18. Area of Circular Region

Let us do 18

1. Question

Today the cow of Amina bibi is fastened to a post with a rope of length 2.1 meter in the vacant field. Let us see by calculating how much maximum area the cow will graze.

Answer

Since the cow is fastened to the post, it can move in the area encircling the post with the length of the rope as the radius of that circular path.

Here, r = 2.1m

Maximum area = area of the circle

$\Rightarrow \pi r^2 = \frac{22}{7} \times (2.1)2$ value of $\pi = \frac{22}{7}$

$= 13.86m^2$

2. Question

Suhana will draw a circle of which perimeter will be 35.2 cm. Let us see by calculating what length of radius Suhana takes to draw a circle and what will be the area of that circular field.

Answer

Given, the perimeter of the circle = 35.2cm

We have to find out the radius and the area of that circle

Perimeter = 2 π r, where r is the radius of the circle

$\Rightarrow 35.2 = 2 \pi r$

$\Rightarrow r = \frac{35.2}{2\pi}$

$\Rightarrow r = \frac{35.2 \times 7}{2 \times 22}$

$= \frac{246.4}{44}$

$= 5.6 \text{ cm}$

Area = πr^2 where r is the radius of circle

$= \frac{22}{7} \times (5.6)^2$

$= \frac{22}{7} \times 31.36 = 98.56 \text{cm}^2$

3. Question

Grandmother of Rekha was made a circular table cover of which area 5544 sq.cm. She wants to paste coloring tape surrounding this circular cover of the table, let us see by calculating how long coloring tape she will buy.

Answer

Given, Area of circular table cloth = 5544cm^2

Since the coloring tape would be placed on the edge of the table cloth so to calculate the length of coloring tape required, we will calculate the perimeter of table cloth.

Now, area of a circle = πr^2, where r is the radius of the circle

$\Rightarrow 5544 = \pi r^2$

$\Rightarrow \frac{5544 \times 7}{22} = r^2 \ (\because \pi = \frac{22}{7})$

$\Rightarrow 1764 = r^2$

$\Rightarrow r = \sqrt{1764} = 42 \text{ cm}^2$

Perimeter = $2\pi r$, where r is the radius of the circle

$\Rightarrow \text{Perimeter} = 2 \times \frac{22}{7} \times 42$

= 264 cm

Therefore, the length of coloring tape required is 264 cm

4. Question

The cost of fencing of our village play ground with railing is Rs. 924 at the rate of Rs. 21 per meter. Let us write by calculating how much sq. meter canvas will be bought for covering the field.

Answer

Given, the cost of fencing = Rs 924

Now cost of fencing = perimeter of playground × rate of fencing ground per meter

⇒ 924 = perimeter × 21

$\Rightarrow \text{perimeter} = \frac{924}{21}$

= 44 m

But, Perimeter = 2πr where r is the radius of the play ground

$\Rightarrow r = \frac{44 \times 7}{2 \times 22}$

= 7 m

Canvas required will be calculated by calculating the area of the circular field

Area of circular field = πr^2 where r is the radius of the play ground

$\Rightarrow \pi r^2 = \frac{22}{7} \times 7 \times 7$

⇒ 154 m^2

5. Question

Faruk will draw a circle of which area will be 616 sq. meter. Let us see by calculating what length of radius Faruk will take to draw a circle and what perimeter he will get.

Answer

Given, area of circle = 616 m^2

But, Area = πr^2, where r is the radius of circle

$\Rightarrow \pi r^2 = 616$

$\Rightarrow r^2 = \frac{616 \times 7}{22}$

$\Rightarrow r^2 = 196$

⇒ r = 14 m

Perimeter of circle = 2π r, where r is the radius of circle

$\Rightarrow 2 \times \frac{22}{7} \times 14$

⇒ 88m

6. Question

Palash and Piyali have drawn two circles the ratio of which length of radius is 4:5. Let us write by calculating the ratio of the area of two circular fields drawn by them.

Answer

let R be the radius of circle drawn by Palash and r be the radius of circle drawn by Piyali

Given the ratio of their radius = 4:5

$\Rightarrow \frac{R}{r} = \frac{4}{5}$

Area of the two circles of Palash and Piyali would be πR^2 and πr^2, respectively.

Ratio of the area = $\frac{\pi R^2}{\pi r^2} = \frac{R^2}{r^2} = \left(\frac{R}{r}\right)^2 = \frac{4^2}{5^2}$

$= \frac{16}{25}$

7. Question

Sumit and Reba have taken two cupper wire having same length. Sumit bent the wire in the form of a rectangular shape of which length and breadth are 48 cm. and 40 cm. But Reba bent the copper wire with same length in the form of a circle. Let us see by calculating which will cover maximum place between the rectangle drawn by Sumit and circle drawn by Reba.

Answer

Perimeter of the rectangular shape figure formed by wire = 2(l + b)

Where, l = length and b = breadth of rectangular figure

Perimeter = 2(48 + 40) = 2 × 88 = 176 m

⇒The length of the wire used for making a circle = 176m

Perimeter of circle = 2πr where r is the radius of circle

⇒2πr = 176

$\Rightarrow r = \frac{176 \times 7}{2 \times 22} = 28\ m$

Area of rectangular shape = l × b = 48 × 40 = 1920 m^2

Area of circular shape = πr^2, where r is the radius of circle

$\Rightarrow \frac{22}{7} \times 28 \times 28$

$= 2464\ m^2$

Since the area enclosed by circular shape is more, wire bend as circle covers maximum place.

8. Question

At the centre of rectangular field of Pioneer atheletic club there is a circular Pool of which length of radius is 14 meter. The length and breadth of rectangular field are 60 meter and 42 meter respectively. Let us see by calculating how much cost it will take for planting grass of remaining place of rectangular field except pool at the rate of Rs. 75 per square meter.

