JEE MAIN 2025

Sample Paper - 2

Time Allowed: 3 hours **Maximum Marks: 300**

General Instructions:

1. There are three subjects in the question paper consisting of Physics (Q. no. 1 to 25), Chemistry (Q, no. 26 to 50), and Mathematics (Q. no. 51 to 75).

2. Each subject is divided into two sections. Section A consists of 20 multiple-choice questions & Section B consists of 5 numerical value-type questions.

3. There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero marks will be awarded for not attempted questions.

4. For Section B questions, 4 marks will be awarded for correct answers and zero for unattempted and incorrect answers.

5. Any textual, printed, or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.

6. All calculations/written work should be done in the rough sheet is provided with the Question Paper.

PHYSICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

1. The velocity, acceleration, and force in two systems of units are related as under

i) $v' = \frac{\alpha^2}{\beta} v$ ii) $a' = (\alpha\beta) a$ iii) $F' = \left(\frac{1}{\alpha\beta}\right) F$

All the primed symbols belong to one system and unprimed ones belong to the other system. α and β are

dimensionless constants. Which of the following is incorrect?

A) Length standards of the systems are related by $L' = \left(\frac{\alpha^3}{\beta^3}\right) L$

B) Mass standards of the two systems are related by $M' = \left(\frac{1}{\alpha^2\beta^2}\right) M$.

C) Time standards of the two systems are related by $T' = \left(\frac{\alpha}{\beta}\right) T$

D) Momentum standards of the systems are related by $P' = \left(\frac{1}{\beta^3}\right) P$.

2. A particle is moving along a circle with velocityV=kt, here k=0.5 SI units. The acceleration of the particle at the moment when it covered $\left(\frac{1}{10}\right)^{th}$ of circle after beginning of motion is ______ (nearly)

A) $1\,ms^{-2}$ B) $1.2\,ms^{-2}$ C) $0.8\,ms^{-2}$ D) $1.4\,ms^{-2}$

3. A ball with velocity of $4ms^{-1}$ impinges at 30^0 with vertical on a smooth horizontal fixed plane. If the coefficient of restitution is 0.5, the velocity and direction of motion with vertical after impact is _______

A) $\sqrt{3}\ ms^{-1}, 60^0$ B) $\sqrt{7}\ ms^{-1}, Tan^{-1}\left(2/\sqrt{3}\right)$

C) $2\ ms^{-1}, 30^0$ D) $1\ ms^{-1}, Tan^{-1}\left(\sqrt{3}/2\right)$

4. A uniform rod of mass m and length L rests on a smooth horizontal surface. One end of the rod is struck by a small ball of same mass in a horizontal direction at right angles to the rod with 'V_0'elastically. The force act on one half of the rod by the other half is…

A) $\frac{9mV_0^2}{4L}$ B) $\frac{9mV_0^2}{2L}$ C) $\frac{3mV_0^2}{4L}$ D) $\frac{3mV_0^2}{2L}$

5. Statement 1 :When there is a thin layer of water between two glass plates there is a strong attraction between them

Statement 2 :The pressure between the plates become less than atmospheric pressure as pressure difference is created due to surface tension.

A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.

B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is not a correct explanation for Statement – 1.

C) Statement -1 is True, Statement – 2 is False.

D) Statement – 1 is False, Statement – 2 is True.

6. Statement 1 :In an adiabatic process the change in internal energy of a gas is equal to negative of the work done by the gas

Statement 2 :Temperature of the gas remains constant during an adiabatic process

A) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.

B) Statement – 1 is True, Statement – 2 is True; Statement – 2 is not a correct explanation for Statement – 1.

C) Statement -1 is True, Statement – 2 is False.

D) Statement – 1 is False, Statement – 2 is True.

7. The mass of a hydrogen molecule is 3.23×10^{-27} kg . If 10^{23} hydrogen molecules strike on $2\,cm^2$ area of a wall per second at an angle 45^0 with normal to the wall with a speed $10^5\,cm\,s^{-1}$, the pressure they exert on the wall is _____ Pa. (Take $\sqrt{2}=1.4$)

A) 3.32×10^3 B) 2.30×10^3 C) 1.27×10^3 D) 1.67×10^3

8. A point mass m is suspended from free end of rod of length ℓ, mass m. Then the time period for small amplitude of oscillations will be:

A) $2\pi\sqrt{\frac{\ell}{g}}$ B) $2\pi\sqrt{\frac{4\ell}{3g}}$ C) $2\pi\sqrt{\frac{8\ell}{9g}}$ D) $2\pi\sqrt{\frac{8\ell}{15g}}$

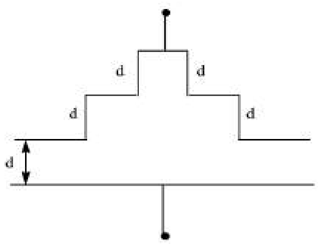
9. A particle of charge –q, mass m moves in a region of space between two plates of a capacitor from a plate at potential –V to the plate at potential +V. The plate separation is d. If K, U, T and E be the respective kinetic energy potential energy, total mechanical energy of the particle and E be the electric field between the plates, then match the facts in Column-I with those in Column-II

Column - I		Column – II	
(A)	K	(P)	constant
(B)	U	(Q)	first increases and then decreases
(C)	T	(R)	increases
(D)	E	(S)	decreases
		(T)	Other than those in (p), (q), (r) or (s)

A) A – S; B – R; C – P; D – P B) A – R; B – S; C – P; D – P

C) A – R; B – S; C – Q; D – P D) A – S; B – R; C – P; D – T

10. The upper plate of parallel plate capacitor of plate area A is modified into 5 equal segments as shown. The equivalent capacitance between the terminals is _____



A) $\frac{10 \epsilon_0 A}{3d}$ B) $\frac{2 \epsilon_0 A}{3d}$ C) $\frac{3 \epsilon_0 A}{10d}$ D) $\frac{3 \epsilon_0 A}{2d}$

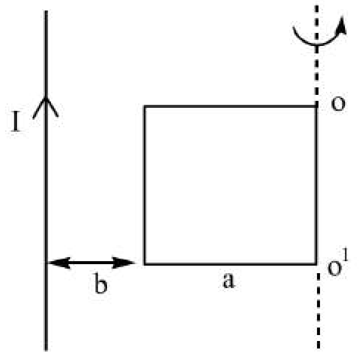
11. A voltage V is applied to a d.c.electric motor of resistance R. The current flowing in the motor to get maximum power produced by the motor is …

A) $\frac{V}{2R}$ B) $\frac{V}{4R}$ C) $\frac{V}{R}$ D) $\frac{4V}{R}$

12. A bar magnet of length 6 cm has a magnetic moment of 4 JT^{-1}. Find the strength of magnetic field at a distance of 200 cm from the center of the magnet along its equatorial line.

A) 4×10^{-8} T B) 3.5×10^{-8} T C) 5×10^{-8} T D) 3×10^{-8} T

13. A square loop of a side a and straight infinite conductor carrying current I are in the same plane as shown, The Resistance of the loop is "R". The frame is turned through 180^0 about the axis oo^1. Find the electric charge that flows in the square loop. (Ignore inductance)



A) $\frac{\mu_0 Ia}{2\pi R}\log\left(\frac{a+b}{2a+b}\right)$ B) $\frac{\mu_0 Ia}{4\pi R}\log\left(\frac{a+2b}{a}\right)$ C) $\frac{\mu_0 Ia}{2\pi R}\log\left(\frac{2a+b}{b}\right)$ D) $\frac{\mu_0 Ia}{4\pi R}\log\left(\frac{2a+b}{a}\right)$

14. In a series LCR circuit the voltages across resistance, capacitance, inductance are 20V each. If the capacitance short-circuited, the voltage across inductance will be ______

A) 20V B) $20\sqrt{2}$V C) $\frac{20}{\sqrt{2}}$V D) 10V

15. A plane electromagnetic wave of wavelength λ has an intensity I. It is propagating along the positive Y-direction. The allowed expressions for the electric and magnetic fields are given by

A) $\vec{E}=\sqrt{\frac{2I}{\epsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y-ct)\right]\hat{k}$; $\vec{B}=+\frac{1}{c}E\hat{i}$

B) $\vec{E}=\sqrt{\frac{2I}{\epsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y+ct)\right]\hat{k}$; $\vec{B}=\frac{1}{c}E\hat{i}$

C) $\vec{E}=\sqrt{\frac{I}{\epsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y-ct)\right]\hat{k}$; $\vec{B}=\frac{1}{c}E\hat{i}$

D) $\vec{E}=\sqrt{\frac{I}{\epsilon_0 c}}\cos\left[\frac{2\pi}{\lambda}(y-ct)\right]\hat{i}$; $\vec{B}=\frac{1}{c}E\hat{k}$

16. A converging lens and a diverging mirror are placed at a separation of 15 cm. The focal length of the lens is 25 cm and that of mirror is 40cm. At what distance from mirror a point source of light placed between two so that, a parallel beam of light comes out from the lens after getting reflected from mirror.

