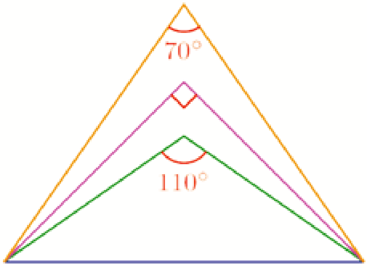
2. Circles

Questions Pg-40

1. Question

Suppose we draw circle with the bottom side of the triangles in the picture as diameter. Find out whether the top corner of each triangle is inside the circle, on the circle or outside the circle.

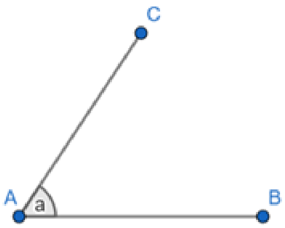


Answer

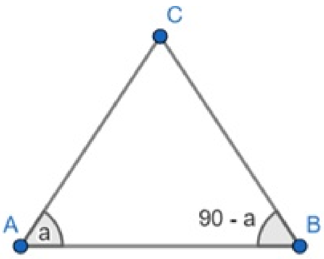
● To see how right triangle makes a circle?

1. Draw a line of 5 cm.

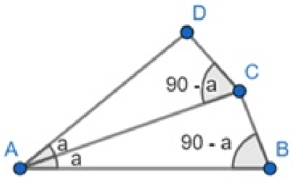
2. Using Line with given length, make a slider from 0 to 180° with increment 5.



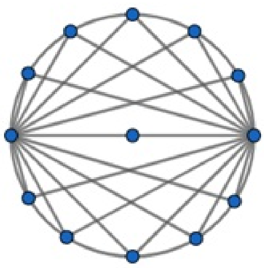
3. Draw an angle ‘a’ at one end and draw ‘90° - a’ clockwise at the other end to make a triangle.



4. Draw several such triangles.



5. Apply trace on to the top vertex and sides of a triangle to get a circle.



● To see angle in a semicircle is right.

1. To show ∠APB = 90° . Firstly join P to the centre of a circle O which splits the angle at P into two parts.

2. Δ AOP and Δ BOP are isosceles triangle.

Thus, ∠A = x° and ∠B = y°

3. In ΔAPB, the sum of angle is 180°

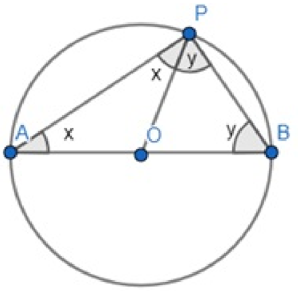
$\Rightarrow x + y + (x + y) = 180°$

$\Rightarrow 2(x + y) = 180°$

$\Rightarrow x + y = 90°$

$\Rightarrow \angle APB = x° + y° = 90°$

Thus, angle in a semicircle is right angle.



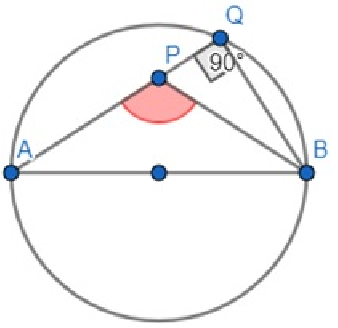
● To see for any point inside the circle, such an angle is larger than right angle.

1. To show ∠APB > 90° . Extend one of the lines to meet the circle. Join this point to the other end of the diameter.

2. APB is the exterior angle at P of triangle AQP.

∠APB = sum of interior angle at Q and B

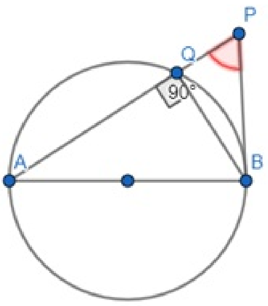
3. The angle at Q is right angle. So, ∠APB is larger than 90°



● To see for any point outside the circle, such an angle is smaller than right angle.

1. To show ∠APB < 90° . APB is the interior angle at P of triangle PQB. The right angle AQB is an exterior angle.

2. So, ∠APB is smaller than 90°



Now, in our given question we can see that, As seen above, point on a circle, such an angle is a right angle.

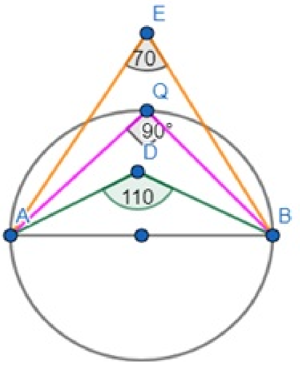
Point outside the circle , such an angle is larger than 90°

Point inside the circle, such an angle is smaller than 90°

Thus, as ∠ACB = 110° > 90° . Therefore point C lies inside the circle.

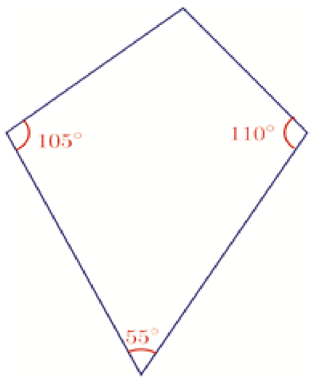
As, ∠ADB = 90° . Therefore point D lies on the circle.

As, ∠AEB = 70° . Therefore point E lies outside the circle.



2. Question

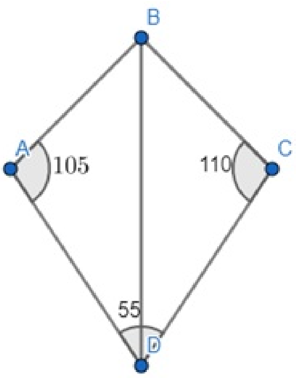
For each diagonal of the quadrilateral shown, check whether the other two corners are inside, on or outside the circle with that diagonal as diameter.



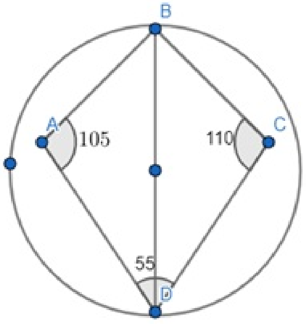
Answer

Now we have been given a quadrilateral ABCD:

<u>Case 1 :</u> let us now draw a diagonal BD in quadrilateral ABCD



● Draw a circle considering, BD as a diagonal.



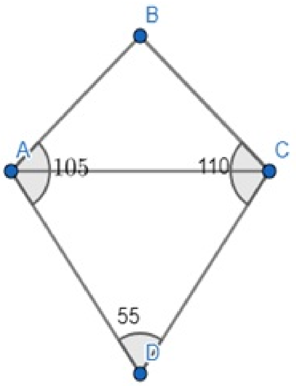
● As ∠A and ∠C are larger then 90° . Thus, point A and C lies inside the circle.

(As seen above, point on a circle, such an angle is a right angle.

Point outside the circle , such an angle is larger than 90°

Point inside the circle, such an angle is smaller than 90°)

<u>Case 2:</u> let us now draw a diagonal AC in quadrilateral ABCD



Also, By angle sum property of quadrilateral

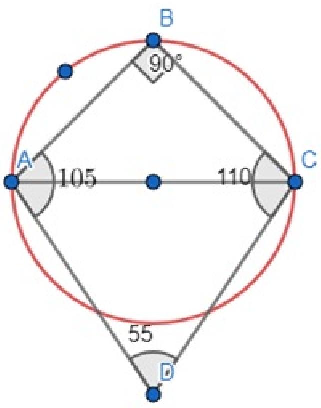
⇒ ∠A + ∠B + ∠C + ∠D = 360°

⇒ 105° + ∠B + 110° + 55° = 360°

⇒ ∠B + 270° = 360°

⇒ ∠B = 360° – 270° = 90°

● Draw a circle considering, AC as a diagonal.



● As ∠B is 90° . Thus, point B lies on the circle.

As ∠D is smaller then 90°. Thus, point D lies outside the circle.

(As seen above, point on a circle, such an angle is a right angle.

Point outside the circle , such an angle is larger than 90°

Point inside the circle, such an angle is smaller than 90°)

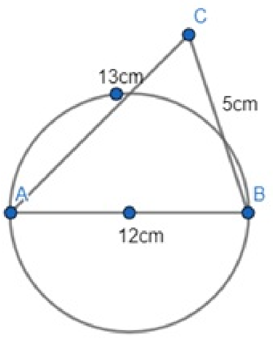
3. Question

If circles are drawn with each side of a triangle of sides 5 centimetres, 12 centimetres and 13 centimetres, as diameters, then with respect to each circle, where would be the third vertex?

Answer

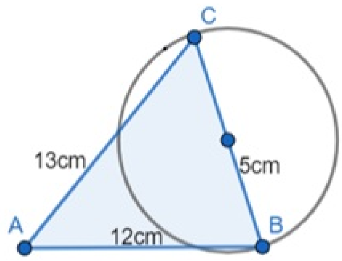
Let, ABC be a triangle with sides AB= 12cm BC = 5cm and AC = 13cm

Case 1: Let us draw a circle considering AB = 12cm as a diameter. From fig (a)



Then, point C lies outside the circle.

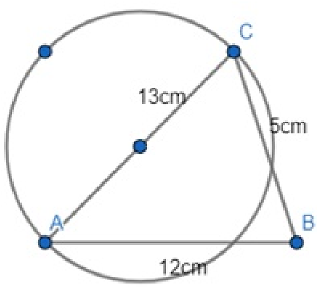
Case 2: Let us draw a circle considering BC = 5cm as a diameter. From fig (b)



Then, point A lies outside the circle.

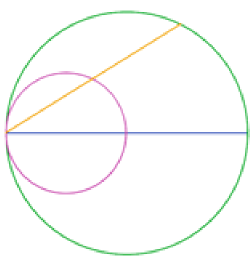
Case 3: Let us draw a circle considering AC = 13cm as a diameter. From fig (c)

Then, point B lies outside the circle.



4. Question

In the picture, a circle is drawn with a line as diameter and a smaller circle with half the line as diameter. Prove that any chord of the larger circle through the point where the circles meet is bisected by the small circle.

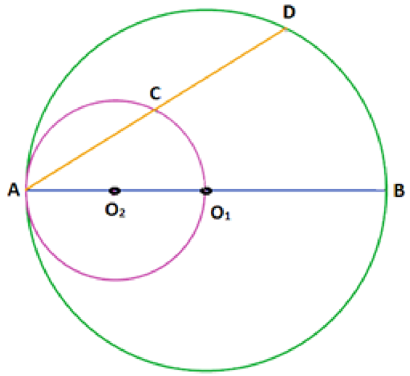


Answer

The figure is given below. O_1 and O_2 are the centres of the big and the small circles respectively.

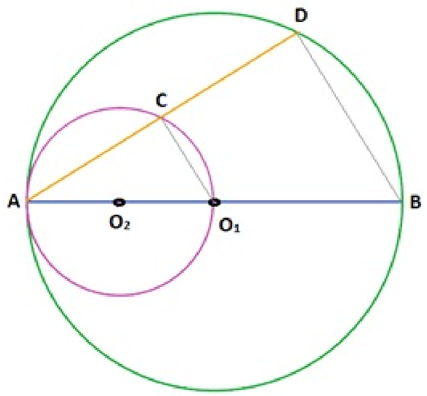
Now, AB is the diameter of big circle.

AO_1 is the diameter of the small circle.



Construction:

Let us join O_1C and DB. The constructed image:



Now, since the diameter subtends a right angle in the circle at any point.

Hence, $\angle ACO_1 = 90°$ And, $\angle ADB = 90°$

Now, in ΔACO_1 and ΔADB,

$\angle A$ is common to both the triangles.

AO �$_1$ = BO_1 (the radii of the big circle)

$\angle ACO_1 = \angle ADB = 90°$

Hence,

in $\Delta ACO_1 \sim \Delta ADB$,

$\Rightarrow \frac{AC}{AD} = \frac{AO_1}{AB}$

Since, the ratio is always equal, we can say that any chord of the larger circle through the point where the circles meet is bisected by the small circle.

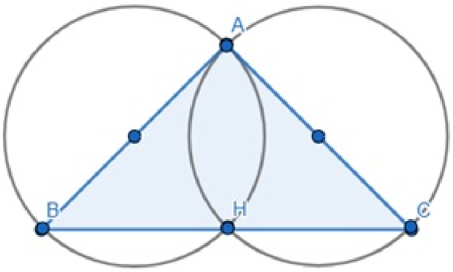
5. Question

Prove that the two circles drawn on the two equal sides of an isosceles triangle as diameters pass through the midpoint of the third side.

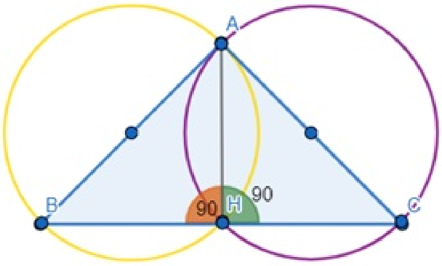
Answer

Let us consider the isosceles triangle ABC with side AB= AC.

i. Draw a circle with diameter AB and draw another circle with diameter AC. Thus, we get



ii. Join AH. We get,



As we can see in yellow circle,

∠AHB = 90°

(∵ angle in a semicircle is right angle)

Similarly, in purple circle,

∠AHC = 90° (∵ angle in a semicircle is right angle)

Consider, Δ AHB and Δ AHC

AB = AC (∵ ABC is isosceles triangle)

AH = AH (∵ common side)

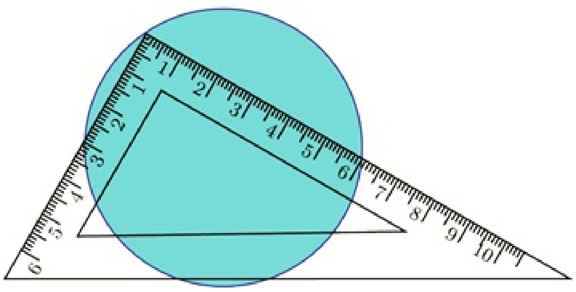
∠AHB = ∠AHC 90°

⇒ Δ AHB ≅ Δ AHC (by RHS congruent rule)

Thus, BH = BC (by CPCT)

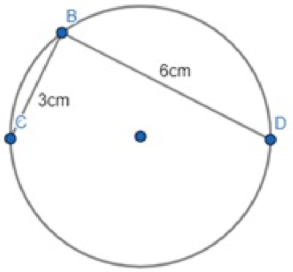
6. Question

Use a calculator to determine upto two decimal places, the perimeter and the area of the circle in the picture.

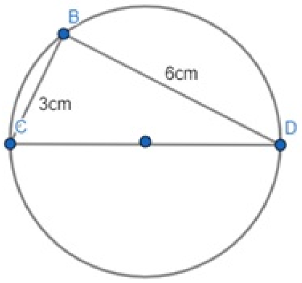


Answer

In the given fig, we can see, BC = 3cm and BD = 6cm



i. Join CD, thus we obtain a diameter CD.



ii. As ∠CBD = 90° (∵ angle in a semi circle is right angle)

Therefore by Pythagoras theorem

$BC^2 + BD^2 = CD^2$

$3^2 + 6^2 = CD^2$

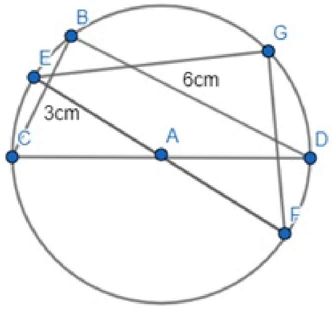
$CD^2 = 9 + 36 = 45$

CD = 3√5

Thus, diameter = CD = 3√5

Radius, $r = \frac{CD}{2} = \frac{3\sqrt{5}}{2}$

iii. Now, change the position of a set square, and draw another diameter to get the center.