Answer

Given, the field is rectangular in shape with length (l) = 60m and breadth (b) = 42m

In the center lies a circular pool with radius (r) = 14m

Since grass is planted within the field we will calculate area of the rectangular field

Area of rectangular field = l × b = 60 × 42 where l and b are length and breadth of rectangular field, respectively.

$= 2520\ m^2$

Area of circular pool = πr^2 where r is the radius of circular pool

$= \frac{22}{7} \times 14 \times 14$

$= 616\ m^2$

The grass will be planted in the area = Area of rectangular field - Area of circular pool

$= 2520 - 616 = 1904\ m^2$

Cost of planting grass per meter is Rs75/ m^2 = area of field × rate of planting per sq. meter

= 1904 × 75 = Rs 1,42, 800

9. Question

A 7 meter wide path runs outside a circular park of Etalgacha Friends association club along perimeter. Let us write by calculating the area of path, if the perimeter of circular park is 352 meter, let us write by calculating how much cost for concreting the path at the rate of Rs. 20 per square meter.

Answer

Given, the perimeter of a circular park = 352 m

Width of a road outside circular park = 7 m

Now,

Perimeter of circle = 2πr where r is the radius of cirlce

$$352 = 2 \times \left(\frac{22}{7}\right) \times r$$

$$\Rightarrow r = \frac{352 \times 7}{44}$$

⇒ 8 × 7 = 56 m

⇒ r = 56m

Inner radius of the park (r) = 56 m

Outer radius of the Park including the road (R) = width of circular path + r

R = 7 + 56 = 63 m

R = 63 m

Area of the road = π (R^2 - r^2) where R is the outer radius of the circle and r is the smaller radius of circle

= π(R + r) (R - r) [a^2 - b^2 = (a-b)(a-b)]

$$= \frac{22}{7}(63 + 56)(63 - 56)$$

$$= \frac{22}{7} \times 119 \times 7$$

= 22 × 119

Area of the road = 2618 m^2

Cost of concreting the path = area of path × rate of concreting the path per square meter

= 2618 × 20

= Rs 52360

10. Question

Anwar bibi has spent Rs. 2664 at the rate of Rs. 18.50 per meter for fencing of her semicircular land. Let us write by calculating how much cost it will take, if she makes the semicircular land plough at the rate of Rs. 32 per sq. meter.

Answer

cost of fencing semicircular land = Rs 2664

Rate is fencing = Rs 18.50/m

Let r be the radius of semicircular field

Then, perimeter of this field = (πr + 2r) m where r is the radius of the circle

Cost of fencing semicircular land = perimeter of the semicircular land × rate of ploughing per sq meter

2664 = (πr + 2r) × 18.50

⇒ 2664 = r(π + 2) × 18.50 (taking r common from the bracket)

$\Rightarrow \frac{2664}{18.50} = r\left(\frac{22}{7} + 2\right)$ (putting π as $\frac{22}{7}$)

$\Rightarrow \frac{266400}{1850} = r \times \frac{36}{7}$

$\Rightarrow r = \frac{266400 \times 7}{1850 \times 36}$

⇒ r = 28 m

Perimeter of the semicircular land = r (π + 2) where r is the radius

$= r\left(\frac{22}{7} + 2\right)$

$= \frac{36}{7} \times 28$

= 144 m

Area of semicircular field = $\frac{1}{2}\pi r^2$ where r is the radius

$= \frac{1}{2} \times \frac{22}{7} \times 28 \times 28$

= 1232 m^2

Cost of ploughing the field = area of field × rate of ploughing per square meter

= 1232 × 32 = Rs 39,424

11. Question

The time which took today my friend Rajat running with uniform speed to round once of a circular field of school is 30 seconds less while he ran diametrically with same speed. Let us write by calculating the area of field of school if his speed is 9 meter/sec.

Answer

Given, speed of Rajat = 9 m /sec

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

Where distance is the perimeter of circular field

But perimeter = 2π r where r is the radius of circle

⇒ Time taken = $\frac{2\pi r}{9}$

According to the given condition

$\frac{2\pi r}{9} - 30 = \frac{2r}{9}$ Where r is radius of circular field and 2r = diameter

$\Rightarrow \frac{2\pi r - 270}{9} = \frac{2r}{9}$

⇒ 2π r – 270 = 2r (taking LCM and cancelling 9 from both side)

⇒ 2π r – 2r = 270

⇒ r (π -1) = 135

$$\Rightarrow r = \frac{135 \times 7}{22 - 7}$$

$\Rightarrow r = \frac{135 \times 7}{15} = 63\text{m}$

Area of the circular field = πr^2 where r is the radius

$= \frac{22}{7} \times 63 \times 63\ \text{m}^2$

$= 12474\ \text{m}^2$

12. Question

A equally wide path runs out-side the circular field of Bakultala. The length of outer circumference exceeds the inner circumference by 132 meter. If the area of path is 14190 sq. meter, let us write by calculating the radius of circular path.

Answer

Let R be the radius of outer circumference and r be the radius of inner circumference

Outer circumference of circular field = 2π R

And inner circumference of circular field = 2π r

Given,

Length of Outer circular field – length of inner circular field = 132m

circumference of circular field - circumference of circular field = 132 m

$\Rightarrow 2\pi (R- r) = 132m$

$\Rightarrow R - r = \frac{132 \times 7}{2 \times 22}$

$\Rightarrow R - r = 21$ m

This is the radius of the circular path

Given, area of path = 14190 m^2

$\Rightarrow \pi (R- r)^2 = 14190$

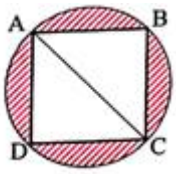
$\Rightarrow (R - r)^2 = \frac{14190 \times 7}{22}$

$\Rightarrow (R-r) = \sqrt{4515}$

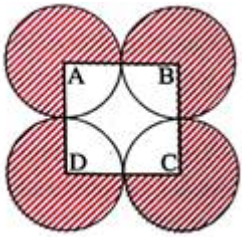
⇒ Radius of circular path = 67.19 m

13. Question

Let us write by calculating the area of shaded region pictures below.

