Operations Research Test 4

Number of Questions 25 **Time:60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The constraints subjected by a flight scheduling problem to be solved as a *L.P* problem are in the form of
   (A) Cubic (B) Quadratic
   (C) Binomial (D) Linear
2. A linear programming model is given as
   Maximize $Z = 3x_1 + 7x_2$
   Subjected to $x_1 + 2x_2 \leq 700$
   $3x_1 + 4x_2 \leq 900$
   How many total number of basic solutions are possible?
   (A) 6 (B) 4
   (C) 12 (D) 8
3. When there is a tie among the minimum non-negative ratios which decides the row of the key element in a simplex method then the solution is said to be
   (A) Unbounded (B) Degenerate
   (C) Infeasible (D) Basic feasible
4. Linear programming can be applied to which of the following problems?
   (A) Transportation problem
   (B) Assignment problem
   (C) Production scheduling problem
   (D) All of the above
5. The optimality of a transportation problem is given by which of these methods
   (A) The stepping-stone method
   (B) The modified distribution method
   (C) The *U-V* method
   (D) All of the above
6. The procedure of calculating the difference between the smallest and the second smallest element in each row and column is done in which of the methods for a transportation problem?
   (A) North-West corner rule
   (B) Column Minima method
   (C) Least-Cost method
   (D) Penalty method
7. Which of these methods requires a square matrix to proceed for the next step?
   (A) North West Method
   (B) Hungarian method
   (C) Vogel's approximation method
   (D) Matrix minima method
8. An assignment problem is a special case of a transportation problem because
   (A) Number of rows are equal to the number of columns.
   (B) All rim conditions are equal to 1 i.e., supply and demand.
   (C) Each row can have only one assignment.
   (D) All of the above
9. In a network analysis problem an activity is found to have zero slack, it implies:
   (A) It is a dummy activity.
   (B) It does not lie on the critical path.
   (C) It lies on the critical path.
   (D) It is the starting activity.
10. An activity of variance 1 has the mean time and most likely time as 10 hrs. What will be the optimistic time in hours?
    (A) 13 (B) 10
    (C) 9 (D) 7
11. By solving for maximization of the objective function using simplex method the maximum value of Z is
    $Z = 4x_1 + 8x_2$
    Subjected to the constraints
    $6x_1 + 2x_2 \leq 10$
    $2x_1 + 3x_2 \leq 20$
    (A) 40 (B) 16
    (C) 20 (D) 32
12. The solution of the *L.P* problem given below is Max $Z = 6x_1 + 7x_2$
    Subjected to $x_1 - 4x_2 \geq 8$
    $x_1 + 2x_2 \geq 6$ and $x_1, x_2 \geq 0$
    (A) Unbounded (B) At two points
    (C) At a single point (D) Infeasible
13. In solving a *L.P* problem by graphical method if the objective function is parallel to a constraint which forms an edge of the feasible region then the solution of the problem is
    (A) Unbounded solution
    (B) Infeasible solution
    (C) Multiple optimal solutions
    (D) Optimal solution
14. A recycling plant produces steel castings using a furnace with the specifications

	Minimum	Maximum
Carbon	0.05%	0.1%
Silicon	2%	5%

The raw materials are procured in two forms with the specifications

	Carbon %	**Silicon %**	**Cost**
Automobile scrap (x_1)	8	16	240/kg
Utensile scrap (x_2)	10	4	300/kg

A *L.P* model is formulated using the given data, then which of the following is not a constraint for a requirement of 10 tonnes of steel castings such that the material used in the mix is at minimum cost.

(A) $4x_1 + 5x_2 \geq 250$ (B) $4x_1 + 5x_2 \leq 500$
(C) $4x_1 + x_2 \geq 500$ (D) $4x_1 + x_2 \leq 12500$

15. The artificial variables are made to leave the base and never to enter in the further iteration by the application of:

(A) Two phase method (B) Big M-method
(C) Both *A* and *B* (D) None

16. A transportation problem has a feasible solution if which of the condition is satisfied?

	1	2	3	…	j	n	Supply
1	C_{11}	C_{12}	C_{13}	…	C_{ij}	C_{1n}	a_1
2	C_{21}	C_{22}	C_{23}	…	C_{2j}	C_{2n}	a_2
:							:
i	C_{i1}	C_{i2}	C_{i3}	…	C_{ij}	C_{in}	a_n
:							:
m	C_{m1}	C_{m2}	C_{m3}	…	C_{mj}	C_{mn}	a_m
Demand	b_1	b_2	b_3	…	b_j	b_n	

(A) m = n (B) $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

(C) $\sum_{i=1}^{m} a_i = m + n - 1$ (D) $\sum_{j=1}^{n} b_j = m + n$

17. How many units are allocated to the cell containing the shipping cost '10' by solving the transportation problem by column minima method?