Thus, A is a centre.

Hence, radius, $CA = AD = r = \frac{CD}{2} = \frac{3\sqrt{5}}{2}$

We need to find perimeter of a circle?

We know that,

Perimeter of a circle = 2πr where r = radius

$= 2\pi \times \frac{3\sqrt{5}}{2} = 21.06 \text{ cm}$

We need to find area of a circle?

We know that,

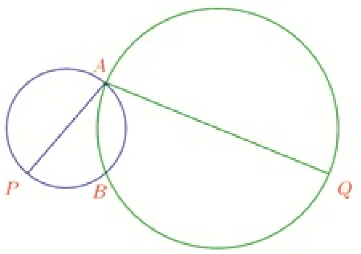
Area of a circle = π r^2 where, r = radius

$$= \pi \left(\frac{3\sqrt{5}}{2}\right)^2$$

$$= \frac{\pi 45}{4} = 35.32 \text{ cm}^2$$

7. Question

The two circles in the picture cross each other at A and B. The points P and Q are the other ends of the diameters through A.

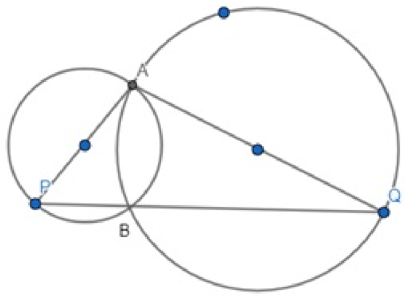


i) Prove that P, B, Q lie on a line.

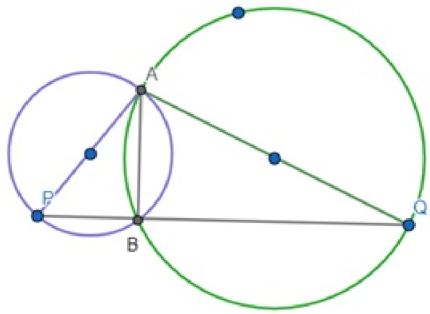
ii) Prove that PQ is parallel to the line joining the centres of the circles and is twice as long as this line.

Answer

i) Construction: Join P to B to Q



Join AB, we get,



In, blue circle, ∠ABP = 90° (since, angle in semicircle is right angle)

In, green circle, ∠ABQ = 90° (since, angle in semicircle is right angle)

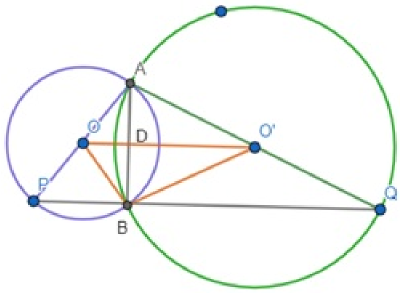
Thus, ∠ABP + ∠ABQ = 90 + 90 =180°

Hence, P, B, Q lie on a line.

ii) In the above diagram,

iv. Join, center O and O’ and OB and O’B, we get

Let D be the point of intersection of OO’ and AB



In Δ AOO’ and Δ BOO’

AO = BO

(radius of a blue circle)

AO’ = BO’

(radius of a green circle)

OO’ = OO’

(common side)

∴ Δ AOO’ ≅ Δ BOO’

(by SSS congruent rule)

∠AOO’ = ∠BOO’ (by CPCT) ….(1)

In Δ AOD and Δ BOD

AO = BO (radius of a blue circle)

∠AOO’ = ∠BOO’ (from (1))

OD = OD (common side)

∴ Δ AOD ≅ Δ BOD (by SAS congruent rule)

∠ODA = ∠ODB (by CPCT) ….(2)

Since, sum of angles on a line is 180°

∴ ∠ODA + ∠ODB = 180 (∵ AB is a line)

⇒ ∠ODA + ∠ODA = 180 (from (2))

⇒ 2∠ODA = 180

⇒ $\angle ODA = \frac{180}{2} = 90°$ …..(3)

⇒ ∠ODB = ∠ODA = 90° (from (2) and (3))

As AB is a transversal line,

Also, We know that, In, blue circle, ∠ABP = 90° (since, angle in semicircle is right angle)

And ∠ODB = 90°

∵ ∠ABP and ∠ODB are co interior angles.

And, as ∠ABP + ∠ODB = 90 + 90 =180°

We know that, if sum of cointerior angle is 180° then the line is parallel.

Therefore, OO’ is parallel to PQ.

⇒ OO’ ∥ PQ …..(4)

To show: PQ = 2 OO’

In Δ APQ, O and O’ is the mid point of line AP and AQ respectively (since, O and O’ are centre of a circle)

Also, from (4)

OO’ ∥ PQ

Thus, by **<u>Mid point theorem:</u>** The line which joins the midpoints of two sides of a triangle is parallel to the third side and is equal to half of the length of the third side.

We get,

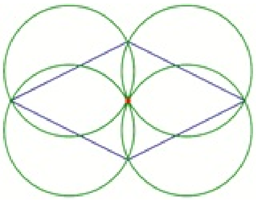
$\Rightarrow OO' = \frac{1}{2}PQ$

⇒ PQ = 2 OO’

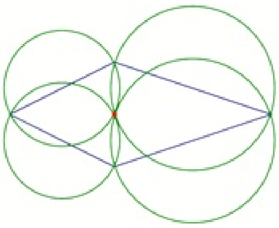
Hence proved.

8. Question

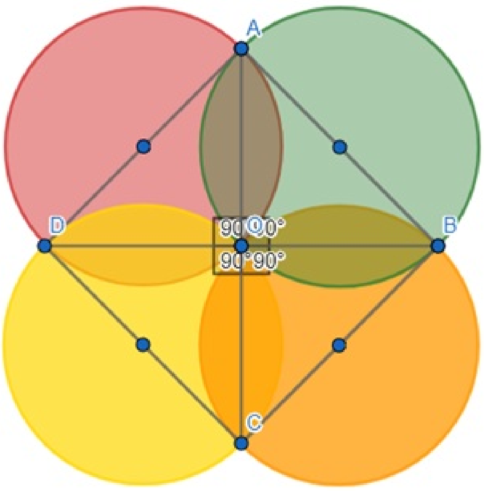
Prove that all four circles drawn with the sides of a rhombus as diameters pass through a common point.



Prove that this is true for any quadrilateral with adjacent sides equal, as in the picture.



Answer



Let ABCD be the rhombus with AC and BD as diagonals intersecting at point O

We know that, diagonals of a rhombus bisect each other at right angles.

∴ ∠AOB = ∠BOC = ∠COD = ∠AOD = 90°

Consider the circle with AB as diameter passes through O. [Since angle in a semi-circle is a right angle]

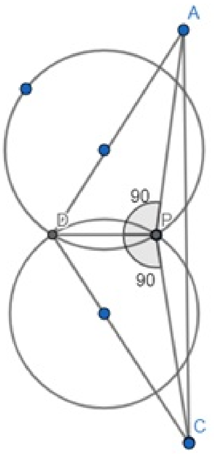
Similarly Consider the circle with BC as diameter passes through O. [Since angle in a semi-circle is a right angle]

Similarly Consider the circle with DC as diameter passes through O. [Since angle in a semi-circle is a right angle]

Similarly Consider the circle with AD as diameter passes through O. [Since angle in a semi-circle is a right angle]

Therefore, the circles with four sides of a rhombus as diameter, pass through the point of intersection of its diagonals.

<u>Next part</u>



First, we will consider the right side of AC

Consider a Δ ACD,

Two circles are drawn with CD and AD as a diameter.

Let they intersect each other at P and let P does not lie on AC

Join DP

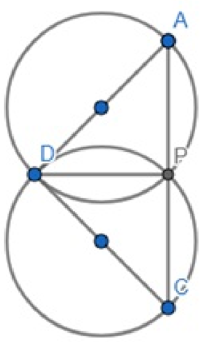
∠DPA = 90° (angle subtended by semi circle)

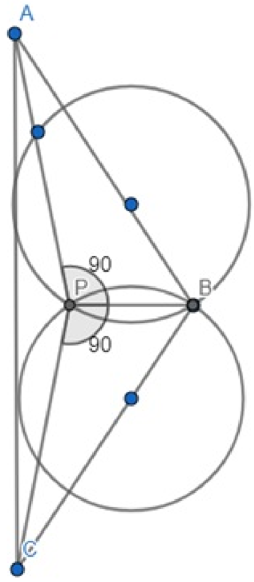
∠DPC = 90° (angle subtended by semi circle)

∠APC = ∠DPA + ∠DPC = 90° + 90° = 180°

Therefore, APC is a straight line and hence our assumption is wrong.

Thus point P lies on third side AC on Δ ACD





Now, we will consider the left side of AC

Consider a Δ ABC

Two circles are drawn with CB and AB as a diameter.

Let they intersect each other at P and let P does not lie on AC

Join BP

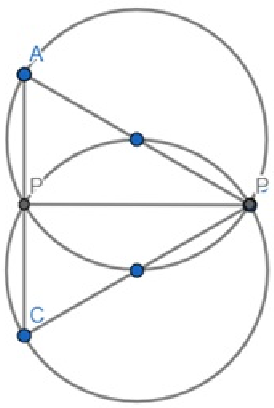
∠BPA = 90° (angle subtented by semi circle)

∠BPC = 90° (angle subtended by semi circle)

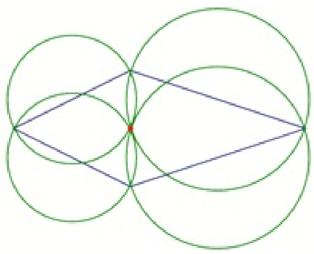
∠APC = ∠BPA + ∠BPC = 90° + 90° = 180°

Therefore, APC is a straight line and hence our assumption is wrong.

Thus, point P lies on third side AC on Δ ABC

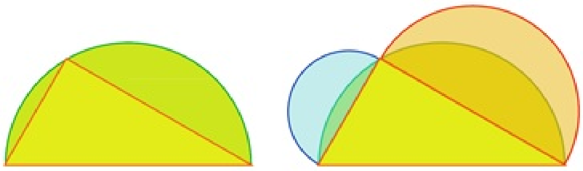


Therefore, the circles with four sides of a quadrilateral as diameter, pass through the point of intersection of its diagonals.



9. Question

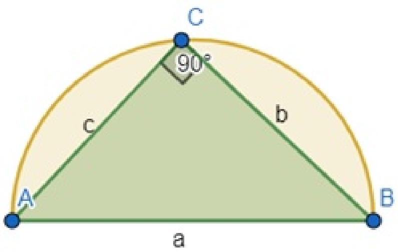
A triangle is drawn by joining a point on a semicircle to the end of the diameter. Then semicircles are drawn with the other two sides as diameter.



Prove that the sum of the areas of the blue and red crescents in the second picture is equal to the area of the triangle.

Answer

From, the given diagram,



∠ACB = 90° (since, angle on a semicircle is right angle)

Thus, by Pythagoras theorem

$AC^2 + CB^2 = AB^2$

$\Rightarrow a^2 = b^2 + c^2$ .....(1)

Also, $\text{area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$\Rightarrow \text{area of } \Delta ABC = \frac{1}{2} \times AC \times CB$

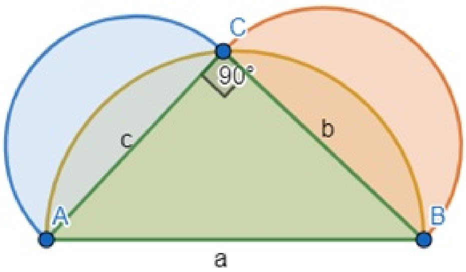
$\Rightarrow \text{area of } \Delta ABC = \frac{1}{2} bc$

Also,$\Rightarrow$ $\text{Area of semicircle on AB } (S_1) = \frac{\pi r^2}{2}$ where, r = radius

Here, $r = \frac{a}{2}$

$\Rightarrow \text{Area of semicircle on AB } (S_1) = \frac{\pi}{2}\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{8}$

Now, by second diagram



We get,

$\Rightarrow \text{Area of semicircle on AC } (S_2) = \frac{\pi r^2}{2}$ where, r = radius

Here, $r = \frac{c}{2}$

$\Rightarrow \text{Area of semicircle on AC } (S_2) = \frac{\pi}{2}\left(\frac{c}{2}\right)^2 = \frac{\pi c^2}{8}$

$\Rightarrow$ Area of semicircle on CB $(S_3) = \frac{\pi r^2}{2}$ where, r = radius

Here, $r = \frac{b}{2}$

$\Rightarrow$ Area of semicircle on CB $(S_3) = \frac{\pi}{2}\left(\frac{b}{2}\right)^2 = \frac{\pi b^2}{8}$

Thus,

Sum of Area of crescent = $S_2 + S_3$ + area of Δ ABC – S_1

$\Rightarrow$ Sum of Area of crescent $= \frac{\pi c^2}{8} + \frac{\pi b^2}{8} +$ area of Δ ABC – $\frac{\pi a^2}{8}$

$\Rightarrow$ Sum of Area of crescent $= \frac{\pi}{8}(c^2 + b^2) +$ area of Δ ABC – $\frac{\pi a^2}{8}$

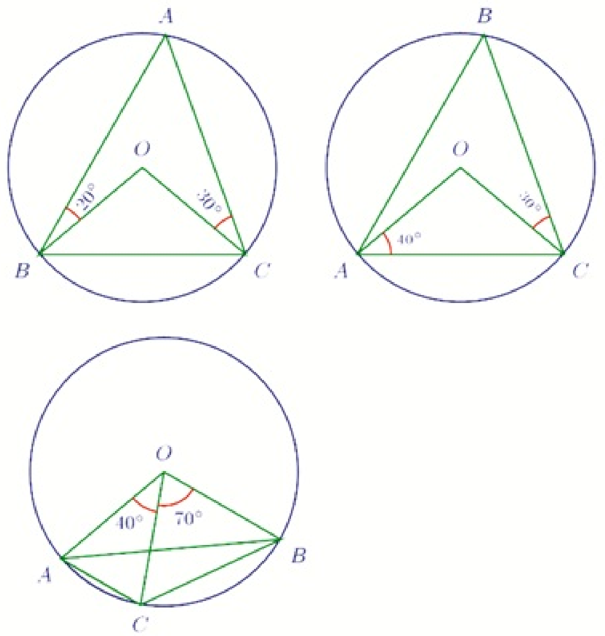
$\Rightarrow$ Sum of Area of crescent $= \frac{\pi}{8}(a^2) +$ area of Δ ABC – $\frac{\pi a^2}{8}$ (from(1))

$\Rightarrow$ Sum of Area of crescent = area of Δ ABC

Questions Pg-51

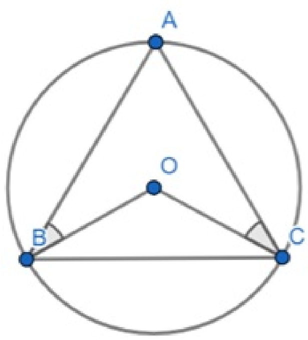
1. Question

In all the pictures given below, O is the centre of the circle and A, B, C are points on it. Calculate all angles of Δ ABC and Δ OBC in each.



