Chapter 1

Number Systems

LEARNING OBJECTIVES

- *Digital circuits*
- *Number system with different base*
- *Conversion of number systems*
- *Complements*
- *Subtraction with complement*
- *Numeric codes*
- *Weighted and non-weighted codes*
- *Error detection and correction code*
- *Sequential, reflective and cyclic codes*
- *Self complementing code*

Digital Circuits

Computers work with binary numbers, which use only the digits '0' and '1'. Since all the digital components are based on binary operations, it is convenient to use binary numbers when analyzing or designing digital circuits.

Number Systems with Different Base

Decimal number system

Decimal numbers are usual numbers which we use in our day-to-day life. The base of the decimal number system is 10. There are ten numbers 0 to 9.

The value of the *n*th digit of the number from the right side = *n*th digit $\times$ $(\text{base})^{n-1}$

Example 1: $(99)_{10} \rightarrow 9 \times 10^1 + 9 \times 10^0$
$= 90 + 9 = 99$

Example 2: $(332)_{10} \rightarrow 3 \times 10^2 + 3 \times 10^1 + 2 \times 10^0$
$= 300 + 30 + 2$

Example 3: $(1024)_{10} \rightarrow 1 \times 10^3 + 0 \times 10^2 + 2 \times 10^1 + \times 10^0$
$= 1000 + 0 + 20 + 4 = 1024$

Binary number system

In binary number system, there are only two digits '0' and '1'. Since there are only two numbers, its base is 2.

Example 4: $(1101)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$= 8 + 4 + 1 = (13)_{10}$

Octal number system

Octal number system has eight numbers 0 to 7. The base of the number system is 8. The number $(8)_{10}$ is represented by $(10)_8$.

Example 5: $(658)_8 = 6 \times 8^2 + 5 \times 8^1 + 8 \times 8^0$
$= 384 + 40 + 8 = (432)_{10}$

Hexadecimal number system

In hexadecimal number system, there are 16 numbers 0 to 9, and digits from 10 to 15 are represented by *A* to *F*, respectively. The base of hexadecimal number system is 16.

Example 6: $(1A5C)_{16} = 1 \times 16^3 + A \times 16^2 + 5 \times 16^1 + C \times 16^0$
$= 1 \times 4096 + 10 \times 256 + 5 \times 16 + 12 \times 1$
$= 4096 + 2560 + 80 + 12 = (6748)_{10}$.

Table 1 *Different number systems*

Decimal	Binary	Octal	Hexadecimal
0	000	0	0
1	001	1	1
2	010	2	2
3	011	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

(*Continued*)

Table 1 (*Continued*)

Decimal	Binary	Octal	Hexadecimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14

1. For a number system with base *n*, the number of different symbols in the number system will be *n*. Example: octal number system will have total of 8 numbers from 0 to 7.
2. The number '*n*' in the number system with base '*n*' is represented as $(10)_n$.
3. The equivalent of number $(a_3a_2a_1a_0 \cdot a_{-1}a_{-2})_n$ in decimal is $a_3 \times n^3 + a_2 \times n^2 + a_1 \times n^1 + a_0 \times n^0 + a_{-1} \times n^{-1} + a_{-2} \times n^{-2}$.

Conversion of Number Systems

The conversion of decimal to any other number system involves successive division by the radix until the dividend reaches 0. At each division, the remainder gives a digit of converted number; and the last one is most significant digit, the remainder of the first division is least significant digit.

The conversion of other number system to decimal involves multiplying each digit of number system with the weight of the position (in the power of radix) and sum the products calculated, the total is the equivalent value in decimal.

Decimal to binary conversion

Example 7: $(66)_{10}$

2	66	
2	33	0
2	16	1
2	8	0
2	4	0
	2	0
	1	0

Reading remainders from bottom to top

$= (1000010)_2$

Example 8: $(928)_{10}$

2	928	
2	464	0
2	232	0
2	116	0
2	58	0
2	29	0
2	14	1
2	7	0
2	3	1
	1	1

$= (1110100000)_2$

Example 9: $(105.75)_{10}$

2	105	
2	52	1
2	26	0
2	13	0
2	6	1
2	3	0
	1	1

$(105)_{10} = (1101001)_2$

$(0.75)_{10}$

Multiply 0.75 by 2 = 1.50

Multiply 0.50 by 2 = 1.00

Reading integers from top to bottom $0.75 = (0.11)_2$

$\therefore (105.75)_{10} = (1101001.11)_2$

Binary to decimal conversion

Example 10: $(10100011)_2$

$= 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 128 + 0 + 32 + 0 + 0 + 0 + 2 + 1$

$= (163)_{10}$

Example 11: $(11010011.101)_2$

$= 1 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3})$

$= 128 + 64 + 0 + 16 + 0 + 0 + 2 + 1 + 0.5 + 0 + 0.125$

$= (211.625)_{10}$

Decimal to octal conversion

Example 12: $(16)_{10}$

8	16	0
	2	

Remainder from bottom to top $= (20)_8$

Example 13: $(347.93)_{10}$

$(.93)_{10}$

$0.93 \times 8 =$	7.44
$0.44 \times 8 =$	3.52
$0.52 \times 8 =$	4.16
$0.16 \times 8 =$	1.28
	……..

