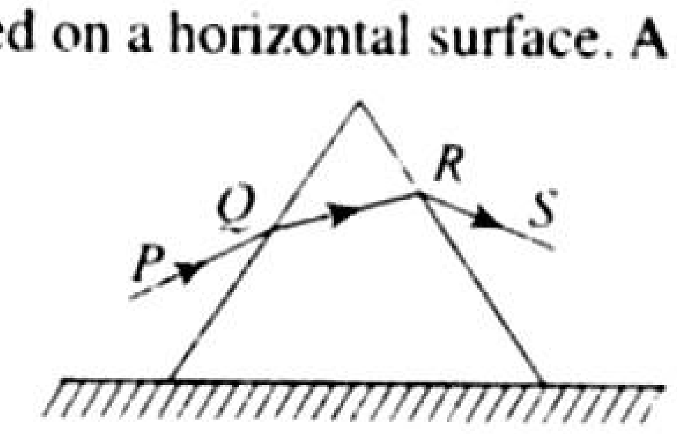
b. All colors except green

c. Yellow, Orange, Red

d. White light

(IIT-JEE 2004)

31. An equilateral prism is placed on a horizontal surface. A ray PQ is incident onto it. For minimum deviation,



a. PQ is horizontal

b. QR is horizontal

c. RS is horizontal

d. any one will be horizontal

32. A point object is placed at the center of a glass sphere of radius 6 cm and refractive index 1.5. The distance of virtual image from the surface is

a. 6 cm b. 4 cm

c. 12 cm d. 9 cm

(IIT-JEE 2004)

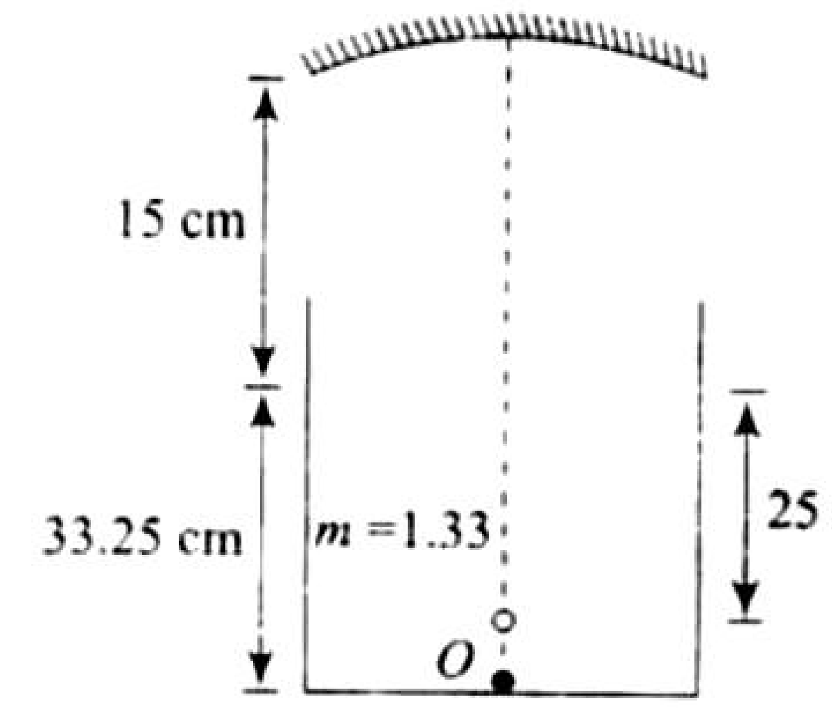
33. A convex lens is in contact with a concave lens. The magnitude of the ratio of their focal lengths is 2/3. Their equivalent focal length is 30 cm. What are their individual focal lengths?

a. −15, 10 b. −10, 15

c. 75, 50 d. −75, 50

(IIT-JEE 2005)

34. A container is filled with water (μ = 1.33) upto a height of 33.25 cm. A convex mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. Focal length of the mirror is

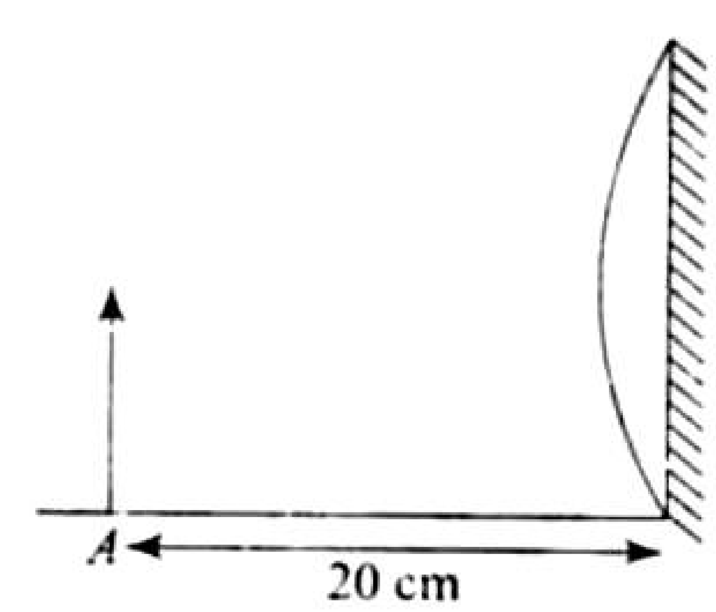


a. 15 cm b. 20 cm

c. −18.31 cm d. 10 cm

(IIT-JEE 2005)

35. Focal length of the plano-convex lens is 15 cm. A small object is placed at A as shown in the figure. The plane surface is silvered. The image will form at



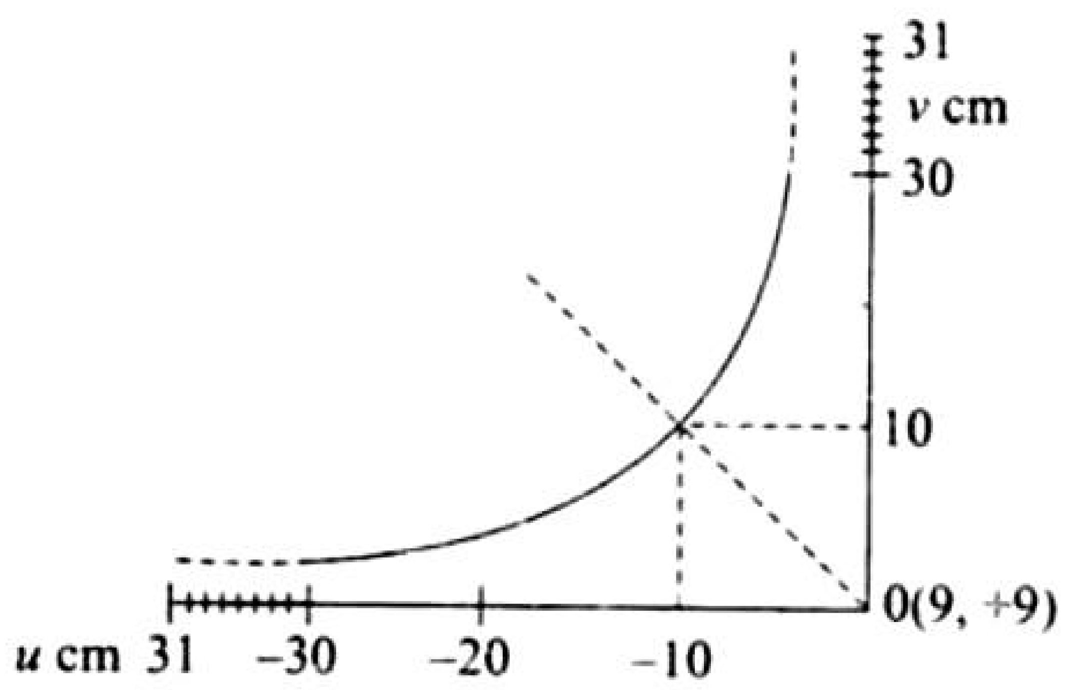
a. 60 cm to the left of lens

b. 12 cm to the left of lens

c. 60 cm to the right of lens

d. 30 cm to the left of lens **(IIT-JEE 2006)**

36. The graph shows relationship between object distance and image distance for an equiconvex lens. Then, focal length of the lens is



a. 0.50 ± 0.05 cm b. 0.50 ± 0.10 cm
c. 5.00 ± 0.05 cm d. 5.00 ± 0.10 cm

(IIT-JEE 2006)

37. A biconvex lens of focal length f forms a circular image of radius r of sun in focal plane. Then, which option is correct?

a. $\pi r^2 \propto f$
b. $\pi r^2 \propto f^2$
c. If lower half part is covered by black sheet, then area of the image is equal to $\pi r^2/2$
d. If f is doubled, intensity will increase

(IIT-JEE 2006)

38. A ray of light travelling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then, there will be

a. only a reflected ray and no refracted ray
b. only a refracted ray and no reflected ray
c. a reflected ray and a refracted ray and the angle between them would be less than $180° - 2\theta$
d. a reflected ray and a refracted ray and the angle between them would be greater than $180° - 2\theta$

(IIT-JEE 2007)

39. In an experiment to determine the focal length (f) of a concave mirror by the u–v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eyes towards left, the image appears to the right of the object pin. Then,

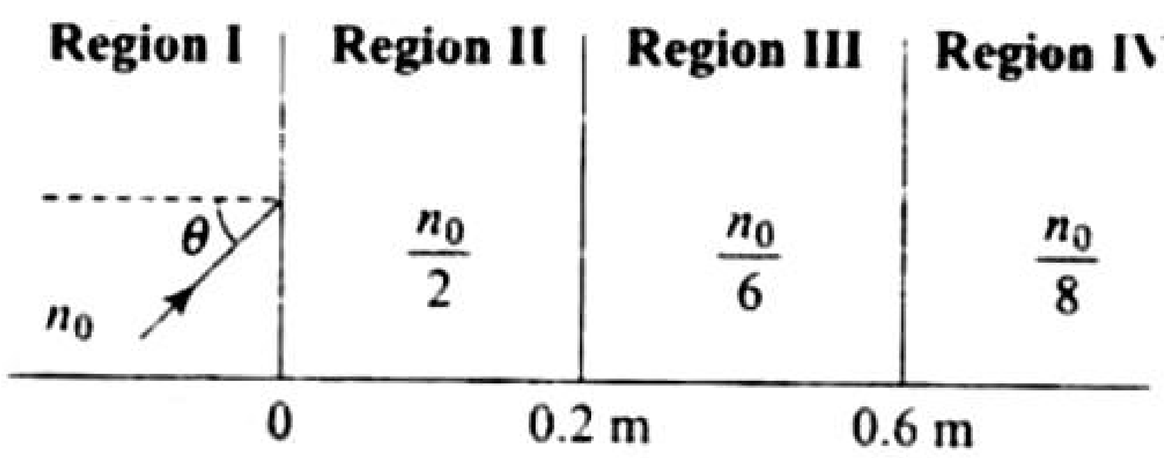
a. $x < f$ b. $f < x < 2f$
c. $x < f$ d. $f > x > 2f$

(IIT-JEE 2007)

40. Rays of light from Sun falls on a biconvex lens of focal length f and the circular image of Sun of radius r is formed on the focal plane of the lens. Then,

a. area of image is πr^2 and area is directly proportional of f
b. area of image is πr^2 and area is directly proportional to f^2
c. intensity of image increases if f is increased
d. If lower half of the lens is covered with black paper, area will become half **(IIT-JEE 2008)**

41. A light beam is traveling from Region I to Region IV (refer figure). The refractive indices in Regions I, II, III, and IV are n_0, $n_0/2$, $n_0/6$, and $n_0/8$, respectively. The angle of incidence θ for which the beam just misses entering Region IV is



a. $\sin^{-1}(3/4)$ b. $\sin^{-1}(1/8)$
c. $\sin^{-1}(1/4)$ d. $\sin^{-1}(1/3)$

(IIT-JEE 2008)

42. Two beams of red and violet colors are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be

a. 30° for both the colors
b. greater for the violet color
c. greater for the red color
d. equal but not 30° for both the colors

(IIT-JEE 2008)

43. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant when the ball is 12.8 m above the water surface, the fish sees the speed of ball as

a. 9 m s^{-1} b. 12 m s^{-1}
c. 16 m s^{-1} d. 21.33 m s^{-1}

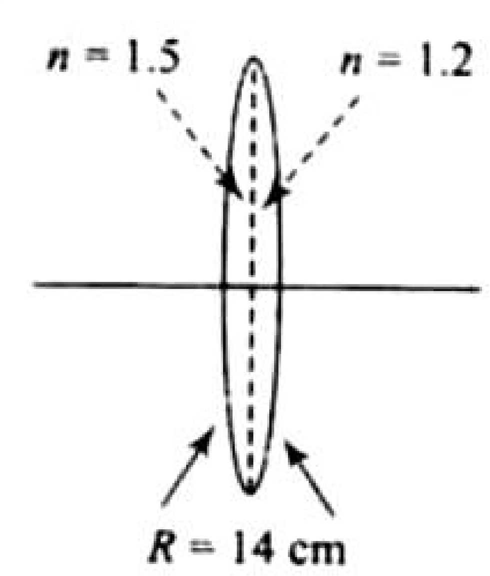
(IIT-JEE 2009)

44. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is

a. virtual and at a distance of 16 cm from the mirror
b. real and at a distance of 16 cm from the mirror
c. virtual and at a distance of 20 cm from the mirror
d. real and at a distance of 20 cm from the mirror

(IIT-JEE 2010)

45. A biconvex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2 Both the curved surfaces are of the same radius of curvature R = 14 cm. For this biconvex lens, for an object distance of 40 cm, the image distance will be



a. −280.0 cm b. 40.0 cm
c. 21.5 cm d. 13.3 cm

(IIT-JEE 2012)

46. A ray of light travelling in the direction $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$. The angle of incidence is

a. 30° **b.** 45° **c.** 60° **d.** 75°

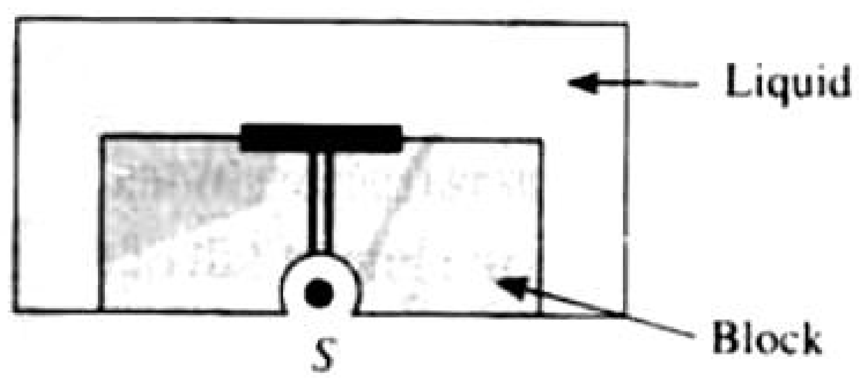
(JEE Advanced 2013)

47. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is

a. 1 m **b.** 2 m **c.** 3 m **d.** 4 m

(JEE Advanced 2013)

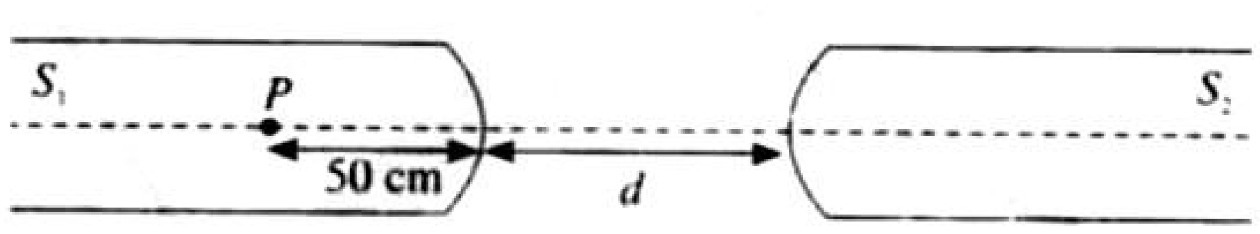
48. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is



a. 1.21 **b.** 1.30 **c.** 1.36 **d.** 1.42

(JEE Advanced 2014)

49. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is:



a. 60 cm **b.** 70 cm **c.** 80 cm **d.** 90 cm

(JEE Advanced 2015)

Multiple Correct Answer Type

1. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen,
   a. half the image will disappear
   b. complete image will be formed
   c. intensity of the image will increase
   d. intensity of the image will decrease

(IIT-JEE 1986)

2. In an astronomical telescope, the distance between the objective and the eyepiece is 36 cm and the final image is formed at infinity. The focal length f_0 of the objective and the focal length f_e of the eyepiece are
   a. f_0 = 45 cm and f_e = – 9 cm
   b. f_0 = 50 cm and f_e = 10 cm
   c. f_0 = 7.2 cm and f_e = 5 cm
   d. f_0 = 30 cm and f_e = 6 cm

(IIT-JEE 1989)

3. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 m and an eyepiece of focal length 2 cm. Then,
   a. the distance between the objective and the eyepiece is 16.02 m
   b. the angular magnification of the planet is –800
   c. the image of the planet is inverted
   d. the objective is larger than the eyepiece

(IIT-JEE 1992)

4. Which of the following form(s) a virtual and erect image for all positions of the object?
   a. Convex lens **b.** Concave lens
   c. Convex mirror **d.** Concave mirror

(IIT-JEE 1996)

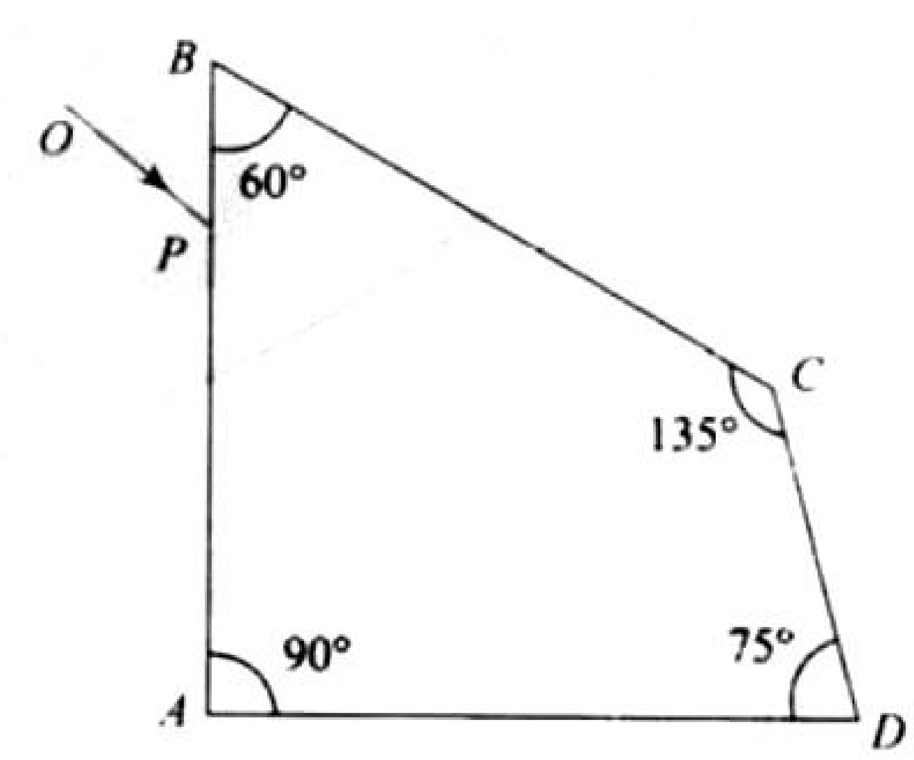
5. A ray of light traveling in a transparent medium falls on a surface separating the medium from air at an angle of incidence of 45°. The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value(s) of n from the following:
   a. 1.3 **b.** 1.4 **c.** 1.5 **d.** 1.6

(IIT-JEE 1998)

6. A student performed the experiment of determination of focal length of a concave mirror by u–v method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are: (42, 56), (48, 48), (60, 40), (66, 33), and (78, 39). The data set(s) that ***cannot*** come from experiment and is (are) incorrectly recorded, is (are)
   a. (42, 56) **b.** (48, 48) **c.** (66, 33) **d.** (78, 39)

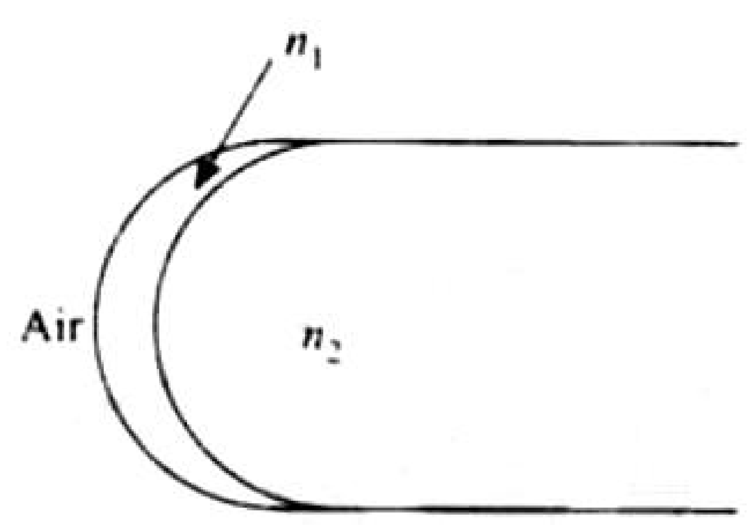
(IIT-JEE 2009)

7. A ray OP of monochromatic light is incident on the face AB of prism $ABCD$ near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct?



a. The ray gets totally internally reflected at face CD
b. The ray comes out through face AD
c. The angle between the incident ray and the emergent ray is 90°
d. The angle between the incident ray and the emergent ray is 120° **(IIT-JEE 2010)**

8. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then



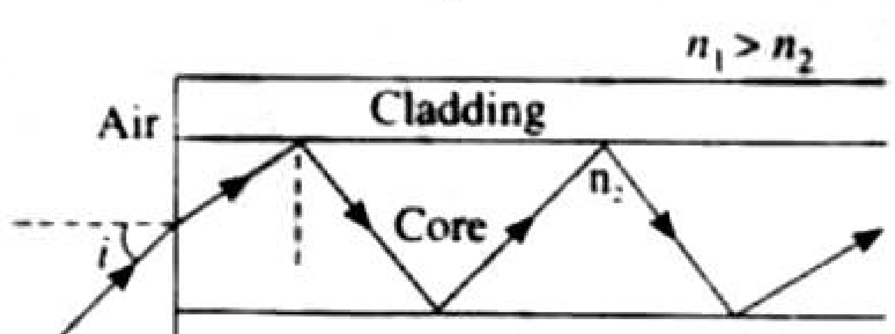
a. $|f_1| = 3R$ b. $|f_1| = 2.8R$
c. $|f_2| = 2R$ d. $|f_2| = 1.4R$

(JEE Advanced 2014)

Linked Comprehension Type

Paragraph Based

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as $\sin i_m$. **(JEE Advanced 2015)**



1. For two structures namely S_1 with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is (are)
   a. NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$
   b. NA of S_1 immersed in liquid of refractive index $\frac{16}{\sqrt{15}}$ is that as that of S_2 immersed in water
   c. NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$
   d. NA of S_1 placed in air is the same as that of S_2 placed in water

2. If two structures of same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is
   a. $\frac{NA_1 NA_2}{NA_1 + NA_2}$ b. $NA_1 + NA_2$
   c. NA_1 d. NA_2

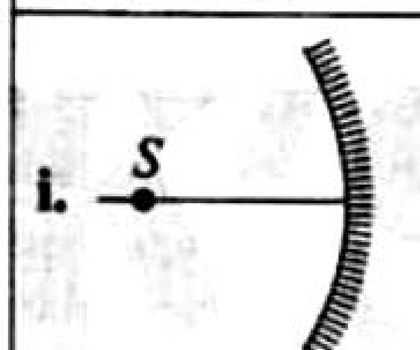
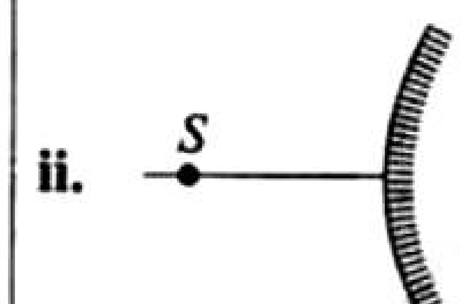
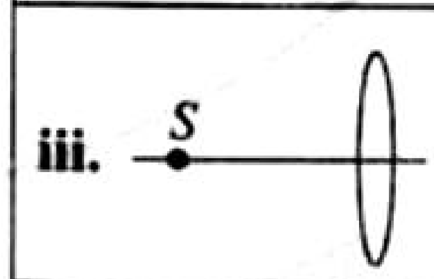
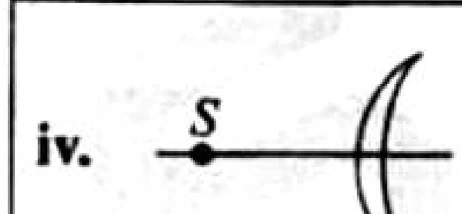
Matching Column Type

1. A simple telescope used to view distant objects has eyepiece and objective lens of focal lengths f_e and f_o respectively. Then,

Column I	Column II
i. Intensity of light received by lens	a. Radius of aperture
ii. Angular magnification	b. Dispersion of lens
iii. Length of telescope lens and eyepiece lens	c. Focal length of objective
iv. Sharpness of image	d. Spherical aberration

(IIT-JEE 2006)

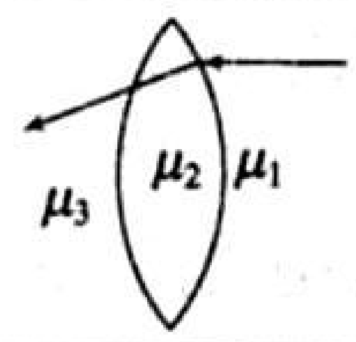
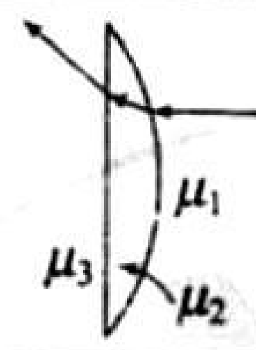
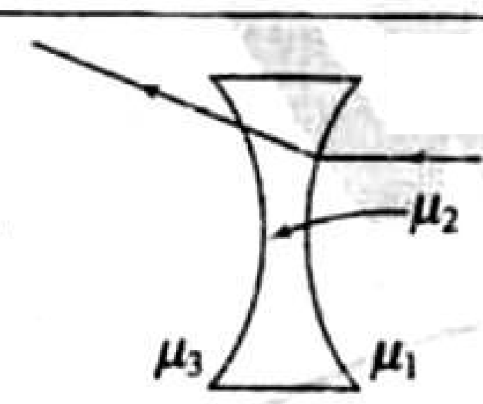
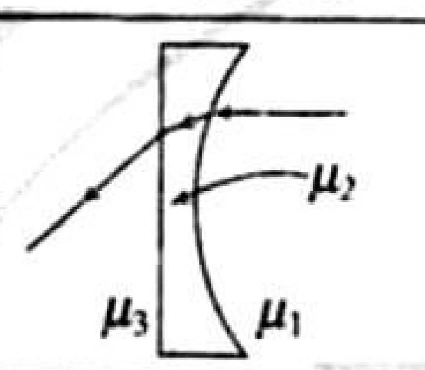
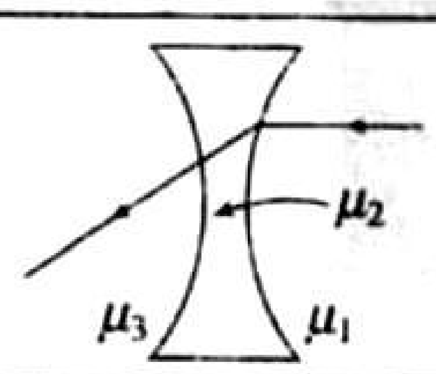
2. An optical component and an object S placed along its optic axis are given in Column I. The distance between the object and the component can be varied. The properties of images are given in Column II. Match all the properties of images from Column II with the appropriate components given in Column I.

Column I	Column II
i. 	a. Real image
ii. 	b. Virtual image
iii. 	c. Magnified image
iv. 	d. Image at infinity

(IIT-JEE 2008)

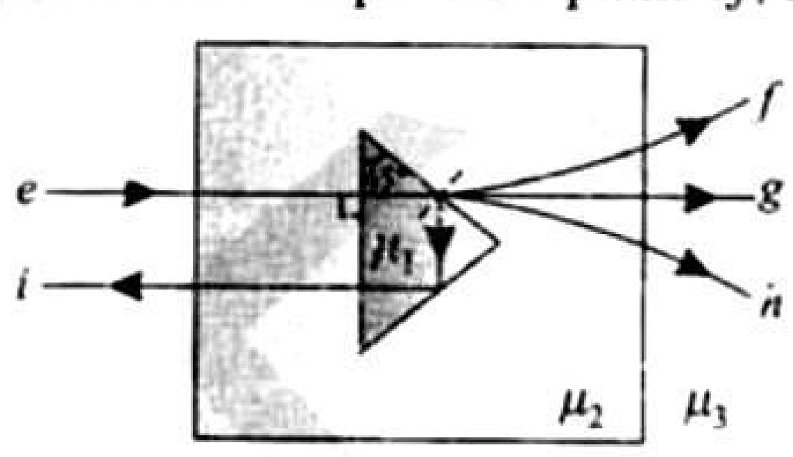
3. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in Column II. A ray traversing these media is also shown in the figures. In

Column I different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagram shown in Column II.

Column I	Column II
i. $\mu_1 < \mu_2$	a. 
ii. $\mu_1 > \mu_2$	b. 
iii. $\mu_2 = \mu_3$	c. 
iv. $\mu_2 > \mu_3$	d. 
	e. 

(IIT-JEE 2010)

4. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light e enters the rectangular block at normal incidence.
Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths ef, eg, eh, or ei.



Match the paths in List I with conditions of refractive indices in List II and select the correct answer using the codes given below the lists:

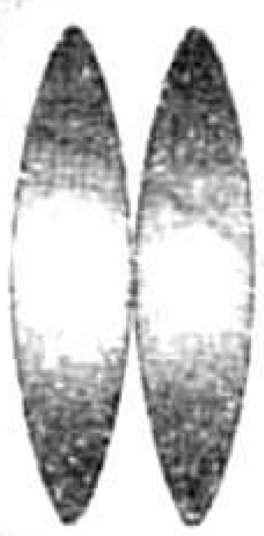
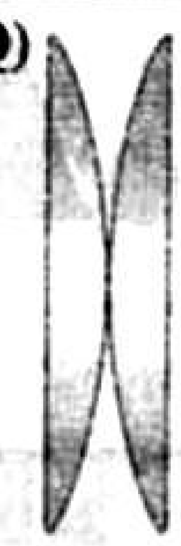
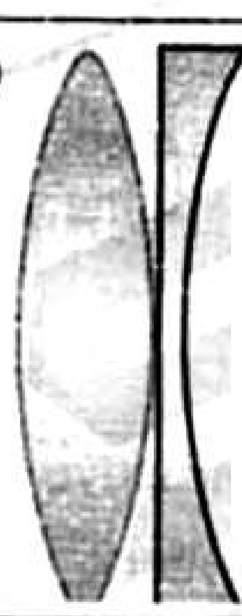
	Column I		Column II
p.	$e \to f$	1.	$\mu_1 > \sqrt{2}\mu_2$
q.	$e \to g$	2.	$\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
r.	$e \to h$	3.	$\mu_1 = \mu_2$
s.	$e \to i$	4.	$\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$

Codes:

	p	q	r	s
a.	2	3	1	4
b.	1	2	4	3
c.	4	1	2	3
d.	2	3	4	1

(JEE Advanced 2013)

5. Four combinations of two thin lenses are given in List I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in List I with their focal length in List II and select the correct answer using the code given below the lists.