Answer

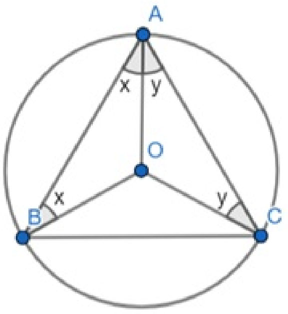
<u>Case1:</u> The angle got by joining any point on the larger part of the circle to the end of the chord is half the angle got by joining the centre of a circle to these end.



● Join A to the centre O of the circle. This line splits the angle at A into two parts, x° and y° .

● Here Δ AOB and Δ AOC are isosceles triangle

Thus, ∠ ABO = ∠ OAB = x° and ∠ ACO = ∠ OAC = y°



● Let us take ∠ BOC = c°

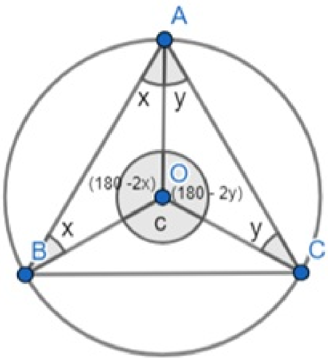
⇒ (180° – 2x) + (180° – 2y) + c° = 360°

⇒ 360° –2 (x+y) +c° = 360°

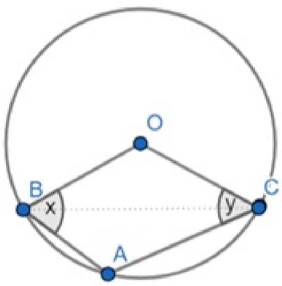
⇒ 2(x+y) = c°

⇒ $(x + y) = \frac{c}{2}$

Thus, $\angle BAC = (x + y) = \frac{c}{2}$



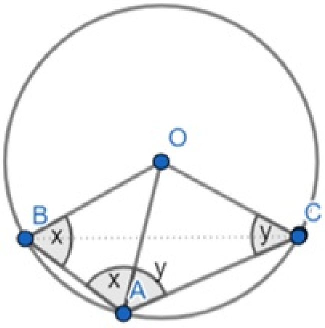
Case2: The angle got by joining any point on the smaller part of the circle to the end of the chord is half the angle at the centre subtracted from 180°.



● Join A to the centre O of the circle. This line splits the angle at A into two parts, x° and y° .

● Here Δ AOB and Δ AOC are isosceles triangle

Thus, ∠ ABO = ∠ OAB = x° and ∠ ACO = ∠ OAC = y°



● Let us take ∠ BOC = c°

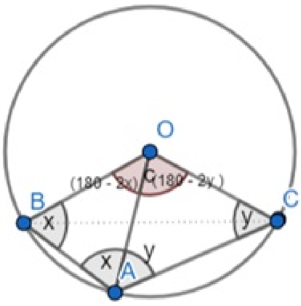
⇒ (180° – 2x) + (180° – 2y) = c°

⇒ 360° –2 (x+y) = c°

⇒ 2(x+y) =360° – c°

⇒ $(x + y) = 180° - \frac{c}{2}$

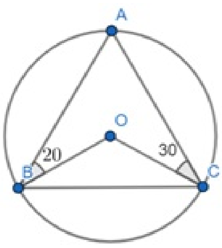
Thus, $\angle BAC = (x + y) = 180° - \frac{c}{2}$

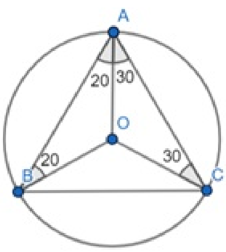


Now, we will consider our question,

We need to find all angles of Δ ABC and Δ OBC

Consider first diagram:





Join OA, Δ AOB and Δ AOC are isosceles triangle

Thus, ∠ ABO = ∠ OAB = 20°

and ∠ ACO = ∠ OAC = 30°

Hence, ∠ BAC = ∠ OAB + ∠ OAC

⇒ ∠ BAC = 20° + 30° = 50°

From case 1, we know that,

2 ∠BAC = ∠BOC

∴ ∠BOC = 2 × 50° = 100° ....(1)

In Δ BOC

Since, OB and OC are radius of a circle

Therefore, ∠ OBC = ∠ OCB (since, angles opposite to equal side are equal) ...(2)

By, angle sum property,

∠ OBC + ∠ OCB + ∠ BOC = 180°

⇒ ∠ OBC + ∠ OBC + 100° = 180° (from (1) and (2))

⇒ 2 ∠ OBC = 180° – 100° = 80°

⇒ ∠ OBC = 40°

Thus, ∠ OBC = ∠ OCB = 40° (from (2))

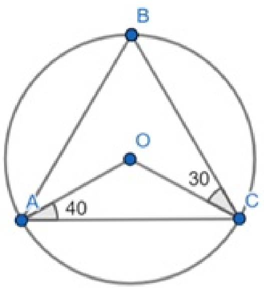
Thus, ∠ ABC = ∠ ABO + ∠ OBC

⇒ ∠ ABC = 20 + 40 = 60°

Thus, ∠ ACB = ∠ ACO + ∠ OCB

⇒ ∠ ABC = 30 + 40 = 70°

Consider second diagram



In Δ AOC

Since, OA and OC are radius of a circle

Therefore, ∠ OAC = ∠ OCA = 40° (since, angles opposite to equal side are equal) ...(1)

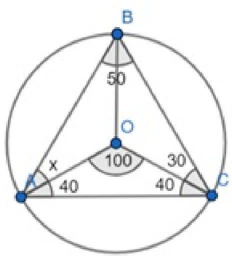
By, angle sum property,

∠ OAC + ∠ OCA + ∠ AOC = 180°

⇒ 40° + 40° + ∠ AOC = 180° (from (1))

⇒ ∠ AOC = 180° – 80° = 100° ...(2)

From case 1, we know that,



2 ∠ABC = ∠AOC

$\therefore\ \angle ABC = \frac{100}{2} = 50°$ (from (2)) ...(3)

Join OB,

As Δ ABO and Δ OBC are isosceles triangle

Thus, ∠ ABO = ∠ OAB = x°

and ∠ BCO = ∠ OBC = 30°

Hence, ∠ ABC = ∠ ABO + ∠ OBC

⇒ 50° = x° + 30°

⇒ x° = 50° –30° = 20°

Thus, ∠ ABO = ∠ OAB = 20°

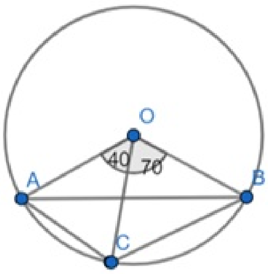
Thus, ∠ BAC = ∠ BAO + ∠ OAC

⇒ ∠ BAC = 20 + 40 = 60°

Thus, ∠ BCA = ∠ ACO + ∠ OCB

⇒ ∠ BCA = 30 + 40 = 70°

Consider third diagram,



Here, ∠ AOB = 40° + 70° = 110°

From case 2, we get,

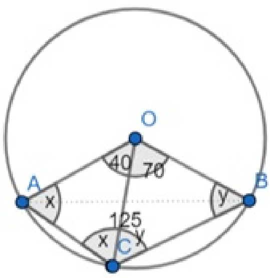
$\angle BCA = 180° - \frac{\angle AOB}{2}$

Thus, $\angle BCA = 180° - \frac{110}{2}$

⇒∠BCA=180°– 55° = 125°

Here Δ AOC and Δ BOC are isosceles triangle

Thus, ∠ CAO = ∠ OCA = x° and ∠ BCO = ∠ OBC = y°



Also, ∠ AOC = 180 – 2x

and ∠ BOC = 180 – 2y (seen in case 2 above)

Thus, 40° = 180 – 2x and 70° = 180 – 2y

⇒ 2x = 180 – 40 = 140

⇒ x = 70°

Thus,∠ CAO = ∠ OCA = x° = 70°

And

⇒ 2y = 180 – 70 = 110

⇒ y = 55°

Thus,∠ BCO = ∠ OBC = y° = 55°

Also, In Δ OAB

∠ AOB = 40 + 70 = 110°

Since, OA and OC are radius of a circle

Therefore, ∠ OAB = ∠ OBA (since, angles opposite to

equal side are equal) ..(1)

By, angle sum property,

∠ OAB + ∠ OBA + ∠ AOB = 180°

⇒ ∠ OAB + ∠ OAB + 110° = 180° (from (1))

⇒ 2 ∠ OAB = 180° – 110° = 70°

⇒ $\angle OAB = \frac{70}{2} = 35°$ ….(2)

Thus, ∠ OAB = ∠ OBA = 35° (from (1) and (2))

Hence, ∠ BAC = ∠ OAC – ∠ OAB

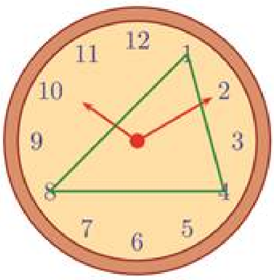
⇒ ∠ BAC = 70 – 35 = 35°

Also, ∠ ABC = ∠ OBC – ∠ OBA

⇒ ∠ BAC = 55 – 35 = 20°

2. Question

The numbers 1, 4, 8 on a clock’s face are joined to make a triangle.



Calculate the angles of this triangle.

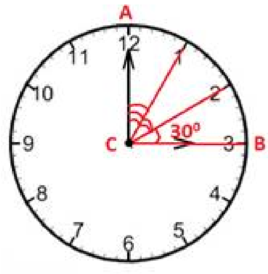
How many equilateral triangles can we make by joining numbers on the clock’s face?

Answer

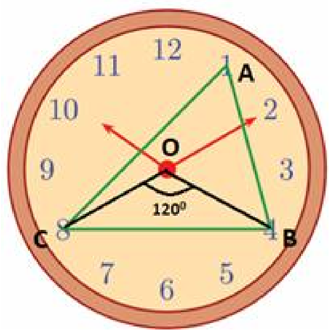
We know that a clock is a circle. And, a circle is made of 360°.

The clock has 12 numbers & each number represents an angle and the separation between them is $\frac{360}{12} = 30°$

Suppose, is the clock reads, 3pm, then the angle subtended at the centre will be equal to 30° × 3 = 90°. (as shown in the figure below)



Now, the figure of the question is given below:



<u>Concept involved:</u>

<u>Theorem (1) : The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.</u>

Now,

∠COB = 120° (30° × 4)

From theorem (1),

The ∠CAB = 1/2 ∠COB = 60°.

And, ∠ABC = 30° × 3 = 90°.

From theorem (1), ∠ACB = 45°

Now in a triangle, all the angles sum is 180°

⇒, ∠1 + ∠4 + ∠8 = 180°

⇒ 60° +∠4 + 45° = 180°

∠4 = 75°

Hence, all the three angles are 60°, 45° and 75°.

3. Question

In each problem below, draw a circle and a chord to divide it into two parts such that the parts are as specified;

i) All angles on one part 80°.

ii) All angles on one part 110°.

iii) All angles on one part half of all angles on the other.

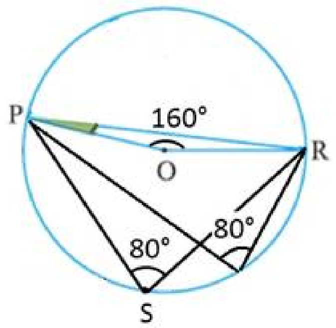
iv) All angles on one part, one and a half times the angles on the other.

Answer

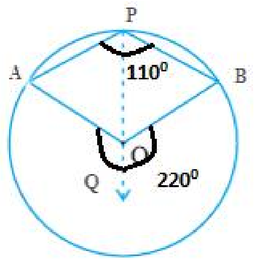
<u>Theorem:</u>

<u>The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.</u>

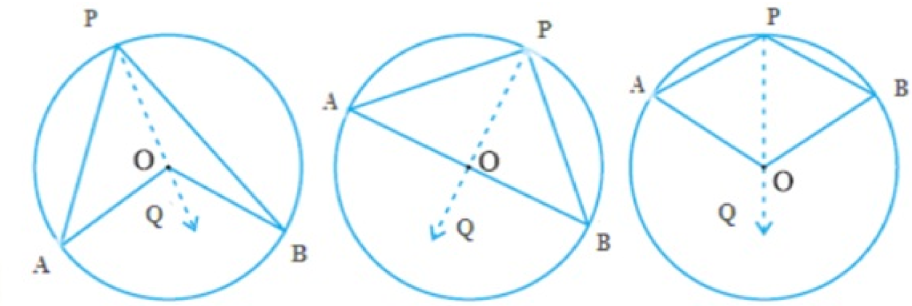
(i) In the circle, let us draw a <u>minor arc</u> PR, which subtends an angle ∠ POR = 160° at the centre. Then, from the theorem, the angle at any other part will be $\frac{160°}{2} = 80°$.



(ii) Here, AB is the major arc. It subtends an ∠ 220° at the centre, then, in its minor segment, the angle at any point will be half of 220°



(iii) <u>The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.</u>



(i) (ii) (iii)

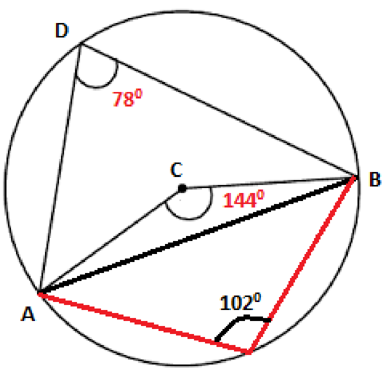
Case (1) $\widehat{AB}$ is a minor arc

Case (2) $\widehat{AB}$ is a semicircle

Case (3) $\widehat{AB}$ is a major arc

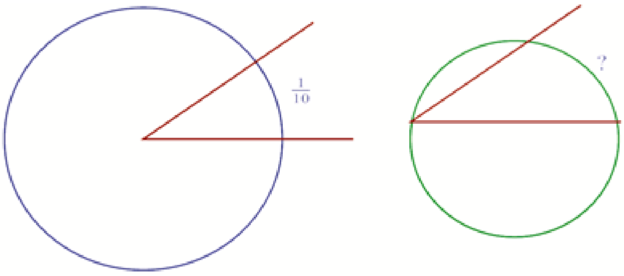
From the figure (i), (ii) and (iii), ∠APB = 1/2 ∠AOB

(iv) Draw an angle 144° at the centre. Then, all angles on one part, one and a half times the angles on the other.

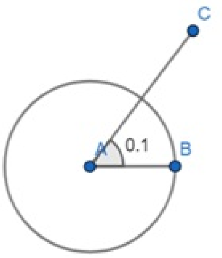


4. Question

A rod bent into an angle is placed with its corner at the centre of a circle and it is found that $\frac{1}{10}$ of the circle lies within it. If it is placed with its corner on another circle, what part of the circle would be within it?