(i) 

ABCD is a square. The length of radius of circle is 7 cm.

(ii) 

The length of radius of each circle is 3.5 cm. The centres of four circles are A, B, C, D respectively.

Answer

NOTE: Area of square = $(side)^2$

Area of circle = πr^2, where 'r' is radius of the circle.

i) Given, ABCD is a square

The length of the radius of circle = 7 cm

Diameter of the circle = 14cm

Let the side of square ABCD = x cm

By Pythagoras theorem (being a square there is angle of 90° between two adjacent sides)

$AB^2 + BC^2 = AC^2$

$\Rightarrow x^2 + x^2 = 14^2$

$\Rightarrow 2x^2 = 196$

$\Rightarrow x^2 = 98$

$\Rightarrow$ Area of square = $x^2 = 98cm^2$

Area of circle = πr^2 where r is the radius of the circle

$= \frac{22}{7} \times 7 \times 7$

$\Rightarrow$ Area of circle = 154 cm^2

Area of shaded region = area of circle – area of square

= 154 – 98 cm^2

= 56 cm^2

ii) Given, Radius of each circle = 3.5cm

And A,B, C, D are center of each circle

Hence, ABCD forms a square with each side of 7cm length

Area of square = $(side)^2$

= 7^2 = 49 cm^2

Area of each circle = πr^2, where r is the radius of circle

$= \frac{22}{7} \times 3.5 \times 3.5$

$= 38.5\ cm^2$

Area of four circle = 4× area of each circle

= 4 × 38.5 = 154 cm^2

Area of shaded region = area of four circles – area of square

= 154 – 49 cm^2

= 105 cm^2

14. Question

Dinesh has made a pie-chart of the students of their class who want to play which game. He has taken length of radius of circle 3.5 cm, let us write by calculating the perimeter and area of each sector of circles.

Answer

Given, radius of circle = 3.5cm

Now, Area of each quarter sector of the circle

Quarter sector means there is angle of 90° at the center of circle

Length of sector = $\frac{90}{360} \times 2 \times \frac{22}{7} \times 3.5$ ($\because$length of sector = $\frac{\theta}{360} \times 2\pi r$)

Where r is the radius of circle

= 269.5 cm

Perimeter of quarter circle = length of sector + 2 × length of radius

= 269.5 + 7

= 276.5 cm

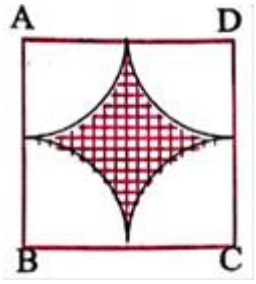
Area of sector = $\frac{90}{360} \times \frac{22}{7} \times 3.5 \times 3.5$ ($\because$ area of sector = $\frac{\theta}{360} \times \pi r^2$)

Where r is the radius of the circle

= 471.62 cm^2

15. Question

Nitu has drawn a square ABCD of which length of each side as 12 cm. My sister has drawn four circular arcs with length of radius 6 cm. centering A, B, C, D like picture besides and she has designed some portion. Let us write by calculating the area of shaded region.



Answer

Given, each side of square = 12cm

⇒ Area of Square ABCD = side^2

⇒ $(12)^2$

⇒ 144cm^2

Let Radius of each quadrant be r

⇒ r = 6 cm (given)

Now, the sum of the area of the four quadrants at the four corners of the square

⇒ 4 × area of each quadrant (area of circle = $\pi r^2$where r is the radius)

⇒ area of quadrant = $\frac{\pi r^2}{4}$

⇒ $4 \times \frac{\pi r^2}{4} = \pi r^2$

⇒ $\frac{22}{7} \times 6 \times 6$

⇒ 113.14cm^2

Now, area of shaded portion

= Area of square, ABCD - sum of the areas of four quadrants at the four corners of the square

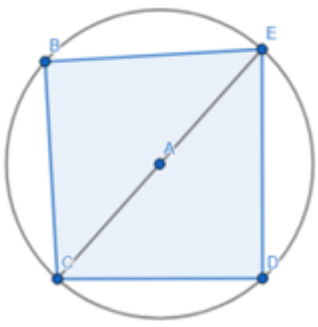
⇒ 144 – 113 .14 cm^2

= 30 .86 cm^2

16. Question

The area of circular field is 154 sq cm. Let us write by calculating the perimeter and area of circumference circular field with square.

Answer



Given: Area of circle = 154 cm^2

We know that

Area of a circle = πr^2

$\Rightarrow \pi r^2 = 154$

$\Rightarrow \frac{22}{7} r^2 = 154$

$\Rightarrow r^2 = 7 \times 7$

$\Rightarrow r = 7$ cm

Also, Diameter of the circle = 2× radius

$\Rightarrow CE = 2 \times 7 = 14$ cm

This acts as the diagonal of the inscribed square.

So, the diagonal of the square BCDE= 14 cm

Let the side of the square be x cm

We know that each angle of a square is 90°.

Using Pythagoras theorem,

$CD^2 + DE^2 = CE^2$

$\Rightarrow x^2 + x^2 = 14^2$

$\Rightarrow 2x^2 = 196$

$\Rightarrow x^2 = 98$

Side of the square $= 7\sqrt{2}$ cm

We know that Area of a Square = side× side

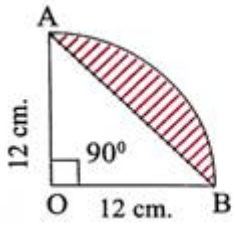
$\Rightarrow$ Area of BCDE $= 98$ cm^2

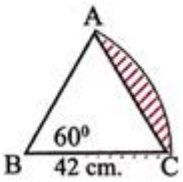
Perimeter of a square = 4× side

$\Rightarrow$ Perimeter of BCDE = $4 \times 7\sqrt{2} = 28\sqrt{2}$ cm

17. Question

Let us write perimeter and area of circular shaded sector below.