List – I	List – I
(P) 	1. $2r$
(Q) 	2. $r/2$
(R) 	3. $-r$
(S) 	4. r

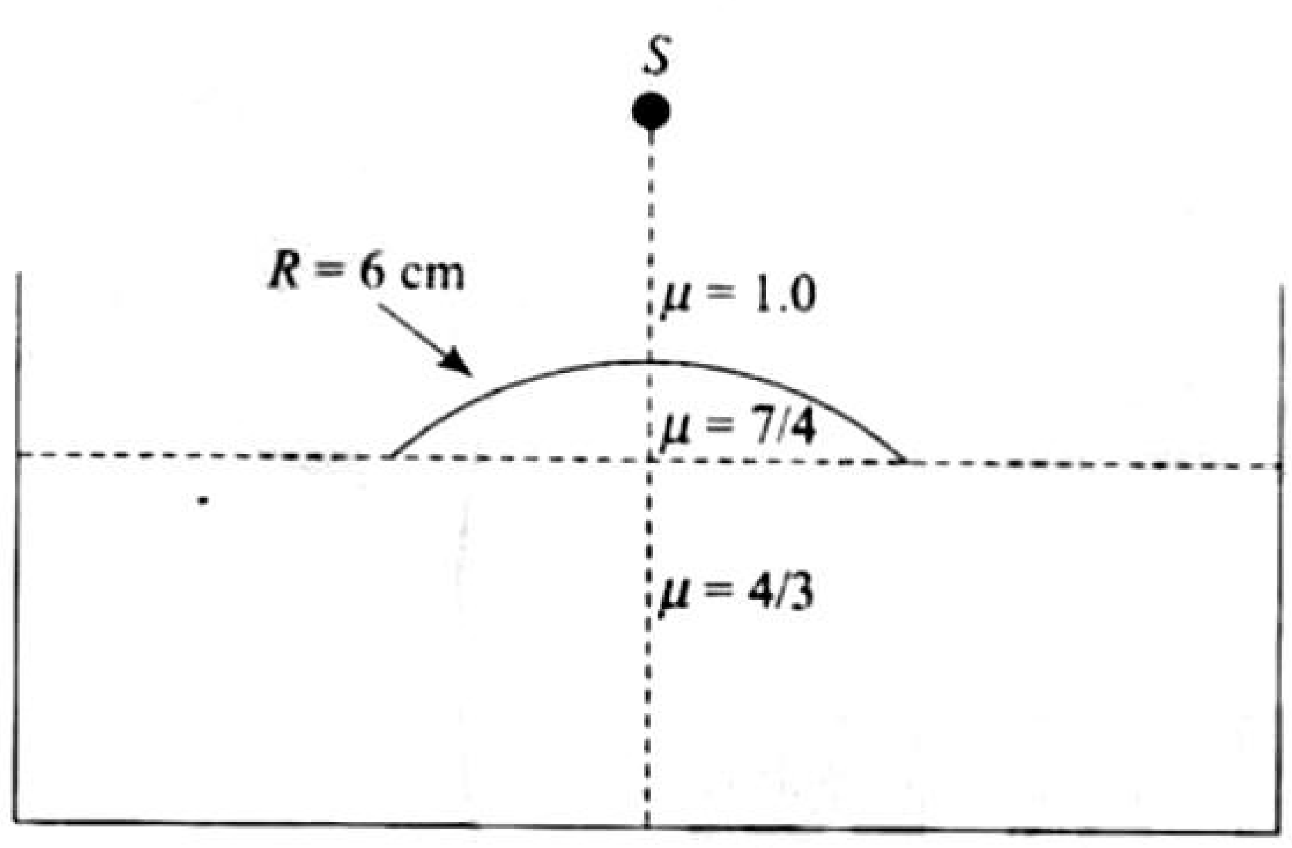
Code:

a. P-1, Q-2, R-3, S-4
b. P-2, Q-4, R-3, S-1
c. P-4, Q-1, R-2, S-3
d. P-2, Q-1, R-3, S-4

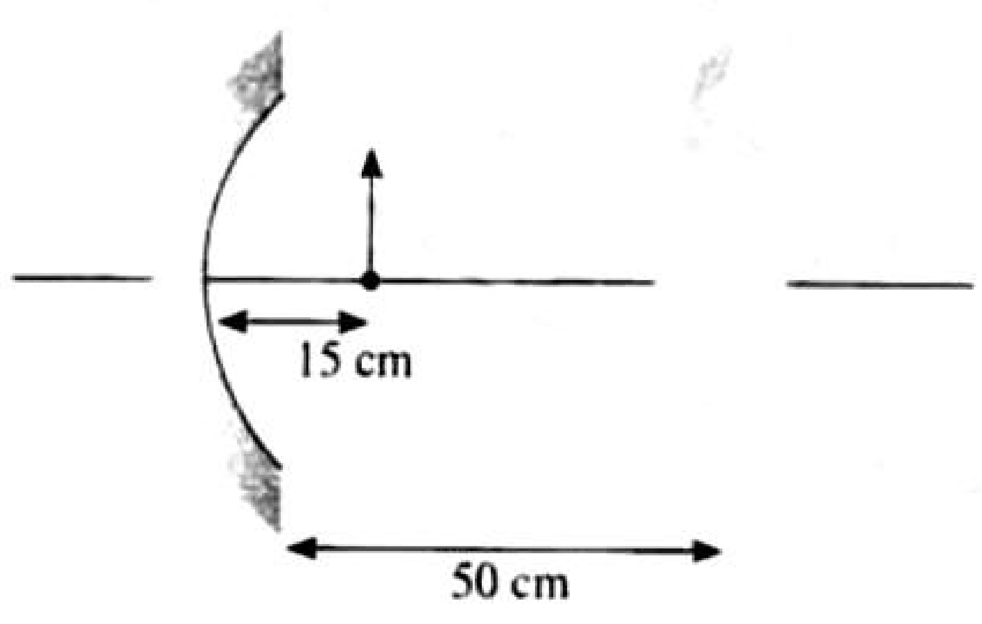
(JEE Advanced 2014)

Integer Answer Type

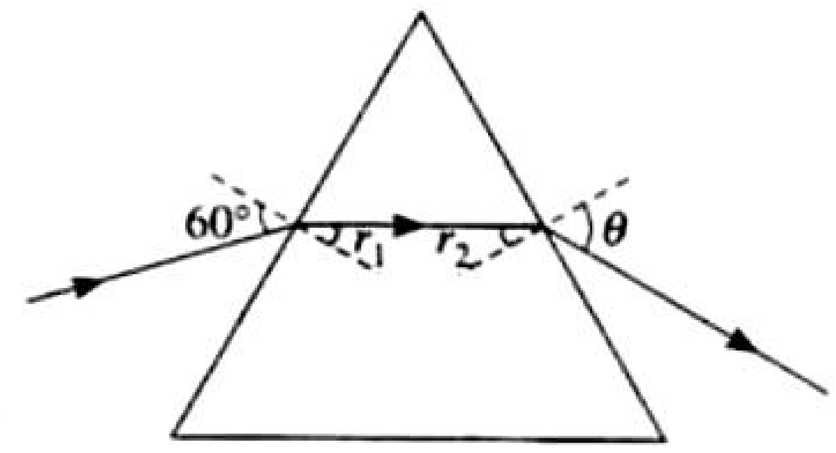
1. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio m_{25}/m_{50} is **(IIT-JEE 2010)**
2. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R? **(IIT-JEE 2010)**
3. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from 25/3 m to 50/7 m in 30 s. What is the speed of the object in km per hour? **(IIT-JEE 2010)**
4. Water (with refractive index = 4/3) in a tank is 18 cm deep. Oil of refractive index 7/4 lies on water making a convex surface of radius of curvature $R = 6$ cm as shown. Consider oil to act as a thin lens. An object S is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then x is **(IIT-JEE 2011)**



5. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index 7/6, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is ________. **(JEE Advanced 2015)**



6. A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see the figure). For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is: **(JEE Advanced 2015)**



Assertion-Reasoning Type

In each of the questions, assertion (A) is given by corresponding statement of reason (R) of the statements. Mark the correct answer.

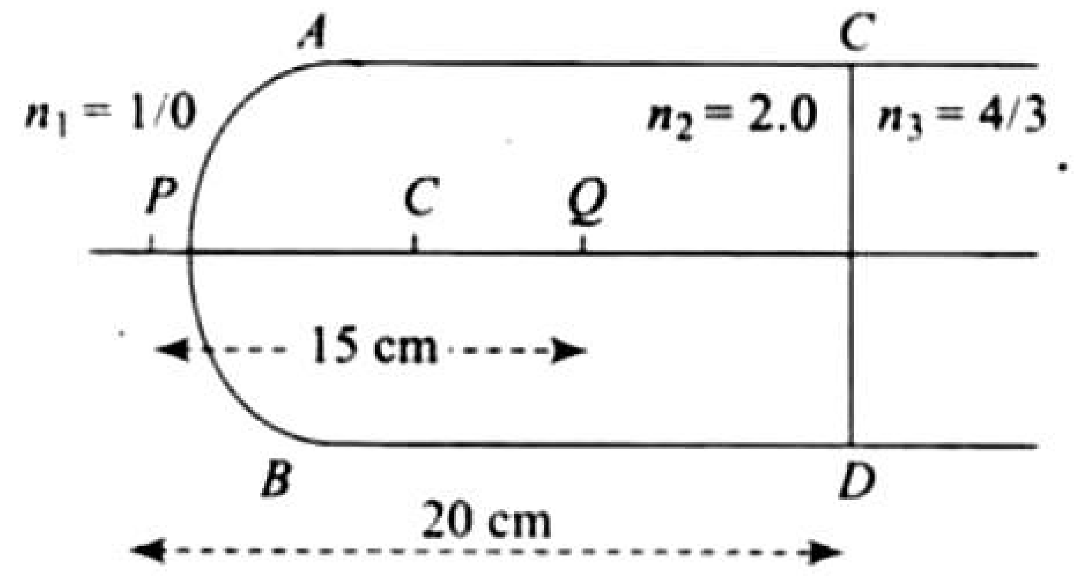
a. If both Statement I and Statement II are true and Statement II is the correct explanation of Statement I.
b. If both Statement I and Statement II are true but Statement II is not the correct explanation of Statement I.
c. If Statement I is true but Statement II is false.
d. If Statement I is false but Statement II is true.

(IIT-JEE 2007)

1. **Statement I:** The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature.
   Statement II: Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.
2. **Statement I:** If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not energy.
   Statement II: When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.

Fill in the Blanks Type

1. A convex lens A of focal length 20 cm and a concave lens B of focal length 5 cm are kept along the same axis with a distance d between them. If a parallel beam of light falling on A leaves B as a parallel beam, then d is equal to ________ cm. **(IIT-JEE 1985)**
2. A thin lens of refractive index 1.5 has a focal length of 15 cm in air. When the lens is placed in a medium of refractive index 4/3, its focal length will become ________ cm. **(IIT-JEE 1987)**
3. A slab of a material of refractive index 2 shown in the figure, has a curved surface APB of radius of curvature 10 cm and a plane surface CD. On the left of APB is air and on the right of CD is water with refractive indices as given in the figure. An object O is placed at a distance of 15 cm from the pole P as shown. The distance of the final image of O from P, as viewed from the left is ________. **(IIT-JEE 1991)**



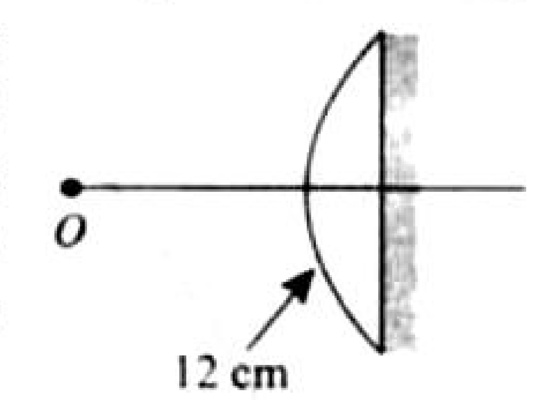
4. A thin rod of length $f/3$ is placed along the optic axis of a concave mirror of focal length f such that its image which is real and elongated, just touches the rod. The magnification is ________. **(IIT-JEE 1991)**
5. A ray of light undergoes deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$. The angle made by the ray inside the prism with the base of the prism is ________. **(IIT-JEE 1992)**
6. The resolving power of an electron microscope is higher than that of an optical microscope because the wavelength of electrons is ________ than the wavelength of visible light. **(IIT-JEE 1992)**
7. If ε_0 and μ_0 are, respectively, the electric permittivity and magnetic permeability of free space, ε and μ the corresponding quantities in a medium, the index of refraction of the medium in terms of the above parameters is ________ **(IIT-JEE 1992)**
8. A slit of width d is placed in front of a lens of focal length 0.5 m and is illuminated normally with light of wavelength 5.89×10^{-7}m. The first diffraction minima on either side of the central diffraction maximum are separated by 2×10^{-3} m. The width d of the slit is ________ m. **(IIT-JEE 1997)**
9. A light of wavelength 6000 Å, in air, enters a medium with refractive index 1.5. Inside the medium, its frequency is ________ Hz and its wavelength is ________ Å. **(IIT-JEE 1997)**
10. Two thin lenses, when in contact, produce a combination of power +10 diopters. When they are 0.25 m apart, the power reduces to +6 diopters. The focal lengths of the lenses are ________ m and ________ m. **(IIT-JEE 1997)**
11. A ray of light is incident normally on one of the faces of a prism of apex angle 30° and refractive index $\sqrt{2}$. The angle of deviation of the ray is ________ degrees. **(IIT-JEE 1997)**

True/False Type

1. The setting sun appears higher in the sky than it really is. **(IIT-JEE 1980)**
2. The intensity of light at a distance 'r' from the axis of a long cylindrical source is inversely proportional to 'r'. **(IIT-JEE 1981)**
3. A convex lens of focal length 1 meter and a concave lens of focal length 0.25 meter are kept 0.75 meter apart. A parallel beam of light first passes through the convex lens, then through the concave lens and comes to a focus 0.5 m away from the concave lens. **(IIT-JEE 1983)**
4. A beam of white light passing through a hollow prism gives no spectrum. **(IIT-JEE 1983)**
5. A parallel beam of white light falls on a combination of a concave and a convex lens, both of the same material. Their focal lengths are 15 cm and 30 cm, respectively for the mean wavelength in white light. On the other side of the lens system, one sees colored patterns with violet color at the outer edge. **(IIT-JEE 1988)**

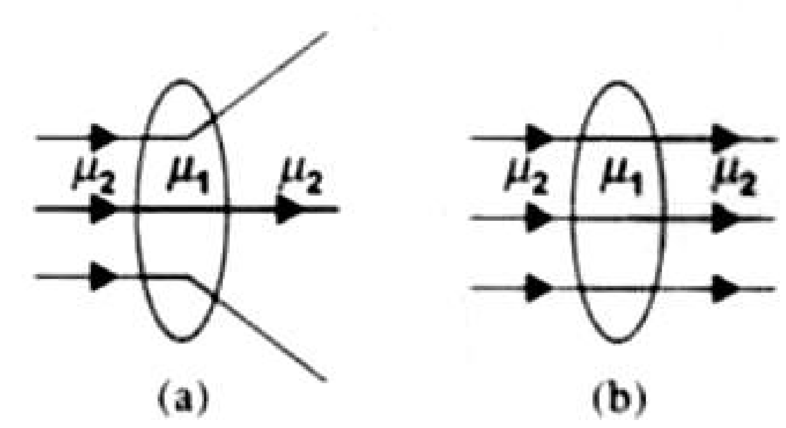
Subjective Type

1. A plano convex lens is silvered on its plane side. The radius of curvature of the other face is 12 cm and the refractive index of the material of the lens is 1.5. An object is placed 24 cm in front of the silvered lens. Where will the image be formed?



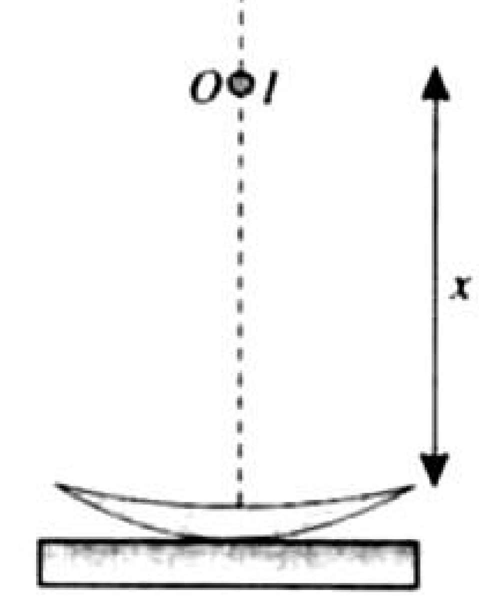
(IIT-JEE 1979)

2. What is the relation between the refractive indices μ, μ_1 and μ_2 if the behaviour of light rays is as shown in the figure?



(IIT-JEE 1979)

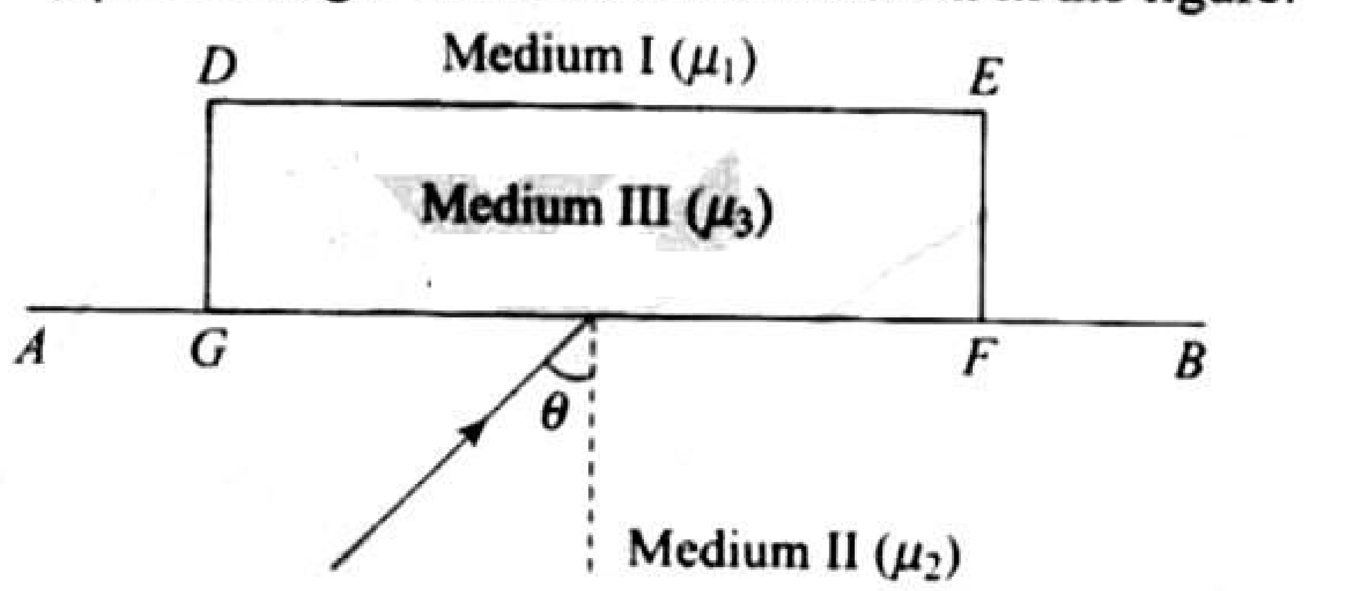
3. A rectangular block of glass is placed on a printed page lying on a horizontal surface. Find the minimum value of the refractive index of glass for which the letters on the page are not visible from any of the vertical faces of the block. **(IIT-JEE 1979)**
4. An object is placed in front of a slab ($\mu = 1.5$) of thickness 6 cm at a distance 28 cm from it. Other face of the slab is silvered. Find the position of final image. **(IIT-JEE 1980)**
5. A telescope has an objective of focal length 50 cm and eyepiece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focused for distinct vision on a scale 200 cm away from the objective. Calculate:
   a. the separation between the objective and eyepiece.
   b. the magnification produced. **(IIT-JEE 1980)**
6. The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface.



Where should be a pin placed on the optic axis such that its image is formed at the same place?
If the concave part is filled with water of refractive index 4/3, find the distance through which the pin should be moved so that the image of the pin again coincides with the pin? **(IIT-JEE 1981)**

7. A plano-convex lens has thickness 4 cm. When placed on a horizontal table with the curved surface in contact with it, the apparent depth of the bottom-most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the center of the plane face of the lens is found to be 25/8 cm. Find the focal length of the lens. **(IIT-JEE 1984)**

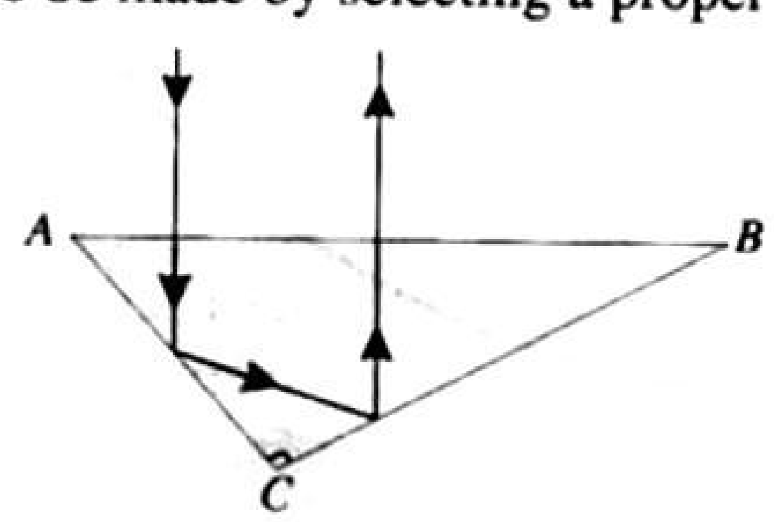
8. A monochromatic light is incident on the plane interface AB between two media of refractive indices μ_1 and μ_2 ($\mu_2 > \mu_1$) at an angle of incidence θ as shown in the figure.



The angle θ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now, if a transparent slab $DEFG$ of uniform thickness and of refractive index μ_3 is introduced on the interface (as shown in the figure), show that for any value of μ_3 all light will ultimately be reflected back into medium II.

9. A ray of light is incident at an angle of 60° on one face of a prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. If the refractive index of the material of the prism is $\mu = \sqrt{a}$, find the value of a. **(IIT-JEE 1987)**

10. A right angled prism is to be made by selecting a proper material and the angles A and B ($B \le A$), as shown in figure. It is desired that a ray of incident on the face AB emerges parallel to the incident direction after two internal reflections.



a. What should be the minimum refractive index n for this to be possible?

b. For $n = \frac{5}{3}$ is it possible to achieve this with the angle B equal to 30 degrees? **(IIT-JEE 1987)**

11. A parallel beam of light travelling in water (refractive index = 4/3) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial.

a. Find the position of image due to refraction at first surface and position of the final image.

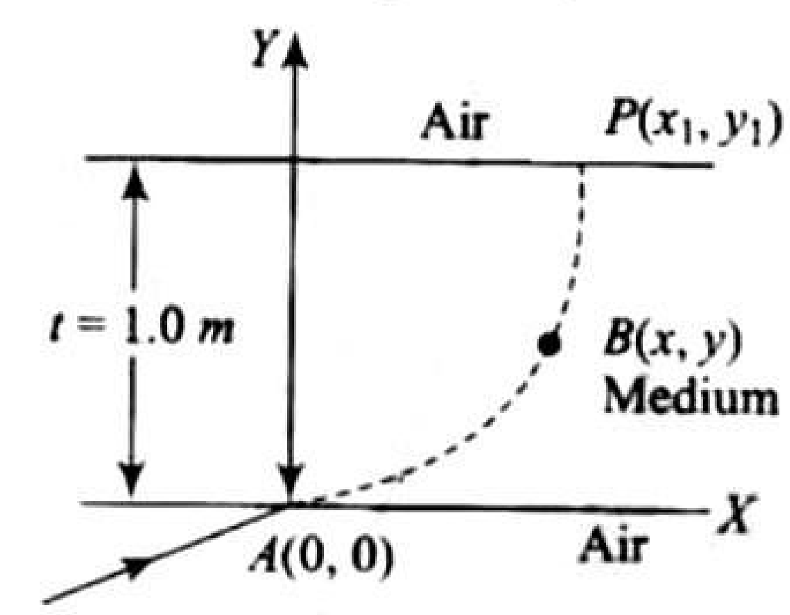
b. Draw a ray diagram showing the position of both images. **(IIT-JEE 1988)**

12. A thin rod of length $f/3$ is placed along the optical axis of a concave mirror of focal length f such that its image which is real and elongated just touches the rod. Calculate the magnification. **(IIT-JEE 1991)**

13. Light is incident at an angle α on one planar end of a transparent cylindrical rod of refractive index n. Determine the least value of n, so that the light entering the rod does not emerge from the curve surface of the rod irrespective of the value of α. **(IIT-JEE 1992)**

14. An image Y is formed of a point object x by a lens whose optic axis is AB as shown in the figure. Draw a ray diagram to locate the lens and its focus. If the image Y of object X is formed by a concave mirror (having the same optic axis AB) instead of lens, draw another ray diagram to locate the mirror and its focus. Write down the steps of construction of the ray diagrams. **(IIT-JEE 1994)**

15. A ray of light travelling in air is incident at grazing angle (incident angle = 90°) on a long rectangular slab of a transparent medium of thickness $t = 1.0$ m. The point of incidence is the origin $A(0, 0)$. The medium has a variable index of refraction $n(y)$ given by



$$n(y) = [ky^{3/2} + 1]^{1/2}$$

where $k = 1.0\ (\text{m})^{-3/2}$

The refractive index of air is 1.0.

a. Obtain a relation between the slope of the trajectory of the ray at point $B(x, y)$ in the medium and the incident angle at the point.

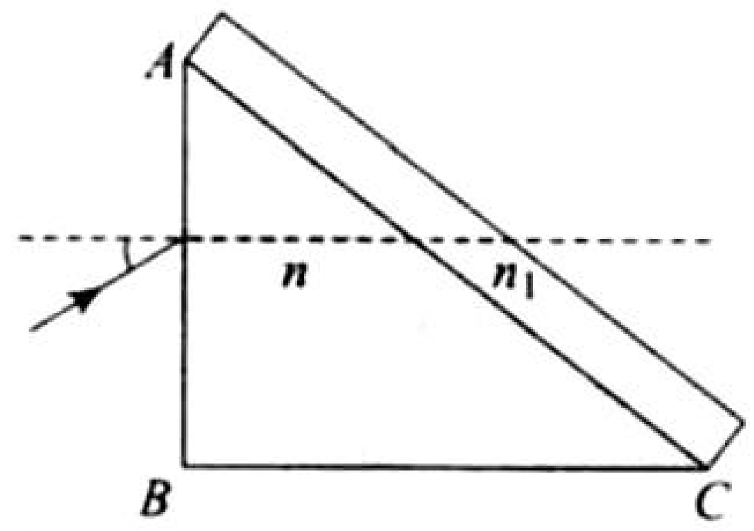
b. Obtain an equation for the trajectory $y(x)$ of the ray in the medium.

c. Determine the coordinates (x_1, y_1) of point P, where the ray intersects the upper surface of the slab–air boundary.

d. Indicate the path of the ray subsequently. **(IIT-JEE 1995)**

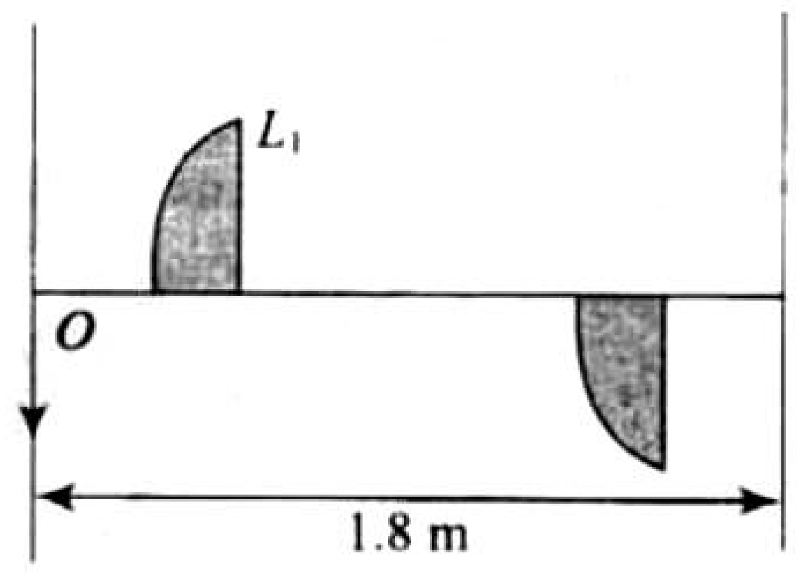
16. A right angled prism (45°, 90°, 45°) of refractive index n has a plate of refractive index ($n_1 < n$) cemented to its diagonal face. The assembly is in air. A ray is incident on AB.

a. Calculate the angle of incidence at AB for which the ray strikes the diagonal face at the critical angle.



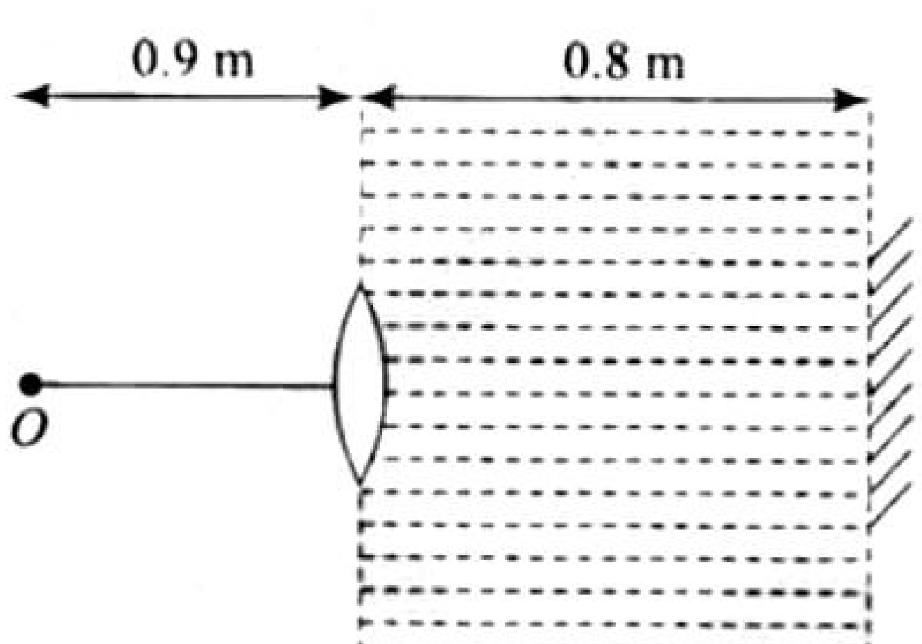
b. Assuming $n = 1.351$, calculate the angle of incidence at AB for which the refracted ray passes through the diagonal face undeviated. **(IIT-JEE 1996)**

17. A thin plano-convex lens of focal length f is split into two halves, one of the halves is shifted along the optical axis. The separation between object and image plane is 1.8 m. The magnification of the image formed by one of the half lenses is 2. Find the focal length of the lens and separation between the halves. Draw the ray diagram for image formation.



(IIT-JEE 1996)

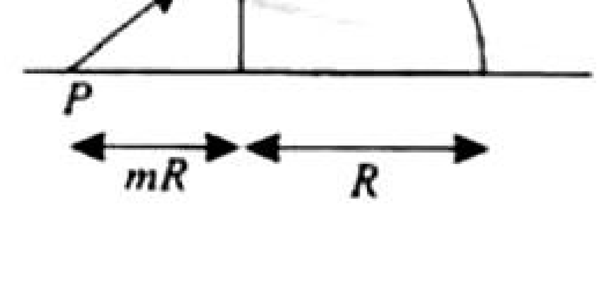
18. A thin equiconvex lens made of glass of refractive index 3/2 and of focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water $\left(\mu = \frac{3}{2}\right)$. On the opposite side of the lens, a mirror is placed inside the tank on the tank wall perpendicular to the lens axis as shown in the figure. The separation between the lens and mirror is 0.8 m. A small object is placed outside the tank in front of the lens at a distance of 0.9 m from the lens along its axis. Find the position (relative to lens) of the image of the object formed by the system.



(IIT-JEE 1997)

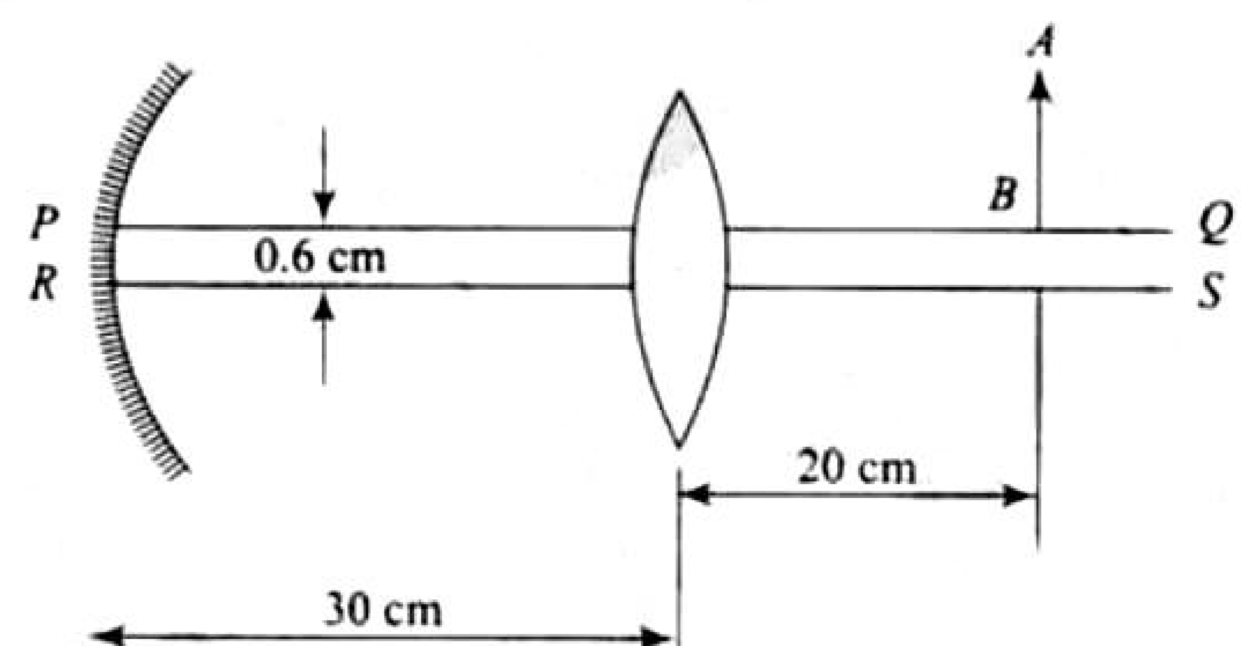
19. A quarter cylinder of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance mR from it.