Centers

Plants	1	2	3	4	5	6	Supply
1	7	6	6	5	3	4	4
2	5	1	7	8	9	7	2
3	10	4	5	12	14	3	10
Demand	6	3	2	1	2	2	

(A) 6 (B) 4
(C) 2 (D) 0

18. What is the total transportation cost of the given problem using VAM? (The cells contain the shipping cost).

Centers

Plants	1	2	3	4	Supply
A	4	4	5	7	8
B	6	3	7	2	7
C	2	1	9	8	9
Demand	4	8	10	2	

(A) 94 (B) 92
(C) 80 (D) 88

19. For the given unbalanced transportation problem the operation done to find a feasible solution is

Stores

Factories	1	2	3	4	Supply
A	4	1	2	3	30
B	8	3	7	9	10
C	9	9	8	6	20
D	10	4	5	7	30
Demand	15	30	45	20	

(A) A factory with supply 20 is created
(B) A store with demand 20 is created
(C) A factory and a store with supply and demand of 20 are created.
(D) Data is insufficient

20. What is the minimum total mileage for transporting four cars to the stations *A*, *B*, *C* and *D*? The mileages between various stations are given in the table.

stations

cars	A	B	C	D
1	10	8	7	12
2	9	4	3	7
3	6	9	8	8
4	4	6	3	10

(A) 20 (B) 29
(C) 23 (D) 25

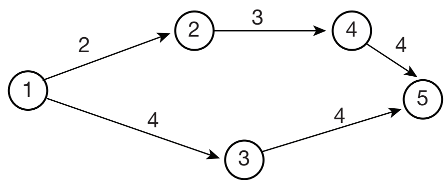
21. A company sells four products in four areas with the profits given in the table
(1 unit = ₹1000)
For maximum profit the assignment is:

Areas

Products	P	Q	R	S
1	4	7	6	3
2	9	4	7	8
3	4	5	6	7
4	8	10	4	8

	P	***Q***	***R***	***S***
(A)	4	2	3	1
(B)	3	2	4	1
(C)	2	4	1	3
(D)	2	1	4	3

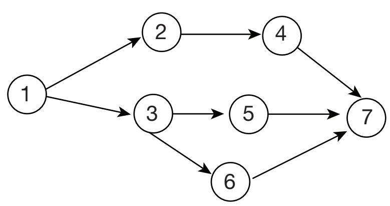
22. Which activity has a non-zero free float in the given network problem? (Durations of activities are given in the diagram)



(A) 1 – 3 (B) 3 – 5
(C) 2 – 4 (D) 4 – 5

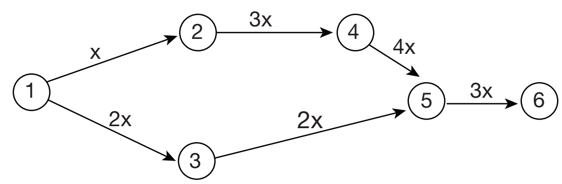
23. What is the standard deviation of the project for the given optimistic, most likely and pessimistic times?

Activity	t_o	t_m	t_p
1 – 2	10	11	12
1 – 3	6	7	8
2 – 4	3	4	11
3 – 5	4	6	8
3 – 6	1	5	9
4 – 7	6	9	12
5 – 7	4	7	10
6 – 7	7	10	13



(A) 1.699 (B) 2.667
(C) 1.334 (D) 0.445

24. The expected times of the activities of a network are given in the figure. The standard deviation of the project is 0.5 and the normal variate for the project completion time of 35 days is 0.48. What is the duration of the activity 3–5? (x is a positive real number).



(A) 9.48 (B) 6.32
(C) 3.16 (D) 12.64

25. The costs of transporting four machines 1, 2, 3 and 4 to five workshops A, B, C, D and E are given in the table. Each workshop requires one machine. If the machines are assigned to minimize the transportation cost then which workshop does not receive a machine?