(i) 

(ii) 

Answer

(i) Given: Radius of the circle = 12 cm and angle subtended by the arc = 90°

We know that the length of the arc $= \frac{\theta}{360^\circ} \times (2\pi r)$

$\Rightarrow$ Length of arc AB$= \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 12 = \frac{132}{7} = 18.857\text{cm}$

Length of AB:

$\because \angle O = 90^\circ$

Using Pythagoras theorem,

$OA^2 + OB^2 = AB^2$

$\Rightarrow 12^2 + 12^2 = AB^2$

$\Rightarrow AB^2 = 288$

$\Rightarrow AB = 12\sqrt{2}$ cm

$\Rightarrow AB = 16.92$ cm

Perimeter of the circular shaded sector = Length of arc AB + length of AB

$\Rightarrow$ Perimeter of the circular shaded sector$= 18.857 + 16.92 = 35.777$ cm

Now, Area of the segment AB = area of sector ABO – area of triangle ABO

We know that the area of the minor sector $= \frac{\theta}{360^\circ} \times (\pi r^2)$

$\Rightarrow$ Area of ABO $= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times 12 \times 12$

$\Rightarrow$ Area of ABO $= 113.14\ \text{cm}^2$

In ΔABO,

$\angle O = 90^\circ$, AO = BO = 12 cm {radius of the circle}

$\text{Area of } \Delta ABO = \frac{1}{2} \times \text{base} \times \text{height}$

Area of $\Delta ABO = \frac{1}{2} \times 12 \times 12$

$\Rightarrow$ Area of $\Delta ABO = 72\ cm^2$

$\therefore$Area of the segment AB = area of sector ABO – area of triangle ABO

$\Rightarrow$Area of the segment AB = 113.14 – 72

$\Rightarrow$ Area of the segment AB = 41.14 cm^2

(ii) Given: Radius of the circle = 42 cm and angle subtended by the arc = 60°

We know that the length of the arc $= \frac{\theta}{360^\circ} \times (2\pi r)$

$\Rightarrow$ Length of arc $AC = \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 42 = 44cm$

Length of AC:

In ΔABC,

$\angle B = 60^\circ$, AB = BC = 42 cm {radius of the circle}

$\Rightarrow \angle ABC = \angle ACB$ {angles opposite to equal sides are equal}

By the angle sum property of the triangle,

$\angle BAC + \angle ACB + \angle B = 180^\circ$

$\Rightarrow 2\angle BAC = 180^\circ - 60^\circ$

$\Rightarrow \angle BAC = 60^\circ$

Hence, Δ ABC is an equilateral triangle.

$\therefore$ AC = 42 cm

Perimeter of the circular shaded sector = Length of arc AC + length of AC

$\Rightarrow$ Perimeter of the circular shaded sector = 42 + 44 = 86 cm

Now, Area of the segment AC = area of sector ACB – area of triangle ACB

We know that the area of the minor sector $= \frac{\theta}{360^\circ} \times (\pi r^2)$

$\Rightarrow$ Area of ACB = $\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 42 \times 42$

$\Rightarrow$ Area of ACB = 924 cm^2

We know that

Area of a equilateral triangle $= \frac{\sqrt{3}}{4}a^2$, where a is the side of it.

$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{4}(42)^2$$

$\Rightarrow$ Area of $\Delta ABC = 763.83 cm^2$

$\therefore$Area of the segment AC = area of sector ABC – area of triangle ABC

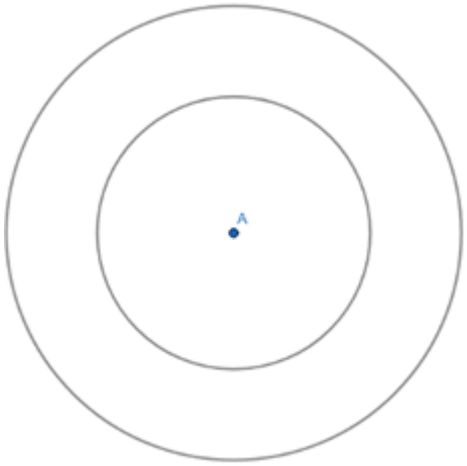
$\Rightarrow$ Area of the segment AC = 924 – 763.83

$\Rightarrow$ Area of the segment AC = 160.17 cm^2

18. Question

Buying a bangle from fair Nila wears in her hand. The Bangle contains 269.5 sq. cm. metal. If the length of outer diameter of bangle is 28 cm, let us write by calculating the length of inner diameter.

Answer



The bangle is like two concentric circles.

Given: Area of the bangle = 269.5 cm^2

Outer diameter of bangle = 28 cm

$\Rightarrow$ Outer radius = 14 cm

Area of the bangle = Area of the larger circle – area of the smaller circle

$\because$The area of a circle = πr^2

$\Rightarrow$ Area of the bangle = $\pi R^2 - \pi r^2$

$$\Rightarrow \text{Area of the bangle} = \frac{22}{7}(R^2 - r^2)$$

$\Rightarrow \frac{22}{7}(14^2 - r^2) = 269.5$

$\Rightarrow \frac{22}{7}(196 - r^2) = 269.5$

$\Rightarrow 196 - r^2 = 85.75$

$\Rightarrow r^2 = 110.25$

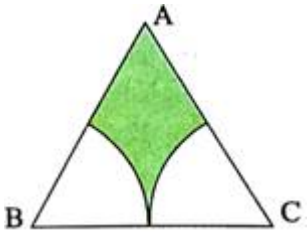
$\Rightarrow r = 10.5$

Inner radius = 10.5 cm

$\Rightarrow$ Inner diameter = 2× 10.5 = 21 cm

19. Question

Protul has drawn an equilateral triangle ABC picture beside of which length of each side is 10 cm. Sumita has drawn three circular arcs centering A, B, C with the length of radius 5 cm. and has coloured some portion at the middle. Let us write by calculating the area of coloured portion.