Read the integers of octal point from top to bottom.

$\therefore (0.93)_{10} = (0.7341)_8$

$(347)_{10}$

8	347	3
8	43	3
	5	

$\therefore (347)_{10} = (533)_8$

Ans: $(533.7341)_8$

Octal to decimal conversion

Example 14: $(33)_8$

$3 \times 8^1 + 3 \times 8^0 = 24 + 3$

$(27)_{10}$

Example 15: $(1023.06)_8$

$1 \times 8^3 + 0 \times 8^2 + 2 \times 8^1 + 3 \times 8^0 + 0 \times 8^{-1} + 6 \times 8^{-2}$

$= 512 + 0 + 16 + 3 + 0 + 0.0937 = (2095.0937)_{10}$

Octal to binary conversion

To convert octal to binary, replace each octal digit with their equivalent 3-bit binary representation.

Example 16: $(7777)_8$

Convert each octal digit to binary

$= \frac{7}{111}\ \frac{7}{111}\ \frac{7}{111}\ \frac{7}{111}$

$= (111\ 111\ 111\ 111)_2$

Example 17: $(567.62)_8$

5	6	7	.	6	2
101	110	111	.	110	010

$= (101110111.110010)_2$

Binary to octal conversion

To convert a binary number to an octal number, starting from the binary point, make groups of 3-bits each on either side of the binary point, and replace each 3-bit binary group by the equivalent octal digit.

Example 18: $(010011101)_2$

$\frac{010}{2}\ \frac{011}{3}\ \frac{101}{5} = (235)_8$

Example 19: $(10010111011.1011)_2$

$\frac{010}{2}\ \frac{010}{2}\ \frac{111}{7}\ \frac{011}{3} \cdot \frac{101}{5}\ \frac{100}{4} = (2273.54)_8$

Decimal to hexadecimal conversion

Example 20: $(527)_{10}$

16	527	
16	32	15
	2	0

Decimal		Hexa
2	→	2
0	→	0
15	→	F

$= (20F)_{16}$

Example 21: $(18.675)_{10}$

$(18)_{10}$

16	18	
	1	2

Decimal		Hexa
1	–	1
2	–	2

$(18)_{10} = (12)_{16}$

$(0.675)_{10}$

0.675×16	10.8
0.800×16	12.8
0.800×16	12.8
0.800×16	12.8

Decimal	Hexa
10	A
12	C
12	C
12	C

$= (0.ACCC)_{16}$

∴ Hexadecimal equivalent is

$= (12.AC\ CC)_{16}$

Hexadecimal to decimal conversion

Example 22: $(A3F)_{16}$

Decimal		Hexa
A	–	10
3	–	3
F	–	15

$\rightarrow 10 \times 16^2 + 3 \times 16^1 + 15 \times 16^0$

$\rightarrow 2560 + 48 + 15 \quad \rightarrow (2623)_{10}$

Example 23: $(1F63.0EB)_{16}$

1	1
F	15
6	6
3	3
0	0
E	14
B	11

$\rightarrow 1 \times 16^3 + 15 \times 16^2 + 6 \times 16^1 + 3 \times 16^0 \times (0 \times 16^{-1}) + (14 \times 16^{-2}) + (11 \times 16^{-3})$

$\rightarrow 4096 + 3840 + 96 + 3 + 0 + 0.0546 + 0.0026$

$\rightarrow (8035.0572)_{10}$

Hexadecimal to binary number system

To represent hexadecimal in binary, represent each HEX number with its 4-bit binary equivalent.

Example 24: $(34F)_{16}$

Hexa	Decimal	Binary
3	3	0011
4	4	0100
F	15	1111

$= (001101001111)_2$

Example 25: $(AFBC \cdot BED)_{16}$

Hexa	Decimal	Binary
A	10	1010
F	15	1111
B	11	1011
C	12	1100
B	11	1011
E	14	1110
D	13	1101

$= (1010111110111100.101111101101)_2$

Binary to hexadecimal number system

To convert binary number to a hexadecimal number, starting from the binary point, make groups of 4-bits each on either side of the binary point and replace each 4-bit group by the equivalent hexadecimal digit.