A) 13.3cm B) 6.66 cm C) 20cm D) 4.44 cm

17. A parallel beam of microwaves of wave length 0.5 mm falls normally on Young's double slit apparatus. The separation between the slits is 1.5 mm and the screen is placed at a distance 1.0 m from the slits. Find the number of maxima in the interference pattern observed on the screen.

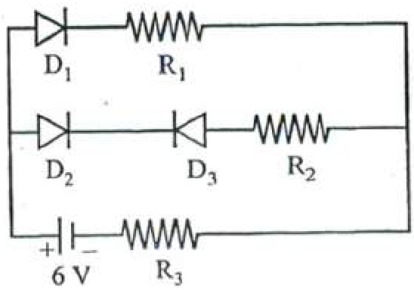
(Excluding maxima formed at infinity)

A) 8 B) 9 C) 5 D) 11

18. An orbital electron in the ground state of hydrogen has magnetic moment μ_1. This orbital electron is excited to 3rd excited state by some energy transfer to the hydrogen atom. The new magnetic moment of the electron is μ_2, then

A) $\mu_1 = 4\mu_2$ B) $2\mu_1 = \mu_2$ C) $16\mu_1 = \mu_2$ D) $4\mu_1 = \mu_2$

19. Figure shows a circuit in which three identical diodes are used. Each diode has forward resistance 20Ω and infinite backward resistance. Resistors $R_1 = R_2 = R_3 = 50\Omega$. Battery voltage is 6 V. The current through R_3 is :



A) 50 mA B) 100 mA C) 60 mA D) 25 mA

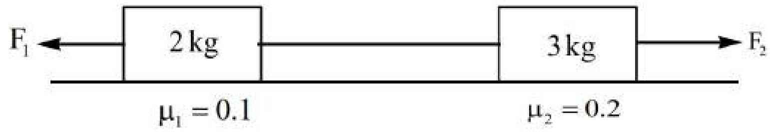
20. In an experiment for measurement of Young`s modulus, following readings are taken : Load = 3.00 kg, length = 2.820 m, diameter = 0.041 cm and extension = 0.87 mm. The percentage error in measurement of Y is around

A) 6% B) 8% C) 1% D) 3%

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

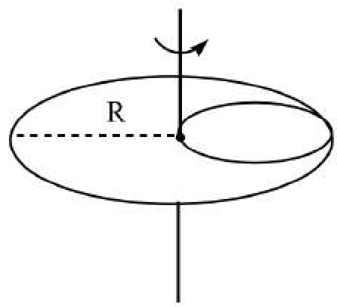
This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**
Marking scheme: +4 for correct answer, -1 in all other cases.

21. The figure shows two blocks placed on a rough horizontal surface, under the action of two forces $F_1 = 3N$ and $F_2 = 12N$. The tension in the string is $\frac{x}{10}N$. Find the value of 'x'(take g=10m/s^2)



22. A Particle moving along the x-axis is acted upon by a single force $F = F_0 e^{-kx}$, here F_0 and k are constants. The particle is released from rest at $x = 0$. It will attain a maximum kinetic energy of $\frac{2F_0}{NK}$, find the value of N.

23. A circular hole of radius $\frac{R}{2}$ is cut from a circular disc of radius R. The radius of gyration of this disc about an axis passing through its original centre and normal to its plane is $\sqrt{\frac{N}{24}}$, find the value of N.



24. If the change in the acceleration of the earth when the position of the moon changes from solar eclipse position to on exactly other side of the earth is $N \times 10^{-5} ms^{-2}$, find the value of N. Ignore the effect of other planets (mass of the moon = 7.36×10^{22} kg, radius of Lunar orbit = $3.8 \times 10^8 m$, distance between the sun and the earth is 150 million kilometers, take $G = 6.7 \times 10^{-11}$ S.I.units) (Mark the nearest integer only)

25. A cylindrical vessel of area of cross-section A and filled with liquid to a height of h_1 has a capillary tube of length l and radius r protruding horizontally at its bottom. If the viscosity of liquid is η and density ρ. Find the time in which the level of water in the vessel falls to h_2 is $\frac{X\eta l A}{\pi\rho g r^Y}\ln\frac{h_1}{h_2}$, find the value of X/Y.

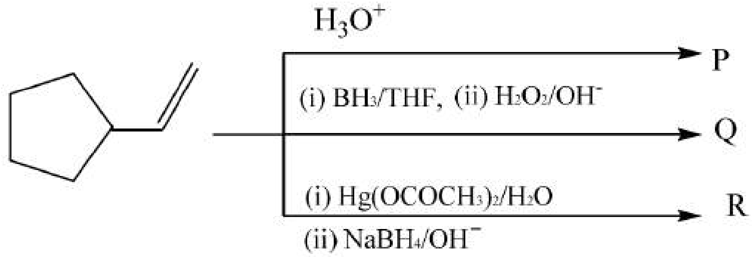
CHEMISTRY **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

26. Which is correct for product, P, Q and R (P, Q, R are major product)



A) Product P & R are identical

B) Product Q & R are identical

C) Product P & Q are functional group isomers

D) Product P, Q & R are different

27. Which of the following statement is incorrect?

A) SRP values of halogens $X_2(g)/X^-(aq) F_2 > Cl_2 > Br_2 > I_2$

B) Bond dissociation enthalpy of $Cl_2 > F_2 > Br_2 > I_2$

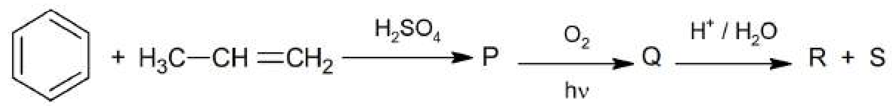
C) Boiling points of $I_2 > Br_2 > Cl_2 > F_2$

D) Reducing power of $I^- > Br^- > Cl^- > F^-$

28. An electron in an atom jumps to the higher energy level in such a way that its kinetic energy changes from 'y' to $\frac{y}{2}$. Then change in its potential energy will be

A) y B) – y C) $+\frac{y}{2}$ D) $-\frac{y}{2}$

29.



If R is aromatic and S is aliphatic, then:

A) Rate of EAS of (R) is more than that in benzene.

B) enol content of "S" is more than the enol content of acetaldehyde

C) R is more acidic than C_2H_5OH

A) A and C are correct B) B and C are correct

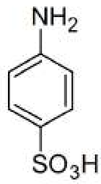
C) Only C is correct D) All A, B and C are correct

30. Which of the following will not give Lassaigne's test for N in sodium extract?

A) $C_6H_5NHNH_2$

B) NH_2CONH_2

C) NH_2-NH_2

D) 

31. The compound which has zero dipole moment is

A) CH_2Cl_2

B) cis But – 2 – ene

C) PCl_3F_2

D) *ortho* - Dichlorobenzene

32. Cerium $(Z = 58)$ is an important member of lanthanoids, which of the following statements about cerium is incorrect?