Answer

Given: ΔABC is an equilateral triangle with side 10 cm, Arcs centered at B and C have radius 5 cm.

We need to find the area of the colored portion.

Area of colored portion = Area of ΔABC – Area of sectors centered at B and C.

∵ ΔABC is an equilateral triangle

We know that

Area of a equilateral triangle $= \frac{\sqrt{3}}{4}a^2$, where a is the side of it.

$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{4}(10)^2$

$\Rightarrow$ Area of ΔABC = 43.3cm^2

For the sectors,

Radius = 5 cm and angle subtended = 60°

{∵ΔABC is an equilateral triangle and each angle of it is 60°}

We know that the area of the minor sector $= \frac{\theta}{360^\circ} \times (\pi r^2)$

$\Rightarrow$ Area of sector centered at B $= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 5 \times 5$

$\Rightarrow$ Area of sector centered at B =13.095 cm^2

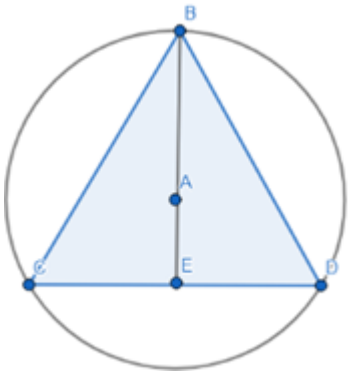
Total area of both the sectors centered at B and C = 13.095 + 13.095 = 26.19 cm^2

$\Rightarrow$ Area of colored portion = 43.3 – 26.19 = 17.11 cm^2

20. Question

Rabeya drew an equilateral triangle with sides 21 cm. on a big paper. Drawing a circle inscribing that triangle coloured the circular region. I write by calculating the area of coloured region.

Answer



Given: Length of side of equilateral triangle = 21 cm

Area of the colored portion = Area of the circum circle - Area of ΔBCD

Height of equilateral triangle $= \frac{\sqrt{3}}{2} \times$ side

$\Rightarrow$ BE $= \frac{\sqrt{3}}{2} \times 21 = 10.5\sqrt{3}$ cm

The centroid of equilateral triangle is at A and lies on height BE.

$\Rightarrow$ BA $= \frac{2}{3}$BE

$\Rightarrow$ BA = $7\sqrt{3}$ cm

So, the radius of the circum circle of this triangle = $7\sqrt{3}$ cm.

∵ Area of a circle = πr^2

Area of the circum circle $= \frac{22}{7}(7\sqrt{3})^2$

$\Rightarrow$ Area of the circum circle = 462 cm^2

We know that

Area of a equilateral triangle $= \frac{\sqrt{3}}{4}a^2$, where a is the side of it.

$$\text{Area of } \Delta BCD = \frac{\sqrt{3}}{4}(21)^2$$

$\Rightarrow$ Area of ΔBCD = 190.96cm^2

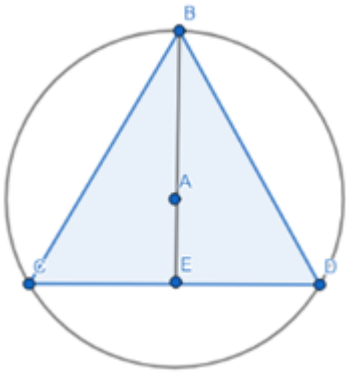
Now, Area of the colored portion = 462 – 190.96

$\Rightarrow$ Area of the colored portion = 271.04 cm^2

21. Question

The area of circumscribing circular region of an equilateral triangle is 462 sq. cm. Let us write by calculating length of each side of this triangle.

Answer



Given: Area of the circumscribing circular region = 462 cm^2

Circle is centered at A.

$\because$ Area of a circle = πr^2

$$\Rightarrow \frac{22}{7}(r)^2 = 462$$

$\Rightarrow r = 7\sqrt{3}$

So, the radius of the circum circle of this triangle = $7\sqrt{3}$ cm.

The centroid of equilateral triangle is at A and lies on height BE.

$$\Rightarrow 7\sqrt{3} = \frac{2}{3}BE$$

⇒ BE = 10.5√3 cm

Height of equilateral triangle = 10.5√3 cm

Height of equilateral triangle $= \frac{\sqrt{3}}{2} \times \text{side}$

$$\Rightarrow \frac{\sqrt{3}}{2} \times \text{side} = 10.5\sqrt{3}\ \text{cm}$$

⇒ Side of the triangle = 21 cm

22. Question

Perimeter of a triangle is 32 cm. and area of inscribing circle is 38.5 sq. cm. Let us write by calculating the area of this triangle.

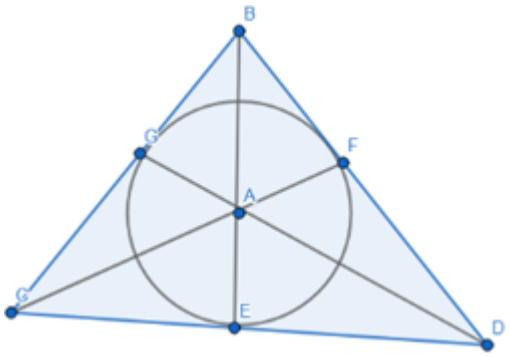
Answer

Given: Area of inscribing circle = 38.5 cm^2

Let BCD is a triangle of perimeter 32 cm. AB, AD and AC are the internal bisectors of respective angles of the triangle. The three internal bisectors meet at A. Perpendiculars drawn from A on sides BC, CD and DB are AG, AE and AF respectively.