Example 26: $(11001001)_2$

$\rightarrow \underline{1100}\ \underline{1001}$

$\quad\ 12 \quad 9$

$\rightarrow (C9)_{16}$

Example 27: $(1011011011.01111)_2$

$$\frac{0010}{2}\ \frac{1101}{D}\ \frac{1011}{B} \cdot \frac{0111}{7}\ \frac{1000}{8} = (2\,DB.78)_{16}$$

Hexadecimal to octal number system

The simplest way to convert hexadecimal to octal is, first convert the given hexadecimal number to binary and the Binary number to Octal.

Example 28: $(C3AF)_{16}$

$\rightarrow 001100001110101111$

$\rightarrow (141657)_8$

Example 29: $(C6.AE)_{16}$

$\rightarrow 0011000110.10101110$

$\rightarrow (306.534)_8$

Octal to hexadecimal number system

The simplest way to convert octal to hexadecimal is first convert the given octal number to binary and then the binary number to hexadecimal.

Example 30: $(775)_8$

$\rightarrow (000111111101)_2$

$\rightarrow (1FD)_{16}$

Example 31: $(34.7)_8$

$\rightarrow (00011100.1110)_2$

$\rightarrow (1C.E)_{16}$

Complements

Complements are used in digital computers to simplify the subtraction operation and for logical manipulation.

There are two types of complements for each base - r-system.

1. Radix complement (or) r's complement: the r's complement of an m digit number N in base r is $r^m - N$ for $N \neq 0$.

   For example, $N = 0$, r's complement is 0.
2. Diminished radix complement: (or) $(r-1)$'s complement: Given a number N in base r having m digits, then $(r-1)$'s complement is $(r^m - 1) - N$.

   For example, decimal number system will have 10's complement and 9's complement.

   Similarly, binary number system will have 2's complement and 1's complement.

Example 32: 10's complement of $(2657)_{10}$ is $(10)^4 - 2657$

10000
2657
7343

Example 33: 9's complement of $(2657)_{10}$ is $(10^4 - 1) - 2657$

10000
– 1
9999
2657
7342

- r's complement can be obtained by adding 1 to $(r-1)$'s complement.

$$r^m - N = \{(r^m - 1) - N\} + 1$$

Example 34: 2's complement of $(101101)_2$ is

$= (2)^6 - 101101$

$(2^6)_{10} = (100000)_2$

2's complement is 100000
–101101
010011

Example 35: 1's complement of $(101101)_2$ is

$2^6 - 1 = 1000000$
– 1
111111
101101

1's complement –010010

The one's complement of a binary number is formed by changing 1's to 0's and 0's to 1's, The 2's complement can be formed by leaving all least significant 0's and the first 1 unchanged, and replacing 1's with zeros and zeros with 1's in all other bits.

If the number M contains radix point, the point should be removed temporarily in order to form r's/$(r-1)$'s complement.

The radix point is then restored to the complemented number in the same relative position.

Example 36: What is 1's complement of $(1001.011)_2$?

$\rightarrow$ Consider without radix point 1001011

Take 1's complement 0110100

Place radix point again $(0110.100)_2$

Example 37: What is 2's complement of $(1001.011)_2$?

Consider without radix point 1001011

Take 2's complement 0110101

Place radix point again $(0110.101)_2$

Complement of a complement is equal to the original number $r^m - (r^m - M) = M$

Subtraction with Complements

Subtraction of two n digit unsigned numbers $A - B$ in base r can be done as follows by r's complement method.

Add A to the r's complement of B. Mathematically $A + (r^n - B) = A - B + r^n$

If $A \geq B$ the sum will produce an end carry r^n; which can be discarded. (Discarding carry is equivalent to subtracting r^n from result). What is left is the result $A - B$?

$A = 1100 \rightarrow$ 1100
$B = 1010$ (2's complement) + 0110
Sum: 10010
discard carry ($-r^n$) − 10000
$A - B$: 0010

If $A < B$, the sum does not produce an end carry and result is $r^n - (B - A)$. Then take r's complement of the sum, and place a negative sign in front.

If $A = 1010$
$B = 1100$
$A - B$ can be done as
$A \rightarrow$ 1010
$B \rightarrow$ 2's complaint + 0100
Sum: 1110

Here, no carry generated, so result is a negative number.
2's complement of result $\rightarrow 0010 = 2$
result = −2

Subtraction of unsigned numbers by using $(r-1)$'s complement can be done in similar way. However, $(r-1)$'s complement is one less than the r's complement. Because of this, the sum produced is one less than the correct difference when an end carry occurs. So end carry will be added to the sum. Removing the end carry and adding 1 to the sum is referred to as an end-around-carry.

