Straight Lines

Quick Revision

Distance Formula

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

The distance of a point $A(x, y)$ from the origin $O(0, 0)$ is given by $OA = \sqrt{x^2 + y^2}$.

Three points A, B and C are collinear i.e. in same straight line, if $AB + BC = AC$ or $AC + CB = AB$ or $BA + AC = BC$.

Section Formulae

- The coordinates of the point which divides the joining of (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ internally, is $\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$ and externally is $\left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}\right)$.
- The coordinates of the mid-point of the joining of (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Area of a Triangle

- If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a ΔABC, then area of ΔABC.

$$= \frac{1}{2}\left|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$$

$$= \frac{1}{2}\left|(x_1y_2 + x_2y_3 + x_3y_1) - (x_1y_3 + x_2y_1 + x_3y_2)\right|$$

- If the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear, then $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$.
- The coordinates of centroid of the triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3), is given by $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

Locus of a Point

The curve described by a moving point under given geometrical conditions is called the locus of that point.

Slope or Gradient of a Line

If θ is the angle of inclination of a line l, then $\tan\theta$ is called the slope or gradient of the line l and it is denoted by m.

i.e. $m = \tan\theta$

The slope of X-axis is zero and slope of Y-axis is not defined.

Slope of a Line Joining Two Points

The slope of a line passing through points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by $m = \tan\theta = \left|\frac{y_2 - y_1}{x_2 - x_1}\right|$

Angle between Two Lines

The angle θ between two lines having slopes m_1 and m_2 is

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

(i) If two lines are **parallel**, then their slopes are equal i.e. $m_1 = m_2$.

(ii) If two lines are **perpendicular** to each other, then product of their slopes is -1, i.e. $m_1 m_2 = -1$.

Note

(i) If $\tan\theta$ is positive, then θ will be an acute angle.

(ii) If $\tan\theta$ is negative, then θ will be an obtuse angle.

Various Forms of the Equation of a Line

(i) If a line is at a distance a and parallel to X-axis, then the equation of the line is $y = \pm a$.

(ii) If a line is parallel to Y-axis at a distance b from Y-axis, then its equation is $x = \pm b$.

(iii) Point-slope form The equation of a line which passes through the point (x_1, y_1) and has the slope m is given by $y - y_1 = m(x - x_1)$.

(iv) **Two points form** The equation of a line passing through the points (x_1, y_1) and (x_2, y_2) is given by $y - y_1 = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$.

(v) **Slope-intercept form** The equation of line with slope m and making an intercept c on Y-axis, is $y = mx + c$.

(vi) If a line with slope m cuts the X-axis at a distance d from the origin i.e. makes x-intercept d. Then, the equation of line is given by $y = m(x - d)$.

(vii) Intercept form The equation of a line which cuts off intercepts a and b respectively on the X and Y-axes is given by $\dfrac{x}{a} + \dfrac{y}{b} = 1$

i.e. $\dfrac{x}{x\text{ - intercept}} + \dfrac{y}{y\text{ - intercept}} = 1$

(viii) **Normal form** The equation of a straight line upon which the length of the perpendicular from the origin is p and angle made by this perpendicular to the X-axis is α, is given by $x\cos\alpha + y\sin\alpha = p$.

General Equation of a Line

Any equation of the form $Ax + By + C = 0$, where A and B are simultaneously not zero, is called the general equation of a line.

Different forms of $Ax + By + C = 0$ are

(i) Slope-intercept form $y = \dfrac{-A}{B}x - \dfrac{C}{B}$, $B \neq 0$

(ii) Intercept form $\dfrac{x}{-C/A} + \dfrac{y}{-C/B} = 1$, $C \neq 0$

(iii) Normal form $x\cos\alpha + y\sin\alpha = p$

where, $\cos\alpha = \pm\dfrac{A}{\sqrt{A^2 + B^2}}$

$\sin\alpha = \pm\dfrac{B}{\sqrt{A^2 + B^2}}$ and $p = \pm\dfrac{C}{\sqrt{A^2 + B^2}}$

Note Proper choice of signs to be made so that p should be always positive.

Angle Between Two Lines, having General Equations

Let general equations of lines be $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$, then slope of given lines are $m_1 = -\dfrac{A_1}{B_1}$ and $m_2 = -\dfrac{A_2}{B_2}$.

Let θ be the angle between two lines, then

$$\tan\theta = \pm\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right) = \pm\left(\frac{-\dfrac{A_2}{B_2} + \dfrac{A_1}{B_1}}{1 + \dfrac{A_1}{B_1}\cdot\dfrac{A_2}{B_2}}\right)$$

Distance of a Point from a Line

The perpendicular distance d of a point $P(x_1, y_1)$ from the line $Ax + By + C = 0$ is given by

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

Distance between Two Parallel Lines

The distance d between two parallel lines $y = mx + C_1$ and $y = mx + C_2$ is given by $d = \dfrac{|C_1 - C_2|}{\sqrt{1 + m^2}}$ and if lines are $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, then, $d = \dfrac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$.

Objective Questions

Multiple Choice Questions

1. The point $(-3, 2)$ is located in the quadrant

(a) quadrant I (b) quadrant II
(c) quadrant III (d) quadrant IV

2. The value of y is, if the distance between points $P(2, -3)$ and $Q\ (10, y)$ is 10 units.

(a) 3 (b) 9
(c) -3 (d) None of these

3. The point on X-axis which is equidistant from the points $(3, 2)$ and $(-5, -2)$ is

(a) (1, 0) (b) (2, 0)
(c) $(-1, 0)$ (d) $(-2, 0)$

4. The coordinates of a point which divides the line segment joining $A\ (1, -3)$ and $B\ (-3, 9)$ internally in the ratio $1 : 3$, are given by

(a) $(-2, 6)$ (b) $(0, 0)$
(c) $\left(\frac{-6}{4}, \frac{18}{4}\right)$ (d) $\left(\frac{10}{4}, \frac{-30}{4}\right)$

5. The coordinates of a point which divides externally the line joining $(1, -3)$ and $(-3, 9)$ in the ratio $1 : 3$ are

(a) $(3, -6)$ (b) $(-6, 3)$
(c) $(3, -9)$ (d) $(-9, 3)$

6. If the vertices of a triangle are $P(1, 3)$, $Q(2, 5)$ and $R(3, -5)$, then the centroid of a ΔPQR is

(a) (1, 2) (b) (1, 3)
(c) (3, 1) (d) (2, 1)

7. The points $(1, -1)$, $(5, 2)$ and $(9, 5)$ collinear.

(a) Yes
(b) No
(c) Cannot say
(d) Insufficient information

8. Area of the triangle whose vertices are $(4, 4)$, $(3, -2)$ and $(-3, 16)$, is

(a) 54 (b) 27
(c) 53 (d) 106

9. The slope of a line whose inclination is $90°$, is

(a) 1 (b) 0
(c) -1 (d) not defined

10. The slope of line, whose inclination is $60°$, is

(a) $\frac{1}{\sqrt{3}}$ (b) 1
(c) $\sqrt{3}$ (d) Not defined

11. The slope of that line, which passes through the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ is

(a) $\frac{2}{t_2 - t_1}$ (b) $\frac{2}{t_2 + 2t_1}$
(c) $\frac{1}{t_2 + t_1}$ (d) $\frac{2}{t_2 + t_1}$

12. The angle between the lines

$$y = (2 - \sqrt{3})(x + 5)$$

and $y = (2 + \sqrt{3})(x - 7)$ is

(a) $30°$ (b) $90°$
(c) $45°$ (d) $120°$

13. The angle between the X-axis and the line joining the points $(3, -1)$ and $(4, -2)$ is

(a) $45°$ (b) $135°$
(c) $90°$ (d) $180°$

14. The tangent of angle between the lines whose intercepts on the axes are $a, -b$ and $b, -a$ respectively, is

(a) $\frac{a^2 - b^2}{ab}$ (b) $\frac{b^2 - a^2}{2}$
(c) $\frac{b^2 - a^2}{2ab}$ (d) None of these

15. The value of y will be, so that the line through $(3, y)$ and $(2, 7)$ is parallel to the line through $(-1, 4)$ and $(0, 6)$.

