CBSE Board
Class X Mathematics

Time: 3 hrs **Total Marks: 80**

General Instructions:

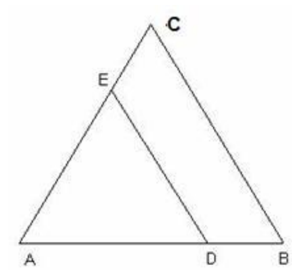
1. **All** questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Question numbers **1 to 6** in **Section A** are multiple choice questions where you are to select **one** correct option out of the given four.
4. Use of calculator is **not** permitted.

Section A
(Questions 1 to 6 carry 1 mark each)

1. What is the probability of getting a prime number when a die is thrown once?

2. The ratio of the length of a pole and its shadow is $\sqrt{3}:1$. Find the angle of elevation of the Sun.

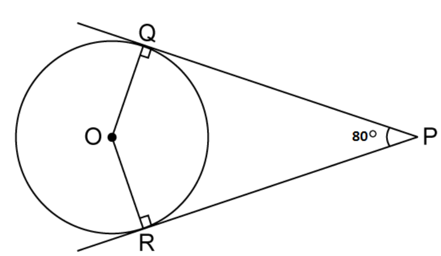
3. In the adjoining figure, DE is parallel to BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value of x.



4. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 + 2x + 1$, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}$.

5. Is 0. 101100101010 an irrational number? Justify your answer.

6. In figure, PQ and PR are two tangents to a circle with centre O. If ∠QPR = 80°, then find ∠QOR.

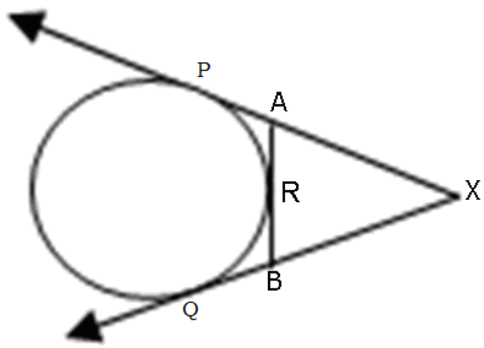


Section B
(Questions 7 to 12 carry 2 marks each)

7. If the zeros of the polynomial $f(x) = x^3 - 3x^2 + x + 1$ are $a - b$, a, $a + b$, find a and b.

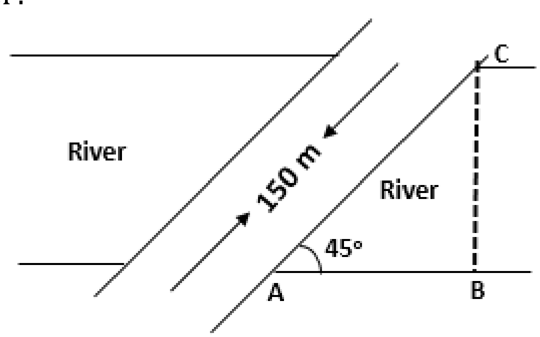
8. Find the H.C.F of 455 and 84 using the division algorithm.

9. In the given figure, XP and XQ are tangents from X to the circle. R is a point on the circle. Prove that XA + AR = XB + BR.



10. A bicycle wheel makes 5000 revolutions in moving 11 km. Find the diameter of the wheel.

11. A bridge across a river makes an angle of 45° with the river bank as shown in the figure. If the length of the bridge across the river is 150 m, what is the width of the river?



12. If $7\sin^2\theta + 3\cos^2\theta = 4$, then find θ and hence prove that $\sec\theta + \operatorname{cosec}\theta = 2 + \frac{2}{\sqrt{3}}$

Section C
(Questions 13 to 22 carry 3 marks each)