A) The common oxidation states for cerium are + 3 and +4

B) The +3 oxidation state of cerium is more stable than +4

C) The +4 oxidation state of cerium is not known in solutions

D) Cerium (IV) acts as an oxidizing agent.

33. The complex $[Co(NH_3)_4(NO_2)_2]Cl$, exhibits

A) Ionization isomerism, geometrical isomerism and optical isomerism

B) Linkage isomerism, geometrical isomerism and optical isomerism

C) Linkage isomerism, geometrical isomerism and ionization isomerism

D) Optical isomerism, ionization isomerism and Linkage isomerism

34. Choose the correct statement(s)

A) If the solubility of Sb_2S_3 is $1.0\times10^{-5}\ mol/L$ at 298 K, its solubility product will be 108×10^{-25}

B) pH of an aqueous solution having $[H^+]=10^{-8}M$ is 8.

C) Ammonia is a leveling solvent for stronger acids like HCl, HBr, HI while glacial acetic acid is differentiating solvent.

A) A and B

B) B and C

C) A and C

D) A, B and C

35. The osmotic pressure of blood at 37^0C is 8.21 atm. The amount of glucose (in gm) that should be added per litre for an intravenous injection so that it is isotonic with blood is (GMW of glucose = 180g and R= 0.082 L atm $mol^{-1}K^{-1}$

A) 20 gm B) 36 gm C) 42 gm D) 58 gm

36. Which of the following statement is correct for an aqueous solution of CH_3COOH with concentration 5×10^{-2}M and having $Ka = 2\times10^{-5}$ ($\log 2 = 0.3$)

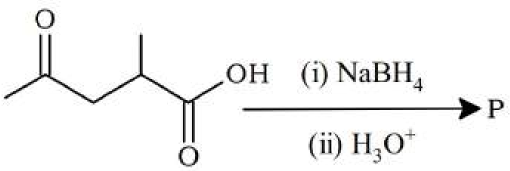
A) Its pH = 3.0

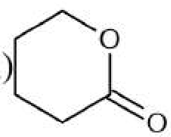
B) If equal moles of NaOH are added then pH =7

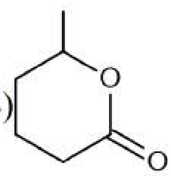
C) It acts as acidic buffer if NaCl is added

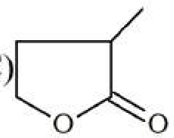
D) It acts as basic buffer on adding NaOH

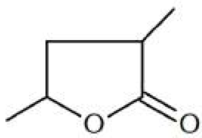
37. The product P in the following reaction is



A) 

B) 

C) 

D) 

38. A compound having the molecular formula $C_6H_4Br_2$ when heated with nitration mixture gave two mono nitro derivatives. The compound is

A) 1, 2–Dibromobenzene B) 1, 4–Dibromobenzene

C) Either 1, 2 or 1, 4–dibromobenzene D) 1,3-di tert butyl benzene

39. The compound of xenon that has the same number of lone pairs as in I_3^- is (on central atom)

A) XeF_2 B) XeO_3 C) XeF_4 D) XeO_4

40. Assertion (A): Aniline on nitration gives meta nitro aniline in maximum yield.

Reason (R) : $-\overset{\oplus}{N}H_3$ acts as meta directing group.

A) Both A and R are true and R is the correct explanation of A

B) Both A and R are true but R is not the correct explanation of A

C) A is true but R is false

D) A is false but R is true

41. According to MO theory which of the list ranks the oxygen species in terms of decreasing Bond order $O_2, O_2^+, O_2^-, O_2^{2-}$

A) $O_2^{2-}, O_2^-, O_2^+, O_2$ B) $O_2^+, O_2, O_2^-, O_2^{2-}$

C) $O_2, O_2^+, O_2^-, O_2^{2-}$ D) $O_2^{2-}, O_2^-, O_2, O_2^+$

42. Which of the following statement is incorrect?

A) $E^0_{AgCl/Ag/Cl^-} = 0.24$ if $E^0_{Ag^+/Ag} = 0.84V$ and K_{sp} $AgCl = 10^{-10}$ (use $\frac{2.303RT}{F} = 0.06$)

B) Λ^0_M for $H^+_{(aq)}$ is highest is aqueous solution

C) In the electrolysis of aqueous Na_2SO_4 if 11.2L of $H_{2(g)}$ is liberated at cathode, then at the anode the volume of $O_{2(g)}$ liberated is 22.4 L at STP

D) In lead – acid battery the equivalent weight of $H_2SO_4 = 98$.

43. In an atom, for a $3p-$ orbital there exist

A) Two spherical nodes

B) Two nonspherical nodes

C) One spherical and one nonspherical nodes

D) One spherical and two nonspherical nodes

44. Which one of the following is incorrect?

A) $\left[Fe(CN)_6\right]^{4-}$ and $\left[Fe(H_2O)_6\right]^{3+}$ have same number of unpaired e^- in central metal ion.

B) A solution of $\left[Ni(H_2O)_6\right]^{2+}$ is green but a solution of $\left[Ni(CN)_4\right]^{2-}$ is colourless

C) $\left[Cr(NH_3)_6\right]^{3+}$ is paramagnetic while $\left[Ni(CN)_4\right]^{2-}$ is diamagnetic

D) d – orbital occupation of the central metal ion in the complex $[CoF_6]^{4-}$ is $t_{2g}^5 e_g^2$

45. Statement-I: Among 13th group elements, Gallium has maximum liquid range.

Statement-II:Oxidation state of Tl in TlI_3 is +3

Choose the correct option.

A) Both Statement-I and Statement-II are correct

B) Both Statement-I and Statement-II are incorrect

C) Statement-I is correct but Statement-II is incorrect

D) Statement-I is incorrect but Statement-II is correct

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**

Marking scheme: +4 for correct answer, -1 in all other cases.

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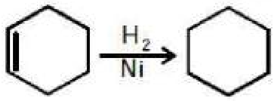
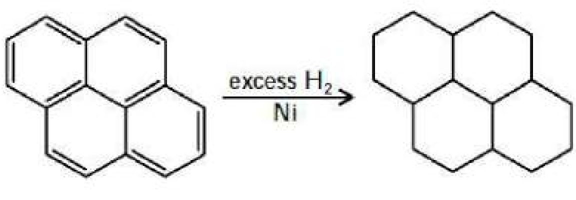
46. Number of $-OH$ groups in one molecule of sucrose is….

47. 100mL of $NaHC_2O_4$ requires 50 mL of 0.1 M $KMnO_4$ solution in acidic medium for its complete oxidation. Volume of 0.1 M NaOH required by 100 mL of same $NaHC_2O_4$ for its complete neutralization is.

48. Given that



$\Delta H = -30\ kcal\ mol^{-1}$;



$\Delta H = -180\ kcal\ mol^{-1}$

Compound ‘A’

What is the resonance energy of‘A’ (in magnitude) is…

49. For a first order reaction $A \longrightarrow B$ the reaction rate at reactant concentration of 0.01 M is found to be 3.0×10^{-5} mol L^{-1} s^{-1}. The half-life period of this reaction in seconds is

50. Consider the following cell reaction

$2Fe(s) + O_2(g) + 4H^+ \longrightarrow 2Fe^{2+}(aq) + 2H_2O(l), E^0 = 1.67\ V$

at $[Fe^{2+}] = 10^{-3} M, P(O_2) = 0.1$ atm and pH = 3, the cell potential (Volts) at 25^0C is $V \times 10^{-3}$.

The value of 'V' is.... ($\frac{2.303RT}{F} = 0.06$)

MATHEMATICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

51. If α,β,γ be the roots of $x^3+\left(a^4+4a^2+1\right)x=x^2+a^2$ (where $a\in R$), then minimum value of $\sum\left\{\frac{\alpha}{\beta}+\left(\frac{\alpha}{\beta}\right)^{-1}\right\}$ is

A) 6 B) 8 C) 4 D) 3

52. The area enclosed by $y=g(x), x-\text{axis}, x=1$ and $x=37$, where $g(x)$ is inverse of $f(x)=x^3+3x+1$ is 297/m. Then value of 'm' will be

A) 4 B) 6 C) 8 D) 2

53. Statement-1: $f(x)=\frac{x^2-5x-9}{3x^2+2x+7}, x\in R$ is not a one-one function.

Statement-2: $f(x)$ is not one-one, if for any $x_1, x_2 \in$ domain of $f(x)$ where $x_1\neq x_2$, $f(x_1)=f(x_2)$.

