JEE MAIN 2024

Sample Paper - 1

Time Allowed: 3 hours **Maximum Marks: 300**

General Instructions:

- All questions are compulsory.
- There are three parts and each part carries 30 questions where the first 20 questions are MCQs and the next 10 questions are numerical.
- Section-A within each part is compulsory. Attempt any 5 questions from section-B within each part.
- You will get 4 marks for each correct response and 1 mark will be deducted for an incorrect answer. However, there is no negative marking for Section-B (Numerical Questions)

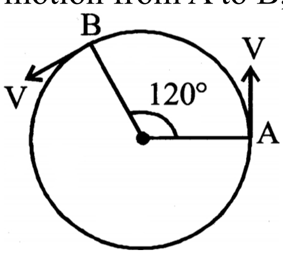
PHYSICS (Section-A)

1. A simple pendulum is being used to determine the value of gravitational acceleration g at a certain place. The length of the pendulum is 25.0 cm and a stop watch with 1s resolution measures the time taken for 40 oscillations to be 50 s. The accuracy in g is: **[4]**

a) 5.40% b) 3.40%

c) 2.40% d) 4.40%

2. As shown in the figure, a particle is moving with constant speed πm/s. Considering its motion from A to B, the magnitude of the average velocity is: **[4]**



a) π m/s b) $2\sqrt{3}$ m/s

c) $\sqrt{3}$ m/s d) $1.5\sqrt{3}$ m/s

3. A ball is projected from the top of a tower at an angle 60° with the vertical. What happens to the vertical component of its velocity? **[4]**

i. Increases continuously

ii. Decreases continuously

iii. Remains unchanged

iv. First decreases and then increases

a) ii and iii
b) i and ii
c) only iv
d) iii and iv

4. The linear velocity of a particle on the equator is nearly: (radius of the earth is 4000 miles). **[4]**

a) zero
b) 10 mile/hr
c) 1000 mile/hr
d) 100 mile/hr

5. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to: **[4]**

a) e^x
b) x^2
c) $\log_e x$
d) x

6. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and body C of mass $\frac{2}{3}M$. The centre of mass of bodies B and C taken together shifts compared to that of body A towards: **[4]**

a) body B
b) does not shift
c) depends on height of breaking
d) body C

7. Two capillary tubes A and B are connected in parallel. A liquid flows through these tubes under the same pressure head. Both the tubes have the same length. The radii of A and B are r and $\frac{r}{2}$ respectively. If the rate of flow of liquid through A is 4 $cm^3\ s^{-1}$, then the rate of flow through the combination is: **[4]**

a) $9.0\ cm^3\ s^{-1}$
b) $4.25\ cm^3\ s^{-1}$
c) $6.50\ cm^3\ s^{-1}$
d) $4\ cm^3\ s^{-1}$

8. A bar of iron is 10 cm at 20^{o} C. At 19^{o} C it will be: (α of iron = 11×10^{-6} /^{o}C) **[4]**

a) 11×10^{-6} cm longer

b) 11×10^{-6} cm shorter

c) 11×10^{-5} cm shorter

d) 11×10^{-5} cm longer

9. Assertion: The melting point of ice decreases with the increase in pressure. **[4]**
Reason: Ice contracts on melting.

a) both Assertion & Reason are true and the reason is the correct explanation of the assertion

b) Assertion is true statement but Reason is false

c) both Assertion and Reason are false statements

d) both Assertion & Reason are true but the reason is not the correct explanation of the assertion

10. A piston of cross-section area A is fitted in cylinder in which gas of volume V at pressure P is enclosed. Gas obeys Boyle's law. What is the angular frequency if piston is displaced slightly? **[4]**

a) $\sqrt{\frac{Ag}{V}}$

b) $\sqrt{\frac{2Ag}{V}}$

c) $\frac{3Ag}{V}$

d) $2\sqrt{\frac{Ag}{V}}$

11. Electric potential at an equatorial point of a small dipole with dipole moment p (r, distance from the dipole) is: **[4]**

a) $\frac{p}{4\pi\varepsilon_0 r^3}$

b) zero

c) $\dfrac{2p}{4\pi\varepsilon_0 r^3}$

d) $\dfrac{p}{4\pi\varepsilon_0 r^2}$

12. An electron is moving with a speed of 10^8 m/s perpendiculars to a uniform magnetic field of intensity B. Suddenly the intensity of the magnetic field is reduced to $\frac{B}{2}$. The radius of the path becomes from the original value of r: **[4]**

