CUET (UG)

Mathematics Sample Paper - 08

Solved

Time Allowed: 50 minutes **Maximum Marks: 200**

General Instructions:

1. There are 50 questions in this paper.
2. Section A has 15 questions. Attempt all of them.
3. Attempt any 25 questions out of 35 from section B.
4. Marking Scheme of the test:

a. Correct answer or the most appropriate answer: Five marks (+5).
b. Any incorrectly marked option will be given minus one mark (-1).
c. Unanswered/Marked for Review will be given zero mark (0).

Section A

1. Let I be an identity matrix, then **[5]**

a) $I = [a_{ij}]_n$, where $a_{ij} = \begin{cases} 0 \text{ if } i = j \\ 1 \text{ if } i \neq j \end{cases}$

b) $I = [a_{ij}]_n$, where $a_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$

c) $I = [a_{ij}]_n$, where $a_{ij} = \begin{cases} 0 \text{ if } i \neq j \\ 1 \text{ if } i \neq j \end{cases}$

d) $I = [a_{ij}]_n$, where $a_{ij} = k \ \forall$ i, j, k $\in R$

2. If A is square matrix, then A is symmetric, if **[5]**

a) $A^2 = A$

b) $A^T = A$

c) $A^T = -A$

d) $A^2 = I$

3. $A = [a_{ij}]_{m \times n}$ is a square matrix, if **[5]**

a) m = n

b) m > n

c) None of these

d) m < n

4. The function $f(x) = x^9 + 3x^7 + 64$ is increasing on **[5]**

a) $(-\infty , 0)$

b) R_0

c) $(0, \infty)$ d) R

5. If m be the slope of a tangent to the curve $e^y = 1 + x^2$ then **[5]**

a) $|m| \leq 1$ b) $|m| < 1$

c) $|m| > 1$ d) $m < 1$

6. The function $f(x) = |x|$ has **[5]**

a) only one maxima b) only one minima

c) no maxima or minima d) none of these

7. $\int \frac{\cot x}{(\,cosec\,x - \cot x)} dx = ?$ **[5]**

a) cosec x + cot x - x + C b) -cosecx + cot x - x + C

c) -cosec x - cot x - x + C d) cosec x - cot x - x + C

8. $\int \frac{x^2}{\left(1 - x^6\right)} dx = ?$ **[5]**

a) $\frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$ b) $\frac{1}{3}\log\left|\frac{1-x^3}{1+x^3}\right| + C$

c) $\frac{1}{6}\log\left|\frac{1-x^3}{1+x^3}\right| + C$ d) None of these

9. The value of $\int_0^{\pi/2} \cos x e^{\sin x}$ dx is **[5]**

a) e - 1 b) 0

c) 1 d) -1

10. The area of the region bounded by the curve $y = x^2$ and the line $y = 16$ **[5]**

a) $\frac{128}{3}$ b) $\frac{64}{3}$

c) $\frac{32}{3}$ d) $\frac{256}{3}$

11. The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is **[5]**

a) $e^x + e^y = \frac{x^3}{3} + c$ b) $e^x - e^y = \frac{x^3}{3} + c$

c) $y = e^{x-y} - x^2 e^{-y} + c$ d) $e^y - e^x = \frac{x^3}{3} + C$

12. The general solution of a differential equation of the type $\frac{dx}{dy} + P_1 x = Q_1$ is **[5]**

a) $xe^{\int P_1 dy} = \int \left(Q_1 e^{\int P_1 dy}\right) dy + C$ b) $ye^{\int P_1 dy} = \int \left(Q_1 e^{\int P_1 dy}\right) dy + C$

c) $y \cdot e^{\int p dx} = \int \left(Q_1 e^{\int P_1 dx}\right) dx + C$ d) $xe^{\int P^1 dx} = \int \left(Q_1 e^{\int P_1 dx}\right) dx + C$