AG = AE = AF

Legth of inner radius of triangle = AG



Let AG be r units.

∵ Area of a circle = πr^2

$$\Rightarrow \frac{22}{7}(r)^2 = 38.5$$

⇒ r = 3.5 cm

Now, Area of ΔBCD = area of ΔBCA + area of ΔDCA +area of ΔBDA

$$\Rightarrow \text{Area of } \Delta BCD = \frac{1}{2} \times BC \times r + \frac{1}{2} \times CD \times r + \frac{1}{2} \times BD \times r$$

$$\Rightarrow \text{Area of } \Delta BCD = \frac{1}{2} \times r(BC + CD + BD)$$

$$\Rightarrow \text{Area of } \Delta BCD = \frac{1}{2} \times 3.5(\text{Perimeter of } \Delta BCD)$$

$$\Rightarrow \text{Area of } \Delta BCD = \frac{1}{2} \times 3.5 \times 32$$

$\Rightarrow$ Area $\Delta BCD = 56cm^2$

23. Question

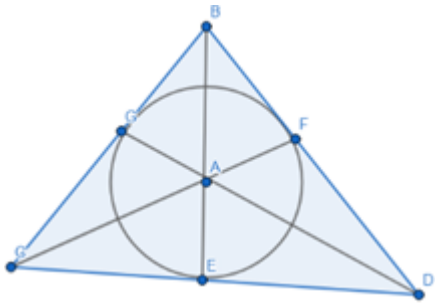
Let us write by calculating the length of radius of incircle and circumcircle of a triangle of which sides are 20 cm, 15 cm, 25 cm. Let us calculate the area of the regions bounded by incricle and circum-circle.

Answer

Let BCD is a triangle of sides BC, BD and DC equal to 20 cm, 15 cm, 25 cm. AB, AD and AC are the internal bisectors of respective angles of the triangle. The three internal bisectors meet at A. Perpendiculars drawn from A on sides BC, CD and DB are AG, AE and AF respectively.

AG = AE = AF

Legth of inner radius of triangle = AG



Let AG be r units.

The given triangle forms a right angled triangle with $\angle B = 90°$

$$\text{Also, } DE = \frac{1}{2}CD = 12.5 \text{ cm}$$

Using Pythagoras theorem,

$BE^2 + DE^2 = BD^2$

$\Rightarrow BE^2 + 12.5^2 = 15^2$

$\Rightarrow BE^2 = 68.75$

$\Rightarrow BE = 8.3$ cm

The centroid of triangle is at A and lies on height BE.

$\Rightarrow AE = \frac{1}{3}BE$

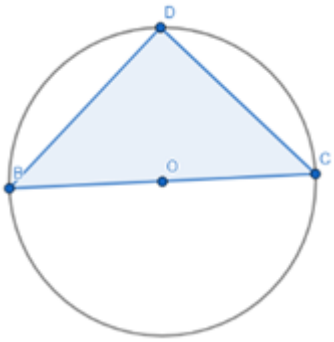
$\Rightarrow$ AE = 2.767 cm

r = 2.767 cm

$\because$ Area of a circle = πr^2

$\Rightarrow$ Area of the incircle $= \frac{22}{7} \times (2.767)^2$

$\Rightarrow$ Area of incircle = 24 cm^2



$\because$The given triangle forms a right angled triangle

$\therefore$ BC will act as the diameter of the circumcircle centered at O.

Diameter = 25 cm

$\Rightarrow$ Radius = 12.5 cm

$\because$ Area of a circle = πr^2

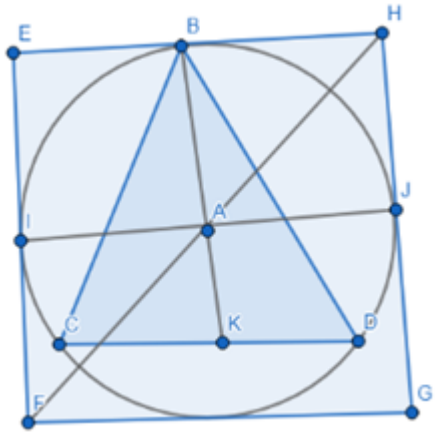
$\Rightarrow$ Area of the circumcircle $= \frac{22}{7} \times (12.5)^2$

$\Rightarrow$ Area of circumcircle = 491 cm^2

24. Question

Jaya drew an incircle of a square. That circle is also circumscribe an equilateral triangle of which each length of side of $4\sqrt{3}$ cm. Let us write by calculating the length of diagonal of square.

Answer



Given: Length of side of equilateral triangle = $4\sqrt{3}$ cm

Height of equilateral triangle $= \frac{\sqrt{3}}{2} \times \text{side}$

$$\Rightarrow BK = \frac{\sqrt{3}}{2} \times 4\sqrt{3} = 6cm$$

The centroid of equilateral triangle is at A and lies on height BK.

$$\Rightarrow BA = \frac{2}{3}BK$$

$\Rightarrow$ BA = 4 cm

So, the radius of the circum circle of this triangle = 4 cm.

Now, diameter of the circle = 2× radius = 8 cm

$\Rightarrow$ IJ = 8 cm

From the figure, IJ = EH = side of the square

$\Rightarrow$ Side of the square = 8 cm

We know that the diagonal of a square =$\sqrt{2}$× side

$\Rightarrow$ FH = $8\sqrt{2}$ cm

25. Question

Sumit cut a wire into two equal parts. One part he bent in the form of square and other part bent in the form of circle. If the area of circle exceeds that of the square by 33 sq. cm. Let us write by calculating the original length of the wire.

Answer

It is given that the wire was cut into two equal parts which means perimeter of square and circle will be equal.

