Chapter 9. Ratio and Proportion

Ex 9.1

Answer 1.

d 2.40

$$= \frac{560}{240} = \frac{56}{24} = \frac{28}{12} = \frac{14}{6} = \frac{7}{3}$$

and 120

$$\frac{216}{60} = \frac{108}{30} = \frac{54}{15} = \frac{18}{5}$$

5.40 and 180 paise

) = 540 paise

$$\frac{54}{18} = \frac{3}{1}$$

$\cdot b^4$ and $a^3 - b^3$

$$\frac{}{} = \frac{(a-b)(a^3 + ab^2 + a^2b + b^3)}{(a-b)(a^2 + ab + b^2)} = \frac{a^3 + ab^2 + a^2b + b^3}{a^2 + ab + b^2}$$

$b^2 + a^2b + b^3 : a^2 + ab + b^2$

$+ 4$ and $x^2 - x - 6$

$$\frac{+4}{\cdot 6} = \frac{(x+2)^2}{(x-3)(x+2)} = \frac{(x+2)}{(x-3)}$$

) : (x-3)

Answer 3.

$$\frac{3}{8}$$

$$2n)\,(5m+n) = \frac{(3m+2n)}{(5m+n)}$$

$$\frac{\dfrac{m}{n}+2}{\dfrac{m}{n}+1}$$

$$\frac{\dfrac{3}{8}+2}{\dfrac{3}{8}+1}$$

$$\frac{+2}{+1}$$

$$= 25:23$$

$$5m+n) = 25:23$$

Answer 4.

(i) Annual expenses to annual income.

Annual income = monthly income × 12 = Rs 5,000 × 12 = Rs 60,000

Monthly expenses = Rs 5,000 − 800 = Rs 4,200

Annual expenses = monthly expenses × 12 = Rs 4,200 × 12 = Rs 50,400

$$\frac{\text{Annual expenses}}{\text{Annual income}} = \frac{50,400}{60,000} = \frac{504}{600} = \frac{21}{25}$$

Therefore, Annual expenses : annual income = 21:25

(ii) monthly savings to monthly expenses.

Monthly expenses = Rs 5,000 − 800 = Rs 4,200

$$\frac{\text{Monthly savings}}{\text{Monthly expenses}} = \frac{800}{4,200} = \frac{8}{42} = \frac{4}{21}$$

Therefore, monthly savings : monthly expenses = 4 : 21

Answer 5.

$a+b : a-b = 11 : 8$

$\Rightarrow \dfrac{a+b}{a-b} = \dfrac{11}{8}$

$\Rightarrow 8a + 8b = 11a - 11b$

$\Rightarrow 3a = 19b$

$\Rightarrow \dfrac{a}{b} = \dfrac{19}{3}$

$\qquad a : b = 19 : 3$

Answer 6.

$p : q = 2 : 5 \Rightarrow \dfrac{p}{q} = \dfrac{2}{5}$

$q : r = 4 : 3 \Rightarrow \dfrac{q}{r} = \dfrac{4}{3}$

$\Rightarrow \dfrac{p}{q} \times \dfrac{q}{r} = \dfrac{2}{5} \times \dfrac{4}{3}$

$\Rightarrow \dfrac{p}{r} = \dfrac{8}{15}$

$\qquad p : r = 8 : 15$

Answer 7.

$a : e = 5 : 12 \Rightarrow \dfrac{a}{e} = \dfrac{5}{12}$

$e : i = 8 : 3 \Rightarrow \dfrac{e}{i} = \dfrac{8}{3}$

$i : u = 9 : 16 \Rightarrow \dfrac{i}{u} = \dfrac{9}{16}$

$\Rightarrow \dfrac{a}{e} \times \dfrac{e}{i} \times \dfrac{i}{u} = \dfrac{5}{12} \times \dfrac{8}{3} \times \dfrac{9}{16}$

$\Rightarrow \dfrac{a}{u} = \dfrac{10}{16} = \dfrac{5}{8}$

$\qquad a : u = 5 : 8$

Answer 8.