13. Prove that: $\frac{\sec A+\tan A}{\sec A-\tan A}=\left(\frac{1+\sin A}{\cos A}\right)^2$

14. Find the area of the quadrilateral ABCD whose vertices are A(1, 1), B(7, −3), C(12, 2) and D(7, 21) respectively.

15. Solve: $\frac{1}{a}+\frac{1}{b}+\frac{1}{x}=\frac{1}{a+b+x}$

16. Solve for x and y:

$\frac{x}{a}+\frac{y}{b}=2;\qquad ax-by=a^2-b^2$

17. Prove that $\frac{3}{2\sqrt{5}}$ is an irrational number.

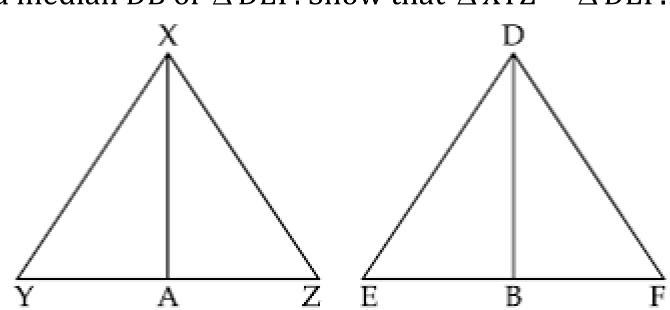
18. Prema invests a certain sum at the rate of 10% per annum of interest and another sum at the rate of 8% per annum to get a yield of Rs. 1640 in one year's time. Next year she interchanges the rates and gets a yield of Rs. 40 less than the previous year. How much did she invest in each type in the first year?

19. If the point (x, y) is equidistant from the points (a + b, b − a) and (a − b, a + b), then prove that bx = ay.

20. Find the mean of following distribution by the step deviation method.

Daily Expenditure:	100-150	150-200	200-250	250-300	300-350
No. of householders:	4	5	12	2	2

21. In the figure, sides XY and YZ and median XA of a triangle XYZ are proportional to sides DE, EF and median DB of ΔDEF. Show that $\Delta XYZ \sim \Delta DEF$.



22. One card is drawn from a pack of 52 cards, each of which is equally likely to be drawn. Find the probability that the card drawn is

i. either red or king

ii. a red face card

iii. '10' of a black suit

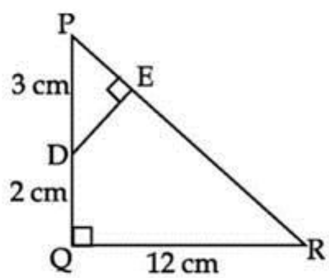
Section D

(Questions 23 to 30 carry 4 marks each)

23. Construct a triangle similar to ΔABC in which AB = 4.6 cm, BC = 5.1 cm, m$\angle$A = 60^o with scale factor 4 : 5.

24. In a school, students thought of planting trees in an around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class in which they are studying, e.g., a section of class-I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students? What value can you infer from the planting the trees?

25. In the given figure, ΔPQR is right-angled triangle right-angled at Q. DE $\perp$ PR. Prove $\Delta PQR \sim \Delta PED$ and find the lengths of PE and DE if PD = 3 cm, QD = 2 cm and QR = 12 cm.



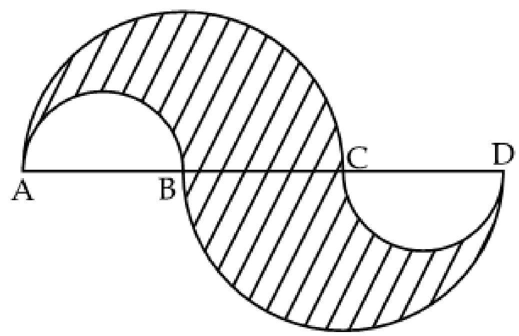
26. Form a pair of linear equations for the following problem, and find the solution graphically.

"10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz."

27. From a window of a house in a street, h metres above the ground, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are α and β respectively. Show that the height of the opposite house is h(1+ tan α . cot β) metres.

28. A tent is of the shape of a right circular cylinder upto a height of 3 metres and conical above it. The total height of the tent is 13.5 metres above the ground. Calculate the cost of painting the inner side of the tent at the rate of Rs. 2 per square metre, if the radius of the base is 14 metres.

29. In the given figure, AC = BD = 7 cm and AB = CD = 1.75 cm. Semicircles are drawn as shown in the figure. Find the area of the shaded region. $\left[\text{Take } \pi = \dfrac{22}{7}\right]$



30. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farms	2	8	12	24	38	16

Change the distribution to a 'more than' type distribution and draw ogive.