(a) 7 (b) 8
(c) 9 (d) 10

16. Using slope of line, till, are $A(4, 4)$, $B(3, 5)$ and $C(-1, 1)$ the vertices of a right angled triangle.

(a) Yes
(b) No
(c) Cannot say
(d) Insufficient Information

17. The points $A\ (x, 4)$, $B\ (3, -2)$ and $C\ (4, -5)$ are collinear in the value of x is

(a) 1 (b) 2
(c) -1 (d) 0

18. The equation of the lines parallel to the X-axis and passing through the point $(-3, 5)$ is

(a) $x = -3$ (b) $y = -3$
(c) $x = 5$ (d) $y = 5$

19. The equation of the line through $(-2, 3)$ with slope -4 is

(a) $x + 4y - 10 = 0$ (b) $4x + y + 5 = 0$
(c) $x + y - 1 = 0$ (d) $3x + 4y - 6 = 0$

20. The equation of the line passing through the point $(1, 2)$ and perpendicular to the line $x + y + 1 = 0$ is

(a) $y - x + 1 = 0$ (b) $y - x - 1 = 0$
(c) $y - x + 2 = 0$ (d) $y - x - 2 = 0$

21. The equation of a line perpendicular to the line $x - 2y + 3 = 0$ and passing through the point $(1, -2)$ is

(a) $y = 2x$ (b) $x = 2y$
(c) $x = -2y$ (d) $y = -2x$

22. The equation of line passing through the points $(-1, 1)$ and $(2, -4)$.

(a) $5x + 2y + 2 = 0$
(b) $5x + 3y - 2 = 0$
(c) $5x + 2y + 3 = 0$
(d) $5x + 3y + 2 = 0$

23. The line passing through the points $(-4, 5)$ and $(-5, 7)$ also passes through the point (l, m), then $2l + m + 3$ is equal to

(a) 1 (b) -1 (c) 2 (d) 0

24. A line cutting off intercept -3 from the Y-axis and the tangent at angle to the X-axis is $\frac{3}{5}$, its equation is

(a) $5y - 3x + 15 = 0$ (b) $3y - 5x + 15 = 0$
(c) $5y - 3x - 15 = 0$ (d) None of these

25. The equations of the line which have slope $1/2$ and cuts-off an intercept 4 on X-axis is

(a) $x - 2y - 4 = 0$ (b) $x + 2y - 4 = 0$
(c) $x + 2y + 4 = 0$ (d) $x - 2y + 4 = 0$

26. Slope of a line which cuts off intercepts of equal lengths on the axes is

(a) -1 (b) 0 (c) 2 (d) $\sqrt{3}$

27. If the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the points $(2, -3)$ and $(4, -5)$, then (a, b) is

(a) $(1, 1)$ (b) $(-1, 1)$
(c) $(1, -1)$ (d) $(-1, -1)$

28. If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is $(3, 2)$, then the equation of the line will be

(a) $2x + 3y = 12$ (b) $3x + 2y = 12$
(c) $4x - 3y = 6$ (d) $5x - 2y = 10$

29. If the normal form of the equation $\sqrt{3}x + y - 8 = 0$ is $x \cos \omega + y \sin \omega = p$, then p and ω respectively are

(a) 4, 45° (b) 4, 30°
(c) 3, 45° (d) 3, 30°

30. Transform the equation of the line $3x + 2y - 7 = 0$ to slope intercept form then the slope and y-intercept will be

(a) $\frac{3}{2}, \frac{7}{2}$ (b) $-\frac{3}{2}, -\frac{7}{2}$
(c) $-\frac{3}{2}, \frac{7}{2}$ (d) None of these

31. Transform the equation of the line $3x + 2y - 7 = 0$ to normal form then the inclination of the perpendicular segment from the origin on the line with the axis and its length is

(a) $\tan^{-1}\left(\frac{1}{3}\right), \frac{7}{\sqrt{13}}$ (b) $\tan^{-1}\left(\frac{2}{3}\right), \frac{7}{\sqrt{13}}$

(c) $\tan^{-1}\left(\frac{3}{4}\right), \frac{7}{\sqrt{13}}$ (d) $\tan^{-1}\left(\frac{1}{5}\right), \frac{7}{\sqrt{13}}$

32. The angle between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$ will not be

(a) $30°$ (b) $150°$

(c) $45°$ (d) None of these

33. The angle between the lines $x - 2y + 3 = 0$ and $3x + y - 1 = 0$ is

(a) $-\tan^{-1}(7)$ (b) $\tan^{-1}\left(\frac{1}{7}\right)$

(c) $\pi - \tan^{-1}(7)$ (d) $2\pi - \tan^{-1}(7)$

34. The equation of line, which passes through point $(4, 3)$ and parallel to the line $2x - 3y = 7$ is

(a) $2x - 3y + 1 = 0$ (b) $2x - 3y - 1 = 0$

(c) $2x + 3y + 1 = 0$ (d) $2x + 3y - 1 = 0$

35. Lines through the points $(-2, 6)$ and $(4, 8)$ is perpendicular to the line through the points $(8, 12)$ and $(x, 24)$. Then, the value of x is

(a) 2 (b) 6

(c) 8 (d) 4

36. The distance of the point $(3, -5)$ from the line $3x - 4y - 26 = 0$ is

(a) $\frac{3}{7}$ (b) $\frac{2}{5}$

(c) $\frac{7}{5}$ (d) $\frac{3}{5}$

37. The perpendicular distance from origin to the line $5x + 12y - 13 = 0$ is

(a) 10 unit (b) 5 unit

(c) 2 unit (d) 1 unit

38. The distance of the point of intersection of the lines $2x - 3y + 5 = 0$ and $3x + 4y = 0$ from the line $5x - 2y = 0$ is

(a) $\frac{130}{17\sqrt{29}}$ (b) $\frac{13}{7\sqrt{29}}$

(c) $\frac{130}{7}$ (d) None of these

39. The distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$, is

(a) $\frac{3}{7}$ (b) $\frac{7}{5}$

(c) $\frac{2}{5}$ (d) $\frac{3}{5}$

40. The distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$ is

(a) $\frac{3}{10}$ (b) $\frac{2}{25}$

(c) $\frac{7}{10}$ (d) $\frac{3}{25}$

Assertion-Reasoning MCQs

Directions (Q.Nos. 41-54) Each of these questions contains two statements : Assertion (A) and Reason (R). Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation for A.

(b) A is true, R is true; R is not a correct explanation for A.

(c) A is true; R is false.

(d) A is false; R is true.

41. **Assertion (A)** The point $(3, 0)$ is at 3 units distance from the Y-axis measured along the positive X-axis and has zero distance from the X-axis.

Reason (R) The point $(3, 0)$ is at 3 units distance from the X-axis measured along the positive Y-axis and has zero distance from the Y-axis.

42. **Assertion (A)** A point $P(h, k)$ lies on the straight line $x+y+1=0$ and is at a distance 5 units from the origin. If k is negative, then h is equal to -3.

Reason (R) The distance formula is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

43. **Assertion (A)** The line $x+y=4$ divides the line joining the points $(-1, 1)$ and $(5, 7)$ in the ratio $1:2$.

Reason (R) Section formula for internal division is $\left(\frac{m_1x_2-m_2x_1}{m_1-m_2}, \frac{m_1y_2-m_2y_1}{m_1-m_2}\right)$ where $p(x, y)$ divides the line segment AB, with $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m_1:m_2$.

44. If the vertices of a triangle are $(1, a)$, $(2, b)$ and $(c^2, -3)$. Then,

Assertion (A) The centroid cannot lie on the Y-axis.

Reason (R) The condition that the centroid may lie on the X-axis is $a+b=3$.

45. **Assertion (A)** Area of the triangle whose vertices are $(4, 4)$, $(3, -2)$ and $(-3, 16)$, is

Reason (R) Area of triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3), is $\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$.

46. **Assertion (A)** Slope of X-axis is zero and slope of Y-axis is not defined.

Reason (R) Slope of X-axis is not defined and slope of Y-axis is zero.