Consider $A = 1100$, $B = 1010$
For $A - B$
$A \rightarrow$ 1100
$B \rightarrow$ (1's complement) + 0101
Sum: 10001
End around carry + 1
$A - B = 0010$

For $B - A$
$B \rightarrow$ 1010
$A \rightarrow$ (1's complement) + 0011
Sum: 1101

There is no end carry, for there result is
$-(B - A) = -$(1's complement of 1101)
$= -0010 = -2$

Signed Binary Numbers

Positive integers can be represented as unsigned numbers; but to represent negative integer, we need a notation for negative values in binary.

It is customary to represent the sign with a bit placed in the left most position of the number. The convention is to make the sign bit 0 for positive and 1 for negative. This representation of signed numbers is referred to as sign-magnitude convention

S Magnitude

+24 is 0 11000
sign magnitude
−24 is 1 11000
sign magnitude

Other notation for representation of signed numbers is signed complement system. This is convenient to use in a computer for arithmetic operations. In this system, a negative number is indicated by its complement (i.e., complement of corresponding positive number) whereas the sign-magnitude system negates a number by changing its sign bit, the signed-complement system negates a number by taking its complement. Positive numbers use same notation in sign-magnitude as well as sign-complement systems.

The signed-complement system can be used either as the 1's complement or the 2's complement.

But 2's complement is the most common.
+24 in 1's/2's complement representation is 011000
−24 in 1's complement representation 100111
−24 in 2's complement representation 101000

Table 2 *Signed binary numbers – (4-bits)*

Decimal	Signed-Magnitude	Signed 1's Complement	Signed 2's Complement
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	1000	1111	–
−1	1001	1110	1111
−2	1010	1101	1110
−3	1011	1100	1101
−4	1100	1011	1100
−5	1101	1010	1011
−6	1110	1001	1010
−7	1111	1000	1001
−8	–	–	1000

The ranges of signed binary numbers with n-bits

Signed-magnitude: $-2^{n-1} + 1$ to $+2^{n-1} - 1$
1's complement representation: $-2^{n-1} + 1$ to $+2^{n-1} - 1$
2's complement representation: -2^{n-1} to $+2^{n-1} - 1$

Signed 2's complement representation can be directly used for arithmetic operations. The carryout of the sign bit position is discarded.

In order to obtain a correct answer, we must ensure that the result has a sufficient number of bits to accommodate the sum/product.

Example 38: $X = 00110$, $Y = 11100$ are represented in 5-bit signed 2's complement system

Then their sum $X + Y$ in 6-bit signed 2's complemented representation is?

Solution: $X = 00110$

$Y = 11100$

are 5-bit numbers

But result needs to be in 6-bit format.

Operands X and Y also should be in 6-bit format

$$\begin{array}{rl} X = & 000110 \\ Y = & \underline{111100} \\ X + Y = & (1)\ 000010 \end{array}$$

The carry out of sign bit position is discarded result is 000010.

Example 39: $(36x\ 70)_{10}$ is 10's complement of $(yzyz0)_{10}$ Then values of x, y, z are?

(A) 4, 5, 2
(B) 4, 6, 3
(C) 3, 6, 3
(D) 3, 5, 4

Solution: (C)

$(36x70)_{10}$ is 10's complement of $(yzyz0)_{10}$.

10's complement of $(yzyz0)_{10}$ is

$$10^5 - yzyz0 = 36 \times 70$$

So $36x70 + yzyz0 = 100000$

$$\begin{array}{r} 36x70 \\ +\underline{yzyz0} \\ 100000 \end{array}$$

so $7 + z = 10$,

$1 + x + y = 10 \quad z = 3$

$1 + 6 + z = 10 \quad y = 6$

$1 + 3 + y = 10$,

$\rightarrow x = 3$

Example 40: The 10's complement of $(843)_{11}$ is?

(A) $(157)_{11}$
(B) $(267)_{11}$
(C) $(156)_{11}$
(D) $(268)_{11}$

Solution: (B)

Given $(843)_{11}$ is base 11 number system and the number in the number system range from 0 to 9 & A ($A = 10$)

10's complement for $(843)_{11}$ means $(r - 1)$'s complement.

So $(r^n - 1) - N = [(11)^n - 1] - N$

$$\begin{array}{rr} (11)^n - 1 \Rightarrow & 1000 \\ & \underline{-\quad 1} \\ & AAA \\ & \underline{-843} \\ & 267 \end{array}$$

10's complement is $(267)_{11}$

Example 41: Consider the signed binary number to be 10111011. What is the decimal equivalent of this number if it is in Sign-Magnitude form, or 1's complement form, or 2's complement form?

Solution: Given binary number = 10111011. As sign bit is 1, it is a negative number. If it is in sign-magnitude format, then MSB is sign bit, and remaining bits represent the magnitude,

$(0111011)_2 = 32 + 16 + 8 + 2 + 1 = 59$. So if the given number is in sign-magnitude format, then the number is –59.