i. does not change

ii. reduces to $\frac{r}{2}$

iii. increases to 2r

iv. the electron stops moving

a) iii

b) ii

c) i

d) iv

13. In an experiment with vibration magnetometer the value of $\frac{4\pi^2 I}{T^2}$ for a short bar magnet is observed as 36×10^{-4}. In the experiment with deflection magnetometer with the same magnet the value of $\left(\frac{4\pi d^3}{2\mu_0}\right)$ is observed as $\frac{10^8}{36}$. The magnetic moment of the magnet used, is: **[4]**

a) 1000 A-m

b) 50 A-m

c) 200 A-m

d) 100 A-m

14. A 50 Hz AC current of peak value 2 A flows through one of the pair of coils. If the mutual inductance between the pair of coils is 150 mH, then the peak value of voltage induced in the second coil is: **[4]**

a) 60 πV b) 30 πV

c) 15 πV d) 300 πV

15. A coil has an inductance of 0.7 H and is joined in series with a resistance of 220 Ω. **[4]**

When an alternating emf of 220 V at 50 cps is applied to it, then the wattless component of the current in the circuit is:

a) 5 amper b) 7 ampere

c) 0.7 ampere d) 0.5 ampere

16. If the potential difference across an X-ray tube is 10,000 volt, then the energy of the electrons striking the anticathode is 1.6×10^{-15} J, the minimum wavelength of the continuous X-ray radiation is nearly: **[4]**

a) 1.23×10^{-8} m b) 1.23×10^{-10} m

c) 1.23×10^{-12} m d) 1.23×10^{-14} m

o **[4]**

17. One milliwatt of light of wavelength 4560 A is incident on a cesium surface of work

function 1.9 eV. Given that quantum efficiency of photoelectric emission is 0.5%, Planck's constant h = 6.62×10^{-34} J-sec, velocity of light c = 3×10^{8} m/s, the photoelectric current liberated is

a) 1.856×10^{-5} amp b) 1.856×10^{-6} amp

c) 1.856×10^{-4} amp d) 1.856×10^{-7} amp

18. The energy required to remove the electron from a singly ionized Helium atom is 2.2 times the energy required to remove an electron from Helium atom. The total energy required to ionize the Helium atom completely is: **[4]**

a) 79 eV b) 34 eV

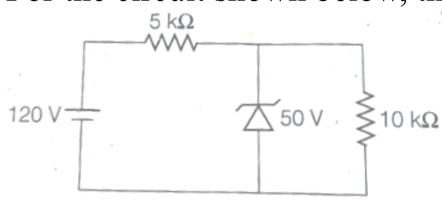
c) 20 eV   d) 109 eV

19. Two nucleons are at a separation of one fermi. Protons have a charge of $+1.6 \times 10^{-19}$ C. The net nuclear force between them is F_1, if both are neutrons, F_2 if both are protons and F_3 if one is proton and the other is neutron. Then: **[4]**

a) $F_1 = F_2 = F_3$ b) $F_1 < F_2 < F_3$

c) $F_1 > F_2 > F_3$ d) $F_1 = F_2 > F_3$

20. For the circuit shown below, the current through the Zener diode is **[4]**



a) 9 mA   b) 14 mA

c) zero   d) 5 mA

PHYSICS (Section-B)

Attempt any 5 questions

21. First, a set of n equal resistors of 10 Ω each are connected in series to a battery of emf 20 V and internal resistance 10 Ω. A current I is observed to flow. Then, the n resistors are connected in parallel to the same battery. It is observed that the current is increased 20 times, then the value of n is ________. **[4]**

22. Two identical conducting spheres with negligible volume have 2.1 nC and -0.1 nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the spheres is ________ $\times 10^{-9}$ N. **[4]**

[Given: $4\pi\varepsilon_0 = \frac{1}{9 \times 10^9}$ SI unit]

23. A coil fo 20 $\times$ 20 cm having 30 turns is making 30 rps in a magnetic field of induction 1 tesla. The peak value of the induced emf is approximately ________ volt. **[4]**

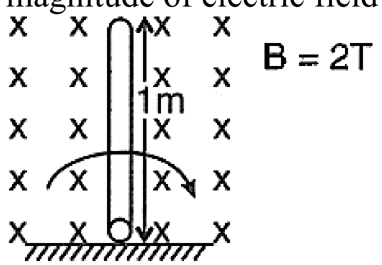
24. A body of mass m is dropped from a height a times the radius of the earth (R) above the surface of the earth. The speed at which the body hits the surface of the earth is given as, **[4]**

$v = \sqrt{2gR}\left(\frac{\alpha+1}{\alpha}\right)^n$. Find the value of n.