13. By graphical method solution of LLP maximize Z = x + y subject to x + y ≤ 2x; y ≥ 0 obtained at **[5]**

a) at infinite number of points

b) only two points

c) only one point

d) at definite number of points

14. A coin is tossed 10 times. The probability of getting exactly six heads is **[5]**

a) $\frac{100}{153}$

b) $\frac{512}{513}$

c) ${}^{10}C_6$

d) $\frac{105}{512}$

15. If $P(A) = \frac{1}{2}$, $P(B) = 0$, then $P(A|B)$ is **[5]**

a) 0

b) not defined

c) $\frac{1}{2}$

d) 1

Section B

Attempt any 25 questions

16. If f and g are two functions from R to R defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, then fog (x) for $x < 0$ is: **[5]**

a) 2x

b) 4x

c) -4x

d) 0

17. The principal value of $\text{cosec}^{-1}(-1)$ is **[5]**

a) 0

b) $\frac{-\pi}{2}$

c) $\frac{\pi}{2}$

d) $\frac{3\pi}{2}$

18. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $A^4 =$ **[5]**

a) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

19. If the points A(3, -2), B(k, 2) and C(8, 8) are collinear then the value of k is **[5]**

a) -3

b) 2

c) -4

d) 5

20. If A is a 2-rowed square matrix and IAI = 6 then A · adj A = ? **[5]**

a) $\begin{bmatrix} \frac{1}{6} & 0 \\ 0 & \frac{1}{6} \end{bmatrix}$

b) None of these

c) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

21. The system of equations, $3x + y - z = 0$, $5x + 2y - 3z = 2$, $15x + 6y - 9z = 5$ has **[5]**

a) a unique solution

b) two distinct solutions

c) no solution

d) infinitely many solutions

22. At x = 2, f(x) = [x] is **[5]**

a) Continuous but not differentiable

b) None of these

c) Continuous as well as differentiable

d) Differentiable but not continuous

23. If $f(x) = \tan^{-1}x$ and $g(x) = \tan^{-1}\left(\frac{x+1}{1-x}\right)$, then **[5]**

a) $f'(x) = g'(x)$

b) $f(x) = g(x)$

c) None of these

d) $D_f = D_g$

24. If $y = \log\sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is given by **[5]**

a) 0

b) ∞

c) $\frac{1}{2}$

d) 1

25. The set of points where the functions f given by f(x) = |x - 3| cos x is differentiable is **[5]**

a) none of these

b) (0, ∞)

c) R - {3}

d) R

26. The function $f(x) = \frac{x-1}{x\left(x^2-1\right)}$ is discontinuous at **[5]**

a) exactly two points

b) exactly one point

c) exactly three points

d) no point

27. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect at an angle of **[5]**

a) $\frac{\pi}{4}$

b) $\frac{\pi}{6}$

c) $\frac{\pi}{3}$

d) $\frac{\pi}{2}$

28. If the tangent to the curve $x = at^2$, $y = 2at$ is perpendicular to x-axis, then its point of contact is **[5]**

a) (a, a)

b) (a, 0)

c) (0, a)

d) (0, 0)

29. The point on the curve $y = (x - 3)^2$ where the tangent is parallel to the chord joining (3, 0) and (4, 1) is **[5]**

a) $(\frac{-5}{2}, \frac{1}{4})$

b) $(\frac{-7}{2}, \frac{1}{4})$

c) $(\frac{7}{2}, \frac{1}{4})$

d) $(\frac{5}{2}, \frac{1}{4})$

30. The normal to the curve $2y = 3 - x^2$ at (1, 1) is **[5]**

a) $x - y = 0$

b) $- y = 0$

c) $x + y + 1 = 0$

d) $x - y + 1 + 0$

31. $\int \frac{dx}{\left(4x^2 - 4x + 3\right)} = ?$ **[5]**

a) None of these

b) $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right)+C$

c) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right)+C$

d) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right)+C$

32. $\int \frac{\sin^2 x}{\cos^4 x}dx =$ **[5]**

a) $\frac{1}{3}\tan^2 x + c\frac{1}{3}\tan^2 x + c$

b) $\frac{1}{2}\tan^2 x + C$

c) none of these

d) $\frac{1}{3}\tan^3 x + c$

33. $\int_0^{\frac{\pi}{2}} \sin x \sin 2x dx =$ **[5]**

a) $\frac{3}{5}$

b) $\frac{2}{3}$

c) $\frac{3}{4}$

d) $\frac{5}{6}$

34. Let [x] denote the greatest integer less than or equal to x. Then, $\int_{-1}^{1}[x]dx =$? **[5]**

a) 2

b) -1

c) 0

d) $\frac{1}{2}$

35. The area bounded by the curves $y = \sqrt{x}$, $2y + 3 = x$ and the x- axis in the first quadrant is **[5]**

a) 25

b) 9

c) none of these

d) 36

36. Find a solution of $\cos\left(\frac{dy}{dx}\right) = a$ $(a \in R)$ which satisfy the condition y = 1 when x = 0. **[5]**