If it is in 1's complement/2's complement form, then the magnitude of negative number can be obtained by taking 1's complement/2's complement for the number, respectively.

10111011 ⇒ 1's complement ⇒ $01000100 = (68)_{10}$.

In 1's complement format, the number is –68.

10111011 ⇒ 2's complement ⇒ $01000101 = (69)_{10}$.

In 2's complement format, the number is –69.

Example: Find $(-9.625)_{10}$ in signed 2's complement representation.

Signed binary fraction can be represented in the same way of signed integer.

$$\begin{array}{r|l} 2 & 9 \\ 2 & 4 - 1 \\ 2 & 2 - 0 \\ & 1 - 0 \end{array} \uparrow$$

$$\begin{array}{l} 0.625 \times 2 = 1.25 \\ 0.25 \times 2 = 0.5 \\ 0.5 \times 2 = 1.0 \\ = 0.101 \end{array} \downarrow$$

$+(9.625) = 01001.101$

$-9.625 = 10110.011$ (by taking 2's complement)

Binary Multipliers

Multiplication of binary number is done in the same way as multiplication of decimal.

The multiplicand (m) is multiplied by each bit of the multiplier (N), starting from the LSB.

Let

$M = B_3\ B_2\ B_1\ B_0$

$N = A_3\ A_2\ A_1\ A_0$

If $M \times N = P$

			A_0B_3	A_0B_2	A_0B_1	A_0B_0	
		A_1B_3	A_1B_2	A_1B_1	A_1B_0		
	A_2B_3	A_2B_2	A_2B_1	A_2B_0			
A_3B_3	A_3B_2	A_3B_1	A_3B_0				
P_7P_6	P_5	P_4	P_3	P_2	P_1	P_0	$=P$

Example: Let $M = 1\ 0\ 1\ 1$

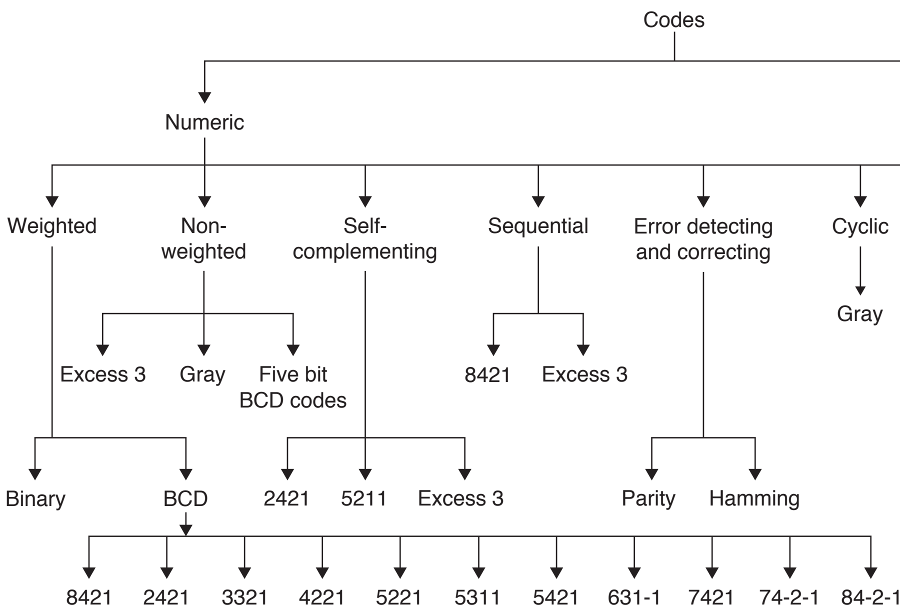
$N = 1\ 1\ 0\ 0$

$M \times N = P$

$$\begin{array}{r} 1011 \\ \underline{1100} \times \\ 0000 \\ 0000 \\ 1011 \\ 1011 \\ \underline{111} \\ 10000100 = P \end{array}$$

Binary Codes

Binary codes can be classified as numeric codes and alphanumeric codes. The figure below shows the classification of codes.



Numeric Codes

Numeric Codes are the codes which represent numericals in binary, i.e., only numbers as a series of 0s and 1s.

Weighted and non-weighted codes

- The weighted codes are those which obey the position-weighting principle. Each position of a number represents a specific weight.
  Example: BCD, Binary, 84-2-1, 2421,
- Non-weighted codes are codes which are not assigned fixed values.
  Example: Excess-3, Gray code
  2421, 5211, 84 – 2 – 1 are examples of weighted codes, in which weight is assigned to each position in the number.
  $(27)_{10}$ in 2421 code → 0010 1101
  $(45)_{10}$ in 5211 code → 0111 1000
  $(36)_{10}$ in 84 – 2 – 1 code → 0101 1010
  Any digit in decimal will be represented by the weights represented by the code.

Error-detecting and correcting codes

Codes which allow only error detection are error-detecting codes.

Example: Parity

Codes which allow error detection as well as correction are called error correcting codes.

Example: Hamming codes

Sequential codes

A sequential code is one in which each succeeding code word is one binary number greater than the preceding code word.
Example: XS–3, BCD

Cyclic codes (unit distance codes)

Cyclic codes are those in which each successive code word differs from the preceding one in only one bit position.
Example: Gray code

Reflective codes

Binary code in which the n least significant bits for code words 2^n through $2^{n+1} - 1$ are the mirror images of than for 0 through $2^n - 1$ is called reflective codes.
Example: Gray Code

Self-complementing codes

A code is said to be self-complementing, if the code word of the 9's complement of number '*N*', i.e., of "9-*N*" can be obtained from the code word of '*N*' by interchanging all the zeros and ones, i.e., by taking 1's complement. In other words, logical complement of number code is equivalent to representation of its arithmetic complement.

Example: 84-2-1, 2421, XS -3.
All weighted BCD codes are self-complementing codes.

Binary-coded decimal (BCD)

In BCD, each decimal digit 0 to 9 is coded by a 4-bit binary number. BCD codes are convenient to convert to/or from decimal number system.

Decimal	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Example 42: $(628)_{10} = (0110\ 0010\ 1000)_{BCD}$

BCD addition

- BCD addition is performed by individually adding the corresponding digits of the decimal number expressed in 4-bit binary groups starting from the LSB.
- If there is no carry and the sum term is not an illegal code, no correction is needed.
- If there is a carry out of one group to the next group or if the sum term is an illegal code, the $(6)_{10}$ is added to the sum term of that group, and the resulting carry is added to the next group.

Example 43: 44 + 12

```
0100   0100  (44 in BCD)
0001   0010  (12 in BCD)
-------------------------
0101   0110  (56 in BCD)
```

Example 44: 76.9+ 56.6

```
 0111   0110 . 1001
 0101   0110 . 0110
 ------------------
 1100   1100 . 1111   (all are illegal
 0110   0110 . 0110    codes)
 ------------------
 0010   0010 . 0101
   +1     +1     +1   (propagate carry)
 ------------------
 0001   0011   0011 . 0101
 1      3      3    .  5
```

BCD subtraction BCD subtraction is performed by subtracting the digits of each 4-bit group of the subtrahend from the corresponding 4-bit group of the minuend in binary starting from the LSB.

Example 45:

```
 42    0100   0010   (42 in BCD)
-12   -0001   0010   (12 IN BCD)
----  -------------
 30    0011   0000   (No borrow, so this is
                      the correct difference)
```

Example 46:

```
 247.7   0010   0100   0111 .  0111   (Borrow
-156.9   0001   0101   0110 .  1001    are
------   ----   ----   ------  ----    present,
  90.8   0000   0111   ·0000 . 1110    subtract
                                       0110)
                -01001  -0110
                ---------------
                1001   000 ·  1000    Corrected
                                      difference
                                      (90.8)
```

Excess-3 (XS-3) code

Excess-3 code is a non-weighted BCD code, where each digit binary code word is the corresponding 8421 code word plus 0011.

Find the XS-3 code of

Example 47: $(3)_{10} \rightarrow (0011)_{BCD} = (0110)_{xS3}$

Example 48: $(16)_{10} \rightarrow (0001\ 0110)_{BCD}$
$\rightarrow (0100\ 1001)_{xS3}$

Gray code

Each gray code number differs from the preceding number by a single bit.

Decimal	Gray Code
0	0000
1	0001
2	0011
3	0010
4	0110
5	0111

Binary to gray conversion

Step I: Shift the binary number one position to the right, LSB of the shifted number is discarded.

Step II: Exclusive or the bits of the binary number with those of the binary number shifted.

Example 49: Convert $(1001)_2$ to gray code

```
Binary          → 1010
Shifted Binary  →  101   ⊕
                ------
Gray            → 1111
```

Gray to binary conversion

(a) Take the MSB of the binary number is same as MSB of gray code number.
(b) X-OR the MSB of the binary to the next significant bit of the gray code.
(c) X-OR the 2nd bit of binary to the 3rd bit of Gray code to get 3rd bit binary and so on.
(d) Continue this till all the gray bits are exhausted.