Find the value of m for which a ray from P will emerge parallel to the table as shown in the figure. **(IIT-JEE 1999)**



20. The XY plane is the boundary between two transparent media. Medium 1 with $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z \leq 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. Find the unit vector in the direction of the refracted ray in medium 2. **(IIT-JEE 1999)**

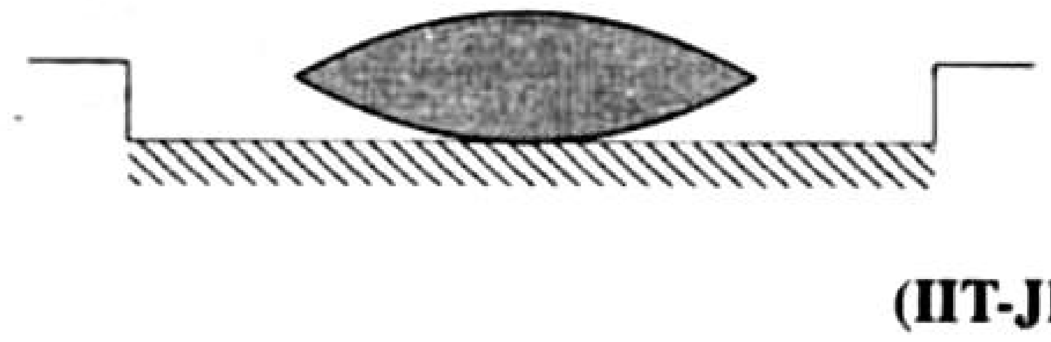
21. A convex lens of focal length 15 cm and a concave mirror of focal length 30 cm are kept with their optic axes PQ and RS parallel but separated in vertical direction by 0.6 cm as shown in the figure. The distance between the lens and mirror is 30 cm. An upright object AB of height 1.2 cm is placed on the optics axis PQ of the lens at a distance of 20 cm from the lens. If $A'B'$ is the image after refraction from the lens and reflection from the mirror, find the distance of $A'B'$ from the pole of the mirror and obtain its magnification. Also, locate positions of A' and B' with respect to the optic axis RS.



(IIT-JEE 2000)

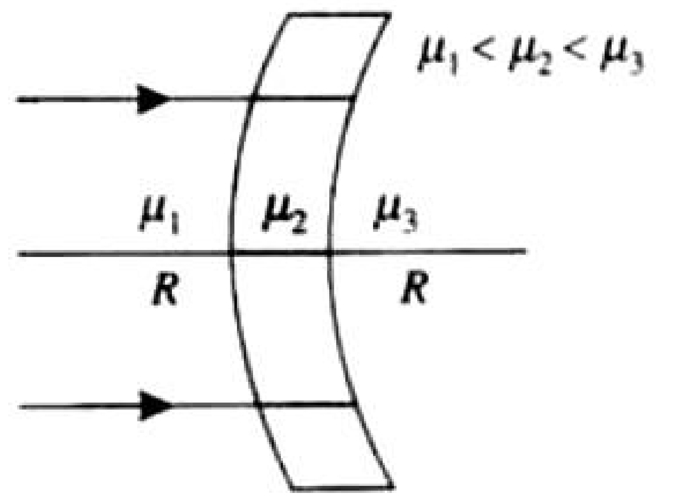
22. The refractive indices of the crown glass for blue and red lights are 1.51 and 1.49, respectively, and those of the flint glass are 1.77 and 1.73, respectively. An isosceles prism of angle 6° is made of crown glass. A beam of white light is incident at a small angle on this prism. The other flint glass isosceles prism is combined with the crown glass prism such that there is no deviation of the incident light. Determine the angle of the flint glass prism. Calculate the net dispersion of the combined system. **(IIT-JEE 2001)**

23. A thin biconvex lens of refractive index 3/2 is placed on a horizontal plane mirror as shown in the figure. The space between the lens and the mirror is then filled with water of refractive index 4/3. It is found that when a point object is placed 15 cm above the lens on its principal axis, the object coincides with its own image. On representing with another liquid, the object and the image again coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid.



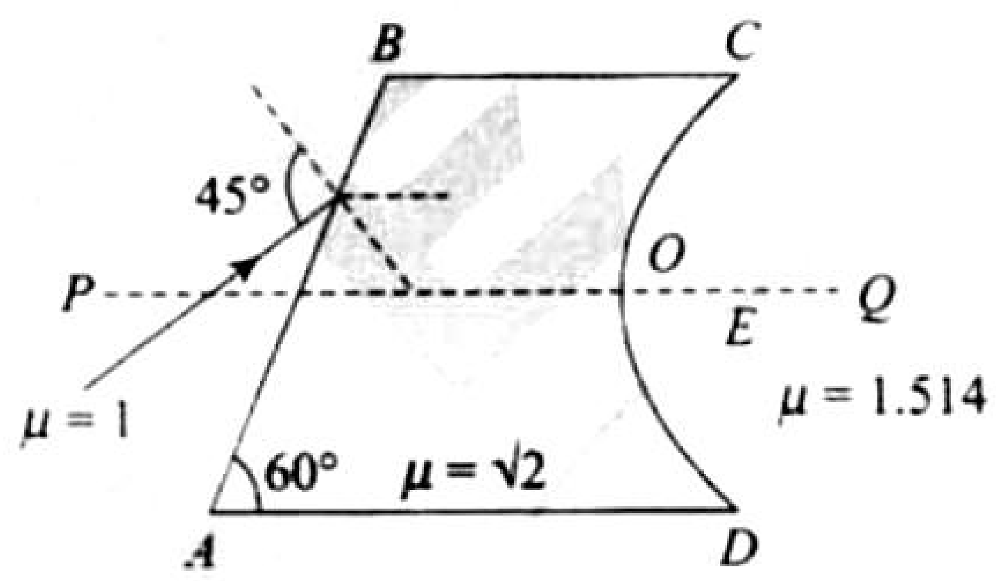
(IIT-JEE 2001)

24. Find the focal length of the lens shown in the figure. The radii of curvature of both the surfaces are equal to R.



(IIT-JEE 2003)

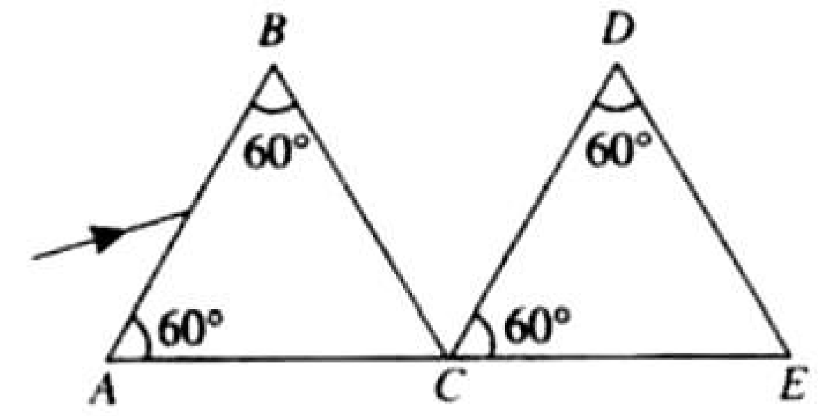
25. Figure below shows an irregular block of material of refractive index $\sqrt{2}$. A ray of light strikes the face AB as shown. After refraction, it is incident on a spherical surface CD of radius of curvature 0.4 m and enters a medium of refractive index 1.514 to meet PQ at E. Find the distance OE up to two places of decimal.



(IIT-JEE 2004)

26. An object is approaching a convex lens of focal length 0.3 m with a speed of 0.01 m s^{-1}. Find the magnitudes of the ratio of change of position and lateral magnification of image when the object is at a distance of 0.4 m from the lens. **(IIT-JEE 2004)**

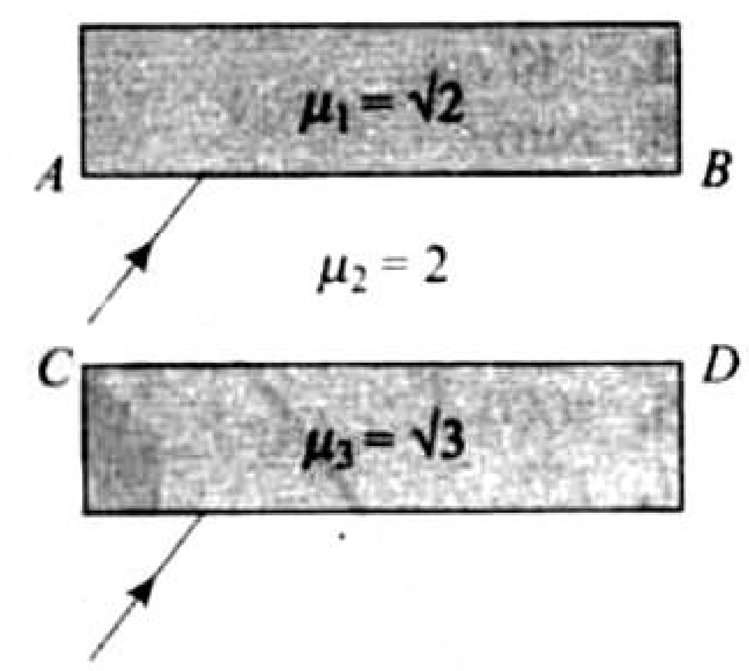
27. Two identical prisms of refractive index $\sqrt{3}$ are kept as shown in the figure. A light ray strikes the first prism at face AB. Find.



a. the angle of incidence, so that the emergent ray from the first prism has minimum deviation.

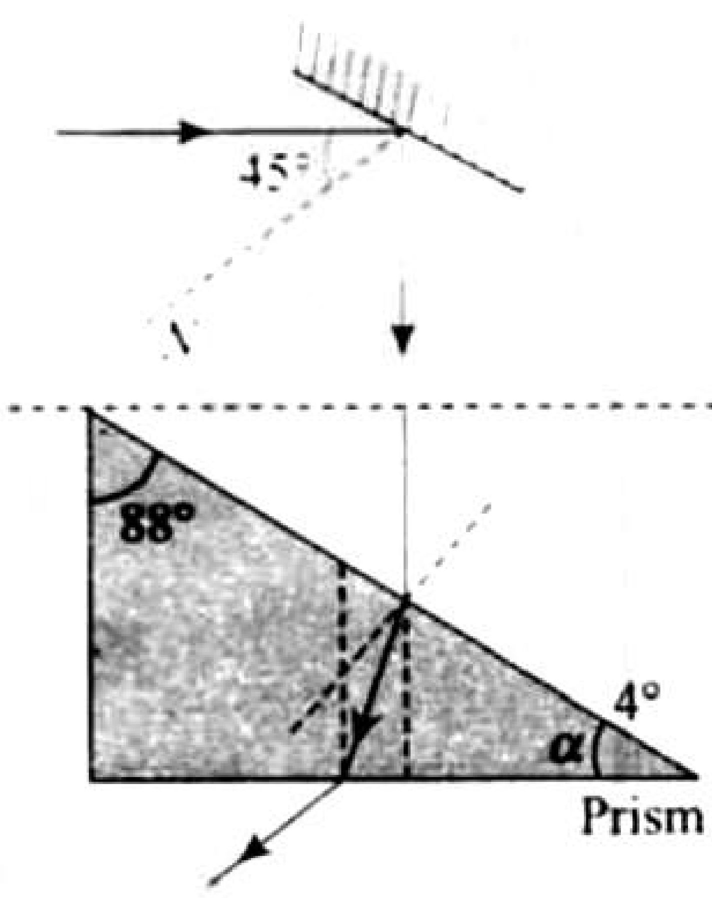
b. through what angle the prism DCE should be rotated about C so that the final ray suffers net minimum deviation. **(IIT-JEE 2005)**

28. AB and CD are surfaces of two slabs as shown in the figure. The medium between the slabs has refractive index 2. Refractive index of the slab above AB is $\sqrt{2}$ and below CD is $\sqrt{3}$. Find the minimum angle of incidence at Q, so that the ray is totally reflected by both the slabs.



(IIT-JEE 2005)

29. A ray of light strikes a plane mirror at an angle of incidence 45° as shown in the figure. After reflection, the ray passes through a prism of refractive index 1.5 whose apex angle is 4°. Through what angle must the mirror be rotated if the total deviation of the ray be 90°?



(IIT-JEE 2006)

ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. a.	**2.** a.	**3.** a.	**4.** a.	**5.** d.
6. a.	**7.** c.	**8.** c.	**9.** c.	**10.** d.
11. c.	**12.** a.	**13.** b.	**14.** b.	**15.** d.
16. a.	**17.** a.	**18.** d.	**19.** d.	**20.** b.
21. a.	**22.** c.	**23.** d.	**24.** c.	**25.** b.
26. c.	**27.** b.	**28.** b.	**29.** b.	**30.** c.
31. b.	**32.** a.	**33.** a.	**34.** c.	**35.** b.
36. c.	**37.** b.	**38.** c.	**39.** b.	**40.** b.
41. b.	**42.** a.	**43.** c.	**44.** b.	**45.** b.
46. a.	**47.** c.	**48.** c.	**49.** b.	

Multiple Correct Answers Type

1. b., d.	**2.** a., b.	**3.** a., b., c., d.
4. b., c.	**5.** c., d.	**6.** c., d.
7. a., b., c.	**8.** a., c.	

Linked Comprehension Type

1. a., c. **2.** d.

Matching Column Type

1. i. → a.; ii. → c.; iiii. → c.; iv. → a., b., d.

2. i. → a., b., c., d.; ii. → b.; iii. → a., b., c. d.
iv. → a., b., c., d.
3. i. → a., c.; ii. → b., d., e.; iii. → a., c., e.; iv. → b., d.;
4. p. → (2); q. → (3); r. → (4); s. → (1)

Integer Answer Type

1. (6) 2. (6) 3. (3) 4. (2) 5. (7)
6. (2)

Assertion–Reasoning Type

1. c. 2. b.

Fill in the Blanks Type

1. +15 cm 2. 60 cm
3. −30 cm 4. 1.5
5. zero 6. smaller
7. $\sqrt{\dfrac{\mu\varepsilon}{\mu_0\varepsilon_0}}$ 8. 2.945×10^{-4} m
9. 4000 Å 10. 0.125 m, 0.5 m
11. 15°

True/False Type

1. True 2. True 3. False 4. True
5. True

Subjective Type

1. 75 cm 2. (a) $\mu_2 < \mu_1$ (b) $\mu_2 = \mu_1$
3. (a) −6 mm, −5 mm
4. Final image is formed 30 cm behind the mirror surface
5. (a) 70.73 cm (b) −2
6. (a) 15 cm (b) 1.15 cm
7. 75 cm 9. a = 3
10. (a) $\sqrt{2}$ (b) $0.5 < 0.6 < 0.86$
11. (a) 5 mm (b) Final image is virtual and is formed at *I*.
12. $m = \dfrac{3}{2}$ 13. $\sqrt{2}$
15. (a) Slope = $\cot i$ (b) $4y^{1/4} = x$ (c) (4m, 1m)
(d) The ray will emerge grazingly.
16. (a) $i_1 = \sin^{-1}\left\{\dfrac{1}{\sqrt{2}}(\sqrt{n^2 - n_1^2} - n_1\right\}$ (b) 73°
17. 0.4 m, 0.6 m
18. 0.9 m from the lens (rightwards) or 0.1 m behind the mirror.
19. $\dfrac{4}{3}$ 20. $\dfrac{1}{\sqrt{2}}(0.6\hat{i} + 0.8\hat{j} - \hat{k})$
21. 15 cm, $-\dfrac{3}{2}$ 22. (a) 4° (b) −0.04°
23. 1.6 24. $\dfrac{\mu_3 R}{\mu_3 - \mu_1}$
25. 6.06 m 26. 0.09 m/s, 0.3 m/s
27. (a) 60° (b) 60° 28. 60°
29. 1°

HINTS AND SOLUTIONS

JEE Advanced

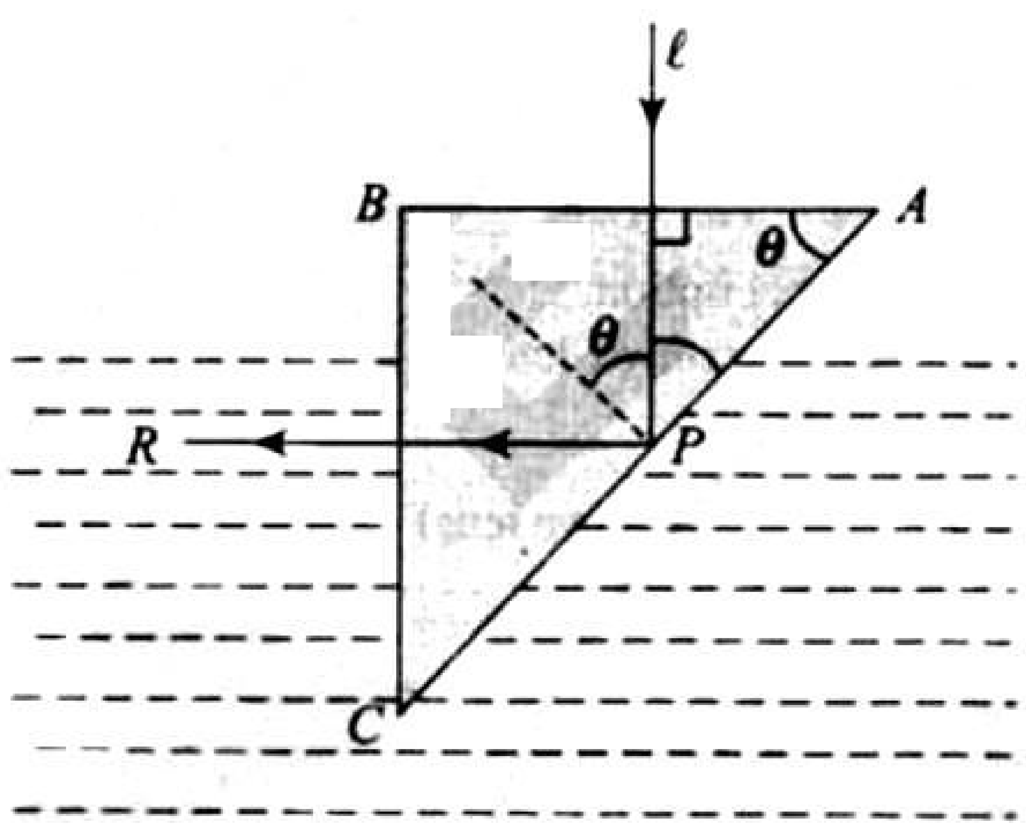
Single Correct Answer Type

1. a. $\lambda = \frac{v}{f}$

In moving from air to glass, f remains unchanged while v decreases. Hence, λ should decrease.

2. a. The phenomenon of total internal reflection takes place during reflection at P.

$$\sin\theta = \frac{1}{{}^w_g\mu} \quad \text{(i)}$$



where θ is the angle of incidence at P.

Now. ${}^w_g\mu = \frac{{}^a_g h}{{}^a_g\mu} = \frac{1.5}{4/3} = 1.125$

Putting in (i).

$$\sin\theta = \frac{1}{1.125} = \frac{8}{9}$$

Therefore. $\sin\theta$ should be greater than $\frac{8}{9}$.

3. a. $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{0.4} + \frac{1}{-0.25} = \frac{1}{0.4} - \frac{1}{0.25}$

$$= \frac{0.25 - 0.4}{0.4 \times 0.25} = \frac{-0.15}{0.4 \times 0.25} = \frac{-0.15}{0.4 \times 0.25} = -1.5$$

$$\Rightarrow \quad P = \frac{1}{f} = -1.5 \text{ D}$$

4. a. ${}^1_2\mu = \frac{\sin 90^\circ}{\sin C} = \frac{1}{\sin C}$ [For critical angle]

$$\therefore \quad C = \sin^{-1} = \left(\frac{1}{{}^1_2\mu}\right) \quad \text{(i)}$$

Applying Snell's law at P, we get

$${}^1_2\mu = \frac{\sin r'}{\sin i} = \frac{\sin(90 - r)}{\sin r}$$

$[\because i = r;\ r' + r = 90^\circ]$

$$\therefore \quad {}^1_2\mu = \frac{\cos r}{\sin r} \quad \text{(ii)}$$

From (i) and (ii), $C = \sin^{-1}(\tan r)$

5. d. From mirror formula $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ (i)

Differentiating equation (i), we obtain

$$0 = -\frac{1}{v^2}dv - \frac{1}{u^2}du \Rightarrow dv = -\left(\frac{v}{u}\right)^2 du \quad \text{(ii)}$$

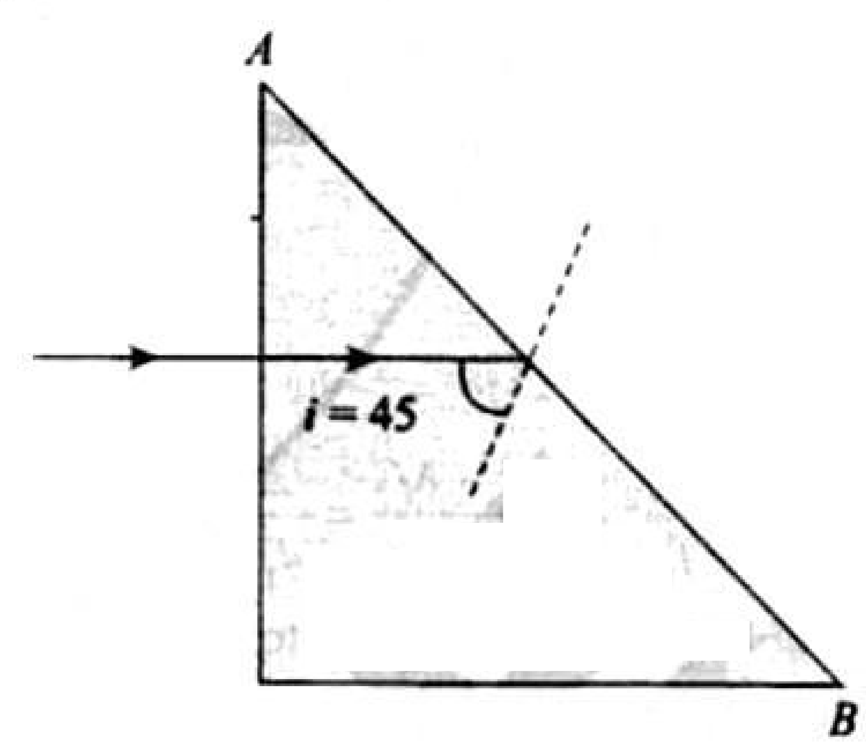
Also from equation (i) $\frac{v}{u} = \frac{f}{u - f}$ (iii)

From equation (ii) and (iii) we get $dv = -\left(\frac{f}{u - f}\right)^2 \cdot l$

Therefore size of image is $\left(\frac{f}{u - f}\right)^2 l$.

6. a. For total internal reflection, $\mu = \frac{1}{\sin C} = \frac{1}{\sin 45^\circ} = 1.414$

i.e., for an angle of incidence of that color will suffer total internal reflection for which the refractive index is less than 1.414.



Therefore, red light will be refracted at interface AB whereas blue and green light will be reflected

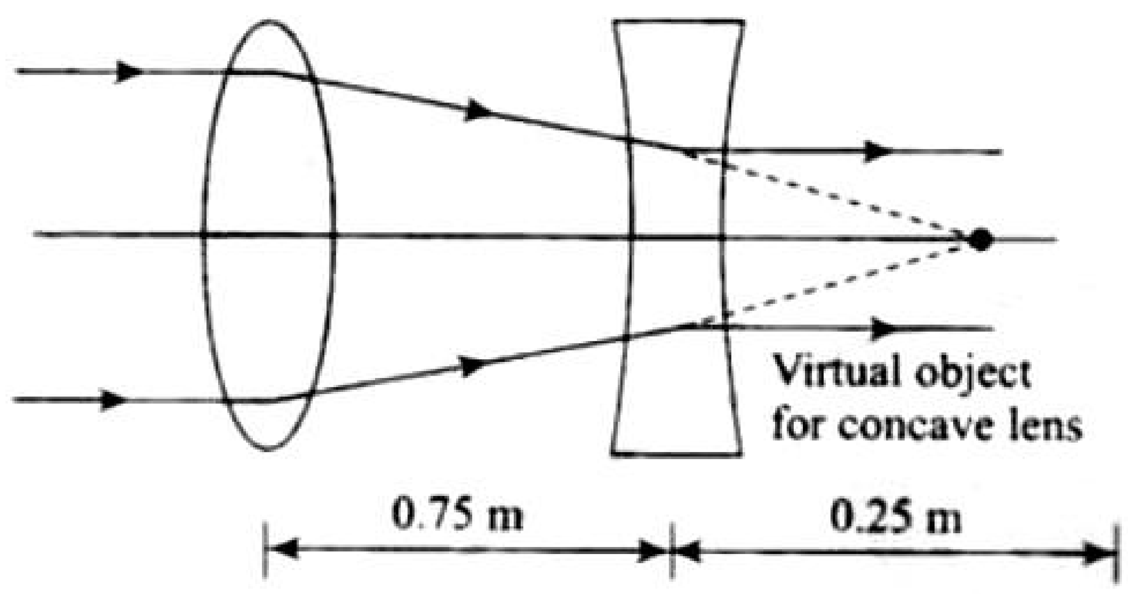
7. c. The angle of deviation for the first prism P_1. $\delta_1 = (\mu_1 - 1)A_1$.
The angle of deviation for the second prism P_2. $\delta_2 = (\mu_2 - 1)A_2$
Since total deviation is to be zero, therefore

$$\delta_1 + \delta_2 = 0 \Rightarrow (\mu_1 - 1)A_1 + (\mu_2 - 1)A_2 = 0$$

$$A_2 = \frac{(1.54-1)}{(1.72-1)}4° = 3°$$

(c) is the correct option.

8. c. The image I' of parallel rays formed by lens 1 will act as a virtual object.



Applying lens formula for lens 2,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{f_1 - d} = \frac{1}{f_2}$$

$$\Rightarrow \quad v = \frac{f_2(f_1-d)}{f_2+f_1-d}$$

The horizontal distance of the image I from O is

$$x = d + \frac{f_2(f_1-d)}{f_2+f_1-d}$$

$$= \frac{df_2 + df_1 - d^2 + f_2 f_1 - df_2}{f_2+f_1-d}$$

$$= \frac{f_1 f_2 + d(f_1-d)}{f_2+f_1-d}$$

To find the y-coordinate, we use magnification formula for lens 2,

$$m = \frac{v}{u} = \frac{\dfrac{f_2(f_1-d)}{f_1+f_2-d}}{f_1-d} = \frac{f_2}{f_1+f_2-d}. \text{ Also}$$

$$m = \frac{h^2}{\Delta} \Rightarrow h_2 = \frac{\Delta \times f_2}{f_1+f_2-d}$$

Therefore, the y-coordinate,

$$y = \Delta - h_2$$

$$= \Delta - \frac{\Delta f_2}{f_1+f_2-d}$$

$$= \frac{\Delta f_1 + \Delta f_2 - \Delta d - \Delta f_2}{f_1+f_2-d} = \frac{\Delta(f_1-d)}{f_1+f_2-d}$$

9. c. Spherical abberation occurs due to the inability of a lens to converge marginal rays of the same wavelength to the focus, as it converges the paraxial rays. This defect can be removed by blocking marginal rays. This can be done by using a circular annular mask over the lens.

10. d. Applying Snell's law at P. $\mu = \frac{\sin r}{\sin 30°}$ $\sin r = \frac{1.44}{2} = 0.72$

$\therefore \quad \delta = r - 30° = \sin^{-1}(0.72) - 30°$

Therefore, the rays make an angle of $2\,\delta = [\sin-1(0.72)-30]$ with each other.

(d) is the correct option.

11. c. A convex mirror and a concave lens always produce semi image for the objects. Therefore, option (b) and (d) are not correct. The image by a convex lens is diminished when the object is placed beyond $2f$.

Let $u = 2f + x$.

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$,

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{-(2f+x)} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{2f+x}$$

$$= \frac{2f+x-f}{f(2f+x)} = \frac{(f+x)}{f(2f+x)}$$

But $u = v = 1$ (given) $(2f + x) + \frac{f(2f+x)}{f+x} \le 1$

$$\Rightarrow \quad 2f = x\left[1 + \frac{f}{f+x}\right] \le 1 \Rightarrow \frac{(2f+x)^2}{f+x} \le 1$$

$$\Rightarrow \quad (2f+x)^2 \le f + x$$

The above is true for $f < 0.25$ m

(c) is the correct answer.

12. a. Here, $f_v = 2$ cm and $f_e = 3$ cm.

Using lens formula for eyepiece, $-\frac{1}{u} + \frac{1}{v_1} = \frac{1}{f_e}$

$$\Rightarrow \quad \frac{-1}{u} + \frac{1}{\infty} = \frac{1}{3} \Rightarrow u_1 = -3 \text{ cm } [\because i = 0]$$

But the distance between objective and eyepiece is 15 cm (given).

Therefore, distance of image formed by the objective, $v = 15 - 3 = 12$ cm. Let u be the object distance from the objective, then

for objective lens $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f_0}$ or $\frac{-1}{u} + \frac{1}{12} = \frac{1}{2}$

$$\Rightarrow \quad \frac{-1}{u} = \frac{1}{2} - \frac{1}{12} = \frac{5}{12} \quad u = -\frac{12}{5} = 2.4 \text{ cm}$$

(a) is the correct option.