47. If $A(-2, -1)$, $B(4, 0)$, $C(3, 3)$ and $D(-3, 2)$ are the vertices of a parallelogram, then

Assertion (A) Slope of AB = Slope of BC and Slope of CD = Slope of AD.

Reason (R) Mid-point of AC = Mid-point of BD.

48. **Assertion (A)** The angle between the lines $x+2y-3=0$ and $3x+y+1=0$ is $\tan^{-1}(1)$.

Reason (R) Angle between two lines is given by $\tan^{-1}\left[\pm\left(\frac{m_2-m_1}{1+m_1m_2}\right)\right]$.

49. **Assertion (A)** Slope of line $3x-4y+10=0$ is $\frac{3}{4}$.

Reason (R) x-intercept and y-intercept of $3x-4y+10=0$ respectively are $\frac{-10}{3}$ and $\frac{5}{2}$.

50. **Assertion (A)** If $x\cos\theta+y\sin\theta=2$ is perpendicular to the line $x-y=3$, then one of the value of θ is $\pi/4$.

Reason (R) If two lines $y=m_1x+c_1$ and $y=m_2x+c_2$ are perpendicular then $m_1=m_2$.

51. **Assertion (A)** The slope of the line $x+7y=0$ is $\frac{1}{7}$ and y-intercept is 0.

Reason (R) The slope of the line $6x+3y-5=0$ is -2 and y-intercept is $\frac{5}{3}$.

52. If the equation of line is $x-y=4$, then

Assertion (A) The normal form of same equation is $x\cos\alpha+y\sin\alpha=p$, where $\alpha=315°$ and $p=2\sqrt{2}$.

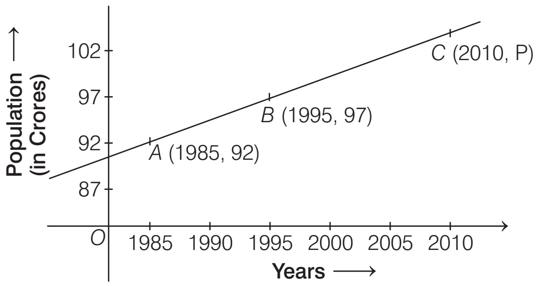
Reason (R) The perpendicular distance of line from the origin is $3\sqrt{2}$.

53. **Assertion (A)** The distance between the lines $4x + 3y = 11$ and $8x + 6y = 15$ is 7/10.

Reason (R) The distance between the lines $ax + by = c_1$ and $ax + by = c_2$ is given by $\left|\dfrac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

Case Based MCQs

54. Population *vs* Year graph given below.



Based on the above information answer the following questions.

(i) The slope of line AB is

(a) 2 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

(ii) The equation of line AB is

(a) $x + 2y = 1791$

(b) $x - 2y = 1801$

(c) $x - 2y = 1791$

(d) $x - 2y + 1801 = 0$

(iii) The population in year 2010 is (in crores)

(a) 104.5 (b) 119.5

(c) 109.5 (d) None of these

(iv) The equation of line perpendicular to line AB and passing through (1995, 97) is

(a) $2x - y = 4087$

(b) $2x + y = 4087$

(c) $2x + y = 1801$

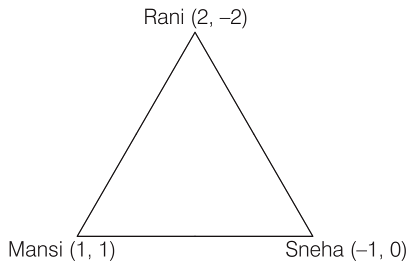
(d) None of the above

(v) In which year the population becomes 110 crores is

(a) 2020 (b) 2019

(c) 2021 (d) 2022

55. Three girls, Rani, Mansi, Sneha are talking to each other while maintaining a social distance due to covid-19. They are standing on vertices of a triangle, whose coordinates are given.



Based on the above information answer the following questions.

(i) The equation of lines formed by Rani and Mansi is

(a) $3x - y = 4$ (b) $3x + y = 4$

(c) $x - 3y = 4$ (d) $x + 3y = 4$

(ii) Slope of equation of line formed by Rani and Sneha is

(a) $\frac{2}{3}$ (b) $\frac{-3}{2}$

(c) $\frac{-2}{3}$ (d) $\frac{1}{3}$

(iii) The equation of median of lines through Rani is

(a) $5x + 4y = 2$ (b) $5x - 4y = 2$

(c) $4x - 5y = 1$ (d) None of these

(iv) The equation of altitude through Mansi is

(a) $3x - 2y = 1$ (b) $2x + 3y = 5$

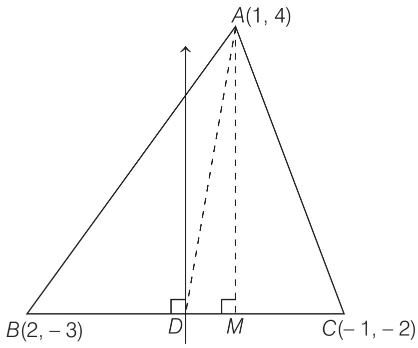
(c) $x + 2y = 3$ (d) None of these

(v) The equation of line passing through the Rani and parallel to line formed by Mansi and Sneha is

(a) $x - 2y = 4$ (b) $x + 2y = 6$

(c) $x - 2y = 6$ (d) $2x + y = 4$

56. Consider the ΔABC with vertices $A(1, 4)$, $B(2, -3)$ and $C(-1, -2)$ as shown in the given figure. AD is the median and AM is the altitude through A.



Based on the above information answer the following questions.

(i) Find the distance between A and C

(a) $\sqrt{40}$ units
(b) $\sqrt{53}$ units
(c) $\sqrt{41}$ units
(d) $\sqrt{29}$ units

(ii) Find the slope of BC.

(a) $-\frac{4}{3}$ (b) $-\frac{1}{3}$
(c) $-\frac{3}{2}$ (d) $-\frac{3}{4}$

(iii) Find the equation of median through A.

(a) $x-13y+9=0$ (b) $x+13y-9=0$
(c) $13x-y-9=0$ (d) $2x-13y+9=0$

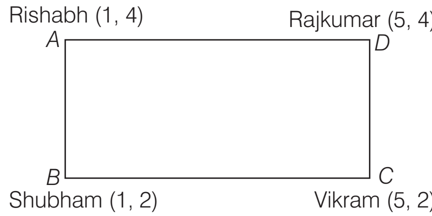
(iv) Find the equation of the altitude through A.

(a) $3x-y+1=0$ (b) $x+2y-3=0$
(c) $x-3y+2=0$ (d) $3x+2y-2=0$

(v) Find the equation of right bisector of side BC.

(a) $x+3y-3=0$ (b) $x-3y+3=0$
(c) $3x-y-4=0$ (d) $3x+y-2=0$

57. Four friends Rishabh, Shubham, Vikram and Rajkumar are sitting on vertices of a rectangle, whose coordinates are given.



Based on the above information answer the following questions.

(i) The equation formed by Shubham and Rajkumar is

(a) $x+2y+3=0$
(b) $x-2y-3=0$
(c) $x-2y+3=0$
(d) None of the above

(ii) The equation formed by Rishabh and Vikram is

(a) $x+2y+9=0$
(b) $x+2y-9=0$
(c) $x-2y-9=0$
(d) None of the above

(iii) The intersection point of above two equations is

(a) (1, 1)
(b) (2, 2)
(c) (3, 3)
(d) (4, 4)

(iv) Slope of equation of line formed by Rishabh and Rajkumar is

(a) zero (b) 1
(c) 2 (d) 3

(v) Pair for the same slope is

(a) Rishabh-Rajkumar and Shubham-Vikram
(b) Rishabh-Rajkumar and Rajkumar-Vikram
(c) Rishabh-Rajkumar and Rishabh-Shubham
(d) None of the above

58. If A and B are two persons sitting at the positions $(2, -3)$ and $(6, -5)$. If C is a third person who is sitting between A and B such that it divides the line AB in 1 : 3 ratio.

Based on the above information, answer the following questions.