25. Two blocks of masses 6 kg and 3 kg are attached to the two ends of a massless spring of spring constant $2\pi^2$ N/m. If the spring is compressed and released on a smooth horizontal surface then the time period (in seconds) of each block. **[4]**

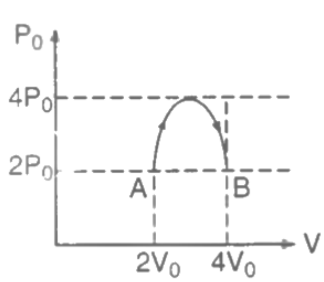
26. A parallel beam of light is allowed to fall on a transparent spherical globe of diameter 30 cm and refractive index 1.5. The distance from the centre of the globe at which the beam of light can converge is ________ mm. **[4]**

27. A rod of length 1 m rotates about one of its end points with an angular velocity 2 rad/sec in a plane perpendicular to the magnetic field B = 2T as shown in the figure. Then find magnitude of electric field (in SI unit) at the mid-point of the rod. **[4]**



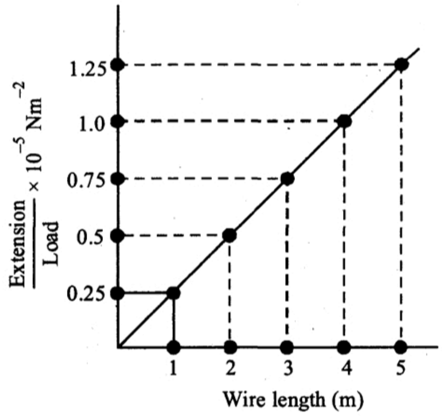
28. Two waves executing simple harmonic motion travelling in the same direction with same amplitude and frequency are superimposed. The resultant amplitude is equal to the $\sqrt{3}$ times of amplitude of individual motions. The phase difference between the two motions is ________ (degree) **[4]**

29. One mole of an ideal monoatomic gas is taken through a thermodynamic process shown in the P - V diagram. The heat supplied to the system in this process is $K \times (\pi + 10)P_0V_0$. The value of K will be ________. **[4]**



30. In an experiment to determine the Young's modulus, steel wires of five different lengths (1, 2, 3, 4 and 5 m) but of same cross-section (2 mm^2) were taken and curves between extension and load were obtained. The slope (extension/load) of the curves were plotted with the wire length and the following graph is obtained. If the Young's modulus of **[4]**

given steel wires is x $\times$ 10^{11} Nm^{-2}, then the value of x is ________.



CHEMISTRY (Section-A)

31. The correct set of four quantum numbers for the valence electron of rubidium atom (Z = 37) is: **[4]**

a) $6, 0, 0, +\frac{1}{2}$

b) $5, 0, 0, +\frac{1}{2}$

c) $5, 1, 1, +\frac{1}{2}$

d) $5, 1, 0, +\frac{1}{2}$

32. Which of the following generally decreases in going down the halogen group? **[4]**

a) Ionisation potential

b) Boiling point

c) Ionic radius

d) Atomic radius

33. Which one of the following diatomic molecular species has only π bonds according of Molecular Orbital Theory? **[4]**

a) O_3

b) N_2

c) Be_2

d) C_2

34. Which of the following is always feasible? **[4]**

a) ΔH (-ve), $T\Delta S$(-ve) and $\Delta H < T\Delta S$

b) $\Delta H = T\Delta S$

c) ΔH (-ve), $T\Delta S$(+ve) and $\Delta H < T\Delta S$

d) ΔH(+ve), $T\Delta S$(-ve) and $\Delta H > T\Delta S$

35. The correct relationship between K_c and K_p in gaseous equilibrium is: **[4]**

a) $\frac{K_p}{RT} = (K_c)^n$

b) $K_p = K_c(RT)^{\Delta n}$

c) $\frac{K_c}{RT} = (K_p)^{\Delta n}$

d) $K_c = K_p(RT)^{\Delta n}$

36. In the reaction, **[4]**
$2CuSO_4 + 4KI \rightarrow Cu_2I_2 + 2K_2SO_4 + I_2$
The ratio of equivalent masses of $CuSO_4$ to its molar mass is:

a) $\frac{1}{2}$

b) 1

c) $\frac{1}{4}$

d) $\frac{1}{8}$

37. Which of the following statement is incorrect regarding the substance known as Plumbago or blacklead? **[4]**

a) It is very soft and slippery

b) It is good conductor of electricity

c) It leaves black mark on paper

d) It’s formula is $PbO.2PbO_2$

38. Ethyl carbocation has ________ hyper conjugative structures. **[4]**

a) five

b) four

c) six

d) three

39. The compound which has one isopropyl group is **[4]**

a) 2,2,3-trimethyl pentane
b) 2,2,3,3-tetramethyl pentane
c) 2,2-dimethyl pentane
d) 2-methyl pentane

40. The boiling point of 0.2 mol kg^{-1} solution of X in water is greater than equimolal solution of Y in water. Which one of the following statements is true in this case? **[4]**

a) Molar mass of X is greater than the molar mass of Y.
b) Molar mass of X is less than the molar mass of Y.
c) X is undergoing dissociation in water.
d) Y is undergoing dissociation in water while X undergoes no change.