13. b. $f_1 = +40$ cm (for convex lens)

$= 0.4$ m

$f_2 = -25$ (for concave lens)

$= -0.25$ m

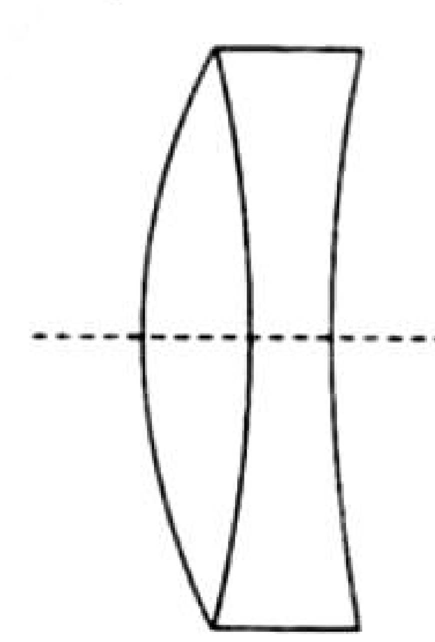
Therefore, focal length (f) of the combination,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= \frac{1}{0.40} - \frac{1}{0.25} = \frac{0.25-0.4}{0.40 \times 0.25}$$

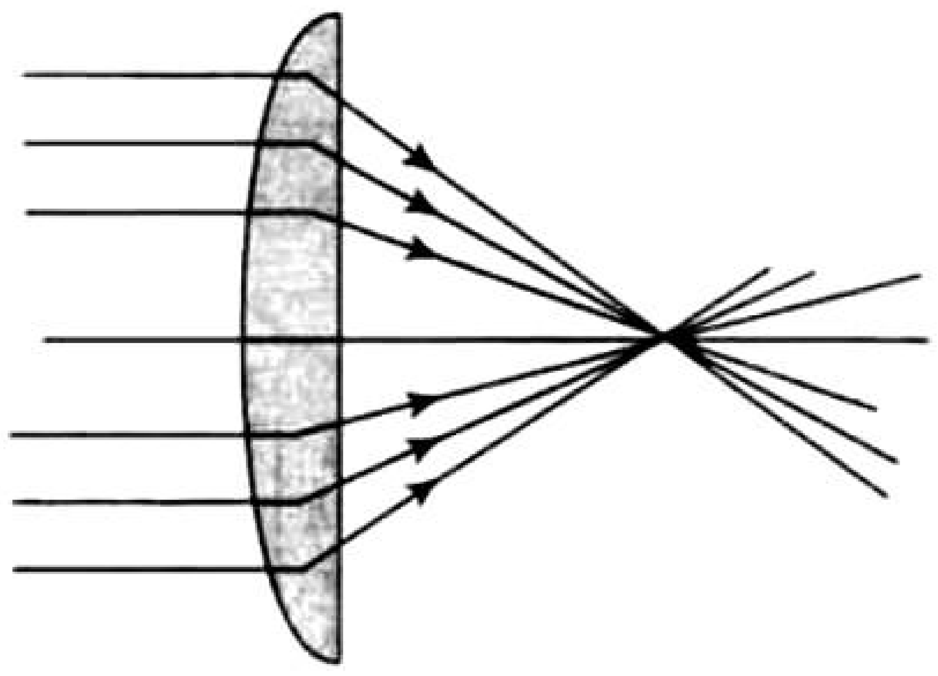
$$= -\frac{0.15}{0.1} = -1.5 \text{ D}$$

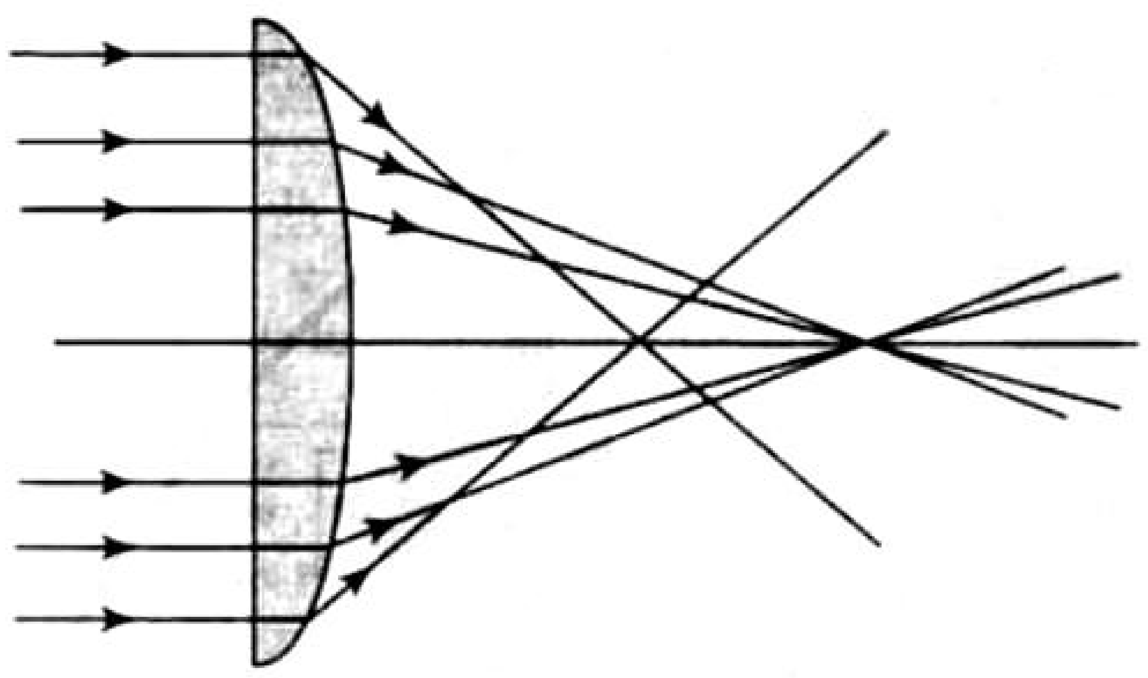
$$\Rightarrow \quad P = \frac{1}{f} = -1.5 \text{ D}$$



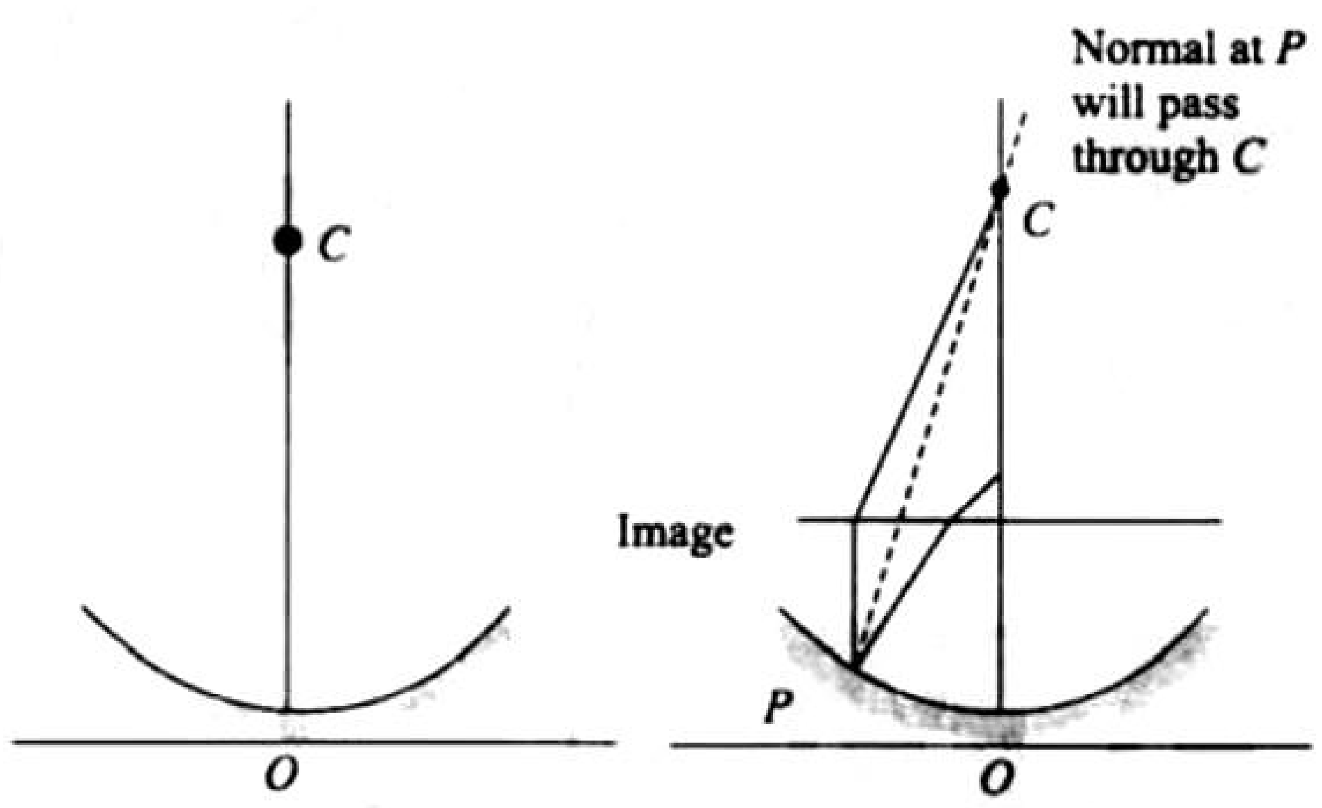
(b) is the correct option.

14. b. In this case, the total deviation is shared between the two surfaces.





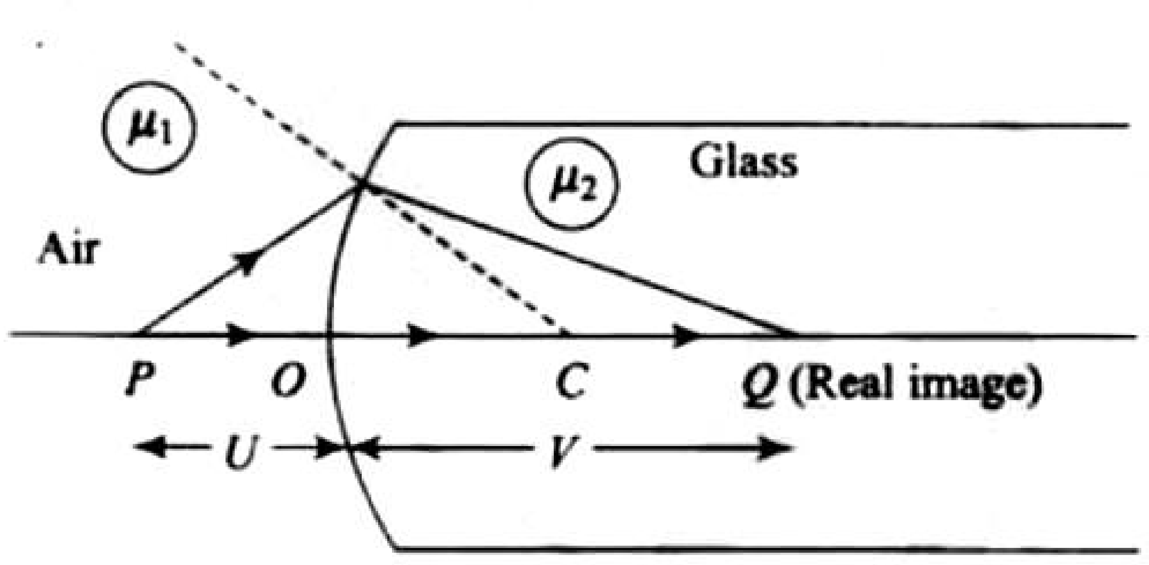
15. d.



The ray diagram is shown in the figure. Therefore, the image will be real and between C and O.

16. a. The formula for spherical refracting surface is

$$\frac{-\mu_1}{\mu}+\frac{\mu_2}{\upsilon}=\frac{\mu_2-\mu_1}{R}$$



Here, $u=-x$, $v=+x$, $R=+R$, $\mu_1=1$, $\mu_2=1.5$

$$\frac{-1}{-x}+\frac{1.5}{x}=\frac{1.5-1}{R}$$

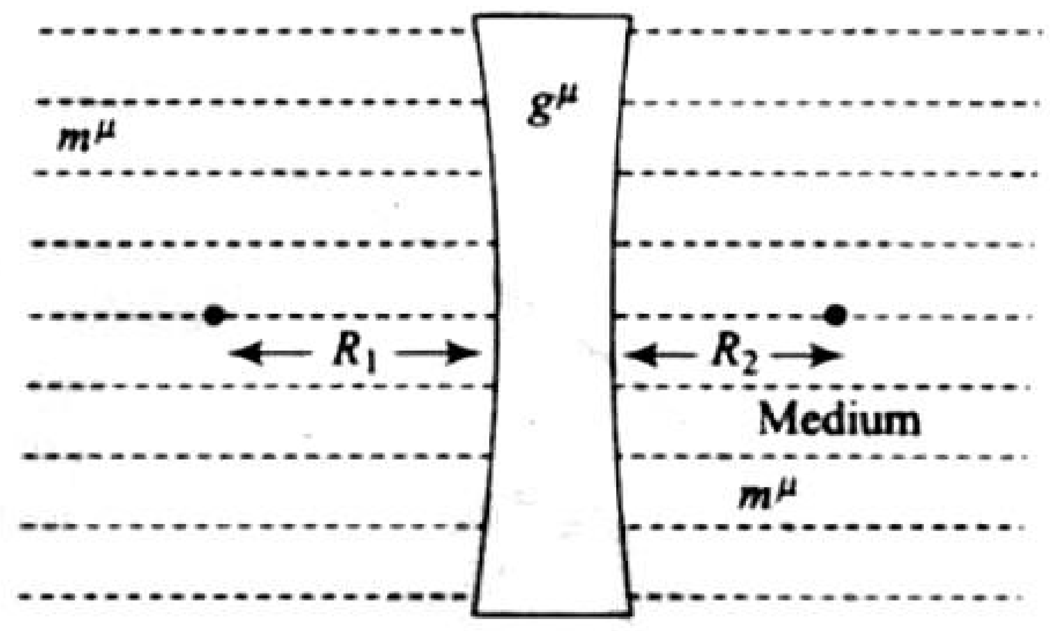
$\Rightarrow \quad x=5R$

17. a. According to lensmaker's formula,

$$\frac{1}{f}=({}_g^m\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Now, ${}_g^m\mu=\frac{{}_g\mu}{{}_m\mu}=\frac{1.5}{1.75}$

For concave lens, as shown in figure, in this case $R_1=-R$ and $R_2=+R$.



$$\therefore \quad \frac{1}{f}=\left(\frac{1.5}{1.75}-1\right)\left(-\frac{1}{R}-\frac{1}{R}\right)=+\frac{0.25\times 2}{1.75R}$$

$\Rightarrow \quad f=+3.5R$

The positive sign shows that the lens behaves as a convergent lens.

(a) is the correct option.

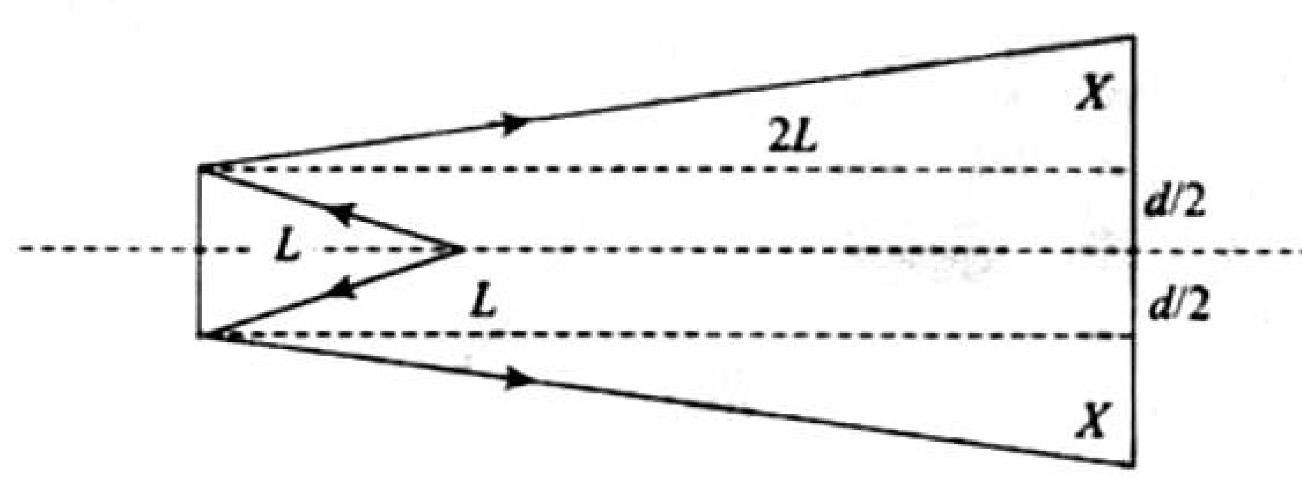
18. d. We know that $\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

For divergence, $\mu_2>\mu_1$

Here, $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ is negative.

Therefore, (d) is the correct option.

19. d. From the ray diagram,



In ΔANM and ΔADC,

$\angle ADC=\angle ANM=90°$ $\qquad$ $[MN \perp AD]$

$\angle MAN=\angle CAN$ $\qquad$ (law of reflection)

$\Rightarrow \quad \Delta ANM$ is similar to ΔADC

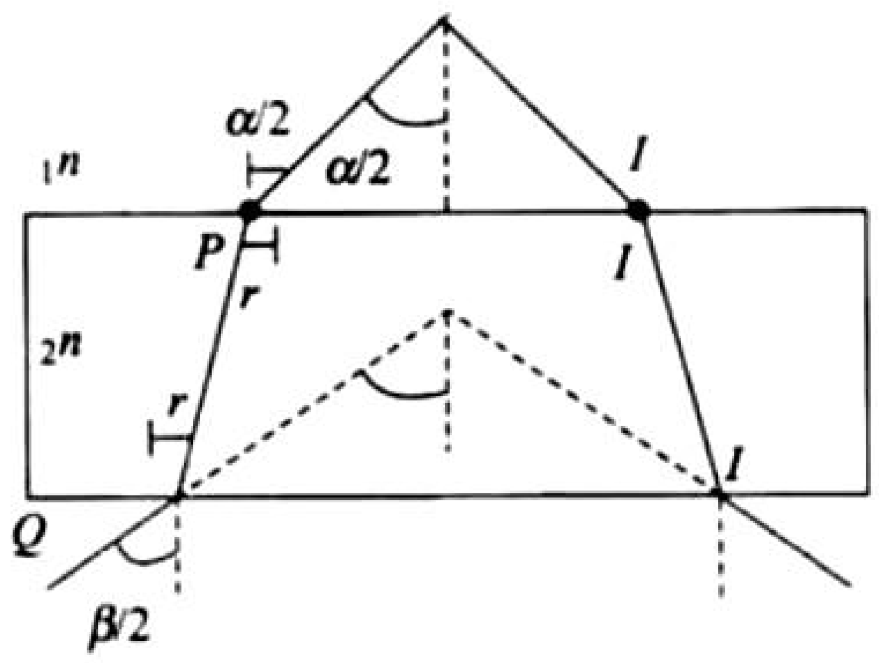
$\therefore \quad \frac{x}{2L}=\frac{d/2}{L}$ or $x=d$

So, required distance $=d+d+d=3d$.

Therefore, (d) is the correct option.

20. b. The path of rays become parallel to initial direction as they emerge.

Applying Snell's law at P, $\frac{1}{2}n=\frac{\sin\alpha/2}{\sin r}$ $\qquad$ (i)

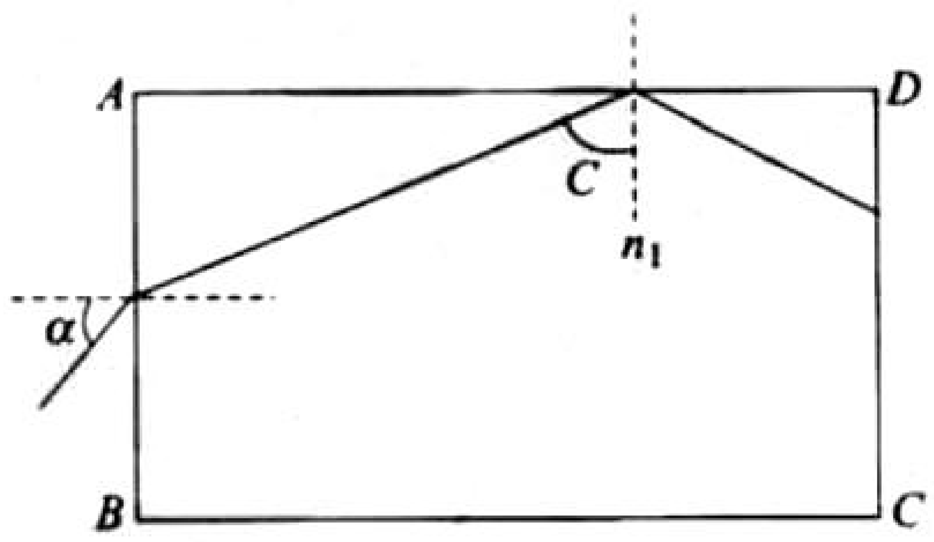


Applying Snell's law at Q, $\frac{1}{2}n = \frac{\sin \beta/2}{\sin r}$ (ii)

From Eqs. (i) and (ii), $\alpha = \beta$

Therefore, (b) is the correct answer.

21. a. See figure. The ray will come out from CD if it suffers total internal reflection at surface AD, i.e., it strikes the surface AD at critical angle C (the limiting case). Applying Snell's law at P, $n_1 \sin C = n_2$ or $\sin C = \frac{n_2}{n_1}$



Applying Snell's law at Q, $n_2 \sin \alpha = n_1 \cos C$;

$$\sin \alpha = \frac{n_1}{n_2}\cos\left\{\sin^{-1}\left(\frac{n_2}{n_1}\right)\right\}$$

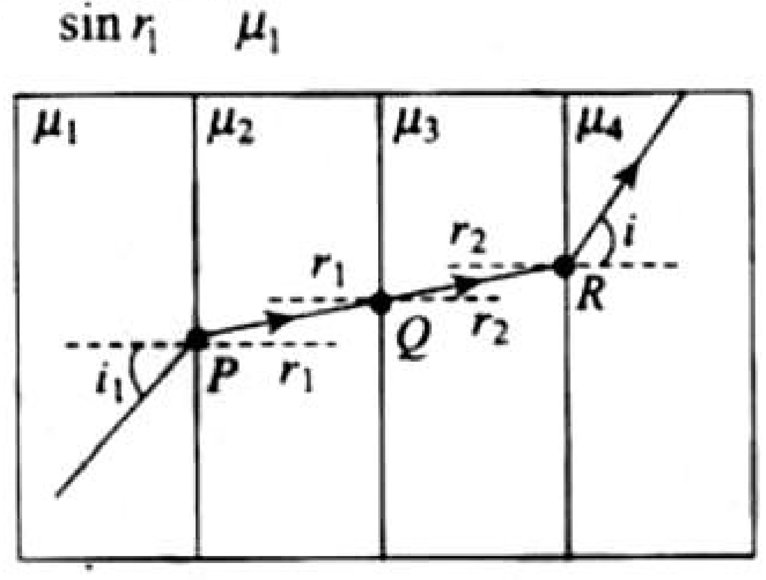
or $$\alpha = \sin^{-1}\left[\frac{n_1}{n_2}\cos\left\{\sin^{-1}\left(\frac{n_2}{n_1}\right)\right\}\right]$$

∴ (a) is the correct option.

22. c. The intermediate image in compound microscope is real, inverted, and magnified.

23. d. Applying Snell's law at P,

$${}^1\mu_2 = \frac{\sin i}{\sin r_1} = \frac{\mu_2}{\mu_1} \quad \text{(i)}$$



Applying Snell's law at Q,

$${}^2\mu_3 = \frac{\sin r_1}{\sin r_2} = \frac{\mu_3}{\mu_2} \quad \text{(ii)}$$

Again, applying Snell's law at R,

$${}^3\mu_4 = \frac{\sin r_2}{\sin i} = \frac{\mu_4}{\mu_3} \quad \text{(iii)}$$

Multiplying (i), (ii), and (iii), we get

$\mu_4 = \mu_1$

Correct option is (d).

24. c. Since there will be no refraction from P to Q and then from Q to R (all being identical). Hence, the ray will now have the same deviation as before the point n, n' being same for the ray. Correct option is (c).

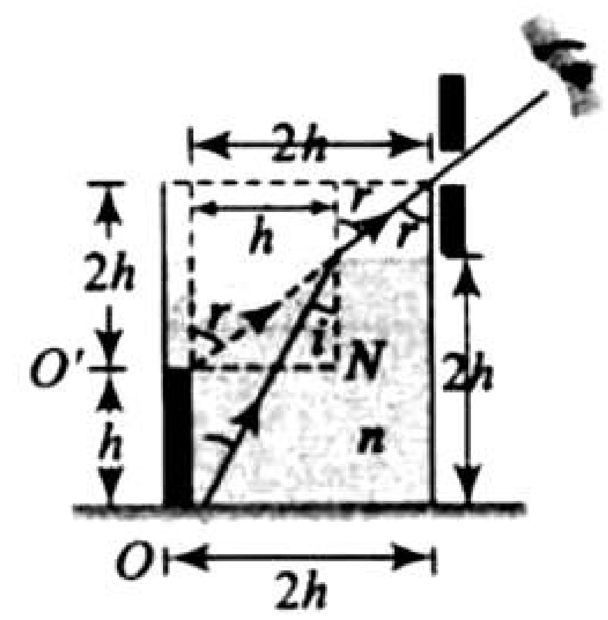
25. b. $\frac{\sin i}{\sin r} = \frac{1}{n}$ (i)

Since $\tan r = \frac{2h}{2h} = 1$

$\Rightarrow \quad r = 45°$

$\Rightarrow \quad \sin i = \frac{h}{h\sqrt{5}}$

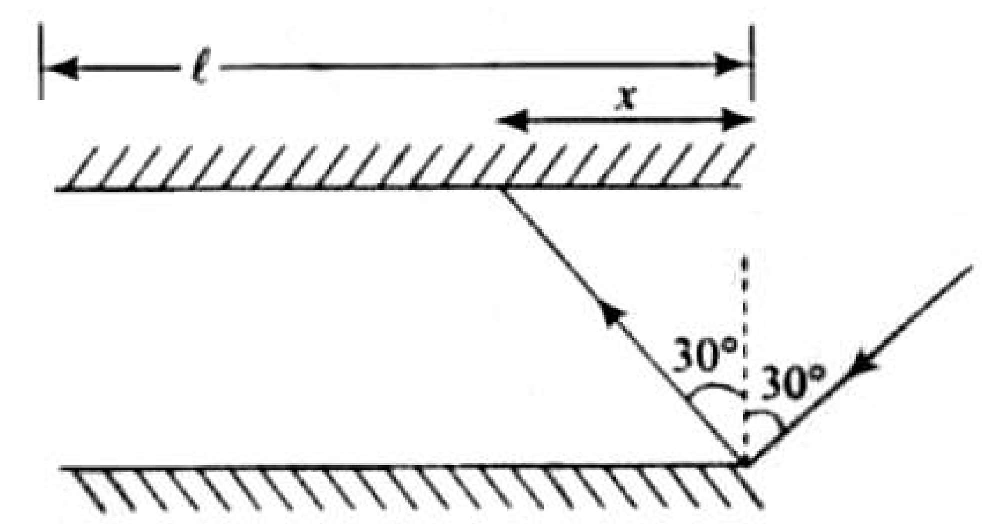
$\Rightarrow \quad \sin i = \frac{1}{\sqrt{5}}$



$\therefore \quad \frac{1}{n} = \frac{1/\sqrt{5}}{1/\sqrt{2}} \Rightarrow n = \sqrt{\frac{5}{2}}$

26. c. Since both surfaces have same radius of curvature on the same side, no dispersion will occur.

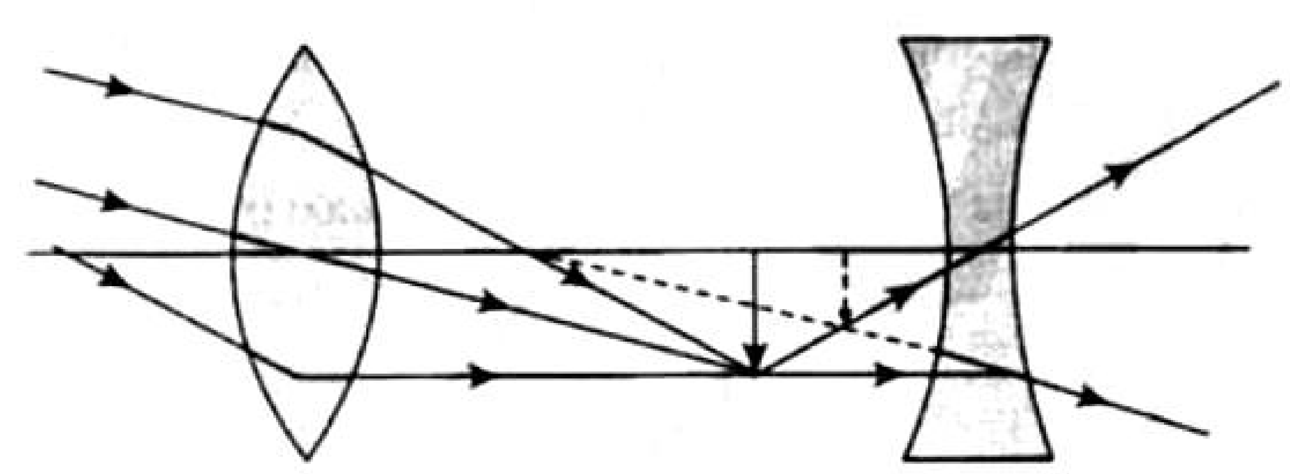
27. b. Maximum number of reflections = $\left[\frac{\ell}{x}\right]$, where $x = 0.2 \tan 30°$



28. b. For the concave lens,

$\frac{1}{v} - \frac{1}{4} = \frac{1}{-20} \Rightarrow v = 5$ cm; $\frac{h_2}{h_1} = \left|\frac{v}{u}\right|$

$\Rightarrow \quad h_2 = 2 \times \frac{5}{4} = 2.5$ cm



29. b. Applying Snell's law at glass–water surface,

$${}^{\omega}_{g}\mu = \frac{\sin r}{\sin i} = \frac{{}_g\mu}{{}_\omega\mu} \quad \text{(i)}$$

Applying Snell's law at water–air surface,

$${}^{\omega}_{g}\mu = \frac{\sin 90°}{\sin r} = \frac{{}_\omega\mu}{{}_a\mu} \quad \text{(ii)}$$

From (i) and (ii),

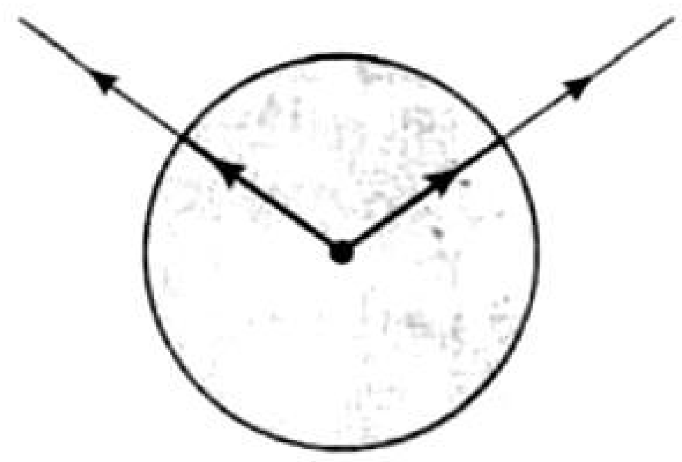
$$\frac{{}_g\mu}{{}_\omega\mu} = \frac{3}{4\sin i} \Rightarrow \frac{3 \times {}_g\mu}{4} = \frac{3}{4\sin i} \Rightarrow {}_g\mu = \frac{1}{\sin i}$$

30. c. We know that $\sin C = \frac{1}{\mu}$ and $\mu \propto \frac{1}{\lambda}$

$\Rightarrow \quad \sin C \propto \lambda$. For more λ, C is more.

31. b. For minimum deviation, incident angle is equal to emerging angle. Therefore, QR is horizontal.

32. a. Here, $u = -6$ cm, $v = ?$, $r = -6$ cm, $\mu_1 = 1$, $\mu = 1.5$



Using the formula of spherical refracting surface,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

We have $\frac{1.5}{v} - \frac{1}{-6} = \frac{1.5-1}{-6} \Rightarrow v = -6$ cm

Alternative method:

The rays coming from the point object fall on the glass–air interface normally and hence pass undeviated. Therefore, if we retrace the path of the refracted rays backward, the image will be formed at the center only.

33. a. $$\frac{|P_1|}{|P_2|} = \frac{2}{3} \Rightarrow \frac{f_2}{f_1} = \frac{2}{3} \qquad \text{(i)}$$

Focal length of their combination, $\frac{1}{f} = \frac{1}{f_1} - \frac{1}{f_2}$

$$\Rightarrow \quad \frac{1}{30} = \frac{1}{f_1} - \frac{1 \times 3}{2f_1} \text{ [from (i)]}$$

$$\Rightarrow \quad \frac{1}{30} = \frac{1}{f_1}\left[1 - \frac{3}{2}\right] = \frac{1}{f_1} \times \left(-\frac{1}{2}\right)$$

$\therefore \quad f_1 = -15$ cm

$\therefore \quad f_2 = \frac{2}{3} \times f_1 = \frac{2}{3} \times 15 = 10$ cm

34. c. The image I' for first refraction (i.e., when the ray comes out of liquid) is at a depth of $= \frac{33.25}{1.33} = 25$ cm

$$\left[\because \text{Apparent depth} = \frac{\text{Real depth}}{\mu}\right]$$

Now, reflection will occur at concave mirror. For this I' behaves as an object

$\therefore \quad u = -(15 + 25) = -40$ cm, $f = f$, $v = ?$

Using mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{v} - \frac{1}{40} \Rightarrow v = \frac{40f}{40+f} \qquad \text{(i)}$$

But $$v = -\left[15 + \frac{25}{1.33}\right] \qquad \text{(ii)}$$

where $\frac{25}{1.33}$ is the real depth of the image.

From Eqs. (i) and (ii),

$$\frac{40f}{40+f} = -\left[15 + \frac{25}{1.33}\right] = -33.79$$

$\Rightarrow \quad f = -18.31$ cm

35. b. Refraction from lens: $\frac{1}{v_1} - \frac{1}{-20} = \frac{1}{15}$

$v = 60$ cm $\quad \overrightarrow{\text{-ve direction}}$

i.e., first image is formed at 60 cm to the right of lens system.

Reflection from mirror:

After reflection from the mirror, the second image will be formed at a distance of 60 cm to the left of lens system.

Refraction from lens:

$$\frac{1}{v_3} - \frac{1}{60} = \frac{1}{15} \leftarrow \text{+ve direction}$$

or $\quad v_3 = 12$ cm

Therefore, the final image is formed at 12 cm to the left of the lens system.