A) Statement-1is True, Statement-2 is True; Statement-2 is a correctExplanation forStatement-1

B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

C) Statement-1 is True, Statement-2 is False

D) Statement-1 is False, Statement-2 is True

54. The largest value of the non-negative integer 'a' for which

$$\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4} \text{ is}$$

A) -2 B) 0 C) $\sqrt{2}$ D) 2

55. In paper of English there are 5 questions such that the sum of marks is 30 and the marks for any question is not less than 2 and not more than 8. If the number of ways in which marks can be awarded is a 3 digit number xyz then the value of $\frac{2}{5}(x+y+z)$ is equal to ? (Given that marks can be allotted in integers only)

A) 5.4 B) 6.4 C) 7.4 D) 8.4

56. If $R = \{(x,y) : x, y \in Z, x^2 + 3y^2 \le 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is :

A) $\{-2,-1,1,2\}$ B) $\{0,1\}$ C) $\{-2,-1,0,1,2\}$ D) $\{-1,0,1\}$

57. In a ΔABC if $\cos A.\cos B.\cos C = \frac{\sqrt{3}-1}{8}$ and $\sin A.\sin B.\sin C = \frac{3+\sqrt{3}}{8}$, then the value of $\tan A.\tan B + \tan B.\tan C + \tan C.\tan A$ is equal to

A) $5-4\sqrt{3}$ B) $5+4\sqrt{3}$ C) $6+\sqrt{3}$ D) $6-\sqrt{3}$

58. For constant number 'a', consider the function $f(x) = ax + \cos 2x + \sin x + \cos x$ on R (the set of real numbers) such that $f(u) < f(\upsilon)$ for all $u < \upsilon$. If the range of 'a' is $\left[\frac{m}{n}, \infty\right)$, then the minimum value of $(m+n)$ is.

A) 25 B) 35 C) 45 D) 15

59. A is one among the 8 horses in a race. A is to be ridden by one of the 3 jockeys P,Q,R. if P rides A all the horses are equally likely to win, if Q rides A his chances are doubled and if R rides A his chances are tripled. A die is thrown if 1 or 2 or 3 appears then P rides A, if 4 or 5 appears then Q rides A other-wise R rides A. Then the probability that A wins is

A) $\frac{1}{12}$ B) $\frac{3}{16}$ C) $\frac{5}{24}$ D) $\frac{7}{48}$

60. If the variance of 1,2,2,3 is λ, then the value of $\log_{1/2} \lambda$

A) 8 B) 1 C) –1 D) –2

61. Let $z \in C$ and if $A = \left\{z, \arg(z) = \frac{\pi}{4}\right\}$ and $B = \left\{z, \arg(z - 3 - 3i) = \frac{2\pi}{3}\right\}$. Then $n(A \cap B)$ is equal to

A) 1 B) 2 C) 3 D) 0

62. A variable line $y = mx - 1$ cuts the lines $x = 2y$ and $y = -2x$ at points A and B. Then locus of centroid of triangle OAB (O being origin) is a curve passing through origin will be

A) $6x^2 - 9xy - 6y^2 - 3x - 4y = 0$ B) $6x^2 - 9xy - 6y^2 - 4x - 3y = 0$

C) $4x^2 - 8xy - 4y^2 - 2x - 3y = 0$ D) $4x^2 - 8xy - 4y^2 - 3x - 2y = 0$

63. If $\int (x^6 + x^4 + x^2)\sqrt{2x^4 + 3x^2 + 6}\, dx = \frac{(\alpha x^6 + \beta x^4 + \gamma x^2)^{3/2}}{18} + C$ where C is constant then, the value of $\frac{1}{4}(\beta + \gamma - \alpha)$ is equal to

A) 1.75 B) 2.75 C) 0.75 D) 3.75

64. Let k be the greatest integer for which $5m^2 - 16, 2km, k^2$ are distinct consecutive terms of an A.P. (arithmetic progression) where $m \in R$. The common difference of the A.P. is equal to :

A) 25.40 B) 25.60 C) 25.80 D) 25.20

65. The locus of the vertex of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is

A) $xy = 105/64$ B) $xy = 3/4$ C) $xy = 35/16$ D) $xy = 64/105$

66. The function $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$ is

A) increasing in $(0, \infty)$

B) decreasing in $(0, \infty)$

C) increasing in $(0, \pi/e)$, decreasing in $(\pi/e, \infty)$

D) decreasing in $(0, \pi/e)$ increasing in $(\pi/e, \infty)$

67. Statement – 1: Coefficient of $a^2b^3c^4$ in the expansion of $(a+b+c)^8$ is $\frac{8!}{2!3!4!}$

Statement – 2: Coefficient of $a^{\alpha}b^{\beta}c^{\gamma}$, where $\alpha+\beta+\gamma=n$, in the expansion of $(a+b+c)^n$ is $\frac{n!}{\alpha!\beta!\gamma!}$.

A) Statement-1is True, Statement-2 is True; Statement-2 is a correct Explanation for Statement-1

B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

C) Statement-1 is True, Statement-2 is False

D) Statement-1 is False, Statement-2 is True

68. Let f be a differentiable function on $(0,\infty)$ and suppose that $\lim\limits_{x\to\infty}\left(f(x)+f'(x)\right)=L$ where L is a finite quantity, then which of the following must be true ?

A) $\lim\limits_{x\to\infty} f(x)=0$ and $\lim\limits_{x\to\infty} f'(x)=L$

B) $\lim\limits_{x\to\infty} f(x)=\frac{L}{2}$ and $\lim\limits_{x\to\infty} f'(x)=\frac{L}{2}$

C) $\lim\limits_{x\to\infty} f(x)=L$ and $\lim\limits_{x\to\infty} f'(x)=0$

D) Nothing definite can be said

69. Given $\frac{x}{a}+\frac{y}{b}=1$ and ax + by = 1 are two variable lines, 'a' and 'b' being the parameters connected by the relation $a^2+b^2=ab$. The locus of the point of intersection has the equation

A) $x^2+y^2+xy-1=0$

B) $x^2+y^2-xy+1=0$

C) $x^2+y^2+xy+1=0$

D) $x^2+y^2-xy-1=0$

70.

Column I		Column II	
(A)	A is a matrix such that $A^2 = A$. If $(I + A)^8 = I + \lambda A$, then $\lambda + 1$ is equal to	(P)	64
(B)	If A is a square matrix of order 3 such that $\|A\| = 2$, then $\left\|\left(\text{adj}A^{-1}\right)^{-1}\right\|$ is equal to	(Q)	1
(C)	Let $\|A\| = \|a_{ij}\|_{3\times 3} \neq 0$. Each element a_{ij} is multiplied by λ^{i-j}. Let $\|B\|$ the resulting determinant, where $\|A\| = \lambda\|B\|$, then λ is equal to	(R)	256
(D)	If A is a diagonal matrix of order 3×3 is commutative with every square matrix of order 3×3 under multiplication and trace (A) = 12, then $\|A\| =$	(S)	4

A) A – R, B – S, C – Q, D – P B) A – P, B – S, C – Q, D – R

C) A – P, B – S, C – R, D – Q D) A – R, B – P, C – Q, D – S

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. T he answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.**

Marking scheme: +4 for correct answer, -1 in all other cases.