Example 50: Convert, gray code 1010 to binary

```
Gray                1     0     1     0
      1010          ⇓  ⊕↗ ||  ⊕↗  ||  ⊕↗  ||
      1100          1     1     0     0
  = (1100)₂
```

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Assuming all the numbers are in 2's complement representation, which of the following is divisible by 11110110?
   (A) 11101010 (B) 11100010
   (C) 11111010 (D) 11100111

2. If $(84)_x$ (in base x number system) is equal to $(64)_y$ (in base y number system), then possible values of x and y are
   (A) 12, 9 (B) 6, 8
   (C) 9, 12 (D) 12, 18

3. Let A = 1111 1011 and B = 0000 1011 be two 8-bit signed 2's complement numbers. Their product in 2's complement representation is

(A) 11001001 (B) 10011100
(C) 11010101 (D) 10101101

4. Let r denotes number system's radix. The only value(s) of r that satisfy the equation $\sqrt[3]{(1331)_r} = (11)_r$ is/are
(A) 10 (B) 11
(C) 10 and 11 (D) any $r > 3$

5. X is 16-bit signed number. The 2's complement representation of X is $(F76A)_{16}$. The 2's complement representation of $8 \times X$ is
(A) $(1460)_{16}$ (B) $(D643)_{16}$
(C) $(4460)_{16}$ (D) $(BB50)_{16}$

6. The HEX number $(CD.EF)_{16}$ in octal number system is
(A) $(315.736)_8$ (B) $(513.637)_8$
(C) $(135.673)_8$ (D) $(531.367)_8$

7. 8-bit 2's complement representation a decimal number is 10000000. The number in decimal is
(A) +256 (B) 0
(C) −128 (D) −256

8. The range of signed decimal numbers that can be represented by 7-bit 1's complement representation is
(A) −64 to + 63 (B) −63 to + 63
(C) −127 to + 128 (D) −128 to +127

9. Decimal 54 in hexadecimal and BCD number system is respectively
(A) 63, 10000111 (B) 36,01010100
(C) 66, 01010100 (D) 36, 00110110

10. A new binary-coded hextary (BCH) number system is proposed in which every digit of a base −6 number system is represented by its corresponding 3-bit binary code. For example, the base −6 number 35 will be represented by its BCH code 011101.
In this numbering system, the BCH code 001001101011 corresponds to the following number in base −6 system.
(A) 4651 (B) 4562
(C) 1153 (D) 1353

11. The signed 2's complement representation of $(-589)_{10}$ in Hexadecimal number system is
(A) $(F24D)_{16}$ (B) $(FDB3)_{16}$
(C) $(F42D)_{16}$ (D) $(F3BD)_{16}$

12. The base of the number system for which the following operation is to be correct $\frac{66}{5} = 13$
(A) 6 (B) 7
(C) 8 (D) 9

13. The solution to the quadratic equation $x^2 - 11x + 13 = 0$ (in number system with radix r) are $x = 2$ and $x = 4$. Then base of the number system is $(r) =$
(A) 7 (B) 6
(C) 5 (D) 4

14. The 16's complement of BADA is
(A) 4525 (B) 4526
(C) ADAB (D) 2141

15. $(11A1B)_8 = (12CD)_{16}$, in the above expression A and B represent positive digits in octal number system and C and D have their original meaning in Hexadecimal, the values of A and B are?
(A) 2, 5 (B) 2, 3
(C) 3, 2 (D) 3, 5

Practice Problems 2

Directions for questions 1 to 20: Select the correct alternative from the given choices.

1. The hexadecimal representation of $(567)_8$ is
(A) 1AF (B) D77
(C) 177 (D) 133

2. $(2326)_8$ is equivalent to
(A) $(14D6)_{16}$ (B) $(103112)_4$
(C) $(1283)_{10}$ (D) $(09AC)_{16}$

3. $(0.46)_8$ equivalent in decimal is?
(A) 0.59375 (B) 0.3534
(C) 0.57395 (D) 0.3435

4. The 15's complement of $(CAFA)_{16}$ is
(A) $(2051)_{16}$ (B) $(2050)_{16}$
(C) $(3506)_{16}$ (D) $(3505)_{16}$

5. 53 in 2's complement from is?
(A) 1001011 (B) 001010
(C) 0110101 (D) 001011

6. Signed 2's complement representation of $(-15)_{10}$ is
(A) 11111 (B) 10001
(C) 01111 (D) 10000

7. $(0.25)_{10}$ in binary number system is?
(A) (0.01) (B) (0.11)
(C) 0.001 (D) 0.101

8. The equivalent of $(25)_6$ in number system with base 7 is?
(A) 22 (B) 23
(C) 24 (D) 26

9. The operation 35 + 26 = 63 is true in number system with radix
(A) 7 (B) 8
(C) 9 (D) 11

10. The hexadecimal equivalent of largest binary number with 14-bits is?
(A) 2FFF (B) 3FFFF
(C) FFFF (D) 1FFFF

11. If x is radix of number system, $(211)_x = (152)_8$, then x is
(A) 6 (B) 7
(C) 9 (D) 5

12. The value of r for which $\sqrt{(224)_r} = (13)_r$ is valid expression, in number system with radix r is?
(A) 5 (B) 6
(C) 7 (D) 8

13. Which of the representation in binary arithmetic has a unique zero?
(A) sign-magnitude (B) 1's compliment
(C) 2's complement (D) All of these

14. For the binary number 101101111 the equivalent hexadecimal number is
(A) 14E (B) 9E
(C) B78 (D) 16F

15. Subtract 1001 from 1110
(A) 0010 (B) 0101
(C) 1011 (D) 1010

16. Which of the following is a positively weighted code?
(A) 8421 (B) 84-2-1
(C) EXS-3 (D) 74-2-1

17. Match the items correctly

Column 1	Column 2
(P) 8421	(1) Cyclic code
(Q) 2421	(2) self-complementing
(R) 5212	(3) sequential code
(S) Gray code	(4) non-sequential code

(A) P–2, Q–4, R–3, S–1
(B) P–1, Q–4, R–3, S–2
(C) P–3, Q–2, R–4, S–1
(D) P–2, Q–4, R–1, S–2

18. Perform the subtraction in XS-3 code 57.6 – 27.8
(A) 0101 1100.1011 (B) 0010 1001.1100
(C) 00011101.1100 (D) 1010 1110.1011

19. The 2's complement representation of –17 is
(A) 101110 (B) 111110
(C) 101111 (D) 110001

20. The decimal 398 is represented in 2421 code by
(A) 110000001000 (B) 001110011000
(C) 001111111110 (D) 010110110010

Previous Years' Questions

1. $(1217)_8$ is equivalent to **[2009]**
(A) $(1217)_{16}$ (B) $(028F)_{16}$
(C) $(2297)_{10}$ (D) $(0B17)_{16}$

2. P is a 16-bit signed integer. The 2's complement representation of P is $(F87B)_{16}$. The 2's complement representation of 8*P is **[2010]**
(A) $(C3D8)_{16}$ (B) $(187B)_{16}$
(C) $(F878)_{16}$ (D) $(987B)_{16}$

3. The smallest integer that can be represented by an 8-bit number in 2's complement form is **[2013]**
(A) –256 (B) –128
(C) –127 (D) 0

4. The base (or radix) of the number system such that the following equation holds is ______ $\frac{312}{20} = 13.1$ **[2014]**

5. Consider the equation $(123)_5 = (x8)_y$ with x and y as unknown. The number of possible solutions is ______. **[2014]**

6. Consider the equation $(43)_x = (y3)_8$ where x and y are unknown. The number of possible solutions is ______ **[2015]**

7. Suppose X_i for $i = 1, 2, 3$ are independent and identically distributed random variables whose probability mass functions are $Pr[X_i = 0] = Pr[X_i = 1] = ½$ for $i = 1, 2, 3$. Define another random variable $Y = X_1 X_2 \oplus X_3$, where $\oplus$ denotes XOR. Then
$Pr[Y = 0|X_3 = 0] =$ ______ **[2015]**

8. The 16-bit 2's complement representation of an integer is 1111 1111 1111 0101; its decimal representation is ____ . **[2016]**

9. Consider an eight - bit ripple - carry adder for computing the sum of A and B, where A and B are integers represented in 2's complement form. If the decimal value of A is one, the decimal value of B that leads to the longest latency for the sum to stabilize is _____ . **[2016]**

10. Let $x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$ where x_1, x_2, x_3, x_4 are Boolean variables, and $\oplus$ is the XOR operator. Which one of the following must always be **TRUE**? **[2016]**
(A) $x_1 x_2 x_3 x_4 = 0$
(B) $x_1 x_3 + x_2 = 0$
(C) $\bar{x}_1 \oplus \bar{x}_3 = \bar{x}_3 \oplus \bar{x}_4$
(D) $x_1 + x_2 + x_3 + x_4 = 0$

11. Consider a quadratic equation $x^2 - 13x + 36 = 0$ with coefficients in a base b. The solutions of this equation in the same base b are $x = 5$ and $x = 6$. Then $b =$ __________. **[2017]**

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** C	**3.** A	**4.** D	**5.** D	**6.** A	**7.** C	**8.** B	**9.** B	**10.** C
11. B	**12.** D	**13.** C	**14.** B	**15.** D					

Practice Problems 2

1. C	**2.** B	**3.** A	**4.** D	**5.** D	**6.** B	**7.** A	**8.** B	**9.** B	**10.** B
11. B	**12.** A	**13.** C	**14.** D	**15.** B	**16.** A	**17.** C	**18.** A	**19.** C	**20.** C

Previous Years' Questions

1. B	**2.** A	**3.** B	**4.** 5	**5.** 3	**6.** 5	**7.** 0.75	**8.** –11	**9.** –1.0	**10.** C
11. 8.0 to 8.0									