: 16 and 8 : 5

ounded ratio of 15:16 and 8:5

$$\frac{15}{16} \times \frac{8}{5} = \frac{3}{2}$$

ounded ratio = 3:2

$b^2):(a^2+b^2)$ and $(a^4-b^4):(a+b)^4$

ounded ratio of $(a^2-b^2):(a^2+b^2)$ and $(a^4-b^4):(a+b)^4$

$$\frac{(a^2-b^2)}{(a^2+b^2)} \times \frac{(a^4-b^4)}{(a+b)^4}$$

$$\frac{(a+b)(a-b)}{(a^2+b^2)} \times \frac{(a^2+b^2)(a^2-b^2)}{(a+b)^2(a+b)^2}$$

$$\frac{(a-b)(a+b)(a-b)(a+b)}{(a+b)^2(a+b)^2}$$

$$\frac{(a-b)^2}{(a+b)^2}$$

ounded ratio = $(a-b)^2 : (a+b)^2$

7:9 and 15: 28

ounded ratio of 3:5, 7:9 and 15: 28

$$\frac{3}{5} \times \frac{7}{9} \times \frac{15}{28}$$

$$\frac{1}{4}$$

pounded ratio = 1:4

(iv) $\sqrt{8} : 4, 3 : \sqrt{5}$ and $\sqrt{20} : \sqrt{27}$

Compounded ratio of $\sqrt{8} : 4, 3 : \sqrt{5}$ and $\sqrt{20} : \sqrt{27}$

$$= \frac{\sqrt{8}}{4} \times \frac{3}{\sqrt{5}} \times \frac{\sqrt{20}}{\sqrt{27}}$$

$$= \frac{2\sqrt{2}}{4} \times \frac{3}{\sqrt{5}} \times \frac{2\sqrt{5}}{3\sqrt{3}}$$

$$= \frac{\sqrt{2}}{\sqrt{3}}$$

Compounded ratio = $\sqrt{2} : \sqrt{3}$

(v) $(m-n):(m+n)$, $(m+n)^2 : (m^2+n^2)$ and $(m^4 - n^4): (m^2-n^2)^2$

Compounded ratio of $(m-n):(m+n)$, $(m+n)^2 : (m^2+n^2)$ and $(m^4 - n^4): (m^2-n^2)^2$

$$= \frac{m-n}{m+n} \times \frac{(m+n)^2}{(m^2+n^2)} \times \frac{(m^4-n^4)}{(m^2-n^2)^2}$$

$$= \frac{m-n}{1} \times \frac{m+n}{(m^2+n^2)} \times \frac{(m^2+n^2)(m^2-n^2)}{(m^2-n^2)^2}$$

$$= \frac{(m^2-n^2)}{(m^2+n^2)} \times \frac{(m^2+n^2)(m^2-n^2)}{(m^2-n^2)^2}$$

$$= \frac{1}{1}$$

Compounded ratio = $1 : 1$

Answer 15.

Let the two numbers be $7x$ and $10x$; since the ratio between numbers is $7 : 10$.

Now,

$$\frac{7x + 8}{10yx8} = \frac{3}{4}$$

$\Rightarrow 28x + 32 = 30x + 24$

$\Rightarrow 2x = 8$

$\Rightarrow x = 4$

$\therefore 7x = 28$

$10x = 40$

Therefore, two numbers are 28 and 40.

Answer 16.

Let the two numbers be 3x and 4x, since the ratio between them is 3:4.

Now,

$(3x)^2 + (4x)^2 = 1225$

$\Rightarrow 9x^2 + 16x^2 = 1225$

$\Rightarrow 25x^2 = 1225$

$\Rightarrow x^2 = 49$

$\Rightarrow x = 7$

$\therefore 3x = 21$

$4x = 28$

Therefore, the two numbers are 21 and 28.