	A	B	C	D	E
1	6	5	7	11	9
2	5	9	12	7	8
3	6	5	3	4	2
4	3	7	5	7	7

(A) *A* (B) *B*
(C) *C* (D) *E*

Answer Keys

1. D	**2.** A	**3.** B	**4.** D	**5.** D	**6.** D	**7.** B	**8.** D	**9.** C	**10.** D
11. A	**12.** A	**13.** C	**14.** C	**15.** C	**16.** B	**17.** D	**18.** D	**19.** A	**20.** C
21. C	**22.** B	**23.** A	**24.** B	**25.** C					

Hints and Explanations

1. The objective function and the set of constraints are all linear expressions. Choice (D)

2. Number of constraints = $m = 2$

Expressing the problem in standard form

Max $Z = 3x_1 + 7x_2 + 0S_1 + 0S_2$

$x_1 + 2x_2 + S_1 + 0S_2 = 700$

$3x_1 + 4x_2 + 0S_1 + S_2 = 900$

Total number of variables is = $m + n = 4$.

(x_1, x_2, S_1 and S_2)

∴ Total number of basic solutions

$$= {}^{m+n}C_m = {}^4C_2 = \frac{4!}{2!.(4-2)!}$$

$$= \frac{4\times3}{2\times1} = 6$$

Choice (A)

3. Choice (B)

4. Choice (D)

5. The two methods commonly used for testing the optimality of a transportation problem are stepping-stone method and modified distribution (MODI) method. MODI method is also known as u–v method.

Choice (D)

10. Variance $= \sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2 = 1$

$\therefore \quad t_p - t_o = 6 - (1)$

$t_e = \frac{t_o + 4t_m + t_p}{6}$ here $t_e = t_m = 10$ hrs

$\therefore \quad 20 = t_o + t_p - (2)$

From (1) and (2)

$t_p = 13$ hrs and $t_o = 7$ hrs Choice (D)

11. Standard form

$Z = 4x_1 + 8x_2 + 0S_1 + 0S_2$

$6x_1 + 2x_2 + S_1 = 10$

$2x_1 + 3x_2 + S_2 = 10$

To find the initial basic feasible solution

C_j C_B Basis	4 0 x_1 S_1	8 0 x_2 S_2	b	θ
0 S_1	6 1	2 0	10	$\frac{10}{2} = 5 \leftarrow$
0 S_2	2 0	3 1	20	$\frac{20}{3} = 6.67$
Z_j $C_j - Z_j$	0 0 4 0	0 0 8 0 ↑		

x_2 is the variable incoming and S_1 is the outgoing variable.

The key element must be made 1 and the intersection elements of the key column are made zero.

The row of S_2:

$2 - (3 \times 3) = -7,\ 3 - (1 \times 3) = 0,\ 0 - (1 \times 3) = -3,$

$1 - (0 \times 3) = 1,\ 20 - (5 \times 3) = 5$

C_j C_B Basis	4 0 x_1 S_1	8 0 x_2 S_2	b	θ
8 x_2	3 ½	1 0	5	
0 S_2	–7 –3 1	0	5	
Z_j $C_j - Z_j$	24 4 –20 –4 0	8 0 0		$C_j - Z_j \le 0$

$\therefore$ The optimal solution is

$x_1 = 0, x_2 = 5, S_1 = 0, S_2 = 5$

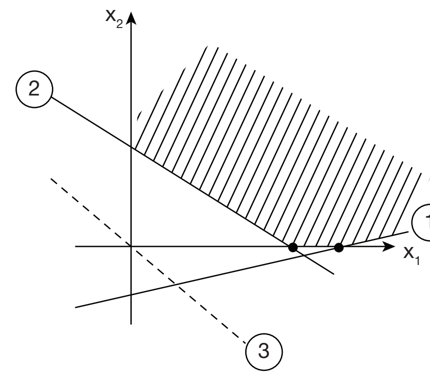
$\therefore \quad Z_{max} = 4 \times 0 + 8 \times 5 = 40$ Choice (A)

12. Max $Z = 6x_1 + 7x_2$

Subjected to $x_1 - 4x_2 \ge 8 - (1)$

$x_1 + 2x_2 \ge 6 - (2)$

$x_1, x_2 \ge 0$



The dotted line (3) indicates the objective function $Z = 0$. When Z is increased from zero the dotted line moves parallel to the line (3) and as we need the maximum value i.e., the farthest point of the shaded region, max z occurs at infinity. Thus the solution is said to be unbounded. Choice (A)

13. Choice (C)

14. Objective is to minimize cost

$Z = 240\, x_1 + 300\, x_2$

Constraints are

$8\, x_1 + 10\, x_2 \ge 0.05 \times 10 \times 10^3$

for a minimum of 0.05% of carbon

i.e., $8x_1 + 10x_2 \ge 500$

$\Rightarrow \quad 4x_1 + 5x_2 \ge 250$

for a minimum of 2% of silicon

$16x_1 + 4x_2 \ge 2 \times 10 \times 10^3$

$16x_1 + 4x_2 \ge 20000$

$\Rightarrow \quad 4x_1 + x_2 \ge 5000$

for a maximum of 0.1% of carbon

$8x_1 + 10x_2 \le 0.1 \times 10 \times 10^3$

$\Rightarrow \quad 4x_1 + 5x_2 \le 500$

for a maximum of 5% of silicon

$16x_1 + 4x_2 \le 5 \times 10 \times 10^3$

$4x_1 + x_2 \le 12500$ Choice (C)

15. The two methods available to solve a L.P problem with constraints of '≥' or '=' are

(1) Two phase method

(2) Big M-method

The artificial variables assume the role of slack variables in the first iteration, only to be replaced at a later iteration.