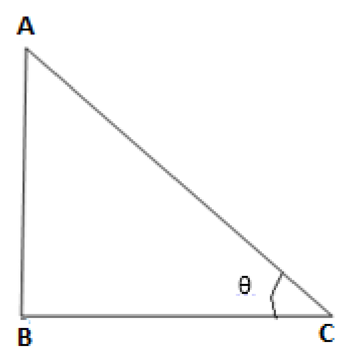
CBSE Board
Class X Mathematics
Solution

Time: 3 hrs **Total Marks: 80**

Section A

1. When a die is thrown once the outcomes are 1, 2, 3, 4, 5 and 6.
Thus, total number of outcomes = 6
Out of these outcomes, 2, 3 and 5 are prime numbers.
Thus, number of favourable outcomes = 3
Therefore, P(prime number) = $\frac{3}{6}=\frac{1}{2}$

2. Consider the following figure.
Let AB be the pole and BC be its shadow.



Given that $\frac{AB}{BC}=\frac{\sqrt{3}}{1}$
Let $\theta=\angle ACB$ be the angle of elevation.
In ΔABC,

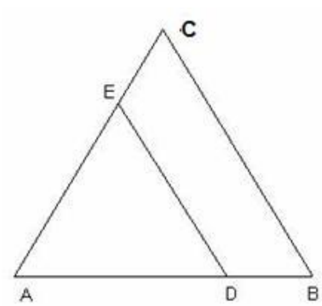
$$\tan\theta=\frac{AB}{BC}$$

$$\Rightarrow \tan\theta=\frac{\sqrt{3}}{1}$$

$$\Rightarrow \tan\theta=\tan 60^{\circ}$$

$$\Rightarrow \theta=60^{\circ}$$

3. In ΔABC, DE ∥ BC.
Then, by Basic Proportionality Theorem,



$$\frac{AD}{DB}=\frac{AE}{EC}$$

$$\Rightarrow \frac{x}{x-2}=\frac{x+2}{x-1}$$

$$\Rightarrow x(x-1)=(x-2)(x+2)$$

$$\Rightarrow x^2-x=x^2-4$$

$$\Rightarrow x=4$$

4. It is given that α and β are the zeros of the quadratic polynomial $f(x) = x^2 + 2x + 1$

$$\therefore \alpha+\beta=-\frac{2}{1}=-2 \text{ and } \quad \text{and } \alpha\beta=\frac{1}{1}=1$$

Now,

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=\frac{-2}{1}=-2$$

5. A real number is an irrational number when it has a non-terminating non repeating decimal representation.
Thus, 0.101100101010……. is an irrational number.

6. The angle between a pair of tangents to a circle which are inclined to each other at an angle is supplementary to the angle between the two radii of the circle.
Thus, the angle between the radii of the circle,
$\angle QOR = 180° - \angle QPR = 180° - 80° = 100°$

Section B

7. Since a – b, a and a + b are the zeros of $f(x) = x^3 - 3x^2 + x + 1$.

$$\therefore (a-b)+a+(a+b)=-\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\Rightarrow 3a=-\frac{-3}{1}$$

$$\Rightarrow 3a=3$$

$$\Rightarrow a=1$$

And, $(a-b)\times a\times(a+b)=-\dfrac{\text{Constant term}}{\text{Coefficient of } x^3}$

$$\Rightarrow a(a^2-b^2)=-\frac{1}{1}$$

$$\Rightarrow 1(1-b^2)=-1$$

$$\Rightarrow b^2=2$$

$$\Rightarrow b=\pm\sqrt{2}$$

8. $455 = 84 \times 5 + 35$
$\Rightarrow 84 = 35 \times 2 + 14$
$\Rightarrow 35 = 14 \times 2 + 7$
$\Rightarrow 14 = 7 \times 2 + 0$
Therefore, H.C.F. = 7

9. Since the lengths of tangents from an exterior point to a circle are equal.

Therefore, XP = XQ (tangents from X) ....(i)