Answer 17.

Let the two numbers be 5x and 7x, since the ratio between them is 5:7.

Now,

$(7x)^2 - (5x)^2 = 600$

$\Rightarrow 49x^2 - 25x^2 = 600$

$\Rightarrow 24x^2 = 600$

$\Rightarrow x^2 = 25$

$\Rightarrow x = 5$

$\therefore 5x = 25$

$7x = 35$

Therefore, the two numbers are 25 and 35.

Answer 18.

Let x be subtracted from each term such that

$$\frac{39 - x}{89 - x} = \frac{2}{5}$$
$$\Rightarrow 195 - 5x = 178 - 2x$$
$$\Rightarrow 3x = 17$$
$$\Rightarrow x = \frac{17}{3}$$

$\frac{17}{3}$ should be subtracted from each term.

Answer 19.

Let x be added from each term such that

$$\frac{19 + x}{51 + x} = \frac{3}{7}$$
$$\Rightarrow 133 + 7x = 153 + 3x$$
$$\Rightarrow 4x = 20$$
$$\Rightarrow x = 5$$

5 should be added to each term.

Answer 20.

Let x be added from each term such that

$$\frac{(p + q) + x}{(p - q) + x} = \frac{(p + q)^2}{(p - q)^2}$$
$$\Rightarrow (p + q + x)(p^2 - 2pq + q^2) = (p^2 + 2pq + q^2)(p - q + x)$$
$$\Rightarrow p^3 + p^2q + p^2x + q^2p + q^3 + q^2x - 2p^2q - 2pq^2 - 2pqx$$
$$= p^3 - p^2q + p^2x + q^2p - q^3 + q^2x + 2p^2q - 2pq^2 + 2pqx$$
$$\Rightarrow p^2q + q^3 - 2p^2q - 2pqx = -p^2q - q^3 + 2p^2q + 2pqx$$
$$\Rightarrow 2q^3 - p^2q = 4pqx$$
$$\Rightarrow 2q(q^2 - p^2) = 4pqx$$
$$\Rightarrow x = \frac{(q^2 - p^2)}{2p}$$

$\frac{(q^2 - p^2)}{2p}$ should be added to each term.

Answer 21.

$$\frac{3x - 4}{2x + 5} = \frac{(3)^2}{(4)^2}$$

$$\Rightarrow \frac{3x - 4}{2x + 5} = \frac{9}{16}$$

$$\Rightarrow 48x - 64 = 18x + 45$$

$$\Rightarrow 30x = 109$$

$$\Rightarrow x = \frac{109}{30}$$

$$x = \frac{109}{30}$$

Answer 22.

$$\frac{5x + 3}{3x + 1} = \frac{(4)^3}{(3)^3}$$

$$\Rightarrow \frac{5x + 3}{3x + 1} = \frac{64}{27}$$

$$\Rightarrow 135x + 81 = 192x + 64$$

$$\Rightarrow 57x = 17$$

$$\Rightarrow x = \frac{17}{57}$$

$$x = \frac{17}{57}$$

Answer 23.

$$\frac{p}{q} = \frac{(p + r)^2}{(q + r)^2}$$

$$\Rightarrow \frac{p}{q} = \frac{p^2 + r^2 + 2pr}{q^2 + r^2 + 2qr}$$

$$\Rightarrow \frac{p}{q} = \frac{p^2 + pq + 2pr}{q^2 + pq + 2qr}$$

$$\Rightarrow pq^2 + p^2q + 2pqr = p^2q + pq^2 + 2pqr$$

Since LHS = RHS, therefore p:q is the duplicate ratio of (p+r) : (q+r)

Answer 24.

$$\frac{2}{3} = \frac{x}{7,47,300}$$

$$\Rightarrow 14,94,600 = 3x$$

$$\Rightarrow x = 4,98,200$$

Sum distributed = Rs (7,47,300+4,98,200) = Rs 12,45,500

Answer 25.

$$\frac{5}{7} = \frac{60}{x}$$

$$\Rightarrow 5x = 420$$

$$\Rightarrow x = 84$$

Number = 84+60 = 144

Answer 26.