36. c. From lens formula:

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, we have

$$\frac{1}{f} = \frac{1}{10} - \frac{1}{-10} \Rightarrow f = +5$$

Further, $\Delta u = 0.1$

and $\Delta v = 0.1$ (from the graph)

Now, differentiating the lens formula, we have

$$\frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}$$

$$\Delta f = \left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right) f^2$$

Substituting the values, we have

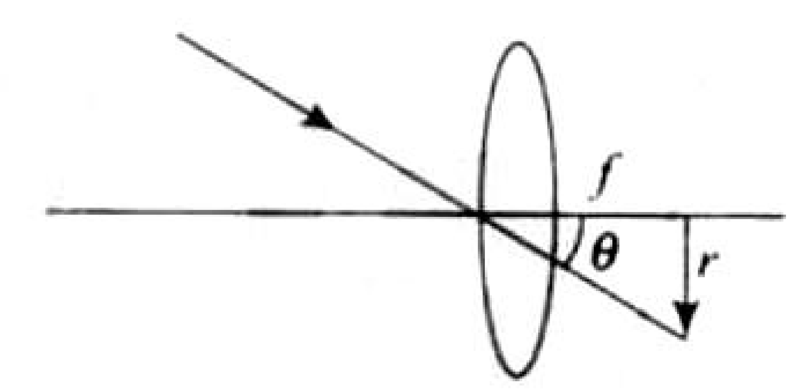
$$\Delta f = \left(\frac{0.1}{10^2} + \frac{0.1}{10^2}\right)(5)^2 = 0.05$$

$\therefore \quad f \pm \Delta f = 5 \pm 0.5$

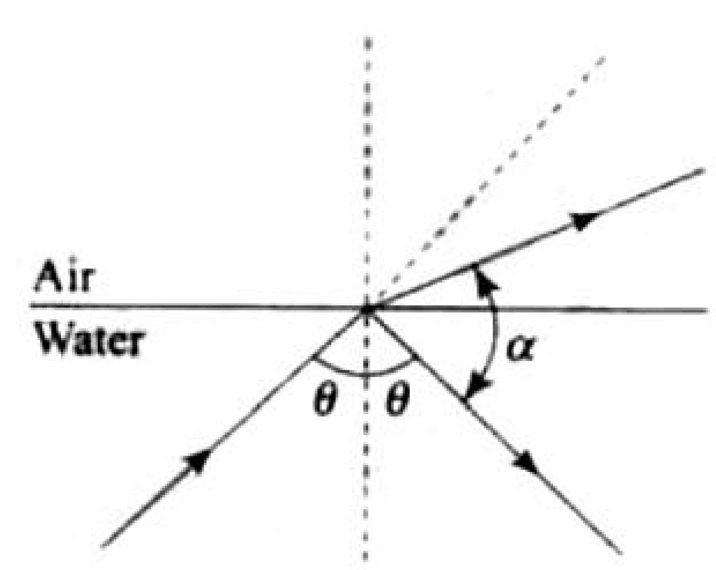
37. b. $\quad r = f \tan \theta$

or $\quad r \propto f$

$\therefore \quad \pi r^2 \propto f^2$

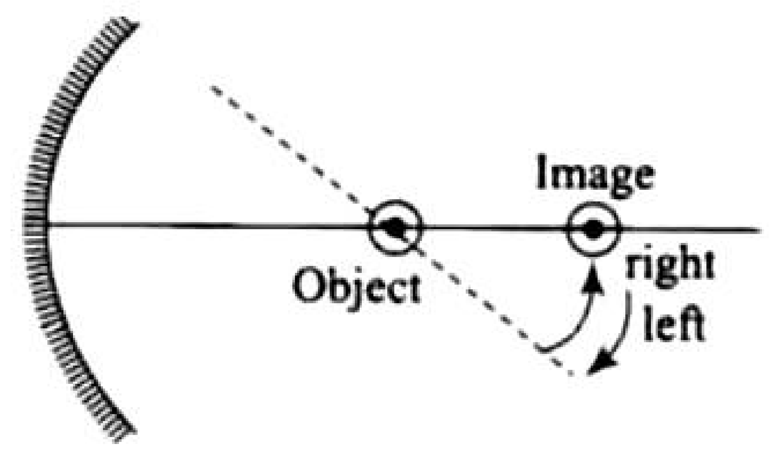


38. c. Since $\theta < \theta_c$ both reflection and refraction will take place. From the figure, we can see that angle between reflected and refracted rays, α, is less than $180° - 2\theta$.



$\therefore \quad$ Option (c) is correct.

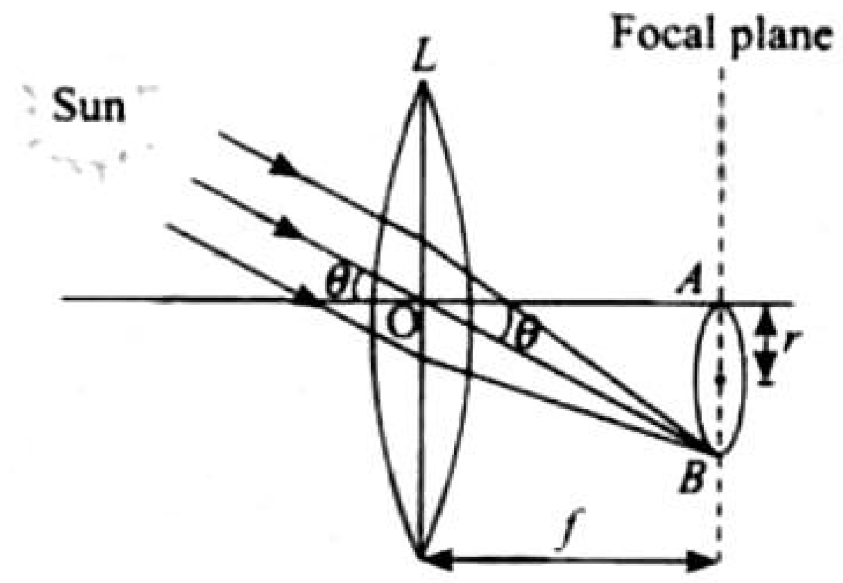
39. b. Since object and image move in opposite directions, the positioning should be as shown in the figure. Object lies between focus and center of curvature, $f < x < 2f$.



$\therefore$ Correct option is (b).

40. b. Rays of light from sun are incident upon a biconvex lens. Sun is at infinity and incident rays from a parallel beam.

The beam is brought to convergence in the focal plane of the lens. AB is image of the sun.



Let r = radius of real image of the sun. Sun is above principal axis of lens.

The real and inverted image of diminished size is formed below the principal axis in the focal plane of the lens as shown in figure.

Area of circular image of sun = πr^2

In the ΔOAB, θ is inclination of sun and is constant.

$$\tan\theta = \frac{r}{f} \Rightarrow r = f\tan\theta$$

or Area of image = $\pi(f\tan\theta)^2$

or Area of image = $(\pi\tan^2\theta)f^2$

From (i) and (iii), area of image = πr^2 and area is proportional to f^2.

41. b. Critical angle from region III to region IV

$$\sin\theta_c = \frac{n_0/8}{n_0/6} = \frac{3}{4}$$

Now, applying Snell's law in region I and region III

$$n_0\sin\theta = \frac{n_0}{6}\sin\theta_c$$

$$\sin\theta = \frac{1}{6}\sin\theta_c = \frac{1}{6}\left(\frac{3}{4}\right) = \frac{1}{8} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{8}\right)$$

$\therefore$ Correct option is (b).

42. a. At minimum deviation ($\delta - \delta_m$):

$$r_1 = r_2 = \frac{A}{2} = \frac{60°}{2} = 30 \text{ (for both colors)}$$

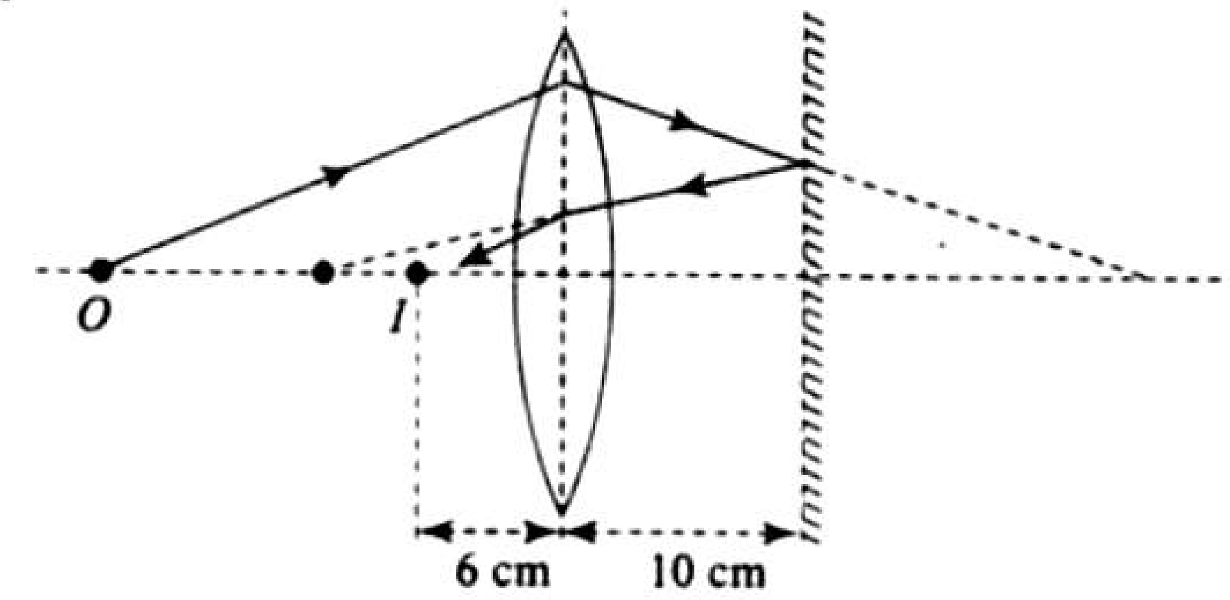
$\therefore$ Correct answer is (a).

43. c. $V_{ball}^2 = 2\times10\times7.2 \Rightarrow v = 12 \text{ m s}^{-1}$

$$X_{\text{image of ball}} = \frac{4}{3}X_{\text{ball}}$$

$$V_{\text{image of ball}} = \frac{4}{3}V_{\text{ball}} = \frac{4}{3}\times 12 = 16 \text{ m s}^{-1}$$

44. b.

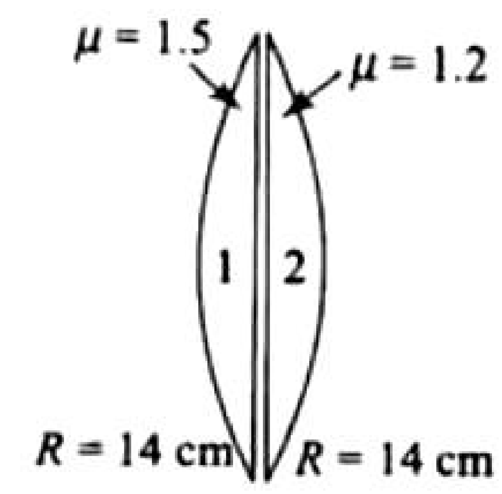


45. b. $\frac{1}{f_1} = (\mu-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (1.5-1)\left[\frac{1}{14} - \frac{1}{\infty}\right] = \frac{0.5}{14}$

$$\frac{1}{f_2} = (1.2-1)\left[\frac{1}{\infty} - \frac{1}{-14}\right]$$

$$\frac{1}{f_1} = \frac{0.2}{14}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{0.5}{14} + \frac{0.2}{14} = \frac{0.7}{14}$$



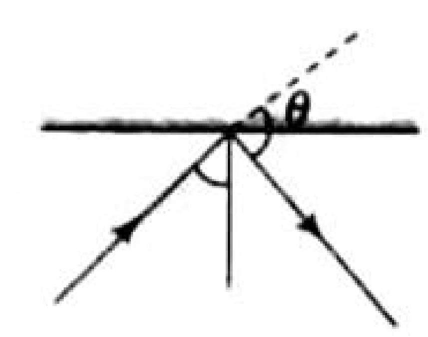
$$\frac{1}{v} = \frac{7}{140} - \frac{1}{40} = \frac{1}{20} - \frac{1}{40}\frac{1}{v} = \frac{2-1}{40}$$

$v = 40$ cm

46. a. Let angle between the directions of incident ray and reflected ray be θ

$$\cos\theta = \frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})\cdot\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$$

$$= -\frac{1}{2}$$



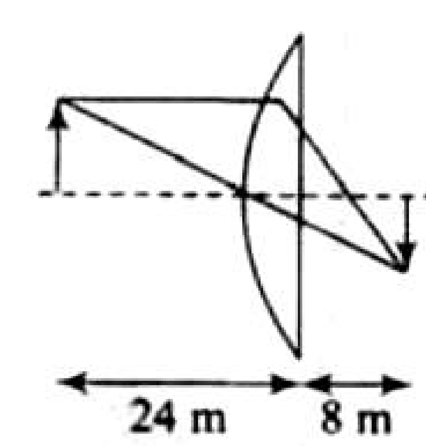
$\theta = 120°$

47. c. $\mu = \frac{\lambda_0}{\lambda_m} = \frac{3}{2}$

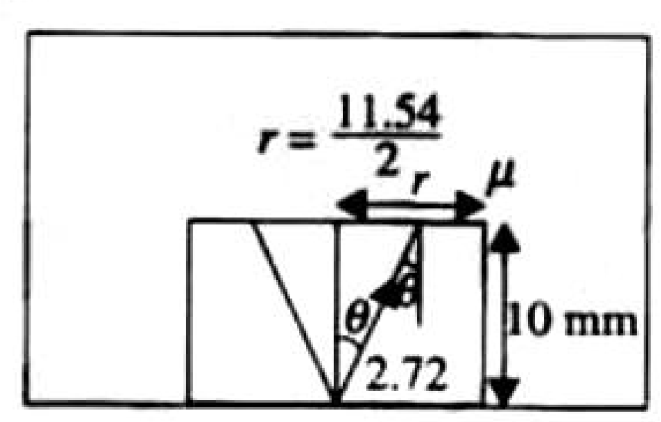
$\Rightarrow \frac{1}{f} = \frac{\mu-1}{R} = \frac{1}{2R}$

$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\Rightarrow \frac{1}{8} - \frac{1}{-24} = \frac{1}{2R} \Rightarrow \frac{3+1}{24} = \frac{1}{2R} \Rightarrow R = 3$ m



48. c. $\tan\theta = \frac{11.54}{2\times10}$



$$\sin\theta = \frac{11.54}{\sqrt{(11.54)^2 + (400)}}$$

$2.72\sin\theta = \mu\times\sin 90°$

$$2.72\times\frac{11.54}{\sqrt{(11.54)^2 + 400}} = \mu$$

$\mu = 1.36$

9. b. $\frac{\mu_2}{V}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

$\frac{1}{V}-\frac{1.5}{-50}=\frac{-0.5}{-10}=\frac{1}{20}$

$\frac{1}{V}=\frac{1}{120}-\frac{3}{100}$

$V = 50$ cm

$\frac{\mu_2}{\infty}-\frac{1.0}{u}=\frac{0.5}{10}$

$-\frac{1}{u}=\frac{1}{20}$

$u = -20$ cm

Hence $d = 50 + 20 = 70$.

Multiple Correct Answer Type

1. b., d.

The image formed will be complete because light rays from all parts of the object will strike on the lower half. But since the upper half light rays are cut off, the intensity will reduce.

2. a., b.

In an astronomical telescope when the object and final image are at infinity. M and L are given as shown.

Angular magnification $M=\frac{f_o}{f_e}$

Separation between lenses $L=f_o+f_e$

$$\frac{f_o}{f_e}=5 \quad \text{or} \quad f_o=5f_o \qquad \text{(i)}$$

$$f_o+f_e=36 \quad \text{or} \quad 5f_e+f_e=36$$

or $\quad f_e = 6$ cm

$\therefore \quad f_o = 5fe$ or $f_o = 30$ cm

Hence $f_o = 30$ cm, $f_e = 6$ cm

3. a., b., c., d.

In case of an astronomical telescope, the distance between the objective lens and eyepiece lens $= f_0 + f_e = 16 + 0.02 = 06.02$ m.

The angular magnification $=\frac{f_{\text{objective}}}{f_{\text{eyepiece}}}=\frac{-16}{0.02}=-800$

The image seen by the astronomical telescope is inverted. Also, the objective lens is larger than the eyepiece lens.

4. b., c.

Concave lens and convex mirror are diverging in nature. Therefore, the refracted/reflected rays do not meet and are produced to make them meet. Therefore, the image formed is virtual and erect.

5. c., d.

For total internal reflection to take place: Angle of incidence, i > critical angle, θ_c $\left[\text{where } \sin\theta_c=\frac{1}{n}\right]$

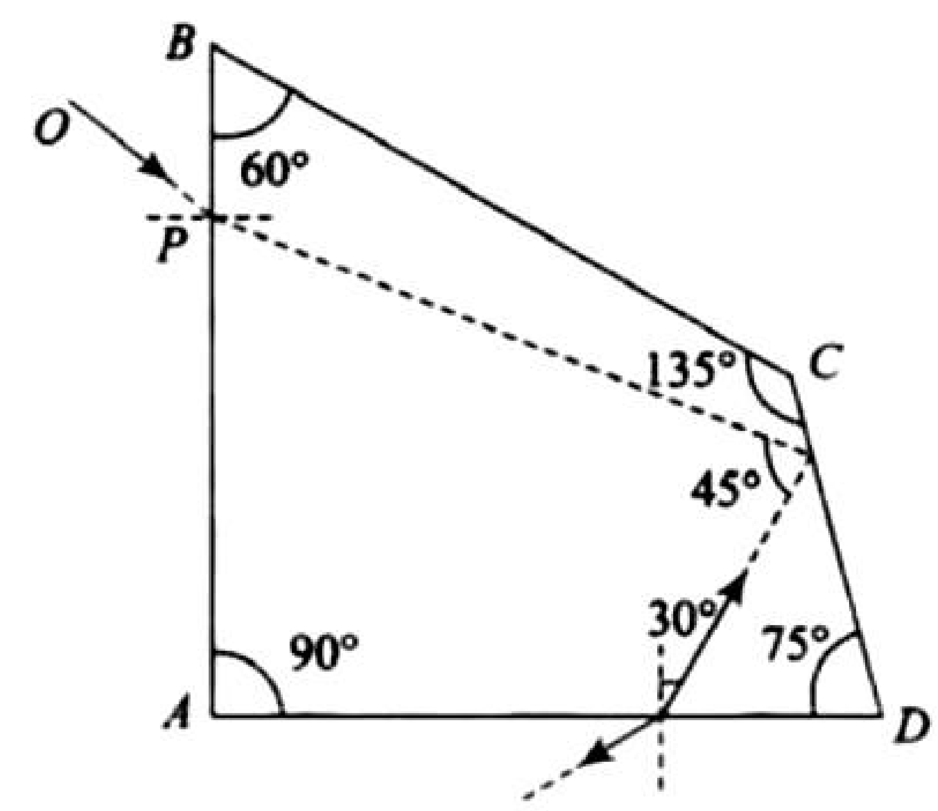
or $\quad \sin 45° > \frac{1}{n}$ or $\frac{1}{\sqrt{2}}>\frac{1}{n}$ or $> \sqrt{2}$ or $n > 1.414$

Therefore, possible values of n can be 1.5 or 1.6 in t he given options.

6. c., d.

$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$ (mirror formula) $f = -24$ cm

7. a., b., c.



Using Snell's law, $\sin^{-1}\frac{1}{\sqrt{3}}<\sin^{-1}\frac{1}{\sqrt{2}}$

Net deviation is 90°

8. a., c. Method 1:

$\frac{1.4}{v}-\frac{1}{\infty}=\frac{1.4-1}{R}$

$\frac{1.4}{v}=\frac{0.4}{R}; v=\frac{7R}{2}$

Film to glass: $\frac{1.5}{v'}-\frac{1.4}{\frac{7R}{2}}=\frac{1.5-1.4}{+R}$

$\frac{1.5}{v'}-\frac{0.4}{R}=\frac{0.1}{R}$

$\frac{1.5}{v'}=\frac{0.5}{R}; \quad v'=3R \Rightarrow f_1=3R$

Glass to film: $\frac{1.4}{v}-\frac{1.5}{\infty}=\frac{1.4-1.5}{-R}$

$\frac{1.4}{v}=\frac{-0.1}{-R}$

$v=-14R$

Film to air: $\frac{1}{v'}-\frac{1.4}{14R}=\frac{1-1.4}{-R}$

$\frac{1}{v'}-\frac{1}{10R}=\frac{+0.4}{+R}$

$\frac{1}{v'}=\frac{0.4}{R}+\frac{0.1}{R}=\frac{0.5}{R}$

$v'=\frac{R}{0.5}=2R \Rightarrow f_2=2R$

Method 2: For air to glass

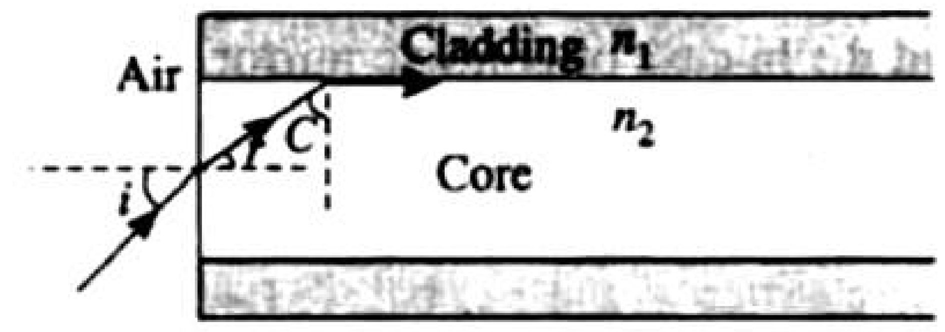
$\frac{1.5}{f_1}=\frac{1.4-1}{R}+\frac{1.5-1.4}{R} \quad \therefore \quad f_1=3R$

For glass to air.

$\frac{1}{f_2}=\frac{1.4-1.5}{-R}+\frac{1-1.4}{-R} \quad \therefore \quad f_2=2R$

Linked Comprehension Type

1. a., c. i_m is the angle of incidence for which $r = 90 - C$



$$\sin C = \frac{n_2}{n_1}$$

$\Rightarrow \quad n_0 \sin(\text{im}) = n_1 \sin(90 - C)$

$$\Rightarrow \quad N_A = \sin(i_m) = \frac{n_1 \cos(C)}{n_0} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

$$\text{For } S_1, NA_1 = \frac{\sqrt{\frac{45}{16} - \frac{9}{4}}}{n_0} = \frac{3}{4n_0}$$

$$\text{For } S_2, NA_2 = \frac{\sqrt{\frac{64}{25} - \frac{49}{25}}}{n_0} = \frac{\sqrt{15}}{5n_0}$$

$$\text{For (a) } NA_1 = \frac{3\times 3}{4\times 4} = \frac{9}{16}$$

$$NA_2 = \frac{\sqrt{15}\times 3\sqrt{15}}{5\times 16} = \frac{9}{16}$$

Hence (a) is correct.

$$\text{For (b) } NA_1 = \frac{3\times\sqrt{15}}{4\times 6} = \frac{\sqrt{15}}{8}$$

$$NA_2 = \frac{\sqrt{15}\times 3}{5\times 4} = \frac{\sqrt{15}\times 3}{20}$$

Hence (b) is incorrect

$$\text{For (c) } NA_1 = \frac{3}{4\times 1} = \frac{3}{4}$$

$$NA_2 = \frac{\sqrt{15}\times\sqrt{15}}{5\times 4} = \frac{3}{4}$$

Hence (c) is correct.

$$\text{For (d) } NA_1 = \frac{3}{4\times 1} = \frac{3}{4}$$

$$NA_2 = \frac{3}{4\times 4} = \frac{9}{16}$$

Hence (D) is incorrect.

2. d. It is given $NA_2 < NA_1$

$\Rightarrow \quad i_{m_1} > i_{m_2}$

Hence if the combination can be placed both ways i.e. 1st structure and then 2nd structure and then reversed also, then the condition of TIR is satisfied for lower i_m then it can be satisfied for all other less angler as well.

Hence NA_2 will be the numerical aperture of the combined structure.

Matching Column Type

1. i. → a.

More the radius of aperture more is the amount of light entering the telescope.

ii. → c. $M = \frac{f_0}{f_e}$ **iii. → c.** $L = f_0 + f_e$ **iv. → a., b., d.**

Depends on dispersion of lens, spherical aberration and radius of aperture.

2. i. → a., b., c., d.; ii. → b.; iii. → a., b., c., d.; iv. → a., b., c., d.

a., c., and d.: In case of concave mirror or convex lens, image can be real, virtual, diminished magnified or of same size.

b.: In case of convex mirror, image is always virtual (for real objects).

3. i. → a., c.; ii. → b., d., e.; iii. → a., c., e.; iv. → b., d.

a. $\mu_2 = \mu_3$, as there is no deviation
$\mu_1 < \mu_2$, as ray bends towards normal

b. $\mu_1 > \mu_2$,as ray bends away from normal
$\mu_2 > \mu_3$, as ray bends away from normal

c. $\mu_2 = \mu_3$, as there is no deviation
$\mu_1 < \mu_2$,as ray bends

d. $\mu_1 > \mu_2$, as ray bends away from normal
$\mu_2 > \mu_3$, as ray bends away from normal

e. $\mu_1 > \mu_2$, as ray bends away from normal
$\mu_2 = \mu_3$, as there is no deviation

4. p. → (2); q. → (3); r. → (4); s. → (1)

p. $\mu_2 > \mu_1$ ... (towards normal)
$\mu_2 > \mu_3$ ... (away from normal)

q. $\mu_1 = \mu_2$ ... (No change in path)
$\angle i = 0 \Rightarrow \angle r = 0$ on the block.

r. $\mu_1 > \mu_2$ ...(Away from the normal)
$\mu_2 > \mu_3$ ...(Away from the normal)

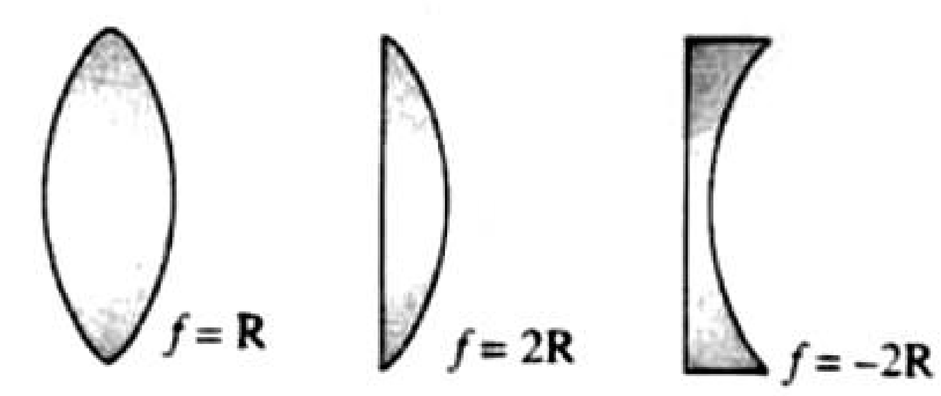
$$\mu_1 \times \frac{1}{\sqrt{2}} = \mu_2 \sin r \quad \Rightarrow \quad \sin r = -\frac{\mu_1}{\sqrt{2}\mu_2}.$$

Since $\sin r < 1 \Rightarrow \mu_1 < \sqrt{2}\mu_2$

s. For TIR: $45° > C \Rightarrow \sin 45° > \sin C$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \quad \Rightarrow \quad \mu_1 > \sqrt{2}\mu_2$$

5. b. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$



Use $\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$

(p) $\frac{1}{f_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}; f_{eq} = \frac{R}{2}$

(q) $\frac{1}{f_{eq}} = \frac{1}{2R} + \frac{1}{2R} = \frac{1}{R}; f_{eq} = R$

(r) $\frac{1}{f_{eq}} = -\frac{1}{2R} - \frac{1}{2R} = -\frac{1}{R}; f_{eq} = -R$

(s) $\frac{1}{f_{eq}} = \frac{1}{R} - \frac{1}{2R} = \frac{1}{2R}; f_{eq} = 2R$

Integer Answer Type

1. (6) $m = \frac{f}{f+u}$

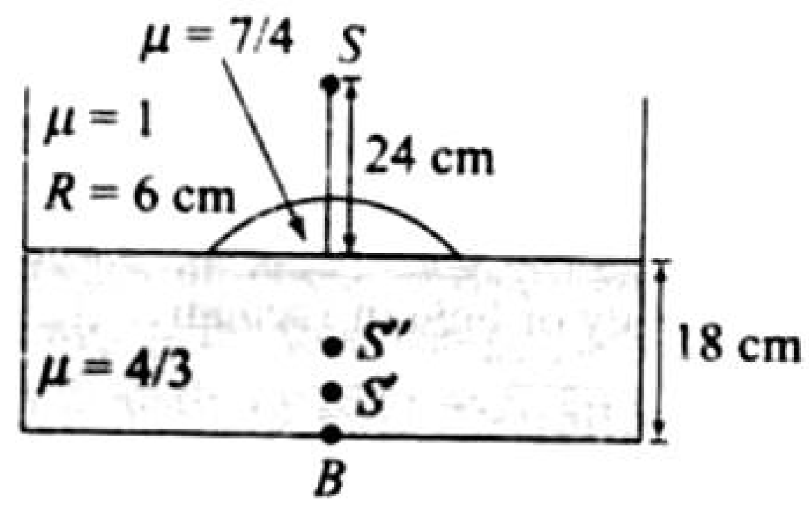
2. (6) $\sin\theta_C = 3/5$

$\therefore \quad R = 6$ cm.

3. (3) For $v_1 = 50/7$ m, $u_1 = -25$ m, $v_2 = 25/3$ m, $u_2 = -50$ m

Speed of object = $\frac{25}{30} \times \frac{18}{3}$ = 3 km h^{-1}

4. (2) $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



$$\frac{7}{4v} - \frac{1}{-24} = \frac{\frac{7}{4} - 1}{6}$$

$$\frac{7}{4v} = \frac{3}{24} - \frac{1}{24} = \frac{2}{24} = \frac{1}{12}$$

$$\frac{7 \times 2}{4} = V = 21 \text{ cm}$$

$$\frac{21}{OS''} = \frac{7/4}{4/3} \Rightarrow OS'' = 16$$

$\therefore \quad BS'' = 2$ cm

5. (7) In air: For mirror $\frac{1}{V} + \frac{1}{-15} = \frac{-1}{10}$

$$\frac{1}{V} = \frac{1}{15} - \frac{1}{10} = \frac{-1}{30}$$

$\therefore \quad V = -30$

$$|m_1| = \frac{30}{15} = 2$$

For lens: $u = -20$

$$\frac{1}{V_2} + \frac{1}{20} = \frac{1}{10}$$

$$\frac{1}{V_2} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$$

$V_2 = 20$ cm

$|m_2| = 1$

$\therefore \quad |M_1| = |m_1|\,|m_2| = 2$

After the medium is changed, by lensmaker's formula:

$$\frac{1}{F} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{10} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{F} = \left(\frac{3 \times 6}{2 \times 7} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Taking ratio, we get F as:

$$F = \frac{70}{4} = \frac{35}{2}$$

Image formation by mirror remains unaffected by changing medium. Applying distance formula with new focal length of lens,

$$\frac{1}{V'} + \frac{1}{20} = \frac{2}{35} \quad \therefore \quad \frac{1}{V'} = \frac{2}{35} - \frac{1}{20} = \frac{5}{20 \times 25}$$

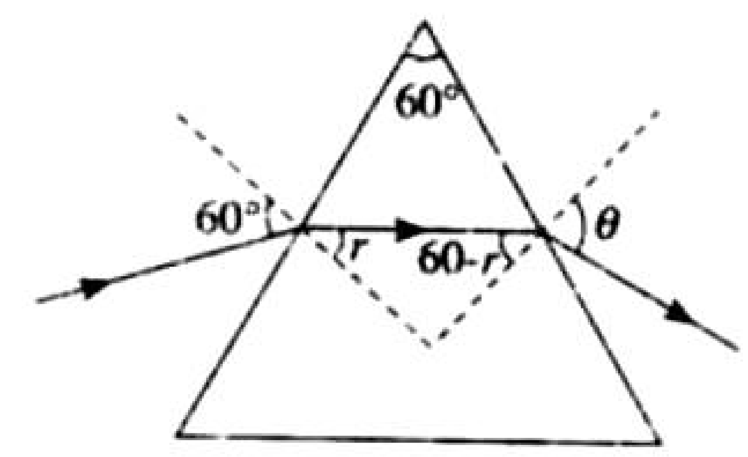
$V' = 140$

$$m_2 = \frac{140}{20} = 7$$

$\therefore \quad M_2 = 7 \times 2 = 14$ So, $\frac{M_2}{M_1} = \frac{14}{2} = 7$

6. (2) $\sin 60 = n \sin r$ (i)

$\sin \theta = n \sin (60 - r)$ (ii)



Differentiating equation (ii).

$$\cos\theta \frac{d\theta}{dt} = -n\cos(60 - r)\frac{dr}{dt} + \sin(60 - r)$$

Differentiating equation (i).

$$n\cos r \frac{dr}{dt} + \sin r = 0$$

$$\cos\theta \frac{d\theta}{dt} = -n\cos(60 - r)\left(\frac{-\tan r}{n}\right) + \sin(60 - r)$$

$$\frac{d\theta}{dt} = \frac{1}{\cos\theta}[+\cos(60 - r)\tan r + \sin(60 - r)]$$

$$\frac{d\theta}{dt} = \frac{1}{\cos 60}(\cos 30 \times \tan 30 + \sin 30)$$

$$= 2\left(\frac{1}{2} + \frac{1}{2}\right) = 2$$

Assertion-Reasoning Type

1. **c.** Laws of reflection can be applied to any type of surface.
2. **b.**

Fill in the Blanks Type

1. Since the beam leaves B as a parallel beam, therefore the effective focal length of the two lenses combination is ∞. For two lenses placed at some distance d apart, the focal length of the combination is given by the formula

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{\infty} = \frac{1}{+20} + \frac{1}{(-5)} - \frac{d}{(+20)(-5)} \quad \Rightarrow \quad d = +15 \text{ cm}$$

2. According to lensmaker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(i)}$$

Given refractive index of the lens, i.e.,

$${}^{a}_{g}\mu = 1.5 = \frac{\mu_g}{\mu_a}$$

Also, given refractive index of medium

$${}^{a}_{m}\mu = \frac{4}{3}\frac{\mu_m}{\mu_a}$$

$$\therefore \quad {}^{m}_{g}\mu = \frac{\mu_g}{\mu_m} = \frac{\mu_g}{\mu_a} \times \frac{\mu_a}{\mu_m} = \frac{1.5}{4/3} = 1.125$$

Applying Eq. (i) for the two cases, we get

$$\frac{1}{15} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

and $$\frac{1}{f_2} = \frac{1.5 - 1}{1.125 - 1} = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

On dividing, we get

$$\frac{f_2}{15} = \frac{1.5-1}{1.125-1} = \frac{0.5}{0.125} = 4 \Rightarrow f_2 = 60 \text{ cm}$$

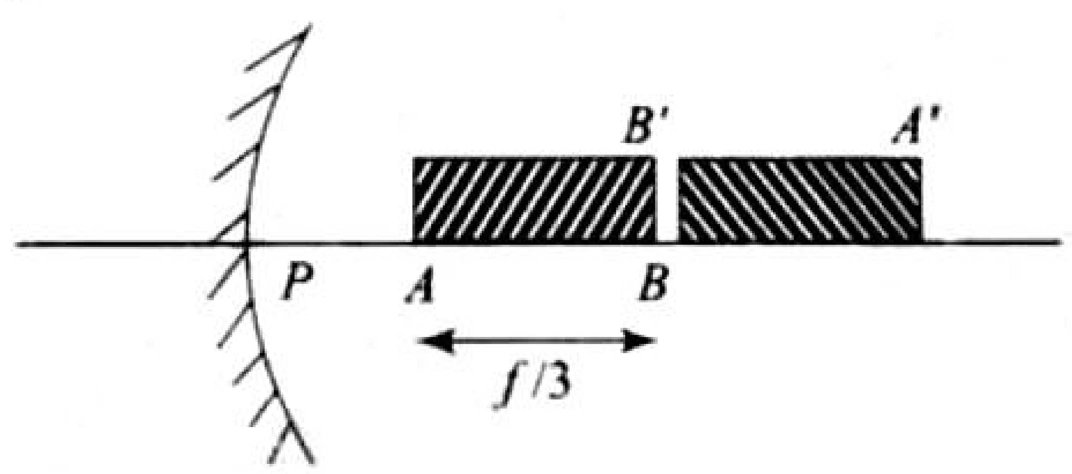
3. For refraction at APB,

$$-\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

$$\Rightarrow \quad \frac{-2}{-15} + \frac{1}{v} = \frac{1-2}{-10} \Rightarrow v' = -30 \text{ cm}$$

$\Rightarrow$ The image O' will be formed at 30 cm to the right at P.

4. Since the image formed is real and elongated, the situation is as shown in the figure. Since the image of B is formed at B' itself, therefore



B is situated at the center of curvature, that is, at a distance $2f$ from the pole.

$$\therefore \quad PA = 2f - \frac{f}{3} = \frac{5f}{3}$$

Let us find the image of A. For point A

$$u = -\frac{5f}{3}, v = ?$$

Applying $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$,

$$\Rightarrow \quad \frac{1}{\frac{-5f}{3}} + \frac{1}{v} = \frac{1}{-f}$$

$$\Rightarrow \quad \frac{1}{v} = -\frac{1}{f} + \frac{3}{5f} \Rightarrow \frac{1}{v} = \frac{-5+3}{5f} = \frac{-2}{5f} \Rightarrow v = -2.5f$$

$$\therefore \quad \text{Image length} = 2.5f - 2f = 0.5f = \frac{f}{2}$$

$$\therefore \quad \text{Magnification} = \frac{\frac{f}{2}}{\frac{f}{3}} = 1.5$$

5. For minimum deviation,

$$\mu = \frac{\sin\left(A + \frac{\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{60+30}{2}\right)}{\sin\left(\frac{60}{2}\right)} = \frac{\sin 45^\circ}{\sin 30^\circ} = \sqrt{2}$$

Since $\mu = \sqrt{2}$ (given for the prism), the condition is for minimum deviation. In this case, the ray inside the prism becomes parallel to the base. Therefore, the angle made by the ray inside the prism with the base of the prism is zero.

6. The resolving power of a magnification device is inversely proportional to the wavelength used. The resolving power of an electron microscope is higher than that of an optical microscope because the wavelength of electrons is smaller than the wavelength of visible light

7. We know that the velocity of light in vacuum,

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Also, the velocity of light in a medium, $v = \frac{1}{\sqrt{\mu\varepsilon}}$

$$\therefore \quad n = \frac{\text{Velocity of light in medium}}{\text{Velocity of light in vacuum}}$$

$$= \frac{v}{c} = \frac{1/\sqrt{\mu_0\varepsilon_0}}{1/\sqrt{\mu\varepsilon}} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0\varepsilon_0}}$$

8. Given that width of central maxima $= 2 \times 10^{-3}$ m

$$\therefore \quad \frac{2\lambda D}{d} = 2 \times 10^{-3}$$

$$\Rightarrow \quad d = \frac{2 \times 5.89 \times 10^{-7} \times 0.5}{2 \times 10^{-3}} = 2.945 \times 10^{-4} \text{ m}$$

9. Frequency remains the same, i.e.,

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{6000 \times 10^{-10}} = 5 \times 10^{14} \text{ Hz}$$

$$\mu = \frac{V_1}{V_2} = \frac{v\lambda_1}{v\lambda_2} \Rightarrow \lambda_2 = \frac{\lambda_1}{\mu}$$

The wavelength gets modified to

$$\lambda_2 = \frac{\lambda_1}{\mu} = \frac{6000 \text{ Å}}{1.5} = 4000 \text{ Å}$$

10. Let P_1 and P_2 be the powers of the two thin lenses, respectively. Power of the two lenses in contact $= P_1 + P_2$. Power of the two lenses at a distance $x = P_1 + P_2 - xP_1P_2$. From the given data, we get

$$P_1 + P_2 = 10 \text{ m}^{-1} \text{ and } P_1 + P_2 - (0.25)P_1P_2 = 6 \text{ m}^{-1}.$$

From these two expressions, we get

$$P_1P_2 = 16 \text{ m}^{-2} \text{ and } P_1 - P_2 = \sqrt{(P_1+P_2)^2 - 4P_1P_2}$$

$$= \sqrt{(10^{-1})^2 - 4(16^{-1})} = 6 \text{ m}^{-1}$$

Thus, $P_1 + P_2 = 10 \text{ m}^{-1}$ and $P_1 - P_2 = 6 \text{ m}^{-1}$, we get

$$P_1 = 8 \text{ m}^{-1} \text{ and } P_2 = 2 \text{ m}^{-1}$$

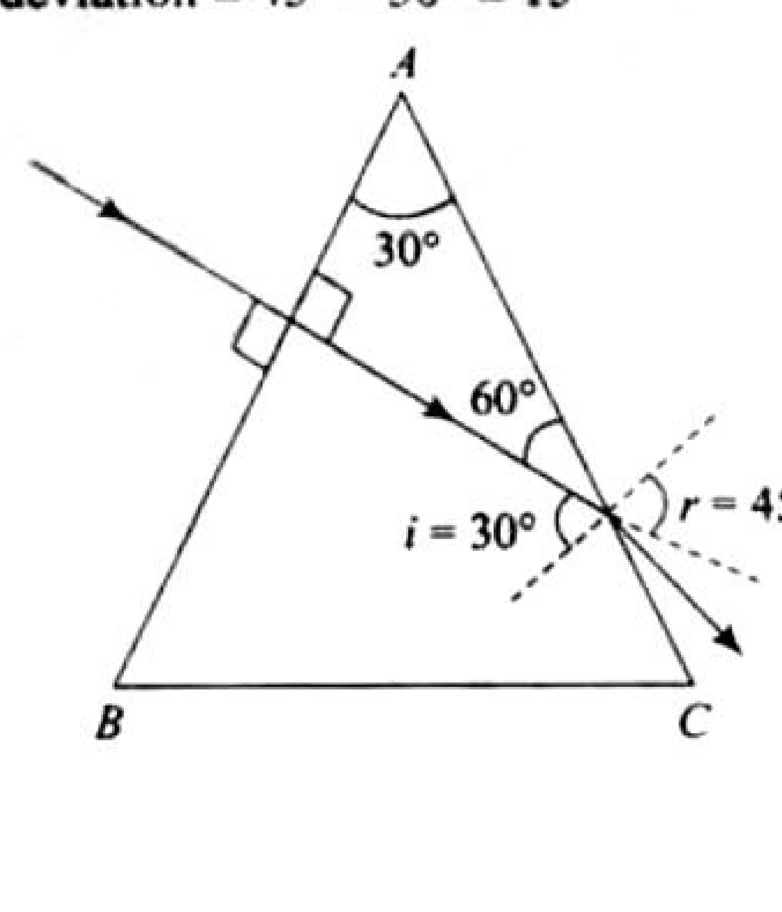
Hence, $f_1 = \frac{1}{P_1} = \frac{1}{8} \text{ m} = 0.125 \text{ m}$

and $\quad f_2 = \frac{2}{P_2} = \frac{1}{2} \text{ m} = 0.5 \text{ m}$

11. Using Snell's law for the refraction at AC, we get $\mu \sin i = (1)\sin r$ $\sqrt{2} \sin 30^\circ = \sin r$.

$$\sin r = \frac{1}{\sqrt{2}} \quad r = 45^\circ.$$

Angle of deviation $= 45^\circ - 30^\circ = 15^\circ$



True/False Type

1. **True.** This is due to atmospheric refraction. The light coming from sun bends toward the normal. Therefore, sun appears higher.
2. **True.** Intensity of light on a surface is the energy falling per second normally on unit area of the surface. The energy falling/second depends on energy density and the velocity of wave. Energy density is the total energy per unit surface area. As we move away from the cylindrical source of light, the surface area of cylindrical wavefront increases. The surface area of cylinder $\propto r$ (the distance from axis)

$\therefore$ Energy density $\propto \frac{1}{r}$

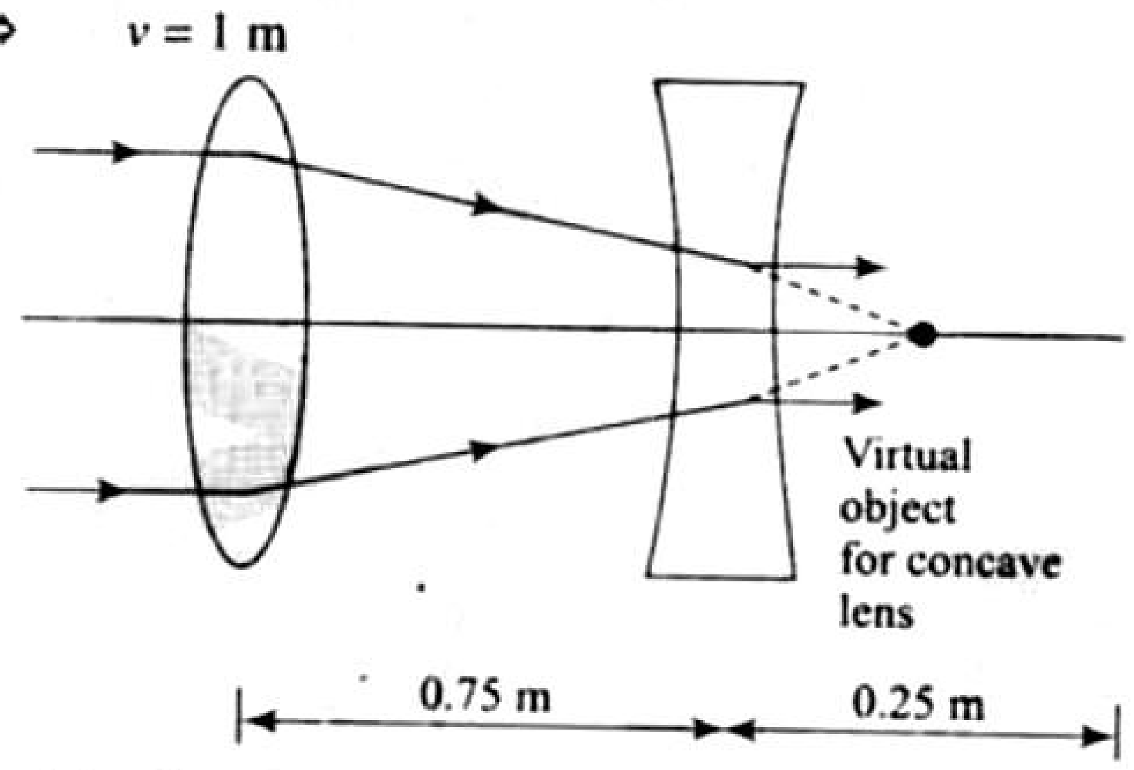
$\therefore$ Energy falling/second normally on unit area $\propto \frac{1}{r}$

$\therefore$ Intensity $\propto \frac{1}{r}$

3. **False.** Applying lens formula for convex lens,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}-\frac{1}{(-\infty)}=\frac{1}{1}$$

$\Rightarrow$ $v = 1$ m



Applying lens formula for concave lens,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}-\frac{1}{0.25}=\frac{1}{-0.25}$$

$$\therefore \quad \frac{1}{v}=\frac{1}{-0.25}+\frac{1}{0.25}=0 \Rightarrow v=\infty$$

The statement is false.

4. **True.** For the light to split, the material should have refractive index greater than 1 through which the light passes. Since the prism is hollow, we get no spectrum.
5. **True.** $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2} \Rightarrow \frac{1}{F}=\frac{1}{-15}+\frac{1}{30}$

$$=\frac{-2+1}{30} \Rightarrow F=-30 \text{ cm}.$$

This combination behaves as a concave lens of focal length 30 cm. Since $F_v < F$, therefore one sees colored pattern with violet color at the outer edge. The statement is true.

Subjective Type

1. Here, $R_1 = +12$ cm, $R_2 = \infty$, $\mu_p = 1.5$, $\mu_a = 1$

$\therefore$ focal length of the lens $\frac{1}{f_1}=(1.5-1)\left[\frac{1}{12}-\frac{1}{\infty}\right]$

or $f_1 = 24$ cm

$\therefore$ the plane side of the lens is silvered, focal length of the mirror

$$f_m=\frac{R_2}{2}=\frac{\infty}{2}=\infty$$

$\therefore$ Effective focal length of the silvered lens is

$$\frac{1}{f_e}=\frac{1}{f_m}-\frac{2}{f_1} \text{ or } f_e=-12 \text{ cm}$$

The object is placed 24 cm in front of the lens.

$u = -24$ cm, $f = -12$ cm

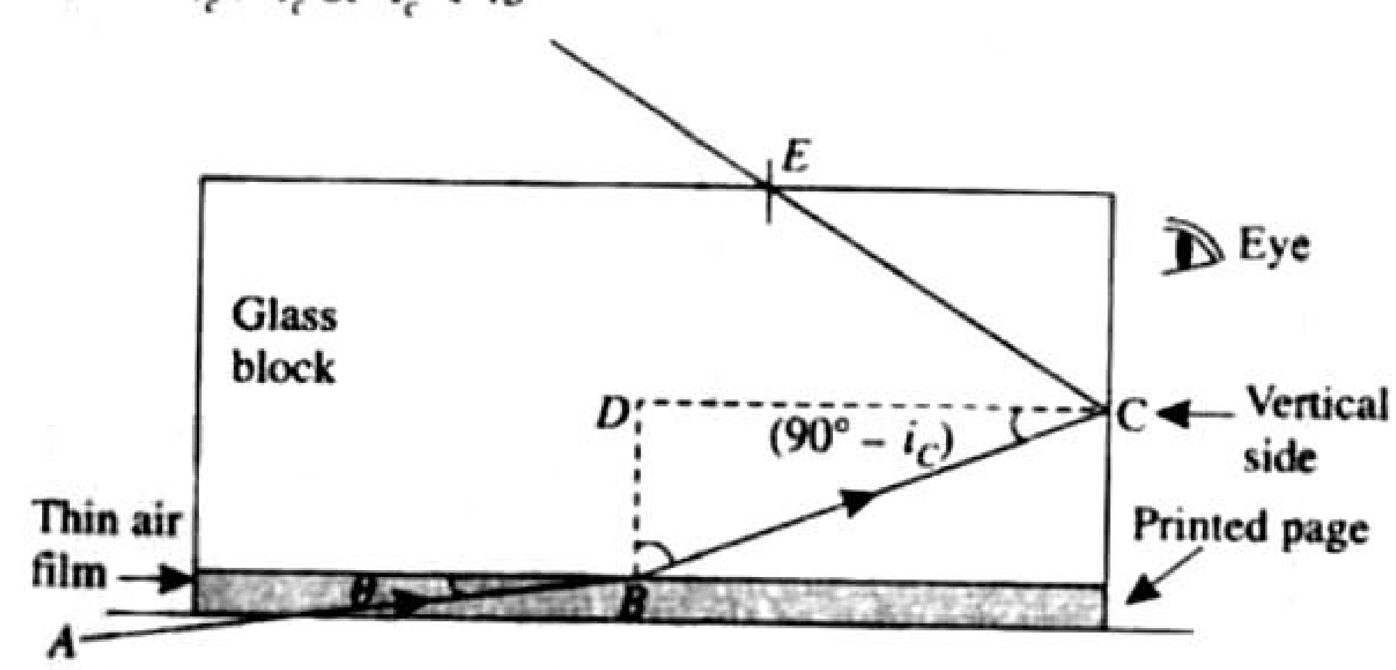
And from the mirror equation (i), we have

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \qquad \text{(i)}$$

or $v = -24$ cm

The final image is formed on the object itself. The behavior is like that of a concave mirror.

2. **a.** $\mu_1 < \mu_2$, because it is behaving like divergence lens.
   b. $\mu_1 = \mu_2$ because no refraction is taking place.
3. When the block is placed on the printed page, a thin air film (shown dotted) is trapped between the block and the page. Looking through the vertical side of the block, the eye will receive rays emerging from this side of the block, if the ray AB starting from a letter A on the printed page falls on the block at grazing incidence, i.e., $\theta \simeq 0°$, in which case the angle of refraction in the block which is angle i_c will be equal to the critical angle for air and glass. The ray BC will not merge from the vertical side if the refracted ray BC falls on the vertical side at an angle of incidence greater than the critical angle, i.e., $\angle BCD$ is greater than i_c. But $\angle BCD = 90° - i_c$. Hence the letter on the printed page will not be visible from the vertical face of the block if the BC suffers total internal reflection at C, i.e., if $90° - i_c > i_c$ or $i_c < 45°$



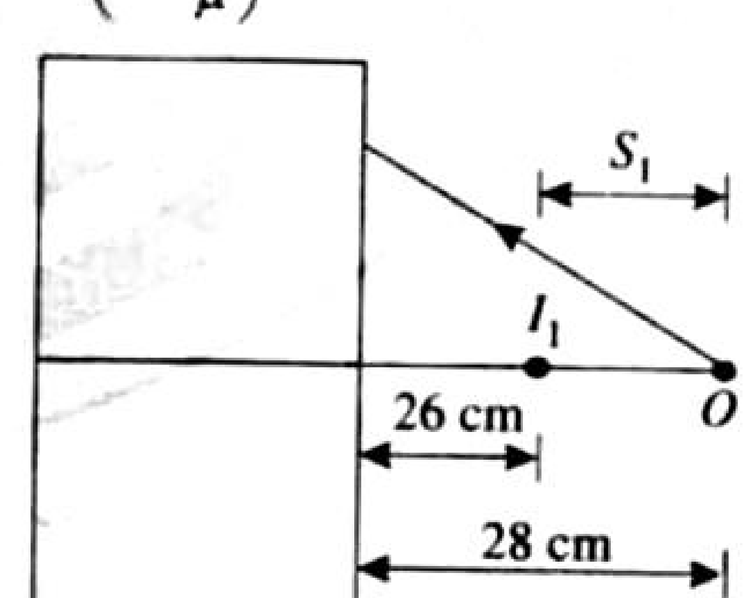
Thus the maximum value of $(i_c)_{max} = 45°$. Hence the minimum value of refractive is

$$n_{min}=\frac{1}{\sin(i_c)_{max}}=\frac{1}{\sin 45°}=\sqrt{2}=1.414$$

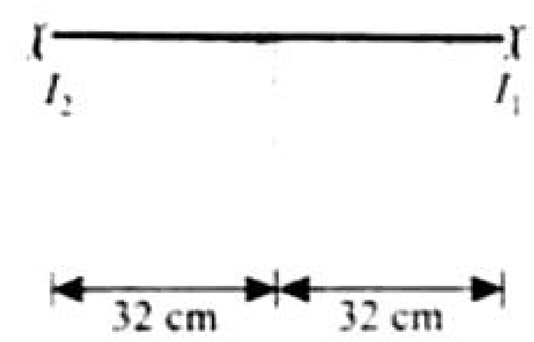
4. **Method 1:**

A ray of light from the object first encounters a glass slab, then a mirror and finally a glass-slab again.

Glass slab: A slab simply shifts the object along the axis by a distance $s_1 = t\left(1-\frac{1}{\mu}\right) = 2$ cm



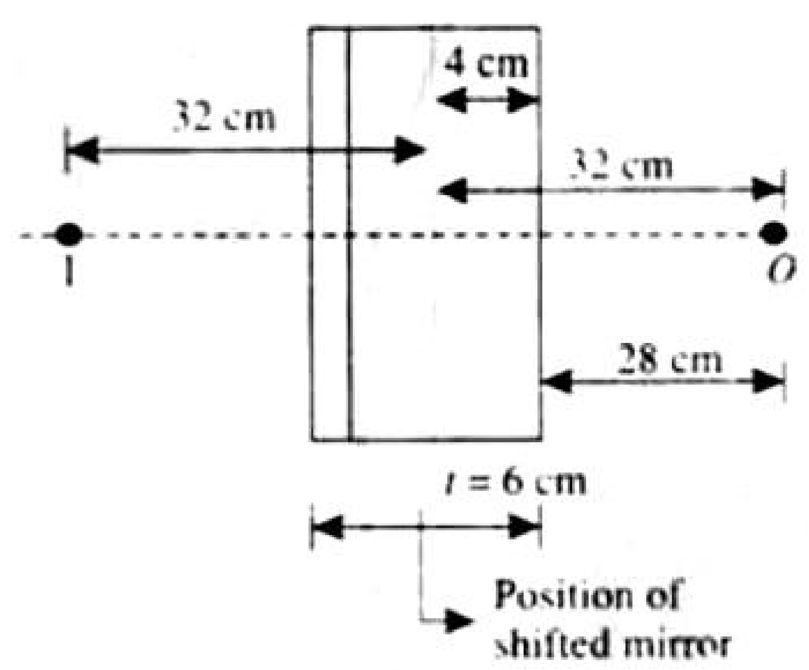
Direction of shift of object is towards left. Therefore the object appears to be at I_1 which is 28 – 2 = 26 cm from the slab.
For mirror, the object for the mirror is the image I_1 formed after shift due to the slab.



Therefore object distance from the mirror is 26 + 6 = 32 cm.
The image will now be formed 32 cm behind the mirror.
Now reflected rays are travelling from left to right.
The ray now travels through the slab again but this time from right to left. Therefore it is shifted again by a distance of 2 cm, but towards the right. Thus final position of the image is 32 – 2 = 30 cm behind the mirror.
Hence final image is formed 30 cm behind the mirror.

Method 2: Shifting of mirror

By the principle of reversiblility of light we can say if light rays are coming from mirror and passes through slab. The mirror will shift 2 m towards right for observer in front of the slab. The position of the object from shifted mirror = 32 cm.



Hence the position of the image formed by shifted mirror will be 32 cm behind it. Hence position of the image from surface 2 is 30 cm left to it and 36 cm left of surface 1.

5. Given $u = -200$ cm, $f = 50$ cm
For image I_1 of object formed by objective lens.

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

We have

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{50} + \frac{1}{200} = \frac{4-1}{200} = \frac{3}{200}$$

$$\Rightarrow \quad v = +\frac{200}{3} \text{ cm}$$

Also, magnification produced by objective lens

$$m_o = \frac{v}{u} = -\frac{200/3}{200} = \frac{1}{3}$$

Image I_1 acts as an object for eye lens.
Here, $v = -25$ cm, $f = 5$ cm

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = -\frac{1}{25} - \frac{1}{5} = -\frac{1+5}{25}$$

$$\therefore \quad u = -\frac{25}{6} \text{ cm}$$

And magnification produced by eye lens.

$$m_e = \frac{v}{u} = \frac{-25}{(-25/6)} = 6$$

a. The separation between objective and eyepiece

$$= |V| + |u| = \frac{200}{3} + \frac{25}{6} = \frac{425}{6} = 70.73 \text{ cm}$$

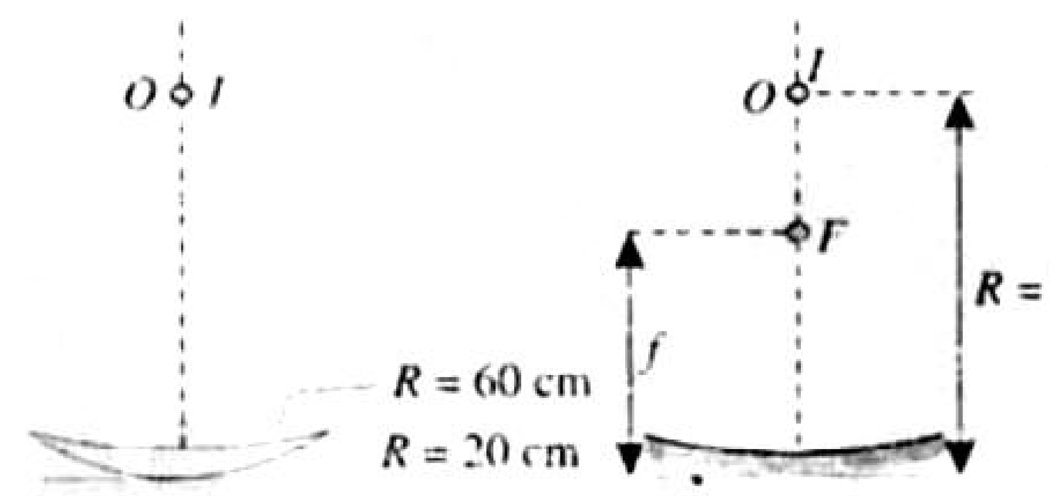
b. Magnification produced, $m = m_0 \times m_e = -\frac{1}{3} \times 6 = -2$

The negative sign shows that the final image is inverted.

6. a. The optical arrangement is equivalent to the concave mirror of focal length F given by

$$\frac{1}{F} = \frac{1}{f_g} + \frac{1}{f_m} + \frac{1}{f_g}$$

where f_g is the focal length of the lens without silvering and f_m is the focal length of the mirror.



$$\frac{1}{f_g} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5-1)\left(\frac{1}{20} - \frac{1}{+60}\right) = \frac{1}{60} \Rightarrow f_g = 60 \text{ cm}$$

$$f_g = R_1/2 = 20/2 = 10 \text{ cm}$$

$$\frac{1}{F} = \frac{1}{60} + \frac{1}{10} + \frac{1}{60} = \frac{8}{60} \Rightarrow F = \frac{60}{8} = 7.5 \text{ cm}$$

For the image to be formed at the place of the object, $X = R = 2F = 7.5 \times 2 = 15$ cm

b. Method 1:

When the concave part is filled with water of refractive index 4/3, the optical arrangement is equivalent to the concave mirror of focal length F such that

$$\frac{1}{F} = \frac{1}{f_w} + \frac{1}{f_g} + \frac{1}{f_m} + \frac{1}{f_g} + \frac{1}{f_w}$$

$$\frac{1}{f_w} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{60} - \frac{1}{\infty}\right) = \frac{1}{180} \Rightarrow f_w = 180 \text{ cm}$$

$f_g = 60$ cm (calculated earlier)

$$\frac{1}{F} = \frac{1}{180} + \frac{1}{60} + \frac{1}{10} + \frac{1}{60} + \frac{1}{180} = \frac{26}{180}$$

$$F = \frac{180}{26} \text{ cm}$$

$$x_1 = R = 2F = \frac{180}{26} \times 2 = \frac{180}{13} = 13.85 \text{ cm}$$

$$\Delta x = 15.0 - 13.85 = 1.15 \text{ cm}$$

Method 2:
We use the equation

$$\frac{n_2}{x_2}-\frac{n_1}{x_1}=\frac{n_2-n_1}{R}$$

For refraction at the interface '1' (air water),

$$\frac{4/3}{x_2}-\frac{1}{x_1}=\frac{4/3-1}{\infty} \quad \text{(i)}$$

The image of interface '1' is the object for the interface '2'.

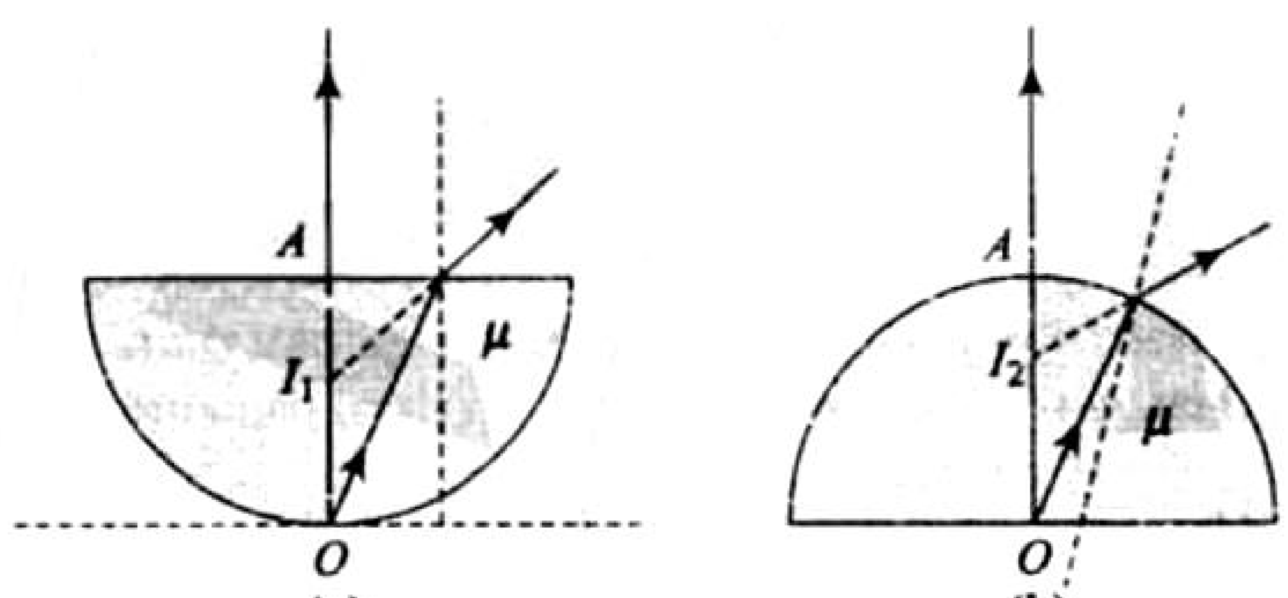
$$\frac{1.5}{+20}-\frac{1}{x_1}=\frac{1.5-4/3}{+60}$$

$$x_1=\frac{360}{26}=13.85$$

$\Delta x = 15.0 - 13.85 = 1.15$ cm

7. When the curved surface of the lens (refractive index μ) is in contact with the table, the image of the bottom-most point of lens (in glass) is formed due to refraction at plane face. The image of O appears at I_1.
Here, $u_1 = AO = -4$ cm, $v_1 = AI_1 = 3$ cm,
$\mu_1 = \mu$, and $\mu_2 = 1$, and $R_1 = \infty$

$$\therefore \quad \frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R_1} \text{ gives } \frac{1}{-3}-\frac{\mu}{-4}=\frac{1-\mu}{\infty} \quad \text{(i)}$$



(a) (b)

When the plane surface of the lens is in contact with the table, the image of center of the plane face is formed due to refraction at curved surface. The image of O is formed at I_2.
Here, $u = AO = -4$ cm, $v = AI_2 = -25/8$ cm
$\mu_1 = \mu$, $\mu_2 = 1$, and $R_2 = -R$

$$\therefore \quad \frac{\mu_2}{v_2}-\frac{\mu_1}{\mu_2}=\frac{\mu_2-\mu_1}{R_2}$$

Gives $\dfrac{1}{\left(-\frac{25}{8}\right)}-\dfrac{\mu}{-4}=\dfrac{1-\mu}{-R}$

From Eq. (i), $\mu = 4/3$, therefore this equation gives

$$-\frac{8}{25}+\frac{4/3}{4}=-\frac{\left(1-\frac{4}{3}\right)}{R} \quad -\frac{8}{25}+\frac{1}{3}=\frac{1}{3R} \text{ or } \frac{1}{75}=\frac{1}{3R}$$

This gives $R = 25$ cm.
The focal length (f) of plano-convex lens ($R_1 = R$ and $R_2 = \infty$) is

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)=\frac{\mu-1}{R}=\frac{\frac{4}{3}-1}{25}=\frac{1}{75}$$

$\Rightarrow \quad f = 75$ cm

8. We will use the symbol $\leq$ to mean 'infinitesimally greater than'. When the slab is not inserted,

$$\theta \leq \theta_c = \sin^{-1}(\mu_1/\mu_2) \text{ or } \sin\theta \geq \mu_1/\mu_2$$

When the slab is inserted, we have two cases

$$\mu_3 \leq \mu_1 \text{ and } \mu_3 > \mu_1.$$

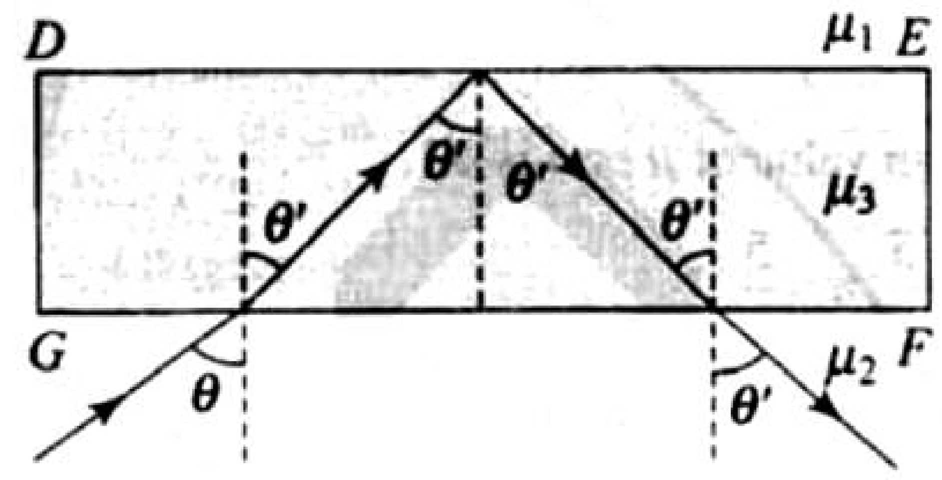
Case I: $\mu_3 < \mu_1$. We have $\sin\theta \geq \mu_1/\mu_2 \geq \mu_3/\mu_2$

Thus, the light is incident on AB at an angle greater than the critical angle $\sin^{-1}(\mu_3/\mu_2)$. It suffers total internal reflection and goes back to medium II.