AP = AR (tangents from A) ....(ii)

BQ = BR (tangents from B) ....(iii)

Now, XP = XQ

$\Rightarrow$ XA + AP = XB + BQ

$\Rightarrow$ XA + AR = XB + BR [Using (ii) and (iii)]

10. Let r be the radius of the wheel.

Distance covered in 1 revolution = $2\pi r$

$\therefore$ Distance covered in 5000 revolutions = $5000 \times 2\pi r$ = 11 km

$$\Rightarrow 5000 \times \frac{22}{7}(2r) = 11 \times 1000 \text{ m}$$

$$\Rightarrow 2r = \frac{7}{10} \text{ m} = 70 \text{ cm}$$

Thus, the diameter of the wheel is 70 cm.

11. In right triangle ABC,

$$\sin 45^\circ = \frac{BC}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{150}$$

$$\Rightarrow BC = \frac{150}{\sqrt{2}}$$

$$\Rightarrow BC = 75\sqrt{2} \text{ m}$$

Thus, the width of the river is $75\sqrt{2}$ metres.

12. $7\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow 7\sin^2\theta + 3(1 - \sin^2\theta) = 4$

$\Rightarrow 7\sin^2\theta + 3 - 3\sin^2\theta = 4$

$\Rightarrow 4\sin^2\theta = 1$

$$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Thus, $\sec 30^\circ + \operatorname{cosec} 30^\circ = \frac{2}{\sqrt{3}} + 2$

Section C

13. $\text{L.H.S.} = \dfrac{\sec A + \tan A}{\sec A - \tan A}$

$$= \frac{\sec A + \tan A}{\sec A - \tan A} \times \frac{\sec A + \tan A}{\sec A + \tan A}$$

$$= \frac{(\sec A + \tan A)^2}{\sec^2 A - \tan^2 A}$$

$$= (\sec A + \tan A)^2 \qquad \left(\because \sec^2\theta = 1 + \tan^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1\right)$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)^2$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)^2$$

$$= \text{R.H.S.}$$

14. Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

Area of triangle = $\dfrac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

Area(ΔABC) = $\dfrac{1}{2}$ [1(−3 − 2) + 7(2 − 1) + 12(1 + 3)] = $\dfrac{1}{2}$ [−5 + 7 + 48] = 25 sq. units

Area(ΔACD) = $\dfrac{1}{2}$ [1(2 − 21) + 12(21 − 1) + 7(1 − 2)] = $\dfrac{1}{2}$ [−19 + 240 − 7] = 107 sq. units

Therefore, area of quadrilateral ABCD = 25 + 107 = 132 sq. units.

15. $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x} = \dfrac{1}{a+b+x}$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{1}{a+b+x} - \frac{1}{x}$$

$$\Rightarrow \frac{a+b}{ab} = \frac{x-a-b-x}{(a+b+x)x}$$

$$\Rightarrow \frac{a+b}{ab} = \frac{-(a+b)}{(a+b+x)x}$$

$$\Rightarrow ax + bx + x^2 = -ab$$

$$\Rightarrow x^2 + ax + bx + ab = 0$$

$$\Rightarrow x(x+a) + b(x+a) = 0$$

$$\Rightarrow (x+a)(x+b) = 0$$

$$\Rightarrow x + a = 0 \text{ or } x + b = 0$$

$$\Rightarrow x = -a, -b$$

16. $\frac{x}{a} + \frac{y}{b} = 2$

$\Rightarrow$ bx + ay = 2ab ....(1)

ax – by = $a^2 - b^2$ ....(2)

Multiplying (1) with a and (2) with b and subtracting, we get

$$\begin{array}{l} \cancel{abx} + a^2y = 2a^2b \\ \cancel{abx} - b^2y = a^2b - b^3 \\ - \quad\; + \qquad\quad - \quad\; + \\ \hline y(a^2+b^2) = a^2b + b^3 \\ \Rightarrow y(a^2+b^2) = b(a^2+b^2) \\ \Rightarrow y = b \end{array}$$

From (1), bx + ab = 2ab

$\Rightarrow$ bx = ab

$\Rightarrow$ x = a

Hence, x = a and y = b.