(i) The distance between A and B is

(a) $\sqrt{5}$ (b) $2\sqrt{5}$
(c) $3\sqrt{5}$ (d) $4\sqrt{5}$

(ii) The equation of AB is

(a) $x+2y+4=0$ (b) $x+2y-4=0$
(c) $x-2y+4=0$ (d) None of these

(iii) Coordinates of point C are

(a) $\left(\frac{7}{2}, -3\right)$ (b) $\left(3, \frac{7}{2}\right)$
(c) $(3, 3)$ (d) $\left(3, -\frac{7}{2}\right)$

(iv) Distance between A and C is

(a) $\sqrt{5}$ (b) $2\sqrt{5}$
(c) $\frac{\sqrt{5}}{2}$ (d) $\sqrt{\frac{5}{2}}$

(v) Distance between C and B is

(a) $\frac{3\sqrt{5}}{2}$ (b) $3\sqrt{5}$
(c) $\frac{2\sqrt{5}}{3}$ (d) None of these

ANSWERS

Multiple Choice Questions

1. (b) 2. (a) 3. (c) 4. (b) 5. (c) 6. (d) 7. (a) 8. (b) 9. (d) 10. (c)
11. (d) 12. (d) 13. (b) 14. (c) 15. (c) 16. (a) 17. (a) 18. (d) 19. (b) 20. (b)
21. (d) 22. (d) 23. (d) 24. (a) 25. (a) 26. (a) 27. (d) 28. (a) 29. (b) 30. (c)
31. (b) 32. (c) 33. (c) 34. (a) 35. (d) 36. (d) 37. (d) 38. (a) 39. (c) 40. (a)

Assertion-Reasoning MCQs

41. (c) 42. (d) 43. (c) 44. (b) 45. (a) 46. (c) 47. (d) 48. (a) 49. (b) 50. (c)
51. (d) 52. (c) 53. (a)

Case Based MCQs

54. (i) - (c); (ii) - (b); (iii) - (a); (iv) - (b); (v) - (c)
55. (i) - (b); (ii) - (c); (iii) - (a); (iv) - (a); (v) - (c)
56. (i) - (a); (ii) - (b); (iii) - (c); (iv) - (a); (v) - (c)
57. (i) - (c); (ii) - (b); (iii) - (c); (iv) - (a); (v) - (a)
58. (i) - (b); (ii) - (a); (iii) - (d); (iv) - (c); (v) - (a)

SOLUTIONS

1. Let $A = (-3, 2)$

Since, x-coordinate of A is negative and its y-coordinate is positive, therefore A lies in the second quadrant.

2. Given, points $P(2, -3)$ and $Q(10, y)$.

According to the question,

$$PQ = 10$$

$$\Rightarrow \quad \sqrt{(10-2)^2 + (y+3)^2} = 10$$

$$\left[\because d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$$\Rightarrow \quad \sqrt{(8)^2 + (y+3)^2} = 10$$

$$\Rightarrow \quad \sqrt{64 + y^2 + 9 + 6y} = 10$$

On squaring both sides, we get

$$64 + y^2 + 9 + 6y = 100$$

$$\Rightarrow \quad y^2 + 6y + 73 = 100$$

$$\Rightarrow \quad y^2 + 6y - 27 = 0$$

$$\Rightarrow \quad y^2 + 9y - 3y - 27 = 0$$

$$\Rightarrow \quad y(y+9) - 3(y+9) = 0$$

$$\Rightarrow \quad (y+9)(y-3) = 0$$

$$\Rightarrow \quad y = 3, -9$$

Hence, value of y is $3, -9$.

3. Let the point on X-axis be $P(x, 0)$, which is equidistant from (say) $A(3, 2)$ and (say) $B(-5, -2)$.

Since, P is equidistant from A and B. So,

$$\therefore \quad PA = PB \Rightarrow PA^2 = PB^2$$

$$\Rightarrow (3-x)^2 + (2-0)^2 = (-5-x)^2 + (-2-0)^2$$

[by distance formula]

$$\Rightarrow \quad 9 + x^2 - 6x + 4 = 25 + x^2 + 10x + 4$$

$$\Rightarrow \quad 16x + 16 = 0$$

$$\Rightarrow \quad x = -1$$

Thus, point on X-axis is $(-1, 0)$.

4. The coordinates of the point which divides the line segment joining $A(1, -3)$ and $B(-3, 9)$ internally in the ratio $1:3$, are given by

$$x = \frac{1\cdot(-3) + 3\cdot 1}{1+3} = 0$$

and
$$y = \frac{1.9 + 3\cdot(-3)}{1+3} = 0$$

5. Let the coordinates of the required point be $P(x, y)$.

Then, $x = \left(\dfrac{1\times(-3) - 3\times 1}{1-3}\right)$

and $\quad y = \left(\dfrac{1\times 9 - 3\times(-3)}{1-3}\right)$

i.e. $\quad x = 3$ and $y = -9$

Hence, the required point is $(3, -9)$.

6. We know that, if the vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then centroid of a triangle is $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$.

Here, $\quad P(1, 3) \equiv P(x_1, y_1)$;

$\quad Q(2, 5) \equiv Q(x_2, y_2)$

and $\quad R(3, -5) \equiv R(x_3, y_3)$

$\therefore$ Centroid of a triangle

$$= \left(\frac{1+2+3}{3}, \frac{3+5-5}{3}\right)$$

$$= \left(\frac{6}{3}, \frac{3}{3}\right) = (2, 1)$$

7. Let $A = (1, -1)$, $B = (5, 2)$ and $C = (9, 5)$

Now, distance between A and B,

$$AB = \sqrt{(5-1)^2 + (2+1)^2}$$

[by distance formula]

$$= \sqrt{(4)^2 + (3)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5$$

Distance between B and C,

$$BC = \sqrt{(5-9)^2 + (2-5)^2}$$

$$= \sqrt{(-4)^2 + (-3)^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25} = 5$$

Distance between A and C,

$$AC = \sqrt{(1-9)^2 + (-1-5)^2}$$

$$= \sqrt{(-8)^2 + (-6)^2}$$

$$= \sqrt{64+36} = 10$$

Clearly, $AC = AB + BC$

Hence, A, B and C are collinear points.

8. Area of the triangle, whose vertices are (4, 4), (3, –2) and (–3, 16)

$$= \frac{1}{2} |4(-2-16) + 3(16-4) + (-3)(4+2)|$$

$$= \frac{|-72+36-18|}{2}$$

$$= \frac{|-54|}{2} = \frac{54}{2} = 27$$

9. The slope of a line whose inclination is 90°, is not defined.

10. Let θ be the inclination of a line, then its slope = $\tan\theta$.

At $\theta = 60°$, slope of a line, $m = \tan 60° = \sqrt{3}$

11. We know that the slope of a line which passes through the points (x_1, y_1) and (x_2, y_2)

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Here, $(x_1, y_1) \equiv (at_1^2, 2at_1), (x_2, y_2) \equiv (at_2^2, 2at_2)$

$$\therefore \text{Slope of a line} = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2}$$

$$= \frac{2a(t_2 - t_1)}{a(t_2 + t_1)(t_2 - t_1)} = \frac{2}{t_2 + t_1}$$

12. Given lines,

$$y = (2-\sqrt{3})(x+5) \quad \ldots(\text{i})$$

Slope of this line,

$$m_1 = (2-\sqrt{3})$$

and $$y = (2+\sqrt{3})(x-7) \quad \ldots(\text{ii})$$

Slope of this line,

$$m_2 = (2+\sqrt{3})$$

Let θ be the angle between lines (i) and (ii), then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{(2-\sqrt{3})-(2+\sqrt{3})}{1+(2-\sqrt{3})(2+\sqrt{3})}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{-2\sqrt{3}}{1+4-3}\right|$$

$$\Rightarrow \quad \tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan\pi/3$$

$$\therefore \quad \theta = \pi/3 = 60°$$

For obtuse angle $= \pi - \pi/3 = 2\pi/3 = 120°$

Hence, the angle between the lines are 60° or 120°.