a) $\cos\frac{y-10}{x} = a$

b) $\cos\frac{y-1}{x} = a$

c) $\cos\frac{y-4}{x} = a$

d) $\cos\frac{y-3}{x} = a$

37. The number of arbitrary constants in the general solution of a differential equation of fourth order are: **[5]**

a) 3

b) 2

c) 1

d) 4

38. The order of the differential equation satisfying $\sqrt{1-x^4} + \sqrt{1-y^4} = a\left(x^2 - y^2\right)$ is **[5]**

a) 4

b) 3

c) 2

d) 1

39. If a vector makes angles α, β and γ with the x axis, y axis and z axis respectviely then the value of $\left(\sin^2\alpha + \sin^2\beta + \sin^2\gamma\right)$ is **[5]**

a) 2

b) 0

c) 3

d) 1

40. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$ then the angle between $\vec{a}$ and $\vec{b}$ is **[5]**

a) $\frac{2\pi}{3}$

b) $\frac{\pi}{3}$

c) $\frac{\pi}{4}$

d) $\frac{\pi}{6}$

41. Find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5). **[5]**

a) $\frac{\sqrt{65}}{3}$

b) $\frac{\sqrt{65}}{2}$

c) $\frac{\sqrt{61}}{3}$

d) $\frac{\sqrt{61}}{2}$

42. The area of the parallelogram whose adjacent sides are $\hat{i} + \hat{k}$ and $2\hat{i} + \hat{j} + \hat{k}$ is **[5]**

a) $\sqrt{2}$

b) 3

c) 4 d) $\sqrt{3}$

43. Find the vector components of the vector with initial point (2, 1) and terminal point (–5, 7). **[5]**

a) $7\hat{i}\ and\ 6\hat{j}$ b) $-7\hat{i}\ and -6\hat{j}$

c) $7\hat{i}\ and -6\hat{j}$ d) $-7\hat{i}\ and\ 6\hat{j}$

44. A vector parallel to the line of intersection of the planes $\vec{r} \cdot (3\hat{i} - \hat{j} + \hat{k}) = 1$ and **[5]**

$\vec{r} \cdot (\hat{i} + 4\hat{j} - 2\hat{k}) = 2$ is

a) $-2\hat{i} - 7\hat{j} + 13\hat{k}$ b) $2\hat{i} + 7\hat{j} + 13\hat{k}$

c) $2\hat{i} + 7\hat{j} - 13\hat{k}$ d) $-2\hat{i} + 7\hat{j} + 13\hat{k}$

45. Find the Cartesian equation of the plane $\vec{r}. \left(2\hat{i} + 3\hat{j} - 4\hat{k}\right) = 1$ **[5]**

a) 2x + 3y – 4 z = 3 b) 2x + 3y – 4 z = 2

c) 2x + 3y – 4 z = 4 d) 2x + 3y – 4 z = 1

46. In vector form, if θ is the angle between the two planes $\vec{r}. \vec{n_1} = d_1$ and $\vec{r} \cdot \vec{n_2} = d_2$, then **[5]**

a) $\theta = \tan^{-1}\frac{\left|\vec{n_1}.\vec{n_2}\right|}{|\vec{n_1}|\left|\vec{n_2}\right|}$

b) $\theta = \cot^{-1}\frac{\left|\vec{n_1}.\vec{n_2}\right|}{|\vec{n_1}|\left|\vec{n_2}\right|}$

c) $\theta = \sin^{-1}\frac{\left|\vec{n_1}.\vec{n_2}\right|}{|\vec{n_1}|\left|\vec{n_2}\right|}$

d) $\theta = \cos^{-1}\frac{\left|\vec{n_1}.\vec{n_2}\right|}{|\vec{n_1}|\left|\vec{n_2}\right|}$

47. If $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then $P(B/A) + P(A/B)$ equals **[5]**

a) $\frac{1}{3}$

b) $\frac{1}{4}$

c) $\frac{5}{12}$

d) $\frac{7}{12}$

48. If the events A and B are independent, then $P(A \cap B)$ is equal to **[5]**

a) $P(A).\ P(B)$

b) $P(A)/P(B)$

c) P(A) − P(B)

d) P(A) + P(B)

49. If P(A) = $\frac{2}{5}$, P(B) = $\frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P\left(\frac{A'}{B'}\right) \cdot P\left(\frac{B'}{A'}\right)$ is equal to **[5]**

a) $\frac{25}{42}$

b) $\frac{5}{6}$

c) $\frac{5}{7}$

d) 1

50. In a binomial distribution, the occurrence and the non-occurrence of an event are equally likely and the mean is 6. The number of trials required is **[5]**

a) 15

b) 6

c) 10

d) 12

Solutions

Section A

1.

(b) I = $[a_{ij}]_n$, where $a_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$

Explanation: By definition of identity matrix

I = $[a_{ij}]_n$, where $a_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$

2.

(b) $A^T = A$

Explanation: Since transpose of a symmetric matrix is equal to the matrix itself so, for a symmetric matrix $A^T = A$

3. **(a)** m = n

Explanation: We know that if a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore, A = $[a_{ij}]_{m \times n}$ is a square matrix if m = n.

4.

(d) R

Explanation: R

5. **(a)** $|m| \leq 1$

Explanation: We have $e^y = 1 + x^2$

$\Rightarrow e^y \frac{dy}{dx} = 2x$

$\Rightarrow \frac{dy}{dx} = \frac{2x}{e^y} = \frac{2x}{1+x^2}$ [$\because$ $e^y = 1 + x^2$]

$\Rightarrow m = \frac{2x}{1+x^2}$ or $|m| = \frac{2|x|}{1+|x|^2}$

As $1 + |x|^2 - 2|x| = (1 - |x|)^2 \geq 0$

$\Rightarrow 1 + |x|^2 \geq 2|x| \Rightarrow 1 \geq \frac{2|x|}{1+|x|^2} = |m|$

$\Rightarrow |m| \leq 1$

6.

(b) only one minima

Explanation: Given, $f(x) = |x| = \begin{cases} -x, & x < 0 \\ x, x > 0 \end{cases}$

$\Rightarrow f'(x) = -1$ when x< 0 and 1 when x > 0

But, we have f'(x) does not exist at x = 0, hence we have x = 0 is a critical point

At x = 0, we get f(0) = 0

For any other value of x, we have f(x) > 0 , hence f(x) has a minimum at x= 0.

7.

(c) -cosec x - cot x - x + C

Explanation: Given : $\int \frac{\cot x}{(cosecx - \cot x)} dx = \int \frac{\cot x(cosecx + \cot x)}{\left((cosecx)^2 - (\cot x)^2\right)} dx$

$= \int \cot x \, cosec\, x + (cot\, x)^2 dx$

$= -cosecx + c_1 + \int (cosec^2 x - 1) dx$

$= -$ cosec x + c_1 - cot x + c_2 - x + c_3

= -cosec x - cot x - x + c ($\therefore$ c = $c_1 + c_2 + c_3$)

8. **(a)** $\frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$

Explanation: $I = \int \frac{x^2}{(1)^2 - \left(x^3\right)^2} dx$

Let $x^3 = t$

$\Rightarrow 3x^2\, dx = dt$

$\Rightarrow x^2 dx = \frac{dt}{3}$

$\therefore I = \frac{1}{3}\int \frac{dt}{1^2 - t^2}$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$= \frac{1}{6}\log\frac{1+t}{1-t} + c$

put $t = x^3$

$= \frac{1}{6}\log\frac{1+x^3}{1-x^3} + c$

9. **(a)** e - 1

Explanation: Let I $= \int_0^{\frac{x}{2}} \cos x e^{\sin x} dx$

Let sin x = t, then cos x dx = dt

When x = 0, t = 0 and x = $\frac{\pi}{2}$, t = 1

Therefore the integral becomes

$I = \int_0^1 e^t dt$

$= \left[e^t\right]_0^1$

= e - 1

10.

(d) $\frac{256}{3}$

Explanation: Since area = $2\int_0^{16} \sqrt{y} dy$, solve the integral to compute the value.

11.

(d) $e^y - e^x = \frac{x^3}{3} + C$

Explanation: We have, $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$\Rightarrow e^y \, dy = (e^x + x^2)dx$

$\Rightarrow \int e^y dy = \int \left(e^x + x^2\right) dx$

$\Rightarrow e^y = e^x + \frac{x^3}{3} + c$

$\Rightarrow e^y - e^x = \frac{x^3}{3} + c$

12. **(a)** $x e^{\int P_1 dy} = \int \left(Q_1 e^{\int P_1 dy}\right) dy + C$

Explanation: The integrating factor of the given differential equation

$\frac{dx}{dy} + P_1 x = Q_1$ is $e^{\int P_1 dy}$

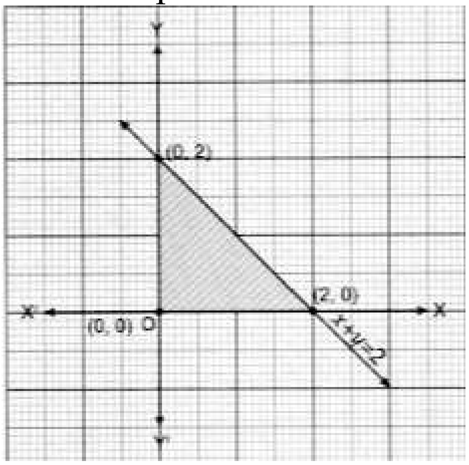
Thus, the general solution of the differential equation is given by,