17. Let $\frac{3}{2\sqrt{5}}$ be a rational number.

$\Rightarrow \frac{3}{2\sqrt{5}} = \frac{a}{b}$, where a and b are co-prime integers and b ≠ 0.

$\Rightarrow \sqrt{5} = \frac{3b}{2a}$

Now, a, b, 2 and 3 are integers.

Therefore, $\frac{3b}{2a}$ is a rational number.

$\Rightarrow \sqrt{5}$ is a rational number.

This is a contradiction as we know that $\sqrt{5}$ is irrational.

Therefore, our assumption is wrong.

Hence, $\frac{3}{2\sqrt{5}}$ is an irrational number.

18. Let us assume that Prema invests Rs. x @10% and Rs. y @8% in the first year.

We know that

Interest = $\frac{P \times R \times T}{100}$

According to given conditions,

$\frac{x \times 10 \times 1}{100} + \frac{y \times 8 \times 1}{100} = 1640$

$\Rightarrow$ 10x + 8y = 164000 ....(i)

After interchanging the rates, we have

$\frac{y \times 10 \times 1}{100} + \frac{x \times 8 \times 1}{100} = 1600$

$\Rightarrow 10y + 8x = 160000$

Or, $8x + 10y = 160000$ ....(ii)

Adding (i) and (ii), we get

$18x + 18y = 324000$

$\Rightarrow x + y = 18000$ ....(iii)

Subtracting (ii) from (i), we get

$2x - 2y = 4000$

$\Rightarrow x - y = 2000$ ....(iv)

Adding (iii) and (iv), we get

$2x = 20000$

$\Rightarrow x = 10000$

Substituting this value of x in (iii), we get

$y = 8000$

Thus, the sums invested in the first year at the rate 10% and 8% are Rs. 10000 and Rs. 8000, respectively.

19. Let P(x, y), Q(a + b, b − a) and R(a − b, a + b) be the given points.

It is given that $PQ = PR \Rightarrow PQ^2 = PR^2$

$\{x - (a + b)\}^2 + \{y - (b - a)\}^2 = \{x - (a - b)\}^2 + \{y - (a + b)\}^2$

$\Rightarrow x^2 - 2x(a + b) + (a + b)^2 + y^2 - 2y(b - a) + (b - a)^2$

$= x^2 + (a - b)^2 - 2x(a - b) + y^2 - 2y(a + b) + (a + b)^2$

$\Rightarrow -2x(a + b) - 2y(b - a) = -2x(a - b) - 2y(a + b)$

$\Rightarrow -ax - bx - by + ay = -ax + bx - ay - by$

$\Rightarrow 2bx = 2ay$

$\Rightarrow bx = ay$

20. Consider the following table:

Let A = 225

$$d_i = \frac{x_i - 225}{50}$$

C.I	f_i	x_i	d_i	$f_i d_i$
100 - 150	4	125	−2	−8
150 - 200	5	175	−1	−5
200 - 250	12	225	0	0
250 - 300	2	275	1	2
300 - 350	2	325	2	4
Total	25			−7

$$\text{Mean} = \bar{x} = A + \frac{\sum f_i d_i}{\sum f_i} \times h = 225 - \frac{7}{\cancel{25}} \times \cancel{50}^{2} = 225 - 14 = 211$$

21. Given: In ΔXYZ and ΔDEF

$$\frac{XY}{DE} = \frac{YZ}{EF} = \frac{XA}{DB} \qquad \text{...(1)}$$

To prove: ΔXYZ ~ ΔDEF

Proof:

Since XA and DB are medians,

2YA = YZ

2EB = EF ...(2)

From (1) and (2)

$$\frac{XY}{DE} = \frac{2YA}{2EB} = \frac{XA}{DB}$$

⇒ΔXYA ~ ΔDEB (BY SSS rule)

⇒∠Y = ∠E ...(3)

Now, in ΔXYZ and ΔDEF,

$$\frac{XY}{DE} = \frac{YZ}{EF} \qquad \text{[From (1)]}$$

∠Y = ∠E [From (3)]

⇒ ΔXYZ ~ ΔDEF (BY SAS rule)

22. Out of 52 cards, one card can be drawn in 52 ways.

So, total number of outcomes = 52

i. There are 26 red cards, including two red kings, in a pack of 52 playing cards.