Perimeter of a square = 4× side

Circumference of a circle = $2\pi r$

Let s be the side of the square and r be the radius of the circle.

According to the question,

$4s = 2\pi r$

$\Rightarrow 2s = \pi r$

$\Rightarrow 2s = \frac{22}{7}r$

$\Rightarrow s = \frac{11}{7}r$ ..(1)

Also, it is given that the area of circle exceeds that of the square by 33 sq. cm.

Area of a circle = πr^2

Area of a square = side× side

$\Rightarrow \pi r^2 = s^2 + 33$

From (1),

$\Rightarrow \frac{22}{7} \times r \times r = (\frac{11}{7}r)^2 + 33$

$\Rightarrow \frac{22}{7}r^2 = \frac{121r^2}{49} + 33$

$\Rightarrow \frac{22}{7}r^2 - \frac{121r^2}{49} = 33$

$\Rightarrow \frac{(154-121)r^2}{49} = 33$

$\Rightarrow 33r^2 = 49 \times 33$

$\Rightarrow r = 7$ cm

Circumference of a circle = $2\pi r$

$\Rightarrow \text{Circumference} = 2 \times \frac{22}{7} \times 7 = 44$ cm

∵ the wire was cut into two equal parts which means perimeter of square and circle will be equal.

Total length of wire = 44 + 44 = 88 cm

26 A. Question

If area of circular field is X sq unit, perimeter is Y unit and length of diameter is Z unit then the value of $\frac{X}{YZ}$ is

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. 1

D. $\frac{1}{8}$

Answer

Given that area of a circle = X sq unit, circumference = Y unit and diameter = Z unit

We know that

Circumference of a circle = π× diameter

⇒ Y = π Z ...(1)

Area of a circle = πr^2

$\text{Radius } = \frac{1}{2}Z$

$\Rightarrow X = \pi\frac{Z^2}{2}$

$\Rightarrow X = \frac{\pi Z^2}{4}$...(2)

$\Rightarrow \frac{X}{YZ} = \frac{\frac{\pi Z^2}{4}}{\pi Z \times Z}$

$\Rightarrow \frac{X}{YZ} = \frac{1}{4}$

26 B. Question

The ratio of area of two square circumscribe and inscribe by a circle is

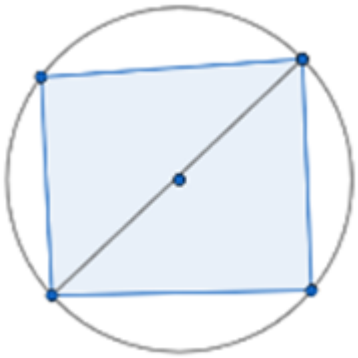
A. 4 : 1

B. 1 : 4

C. 2 : 1

D. 1 : 2

Answer



For a square inscribing the circle,

Diagonal of the square = diameter of the circle

Let the radius of the circle be r

⇒ Diagonal of the circle = 2r

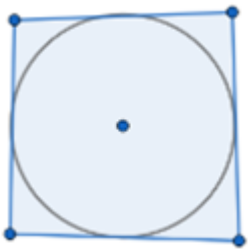
We know that the diagonal of the square = $\sqrt{2}$ side

⇒$\sqrt{2}$ side = 2r

⇒Side of the inscribed circle = $\sqrt{2}$r

Area of the inscribed square = side× side

⇒ Area of the inscribed circle = $2r^2$



For a square circumscribing the circle,

Side of the square = diameter of the circle

⇒ Side of the circumscribed circle = 2r

Area of the circumscribed square = side× side

⇒Area of the circumscribed circle = $4r^2$

Ratio of area of two square circumscribe and inscribe by a circle $= \frac{4r^2}{2r^2}$

⇒ Ratio = 2:1

26 C. Question

The numerical value of perimeter and area of a circular field is equal. The length of diagonal of square circumscribe by a circle is

A. 4 unit

B. 2 unit

C. $4\sqrt{2}$ unit

D. $2\sqrt{2}$ unit

Answer

Given that the perimeter and area of a circular field are equal.

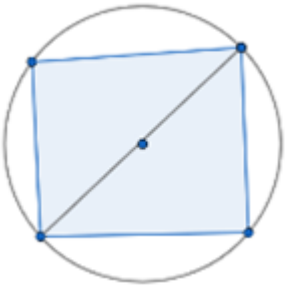
We know that circumference of the circle = $2\pi r$

Also, the area of a circle = πr^2

According to the question,

$2\pi r = \pi r^2$

⇒ r = 2 units



For a square inscribing the circle,

Diagonal of the square = diameter of the circle

⇒ Diagonal of the circle = 2r = 4 units

26 D. Question

The ratio of the area of an equilateral triangle inscribing a circle is

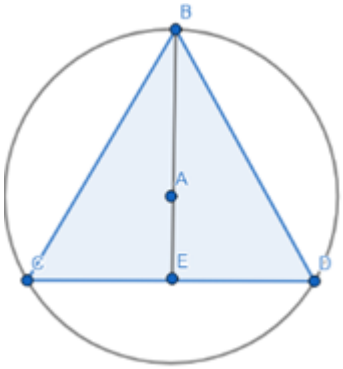
A. 4 : 1

B. 1 : 4

C. 2 : 1

D. None of the above

Answer



Let the length of side of equilateral triangle be s units.

Height of equilateral triangle $= \frac{\sqrt{3}}{2} \times$ side

$\Rightarrow BE = \frac{s\sqrt{3}}{2}$ units

The centroid of equilateral triangle is at A and lies on height BE.

$\Rightarrow BA = \frac{2}{3}BE$

$\Rightarrow BA = \frac{s\sqrt{3}}{3}$ units $=$ Radius of the circle

$\because$ Area of a circle = πr^2

Area of the circum circle $= \frac{22}{7}(\frac{s\sqrt{3}}{3})^2$

$\Rightarrow$ Area of the circum circle $= \frac{22s^2}{21}$ sq units

We know that

Area of a equilateral triangle $= \frac{\sqrt{3}}{4}a^2$, where a is the side of it.