71. If r_1 and r_2 are the maximum and minimum distance of a points on the curve

$10\left(z\bar{z}\right) - 3i\left\{z^2 - \left(\bar{z}\right)^2\right\} - 16 = 0$ from origin, then value of $(r_1 + r_2)$ will be

72. Consider three matrices $A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$. Then the value of

the sum $\text{tr}(A) + \text{tr}\left(\dfrac{ABC}{2}\right) + \text{tr}\left(\dfrac{A(BC)^2}{4}\right) + \text{tr}\left(\dfrac{A(BC)^3}{8}\right) + \ldots + \infty$ is

73. If $f(x) = g(x)|(x-1)(x-2)\ldots\ldots\ldots(x-10)| - 2$ is derivable for all $x \in R$, where

$g(x) = ax^9 + bx^6 + cx^3 + d$, $a, b, c, d \in R$, then value of $f'(-1)$ is

74. The numbers 1,1,1,2,2,2,3,3,3 are placed randomly in a 3×3 matrix. The probability that each row and each column contain all three different numbers is given by $\frac{p}{q}$, where p and q are coprime then value of (p+q) is :

75. The number of real solutions of the equation $\sqrt{1+\cos 2x}=\sqrt{2}\sin^{-1}(\sin x)$ in $-\pi\le x\le\pi$ is

KEY SHEET

PHYSICS

1	**C**	2	**C**	3	**B**	4	**B**	5	**A**
6	**C**	7	**B**	8	**C**	9	**B**	10	**B**
11	**A**	12	**C**	13	**C**	14	**C**	15	**A**
16	**A**	17	**C**	18	**A**	19	**A**	20	**A**
21	**54**	22	**2**	23	**13**	24	**7**	25	**2**

CHEMISTRY

26	**D**	27	**B**	28	**A**	29	**D**	30	**C**
31	**C**	32	**C**	33	**C**	34	**D**	35	**D**
36	**A**	37	**D**	38	**A**	39	**A**	40	**D**
41	**B**	42	**C**	43	**C**	44	**A**	45	**C**
46	**8**	47	**125**	48	**60**	49	**231**	50	**1565**

MATHEMATICS

51	**D**	52	**A**	53	**A**	54	**D**	55	**B**
56	**D**	57	**B**	58	**A**	59	**C**	60	**B**
61	**D**	62	**A**	63	**A**	64	**B**	65	**A**
66	**B**	67	**D**	68	**C**	69	**A**	70	**A**
71	**3**	72	**6**	73	**0**	74	**141**	75	**2**

SOLUTIONS
PHYSICS

1. $L' = \frac{v'^2}{a'}, L = \frac{v^2}{a}$

$$\Rightarrow \frac{L'}{L} = \left(\frac{v'}{v}\right)^2 \left(\frac{a}{a'}\right) = \left(\frac{\alpha^2}{\beta}\right)\frac{1}{\alpha\beta} = \alpha^3 / \beta^3$$

$$\frac{m'}{m} = \frac{F'}{F}\frac{a}{a'} = \frac{1}{\alpha\beta} \times \frac{1}{\alpha\beta} = \frac{1}{\alpha^2\beta^2}$$

Time =Velocity /Acceleration , i.e.,
Momentum =Mass X Velocity

2. $a_c = \frac{V^2}{r}$ & $a_t = \frac{dv}{dt}$, $a_{Net} = \sqrt{a_c{}^2 + a_t{}^2}$

3. $V_x = 4\sin 30^0$ and $V_y = e\,U_y = 0.5\left(4\cos 30^0\right)$

4. $mv_0 \frac{L}{2} = \frac{ml^2}{12} w \Rightarrow w = \frac{6v_0}{\ell}$ & $F = \int_0^{\ell/2} \frac{m}{\ell}.\, w^2\, x\, dx$

5. Conceptual

6. Conceptual

7. $\rho = \frac{2mVN\cos\theta}{A}$

8. $T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{4}{3}ml^2}{2mg.3\frac{l}{4}}} = 2\pi\sqrt{\frac{8l}{9g}}$

9. E is uniform and conservative, hence total energy 'T' is constant, K increases, U decreases.

10. $C = \frac{\epsilon_0 A}{d}$ & $C = 2C_1 + 2C_2 + C_3$

11. $i = \frac{\epsilon}{R+r} = \frac{V}{2R}$

12. $B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$

13. $d\phi = \frac{\mu_0}{2\pi}\frac{I}{x}.a\,d_x$ & $q = \frac{\Delta\phi}{R}$

14. $V = \sqrt{V_R{}^2 + \left(V_L - V_C\right)^2}$ & for LR circuit $Z = \sqrt{R^2 + X_L{}^2}$

15. $I = \frac{1}{2}\epsilon_0\, E^2 C$ & $\hat{S} = \hat{E} \times \hat{B}$

16. $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

17. $-\frac{d}{\lambda} \le n \le +\frac{d}{\lambda}$

18. $m = \frac{e}{2m}(L) = \frac{e}{2m}\left(\frac{nh}{2\pi}\right)$

19. $i = \frac{E}{R_1 + R_3 + R_d}$

20.

If Y = Young's modulus of wire, M = mass of wire, g = acceleration due to gravity, x = extension in the wire, A = area of cross-section of the wire and l = length of the wire.

$$Y = \frac{Mgx}{Al} \Rightarrow \frac{\Delta Y}{Y} = \frac{\Delta M}{M} + \frac{\Delta x}{x} + \frac{\Delta A}{A} + \frac{\Delta l}{l}$$

$$\Rightarrow \quad \frac{\Delta Y}{Y} = \frac{0.01}{3.00} + \frac{0.01}{0.87} + \frac{2 \times 0.001}{0.041} + \frac{0.001}{2.820}$$

$$= 0.064 \Rightarrow \frac{\Delta Y}{Y} \times 100 = \pm 6.4\% \simeq 6\%$$

21. $a = \frac{12 - (3 + 6 + 2)}{5} = 0.2$ & for 2kg

$T - 5 = 0.4 \Rightarrow T = 5.4N$

22. W – E theorem, $W = \int_0^{\infty} f.dx$

23. $K = \sqrt{\frac{I}{M}}$

24. $\left.\begin{matrix} F_s + F_m = M_e a_1 \\ F_s - F_m = Mea_2 \end{matrix}\right\} a_1 - a_2 = \frac{2G(Mm)}{r^2}$

25. $A\left(-\frac{dh}{dt}\right) = \frac{\pi hdgr^4}{8\eta\ell}$

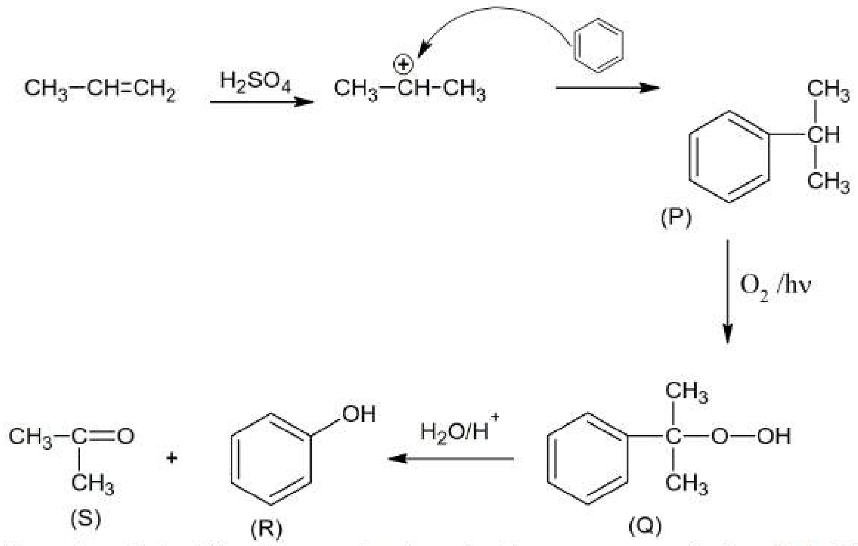
CHEMISTRY

26. P: Markonikoff product with rearrangement
Q: Antimarkonikoff product
R: Markonikoff product without rearrangement

27. Conceptual

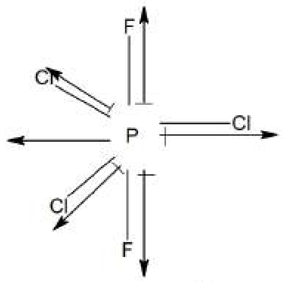
28. Conceptual

29.