   Also, there are 4 kings, two red and two black.

   Therefore, card drawn will be either a red card or a king if it is any one of 28 cards (26 red cards and 2 black kings).

   So, favourable number of elementary events = 28

   Hence, required probability = $\frac{28}{52} = \frac{7}{13}$

ii. There are 6 red face cards, 3 each from diamonds and hearts.

   Out of these 6 red face cards, one card can be chosen in 6 ways.

   So, favorable number of elementary events = 6

   Hence, required probability = $\frac{6}{52} = \frac{3}{26}$

iii. There are two suits of black cards, viz., spades and clubs.

   Each suit contains one card bearing number 10.

   So, favorable number of elementary events = 2

   Hence, required probability = $\frac{2}{52} = \frac{1}{26}$

Section D
(Questions 23 to 30 carry 4 marks each)

23. Steps of construction:-

(1) Draw a line segment AB of length 4.6 cm.

(2) At A draw an angle BAY of 60°.

(3) With centre B and radius 5.1 cm, draw an arc which intersects line AY at point C.

(4) Join BC.

(5) At A draw an acute angle BAX of any measure.

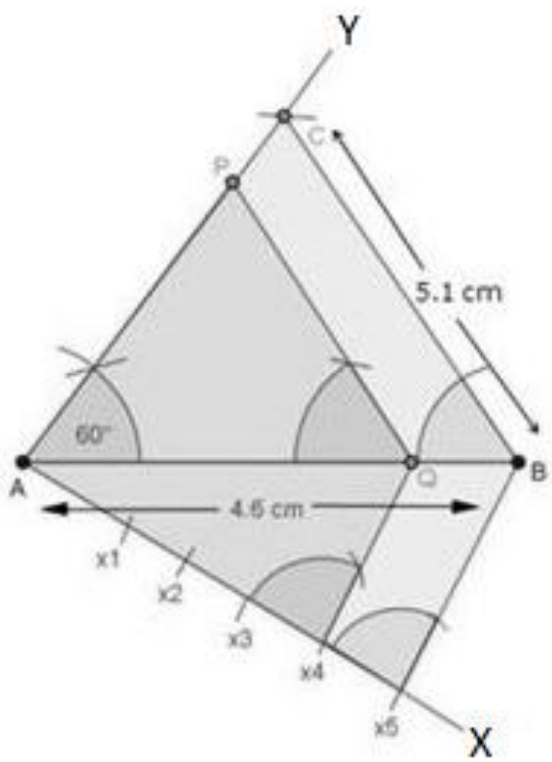
(6) Starting from A, cut 5 equal parts on AX.

(7) Join X_5B

(8) Through X_4, Draw $X_4Q \parallel X_5B$

(9) Through Q, Draw $QP \parallel BC$

$\therefore \Delta PAQ \sim \Delta CAB$



24. There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.

The number of trees planted by class I = number of sections × 1 = 3 × 1 = 3

The number of trees planted by class II = number of sections × 2 = 3 × 2 = 6

The number of trees planted by class III = number of sections × 3 = 3 × 3 = 9

Therefore, we have the sequence: 3, 6, 9, ..., (12 terms)

To find total number of trees planted by all the students, we need to find sum of the 12 terms of the sequence.

First term = a = 3

Common difference = d = 6 – 3 = 3

n = 12

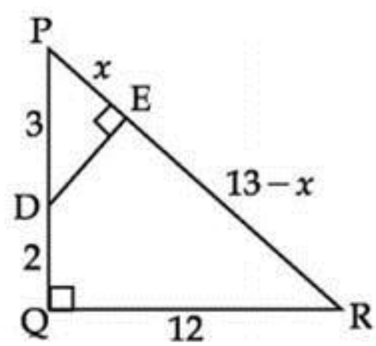
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{12} = \frac{12}{2}[2\times3 + (12-1)3] = 6[6+33] = 6\times39 = 234$$

Thus, in total 234 trees will be planted by the students.