13. Given, $(x_1, y_1) = (3, -1)$

and $(x_2, y_2) = (4, -2)$

$$\text{Slope of line } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2+1}{4-3}$$

$$\tan\theta = \frac{-1}{1} \Rightarrow \tan\theta = -1$$

$$\Rightarrow \quad \tan\theta = -\tan 45° \quad (\because 1 = \tan 45°)$$

$$\Rightarrow \quad \tan\theta = \tan(180° - 45°)$$

$$[\because \tan(180° - \theta) = -\tan\theta]$$

$$\Rightarrow \quad \tan\theta = \tan 135° \Rightarrow \theta = 135°$$

14. Since, intercepts on the axes are a, $-b$ then equation of the line is $\frac{x}{a} - \frac{y}{b} = 1$.

$$\Rightarrow \quad \frac{y}{b} = \frac{x}{a} - 1$$

$$\Rightarrow \quad y = \frac{bx}{a} - b$$

So, the slope of this line i.e. $m_1 = \frac{b}{a}$.

Also, for intercepts on the axes as b and $-a$, then equation of the line is

$$\frac{x}{b} - \frac{y}{a} = 1$$

$$\Rightarrow \quad \frac{y}{a} = \frac{x}{b} - 1$$

$$\Rightarrow \quad y = \frac{a}{b}x - a$$

and slope of this line i.e. $m_2 = \frac{a}{b}$

$$\therefore \tan\theta = \frac{\frac{b}{a} - \frac{a}{b}}{1 + \frac{a}{b}\cdot\frac{b}{a}} = \frac{\frac{b^2 - a^2}{ab}}{2}$$

$$= \frac{b^2 - a^2}{2ab}$$

15. Let $A(3, y)$, $B(2, 7)$, $C(-1, 4)$ and $D(0, 6)$ be the given points.

Then, m_1 = Slope of the line

$$AB = \frac{7-y}{2-3} = (y-7)$$

and m_2 = Slope of the line

$$CD = \frac{6-4}{0-(-1)} = 2$$

Since, AB and CD are parallel.

$$\therefore \quad m_1 = m_2 \Rightarrow y - 7 = 2 \Rightarrow y = 9$$

16. In ΔABC, we have

$$m_1 = \text{Slope of } AB = \frac{4-5}{4-3} = -1$$

and $m_2 = \text{Slope of } BC = \dfrac{5-1}{3-(-1)} = \dfrac{4}{4} = 1$

Clearly, $m_1 m_2 = -1$

This shows that AB is perpendicular to BC.

i.e. $\angle ABC = \pi / 2$

Hence, the given points are the vertices of a right angled triangle.

Option (a) is correct.

17. Given points are $A(x, 4)$, $B(3, -2)$ and $C(4, -5)$.

From the condition of collinearity of three points A, B and C, we should have

Slope of AB = Slope of BC

i.e. $\dfrac{-2-4}{3-x} = \dfrac{-5+2}{4-3}$ $\left[\because \text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}\right]$

$\Rightarrow \dfrac{-6}{(3-x)} = \dfrac{-3}{1}$

$\Rightarrow \dfrac{2}{3-x} = \dfrac{1}{1}$

$\Rightarrow 2 = 3 - x$

$\Rightarrow x = 3 - 2 = 1$

$\Rightarrow x = 1$

18. Clearly, the equation of a line parallel to the X-axis and passing through $(-3, 5)$ is $y = 5$.

19. Here, $m = -4$ and given point (x_0, y_0) is $(-2, 3)$.

By slope-point form,

Equation of the given line is

$$y - 3 = -4(x+2)$$

or $4x + y + 5 = 0$, which is the required equation.

20. Given point is (1, 2) and slope of the required line is 1.

$\because \quad x + y + 1 = 0 \Rightarrow y = -x - 1 \Rightarrow m_1 = -1$

$\therefore \quad$ Slope of the line $= \dfrac{-1}{-1} = 1$

$\therefore$ Equation of required line is

$$y - 2 = 1\,(x-1)$$

$\Rightarrow \quad y - 2 = x - 1$

$\Rightarrow \quad y - x - 1 = 0$

21. Given, line $x - 2y + 3 = 0$ can be written as

$$y = \frac{1}{2}x + \frac{3}{2} \qquad \ldots\text{(i)}$$

Slope of the line (i) is $m_1 = \dfrac{1}{2}$. Therefore, slope of the line perpendicular to line (i) is

$m_2 = -\dfrac{1}{m_1} = -2.$

Equation of the line with slope -2 and passing through the point $(1, -2)$ is

$$y - (-2) = -2(x-1) \text{ or } y = -2x$$

which is the required equation.

22. Let the given points are $A(x_1, y_1) \equiv A(-1, 1)$ and $B(x_2, y_2) \equiv B(2, -4)$, then equation of line AB is

$$y - 1 = \frac{-4-1}{2+1}(x+1)$$

$$\left[\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

$\Rightarrow \quad y - 1 = \dfrac{-5}{3}(x+1)$

$\Rightarrow \quad 3y - 3 = -5x - 5$

$\Rightarrow \quad 5x + 3y + 2 = 0$

23. Let the given points be $A(-4, 5)$ and $B(-5, 7)$.

We know that, equation of line passing through two points is

$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1).$

Then, equation of line passing through the points A and B is

$$y - 5 = \frac{7-5}{-5+4}(x+4)$$

$\Rightarrow \quad y - 5 = \dfrac{2}{-1}(x+4)$

$\Rightarrow \quad y - 5 = -2(x+4)$

$\Rightarrow \quad y - 5 = -2x - 8$

$\Rightarrow \quad y + 2x = -8 + 5$

$\Rightarrow \quad y + 2x = -3$

$\Rightarrow \quad 2x + y + 3 = 0 \qquad \ldots\text{(i)}$

Now, line (i) passes through the point (l, m), so point (l, m) will satisfy Eq. (i).

Put $x = l$, $y = m$ in Eq. (i),

$$2l + m + 3 = 0$$

Hence, option (d) is correct.

24. Given that,

$$c = -3 \text{ and } m = \frac{3}{5}$$

$\therefore$ Equation of the line is $y = mx + c$

$$y = \frac{3}{5}x - 3$$

$\Rightarrow$ $5y = 3x - 15 \Rightarrow 5y - 3x + 15 = 0$

25. Given, m = Slope of the line $= \frac{1}{2}$

Here, d = Intercept of the line on X-axis = 4.
Hence, required equation of the line is

$$y = \frac{1}{2}(x - 4)$$

$\Rightarrow$ $x - 2y - 4 = 0$ $\quad [\because y = m(x - d)]$

26. Let equation of line be

$$\frac{x}{a} + \frac{y}{a} = 1$$

$\Rightarrow$ $x + y = a \Rightarrow y = -x + a$

$\therefore$ Required slope $= -1$

27. Given, line is $\frac{x}{a} + \frac{y}{b} = 1$ $\quad \ldots$(i)

Since, the points (2, –3) and (4, –5) lies on this line.

$\therefore$ $\frac{2}{a} - \frac{3}{b} = 1$ $\quad \ldots$(ii)

and $\frac{4}{a} - \frac{5}{b} = 1$ $\quad \ldots$(iii)

On multiplying by 2 in Eq. (ii) and then subtracting Eq. (iii) from Eq. (ii), we get

$$-\frac{6}{b} + \frac{5}{b} = 1 \Rightarrow \frac{-1}{b} = 1$$

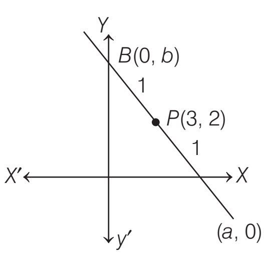
$\therefore$ $b = -1$

On putting $b = -1$ in Eq. (ii), we get

$$\frac{2}{a} + 3 = 1 \Rightarrow \frac{2}{a} = -2 \Rightarrow a = -1$$

$\therefore$ $(a, b) = (-1, -1)$

28. Since, the coordinates of the middle point are $P(3, 2)$.



$\therefore$ $3 = \frac{1 \cdot 0 + 1 \cdot a}{1 + 1}$

$\Rightarrow$ $3 = \frac{a}{2} \Rightarrow a = 6$

Similarly, $b = 4$

$\therefore$ Equation of the line is $\frac{x}{6} + \frac{y}{4} = 1$

$\Rightarrow$ $2x + 3y = 12$

29. Given equation is

$$\sqrt{3}x + y - 8 = 0 \quad \ldots\text{(i)}$$

Dividing Eq. (i) by $\sqrt{(\sqrt{3})^2 + (1)^2} = 2$, we get

$$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 4$$

or $\cos 30° x + \sin 30° y = 4$ $\quad \ldots$(ii)