Let the number of Re 1, Rs 2 and Rs 5 coins be 3x, 7x and 11x

The sum of their value =

$$3x + (2 \times 7x) + (5 \times 11x) = Rs1800$$
$$3x + 14x + 55x = 1800$$
$$72x = 1800$$
$$x = 25$$

$$3x = 3 \times 25 = 75$$
$$7x = 7 \times 25 = 175$$
$$11x = 11 \times 25 = 275$$
$$\text{total} = 75 + 175 + 275 = 525$$

Total number of coins = 525

Answer 27.

Let present ages be 4x and 3x.

$$\frac{4x + 9}{3x + 9} = \frac{23}{18}$$
$$\Rightarrow 72x + 162 = 69x + 207$$
$$\Rightarrow 3x = 45$$
$$\Rightarrow x = 15$$

$$\therefore 3x = 45$$
$$4x = 60$$

Hence, their present ages are 60 years and 45 years.

Answer 28.

Let money saved by the boy and her sister be 5x and 4x and let the sister need to save Rs y more

$$\frac{5x + 100}{4x + y} = \frac{5}{4}$$
$$\Rightarrow 20x + 400 = 20x + 5y$$
$$\Rightarrow 5y = 400$$
$$\Rightarrow y = 80$$

The sister needs to save Rs 80 more.

Answer 29.

Quantity of milk $= 60 \times \frac{11}{15} = 44$ litres

Quantity of water in it $= 60 - 44 = 16$ litres

New ratio $= 11:6$

Let quantity of water to be added further be x litres.

Then, milk : water $= \frac{44}{16 + x}$

Now,

$$\frac{44}{16 + x} = \frac{11}{6}$$
$$\Rightarrow 264 = 176 + 11x$$
$$\Rightarrow 11x = 88$$
$$\Rightarrow x = 8$$

Therefore, 8 litres of water need to be added further.

Answer 30.

Let successful candidates be 7x and unsuccessful candidates be 5x.

Now,

$$\frac{7x + 10}{5x + 20} = \frac{4}{3}$$

$\Rightarrow 21x + 30 = 20x + 80$

$\Rightarrow x = 50$

$7x = 7 \times 50 = 350$

$5x = 5 \times 50 = 250$

$\therefore 7x + 5x = 350 + 250 = 600$

Therefore, there were 600 candidates originally.

Ex 9.2

Answer 5.

Since x, 12 and 16 are in continued proportion

$\Rightarrow x : 12 :: 12 : 16$

$\Rightarrow 16x = 12 \times 12$

$\Rightarrow 16x = 144$

$\Rightarrow x = 9$

$x = 9$

Answer 6.

Since $\dfrac{1}{12}$, x and $\dfrac{1}{75}$ are in continued proportion

$\Rightarrow \dfrac{1}{12} : x :: x : \dfrac{1}{75}$

$\Rightarrow x^2 = \dfrac{1}{12} \times \dfrac{1}{75}$

$\Rightarrow x = \sqrt{\dfrac{1}{900}}$

$\Rightarrow x = \dfrac{1}{30}$

$x = \dfrac{1}{30}$

Answer 7.

Since y is the mean proportion between x and z

$y^2 = xz$

LHS

$xyz(x + y + z)^3$

$= y \, y^2 (x + y + z)^3$

$= y^3 (x + y + z)^3$

$= [y(x + y + z)]^3$

$= (xy + y^2 + yz)^3$

$= (xy + yz + xz)^3 = RHS$

LHS = RHS

Answer 8.

Since y is the mean proportion between x and z

Therefore, $y^2 = xz$

Now, we have to prove that $xy + yz$ is the mean proportional between $x^2 + y^2$ and $y^2 + z^2$.