Values inferred are environmental friendly and social.

25. In right ΔPQR,



$PR^2 = PQ^2 + QR^2 = 5^2 + 12^2 = 25 + 144 = 169$

$\therefore$ PR = 13 cm

Let PE = x, then ER = 13 – x

In ΔPQR and ΔPED,

$\angle PQR = \angle PED$

$\angle QPR = \angle EPD$

$\therefore$ ΔPQR ~ ΔPED [AA similarity]

$$\therefore \frac{PQ}{PE} = \frac{QR}{ED} = \frac{PR}{PD}$$

$$\Rightarrow \frac{5}{x} = \frac{12}{ED} = \frac{13}{3}$$

$$\therefore PE = x = \frac{5\times3}{13} = \frac{15}{13} = 1\frac{2}{13} \text{ cm}$$

$$ED = \frac{12\times3}{13} = \frac{36}{13} = 2\frac{10}{13} \text{ cm}$$

26. Let the number of girls and boys in the class be x and y, respectively.

According to the given conditions, we have:

x + y = 10

$\Rightarrow$ x = 10 – y

Three solutions of this equation can be written in a table as follows:

x	5	4	6
y	5	6	4

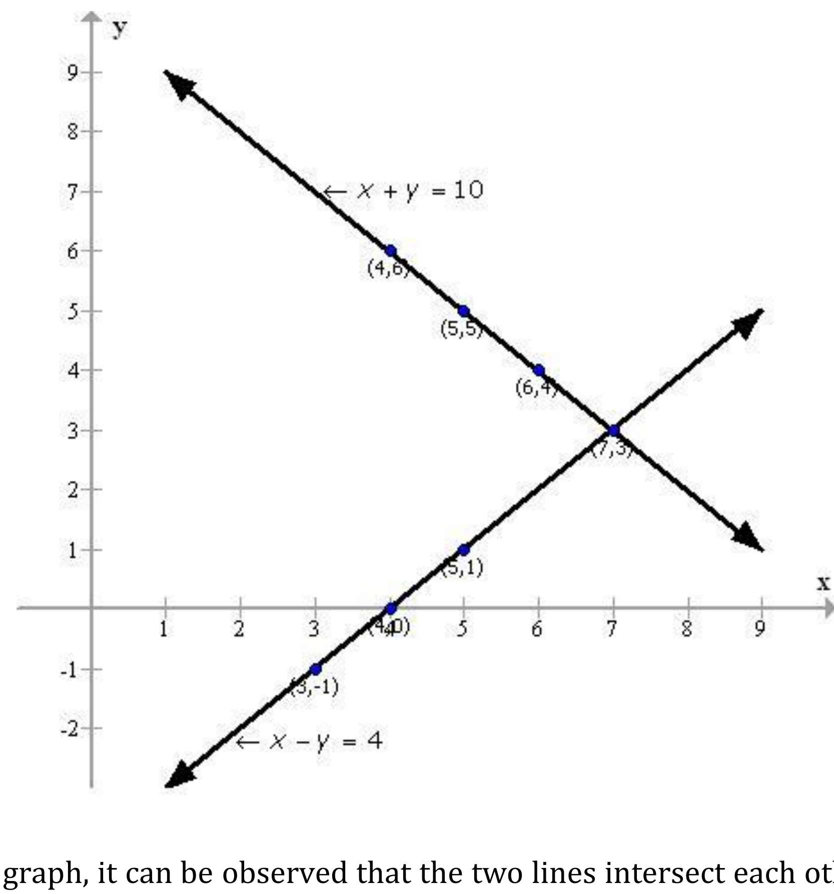
x – y = 4

$\Rightarrow$x = 4 + y

Three solutions of this equation can be written in a table as follows:

x	5	4	3
y	1	0	–1

The graphs of the two equations can be drawn as follows:



From the graph, it can be observed that the two lines intersect each other at the point (7, 3).

So, x = 7 and y = 3 is the required solution of the given pair of equations.

27. Let B be the window of a house AB and let CD be the other house.

Then, AB = EC = h metres.

Let CD = H metres.