Choice (C)

16. A transportation problem is said to be balanced when

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

This type of problem will have a feasible solution. Choice (B)

17.

Centers

Plants	1	2	3	4	5	6	Supply
1	7 (4)	6	6	5	3	4	4/0
2	5 (2)	1	7	8	9	7	2/0
3	10 (0)	4	5	12	14	3	10
Demand	6/4/0	3	2	1	2	2	

The first allocation is done to the cell containing shipping cost 5 in the first column with 2 units and the supply of plant 2 is exhausted. The next allocation is done to the cell containing shipping cost 7 with 4 units and the supply of plant 1 is exhausted. The demand at the center 1 is completed thus the cell containing shipping cost 10 is allocated with 0 units. Choice (D)

18.

Centers

Plants	1	2	3	4	Supply	Penalty
A	4	4	5	7	8	0
B	6	3	7	2 (2)	7/5	1
C	2	1	9	8	9	1
Demand	4	8	10	2/0		
Penalty	2	2	2	(5)		

Centers

Plants	1	2	3	Supply	Penalty
A	4	4	5	8	0
B	6	3 (5)	7	5/0	(3)
C	2	1	9	9	1
Demand	4	8/3	10		
Penalty	2	2	2		

Center

Plants	1	2	3	Supply	
A	4	4	5 (8)	8/0	0
C	2 (4)	1 (3)	9 (2)	9/7/0	1
Demand	4	3	10/2/		
Penalty	2	3	(4)		

∴ The allocations using Vogel's approximation method are:

Centers

Plants	1	2	3	4
A	4	4	5 (8)	7
B	6	3 (5)	7	2 (2)
C	2 (4)	1 (3)	9 (2)	8

∴ The total transportation cost is
$= (5 \times 8) + (3 \times 5) + (2 + 2) + (2 \times 4) + (1 \times 3) + (9 \times 2) = 88.$ Choice (D)

19. The total demand = 110, whereas the supply is only 90. Therefore a factory of supply 20 with zero cost coefficients is added to make the problem balanced.

	A	B	C	D
1	3	0	0	3
2	6	0	0	2
3	0	2	2	0
4	1	2	0	5

Choice (A)

20. As it is a square matrix, we shall start reducing the matrix.
By subtracting the minimum elements of each row from the elements of the row we get:

	A	B	C	D
1	3	1	0	5
2	6	1	0	4
3	0	3	2	2
4	1	3	0	7

As 2nd and 4th column does not have zeros, subtract the minimum element from all the column elements.

	A	B	C	D
1	3	0	0	3
2	6	0	0	2
3	0	2	2	0
4	1	2	0	5

An optimum assignment is not possible for the obtained reduced matrix.
Therefore by drawing the minimum number of lines the obtained uncovered cells are subtracted by the minimum value among them and the minimum value is added to cells where two lines intersect.

	A	B	C	D
1	2	0	[0]	2
2	5	[0]	0	1
3	0	3	3	[0]
4	[0]	2	0	5

The minimum number of lines crossing all the zeros is 4 hence an optimal assignment can be made.

∴ The minimum mileage = 7 + 4 + 8 + 4 = 23.

Choice (C)

21. Firstly convert the maximization problem to minimization.

Subtract all the elements from the highest element.

Now reduce the matrix

	P	Q	R	S
1	3	0	1	4
2	0	5	2	1
3	3	2	1	0
4	2	0	6	2

As column 3 does not contain a zero, subtract the column elements with the minimum element in the column.