Case II: $\mu_3 > \mu_1$

$$\sin\theta \geq \mu_1/\mu_2 < \mu_3/\mu_2$$

Thus, the angle of incidence θ may be smaller than the critical angle $\sin^{-1}(\mu_3/\mu_2)$ and hence it may enter medium III. The angle of refraction θ' is given by (figure).



$$\frac{\sin\theta}{\sin\theta'}=\frac{\mu_3}{\mu_2} \quad \text{(i)}$$

$$\Rightarrow \quad \sin\theta'=\frac{\mu_2}{\mu_3}\sin\theta \leq \frac{\mu_2}{\mu_3}\cdot\frac{\mu_1}{\mu_2}$$

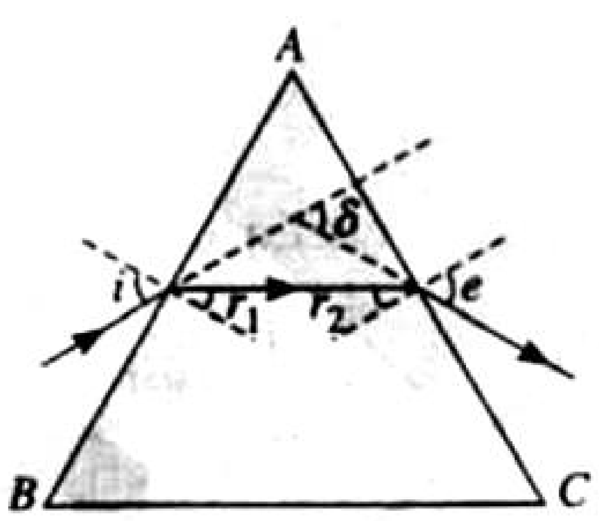
Thus, $\sin\theta' \geq \dfrac{\mu_1}{\mu_3} \quad \Rightarrow \quad \theta' \geq \sin^{-1}\left(\dfrac{\mu_1}{\mu_3}\right)$ (ii)

As the slab has parallel faces, the angle of refraction at the face FG is equal to the angle of incidence at the face DE. Equation (ii) shows that this angle is infinitesimally greater than the critical angle here. Hence, the light suffers total internal reflection and falls at the surface FG at an angle of incidence θ'. At this face, it will refract into medium II and the angle of refraction will be θ as shown by Eq. (i). Thus, the total light energy is ultimately reflected back into medium II.

9. Given $i = 60°$, $\delta = 30°$ and $A = 30°$. We have

$$\delta = i + e - A \quad \text{(i)}$$

From equation (i), we get $30° = 60° + e - 30°$ or $e = 0$
Here also $i = 60°$ and $\delta = 30°$.

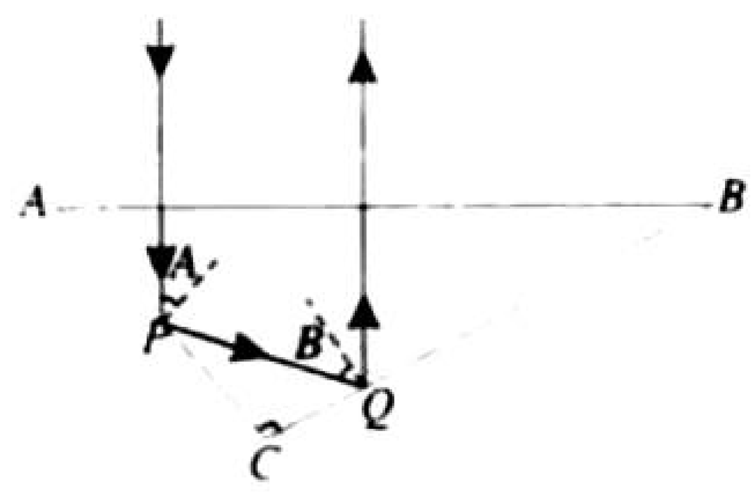


Therefore, $r_1 = i - \delta = 60° - 30° = 30°$. Hence

$$\mu=\frac{\sin i}{\sin r_1}=\frac{\sin 60°}{\sin 30°}=\sqrt{3}$$

Hence the value of $a = 3$.

10. a. At P, angle of incidence $i_A = A$
If TIR satisfies for the smaller angle of incidence than for larger angle of incidence is simultaneously satisfied.



$B \le A \quad \therefore i_B \le i_A$

Maximum value of B can be 45°. Therefore, if condition of TIR is satisfied, then condition of TIR will be satisfied for all value of i_A and i_B.

Thus, $45 \ge \theta_c$

$\sin 45° \ge \theta_c$

$$\frac{1}{\sqrt{2}} \ge \frac{1}{\mu} \Rightarrow \mu \ge \sqrt{2}$$

Minimum value of μ or n is $\sqrt{2}$.

b. For $n = \frac{5}{3}, n = \frac{5}{3}$

If $B = 30°$, then $i_B = 30°$

Then $A = 60°$ or $i_A = 60°$

$i_A < \theta_c$ but $i_B < \theta_c$

i.e., TIR will take placed at A but not at B.

or we write: $\sin i_B < \sin i_C < \sin i_A$

$$\sin 30° < \frac{3}{5} < \sin 60° \quad \text{or} \quad 0.5 < 0.6 < 0.86$$

11. a. For refraction at spherical surface,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \qquad \text{(i)}$$

For refraction at the first surface,

$\mu_2 = 1, \mu_1 = 4/3, u = \infty, R_1 = +2$ mm, $v = v'$ (say)

Position of image due to refraction at the first surface is given by

$$\frac{1}{v'} - \frac{4/3}{\infty} = \frac{1-(4/3)}{2}$$

This given $v' = -6$ mm

That is the image is formed at a distance of 6 mm to the left of the first surface.

For refraction at the second surface,

$u' = u = -(6+4) = -10$ mm, $\mu_1 = 1, \mu_2 = 4/3$

$R_2 = -2$ mm

Substituting these values in Eq. (i), we get

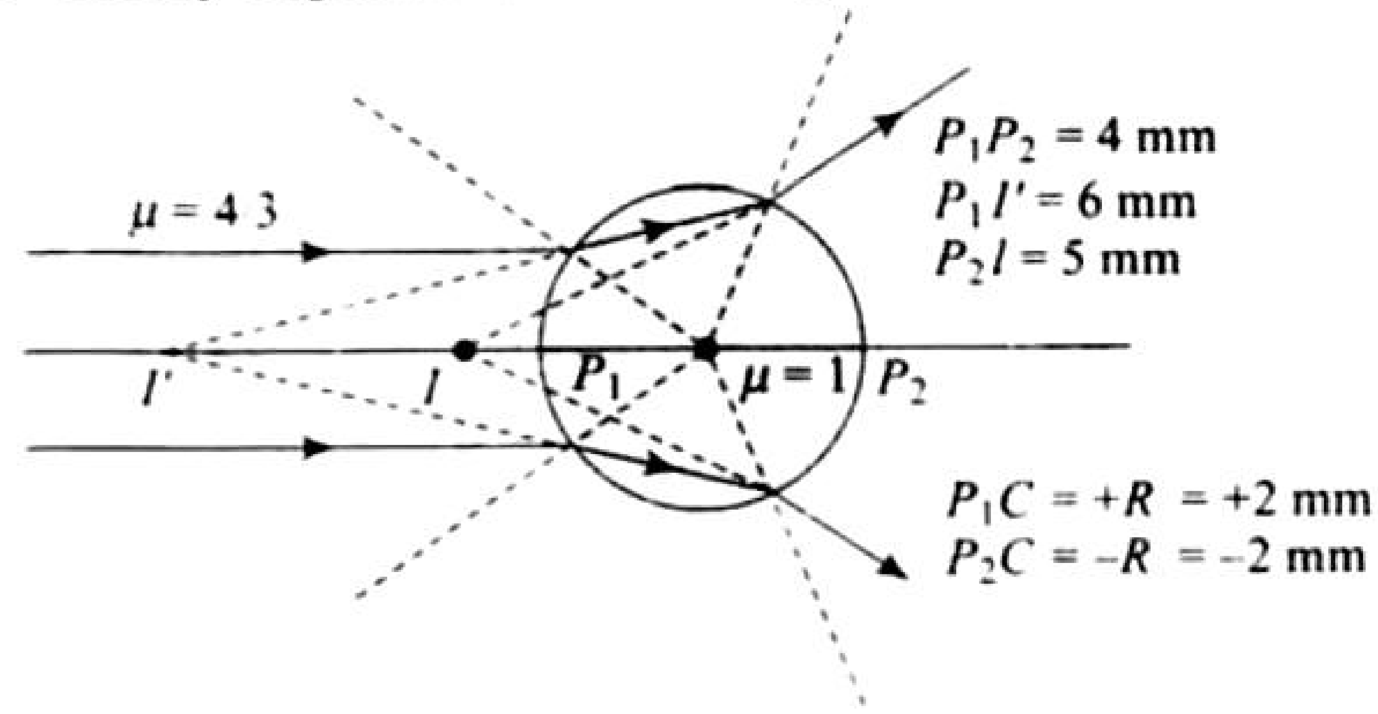
$$\frac{(4/3)}{v} - \frac{1}{(-10)} = \frac{\frac{4}{3}-1}{(-2)}$$

$$\Rightarrow \frac{(4/3)}{v} - \frac{1}{6} - \frac{1}{10} = \frac{-10-6}{60}$$

$$\Rightarrow v = -\frac{60 \times \left(\frac{4}{3}\right)}{16} = -5 \text{ mm}$$

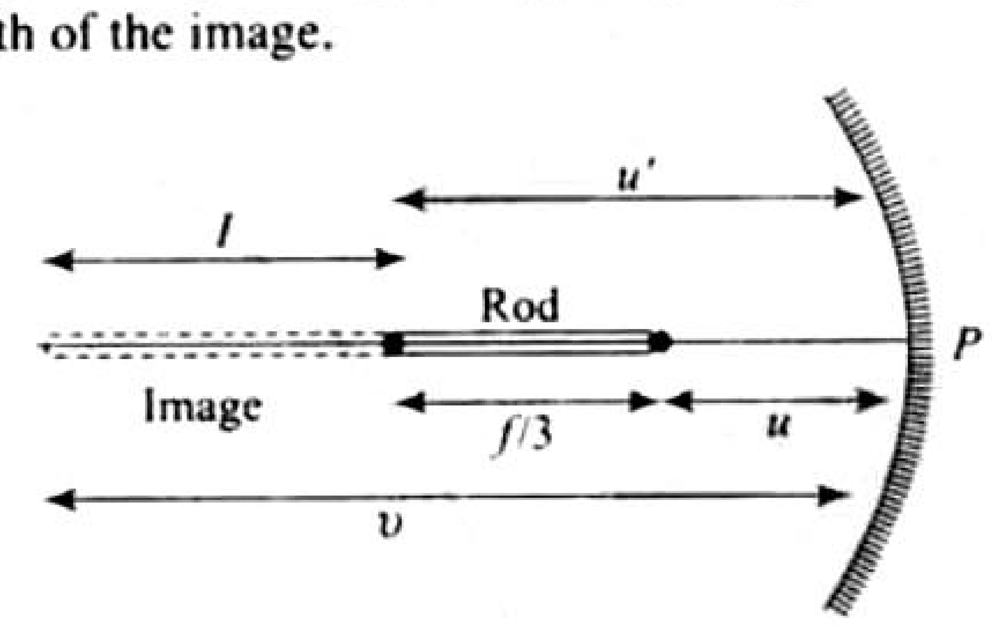
The final image I is at a distance of 5 mm to the left of the second surface.

b. The ray diagram is shown in the figure



I' is the virtual image formed by the first surface and the final image is virtual and is formed at I.

12. As in question, image touches the rod, i.e., image and object coincides, hence one end of the rod should be at the center of curvature. It is also written that image is enlarged, it indicates that the orientation of rod should be toward focus then only we can get enlarged image along the principal axis. Let l be the length of the image.



Then, $m = \frac{l}{f/3} \Rightarrow l = \frac{mf}{3}$

Also, one end of the image coincides with the object, $u' = 2f$.

Now, $u' = u + \frac{f}{3} \Rightarrow u = 2f - \frac{f}{3} = \frac{5f}{3}$

$$v = -\left(u + \frac{f}{3} + \frac{mf}{3}\right).$$

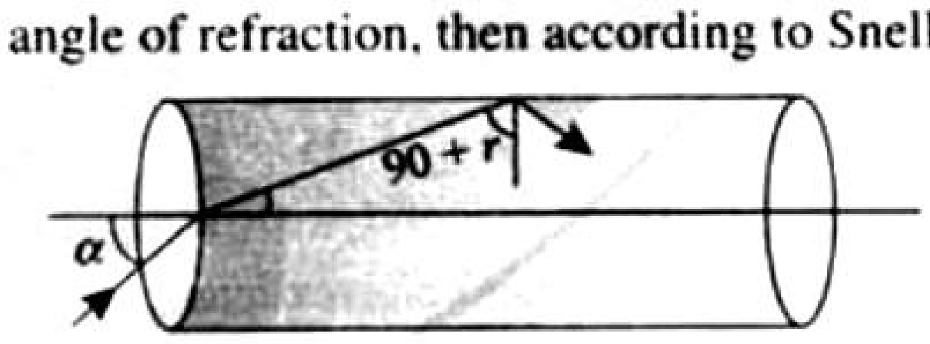
Putting in mirror formula, we get

$$\frac{1}{u + f/3 + mf/3} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{3}{5f + f + mf} + \frac{3}{5f} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{m+6} = \frac{2}{15} \Rightarrow m = \frac{3}{2}$$

13. Let α be angle of incidence on a plane face of cylindrical rod. if r is angle of refraction, then according to Snell's law



$$n = \frac{\sin\alpha}{\sin r} \text{ or } \sin r = \frac{\sin\alpha}{n} \qquad \text{(i)}$$

The angle of incidence at curved surfacter is $(90° = r)$. The ray is now passing from denser to rarer medium, therefore for no $(90° - r) \ge C$, where C is critical angle

$\text{Sin}(90° - r) \ge \sin C$

$\cos r \geq \sin C$

$\sqrt{(1-\sin^2 r)} \geq \sin C$

$1 - \dfrac{\sin^2 \alpha}{n^2} \geq \sin^2 C$ [using (i)]

$A \sin C = \dfrac{1}{n}$ $\therefore$ $1 - \dfrac{\sin^2 \alpha}{n^2} \geq \dfrac{1}{n^2}$

This gives $\left(\dfrac{1}{n^2} + \dfrac{\sin^2}{n^2}\right) \leq 1$

$\dfrac{1}{n^2}$ $[1 \sin^2 \theta] \leq 1$ or $n^2 \geq (1 + \sin^2 \alpha)$.

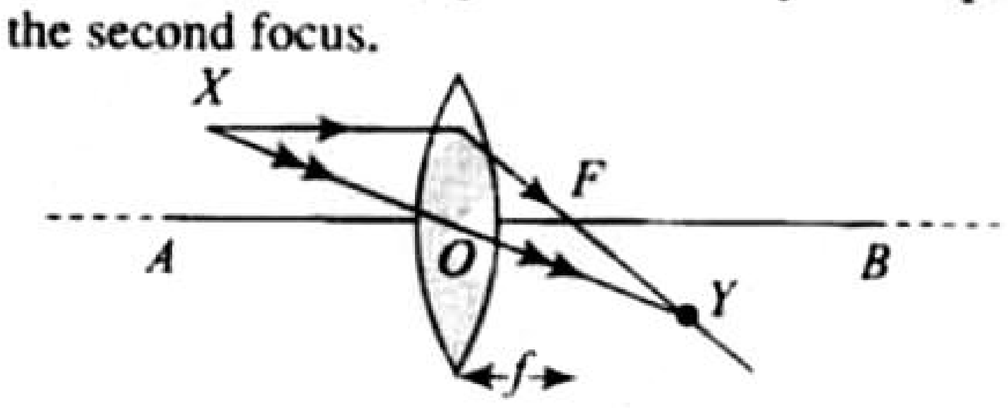
Maximum value of $\sin^2 \alpha = 1$

$\therefore$ $n^2 \geq 2$ or $\sqrt{2}$

Hence $n_{min} = \sqrt{2}$

14. a. When a convex lens is used to form image of X at Y:

i. A ray starting from X parallel to the optic axis passes through the second focus.

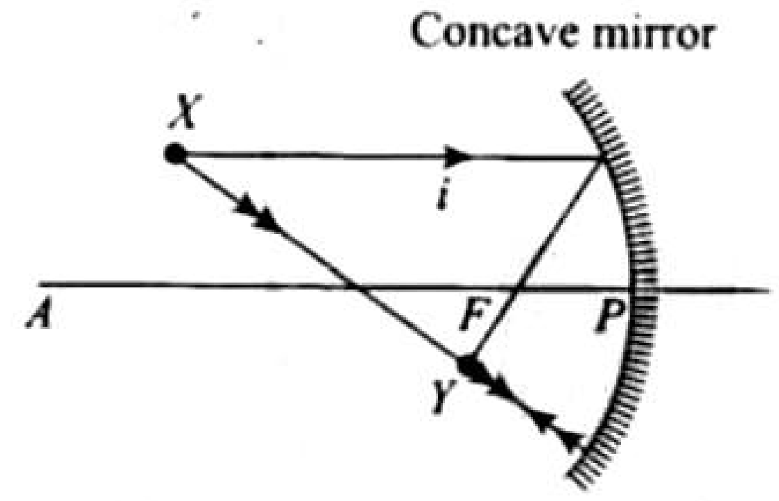


ii. A rays starting from X directed toward the optic axis passes undeviated.

Both the rays meet at Y, which is the real image of X.

b. When a concave mirror is used to form image of X and Y:

i. A ray starting from X parallel to the optic axis passes through principal focus F.



ii. A ray starting from X, directed toward the center of curvature C, falls normally on the mirror and retraces its path.

Both the rays meet at Y, which is the real image of X.

15. a. If i is the angle of incidence at $B(x, y)$, then slope of trajectory at B,

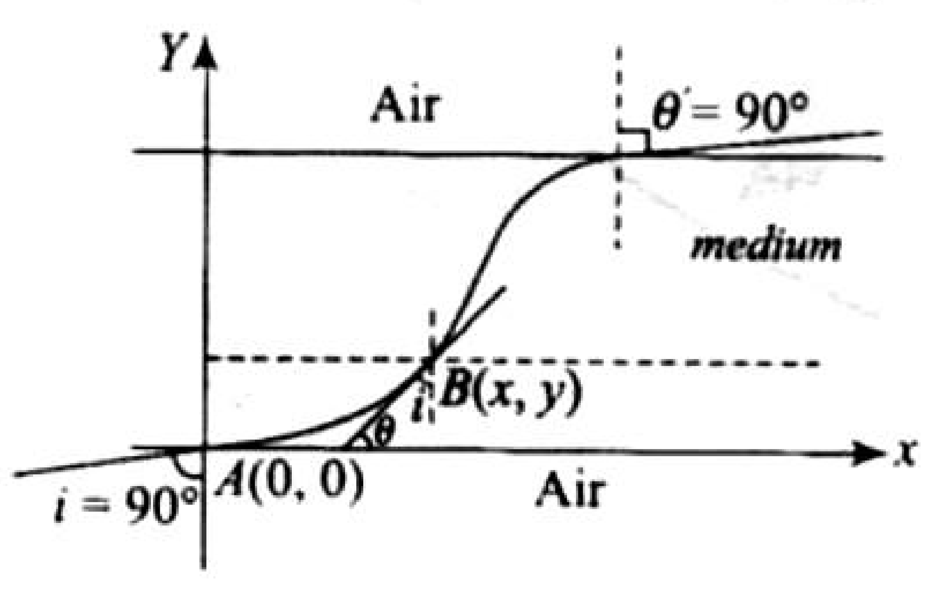
$$dy/dx = \tan \theta = \tan (90° - i) = \cot i \quad \text{(i)}$$

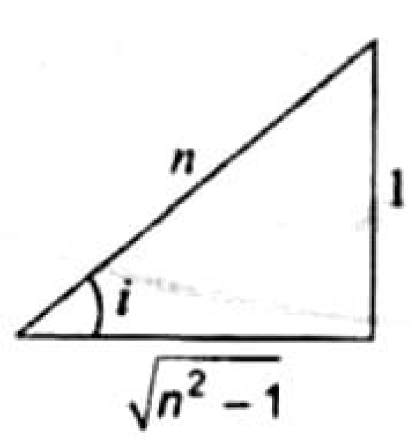
b. From Snell's law, $n \sin i$ = constant C.

From Snell's law at $A(0, 0)$,

$n \sin i = 1 \times \sin 90° = 1 \Rightarrow n \sin i = 1$

$\sin i = 1/n$ or $i = \sin^{-1}(1/n)$





$\therefore$ $\cot i = \dfrac{\cos i}{\sin i} = \dfrac{\sqrt{(1-1/n)^2}}{(1/n)} = \sqrt{n^2 - 1}$

From Eq. (i),

or $n^2 \sin^2 i$ or $n^2 \dfrac{1}{1 + \cot^2 i} = 1$

or $\dfrac{n^2}{1 + (dy/dx)^2} = 1$

Given $n = [ky^{3/2} + 1]^{1/2} \Rightarrow n^2 = ky^{3/2} + 1$

$\therefore$ $\dfrac{ky^{3/2} + 1}{1 + (dy/dx)^2} = 1 \Rightarrow ky^{3/2} + 1 = 1 + \left(\dfrac{dy}{dx}\right)^2$

or $(dy/dx)^2 = ky^{3/2} \Rightarrow dy/dx - k^{1/2}y^{3/4}$

or $dy/y^{3/4} = k^{1/2}\, dx$.

Integrating, we get $4y^{1/4} = k^{1/2} x + C$

where c is constant of integration.

At $x = 0, y = 0 \Rightarrow C = 0$.

$\therefore$ $4y^{1/4} = k^{1/2}x$

As $k = 1.0$ (given)

$\therefore$ $y^{1/4} = (1/4)x$ (iii)

This is the required equation of trajectory.

c. At $y = 1.0$ m, Eq. (ii) gives $x = 4$ m.

$\therefore$ $B(x_1, y_1) = P(4, 1)$

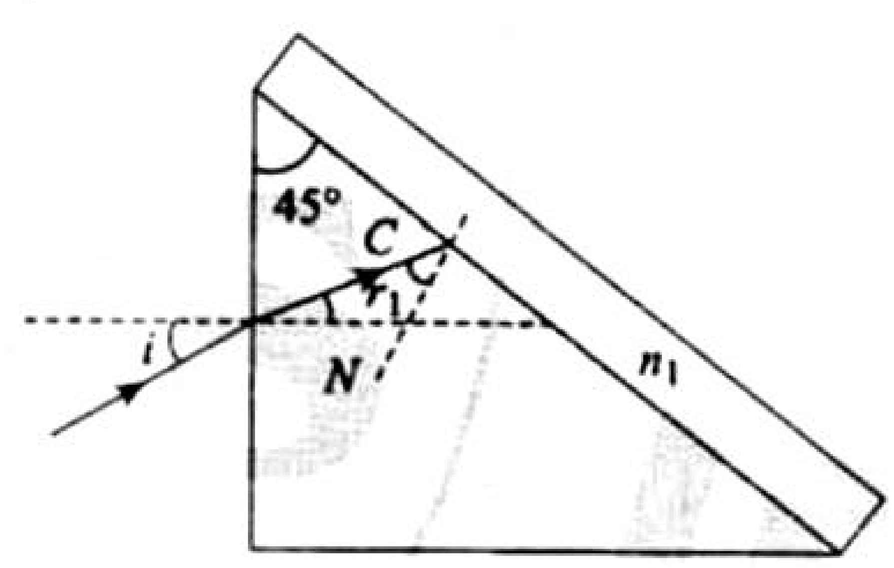
d. The path of the ray subsequently will be the grazing angle of emergence since

$n \sin e = 1$ or $1 \sin e = 1 \Rightarrow e = 90°$.

16. a. $\sin C = \dfrac{n_1}{n}$

From the figure, $(90° - r_1) + 45° + (90° - C) = 180°$

$\Rightarrow$ $r_1 = 45° - C$



From Snell's law,

$\dfrac{\sin i}{\sin r_1} = n$

$\therefore$ $\sin i = n \sin r_1$

$= n \sin (45° - C)$

$= n (\sin 45° \cos C - \cos 45° \sin C)$

$= \dfrac{n}{\sqrt{2}}(\cos C - \sin C)$

$= \dfrac{n}{\sqrt{2}}\left[\sqrt{1 - \sin^2 C} - \sin C\right]$

$= \left[\sqrt{1 - \left(\dfrac{n_1}{n}\right)^2} - \dfrac{n_1}{n}\right]$

$\dfrac{1}{\sqrt{2}}\left[\sqrt{(n^2 - n_1^2)} - n_1\right]$

$$\Rightarrow \quad i = \sin^{-1}\left\{\frac{1}{\sqrt{2}}\left(\sqrt{n^2 - n_1^2}\right) - n_1\right\}$$

b. When refracted ray passes through the diagonal face undeflected, the incidence at diagonal face is perpendicular.

$$r_2 = 0 \quad \text{so} \quad r_1 + r_2 = 45° \quad \Rightarrow \quad r = 45°$$

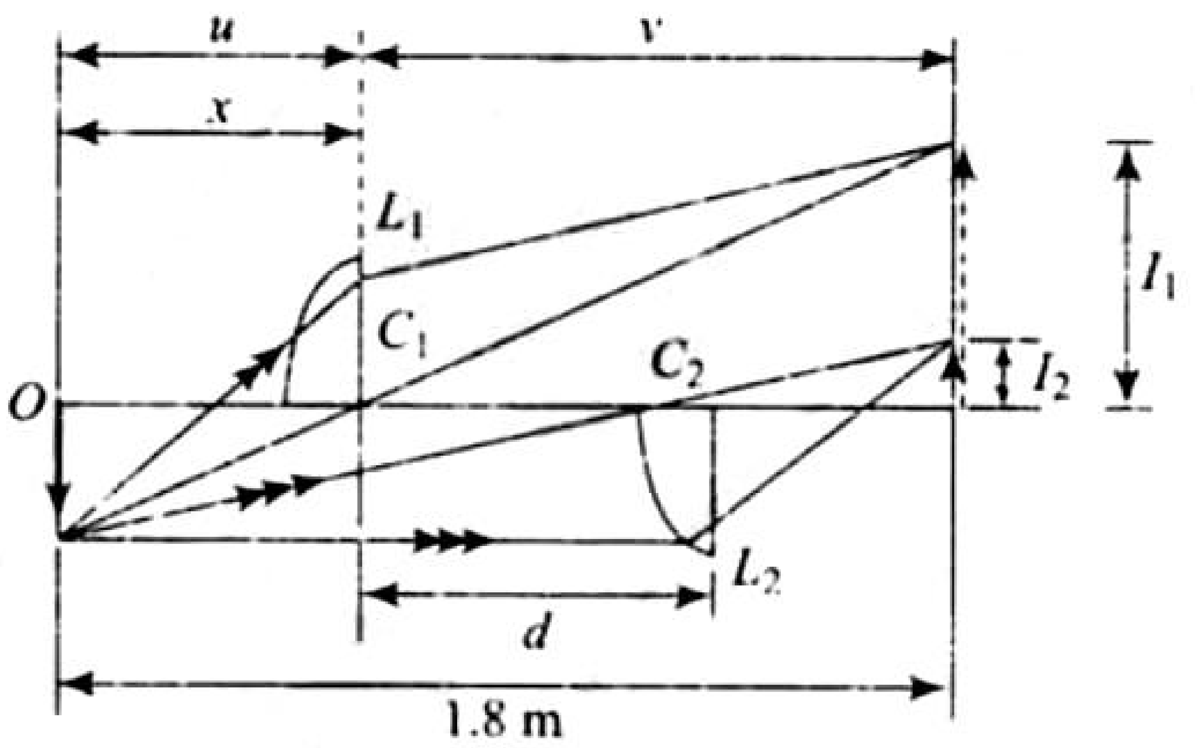
Again, $\dfrac{\sin i}{\sin r_1} = n \Rightarrow \sin i = n \sin r_1 = 1.352 \sin 45°$

or $\quad \sin i = 1.3520020\ \dfrac{1}{\sqrt{2}} = 0.956$

or $\quad i = \sin^{-1}(0.956) = 72°58'$.

17. Let magnification caused by the first lens be 2 and distance $OL_1 = x$. Distance v of image from first lens L_1 is given by

$$m = \frac{v}{u} = 2 \quad \Rightarrow \quad v = 2u = 2x.$$



Clearly, $u + v = 1.8$ m $\Rightarrow$ $x + 2x = 1.8$ m

or $\quad 3x = 1.8$ m $\Rightarrow$ $x = \dfrac{1.8}{3} = 0.6$ m

By sign convention,

$$u = -x = -0.6 \text{ m}, \ v = 2x = 1.2 \text{ m}$$

Lens formula $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$ gives

$$\frac{1}{f} = \frac{1}{1.2} + \frac{1}{0.6} = \frac{1+2}{1.2}$$

$\therefore \quad$ Focal length $f = \dfrac{1.2}{3} = 0.4$ m

For real image, lens formula takes the form

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

Clearly, u and v are interchangeable. Therefore, for lens L_2

$u' = v = 1.2$ m and $v' = 0.6$ m

$OL_1 = L_2I_2 = x$

If d is the separation between the lenses, then

$x + d + x = 1.8$ m

$\therefore \quad d = 1.8 - 2x = 1.8 - 2 \times 0.6 = 0.6$ m

Method 2 Since the magnification for L_1 is 2

$$\Rightarrow \quad \frac{v}{u} = -2 \Rightarrow \frac{\frac{D+d}{2}}{-\frac{D-d}{2}} = -2$$

$$\Rightarrow \quad \frac{D+d}{D-d} = 2 \quad \Rightarrow \quad D = 1.8 \text{ m}, \ d = 0.6 \text{ m}.$$

$$f = \frac{D^2 - d^2}{4D} = \frac{(1.8+0.6)(1.8-0.6)}{4 \times 1.8} = 0.4 \text{ m}$$

18. If R is radius of curvature of each lens surface, then for air on either side of lens, we have

$$\frac{1}{f} = ({}_a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{R} \Rightarrow f = R \Rightarrow R = f = 0.3 \text{ m}$$

If μ_2 is refractive index of lens material and μ_1, μ_3 are refractive indices on either side of the lens, then the formula is

$$\frac{\mu_3}{v} - \frac{\mu_1}{\mu} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2} \qquad \text{(i)}$$

Here $R_1 = 0.3$ m, $R_2 = -0.3$ m, $\mu_1 = \mu_{air} = 1$,

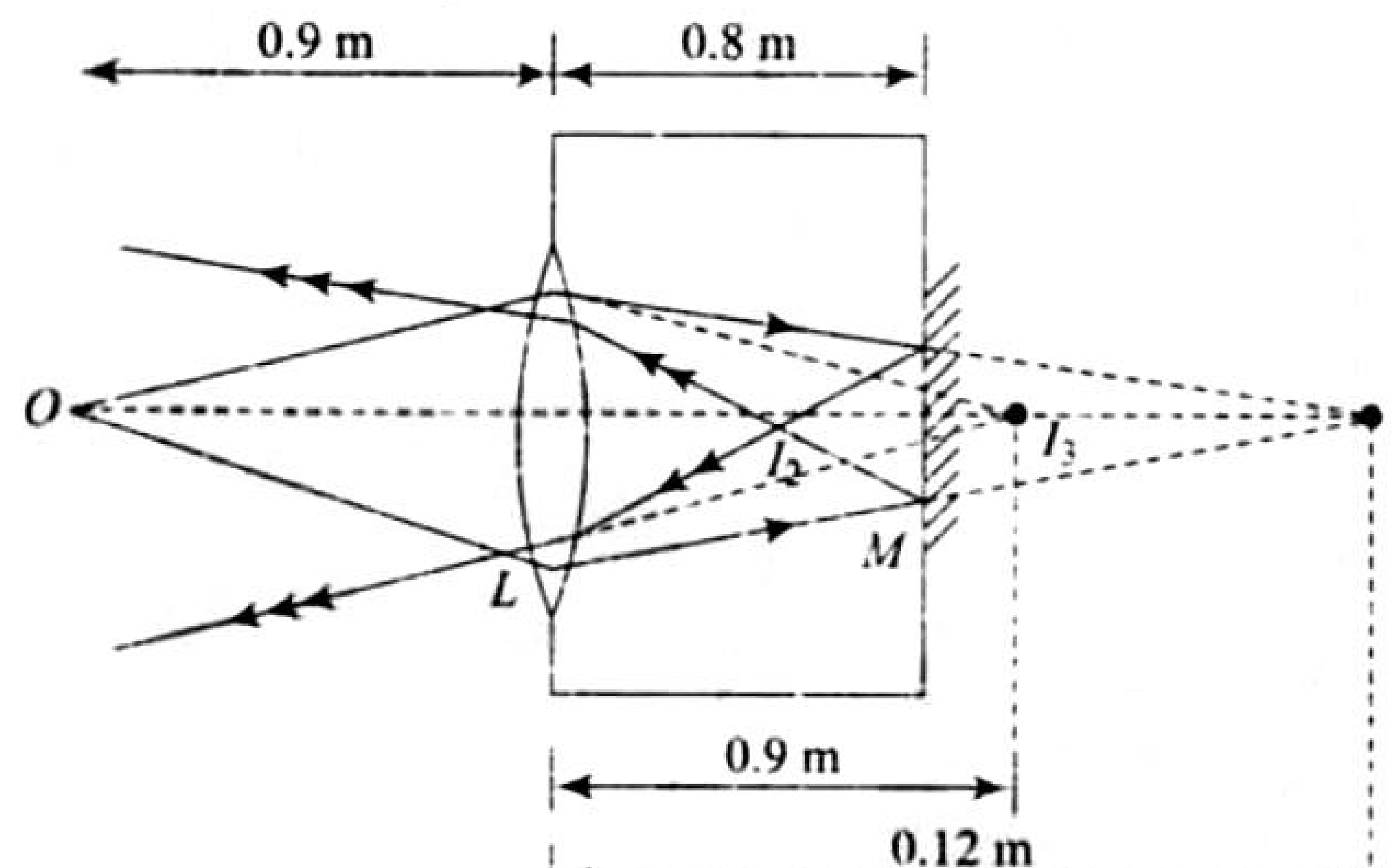
$$\mu_2 = \mu_{glass} = \frac{3}{2}, \ \mu_3 = \mu_{water} = \frac{4}{3}, \ u = -0.9 \text{ m}, \ v = ?$$

$$\therefore \quad \frac{4/3}{v} - \frac{1}{(-0.9)} = \frac{\left(\frac{3}{2} - 1\right)}{0.3} + \frac{\left(\frac{4}{3} - \frac{3}{2}\right)}{-0.3}$$

or $\quad \dfrac{4}{3v} + \dfrac{1}{0.9} = \dfrac{1}{0.6} + \dfrac{1}{1.8}$

or $\quad \dfrac{4}{3v} = \dfrac{1}{0.6} + \dfrac{1}{1.8} - \dfrac{1}{0.9} = \dfrac{1}{0.9}$

or $\quad v = \dfrac{0.9 \times 4}{3} = 1.2$ m



That is image I_1 is at a distance 1.2 m from lens L to the right or at a distance $(1.2 - 0.8) = 0.4$ m to the right of mirror M. This image I_1 acts as a virtual source for the plane mirror and forms real image I_2 at a distance 0.4 m on to the left of mirror and hence at a distance $(0.8 - 0.4) = 0.4$ m to the right of convex lens. This image I_2 acts as an object for the lens.