Comparing Eq. (ii) with $x\cos\omega + y\sin\omega = p$, we get

$$p = 4 \text{ and } \omega = 30°$$

30. Given equation is $3x + 2y - 7 = 0$.

It can be rewritten as

$$2y = -3x + 7$$

or $y = -\frac{3}{2}x + \frac{7}{2}$ $\quad \ldots$(i)

which is the required slope intercept form of the given line.
On comparing Eq. (i) with $y = mx + c$, we get
Slope, $m = -\frac{3}{2}$ and y-intercept, $c = \frac{7}{2}$

31. Given equation is $3x + 2y - 7 = 0$. $\quad \ldots$(i)

On comparing Eq. (i) with $Ax + By + C = 0$, we get

$$A = 3, B = 2 \text{ and } C = -7$$

$\therefore$ $\sqrt{A^2 + B^2} = \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}$

On dividing both sides of Eq. (i) by $\sqrt{13}$, we get

$$\frac{3}{\sqrt{13}}x + \frac{2}{\sqrt{13}}y - \frac{7}{\sqrt{13}} = 0$$

$\Rightarrow$ $\frac{3}{\sqrt{13}}x + \frac{2}{\sqrt{13}}y = \frac{7}{\sqrt{13}}$ $\quad \ldots$(ii)

which is the required normal form of the given line.
On comparing Eq. (ii) with $x\cos\omega + y\sin\omega = p$, we get

$$\cos\omega = \frac{3}{\sqrt{13}}, \ \sin\omega = \frac{2}{\sqrt{13}} \text{ and } p = \frac{7}{\sqrt{13}}$$

Since, $\cos\omega$ and $\sin\omega$ both are positive, therefore ω is in the first quadrant and is equal to $\tan^{-1}\frac{2}{3}$.

Hence, for the given line, we have

$$\omega = \tan^{-1}\frac{2}{3} \quad \text{and } p = \frac{7}{\sqrt{13}}.$$

32. Given lines are

$$y - \sqrt{3}x - 5 = 0 \text{ or } y = \sqrt{3}x + 5 \qquad \dots\text{(i)}$$

$$\sqrt{3}y - x + 6 = 0 \text{ or } y = \frac{1}{\sqrt{3}}x - 2\sqrt{3} \qquad \dots\text{(ii)}$$

Slope of line (i) is $m_1 = \sqrt{3}$ and slope of line (ii) is $m_2 = \frac{1}{\sqrt{3}}$.

The acute angle (say) θ between two lines is given by

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| \qquad \dots\text{(iii)}$$

Putting the values of m_1 and m_2 in Eq. (iii), we get

$$\tan\theta = \left|\frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}\right| = \left|\frac{1-3}{2\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$$

which gives $\theta = 30°$. Hence, angle between two lines is either $30°$ or $180° - 30° = 150°$.

33. Let m_1 and m_2 be the slopes of the straight lines $x - 2y + 3 = 0$ and $3x + y - 1 = 0$.

Then, $m_1 = -\frac{1}{-2} = \frac{1}{2}$ and $m_2 = -\frac{3}{1} = -3$

Let θ be the angle between the given lines.

Then,

$$\tan\theta = \pm\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right) = \pm\left(\frac{-3 - \frac{1}{2}}{1 - \frac{3}{2}}\right) = \pm 7$$

$\Rightarrow \quad \theta = \tan^{-1}(7)$ or $\pi - \tan^{-1}(7)$

34. Let the equation of line parallel to the given line is

$$2x - 3y = \lambda \qquad \dots\text{(i)}$$

[$\because$ In two parallel lines, $m_1 = m_2$]

Since, the line (i), passes through the point (4, 3).

So, this point will satisfy the equation of line.

$\therefore \quad 2 \times 4 - 3 \times 3 = \lambda$

$\Rightarrow \quad 8 - 9 = \lambda \Rightarrow \lambda = -1$

Put the value of λ in Eq. (i), we get

$$2x - 3y = -1$$

$\Rightarrow \quad 2x - 3y + 1 = 0$

which is the required equation of line.

35. Slope of the line through the points (– 2, 6) and (4, 8) is

$$m_1 = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$$

Slope of the line through the points (8, 12) and $(x, 24)$ is

$$m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$$

Since, two lines are perpendicular.

So, $\quad m_1 m_2 = -1$

$\Rightarrow \quad \frac{1}{3} \times \frac{12}{x-8} = -1$

$\Rightarrow \quad 8 - x = 4$ or $x = 4$

36. Given line is $3x - 4y - 26 = 0 \qquad \dots\text{(i)}$

On comparing Eq. (i) with general equation of line $Ax + By + C = 0$, we get

$$A = 3, B = -4 \text{ and } C = -26$$

Given point is $(x_1, y_1) = (3, -5)$.

The distance of the given point from given line is

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

$$= \frac{|3 \cdot 3 + (-4)(-5) - 26|}{\sqrt{3^2 + (-4)^2}} = \frac{3}{5}$$

37. Given equation of line is

$$5x + 12y - 13 = 0 \qquad \dots\text{(i)}$$

Length of perpendicular from origin to the line (i) is

$$p = \frac{|-13|}{\sqrt{5^2 + 12^2}}$$

[$\because$ perpendicular length from origin to the line $ax + by + c = 0$ is $p = \frac{|c|}{\sqrt{a^2 + b^2}}$]

$$= \frac{|-13|}{\sqrt{25 + 144}} = \frac{|-13|}{\sqrt{169}} = \frac{13}{13} = 1$$

[$\because |x| = -x, x < 0$]

$\therefore$ Required length of perpendicular is 1 unit.

38. Given equation of lines

$$2x - 3y + 5 = 0 \quad \ldots(\text{i})$$

and $$3x + 4y = 0 \quad \ldots(\text{ii})$$

From Eq. (ii), put the value of $x = \frac{-4y}{3}$ in Eq. (i), we get

$$2\left(\frac{-4y}{3}\right) - 3y + 5 = 0$$

$$\Rightarrow \quad -8y - 9y + 15 = 0 \Rightarrow y = \frac{15}{17}$$

From Eq. (ii), $3x + 4 \cdot \frac{15}{17} = 0$

$$\Rightarrow \quad x = \frac{-60}{17 \cdot 3} = \frac{-20}{17}$$

So, the point of intersection is $\left(\frac{-20}{17}, \frac{15}{17}\right)$.

$\therefore$ Required distance from the line $5x - 2y = 0$ is,

$$d = \frac{\left|-5 \times \frac{20}{17} - 2\left(\frac{15}{17}\right)\right|}{\sqrt{25 + 4}}$$

$$= \frac{\left|\frac{-100}{17} - \frac{30}{17}\right|}{\sqrt{29}} = \frac{130}{17\sqrt{29}}$$

$\left[\because \text{distance of a point } p(x_1, y_1) \text{ from the line } ax + by + c = 0 \text{ is } d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}\right]$

39. Here, $A = 3, B = -4, C_1 = 7$, and $C_2 = 5$.

Therefore, the required distance is

$$d = \frac{|7 - 5|}{\sqrt{3^2 + (-4)^2}} = \frac{2}{5}$$

40. The equations of lines $3x + 4y = 9$ and $6x + 8y = 15$ may be rewritten as

$$3x + 4y - 9 = 0$$

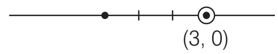
and $$3x + 4y - \frac{15}{2} = 0$$

Since, the slope of these lines are same and hence they are parallel to each other.

Therefore, the distance between them is given by

$$\left|\frac{9 - \frac{15}{2}}{\sqrt{3^2 + 4^2}}\right| = \frac{3}{10}$$

41. **Assertion** The point (3, 0) is at 3 units distance from the Y-axis measured along the positive X-axis and has zero distance from the X-axis.



Hence, Assertion is true but Reason is false.