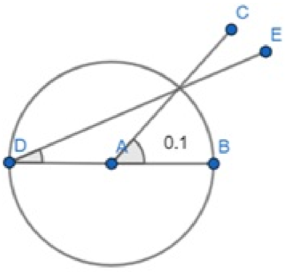
Answer



The corner of an angle is the centre 'A' of a circle. Now extend one side of a angle to meet the circle and join the point to the point where the other side cuts the circle. This gives us half the angle.

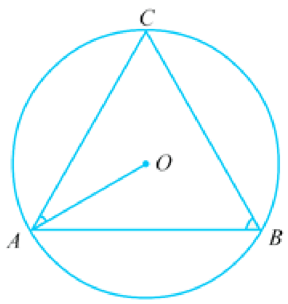
Thus, $\angle \mathrm{CDB} = \frac{1}{2} \angle \mathrm{CAB}$

$\Rightarrow \angle \mathrm{CDB} = \frac{1}{2} \times \frac{1}{10} = \frac{1}{20}$



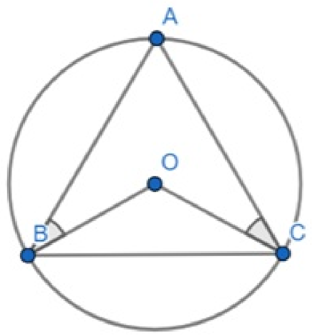
5. Question

In the picture, O is the centre of the circle and A, B, C are points on it. Prove that $\angle$ OAC + $\angle$ ABC = 90°.



Answer

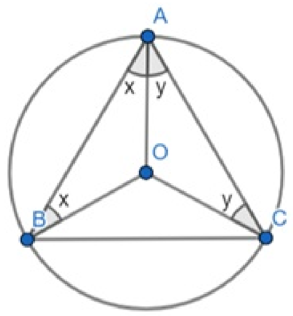
Case1: The angle got by joining any point on the larger part of the circle to the end of the chord is half the angle got by joining the centre of a circle to these end.



● Join A to the centre O of the circle. This line splits the angle at A into two parts, x° and y° .

● Here Δ AOB and Δ AOC are isosceles triangle

Thus, ∠ ABO = ∠ OAB = x° and ∠ ACO = ∠ OAC = y°



● Let us take ∠ BOC = c°

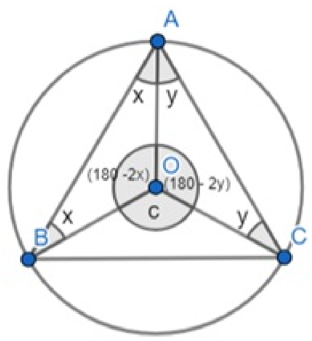
⇒ (180° – 2x) + (180° – 2y) + c° = 360°

⇒ 360° –2 (x+y) +c° = 360°

⇒ 2(x+y) = c°

⇒ $(x + y) = \frac{c}{2}$

Thus, $\angle BAC = (x + y) = \frac{c}{2}$



Consider, the diagram given in question.

Join, OA and OB

In, Δ OAB

Since, OA and OB are radius of a circle

Therefore, ∠ OAB = ∠ OBA (since, angles opposite to

equal side are equal) ..(1)

By, angle sum property,

∠ OAB + ∠ OBA + ∠ AOB = 180°

⇒ ∠ OAB + ∠ OAB + ∠ AOB = 180° (from (1))

⇒ 2 ∠ OAB = 180° – ∠ AOB

$\Rightarrow \angle OAB = \frac{180 - \angle AOB}{2} = 90° - \frac{\angle AOB}{2}$

$\Rightarrow \angle OAB = \angle OBA = 90° - \frac{\angle AOB}{2}$ ....(2)

As, Δ OAC and Δ OBC are isosceles

∴ ∠ ACO = ∠ CAO and ∠ OBC = ∠ BCO .....(3)

Also, by case1 we get,

$\angle ACB = \angle ACO + \angle BCO = \frac{\angle AOB}{2}$ ...(4)

Consider,

∠ CAO + ∠ ABC = ∠ CAO + ∠ CBO + ∠ OBA

$= \angle ACO + \angle BCO + 90° - \frac{\angle AOB}{2}$ (from (2) and (3))

$= \frac{\angle AOB}{2} + 90° - \frac{\angle AOB}{2}$ (from (4))

= 90°

6. Question

Draw a triangle of circumradius 3 centimetres and two of the angles $32\frac{1°}{2}$ and $37\frac{1°}{2}$.

Answer

We have to construct a triangle with ∠A = 32.5° ,

∠B = 37.5°

And, we know in a triangle, the sum of all angles is 180° .

Hence, the ∠C = 180 – 32.5 – 37.5

∴ ∠C = 110 degrees.

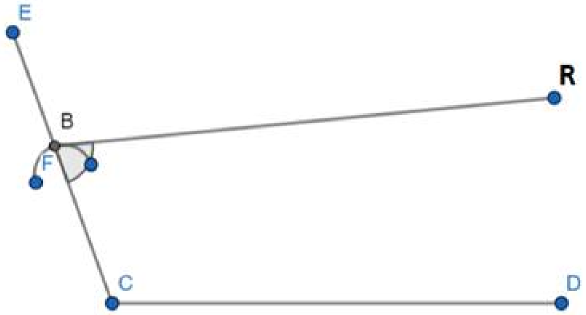
The circumradius = 3 cm.

Let’s say a = 3 cm, this is side BC.

1) First draw a line CD. At C and with a protractor make an angle of 110° , as shown,

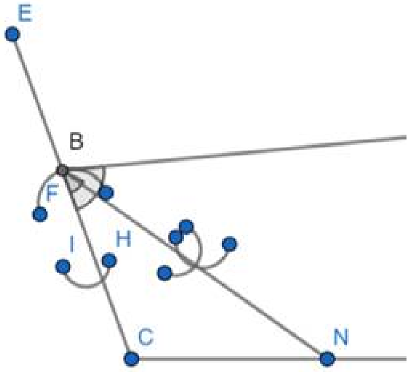
2) With C as the centre and a radius of 3 cm cut the line CE. This point will be B.

3) At B, and with a protractor, make an angle of 75° with BC, as shown below:



Now, bisect the ∠CBR to get ∠CBN = 37/5°

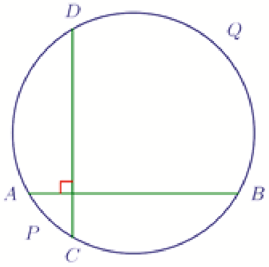
4) The other angles can be measured with the help of the protractor.



This will be the required triangle.

7. Question

In the picture, AB and CD are mutually perpendicular chords of the circle. Prove that arcs APC and BQD joined together would make half the circle.



Answer

Let O be the center of a circle

Join BC, OA, OB, OC and OD

Given: AB⊥ CD

In Δ ABK

$\angle$ ABC + $\angle$ BCD + $\angle$ BKC = 180° (Angle sum property)

⇒ $\angle$ ABC + $\angle$ BCD + 90° = 180°

⇒ $\angle$ ABC + $\angle$ BCD = 90° .....(1)

$\angle$AOC = 2$\angle$ ABC (∵ angle at center twice angle in

segment) ...(2)

$\angle$BOD = 2$\angle$ BCD (∵ angle at center twice angle in

segment) ...(3)

Adding (2) and (3)

$\angle$AOC + $\angle$BOD = 2$\angle$ ABC + 2$\angle$ BCD

$\angle$AOC + $\angle$BOD = 2 ($\angle$ ABC + $\angle$ BCD)

$\angle$AOC + $\angle$BOD = 2 × 90° (from (1))

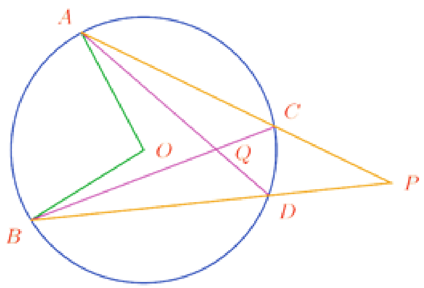
$\angle$AOC + $\angle$BOD = 180°

$\angle AOC + \angle BOD = \frac{1}{2} \times 360°$

Hence, $\text{arc APC} + \text{arc BQD} = \frac{1}{2} \times \text{circumference}$ (angle at center is half of 360°)

8. Question

In the picture, A, B, C, D are points on a circle centred at O. The line AC and BD are extended to meet at P. The line AD and BC intersect at Q. Prove that the angle which the small arc AB makes at O is the sum of the angles it makes at P and Q.



Answer

Given: In the picture, A, B, C, D are points on a circle centred at O. The line AC and BD are extended to meet at P. The line AD and BC intersect at Q

To Prove: The angle which the small arc AB makes at O is the sum of the angles it makes at P and Q i.e.

$\angle$AOB = $\angle$APB + $\angle$AQB

We know that,

The angle made by an arc of circle on the alternate arc is half the angle made at center.

Therefore,

$\angle ACB = \frac{1}{2}\angle AOB$ ...[1] and

$\angle ADB = \frac{1}{2}\angle AOB$ ...[2]

Also, By linear pair

$\angle$ACB + $\angle$PCB = 180°

$\angle$ACB = 180° – $\angle$PCB ...[3]

And

∠ADB + ∠ADP = 180°

∠ADB = 180° – ∠ADP ...[4]

Also, By angle sum property of quadrilateral of CQDP

∠CQD + ∠QCP + ∠CPD + ∠PDQ = 360°

⇒ ∠AQB + ∠PCB + ∠APB + ∠ADP = 360°

⇒ ∠PCB + ∠ADP = 360°– (∠AQB + ∠APB) ...[5]

[Here, ∠CQD = ∠AQB, vertically opposite angles]

Adding [1] and [2]

$\Rightarrow \angle ACB + \angle ADB = \frac{1}{2}\angle AOB + \frac{1}{2}\angle AOB$

⇒ ∠ACB + ∠ADB = ∠AOB

⇒ 180° – ∠PCB + 180° – ∠ADP = ∠AOB [Using [3] and [4]]

⇒ 360° – (∠PCB + ∠ADP) = ∠AOB

⇒ 360° – (360° – (∠AQB + ∠APB)) = ∠AOB

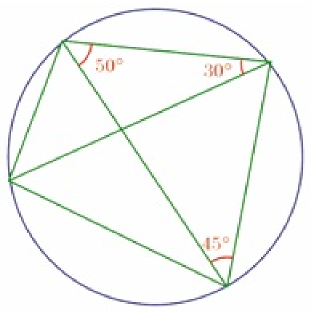
⇒ ∠AOB = ∠AQB + ∠APB

Hence Proved.

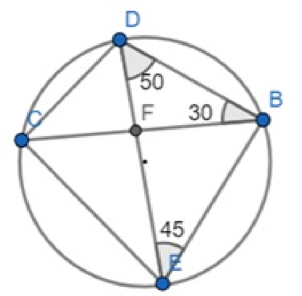
Questions Pg-57

1. Question

Calculate the angles of the quadrilateral in the picture and also the angles between their diagonals:



Answer



In Δ FBD

∠ BFD + ∠ FBD + ∠ BDF = 180° (angle sum property)

⇒ ∠ BFD + 50 + 30 = 180°

⇒ ∠ BFD = 180° – 80° = 100°

Also, ∠ CFE = ∠ BFD =100° (∵ ∠ BFD and ∠ CFE are vertically opp. Angle)

Since, EFD is a line

∠ BFD + ∠ BFE = 180° (sum of angle on a straight line is 180°)

⇒ 100 + ∠ BFE = 180°

⇒ ∠ BFE = 180° – 100° = 80°

Also, ∠ CFD = ∠ BFE = 80° (∵ ∠ BFE and ∠ CFD are vertically opp. Angle)

In Δ FBE

∠ BFE + ∠ FBE + ∠ BEF = 180° (angle sum property)

⇒ ∠ FBE + 80 + 45 = 180°

⇒ ∠FBE = 180° – 125° = 55°

Thus, ∠ DBE = ∠ FBD + ∠FBE

⇒ ∠ DBE = 30 + 55 = 85°

In quad CDBE

∠ DCE + ∠ DBE = 180° (∵ if all four vertices of a quadrilateral are

on circle then opposite angle are supplementary)

⇒ ∠ DCE + 85 = 180°

⇒ ∠ DCE = 180° – 85° = 95°

Also,

∠ CBD = ∠ DEC = 30° (∵ angle in a same segment are equal)

∠ CBE = ∠ CDE = 55° (∵ angle in a same segment are equal)

Thus, ∠ CDB = ∠ CDE + ∠ BDE

⇒ ∠ CDB = 55 + 50 = 105°

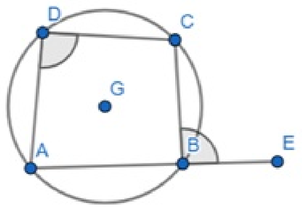
Thus, ∠ CEB = ∠ CED + ∠ BED

⇒ ∠ CEB = 30 + 45 = 75°

2. Question

Prove that any exterior angle of a cyclic quadrilateral is equal to the interior angle at the opposite vertex.

Answer



Given : A cyclic quadrilateral ABCD one of whose side AB is produced to E.Prove that : ∠CBE = ∠ADCProof:

∠ ABC + ∠ ADC = 180° [Opposite angles of cyclic quadrilateral]

∠ ABC + ∠ CBE = 180° [Linear Pair angles.]

∠ABC + ∠ADC = ∠ABC + ∠CBE [From the above equations,]

∠ADC = ∠CBE [Subtraction property]

3. Question

Prove that a parallelogram which is not a rectangle is not cyclic.

Answer

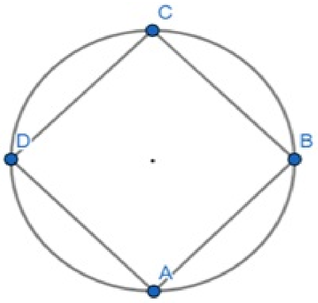
We will prove by negation

i.e. If ABCD is a cyclic parallelogram then it is a rectangle

Proof:

$\angle A + \angle C = 180°$ (ABCD is a cyclic quadrilateral) ...(1)

Since $\angle A = \angle C$ (Opposite angles of a parallelogram) ...(2)



$\Rightarrow \angle A + \angle A = 180°$ (from (1) and (2))

$\Rightarrow 2\angle A = 180°$

$\Rightarrow \angle A = 90°$

Thus, ABCD is a rectangle.

That is a parallelogram which is not a rectangle is not cyclic.

4. Question

Prove that a non-isosceles trapezium is not cyclic.