30. Lassaigne's test for nitrogen is given by those compounds in which N is bonded to carbon.

31. CH_2Cl_2, NF_3 and ClO_2 have non–zero dipole moment.
PCl_3F_2 has zero dipole moment



Obviouslly $\mu_{res.} = 0$

32. Even though Ce^{+4} is favoured by its noble gas configuration, it is strong oxidant, reverting to common oxidation state of +3. E^0 of Ce^{4+}/Ce^{+3}= 1.74V suggests that Ce^{4+} can oxidize even water(but reaction is slow)

33. $\left[Co(NH_3)_2(NO_2)_2\right]^+$ and $\left[Co(NH_3)_4(NO_2)(ONO)\right]^+$ are linkage isomers.
$\left[Co(NH_3)_4(NO_2)_2\right]^+$ exhibits geometrical isomerism but both the geometrical isomers are optically inactive $\left[Co(NH_3)_4(NO_2)_2\right]Cl$ and $\left[Co(NH_3)_4(NO_2)Cl\right]NO_2$ are ionisation isomers.

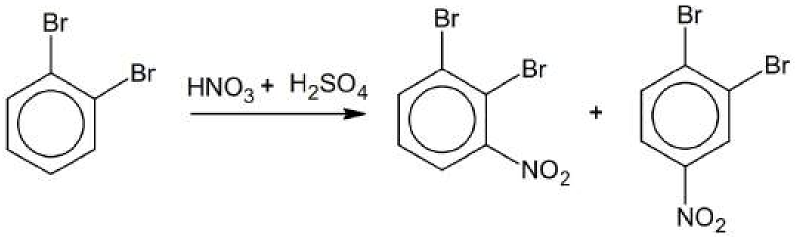
34. $Sb_2S_3 \rightleftharpoons 2Sb^{3+} + 3s^{2-}$
Eq. Conc. 2S 3S (Suppose solubility of Sb_2S_3 is S moles L^{-1})
$K_{sp} = (2s)^2(3s)^3 = 108s^5 = 108 \times \left(10^{-5}\right)^5$
$\Rightarrow K_{sp} = 108 \times 10^{-25}$

35. For isotonic sol $\pi_1 = \pi_2$
8.21 = C x 0.0821 x 310 .
C = 0.323 mol/lit ; wt of glucose = 0.323 x 180 = 58.14

36.

37. Conceptual

38.



39. Conceptual

40. $-NH_2$ acts as both ortho, para and meta directing group in the presence of acid due to salt formation.

41. $O_2^{2-}(BO=1)\, O_2^{-}(BO=1.5)\, O_2(BO=2)\, O_2^{+}(BO=2.5)$

42. Conceptual

43. No. of angular nodes $= \ell$
No. of radial (spherical) nodes $= n-\ell-1$
The no of peaks in radial probability distribution curves$= n-\ell$

44. Conceptual

45. Inert pair effect

46. Conceptual

47. $NaHC_2O_4$ & $KMnO_4$

$$\frac{M_1 \times 100}{5} = \frac{50 \times 0.1}{2}$$

$NaHC_2O_4$ & $NaOH$

$$\frac{M_1 \times 100}{1} = \frac{0.1 \times V}{1}$$

48 $-180 = -RE - 240$

$RE = 180 - 240 = -60 \text{Kcal mol}^{-1}$

49. $k = \frac{Rate}{[A]} = 3 \times 10^{-3} = \frac{0.693}{t_{1/2}}$

50. $E_{cell} = E_{cell}^0 - \frac{0.06}{4} \log \frac{[Fe^{2+}]^2}{(P_{O_2})[H^+]^4}$

MATHS

51. Given equation can be written as

$$x^3 - x^2 + \left(a^4 + 4a^2 + 1\right)x - a^2 = 0$$

$\because \alpha, \beta, \gamma$ are roots $\therefore \sum\alpha = 1, \sum\alpha\beta = a^4 + 4a^2 + 1, \alpha\beta\gamma = a^2$

Now,

$$(\alpha,\beta,\gamma)\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)=3+\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)+\left(\frac{\alpha}{\gamma}+\frac{\gamma}{\alpha}\right)+\left(\frac{\beta}{\gamma}+\frac{\gamma}{\beta}\right)\Rightarrow\left(\sum\alpha\right)\left(\frac{\sum\alpha\beta}{\alpha\beta\gamma}\right)=3+\sum\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$$

$$\Rightarrow \sum\left\{\frac{\alpha}{\beta}+\left(\frac{\alpha}{\beta}\right)^{-1}\right\}=\frac{1\cdot\left(a^4+4a^2+1\right)}{a^2}-3=a^2+\frac{1}{a^2}+1\geq 3$$

52.

Required area, $A = \int_1^{37} g(x)\,dx = \int_1^{37} f^{-1}(x)\,dx$.

Let $f^{-1}(x) = t$ or $x = f(t)$

Using intelligent guessing, $f(3) = 37$ and $f(0) = 1$

$$\therefore \quad A = \int_0^3 t f'(t)\,dt = \left[t f(t)\right]_0^3 - \int_0^3 f(t)\,dt$$

$$= 3f(3) - \int_0^3 (t^3 + 3t + 1)\,dt$$

$$= 111 - \frac{147}{4} = \frac{297}{4}$$

Alternative method:

$f(x) = x^3 + 3x + 1.$

$\therefore \quad f'(x) = 3x^2 + 3 > 0, \forall \in R.$

$\therefore \quad f(x)$ is an increasing function.

Also, $x^3 + 3x + 1 = x$ or $x^3 + 2x + 1 = 0$ has no positive root.

So, line $y = x$ never meet curve $y = f(x)$ for $x > 0$.

Graph of $y = f(x)$ and $y = f^{-1}(x)$ are as shown in the following figure.

When $y = 1, x^3 + 3x + 1 = 1, x = 0.$

When $y = 37, x^3 + 3x = 36, x = 3$

53. Statement-2 is true.

Consider Statement-1.

Let α and β denote the roots of the quadratic $x^2 - 5x - 9 = 0$.

Then, $\alpha \neq \beta$, but $f(\alpha) = f(\beta) = 0$

$\Rightarrow f(x)$ is not one one

$\Rightarrow$ Statement-1 is true.

54. Given

$$\lim_{x\to 1}\left\{\frac{\sin(x-1)+a(1-x)}{(x-1)+\sin(x-1)}\right\}^{\frac{(1+\sqrt{x})(1-\sqrt{x})}{1-\sqrt{x}}}=\frac{1}{4}$$

$$\Rightarrow \lim_{x\to 1}\left\{\frac{\frac{\sin(x-1)}{(x-1)}-a}{1+\frac{\sin(x-1)}{(x-1)}}\right\}^{1+\sqrt{x}}=\frac{1}{4}$$

$$\Rightarrow \left(\frac{1-a}{2}\right)^2=\frac{1}{4}$$

$$\Rightarrow (a-1)^2=1$$

$$\Rightarrow \quad a=2 \text{ or } 0$$

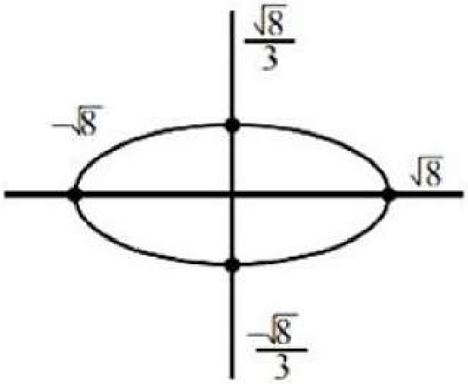
Hence, the maximum value of a is 2.

55. Required no.of ways = coeff of x^{30} in $\left(x^2+x^3+......+x^8\right)^5$ = coeff of x^{30} in $\left(\frac{x^2\left(1+x^7\right)}{1-x}\right)^5$

$= \text{coeff of } x^{20} \text{ in} \left(1-x^7\right)^5(1-x)^{-5} = {}^{24}C_{20}-5\times{}^{17}C_{13}+10\times{}^{10}C_6=826$

56. $\{-1,0,1\}$

$R=\left\{(x,y):x,y\in z, x^2+3y^2\le 8\right\}$



For domain of R^{-1}

Collection of all integral of $y's$

For $x=0, 3y^2\le 8$

$\Rightarrow y\in\{-1,0,1\}$

57. $\sum \tan A \tan B=\sum\frac{\sin A.\sin B.\cos C}{\cos A.\cos B.\cos C}$

$$=\frac{1}{\cos A.\cos B.\cos C}(\sin A.\sin B.\cos C+\cos A.\sin B.\sin C+\sin A.\cos B.\sin C)$$

$$=\frac{1}{\cos A.\cos B.\cos C}\left(\sin B.\left(\sin\left(A+C\right)\right)+\sin A.\cos B.\sin C\right)$$
$$=\frac{1}{\cos A.\cos B.\cos C}\left(1-\cos^2 B+\cos B\sin A.\sin C\right)$$
$$=\frac{1}{\cos A.\cos B.\cos C}\left(1+\cos b\left(\sin A\sin C-\cos B\right)\right)=\frac{1+\cos A.\cos B.\cos C}{\cos A.\cos B.\cos C}$$
$$=\frac{8}{\sqrt{3}-1}+1=4\left(\sqrt{3}+1\right)+1$$

58. We have $f\left(x\right)=ax+\cos 2x+\sin x+\cos x$

As $f'\left(x\right)\geq 0$ for any real number $x\Rightarrow a\geq 2\sin 2x+\sin x-\cos x\ldots.$

$$\text{Let } t=\sin x-\cos x=\sqrt{2}\sin\left(t-\frac{\pi}{4}\right)\Rightarrow -\sqrt{2}\leq t\leq\sqrt{2}.$$

So the inequality can be written as $a>-2t^2+t+2$

Let $g\left(t\right)=-2t^2+t+2=-2\left(t-\frac{1}{4}\right)^2+\frac{17}{8}$

The range of $g\left(t\right)$ for $-\sqrt{2}\leq t\leq\sqrt{2}$ is $g\left(-\sqrt{2}\right)\leq g\left(t\right)\leq g\left(\frac{1}{4}\right)\Rightarrow -2-\sqrt{2}\leq g\left(t\right)\leq\frac{17}{8}$

So, the range of a can be $a\geq \max_{|t|\leq\sqrt{2}}\Rightarrow a\geq\frac{17}{8}\Rightarrow a\in\left[\frac{17}{8},\infty\right)$ Hence,

$\left(m+n\right)_{least}=17+8=25$

59. Let E_1,E_2,E_3 be respectively Events that P,Q,R ride the horse A.

A = Event that horse A win the race

$$P\left(E_1\right)=\frac{1}{2};P\left(E_2\right)=\frac{1}{3};P\left(E_3\right)=\frac{1}{6}$$
$$P\left(A/E_1\right)=\frac{1}{8};P\left(A/E_2\right)=\frac{2}{8};P\left(A/E_3\right)=\frac{3}{8}$$
$$P\left(A\right)=\sum_{i=1}^{3}P\left(E_i\right)P\left(\frac{A}{E_i}\right)=\frac{5}{24}$$

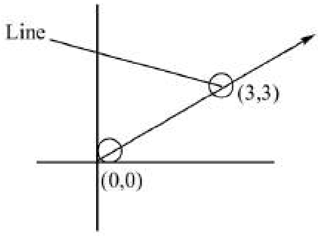
60.

x_i	x_i^2
1	1
2	4
2	4
3	9

$$\therefore \sum x_i=8;\sum x_i^2=18$$

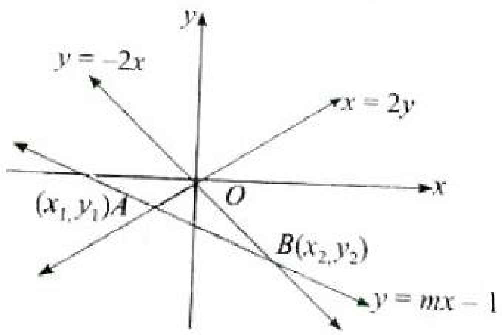
$$\text{Variance }=\frac{\sum x_i^2}{n}-\left(\frac{\sum x_i}{n}\right)^2=\frac{18}{4}-\left(\frac{8}{4}\right)^2=\frac{9}{2}-4=\frac{1}{2}\Rightarrow \qquad \log_{1/2}\frac{1}{2}=1$$

61. We can observe that $3+3i \in A$ but $\notin B$



$\therefore \quad n(A \cap B) = 0$

62.



Solving the variable line $y = mx - 1$ with $x = 2y$, we get

$$x_1 = \frac{2}{2m-1} \qquad (1)$$

Solving with $y = -2x$, we get

$$x_2 = \frac{1}{m+2} \qquad (2)$$

Now, $y_1 + y_2 = m(x_1 + x_2) - 2$

Let the centroid of triangle OAB be (h,k). Then,

$$h = \frac{x_1 + x_2}{3}$$

and $k = \dfrac{y_1 + y_2}{3} = \dfrac{m(x_1 + x_2) - 2}{3}$

or $m = \dfrac{3k+2}{3h}$

So, $3h = x_1 x_2 = \dfrac{2}{2\left(\dfrac{3k+2}{3h}\right) - 1} + \dfrac{1}{\left(\dfrac{3k+2}{3h}\right) + 2}$

[Using (1) and (2)]

or $\dfrac{2}{6k - 3h + 4} + \dfrac{1}{6h + 3k + 2} = 2$

Simplifying, we get the final locus as $6x^2 - 9xy - 6y^2 - 3x - 4y = 0$ which is a hyperbola passing through the origin, as $h^2 > ab$ and $\Delta \neq 0$.

63.

$$\int (x^5 + x^3 + x)\sqrt{2x^6 + 3x^4 + 6x^2}\, dx$$

Let $2x^6 + 3x^4 + 6x^2 = t^2 \Rightarrow 12(x^5 + x^3 + x)\, dx = 2t\, dt$

$$= \frac{1}{12}\int 2t^2 dt = \frac{1}{18}(2x^6 + 3x^4 + 6x^2)^{3/2} + C$$

64. $4km = 5m^2 - 16 + k^2 \qquad k \Rightarrow 5m^2 - 4km + \left(k^2 - 16\right) = 0 \;\; ; m \in R$

$\Delta \geq 0 \Rightarrow 16k^2 - 20\left(k^2 - 16\right) \geq 0 \Rightarrow -4k^2 + 320 \geq 0 \Rightarrow k^2 \leq 80 \Rightarrow k = 8 \Rightarrow m = \frac{12}{5}$ & $m = 4$ for

$m = 4$; common difference = 0 & for $m = \frac{12}{5}$; common difference $= \frac{128}{5} = 25.60$

65. The family of parabolas is

$$y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a = Ax^2 + Bx + C$$

and the vertex is $P\left(-B/2A, -D/4A\right) \equiv \left(h, k\right)$. Therefore,

$$h = -\frac{a^2/2}{2\left(a^3/3\right)} = -\frac{3}{4a}$$

and $k = -\dfrac{\left(a^2/2\right)^2 - \left\{4a^3\left(-2a\right)/3\right\}}{4\left(a^3/3\right)}$

or $h = -\dfrac{3}{4a}$ and $k = -\dfrac{35a}{16}$

Eliminating a, we have $hk = 105/64$.

Hence, the required locus is $xy = 105/64$.