$\text{Area of } \Delta BCD = \frac{\sqrt{3}}{4}(s)^2$

So the ratio of the area of the equilateral triangle and the circle

$= \frac{\frac{\sqrt{3}}{4}(s)^2}{\frac{22s^2}{21}} = \frac{21\sqrt{3}}{88}$

26 E. Question

The inner diameter and external diameter of an Iron ring plate are 20 cm and 22 cm. The quality of iron plate in the ring is

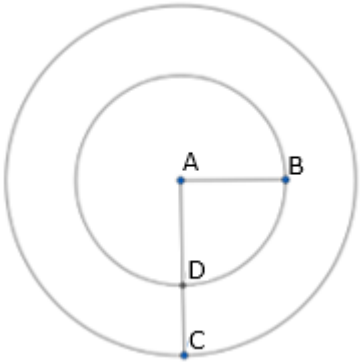
A. 22 sq.cm.

B. 44 sq.cm.

C. 66 sq.cm.

D. 88 sq.cm.

Answer



Given: Inner diameter = 20 cm

Outer diameter = 22 cm

⇒ Inner radius = 10 cm and outer radius = 11 cm

Area of such concentric circles = Area of the larger circle – area of the smaller circle

∵The area of a circle = πr^2

⇒ Area of the iron plate = πR^2 - πr^2

⇒ Area of the iron plate = $\frac{22}{7}(R^2 - r^2)$

⇒ Area of the iron plate = $\frac{22}{7}(11^2 - 10^2)$

⇒ Area of the iron plate = $\frac{22}{7}(121 - 100)$

⇒ Area of the iron plate = $\frac{22}{7} \times 21$

⇒ Area of the iron plate = 22× 3 = 66cm^2

27 A. Question

If the length of radius of a circular field was increased by 10%, let us write by calculating what per cent it increase the area of circular field.

Answer

Let the radius of the circular field be r units.

The area of a circle = πr^2

Now, increasing the radius to 10%

$\Rightarrow$ New radius $= (1 + \frac{10}{100})r$

New radius =1.1r

Area of new circle = $\pi(1.1r)^2$

$\Rightarrow$ Area of new circle = $1.21\pi r^2$

Increase in area =$1.21\pi r^2$ - πr^2 =$0.21\pi r^2$

Per centage increase in the area of circular field $= \frac{0.21\pi r^2}{\pi r^2} \times 100$

$\Rightarrow$ Percentage increase = 21%

27 B. Question

If the perimeter of a circular field was decreased by 50%, let us write by calculating what per cent it decreases the area of circular field.

Answer

Let the radius of the circular field be r units.

Circumference = 2π r

The area of a circle = πr^2

Now, decreasing the perimeter to 50%

$\Rightarrow$ New perimeter $= \left(1 + \frac{50}{100}\right)2\pi r$

New perimeter =$3\pi r$

$\Rightarrow$ New radius = 1.5r

Area of new circle = $\pi(1.5r)^2$

$\Rightarrow$ Area of new circle = $2.25\pi r^2$

Increase in area =$2.25\pi r^2$ - πr^2 =$1.25\pi r^2$

Per centage increase in the area of circular field $= \frac{1.25\pi r^2}{\pi r^2} \times 100$

$\Rightarrow$ Percentage increase = 125%

27 C. Question

The length of radius of a circular field is r meter. If the area of other circle is x times of first circle, let us see by calculating how length is of radius of other circle.

Answer

Given that the length of the circular field = r meter

The area of the circular field = πr^2

Also, given that the area of the other circle = $x\pi r^2$

We know that area of the circle = $\pi(\text{radius})^2$

Then, $\pi \times(\text{radius of other circle})^2 = x\pi r^2$

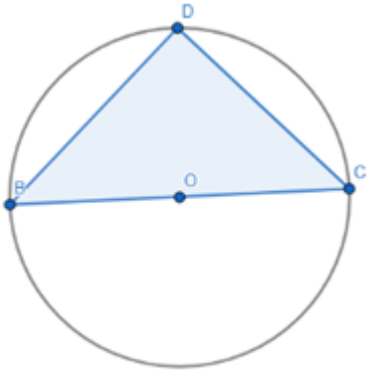
$\Rightarrow$ $(\text{Radius of the other circle})^2 = x\ r^2$

$\Rightarrow$ Radius of the other circle = $r\sqrt{x}$ meter

27 D. Question

Let us calculate the area of a circular region circumscribe a triangle of which sides are 3 cm, 4 cm and 5 cm.

Answer



By the pythagoras theorem,

$3^2 + 4^2 = 9+16 = 25$

And $5^2 = 25$

$\Rightarrow$ The given triangle forms a right angled triangle

$\therefore$ BC will act as the diameter of the circumcircle centered at O.

Diameter = 5 cm

$\Rightarrow$ Radius = 2.5 cm

$\because$ Area of a circle = πr^2

⇒ Area of the circumcircle $= \frac{22}{7} \times (2.5)^2$

⇒ Area of circumcircle = 19.64 cm^2

27 E. Question

Three circular plate were cut off from a tin plate with equal width if the ratio of length of diameter of three circles is 3:5:7, let us see by calculating the ratio of their weight.

Answer

Given ratio of diameter of three circles =3:5:7

$\Rightarrow$ Ratio of their radius $= \frac{3}{2} : \frac{5}{2} : \frac{7}{2}$

⇒ Ratio of their radius = 3:5:7

∵ Area of a circle = πr^2

⇒ Ratio of their areas = 3× 3 : 5× 5 : 7× 7

⇒ Ratio of their areas = 9 : 25 : 49