Then, ED = (H − h) m

In ΔBED,

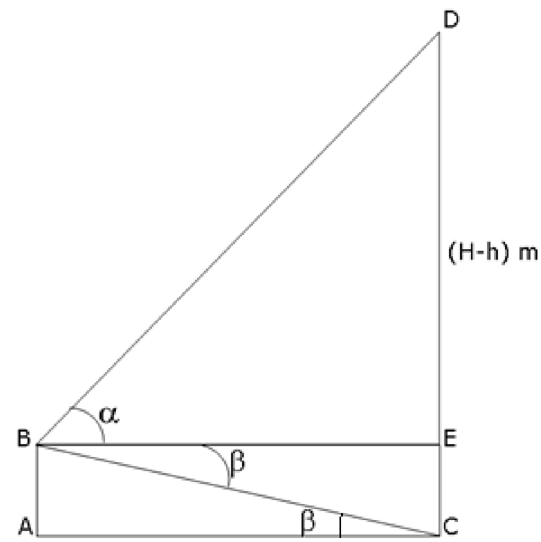
$\cot\alpha = \dfrac{BE}{ED}$

BE = (H − h) cot α ... (a)

In ΔACB,

$\dfrac{AC}{AB} = \cot\beta$

AC = h.cot β .... (b)



But BE = AC

∴ (H − h) cot α = hcot β ....[From (a) and (b)]

$H = h\,\dfrac{(\cot\alpha+\cot\beta)}{\cot\alpha}$

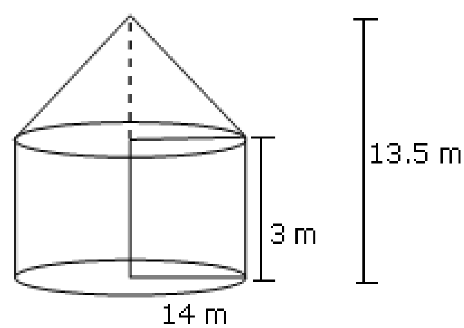
H = h(1 + tanα cotβ)

Thus, the height of the opposite house is h(1 + tanα.cotβ) metres.

28. Radius of conical portion = Radius of cylindrical portion = 14 m

Height of cylindrical portion = 3 m

Height of conical portion = 13.5 m − 3 m = 10.5 m



C.S.A. of tent = C.S.A. of cylinder + C.S.A. of cone

$= 2\pi rh + \pi rl$

$= 2\pi(14)(3) + \pi(14)\sqrt{14^2 + 10.5^2}$

$= 264 + 44\sqrt{306.25}$

$= 264 + 44(17.5)$

$= 264 + 770$

$= 1034\ m^2$

Cost of painting the inside of tent,

i.e. 1034 m^2 at the rate of Rs. 2 per sq. m = Rs. 1034 × 2 = Rs. 2068

29. Given, AC = BD = 7 cm and AB = CD = 1.75 cm = $\frac{7}{4}$ cm

Area of shaded region

= 2(area of semi-circle of radius $\frac{7}{2}$ cm) − 2(area of semi-circle of radius $\frac{7}{8}$ cm)

$$= 2\left[\frac{1}{2}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right] - 2\left[\frac{1}{2}\times\frac{22}{7}\times\frac{7}{8}\times\frac{7}{8}\right]$$

$$= \left(\frac{77}{2} - \frac{77}{32}\right)$$

$$= \frac{77}{2}\left[1 - \frac{1}{16}\right] = \frac{77}{2}\times\frac{15}{16}\text{ cm}^2 = \frac{1155}{32}\text{ cm}^2 = 36.1\text{ cm}^2$$

30. We can obtain cumulative frequency distribution of more than type as following:

Production yield (lower class limits)	Cumulative frequency
More than or equal to 50	100
More than or equal to 55	100 – 2 = 98
More than or equal to 60	98 – 8 = 90
More than or equal to 65	90 – 12 = 78
More than or equal to 70	78 – 24 = 54
More than or equal to 75	54 – 38 = 16

Now, taking lower class limits on x-axis and their respective cumulative frequencies on y-axis, we can obtain the ogive as follows: