18. Trigonometry

(i) **Measurement of Angles:** Mainly there are three systems for measuring angles.

Sexagesimal System: In this system, angle is measured in degrees.

1 right angle = 90^0, $1^0 = 60'$ and $1' = 60''$.

Centesimal System: In this system angle is measured in grades.

1 right angle = 100^g, $1^g = 100'$ and $1' = 100''$.

Circular Measure: In this system, angle is measured in radius.

The angle subtended at the centre of a circle by an arc of length equal to its radius is 1 radian, written as 1^c. π^c = 180^0 = 200^R = 2 right angles.

(ii) Trigonometric Ratios:

$$\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{PM}{AP}; \cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{AM}{AP};$$

$$\tan\theta = \frac{\text{Perp.}}{\text{Base}} = \frac{PM}{AM};$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta}; \sec\theta = \frac{1}{\cos\theta}; \cot\theta = \frac{\cos\theta}{\sin\theta}.$$

(iii) Identities:

(i) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (ii) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

(iii) $\sin^2\theta + \cos^2\theta = 1$

(iv) $1 + \tan^2\theta = \sec^2\theta$ (v) $1 + \cot^2\theta = \text{cosec}^2\theta$

(iv) Values Of T-Ratios:

Angle	$\sin\theta$	$\cos\theta$	$\tan\theta$
0^0	0	1	0
$30^0 = \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45^0 = \frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60^0 = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90^0 = \frac{\pi}{2}$	1	0	∞
$180^0 = \pi$	0	-1	0

(v) Sign Of T-Ratios:

1st Quadrant: All positive.

2nd Quadrant: $\sin\theta$ & $\operatorname{cosec}\theta$ positive.

3rd Quadrant: $\tan\theta$ & $\cot\theta$ positive.

4^{th} Quadrant: cos θ & sec θ positive.

Remember: I II III IV

All sin tan cos

(vi) Range Of T-Ratios:

(i) $-1 \leq \sin\theta \leq 1$ and $-1 \leq \cos\theta \leq 1$

Thus, $|\sin\theta| \leq 1$ and $|\cos\theta| \leq 1$.

The value of sin θ is never greater than 1 and never less than – 1.

The value of cos θ is never greater than 1 and never less than – 1.

(ii) cosec $\theta \geq 1$ and cosec $\theta \leq -1$.

(iii) sec $\theta \geq 1$ and sec $\theta \leq -1$.

(iv) tan θ may assume any value.

(vii) Increasing & Decreasing Functions:

(i) The value of sin θ increase from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$. As θ increases in this interval, then sin θ also increases.

(ii) 1st Quadrant: sin θ increases from 0 to 1 ; cos θ decreases from 1 to 0 and tan θ increases from 0 to ∞.

(iii) 2nd Quadrant: sin θ decreases from 1 to 0 ; cos θ decreases from 0 to – 1; tan θ decreases from ∞ to 0.

(iv) 3rd Quadrant: sin θ decreases from 0 to – 1 ; cos θ increases from – 1 to 0; tan θ increases from 0 to ∞.

(v) 4th Quadrant: sin θ increases from – 1 to 0; cos θ increases from 0 to 1; tan θ decreases from ∞ to 0.

(viii) T-Ratios Of Negative, Complementary, Supplementary Angles etc.

(i) $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$, $\tan(-\theta) = -\tan\theta$.

(ii) $\sin(90^0 - \theta) = \cos\theta$; $\cos(90^0 - \theta) = \sin\theta$; $\tan(90^0 - \theta) = \cot\theta$.

(iii) $\sin(90^0 + \theta) = \cos\theta$; $\cos(90^0 + \theta) = -\sin\theta$; $\tan(90^0 + \theta) = -\cot\theta$.

(iv) $\sin(180^0 - \theta) = \sin\theta$; $\cos(180^0 - \theta) = -\cos\theta$; $\tan(180^0 - \theta) = -\tan\theta$.

(v) $\sin(180^0 + \theta) = -\sin\theta$; $\cos(180^0 + \theta) = -\cos\theta$; $\tan(180^0 + \theta) = \tan\theta$.

(vi) $\sin(360^0 - \theta) = -\sin\theta$, $\cos(360^0 - \theta) = \cos\theta$; $\tan(360^0 - \theta) = -\tan\theta$.

(vii) $\sin(360^0 + \theta) = \sin\theta$, $\cos(360^0 + \theta) = \cos\theta$, $\tan(360^0 + \theta) = \tan\theta$.

(ix) Sum & Difference Formula:

(i) $\sin (x + y) = \sin x \cos y + \cos x \sin y$.

(ii) $\sin (x - y) = \sin x \cos y - \cos x \sin y$.

(iii) $\cos (x + y) = \cos x \cos y - \sin x \sin y$

(iv) $\cos (x - y) = \cos x \cos y + \sin x \sin y$.

(v) $2 \sin x \cos y = \sin (x + y) + \sin (x - y)$.

(vi) $2 \cos x \sin y = \sin (x + y) - \sin (x - y)$.

(vii) $2 \cos x \cos y = \cos (x + y) + \cos (x - y)$.

(viii) $2 \sin x \sin y = \cos (x - y) - \cos (x + y)$.

(ix) $\sin^2 x - \sin^2 y = \sin (x + y). \sin (x - y)$.

(x) $\cos^2 x - \sin^2 y = \cos (x + y). \cos (x - y)$.

(x) Some More Formulae:

(i) $\sin C + \sin D = 2 \sin \left(\frac{C+D}{2}\right) \cos \left(\frac{C-D}{2}\right)$

(ii) $\sin C - \sin D = 2 \cos \left(\frac{C+D}{2}\right) \sin \left(\frac{C-D}{2}\right)$

(iii) $\cos C + \cos D = 2 \cos \left(\frac{C+D}{2}\right) \cos \left(\frac{C-D}{2}\right)$

(iv) $\cos C - \cos D = - 2 \sin \left(\frac{C+D}{2}\right) \sin \left(\frac{C-D}{2}\right)$

(xi) Tangent Formulae:

(i) $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

(ii) $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

(xii) T-Ratios Of Multiple Angles:

(i) $\sin 2x = 2 \sin x \cos x$

(ii) $\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$.

(iii) $1 + \cos 2x = 2 \cos^2 x$ and $1 - \cos 2x = 2 \sin^2 x$.

(iv) $\sin 3x = 3 \sin x - 4 \sin^3 x$

(v) $\cos 3x = 4 \cos^3 x - 3 \cos x$

(vi) $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

(vii) $\tan 3x = \left(\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} \right)$

(viii) $\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$

(ix) $\sin 2x = \frac{2\tan x}{1 + \tan^2 x}$

(xiii) T-Ratios Of Sub-multiple Angles:

(i) $\sin x = 2 \sin (x/2) \cos (x/2)$

(ii) $\cos x = \cos^2$

$$\frac{x}{2} - \sin^2 \frac{x}{2} = 2\cos^2 \frac{x}{2} - 1 = 1 - 2\sin^2 \frac{x}{2}$$

(iii) $\tan x = \frac{2\tan(x/2)}{1 - \tan^2(x/2)}$

(iv) $1 - \cos x = 2 \sin^2 \frac{x}{2}$ and $1 + \cos x =$

$2 \cos^2 \frac{x}{2}$

(v) $\sin x = \frac{2\tan(x/2)}{1 + \tan^2(x/2)}$

(vi) $\cos x = \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}$

(xiv) T-Ratios Of Some Special Angles:

(i) $\sin 15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^0$

(ii) $\cos 15^0 = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^0$

(iii) $\sin 18^0 = \frac{\sqrt{5}-1}{4} = \cos 72^0$

(iv) $\cos 18^0 = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72^0$

(v) $\sin 36^0 = \frac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54^0$

(vi) $\cos 36^0 = \frac{\sqrt{5}+1}{4} = \sin 54^0$

(vii) $\sin 22\frac{1^0}{2} = \frac{\sqrt{2-\sqrt{2}}}{2}$

(viii) $\cos 22\frac{1^0}{2} = \frac{\sqrt{2+\sqrt{2}}}{2}$