$x(I.F.) = \int (Q_1 \times I.F.) dy + C$

$\Rightarrow x.\, e^{\int P_1 dy} = \int \left(Q_1 e^{\int P_1 dy}\right) dy + C$

13. **(a)** at infinite number of points

Explanation:



Feasible region is shaded region with corner points (0, 0), (2, 0) and (0, 2)

Z(0, 0) = 0

Z(2, 0) = 2 ⟵ maximise

Z(0, 2) = 2 ⟵ maximise

Z_{max} = 2 obtained at (2, 0) and (0, 2) so is obtained at any point on line segment joining (2, 0) and (0, 2).

14.

(d) $\frac{105}{512}$

Explanation: n = 10, X = 6, p = q = $\frac{1}{2}$

$$P(X = 6) = {}^{10}C_6\left(\frac{1}{2}\right)^{10} = \frac{105}{512}$$

15.

(b) not defined

Explanation: We know that :

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{0}$$

which is not defined

Section B

16.

(c) -4x

Explanation: f(x) = |x| + x

$$f(x) = \begin{cases} 2x & \text{if } x > 0 \\ 0 & \text{if } x \le 0 \end{cases}$$

Similarly,

$$g(x) = \begin{cases} 0 & \text{if } x > 0 \\ -2x & \text{if } x \le 0 \end{cases}$$

for x < 0

fog(X) = f(g(x)) = f(-2x) = -4x

17.

(b) $\frac{-\pi}{2}$

Explanation: Let cosec^{-1} (-1) = α $\Rightarrow$ cosec α = -1 = cosec $(\frac{-\pi}{2})$

$$\Rightarrow \alpha = -\frac{\pi}{2} \in \left|-\frac{\pi}{2}, \frac{\pi}{2}\right| - \{0\}$$

$\therefore$ Principal value of cosec^{-1} (-1) is $\frac{-\pi}{2}$.

18.

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Explanation: In the given question the given matrix is an Identity matrix, and I^n = I.I.I .... .I(n times) = I.

19.

(d) 5

Explanation: As given parts are collinear

So, value of determinant = 0

$$\therefore \begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} \Rightarrow 3(2 - 8) + 2(k - 8) + 1(8k - 16) = 0$$

-18 + 2k - 16 + 8k - 16 = 0

10k - 50 = 0

$\therefore$ k = 5

20.

(d) $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

Explanation: A.(adj A) = |A|I

$$= 6\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

21.

(c) no solution

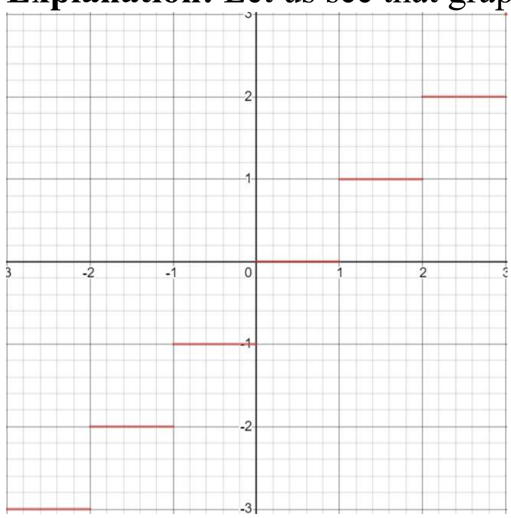
Explanation: The given system of equations does not has a solution if:

$$\begin{vmatrix} 3 & 1 & -1 \\ 5 & 2 & -3 \\ 15 & 6 & -9 \end{vmatrix} = 0 \Rightarrow 3(-18 + 18) - 1(-45 + 45) -1(30 - 30) = 0$$

22.

(b) None of these

Explanation: Let us see that graph of the floor function, we get



We can see that f(x) = [x] is neither continuous and non differentiable at x = 2.

23. **(a)** f ' (x) = g ' (x)

Explanation:

$$g(x) = \tan^{-1}\left(\frac{1+x}{1-x}\right) \Rightarrow g'(x) = \frac{1}{1+\left(\frac{1+x}{1-x}\right)^2}\frac{(1-x).1-(1+x).(-1)}{(1-x)^2} = \frac{1}{(1+x^2)}$$

24.

(d) 1

Explanation: $y = \log\sqrt{\tan x}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{\tan x}} \times \frac{1}{2\sqrt{\tan x}}\sec^2 x$$

$$\frac{dy}{dx} = \frac{\sec^2 x}{2\tan x}$$

$$\left|\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = \frac{\sec^2\frac{\pi}{4}}{2\tan\frac{\pi}{4}} = \frac{2}{2\times 1} = 1$$

25.

(c) R - {3}

Explanation: R - {3}

26. **(a)** exactly two points

Explanation: exactly two points

27.

(d) $\frac{\pi}{2}$

Explanation: Given equation of curves are

$x^3 - 3xy^2 + 2 = 0$ ...(i)

and $3x^2y - y^3 - 2 = 0$ ...(ii)

Differentiating equation (i) w.r.t. x, we get

$$3x^2 - 3x \cdot 2y\frac{dy}{dx} - 3y^2 \cdot 1 + 0 = 0$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_1 = \frac{x^2 - y^2}{2xy}$$

Differentiating equation (ii) w.r.t. x, we get

$$3x^2\frac{dy}{dx} + 6xy - 3y^2\frac{dy}{dx} + 0 = 0$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_2 = \frac{2xy}{y^2 - x^2}$$

Now $\left(\frac{dy}{dx}\right)_1\left(\frac{dy}{dx}\right)_2 = -1$

Hence, the curves intersect at right angle.

28.

(d) (0, 0)

Explanation: $x = at^2$ and $y = 2at$

$$\Rightarrow \frac{dx}{dt} = 2at \text{ and } \frac{dy}{dt} = 2a$$

$$\Rightarrow \frac{dy}{dx} = \frac{2a}{2at} = \frac{2a}{y}$$

Slope of tangent $= \frac{2a}{y}$

Tangent is perpendicular to y-axis.

$\Rightarrow$ Tangent is parallel to x-axis.

Slope of tangent = Slope of x-axis

$$\frac{2a}{y} = 0$$

a = 0

$\Rightarrow$ x = 0 and y = 0

Point is (0, 0)

29.

(c) $(\frac{7}{2}, \frac{1}{4})$

Explanation: Given, $y = (x - 3)^2$

$\Rightarrow \frac{dy}{dx} = 2(x-3)$

Let (α, β) be the required point, then $\beta = (\alpha - 3)^2$...(i)

and $\left(\frac{dy}{dx}\right)_{(\alpha, \beta)}$ = slope of the line joining (3,0) and (4,1)

$\Rightarrow 2(\alpha - 3) = \frac{1-0}{4-3} = 1$

$\Rightarrow \alpha = \frac{7}{2}$

$\Rightarrow (\frac{7}{2} - 3)^2 = \frac{1}{4}$

Hence, the required point is $\left(\frac{7}{2}, \frac{1}{4}\right)$

30. **(a)** x – y = 0

Explanation: $2y = 3 - x^2$

$\Rightarrow 2\frac{dy}{dx} = -2x$

$\Rightarrow \frac{dy}{dx} = -x$

$\Rightarrow \frac{dy}{dx} at (1, 1) = -1$

Slope of tangent = m = -1

Hence, equation of normal is $y - y_1 = \frac{-1}{m}\left(x - x_1\right)$

$\Rightarrow y - 1 = 1(x - 1)$

$\Rightarrow x - y = 0$

31.

(b) $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$

Explanation: Consider $\int \frac{dx}{4x^2 - 4x + 3}$,

Completing the square

$4x^2 - 4x + 3 = 4\left(x^2 - x + \frac{3}{4}\right)$

$= 4\left(x^2 - x + \frac{3}{4} + \frac{1}{4} - \frac{1}{4}\right)$

$= 4\left(\left(x - \frac{1}{2}\right)^2 + \frac{1}{2}\right)$

$= \frac{1}{4}\int \frac{dx}{\left(\left(x - \frac{1}{2}\right)^2 + \frac{1}{2}\right)}$

Let $x - \frac{1}{2} = t$

dx = dt

$\therefore I = \frac{1}{4}\int \frac{dt}{t^2 + \frac{1}{\sqrt{2}}^2}$

We know, $\int \frac{1}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$\Rightarrow I = \frac{\sqrt{2}}{4}\tan^{-1}\frac{t}{\frac{1}{\sqrt{2}}} + c$

$= \frac{1}{2\sqrt{2}}\tan^{-1}\sqrt{2}t + c$

put t = x - 1

$= \frac{1}{2\sqrt{2}}\tan^{-1}\frac{2x - 1}{\sqrt{2}} + c$

32.

(d) $\frac{1}{3}\tan^3 x + c$

Explanation: $I = \int \frac{\sin^2 x}{\cos^4 x} dx$

$I = \int \tan^2 x \sec^2 x dx$

Put tanx $= t \Rightarrow \sec^2 x dx = dt$

Put tan$x = t \rightarrow \sec^2 x dx = dt$

$I = \int t^2 dt$

$I = \frac{t^3}{3} + c$

$I = \frac{\tan^3 x}{3} + c$

33.

(b) $\frac{2}{3}$

Explanation: $I = \int_0^{\frac{\pi}{2}} \sin x(2\sin x\cos x)dx$

$= 2\int_0^{\frac{\pi}{2}} \sin^2 x\cos x dx$

Let, sin x = t

Differentiating both side with respect to t

$\cos x \frac{dx}{dt} = 1$

$\Rightarrow$ cos x dx = dt

At x = 0, t = 0

At $x = \frac{\pi}{2}, t = 1$

$I = 2\int_0^1 t^2 dt$

$= 2\left(\frac{t^3}{3}\right)_0^1$

$= \frac{2}{3}$

34.

(b) -1

Explanation: $\int_{-1}^{1} [x]dx = \int_{-1}^{0} [x]dx + \int_0^1 [x]dx$

$= \int_{-1}^{0} - 1\text{dx} + \int_0^1 0\text{dx}$

$= -1 - 0 + 0$
$= -1$

35.

(b) 9

Explanation: To find area the curves y = $\sqrt{x}$ and x = 2y + 3 and x – axis in the first quadrant., We have ;

$y^2 - 2y - 3 = 0$,(y – 3) (y + 1) = 0 . y = 3 , - 1. In first quadrant , y = 3 and x = 9.
Therefore , required area is ;

$$\int_0^9 \sqrt{x}dx - \int_3^9 \left(\frac{x-3}{2}\right) = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^9 - \frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^9 = 9$$

36.

(b) $cos\frac{y-1}{x} = a$

Explanation: $\frac{dy}{dx} = cos^{-1}a$

$\int dy = cos^{-1}a\int dx$

$y = xcos^{-1}a + c$

When y = 1 , x = 0 , then 1=0 $cos^{-1}a + c \quad c = 1$

$\therefore y = xcos^{-1}a + 1$

$\therefore \frac{y-1}{x} = cos^{-1}a$

37.

(d) 4

Explanation: 4, because the no. of arbitrary constants is equal to order of the differential equation.

38.

(d) 1

Explanation: The differential equation contains only one constant.
Therefore, Order of differential equation is 1.

39. **(a)** 2

Explanation: From the identity, we know that,
If cosα, cosβ and cosγ be the direction cosines of a vector, then,

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Using, $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 + 1 + 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 1 = 3$

On simplifying, we get,

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

40.

(c) $\frac{\pi}{4}$

Explanation: Given $|\vec{a}| = \sqrt{3}$ $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{6}$

let θ be the angle between the vectors $\vec{a}$ *and* $\vec{b}$

Hint

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{\sqrt{6}}{\sqrt{3}\times 2} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$$

41.

(d) $\frac{\sqrt{61}}{2}$

Explanation: Given position vector of A ,$\overrightarrow{OA} = \hat{i} + \hat{j} + 2\hat{k}$ position vector of B, $\overrightarrow{OB} = 2\hat{i} + 3\hat{j} + 5\hat{k}$ and that of C,$\overrightarrow{OC} = \hat{i} + 5\hat{j} + 5\hat{k}$ therefore, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right) - \left(\hat{i} + \hat{j} + 2\hat{k}\right) = \hat{i} + 2\hat{j} + 3\hat{k}$ (by triangle law of vector addition) thus we may write

$\overrightarrow{AB} = \hat{i} + 2\hat{j} + 3\hat{k}, \overrightarrow{AC} = 4\hat{j} + 3\hat{k},$

$$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = -6\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\Rightarrow |\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{61}$$

$$\Rightarrow \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{61}$$

Therefore, the area of triangle ABC is $=\frac{1}{2}\sqrt{61}$

42.

(d) $\sqrt{3}$

Explanation: $\sqrt{3}$ is the correct answer. Area of the parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \hat{b}|$.

43.

(d) $-7\hat{i}$ *and* $6\hat{j}$

Explanation: The scalar and vector components of the vector with initial point (2, 1) and terminal point (– 5, 7) is given by : (- 5 – 2) i.e. – 7 and (7 – 1) i.e. 6. Therefore, the scalar components are – 7 and 6 .,and vector components are - $7\hat{i}$ *and* $6\hat{j}$.

44.

(d) $-2\hat{i} + 7\hat{j} + 13\hat{k}$

Explanation: Let the required vector be a $\hat{i} + b\hat{j} + c\hat{k}$ ...(i)

Since the vector is parallel to the line of intersection of the given planes,

3a - b + c = 0 ...(ii)

a + 4b - 2c = 0 ...(iii)

Solving (i) and (iii), we get

$$\frac{a}{-2} = \frac{b}{7} = \frac{c}{13}$$

Substituting these values in (i), we get

$-2\hat{i} + 7\hat{j} + 13\hat{k}$ which is the required vector.

45.

(d) 2x + 3y – 4 z = 1

Explanation: On putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$,we get:

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(2\hat{i} + 3\hat{j} - 4\hat{k}\right) = 1 \Rightarrow 2x + 3y - 4z = 1$$

46.

(d) $\theta = \cos^{-1}\frac{\left|\vec{n_1}.\vec{n_2}\right|}{|\vec{n_1}|\left|\vec{n_2}\right|}$

Explanation: In vector form, if θ is the angle between the two planes $\vec{r}.\vec{n_1} = d_1$ and

$\vec{r} \cdot \vec{n_2} = d_2$, then, the cosine of the angle between these two lines is given by:

$$\theta = \cos^{-1} \frac{\left|\vec{n_1} . \vec{n_2}\right|}{|\vec{n_1}| \left|\vec{n_2}\right|}.$$

47.

(d) $\frac{7}{12}$

Explanation: Here, $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$

$$P(B/A) + P(A/B) = \frac{P(B \cap A)}{P(A)} + \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A) + P(B) - P(A \cup B)}{P(A)} + \frac{P(A) + P(B) - P(A \cup B)}{P(B)}$$

$$= \frac{\frac{3}{10} + \frac{2}{5} - \frac{3}{5}}{\frac{3}{10}} + \frac{\frac{3}{10} + \frac{2}{5} - \frac{3}{5}}{\frac{2}{5}}$$

$$= \frac{\frac{1}{10}}{\frac{3}{10}} + \frac{\frac{1}{10}}{\frac{2}{5}} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

48. **(a)** $P(A). P(B)$

Explanation: If A and B are independent, then $P(A \cap B) = P(A) \cdot P(B)$

49. **(a)** $\frac{25}{42}$

Explanation: Here, P(A) = $\frac{2}{5}$, P(B) = $\frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$

$$P\left(A'/B'\right) = \frac{P\left(A' \cap B'\right)}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)}$$

$$= \frac{1 - [P(A) + P(B) - P(A \cap B)]}{1 - P(B)}$$

$$= \frac{1 - \left(\frac{2}{5} + \frac{3}{10} - \frac{1}{5}\right)}{1 - \frac{3}{10}}$$

$$= \frac{1 - \left(\frac{4+3-2}{10}\right)}{\frac{7}{10}} = \frac{1 - \frac{1}{2}}{\frac{7}{10}} = \frac{5}{7}$$

And $P\left(B'/A'\right) = \frac{P\left(B' \cap A'\right)}{P\left(A'\right)} = \frac{1 - P(A \cup B)}{1 - P(A)}$

$$= \frac{1 - \frac{1}{2}}{1 - \frac{2}{5}} = \frac{1/2}{3/5} = \frac{5}{6}\left[\because P(A \cup B) = \frac{1}{2}\right]$$

$$\therefore P\left(A'/B'\right) \cdot P\left(B'/A'\right) = \frac{5}{7} \cdot \frac{5}{6} = \frac{25}{42}$$

50.

(d) 12

Explanation: Given, p = q = $\frac{1}{2}$ and mean of Binomial distribution = np = 6

$\Rightarrow$ n $\times \frac{1}{2}$ = 6 $\Rightarrow$ n = 12