42. **Assertion** Since, the point (h, k) lies on $x + y + 1 = 0$.

$$\Rightarrow \quad h + k + 1 = 0$$

and $$h^2 + k^2 = 25$$

$$\Rightarrow \quad (-1 - k)^2 + k^2 = 25$$

$$\Rightarrow \quad 2k^2 + 2k - 24 = 0$$

$$\Rightarrow \quad k^2 + k - 12 = 0 \Rightarrow k = -4 \text{ or } k = 3$$

$[k = 3 \text{ rejected as } k < 0]$

$$\therefore \quad h = -1 - (-4) = 3$$

Hence, Assertion is false and Reason is true.

43. **Assertion** Let required ratio be $\lambda : 1$.

Then, the coordinates of point which divides the line joining $(-1, 1)$ and $(5, 7)$ in the ratio $\lambda : 1$, is $\left(\frac{5\lambda - 1}{\lambda + 1}, \frac{7\lambda + 1}{\lambda + 1}\right)$.

But it lies on $x + y = 4$

$$\therefore \frac{5\lambda - 1}{\lambda + 1} + \frac{7\lambda + 1}{\lambda + 1} = 4$$

$$\Rightarrow \quad 12\lambda = 4\lambda + 4 \Rightarrow \lambda = 1/2$$

$\therefore$ Required ratio $= 1 : 2$

Hence, Assertion is true and Reason is false.

44. **Assertion** Centroid of the triangle is

$$G \equiv \left(\frac{1 + 2 + c^2}{3}, \frac{a + b - 3}{3}\right)$$

i.e. $$\left(\frac{3 + c^2}{3}, \frac{a + b - 3}{3}\right)$$

$\because$ G will lie on Y-axis, then

$$\frac{3 + c^2}{3} = 0 \Rightarrow c^2 = -3 \text{ or } c \equiv \pm i\sqrt{3}$$

$\because$ Both values of c are imaginary.

Hence, G cannot lie on Y-axis.

Reason $\because$ G will lies on X-axis, then

$$\frac{a + b - 3}{3} = 0$$

$$\Rightarrow \quad a + b - 3 = 0 \text{ or } a + b = 3$$

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

45. Assertion Area of the triangle, whose vertices are $(4, 4)$, $(3, -2)$ and $(-3, 16)$

$$= \frac{1}{2}|4(-2-16) + 3(16-4) + (-3)(4+2)|$$

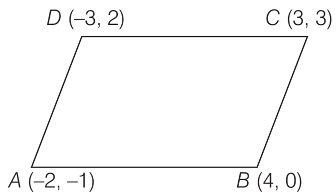
$$= \frac{|-72+36-18|}{2} = \frac{|-54|}{2} = \frac{54}{2} = 27$$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

46. Assertion Slope of X-axis is zero and slope of Y-axis is not defined.

Hence, Assertion is true but Reason is false.

47. Assertion



$\because$ $ABCD$ is a parallelogram.

$\therefore$ $AB \parallel CD \Rightarrow$ Slope of AB = Slope of CD

and $BC \parallel AD \Rightarrow$ Slope of BC = Slope of AD

Reason Mid-point of

$$AC = \left(\frac{-2+3}{2}, \frac{-1+3}{2}\right)$$

$$= \left(\frac{1}{2}, \frac{2}{2}\right) = \left(\frac{1}{2}, 1\right)$$

and mid-point of $BD = \left(\frac{4-3}{2}, \frac{0+2}{2}\right)$

$$= \left(\frac{1}{2}, 1\right)$$

$\Rightarrow$ Mid-point of AC = Mid-point of BD

Hence, Assertion is false and Reason is true.

48. Assertion Let m_1 and m_2 be the slopes of the straight lines $x + 2y - 3 = 0$ and $3x + y + 1 = 0$.

Then, $m_1 = -\frac{1}{2}$ and $m_2 = -3$

Let θ be the angle between the given lines.

Then, $\tan\theta = \pm\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$

$$= \pm\left(\frac{-3+\frac{1}{2}}{1+\frac{3}{2}}\right) = \pm 1$$

$$\Rightarrow \quad \theta = \tan^{-1}(1) \text{ or } \pi - \tan^{-1}(1)$$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

49. Assertion Given equation $3x - 4y + 10 = 0$ can be written as

$$y = \frac{3}{4}x + \frac{5}{2} \qquad \ldots\text{(i)}$$

Comparing Eq. (i) with $y = mx + c$, we have slope of the given line as $m = \frac{3}{4}$.

Reason Equation $3x - 4y + 10 = 0$ can be written as

$$3x - 4y = -10 \quad \text{or} \quad \frac{x}{-\frac{10}{3}} + \frac{y}{\frac{5}{2}} = 1 \qquad \ldots\text{(ii)}$$

Comparing Eq. (ii) with $\frac{x}{a} + \frac{y}{b} = 1$, we have x-intercept as $a = -\frac{10}{3}$ and y-intercept as $b = \frac{5}{2}$.

Hence, Assertion and Reason both are true and Reason is not the correct explanation of Assertion.

50. Assertion Since, slope of line $x\cos\theta + y\sin\theta = 2$ is $-\cot\theta$ and slope of line $x - y = 3$ is 1.

Also, these lines are perpendicular to each other.

$\therefore \quad (-\cot\theta)(1) = -1$

$$\Rightarrow \quad \cot\theta = 1 = \cot\frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}$$

Reason Condition of perpendicularity of two lines is $m_1 \cdot m_2 = -1$.

Hence, Assertion is true and Reason is false.

51. Assertion Given equation is $x + 7y = 0$

$$\Rightarrow \quad y = \frac{-x}{7} + 0$$

On comparing with $y = mx + c$, we get

Slope $(m) = \frac{-1}{7}$, y-intercept $= 0$

Reason Given equation is $6x + 3y - 5 = 0$

$$\Rightarrow \quad y = -2x + \frac{5}{3}$$

On comparing with $y = mx + c$, we get

Slope $(m) = -2$, y-intercept $= \frac{5}{3}$

Hence, Assertion is false and Reason is true.

52. **Assertion** Given equation of line is

$$x - y = 4$$

On dividing above equation by $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$

i.e. $\sqrt{(1)^2 + (-1)^2} = \sqrt{2}$, we get

$$\frac{1}{\sqrt{2}} x - \frac{1}{\sqrt{2}} y = \frac{4}{\sqrt{2}}$$

$\Rightarrow$ $\cos 45° \, x - \sin 45° \, y = 2\sqrt{2}$

[$\because$ cos x is positive and sin x is negative, it is possible only in fourth quadrant]

$\Rightarrow$ $x \cos(360° - 45°) + y \sin(360° - 45°) = 2\sqrt{2}$

$\left[\because \cos(360° - \theta) = \cos\theta \text{ and } \sin(360° - \theta) = -\sin\theta\right]$

$\Rightarrow$ $x \cos 315° + y \sin 315° = 2\sqrt{2}$

On comparing with $x \cos\alpha + y \sin\alpha = p$, we get

$$\alpha = 315°$$

and $p = 2\sqrt{2}$

Hence, Assertion is true but Reason is false.

53. **Assertion** Given lines are $4x + 3y = 11$ and $4x + 3y = 15/2$.

Distance between them

$$= \left| \frac{11 - \frac{15}{2}}{\sqrt{16 + 9}} \right|$$

$$= \left| \frac{7}{2 \times 5} \right| = \frac{7}{10}$$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

54. (i) Slope of line $AB = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{97 - 92}{1995 - 1985}$

$$= \frac{5}{10} = \frac{1}{2}$$

(ii) Equation of line AB is

$$y - y_1 = m(x - x_1)$$

$\therefore$ $y - 92 = \dfrac{1}{2}(x - 1985)$

$$2y - 184 = x - 1985$$

$$x - 2y = 1801$$

(iii) Let the population in year 2010 is P.