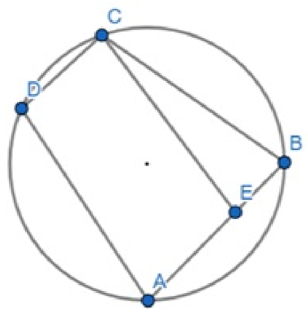
Answer

We will prove by negation

Let ABCD be the cyclic trapezium with AB ∥ CD

Through C draw CE parallel to AD meeting AB in E

Thus, AECD is a parallelogram



Thus, $\angle D = \angle AEC$ (opp. Angle of parallelogram are equal) ...(1)

But, $\angle D + \angle ABC = 180°$ (opp. Angle of a cyclic quadrilateral are

Supplementary) ....(2)

From (1) and (2)

$\angle AEC + \angle ABC = 180°$

But, $\angle AEC + \angle CEB = 180°$ (linear pair)

Thus, ∠ AEC + ∠ ABC = ∠ AEC + ∠ CEB

⇒ ∠ ABC = ∠ CEB ...(3)

⇒ CE = CB (side opposite to equal angle are equal) ...(4)

But, CE = AD (opp. Sides of parallelogram AECD)

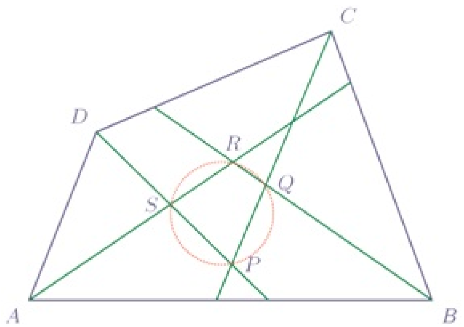
From (4) we get,

AD = CB

Thus, cyclic quadrilateral ABCD is isosceles

5. Question

In the picture, bisectors of adjacent angles of the quadrilateral ABCD intersect at P, Q, R, S.



Prove that PQRS is a cyclic quadrilateral.

Answer

In the given figure,

∠ASD = ∠PSR [Vertically opposite angles]...[1]

In ΔASD

∠ASD + ∠ADS + ∠DAS = 180° [By angle sum property]

Also, As AS and DS are angle bisectors, therefore

$\angle ADS = \frac{1}{2}\angle D$ and

$\angle DAS = \frac{1}{2}\angle A$

Using these and from equation [1] we have,

$\angle PSR + \frac{1}{2}\angle D + \frac{1}{2}\angle A = 180°$

$\Rightarrow \angle PSR = 180° - \frac{1}{2}(\angle A + \angle D)$ ...[2]

Similarly,

$\angle PQR = 180° - \frac{1}{2}(\angle B + \angle C)$ ...[3]

Adding [2] and [3]

$\angle PSR + \angle PQR = 360° - \frac{1}{2}(\angle A + \angle B + \angle C + \angle D)$

Also, ∠A + ∠B + ∠C + ∠D = 360° [By angle sum property of quadrilateral]

$\Rightarrow \angle PSR + \angle PQR = 360° - \frac{360°}{2}$

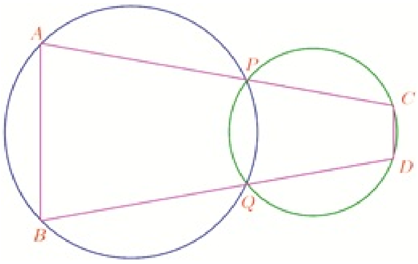
⇒ ∠PSR + ∠PQR = 180°

⇒ PQRS is a cyclic quadrilateral.

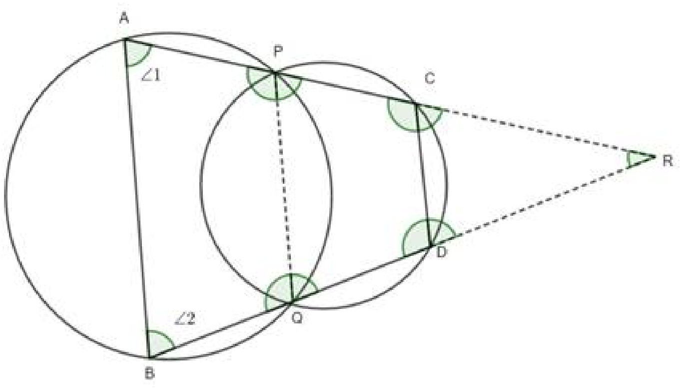
[As in a cyclic quadrilateral, sum of any pair of opposite angles is 180°].

6 A. Question

The two circles below intersect at P, Q and lines through these points meet the circles at A, B, C, D. Prove that if AC and BD are of equal length, then ABCD is a cyclic quadrilateral.



Answer



<u>Given:</u> two intersecting circles, which intersect each other at P and Q, A quadrilateral ABCD is drawn, such that sides AC and BD are passing through P and Q. Also, AC = BD

<u>To prove:</u> ABCD is cyclic

Construction: Join PQ, and extend AC and BD to meet at R.

Proof:

As, APBQ is a cyclic quadrilateral

∠BAP + ∠BQP = 180°

⇒ ∠BQP = 180° – ∠1 ...[1]

Also, By linear pair

∠BQP + ∠PQD = 180°

⇒ 180° – ∠1 + ∠PQD = 180°

⇒ ∠PQD = ∠1

Also, CDQP is also a cyclic quadrilateral

⇒ ∠PQD + ∠PCD = 180°

⇒ ∠1 + ∠PCD = 180°

⇒ ∠PCD = 180° – ∠1 ...[2]

Similarly, we can prove

∠QDC = 180° – ∠2 ...[3]

Also, By linear pair

⇒ ∠PCD + ∠RCD = 180°

⇒ 180 – ∠1 + ∠RCD = 180° [From 2]

⇒ ∠RCD = ∠1

Hence, AB || CD [As, corresponding angles are equal through transversal AR]

Now,

In ΔABR, AB || CD and CD intersects AR and BR, therefore by basic proportionality theorem

$$\frac{AR}{AC} = \frac{BR}{BD}$$

As, AC = BD is given, we have

AR = BR

⇒ ∠1 = ∠2 [Angles opposite to equal sides are equal] ...[4]

Now,

∠A + ∠D

= ∠1 + ∠QDC [From 2]

= ∠1 + 180° – ∠2

= ∠1 + 180° – ∠1 [From 4]

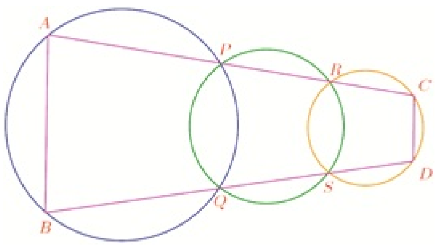
= 180°

Hence, ABCD is cyclic.

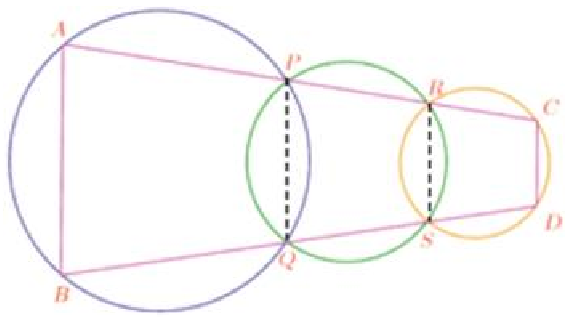
[As in a cyclic quadrilateral, sum of any pair of opposite angles is 180°].

6 B. Question

In the picture, the circles on the left and right intersect the middle circle at P, Q, R, S; the lines joining them meet the left and right circles at A, B, C, D. Prove that ABCD is a cyclic quadrilateral.



Answer



<u>Given:</u> In the given figure, the circles on the left and right intersect the middle circle at P, Q, R, S; the lines joining them meet the left and right circles at A, B, C, D

<u>To Prove:</u> ABCD is cyclic

Construction: Join PQ and RS.

Proof:

As, APBQ is a cyclic quadrilateral,

∠BAP + ∠BQP = 180° [Opposite angles in a cyclic quadrilateral are supplementary]

∠BQP = 180° – ∠BAP

Now, ∠BQP + ∠SQP = 180° [linear pair]

⇒ 180° – ∠SQP = 180° – ∠BAP

⇒ ∠SQP = ∠BAP

But PQRS is a cyclic quadrilateral

⇒ ∠SQP + ∠PRS = 180°

⇒ ∠BAP + ∠PRS = 180°

Now, ∠PRS + ∠CRS = 180° [linear pair]

⇒ ∠BAP + 180° – ∠CRS = 180°

⇒ ∠BAP = ∠CRS

But CDSR is also cyclic

⇒ ∠CRS + ∠CDS = 180°

⇒ ∠BAP + ∠CDS = 180°

Or

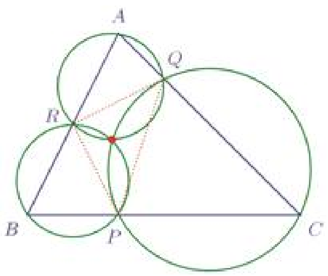
⇒ ∠A + ∠D = 180°

⇒ ABCD is a cyclic quadrilateral

[As in a cyclic quadrilateral, sum of any pair of opposite angles is 180°].

7. Question

In the picture, points P, Q, R are marked on the sides BC, CA, AB of Δ ABC and the circumcircles, of Δ AQR, Δ BRP, Δ CPQ are drawn.



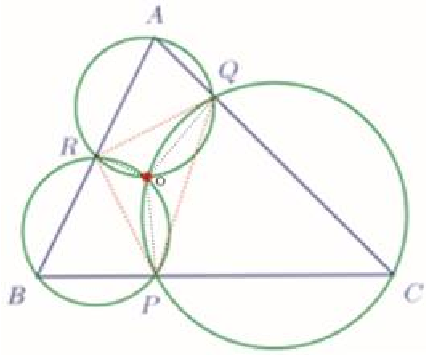
Prove that they pass through a common point.

Answer

Given: In the picture, points P, Q, R are marked on the sides BC, CA, AB of Δ ABC and the circumcircles, of Δ AQR, Δ BRP, Δ CPQ are drawn.

To Prove: All the circles pass through a common point.

Proof:



Let us assume a point O, which passes through all the circles.

and Join OP, OQ and OR.

If we prove that such a point O exists, then we are done.

Now, OQAR, OPCQ and OPBR are cyclic quadrilaterals, and in a cyclic quadrilateral, sum of any pair of opposite angles is 180°.

⇒ ∠QOR + ∠A = 180° ...[1]

⇒ ∠POQ + ∠C = 180° ...[2]

⇒ ∠POR + ∠B = 180° ...[3]

Also, By angle sum property of triangle

∠A + ∠B + ∠C = 180° ...[4]

Now, Adding [1] [2] and [3]

⇒ ∠QOR + ∠POQ + ∠POR + ∠A + ∠C + ∠B = 540°

⇒ ∠POQ + ∠QOR + ∠POR + 180° = 540° [From 4]

⇒ ∠POQ + ∠QOR + ∠POR = 360°

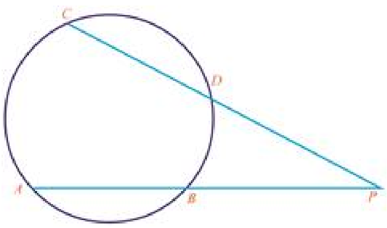
Now, sum of all the angles around O is 360°,

⇒ O exists and is common to all circles.

Questions Pg-64

1. Question

In the picture, chords AB and CD of the circle are extended to meet at P.



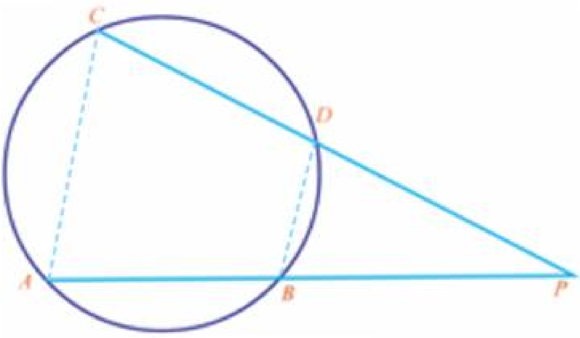
i) Prove that the angles of □ APC and □ PBD, formed by joining AC and BD, are the same.

ii) Prove that PA × PB = PC × PD.

iii) Prove that if PB = PD, then ABCD is an isosceles trapezium.

Answer

(i)



As, ABDC is a cyclic quadrilateral

⇒ ∠ACD + ∠ABD = 180° [Sum of opposite pair of angles in a cyclic quadrilateral is 180°]

⇒ ∠ABD = 180° – ∠ACD

Also,

∠ABD + ∠PBD = 180° [linear pair]

⇒180 – ∠ACD + ∠PBD = 180°

⇒∠ACD = ∠PBD ...[1]

Similarly

⇒∠BAC = ∠PDB ...[2]

And, clearly

∠BPD = ∠APC [Common] ...[3]

From [1], [2] and [3]

It's clear that all angles of both the triangles are equal.

And ΔAPC ~ ΔBPD [By AAA similarity criterion]

(ii) As,

ΔAPC ~ ΔBPD

$\Rightarrow \frac{PD}{PA} = \frac{PB}{PC}$

⇒ PD × PC = PB × PA

Or

⇒ PA × PB = PC × PD

(iii) As, PD = PB ...[4]

∠PBD = ∠PDB [Angles opposite to equal sides are equal] ...[5]

and also, as ΔAPC ~ ΔBPD

⇒ ∠ACD = ∠PBD

⇒ ∠ACD = ∠PDB [From 5]

⇒ AC || BD [As corresponding angles are equal]

Also,

⇒ PD × PC = PB × PA [from ii part]

⇒ PD × PC = PD × PA

⇒ PC = PA ...[6]

On subtracting [6] from [5]

⇒ PC – PD = PA – PB

⇒ CD = AB

Hence, in quadrilateral, ABDC one pair of opposite sides is parallel, and sides of another pair are equal,

Therefore, ABDC is an isosceles trapezium.

2. Question

Draw a rectangle of width 5 centimetres and height 3 centimetres.

i) Draw a rectangle of the same area with width 6 centimetres.

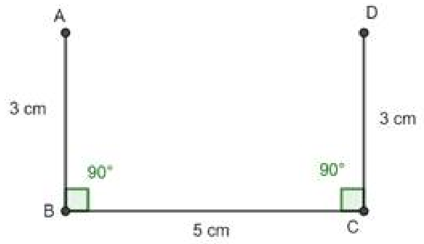
ii) Draw a square of the same area.

Answer

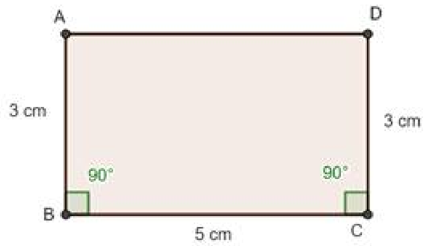
Steps of construction:

1. Draw a line BC = 5 cm, taking B and C as centres, draw

∠ABC = ∠BCD = 90°, such that AB = CD = 3 cm

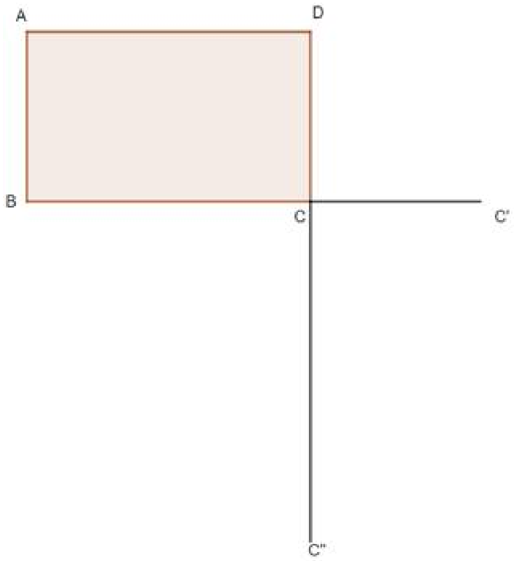


2. Join AD to get rectangle ABCD.

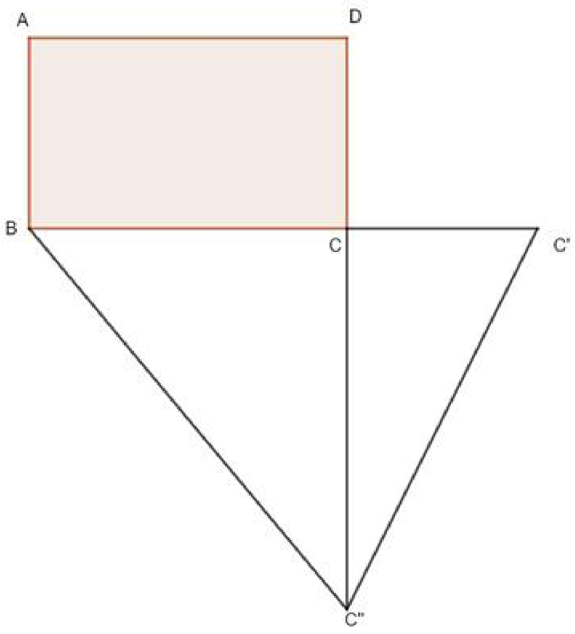


(i) Steps of construction:

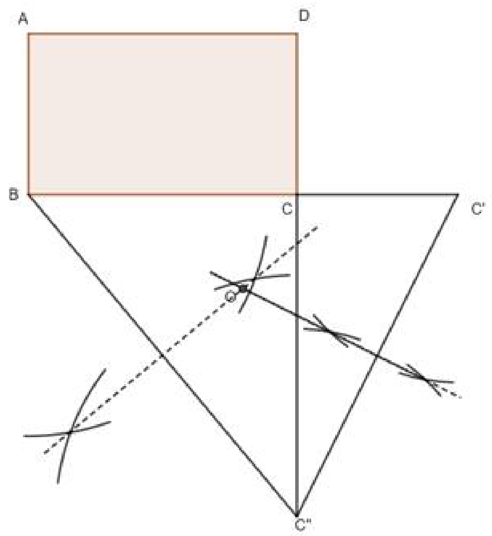
1. In the previous rectangle, Extend DC by 6 cm to DC' and and extend BC by 3 cm to BC''.



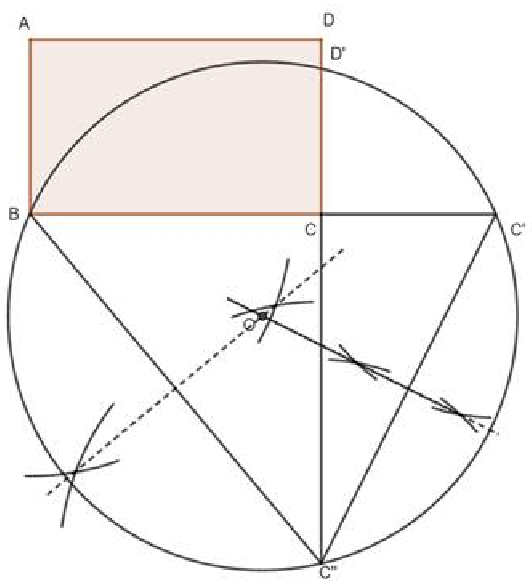
2. Join C''C' and BC'



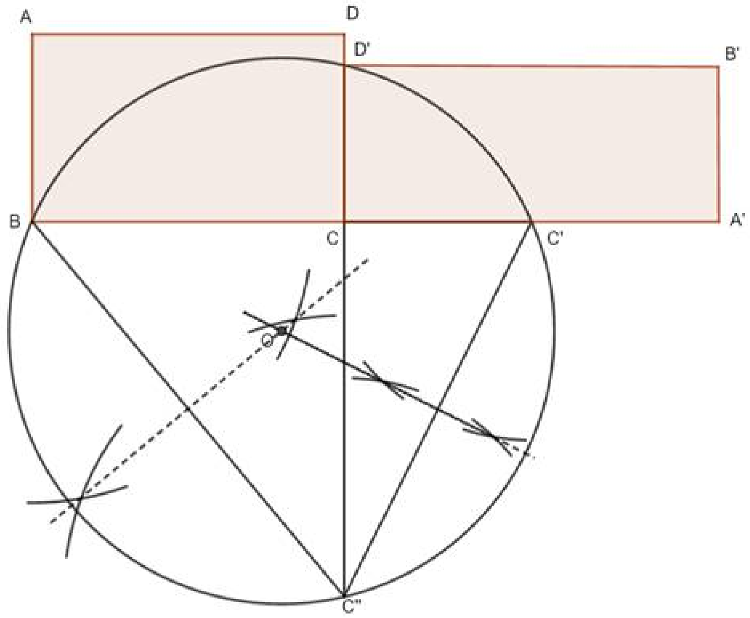
3. Draw the perpendicular bisector of C''C' and BC', they intersect each other at O.



4. Taking O as centre and OC' as radius draw a circle which intersect the previous rectangle at D'.



5. Now, taking CD' as height and CA'=CC'' as length, draw a rectangle A'B'D'C, which is required here.



Concept used: If the chords of a circle intersect each other within a circle, then the areas of rectangle formed by the parts of the same chord are equal.

And in the diagram, we made

BC' and DC" are two chord,

Therefore

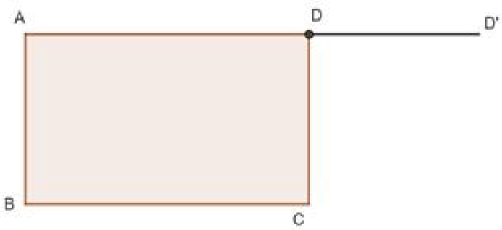
Area(ABCD) = Area(A'B'D'C) and

CA' = CC'' = 6 cm

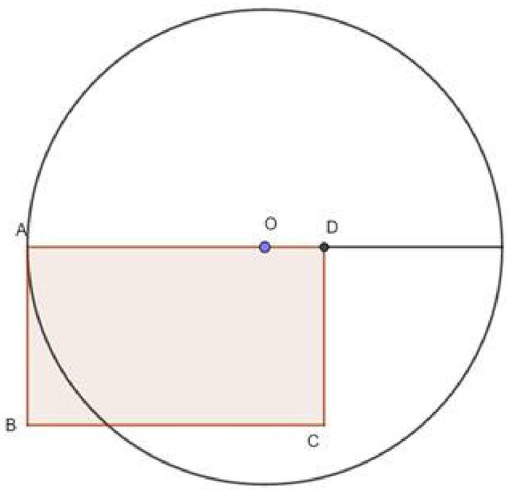
Therefore, all the required conditions are verified here.

(ii) Steps of construction:

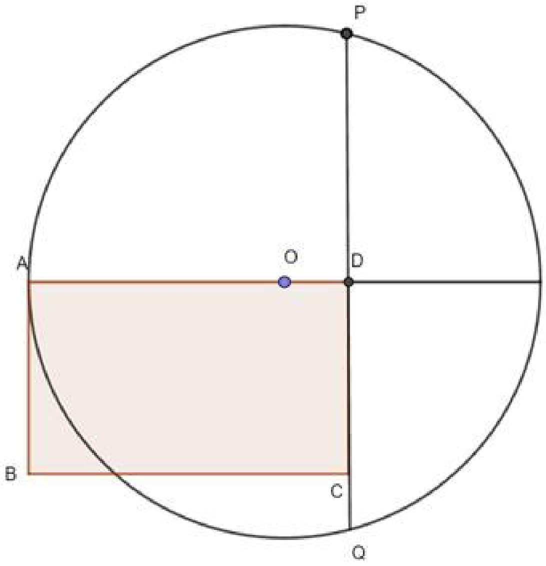
1. From the rectangle drawn, extend AD by 3 cm to D'.



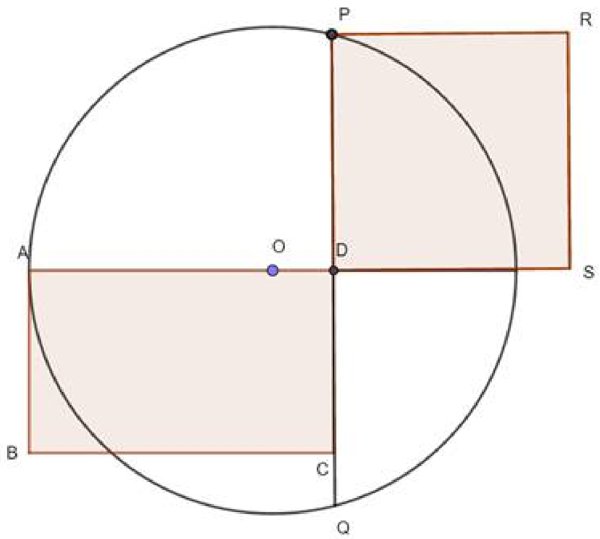
2. From the mid–point of AD'(Let it be O such that OA = OD'=4 cm) Draw a circle taking OA = OD' as radius.



3. Now, draw a chord PQ ⊥ AD', passing through D.



4. Taking DP as side, draw a square DPRS, which is required.



Concept used:

The area of rectangle form of parts into which a diameter of a circle is cut by a perpendicular chord is equal to the area made by the square of half the chord.

And in the diagram, we made, AD' is diameter and PQ is perpendicular chord, therefore

Area(ABCD) = ar(DPRS), and DPRS is a square

Hence, all the required conditions are verified here.

3. Question

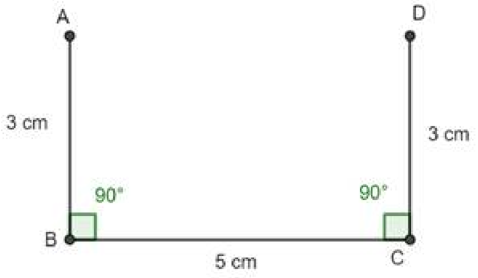
Draw a square of area 15 square centimetres.

Answer

Steps of construction:

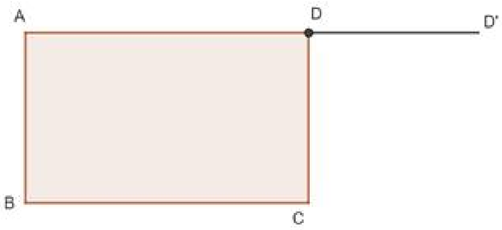
1. Draw a line BC = 5 cm, taking B and C as centres, draw

∠ABC = ∠BCD = 90°, such that AB = CD = 3 cm

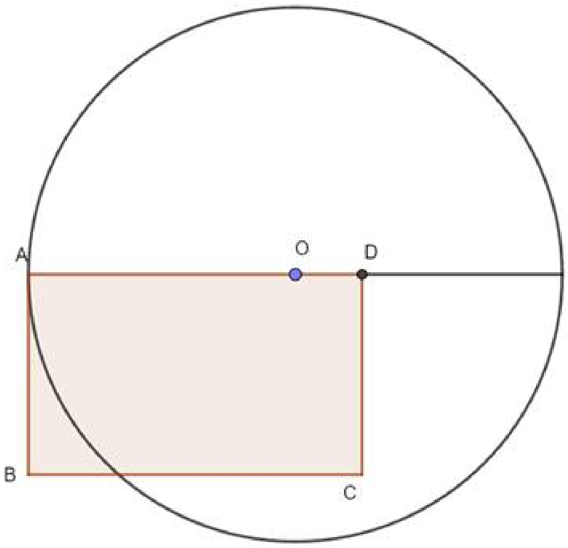


2. Join AD to get rectangle ABCD, here area(ABCD) = length × breadth = 5 × 3 = 15 cm.

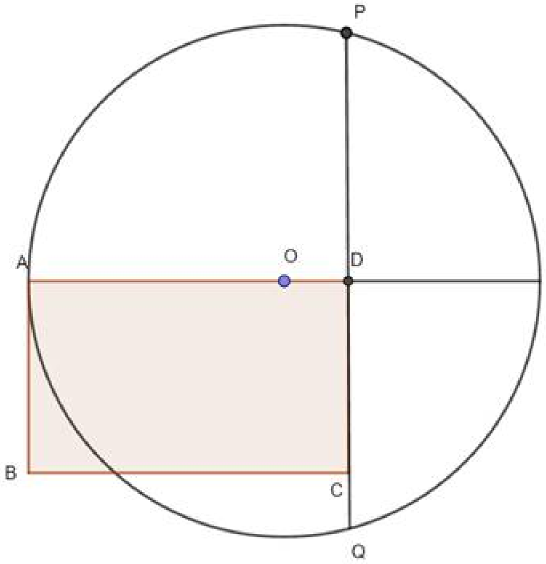
3. From the rectangle drawn, extend AD by 3 cm to D'.



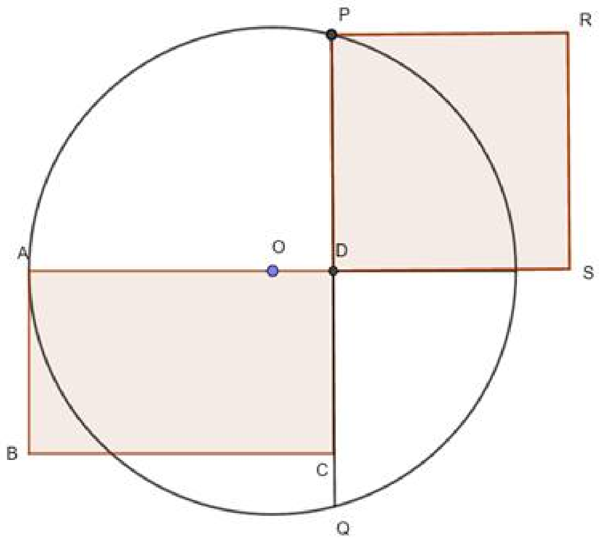
4. From the mid–point of AD'(Let it be O such that OA = OD'=4 cm) Draw a circle taking OA = OD' as radius.



3. Now, draw a chord PQ ⊥ AD', passing through D.



6. Taking DP as side, draw a square DPRS, which is required.



Concept used:

The area of rectangle form of parts into which a diameter of a circle is cut by a perpendicular chord is equal to the area made by the square of half the chord.

And in the diagram, we made, AD' is diameter and PQ is perpendicular chord, therefore

Area(DPRS) = ar(ABCD) = 15 cm^2

And DPRS is a square

4. Question

Draw a square of area 5 square centimetres in three different ways.

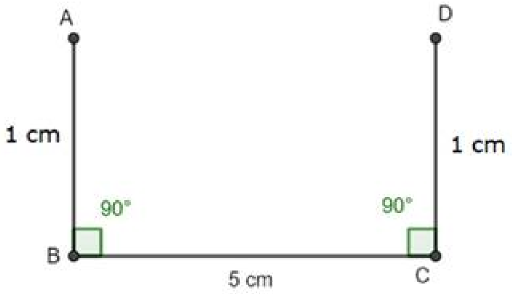
Answer

Way 1:

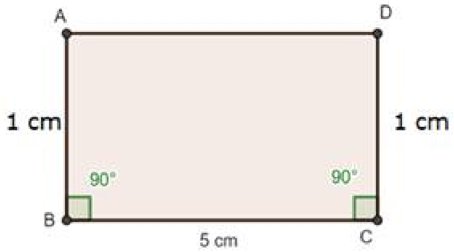
Steps of construction:

1. Draw a line BC = 5 cm, taking B and C as centres, draw

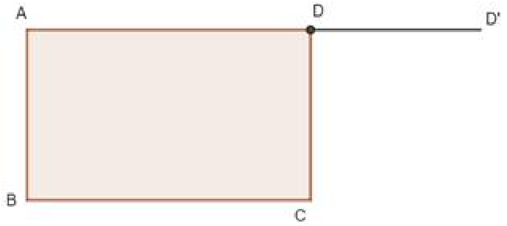
∠ABC = ∠BCD = 90°, such that AB = CD = 1 cm



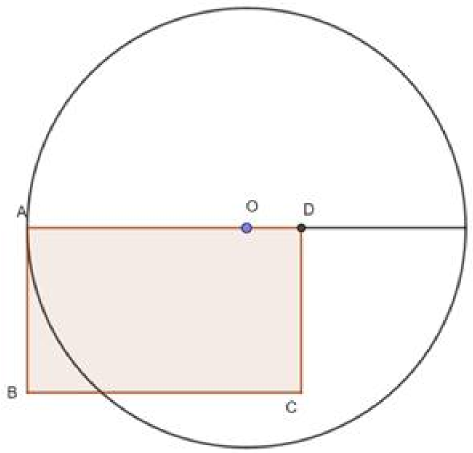
2. Join AD to get rectangle ABCD, here area(ABCD) = length × breadth = 5 × 1 = 5 cm.



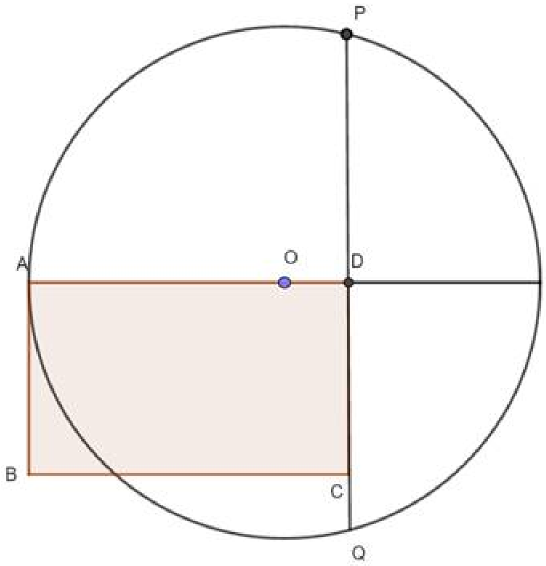
3. From the rectangle drawn, extend AD by 3 cm to D'.



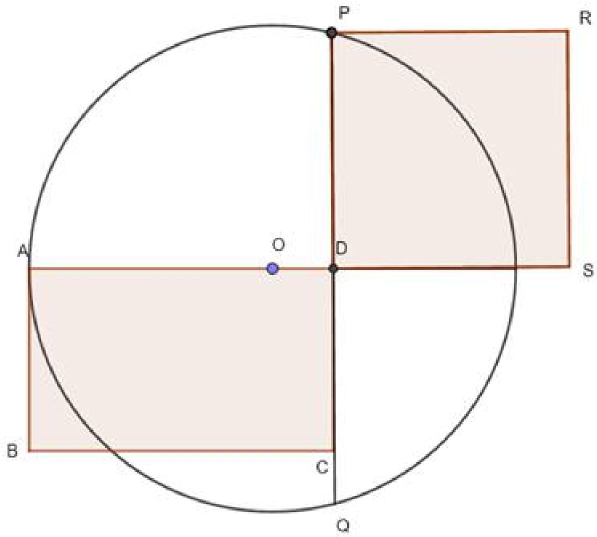
4. From the mid–point of AD'(Let it be O such that OA = OD'=4 cm) Draw a circle taking OA = OD' as radius.



3. Now, draw a chord PQ ⊥ AD', passing through D.



6. Taking DP as side, draw a square DPRS, which is required.



Concept used:

The area of rectangle form of parts into which a diameter of a circle is cut by a perpendicular chord is equal to the area made by the square of half the chord.

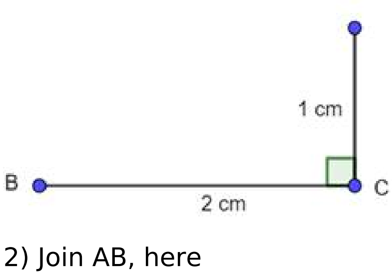
And in the diagram, we made, AD' is diameter and PQ is perpendicular chord, therefore

Area(DPRS) = ar(ABCD) = 5 cm^2

Way 2:

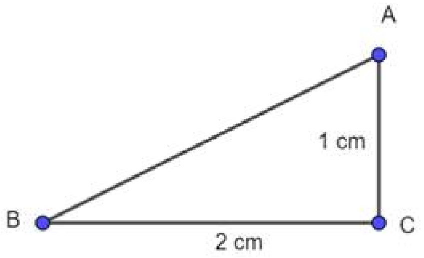
Steps of construction:

1) Make a line segment BC = 2 cm, and C draw AC ⊥ BC such that AC = 1 cm



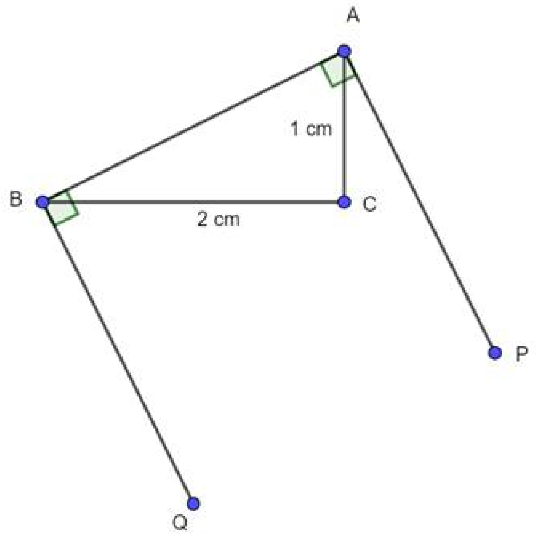
2) Join AB, here

AB = $\sqrt{(2^2 + 1^2)} = \sqrt{5}$ cm

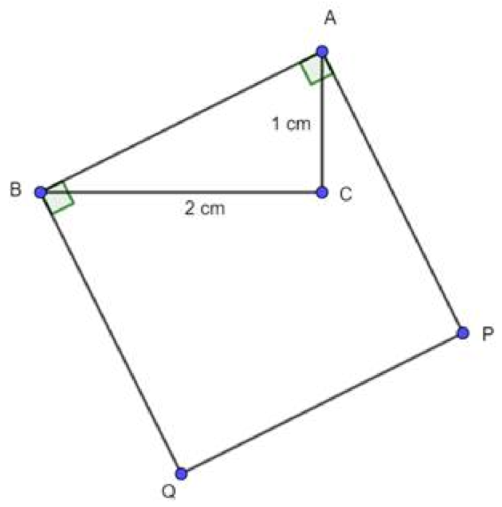


3) At points A and B, draw perpendiculars AP ⊥ AB and BQ ⊥ AB, such that

AP = BQ = AB.



4) Join PQ, ABQP is required square and area(ABQP) = $(\sqrt{5})^2$ = 5 cm.

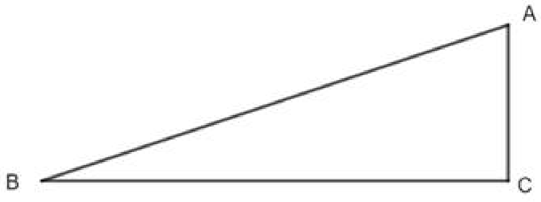


Way 3

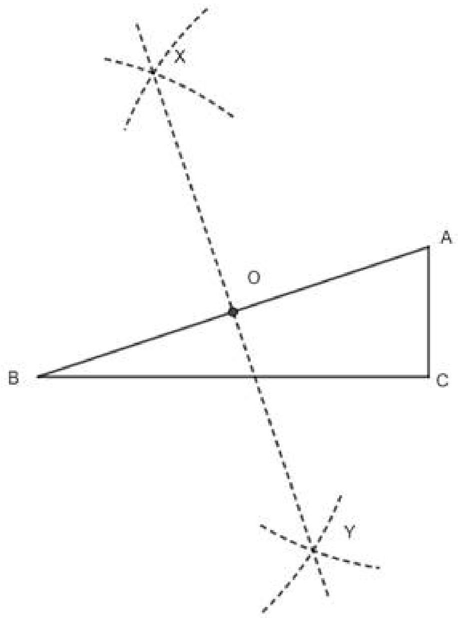
1) Make a line segment BC = 3 cm, and C draw AC ⊥ BC such that AC = 1 cm

2) Join AB, here

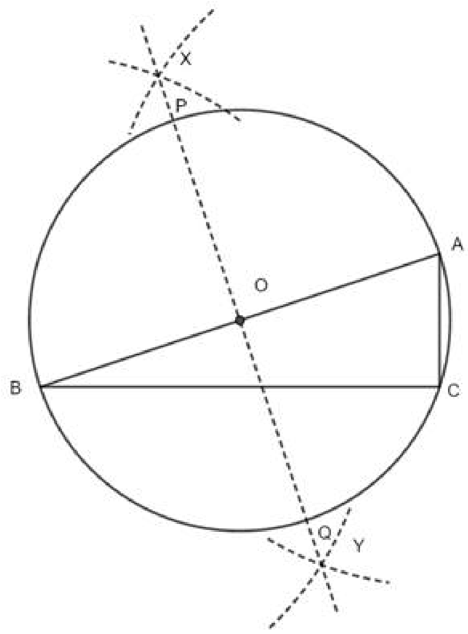
AB = $\sqrt{(3^2 + 1^2)} = \sqrt{10}$ cm



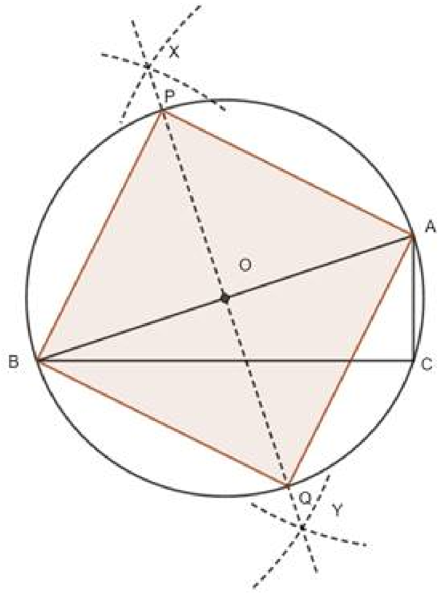
3) Draw the perpendicular bisector of AB, and name it as XY, XY intersects AB at O.



4) Taking O as centre, and OA = OB as radius, draw a circle which intersects XY at P and Q.



5) Join AP, PB, BQ and QA to get the required square.



Verification:

Here, AB = √10 cm is the diagonal of square, and we know, diagonal of a square of side a is equal to √2 a

⇒ √2 a =√10

⇒ a = √5 cm

5. Question

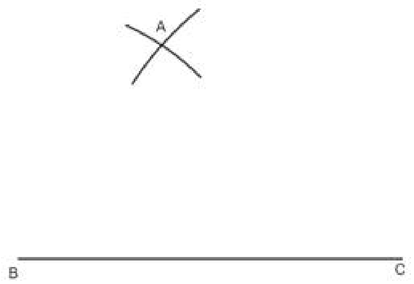
Draw a triangle of sides, 4, 5, 6 centimetres and draw a square of the same area.

Answer

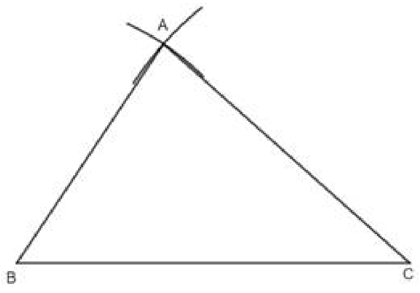
Steps of construction:

PART A: Construction of triangle

1. Draw a line BC = 6 cm, From B as center, draw an arc of radius 4 cm and from C as center, draw an arc of 5 cm, an d the both arcs intersect each other at A.



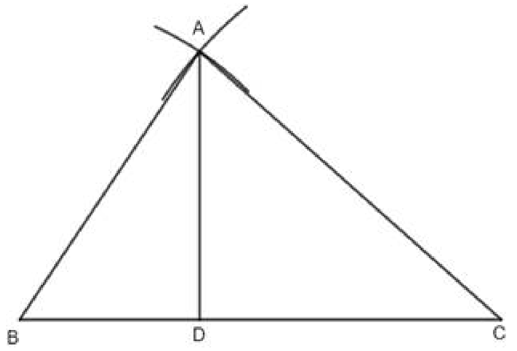
2. Join AB and AC to get the required triangle.



PART B: Construction of square

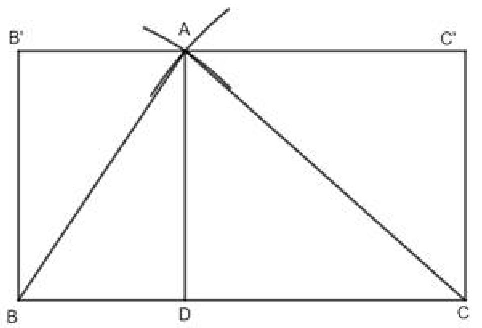
For making the square of equal area, we first make a rectangle having the double area as of triangle, then we will half the area of rectangle and finally, we will make our square of area of same as half part of rectangle which will be equal to area of triangle.