$(xy + yz)^2 = (x^2 + y^2)(y^2 + z^2)$

$LHS = (xy + yz)^2$

$= [y(x + z)]^2$

$= y^2(x + z)^2$

$= xz(x + z)^2$

$RHS = (x^2 + y^2)(y^2 + z^2)$

$= (x^2 + xz)(xz + z^2)$

$= x(x + z)z(x + z)$

$= xz(x + z)^2$

LHS = RHS

Answer 9.

Let x, y and z are the three quantities which are in continued proportion.

Then, $x : y :: y : z \Rightarrow y^2 = xz$

Now, we have to prove that

$x : z = x^2 : y^2$

$\Rightarrow \quad xy^2 = x^2 z$

LHS

$= xy^2 = x(xz) = x^2z = RHS$

LHS = RHS

Answer 11.

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{ax}{b} = \frac{cx}{d}$$

Adding y to both sides:

$$\Rightarrow \frac{ax}{b} + y = \frac{cx}{d} + y$$

$$\Rightarrow \frac{ax + by}{b} = \frac{cx + dy}{d}$$

$$\Rightarrow (ax + by) : b = (cx + dy) : d$$

Proved.

Answer 12.

$$\frac{a + c}{b} = \frac{5}{1}$$

$$\Rightarrow a + c = 5b$$

$$\frac{bc + cd}{bd} = \frac{5}{1}$$

$$\Rightarrow bc + cd = 5bd$$

$$\Rightarrow bc + cd = (a + c)d$$

$$\Rightarrow bc + cd = ad + cd$$

$$\Rightarrow bc = ad$$

$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

Hence a:b = c:d

Answer 13.

Let x be subtracted from each number so that 20-x, 29-x, 84-x and 129-x are in proportion.

$$\therefore \frac{20 - x}{29 - x} = \frac{84 - x}{129 - x}$$

$$\Rightarrow (20 - x)(129 - x) = (29 - x)(84 - x)$$

$$\Rightarrow 2580 - 129x - 20x + x^2 = 2436 - 84x - 29x + x^2$$

$$\Rightarrow 2580 - 149x = 2436 - 113x$$

$$\Rightarrow 36x = 144$$

$$\Rightarrow x = 4$$

Hence, 4 is to be subtracted from 20, 29, 84 and 129 for them to be in proportion.

Answer 14.

Let a and b be the two numbers , whose mean proportional is 12.

$$\therefore ab = 12^2 \Rightarrow ab = 144 \Rightarrow b = \frac{144}{a} \quad \ldots\ldots\ldots(i)$$

Now, third proportional is 324

$$\therefore a : b :: b : 324$$
$$\Rightarrow b^2 = 324a$$
$$\Rightarrow \left(\frac{144}{a}\right)^2 = 324a$$
$$\Rightarrow \frac{(144)^2}{a^2} = 324a$$
$$\Rightarrow a^3 = \frac{144 \times 144}{324}$$
$$\Rightarrow a^3 = 64$$
$$\Rightarrow a = 4$$

$$b = \frac{144}{a} = \frac{144}{4} = 36$$

Therefore, numbers are 4 and 36

Answer 15.

Let a and b be the two numbers , whose mean proportional is 18.

$$\therefore ab = 18^2 \Rightarrow ab = 324 \Rightarrow b = \frac{324}{a} \quad \ldots\ldots\ldots(i)$$

Now, third proportional is 486

$$\therefore a : b :: b : 486$$
$$\Rightarrow b^2 = 486a$$
$$\Rightarrow \left(\frac{324}{a}\right)^2 = 486a$$
$$\Rightarrow \frac{(324)^2}{a^2} = 486a$$
$$\Rightarrow a^3 = \frac{324 \times 324}{486}$$
$$\Rightarrow a^3 = 216$$
$$\Rightarrow a = 6$$

$$b = \frac{324}{a} = \frac{324}{6} = 54$$

Therefore, numbers are 6 and 54