Again, the formula is

$$\frac{\mu_2'}{v''} - \frac{\mu_1'}{u''} = \frac{\mu_2' - \mu_1'}{R_1} + \frac{\mu_3' - \mu_2'}{R_2}$$

But now

$$\mu_1' = \mu_{water} = \frac{4}{3}, \quad \mu_2' = \mu_{glass} = \frac{3}{2}, \quad \mu_3' = \mu_{air} = 1$$

$R_1 = -0.3$ m, $R_2 = +0.3$ m, $u'' = 0.4$ m, $v'' = ?$

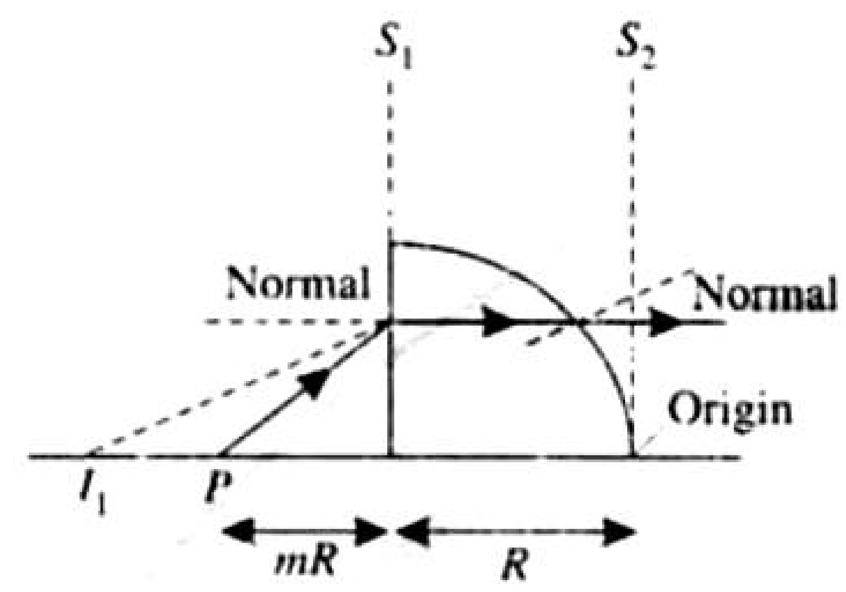
$$\therefore \quad \frac{1}{v''} - \frac{1}{0.3} = -\frac{1}{1.8} - \frac{1}{0.6}$$

or $\quad \dfrac{1}{v''} = \dfrac{1}{0.3} - \dfrac{1}{1.8} - \dfrac{1}{0.6} - \dfrac{1}{0.9} = \dfrac{1}{0.9} \quad \Rightarrow \quad v'' = 0.9$ m

That is image I_3 is formed to the right at a distance 0.9 m from the lens and is virtual. That is position of final image will be 0.9 m to the right of lens.

19. We will consider two refractions at plane surface S_1 and curved surface S_2. Refraction at S_1,

$$\frac{n}{v_i} - \frac{1}{(-mR)} = 0$$



$v = -nmR$

The image is virtual and on left of surface S_1. It acts as an object for refraction at the curved surface

Refraction at S_2:

$$\frac{1}{v^2} - \frac{n}{-(nmR + R)} = \frac{1-n}{(-R)}$$

$$\frac{n}{(nmR + R)} = \frac{1-n}{(-R)}$$

as emergent rays are parallel to principal axis $v_2 = \infty$. On solving for m, we get

$$m = \frac{1}{n^2 - n} = \frac{1}{(3/2)^2 - (3/2)} = \frac{4}{3}$$

20. Unit vector representing the normal to the plane $\hat{e}_n = \hat{k}$.

Component of the incident ray along the normal is $-1\hat{k}$.

The unit vector that represents the plane of the incident ray and the normal

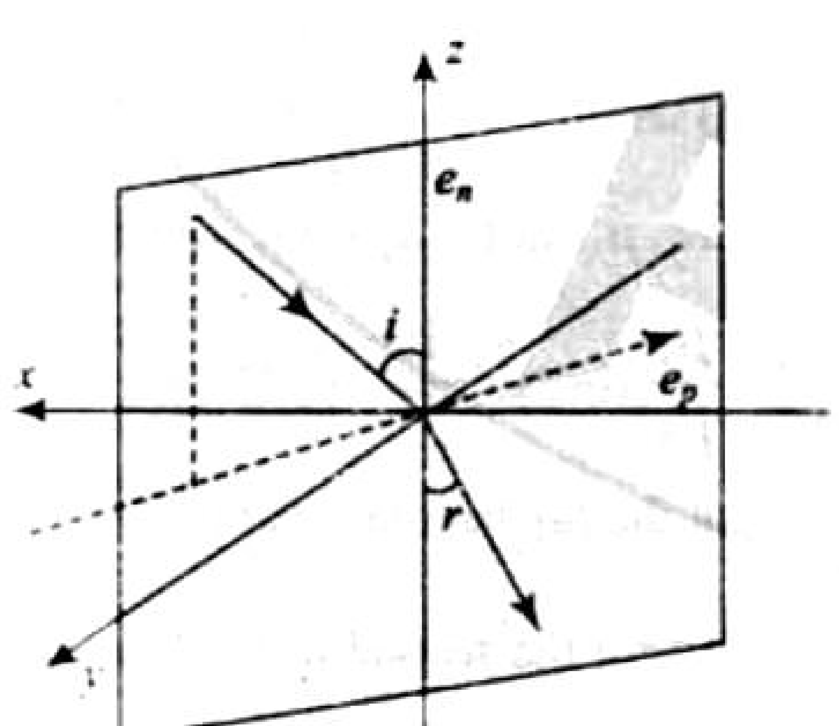
$$\hat{e}_p = \frac{(6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j})}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2}} = 0.6\hat{i} + 0.8\hat{j}$$

Angle between the incident ray and the normal is given by

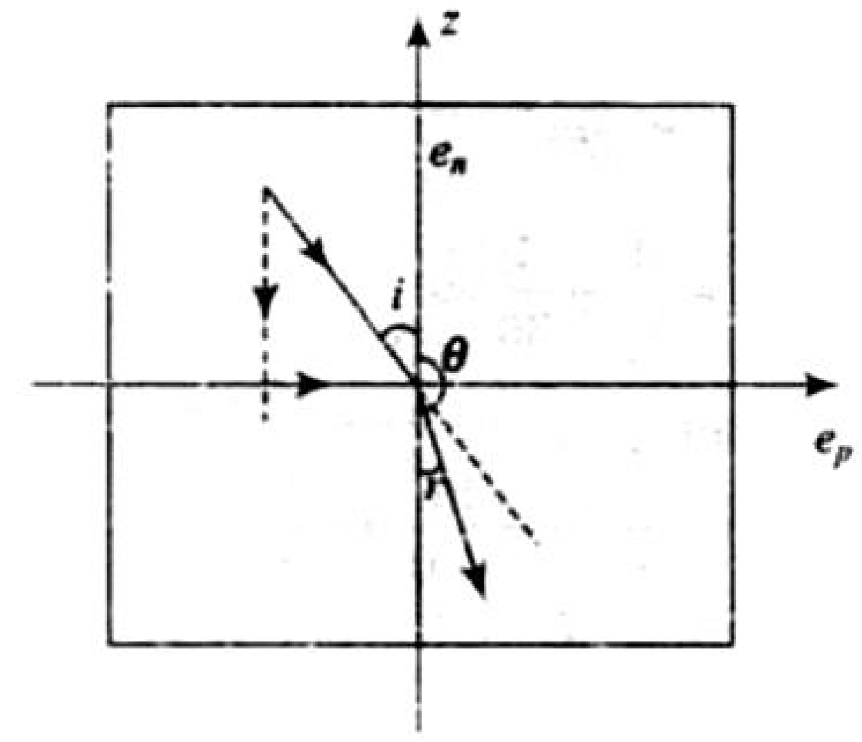
$$\cos\theta = (6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k})\,\hat{k} \Big/ \sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + 10^2}$$

or $\quad \cos\theta = -0.5$

Therefore, the angle $\theta = 120°$



(a)



(b)

The angle of incidence is $i = 180° - 120° = 60°$

The angle of the refracted beam is given by

$$\sqrt{2}\sin(i) = \sqrt{3}\sin(r) \quad \text{or } r = 45°$$

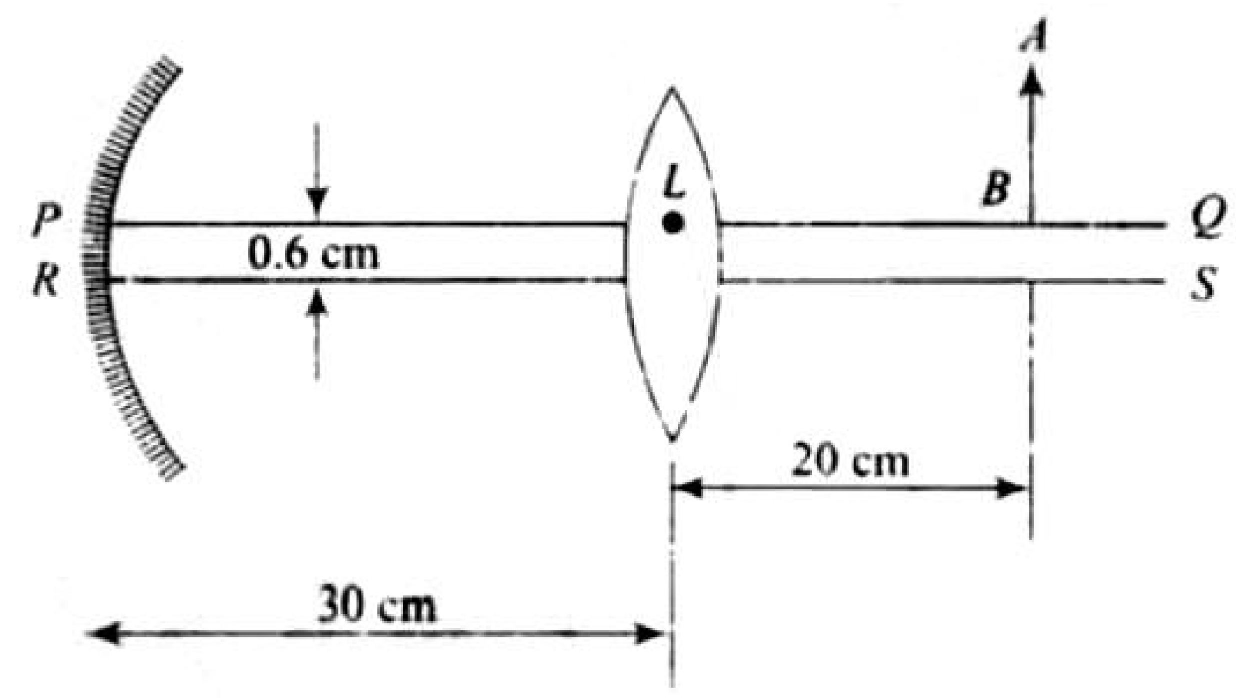
The equation of the emergent ray is $\cos(r)\hat{e}_n + \sin(r)\hat{e}_p$

$$= \cos(45°)(-\hat{k}) + \sin(45°)\cdot(0.6\hat{i} + 0.8\hat{j})$$

$$= \frac{1}{\sqrt{2}}(0.6\hat{i} + 0.8\hat{j} - \hat{k})$$

21. For convex lens using sign convection of coordinate geometry,

$u = +20$ cm, $f = -15$ cm

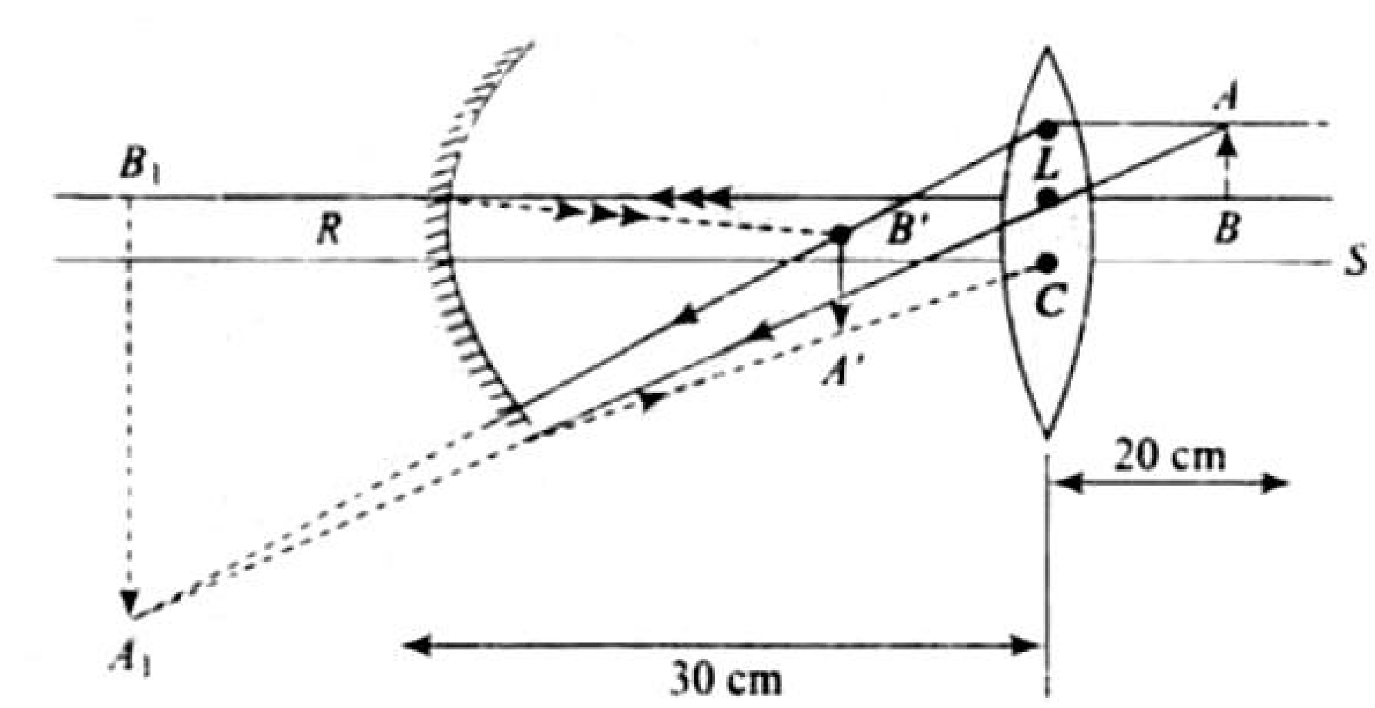


So, $\quad \dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u} \quad \Rightarrow \quad -\dfrac{1}{15} = \dfrac{1}{v_1} - \dfrac{1}{20}$

$\Rightarrow \quad \dfrac{1}{v_1} = \dfrac{1}{20} - \dfrac{1}{15} = \dfrac{3-4}{60} \quad \Rightarrow \quad v_1 = -60$ cm

i.e., image is formed at a distance 60 cm to the left of lens L.

$$\text{Magnification, } m_1 = \frac{v_1}{u_1} = -\frac{60}{20} = -3$$



This image is real and inverted. It is intercepted by the mirror.

For concave mirror, $u_2 = -60 + 30 = -30$ cm, $f_2 = +30$ cm

So, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ gives

$$\frac{1}{30} = \frac{1}{v_2} - \frac{1}{30} \Rightarrow \frac{1}{v_2} = \frac{1}{30} + \frac{1}{30} = \frac{2}{30} \Rightarrow v_2 = 15 \text{ cm}$$

Magnification,

$$m_2 = -\frac{v_2}{u_2} = -\frac{15}{-30} = +\frac{1}{2}$$

$\therefore$ Net magnification,

$$m = m_1 \times m_2 = (-3) \times \left(\frac{1}{2}\right) = -1.5$$

Size of image $A'B' = -1.5 \times 1.2$ cm $= -1.8$ cm.

Magnification of mirror is half and image of B formed by convex lens is 0.6 cm above RS, so the length of image will be 1.5 cm below RS.

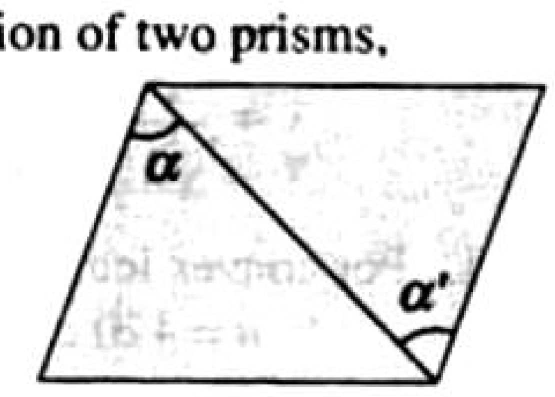
Thus, B' will be 0.3 cm above RS and A' will be 1.5 cm below RS.

22. Deviation produced by a prism $= (\mu_y - 1)\,\alpha$.

For no deviation caused by combination of two prisms,

$(\mu_y - 1)\alpha + (\mu'_y - 1)\,\alpha' = 0$

$$\Rightarrow \quad \alpha = -\frac{\mu_y - 1}{\mu'_y - 1}\alpha$$



Here $\alpha = 60°$

$$\mu'_y = \frac{\mu'_b + \mu'_r}{2} = \frac{1.77 + 1.73}{2} = 1.75$$

Refracting angle of flint glass prism,

$$\alpha' = \frac{\mu_y - 1}{\mu'_y - 1}\alpha = -\left[\frac{(1.50 - 1)}{(1.75 - 1)}\right] \times 6°$$

$$= -\frac{0.50}{0.75} \times 6° = -4°$$

The negative sign shows that the bases of prisms are opposite.

Therefore, refracting angle of flint glass prism = 4°

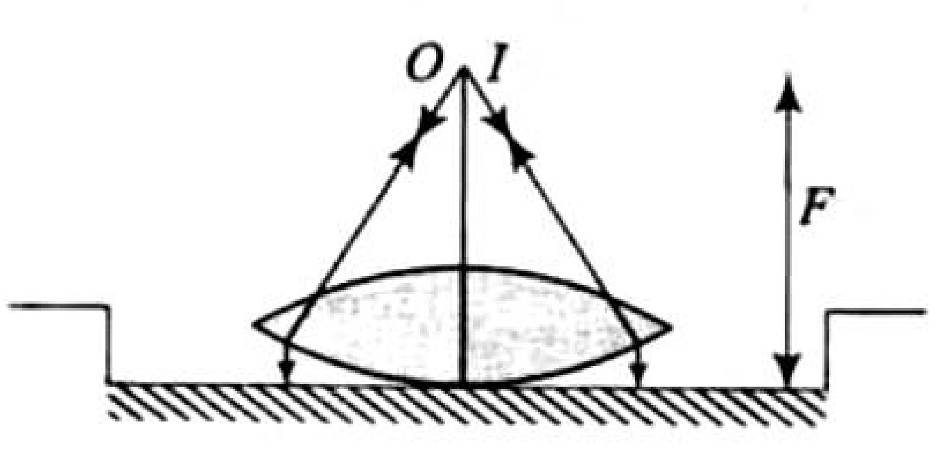
Net dispersion produced

$$\Delta\theta = (\delta_b - \delta_r) + (\delta'_b - \delta'_r)$$
$$= (\mu_b - \mu_r)\alpha + (\mu'_b - \mu'_r)\alpha'$$
$$= (1.51 - 1.49) \times 6° - (1.77 - 1.73) \times 4°$$
$$= 0.12° - 0.16° = -0.04° = 0.04° \text{ (numerically)}$$

23. Let f_1 be the focal length of convex lens; radius of curvature of each curved face is R.

$$\frac{1}{f_1} = (m - 1)\left\{\frac{1}{R} - \left(\frac{1}{-R}\right)\right\} = (\mu - 1)\frac{2}{R}$$

$$\Rightarrow \quad f_1 = \frac{R}{2(\mu - 1)} = \frac{R}{2\left(\frac{3}{2} - 1\right)} = R$$



When the space between the lens and mirror is filled by water of refractive index $\mu_1 = 4/3$, then the focal length of liquid concave lens f_2 is

$$\frac{1}{f_2} = (\mu_1 - 1)\left(-\frac{1}{R} - \infty\right)$$

$$\Rightarrow \quad f_2 = \frac{-R}{\mu_1 - 1} = -\frac{R}{\left(\frac{4}{3} - 1\right)} = -3R$$

The combined focal length of lenses is $F_1 = 15$ cm

$$\therefore \quad \frac{1}{F_1} = \frac{1}{f_1} + \frac{1}{f_2} \quad \Rightarrow \quad \frac{1}{15} = \frac{1}{R} - \frac{1}{3R} = \frac{3 - 1}{3R}$$

$\Rightarrow \quad 3R = 30 \quad \Rightarrow \quad R = 10$ cm

In the second case,

$F_2 = 25$ cm. Let $\mu_1 = \mu_2$.

$$\therefore \quad \frac{1}{F_2} = \frac{1}{f_1} + \frac{1}{f_2} \quad \Rightarrow \quad \frac{1}{25} = \frac{1}{10} + \frac{1}{f'_2}$$

$$\Rightarrow \quad \frac{1}{f'_2} = \frac{1}{25} - \frac{1}{10} = \frac{2 - 5}{50}$$

$$\therefore \quad f'_2 = \frac{-50}{3} \text{ cm}$$

$$f'_2 = \frac{R}{\mu_2 - 1} \Rightarrow \mu_2 - 1 = -\frac{R}{f'_2} = \frac{-10}{(-50/3)} = \frac{3}{5} = 0.6$$

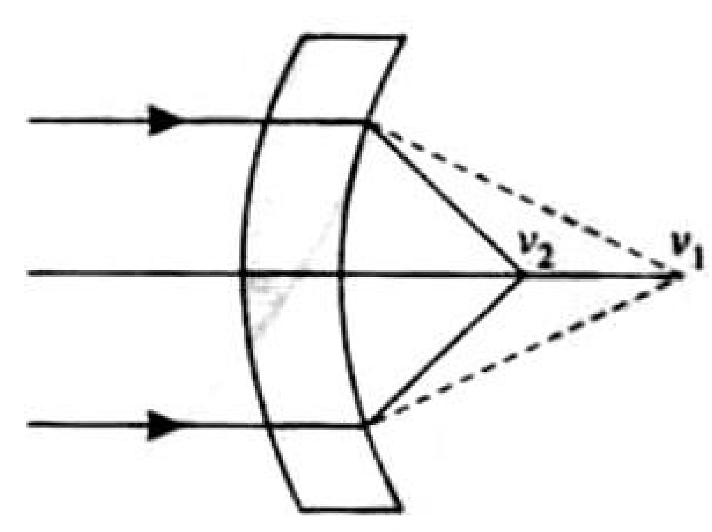
$\Rightarrow \quad \mu_2 = 1 + 0.6 = 1.6$

24. For an object placed at infinity the image after first reflection will be formed at v_1

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{-\infty} = \frac{\mu_2 - \mu_1}{+R} \qquad \text{(i)}$$

The image after second refraction will be found to v_2

$$\frac{\mu_3}{v_2} - \frac{\mu_2}{v_1} = \frac{\mu_3 - \mu_2}{+R} \qquad \text{(ii)}$$



Adding (i) and (ii) $\frac{\mu_3}{v_2} = \frac{\mu_3 - \mu_1}{R} \Rightarrow v_2 = \frac{\mu_3 R}{\mu_3 - \mu_1}$

Therefore focal length will be $\frac{\mu_3 R}{\mu_3 - \mu_1}$.

25. From Snell's law, $\mu_1 \sin i = \mu_2 \sin r$, we have

$$\sin r = \frac{\mu_1}{\mu_2}\sin i = \frac{1}{\sqrt{2}} \sin 45° = \frac{1}{2} \Rightarrow r = 30°.$$

This means that the ray becomes parallel to side AD inside the slab.

This implies that for the second face CD, $u = \infty$

Given $R = 0.4$ m

Now, we have

$$\frac{\mu_3}{v} - \frac{\mu_2}{u} = \frac{\mu_3 - \mu_2}{R}$$

$$\Rightarrow \quad \frac{1.514}{v} - \frac{\sqrt{2}}{\infty} = \frac{1.514 - \sqrt{2}}{R}$$

$$\Rightarrow \quad v = \frac{1.514 \times 0.4}{1.514 - 1.414} = \frac{1.514 \times 0.4}{0.1}$$

$= 6.056$ m $= 6.06$ m (up to two decimal places).

26. Given $f = 0.3$ m, $u = -0.4$ m

From lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \text{(i)}$$

We have,

$$\frac{1}{0.3} = \frac{1}{v} + \frac{1}{0.4} \quad \Rightarrow \quad \frac{1}{v} = \frac{1}{0.3} - \frac{1}{0.4} \quad \text{or } v = 1.2 \text{ m}$$

Differentiating Eq. (i) with respect to time t, we get

$$0 = -\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} \quad \Rightarrow \quad \frac{dv}{dt} = \frac{v^2}{u^2}\frac{du}{dt} \quad \text{(ii)}$$

Given $\frac{du}{dt} = 0.01 \text{ m s}^{-1}$

$$\Rightarrow \quad \frac{dv}{dt} = \left(\frac{1.2}{0.4}\right)^2 \times 0.01 = 0.09 \text{ m s}^{-1}.$$

Therefore, rate of change of position of image $= 0.09 \text{ m s}^{-1}$.

Magnification, $m = \frac{v}{u}$

Differentiating with respect to time t,

$$\frac{dm}{dt} = \frac{u\frac{dv}{dt} - v\frac{du}{dt}}{u^2} = \frac{(-0.4)\times 0.09 - 1.2 \times (0.01)}{(-0.4)^2}$$

$$= \frac{-0.036 - 0.012}{0.16} = -\frac{0.048}{0.016} = -0.3 \text{ s}^{-1}$$

27. a. For minimum deviation of emergent ray from the first prism MN is parallel to AC

$\angle BMN - 90°$

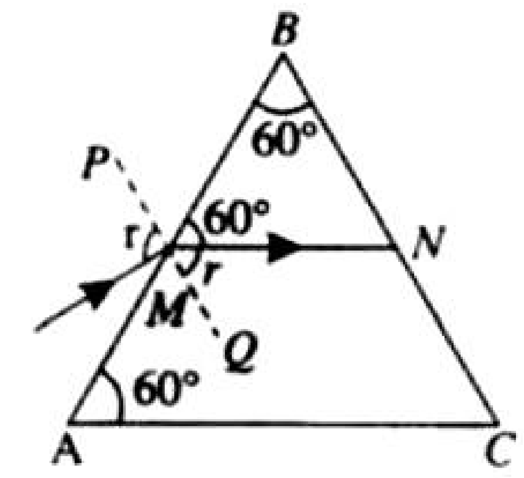
$\angle r = 30°$

Applying Snell's law at M

$$\mu = \frac{\sin i}{\sin r}$$

$$\sin i = m \sin r$$

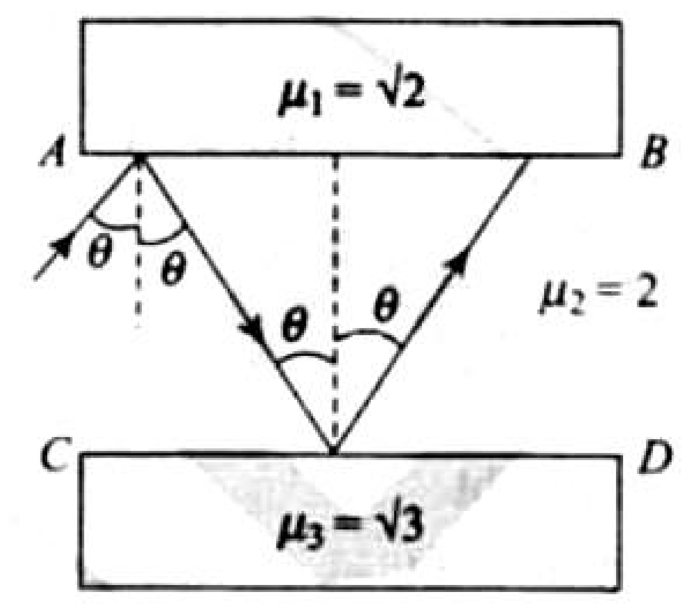
$$\sin i = \sqrt{3} \times \sin 30° = \frac{\sqrt{3}}{2}$$



$i = 60°$

b. When the prism DCE is rotated about C in anticlockwise direction, as shown in the figure, then the final emergent ray SR becomes parallel to the incident TM. Thus, the angle of deviation becomes zero.

28. Let θ be the angle of incidence at face AB, then for total internal reflection at face AB



$$\sin\theta > C_1 = \frac{\mu_1}{\mu_2} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \quad \sin\theta > \frac{1}{\sqrt{2}} \quad \text{or} \quad \sin\theta > \sin 45° \quad \Rightarrow \quad \theta > 45°$$

For total internal reflection at face CD,

$$\sin\theta > \sin C_2 = \frac{\mu_3}{\mu_2} = \frac{\sqrt{3}}{2} \Rightarrow \sin\theta > \frac{\sqrt{3}}{2}$$

or $\sin\theta > \sin 60°$ or $\theta > 60°$

Therefore, for total internal reflection at both the surfaces,

$\theta_{min} = 60°$.

29. The deviation produced by small angled prism,

$\delta_1 = (\mu - 1)\alpha = (1.5 - 1)4° = 2°$ (always)

Deviation caused by mirror,

$\delta_2 = 180° - 2i = 180° - 2 \times 45° = 90°$

Net deviation produced by system $= \delta_1 + \delta_2 = 2° + 90° = 92°$

This is more than 90°.

Greater is angle of incidence on the mirror, smaller is the deviation.

If β is the angle of rotation of mirror in clockwise direction to increase angle of incidence, then deviation produced by the mirror will be $180° - 2(45° + \beta) = 90° - 2\beta$

Therefore, total deviation produced $= 90° - 2\beta + 2° = 92° - 2\beta$

Given, $92° - 2\beta = 90° \Rightarrow \beta = 1°$