3. Draw AD ⊥ BC, it divides ΔABC into two right–angled triangles, ΔABD and ΔADC.

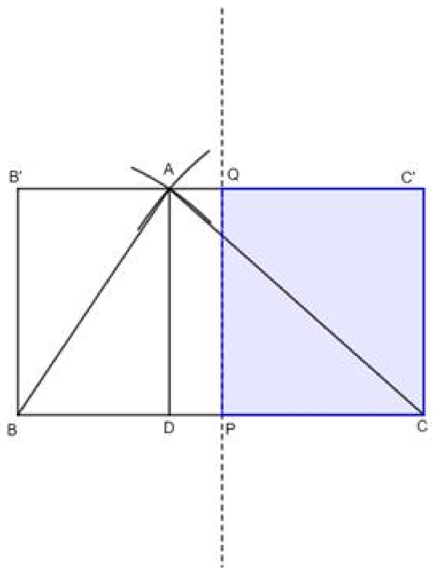


4. For each right angle draw a reflection, as shown in figure, it will construct a rectangle BB'C'C.

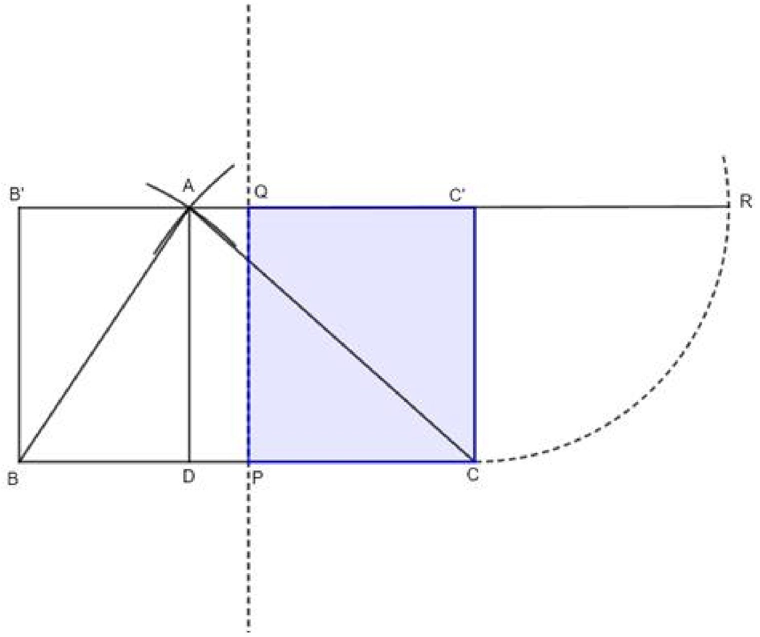
Clearly, area of rectangle is double of the area of triangle.



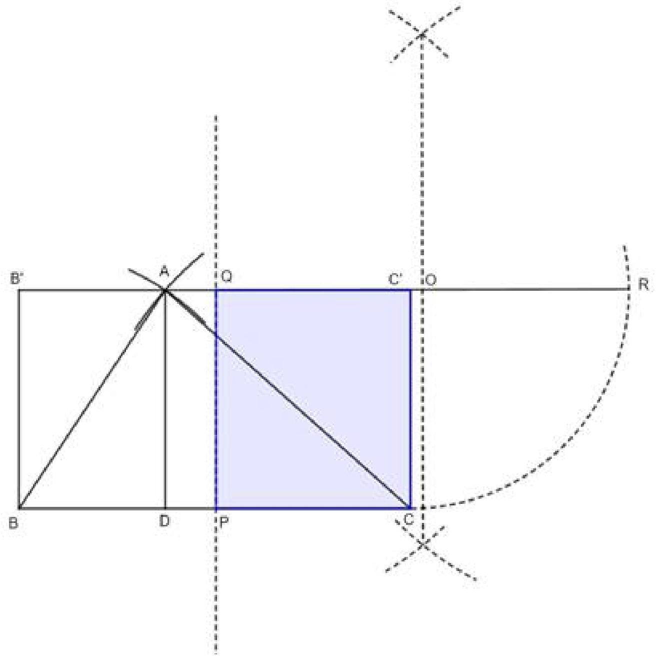
5. Now, we bisect the rectangle [By bisecting BC], which gives a new rectangle PCC'Q.



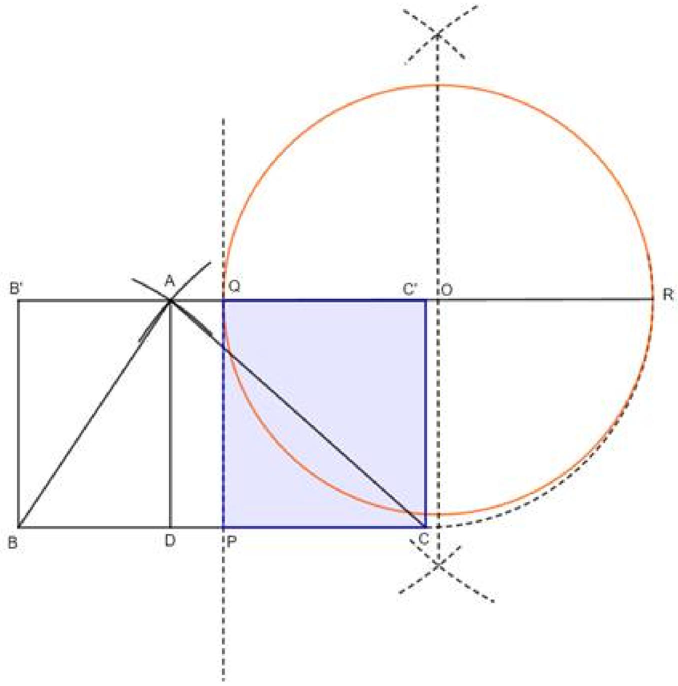
6. Now, extend QC' to QR, such that C'R = C'C



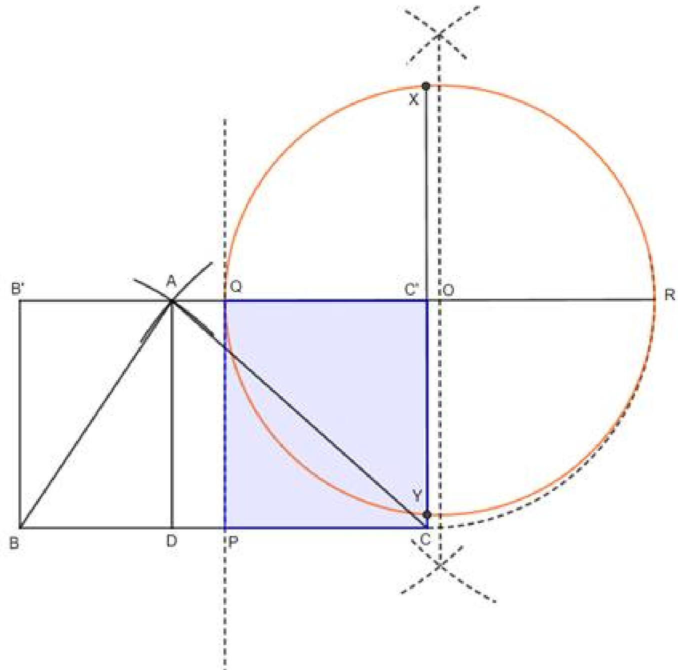
7. Draw the perpendicular bisector of QR, which intersects QR at O.



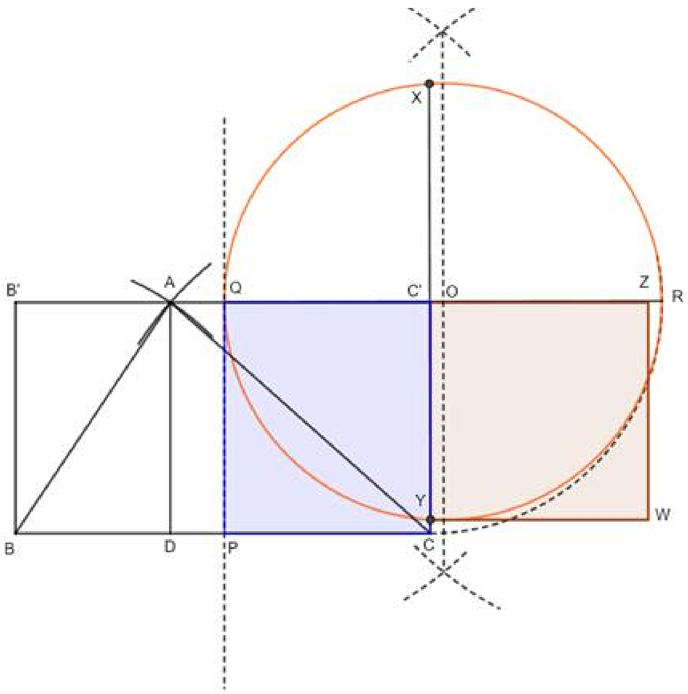
8. Taking O as center and OQ = OR as radius draw a circle.



9. Now, draw a chord XY ⊥ QR, such that XY passes through C'.



10. Now taking, XC' or C'Y as a side, Draw a square C'XYZ, which is required here.



Verification:

Clearly,

$$\frac{1}{2}(\text{area of rectangle } BB'CC') = \text{area of triangle}$$

And

$$\frac{1}{2}(\text{area of rectangle } BB'CC) = \text{area of rectangle } PCC'Q = \text{area of square}$$

[As, The area of rectangle form of parts into which a diameter of a circle is cut by a perpendicular chord is equal to the area made by the square of half the chord]

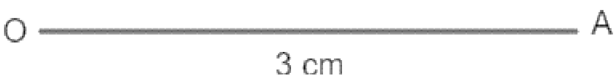
⇒ area of square = area of triangle

6. Question

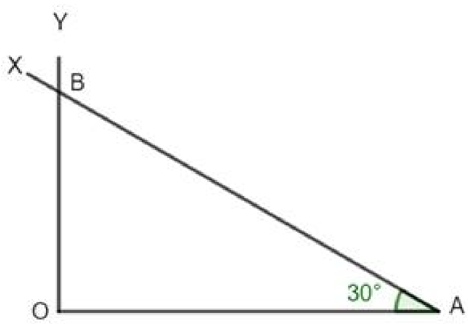
Draw an equilateral triangle of height 3 centimetres.

Answer

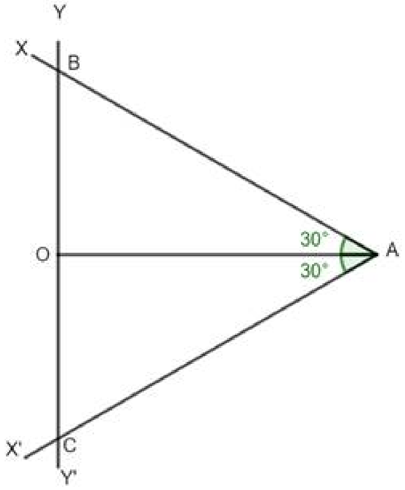
1) Draw a line OA = 3 cm



2) From O draw ∠AOY = 90° and From A draw ∠OAX = 30° above the horizontal, such that OY and AX intersect each other at B.



3) From O draw ∠AOY' = 90° and From A draw ∠OAX' = 30° below the horizontal, such that OY' and AX' intersect each other at C.



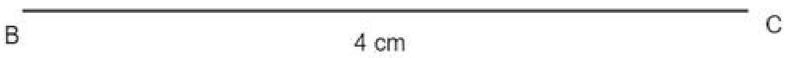
ABC is required equilateral triangle.

7. Question

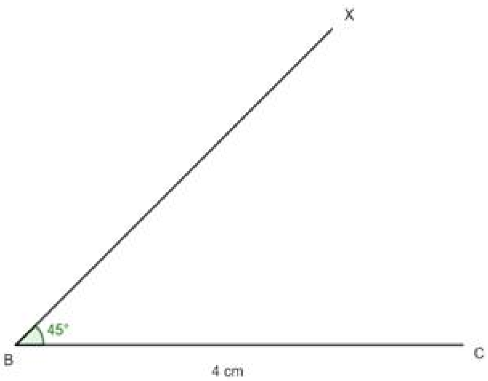
Draw an isosceles right triangle of hypotenuse 4 centimetres.

Answer

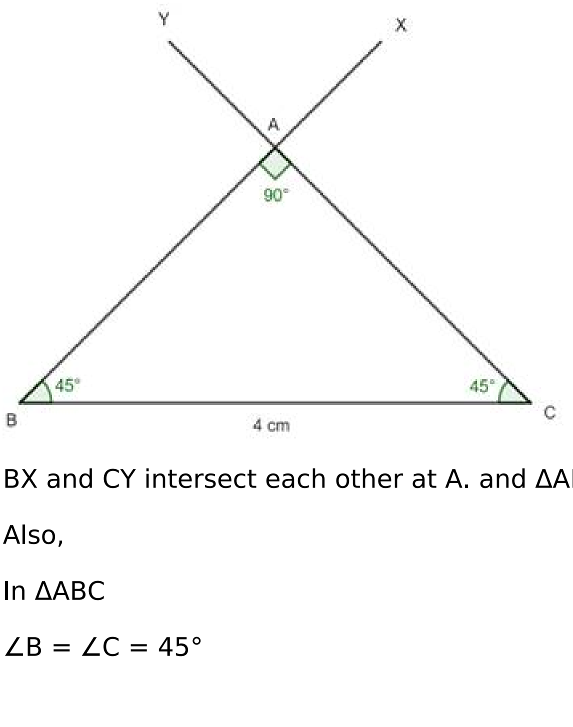
1) Draw a line BC 4 cm.



2) At B, draw ∠XBC = 45°



3) At C, draw ∠YCB = 45°



BX and CY intersect each other at A. and ∆ABC is required triangle,

Also,

In ∆ABC

∠B = ∠C = 45°

Therefore, AB = AC [i.e. ABC is isosceles]

And By angle sum property of triangle,

∠A + ∠B + ∠C = 180°

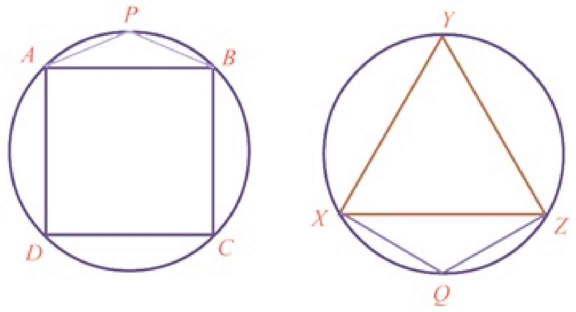
⇒ ∠A + 45 + 45 = 180

⇒ ∠A = 90°

Hence, ABC is isosceles.

8. Question

In the picture below, ABCD is a square with vertices on a circle and XYZ is such an equilateral triangle. P and Q are points on the circules:

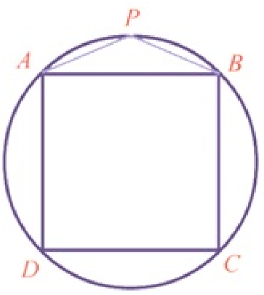


i) How much is □ APB?

ii) How much is □ XQZ?

Answer

(i)



Join AC.

Now, APBC is a cyclic quadrilateral, as it lies on a circle

Therefore,

∠APB + ∠ACB = 180° [Sum of opposite angles of a cyclic quadrilateral is 180°]

Now, Diagonals of the square bisects the angles of a square

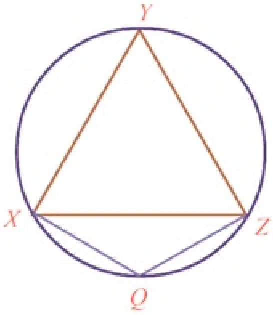
$\Rightarrow \angle ACB = \frac{1}{2}\angle BCD$

$\Rightarrow \angle ACB = \frac{1}{2}(90°) = 45°$

⇒ ∠ APB + 45° = 180°

⇒ ∠APB = 135°

(ii)



As, XYZQ is a cyclic quadrilateral

$\angle XYZ + \angle XQZ = 180°$ [Sum of opposite angles of a cyclic quadrilateral is 180°]

Now, $\angle XYZ = 60°$, as XYZ is an equilateral triangle

$\Rightarrow \angle XQZ = 180° - 60° = 120°$