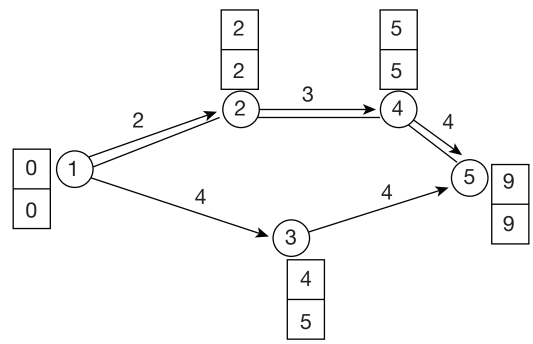
	P	Q	R	S
1	3	0	[0]	4
2	[0]	5	1	1
3	3	2	0	[0]
4	2	[0]	5	2

As the minimum number of lines passing through all the zeros is equal to the order of the matrix an optimal assignment can be made.

∴ The optimal assignment is

$P-2, Q-4, R-1, S-3.$ Choice (C)

22.



The critical path for the network is 1 – 2 – 4 – 5

Activity	Duration (t)	Start time		Finish time		Total float	Free float
		Earliest	Latest	Earliest	Latest		
1 – 2	2	0	0	2	2	0	0
1 – 3	4	0	1	4	5	1	0
2 – 4	3	2	2	5	5	0	0
3 – 5	4	4	5	8	9	1	1
4 – 5	4	5	5	9	9	0	0

Earliest start, E is given by the start time of the tail even of an activity.

Latest finish, L is given by the finish time of the head even of an activity.

LS = Latest start = $L - t$

EF = Earliest finish = $E + t$

Total float = $LS - E = L - EF$

Free float = TF – (head event slack)

Head even slack = L-E for the head event.

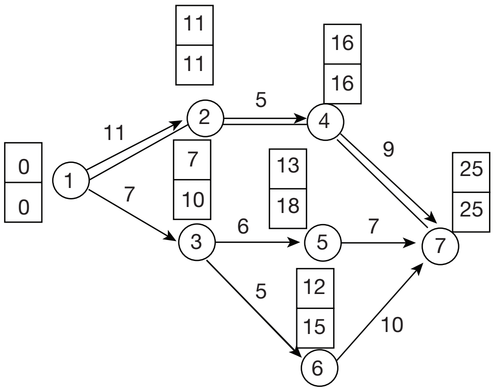
For activity 1 – 3 the head event slack is 1 and rest all activities have zero head even slacks.

∴ Free float of activity 3 – 5 is 1.

Choice (B)

23.

Activity	t_o	t_m	t_p	t_e	$\sigma = \frac{t_p - t_o}{6}$	σ^2
1 – 2	10	11	12	11	0.334	0.112
1 – 3	6	7	8	7	0.334	0.112
2 – 4	3	4	11	5	1.334	1.778
3 – 5	4	6	8	6	0.667	0.445
3 – 6	1	5	9	5	1.334	1.778
4 – 7	6	9	12	9	1	1
5 – 7	4	7	10	7	1	1
6 – 7	7	10	13	10	1	1



$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

∴ The critical path for the project is 1 – 2 – 4 – 7.

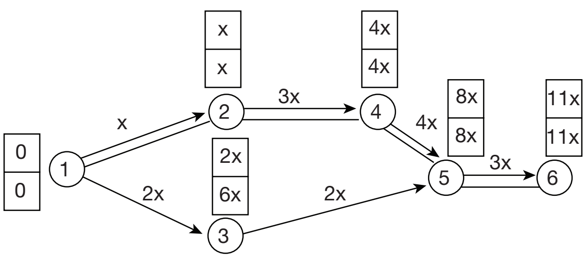
∴ The standard deviation of the project is

$\sigma = \sqrt{\Sigma\sigma_{ij}^2}$, for all i-j on critical path.

∴ $\sigma = \sqrt{0.112 + 1.778 + 1}$

$\sigma = 1.7$ Choice (A)

24.



∴ The critical path time is $11x$.

Normal variate $Z = 0.48 = \dfrac{T - T_{cp}}{\sigma}$

$T_{cp} = 11x$, $T = 35$, $\sigma = 0.5$

$\Rightarrow \dfrac{35 - 11x}{0.5} = 0.48$

$\Rightarrow$ $x = 3.16$ days

∴ for the activity 3 – 5, duration $= 2x = 6.32$ days Choice (B)

25. Prepare a square matrix and reduce it

	A	B	C	D	E
1	1	0	2	6	4
2	0	4	7	2	3
3	4	3	1	2	0
4	0	4	2	4	4
d	0	0	0	0	0

d – dummy

As the minimum number of lines passing over all the zeros is less than the order of the matrix the uncovered cells are subtracted with the least among them.

	A	B	C	D	E
1	1	[0]	2	6	4
2	0	2	5	[0]	3
3	7	3	1	2	[0]
4	[0]	2	0	2	2
d	3	0	[0]	0	0

∴ The work shop *C* does not receive any machine. Choice (C)