66. $$f'\left(x\right) = \frac{\log\left(e + x\right) \times \frac{1}{\pi + x} - \log\left(\pi + x\right)\frac{1}{e + x}}{\left(\log\left(e + x\right)\right)^2}$$

$$= \frac{\log\left(e + x\right) \times \left(e + x\right) - \left(\pi + x\right)\log\left(\pi + x\right)}{\left(\pi + x\right)\left(e + x\right)\left(\log\left(e + x\right)\right)^2}$$

Since log function is an increasing function and $e < \pi$,

$\log\left(e + x\right), \log\left(\pi + x\right)$

Thus, $\left(e + x\right)\log\left(e + x\right) < \left(e + x\right)\log\left(\pi + x\right) < \left(\pi + x\right)\log\left(\pi + x\right)$ for all $x > 0$.

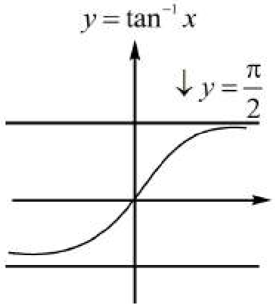
Thus, $f'\left(x\right) < 0$.

Therefore, $f\left(x\right)$ decrease on $\left(0, \infty\right)$

67. $\because \qquad (a + b + c)^n = \sum \frac{n!}{p!.q!.r!}.a^p b^q c^r,\ p + q + r = n$

In statement – 1 p + q + r exceeds n

68.



At $x \to \infty$

$\tan^{-1}x \to \frac{\pi}{2}$

and

$f'(x) \to 0$

$\lim_{x\to\infty}\left(f(x)+f'(x)\right)=L$

69. Let point of intersection be (h,k)

$\Rightarrow \frac{h}{a}+\frac{k}{b}=1 \text{and} \, ah+kb=1 \text{and} \frac{a}{b}+\frac{b}{a}=1$

$\left(\frac{h}{a}+\frac{k}{b}\right)(ah+bk)=1$

$h^2+k^2+hk\left(\frac{b}{a}+\frac{a}{b}\right)=1$

70. (A) $(I+A)^8={}^8C_oI+{}^8C_1IA+{}^8C_2IA^2+.........+{}^8C_8IA^8$

$={}^8C_0I+{}^8C_1A+{}^8C_2A+......+{}^8C_8A^8$

$=I+A\left({}^8C_1+{}^8C_2+......+{}^8C_8\right)$

$=I+A(2^8-1) \;\; \Rightarrow \lambda=2^8-1$

(B) $\left|adj\left(A^{-1}\right)\right|=|A^{-1}|^2=\frac{1}{|A|^2}$

$\left|\left(adj\left(A^{-1}\right)\right)^{-1}\right|=\frac{1}{|adjA^{-1}|}=|A|^2=2^2=4$

(C) $|A|=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$\Rightarrow |B|=\begin{vmatrix} a_{11} & \lambda^{-1}a_{12} & \lambda^{-2}a_{13} \\ \lambda a_{21} & a_{22} & \lambda^{-1}a_{23} \\ \lambda^2 a_{31} & \lambda a_{32} & a_{33} \end{vmatrix} = \frac{1}{\lambda^3}\begin{vmatrix} \lambda^2 a_{11} & \lambda a_{12} & a_{13} \\ \lambda^2 a_{21} & \lambda a_{22} & a_{23} \\ \lambda^2 a_{31} & \lambda a_{32} & a_{33} \end{vmatrix}=|A|$

Hence, $|A|=|B| \Rightarrow \lambda=1$.

(D) A diagonal matrix is commutative with every square matrix, if it is a scalar matrix.

So every diagonal element is 4.

$\therefore \;\; |A|=64$.

$10\ z\bar{z}\ \ -3i\ \ z^2 - \ \bar{z}^{\ 2}\ \ -6=0$

$\text{or}\, 5\left(x^2+y^2\right)+6xy-8=0 \qquad ....(1)$

Let $\left(r\cos\theta, r\sin\theta\right)$ be a point on (1), then

$$5r^2+6r^2\sin\theta\cos\theta-8=0 \Rightarrow r^2=\frac{8}{5+3\sin 2\theta}$$

$\text{Clearly}\, 1\le r^2\le 4 \Rightarrow \le |r| \le 2$

$\therefore r_1 |r|_{\max} = 2\, \text{and}$

$\therefore r_1 |r|_{\max} = 1 \Rightarrow r_1 + r_2 = 3$

72. $$f(\alpha)=\int_{\alpha^{-1}}^{\alpha}\frac{1}{x}\cot^{-1}\left(\frac{x^2-x+1}{2x-3x^2}+\frac{x^2-x+1}{3-2x}\right)dx \qquad ....(1)$$

$$x=\frac{1}{t} \ \Rightarrow dx=-\frac{1}{t^2}dt$$

$$f(\alpha)=\int_{\alpha}^{\frac{1}{\alpha}} t\cot^{-1}\left(\frac{t^2-t+1}{2t-3}+\frac{t^2-t+1}{3t^2-2t}\right)\left(\frac{-1}{t^2}\right)dt=\int_{\frac{1}{\alpha}}^{\alpha}\frac{1}{t}\cot^{-1}\left(\frac{t^2-t+1}{2t-3}+\frac{t^2-t+1}{3t^2-2t}\right)dt$$

$$=\int_{\frac{1}{\alpha}}^{\alpha}\frac{1}{t}\left\{\pi-\cot^{-1}\left(\frac{t^2-t+1}{3-2t}+\frac{t^2-t+1}{2t-3t^2}\right)\right\}dt \qquad ...(2)$$

Equation (1) + (2)

$$2f(\alpha)=\int_{\frac{1}{\alpha}}^{\alpha}\frac{\pi}{t}=\pi\left(\ln\alpha-\ln\left(\frac{1}{\alpha}\right)\right)=2\pi\ln\alpha \Rightarrow \boxed{f(\alpha)=\pi\ln\alpha}$$

Now

$$g(x)=\int_{\ln\frac{1}{\alpha}}^{\ln\alpha}\left(\underbrace{\frac{\left|x^2-3x+2\right|-\left|(x+1)(x+2)\right|}{|x+1|+|x-1|}}_{\text{Odd function i.e } f(-x)=-f(x)}+1\right)dx=\int_{\ln\left(\frac{1}{\alpha}\right)}^{\ln\alpha}1.dx=\ln\alpha-\ln\left(\frac{1}{\alpha}\right)=2\ln\alpha$$

$$f(200)-\frac{\pi}{2}g(50)=\pi\ln(200)-\pi\ln(50)=\pi\ln 4=3\cdot\frac{\pi}{3}\ln 4 \Rightarrow a=3, b=4.$$

73. Clearly, $g(x)=0 \,\forall x\in R$

$\therefore \quad f(x)=-2$

$\therefore \quad f'(x)=0$

74. Total no.of ways $= \dfrac{\lfloor\underline{9}}{\lfloor\underline{3}\lfloor\underline{3}\lfloor\underline{3}}$

Favourable cases $\Rightarrow \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$

$\Rightarrow \lfloor\underline{3} = 6$

$\Rightarrow \lfloor\underline{3}\left(\dfrac{1}{\lfloor\underline{2}} - \dfrac{1}{\lfloor\underline{3}}\right) = 2$

$\Rightarrow$ only one way

$= 6 \times 2 \times 1 = 12$

$$\therefore \text{Probability} = \frac{12}{\dfrac{\lfloor\underline{9}}{\lfloor\underline{3}\lfloor\underline{3}\lfloor\underline{3}}} = \frac{12 \times 6 \times 6 \times 6}{9 \times 8 \times 7 \times 6 \times 120} = \frac{1}{140} \Rightarrow p + q = 141$$

75. $\sqrt{1 + \cos 2x} = \sqrt{2}\sin^{-1}(\sin x)$

$\Rightarrow \quad \sqrt{2}\left|\cos x\right| = \sqrt{2}\sin^{-1}(\sin x)$

$\Rightarrow \quad \left|\cos x\right| = \sqrt{2}\sin^{-1}(\sin x)$

When we draw the graph both functions (shown below) we can actually see that they intersect only at two points $\forall x \in -\pi \le x \le \pi$