Since, A, B, C are collinear

$\therefore$ Slope of AB = slope of BC

$$\frac{97 - 92}{1995 - 1985} = \frac{P - 97}{2010 - 1995}$$

$\Rightarrow$ $\dfrac{1}{2} = \dfrac{P - 97}{15}$

$\Rightarrow$ $7.5 = P - 97$

$\Rightarrow$ $P = 97 + 7.5$

$= 104.5$ crores

(iv) $\because$ Slope of $AB = \dfrac{1}{2}$

Slope of line perpendicular to AB

$$= \frac{-1}{\frac{1}{2}} = -2$$

$\therefore$ Equation of line perpendicular to AB passing through (1995, 97) is

$$y - 97 = -2(x - 1995)$$

$\Rightarrow$ $y - 97 = -2x + 3990$

$\Rightarrow$ $2x + y = 4087$

(v) Equation of line AB is

$$x - 2y = 1801$$

Putting $y = 110$

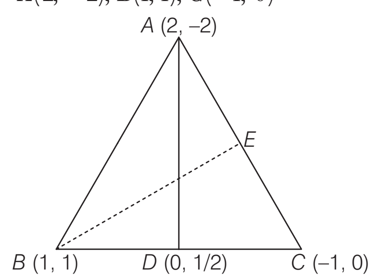
$\therefore$ $x = 1801 + 220$

$\Rightarrow$ $x = 2021$

$\therefore$ Population becomes 110 crores in 2021.

55. Let the point on Rani, Mansi and Sneha stand on a vertices of triangle be A, B, C.

$\therefore$ $A(2, -2)$, $B(1, 1)$, $C(-1, 0)$



(i) The equation of line AB is

$$y-1=\frac{-2-1}{2-1}(x-1)$$

$$\left[\because\ y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)\right]$$

$\Rightarrow \quad y-1=-3x+3 \Rightarrow 3x+y=4$

(ii) Slope of equation of line AC is

$$m=\frac{0+2}{-1-2}=\frac{2}{-3}$$

(iii) Let D be the mid-point of BC.

Coordinates of D are $\left(\frac{1-1}{2}, \frac{0+1}{2}\right)=\left(0, \frac{1}{2}\right)$

$\therefore$ Equation of AD is $y+2=\dfrac{\frac{1}{2}+2}{0-2}(x-2)$

$\Rightarrow \quad y+2=\frac{-5}{4}(x-2)$

$\Rightarrow \quad 4y+8=-5x+10$

$\Rightarrow \quad 5x+4y=2$

(iv) Slope of $AC=\frac{-2}{3}$

$\therefore$ Slope of $BE=\frac{3}{2}$ $\quad [\because BE \perp AC]$

Equation of altitude through B is

$y-1=\frac{3}{2}(x-1) \Rightarrow 3x-2y=1$

(v) Slope of line $BC=\frac{0-1}{-1-1}=\frac{1}{2}$

Equation of line passing through A and parallel to BC is

$$y+2=\frac{1}{2}(x-2)$$

$\Rightarrow \quad 2y+4=x-2$

$\Rightarrow \quad x-2y=6$

56. (i) $AC=\sqrt{(-1-1)^2+(-2-4)^2}$

[using distance formula]

$=\sqrt{(-2)^2+(-6)^2}$

$=\sqrt{4+36}=\sqrt{40}$ units

(ii) Slope of $BC=\frac{y_2-y_1}{x_2-x_1}=\frac{-2-(-3)}{-1-2}$

$=\frac{-2+3}{-3}=-\frac{1}{3}$

(iii) Since D is the mid-point of BC.

$\therefore$ Coordinates of D are $\left(\frac{2-1}{2}, \frac{-3-2}{2}\right)$

$=\left(\frac{1}{2}, -\frac{5}{2}\right)$

$\therefore$ Slope of $AD=\dfrac{-\frac{5}{2}-4}{\frac{1}{2}-1}=\dfrac{-\frac{13}{2}}{-\frac{1}{2}}=13$

$\therefore$ Equation of the median AD is

$$y-4=13(x-1)$$

$\Rightarrow \quad 13x-y-9=0$

(iv) Since AM is the altitude through A.

$\therefore$ Slope of $AM=-\dfrac{1}{\text{slope of } BC}=-\dfrac{1}{-\frac{1}{3}}=3$

$\therefore$ Equation of the altitude through A is given by

$$y-4=3(x-1)$$

$\Rightarrow \quad y-4=3x-3 \Rightarrow 3x-y+1=0$

(v) Equation of the right bisector of BC is a line which passes through D and having slope is 3.

$\therefore \quad y-\left(-\frac{5}{2}\right)=3\left(x-\frac{1}{2}\right)$

$\Rightarrow \quad y+\frac{5}{2}=3x-\frac{3}{2} \Rightarrow 3x-y-4=0$

57. (i) We have the positions,

Shubham $B(1, 2)$ and Rajkumar $D(5, 4)$

Slope, $m_1=\frac{4-2}{5-1}=\frac{2}{4}=\frac{1}{2}$

Taking point $(1, 2)=(x_1, y_1)$ and $m_1=\frac{1}{2}$

Equation of line joining points B and D is

$$(y-y_1)=m_1(x-x_1)$$

$\Rightarrow \quad (y-2)=\frac{1}{2}(x-1)$

$\Rightarrow \quad 2y-4=x-1$

$\Rightarrow \quad x-2y+3=0$

(ii) We have the positions,

Rishabh $A(1, 4)$ and Vikram $C(5, 2)$

Slope, $m_2=\frac{2-4}{5-1}=\frac{-2}{4}=-\frac{1}{2}$

Taking point $(x_1, y_1) = (1, 4)$ and $m_2 = -\frac{1}{2}$

Equation of line is

$$(y-4) = -\frac{1}{2}(x-1)$$

$\Rightarrow \quad 2y - 8 = -x + 1 \Rightarrow x + 2y - 9 = 0$

(iii) We have, $x - 2y + 3 = 0$ ...(i)

and $x + 2y - 9 = 0$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2x - 6 = 0 \Rightarrow x = 3$$

$\therefore \quad 3 + 2y - 9 = 0$ [From Eq. (ii)]

$\Rightarrow \quad 2y = 6 \Rightarrow y = 3$

Hence, point of intersection is (3, 3).

(iv) We have positions,

Rishabh $A(1, 4)$ and Rajkumar $D(5, 4)$

Slope $AD = \frac{4-4}{5-1} = 0$

Hence, slope is zero.

(v) The line formed by Rishabh-Rajkumar is opposite and parallel to the line formed by Shubham and Vikram. Hence, first pair have same slope.

58. (i) Given positions of person A and B are as follows

$A(2, -3)$ and $B(6, -5)$

$$d = \sqrt{(6-2)^2 + (-5+3)^2}$$ [using distance formula]

$$= \sqrt{(4)^2 + (-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

(ii) We have, $A(2, -3)$ and $B(6, -5)$

Slope, $m = \frac{-5-(-3)}{6-2} = \frac{-5+3}{4} = \frac{-2}{4} = -\frac{1}{2}$

Taking point $A(2, -3) = (x_1, y_1)$ and $m = -\frac{1}{2}$

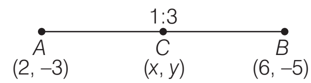
Equation of line AB is

$$(y-(-3)) = -\frac{1}{2}(x-2)$$

$\Rightarrow \quad 2(y+3) = -(x-2)$

$\Rightarrow \quad 2y + 6 = -x + 2 \Rightarrow x + 2y + 4 = 0$

(iii) Let point C divides AB in the ratio m_1 and m_2.



Then, $(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right)$

$$= \left(\frac{1\times 6 + 3\times 2}{1+3}, \frac{1\times(-5) + 3(-3)}{1+3}\right)$$

$$= \left(\frac{12}{4}, \frac{-14}{4}\right) = \left(3, -\frac{7}{2}\right)$$

(iv) We have, $A(2, -3)$ and $C\left(3, -\frac{7}{2}\right)$

$$AC = \sqrt{(3-2)^2 + \left(-\frac{7}{2}+3\right)^2}$$

$$= \sqrt{1^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2}$$

(v) We have, $C\left(3, -\frac{7}{2}\right)$ and $B(6, -5)$

$$CB = \sqrt{(6-3)^2 + \left(-5+\frac{7}{2}\right)^2}$$

$$= \sqrt{3^2 + \left(-\frac{3}{2}\right)^2} = \sqrt{9 + \frac{9}{4}} = \sqrt{\frac{45}{4}} = \frac{3\sqrt{5}}{2}$$