NUMBER SYSTEM

Learning Objectives

- Introduction
- Classification of Numbers
- LCM
- HCF

Introduction

Numbers are the basic unit of Mathematics, after all, it with numbers that we perform the various functions which constitute Mathematics. For example: Addition, Subtraction,
Multiplication & Division. The Number system is the backbone of any competitive exam. The correct understanding will help you to solve different and complex problems that appear in these examinations. First and for most, let us have a look at the basic classification of numbers and its various kinds.

Classification of Numbers

Natural Numbers

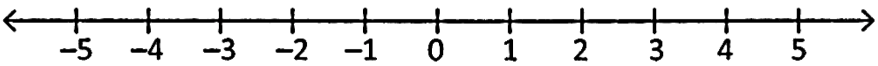
Natural numbers are all of the whole numbers EXCEPT zero. 1, 1, 3, 4, 5, 6, 7, 8, 9, 10, 11…....
They are also called counting numbers. The lowest natural number is 1.

Whole Numbers

Whole numbers are those numbers which start by 0 or we can say when 0 is Included in the list of natural numbers then we call it whole numbers, for example 0, 1, 2, 3, 7, 4, 5.......

Integers

It is the series of both positive and negative numbers lying on the number line. It is the combination of both positive and negative whole and natural numbers.



Rational Numbers

Rational numbers are those numbers that can be written in the form of a ratio of x and y, where the denominator is not zero.

Real Numbers

The number which lies on the number line is a real number .The number can be positive or negative in nature, for example it may be like as 3, 4, 5, 6, -6, -5, -4, -3, -2.....

Irrational Numbers

Irrational numbers are those which are not rational, that is those numbers that cannot be written in the form of a ratio.

Counting Numbers

Counting numbers are those numbers which are well managed on the number line with the difference of 1. The smallest counting number in the number line is 1.

Complex Numbers

Includes real numbers and imaginary numbers are called complex numbers, eg. a + ib.

Prime numbers

The numbers which don't have any factor other than 1 or itself.
For example: 2, 3, 5, 7, 9, 29, 31, 43 .... or we can say that the numbers which are not divisible by any number are called prime numbers. There are 24 prime numbers between 1 and 100.

- 2 is the only even prime number and the least prime number.
- 1 is neither a prime nor a composite number.

List of Prime Numbers

Number Range	Number of Primes
1 - 100	25
101 - 200	21
201 - 300	16
301 - 400	16
401 - 500	17
501 - 600	17
601 - 700	16
701 - 800	14
801 - 900	15
901 - 1000	14

Co-prime Numbers

Two natural numbers are called co-prime numbers if they have no common factor other than
1. The highest common factor (HCF) between co-prime numbers is 1. eg. (8, 9), (13, 15), (14, 25), (15, 16) etc.

Composite Numbers

All whole numbers that are not prime are composite except for 1 or 0.
The number which is the product of two or more than two distinct or same prime numbers. Example 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21...........

Some points to remember

- 1 is neither prime nor composite.
- 0 is neither positive nor negative.

Number system at a glance

N	Natural	0, 1, 1, 3, 4,... or 1, 2, 3, 4,...
Z	Integers	..., – 5, – 4, – 3, – 2, – 1, 0, 1, 2, 3, 4, 5,...
Q	Rational	a/b where a and b are integers and b is not 0
R	Real	The limit of a convergent sequence of rational numbers
C	Complex	a + bi or a + ib where a and b are real numbers and i is the square root of – 1

Some Properties of Nature Numbers

- $1+2+3+------------+n=\frac{n(n+1)}{2}$

- $1^2+2^2+3^2+------------+n^2=\frac{n(n+1)(2n+1)}{6}$
- $1^3+2^3+3^3+------------+n^3=\left[\frac{n(n+1)}{2}\right]^2$
- x^n+y^n is divisible by $(x+y)$ for all values of n
- x^n-y^n is divisible by $(x-y)$ for all values of n
- If a number is divisible by m and n, then it is always divisible by the LCM of m and n.
- x^n-y^n is divisible by $(x+y)$ if n is even

LCM
LCM of a given set of numbers is the least number which is exactly divisible by every number of the given set.

HCF
HCF of a given set of numbers is the highest number which divides exactly every number of the given set.

Combined Formula of LCM and HCF

- $\text{HCF of numerators}=\frac{\text{HCF of fractions}}{\text{LCM of fractions}}$
- $\text{LCM of numerators}=\frac{\text{LCM of fractions}}{\text{HCF of fractions}}$
- $\text{One of number}=\frac{\text{LCM}\times\text{HCF}}{\text{2nd number}}$
- $\text{LCM = of two numbers}=\frac{\text{Pruduct of the numbers}}{\text{HCF}}$
- $\text{HCF}=\frac{\text{Pruduct of the numbers}}{\text{LCM}}$

Commonly Asked Questions

- If the HCF and LCM of two numbers are 9 and 924 respectively, then find the product of two numbers.
  (a) 8216
  (b) 8968
  (c) 8316
  (d) 8896
  (e) None of these

  Answer: (c)
  Explanation: Product of the numbers $=\text{LCM}\times\text{HCF}=9\times 924=8316$

- Find the largest positive number that will divide 396, 434, and 540 leaving the remainder 5, 9, and 13 respectively.
  (a) 15 (b) 13
  (c) 17 (d) 19
  (e) None of these

  Answer (c)
  Explanation: $396-5=391$; $434\ -9-425$;
  $540-13=527$
  $391=23\times 17$
  $425=25\times 17$ *and* $527=31\times 17$
  Hence the required number will be 17

- Find the smallest number which, when divided by 32, 45, and 68 leaves remainder 5 in each case.

  (a) 24480 (b) 7560
  (c) 24485 (d) 7565
  (e) None of these

  Answer (c)

  Explanation: First we have to find the LCM of 32, 45 and 68

  $32 = 2\times2\times2\times2\times2$

  $45 = 3\times3\times5$

  $68 = 2\times2\times17$

  Therefore required number will be:

  $(2\times2\times2\times2\times2\times3\times3\times17)+5 = 24485$

- A boy draws n squares with sides 1, 2, 3, 4, 5........ in inches. The average area covered by these n squares will be:

  (a) $\frac{n+1}{2}$ (b) $\left(\frac{n+1}{2}\right)\left(\frac{2n+1}{3}\right)$
  (c) $\left(\frac{n+1}{2}\right)\left(\frac{2n+1}{3}\right)$ (d) $\left(\frac{n+1}{2}\right)-1\left(\frac{2n+1}{3}\right)$
  (e) None of there

  Answer: (b)

  Explanation: $Average\,area = \frac{1^2+2^2+3^2+.....+n^2}{n} = \frac{\left(\frac{n(n+1)(2n+1)}{6}\right)}{n}$

  $= \frac{(n+1)(2n+1)}{6} = \left(\frac{n+1}{2}\right)\left(\frac{2n+1}{n}\right)$