41. The number of water molecules present in a drop of water (volume 0.0018 ml) at room temperature is **[4]**

a) 4.84×10^{17}
b) 6.023×10^{19}
c) 1.084×10^{18}
d) 6.023×10^{23}

42. If $E^{\circ}_{Fe^{2+}/Fe} = -0.441V$ and $E^{\circ}_{Fe^{3+}/Fe^{2+}} = 0.771$ V then E^{o} for the reaction: **[4]**
$Fe + 2Fe^{3+} \longrightarrow 3Fe^{2+}$

a) 1.653 V
b) 0.330 V
c) 0.111 V
d) 1.212 V

43. If 50% of a reaction occurs in 100 seconds and 75% of the reaction occurs in 200 seconds, the order of this reaction is: **[4]**

a) 0
b) 2
c) 3
d) 1

44. Which of the following order is correct with respect to dipole moment? **[4]**

a) $CH_3Br > CH_3F > CH_3Cl$
b) $CH_4Br > CH_4F > CH_4Cl$
c) $CH_3Cl > CH_3F > CH_3Br$
d) $CH_3Cl > CH_3Br > CH_3F$

45. Which one of the following reactions of xenon compounds are not feasible? **[4]**

a) $XeO_3 + 6HF \rightarrow XeF_6 + 3H_2O$

b) $2XeF_2 + 2H_2O \rightarrow 2Xe + 4HF + O_2$

c) $XeF_6 + RbF \rightarrow Rb\left[XeF_7\right]$

d) $3XeF_4 + 6H_2O \rightarrow 2Xe + XeO_3 + 12HF + 1.5C$

46. When potassium hexachloridoplatinate (IV) is dissolved in water, the solution: **[4]**

a) contains 6 ions per molecule

b) does not contain any Cl^- ion

c) reacts with $AgNO_3$ to give 6 moles of AgCl

d) contains K^+, Pt^{4+} and Cl^- ions

47. Monochlorination of 2,4-dimethylpentane gives ________ derivatives. **[4]**

a) 3

b) 4

c) 5

d) 2

48. When glycerol is treated with excess of HI, it produces: **[4]**

a) allyl iodide

b) glycerol triiodide

c) propene

d) 2-iodopropane

49. The IUPAC name of the formula $CH_3 - \overset{\overset{CH_3}{|}}{C} = \overset{\overset{H}{|}}{C} - COOH$ is: **[4]**

a) 2-methylbut-2-enoic acid

b) 3-methylbut-3-enoic acid

c) 3-methylbut-2-enoic acid

d) 2-methylbut-3-enoic acid

50. When methylamine reacts with HCl, the product is________. **[4]**

a) methyl ammonium chloride

b) methane and methyl chloride

c) methonoate chloride   d) methylammonia

CHEMISTRY (Section-B)

Attempt any 5 questions

51. The radius of the second Bohr orbit for hydrogen atom is (Planck's constant (h) = 6.6262×10^{-34} Js; mass of electron = 9.1091×10^{-31} kg ; charge of electron (e) = 1.60210×10^{-19} C; permitivity of vacuum(ε_o) = 8.854185×10^{-12} $kg^{-1}m^{-3}A^2$) **[4]**

o

________ A.

52. An aqueous solution of a metal bromide MBr_2 (0.05M) is saturated with H_2S. What is the minimum pH at which MS will precipitate? **[4]**
K_{sp} for MS = 6.0×10^{-21}; concentration of saturated H_2S = 0.1 M
$K_1 = 10^{-7}$ and $K_2 = 1.3 \times 10^{-13}$, for H_2S.

53. Find number of valence electrons of $B_3N_3H_6$ which are NOT involve in π-bond formation. **[4]**

54. From the given compounds if X number of compounds are acidic in water. **[4]**
CaO, SO_2, SO_3, Fe_2O_3, Cl_2O_7, CO_2, Na_2O
Find the value of X.

55. The spin only magnetic moment of $[Mn(H_2O)_6]^{2+}$ complexes is ________ B.M. (Nearest integer) (Given: Atomic no. of Mn is 25) **[4]**

56. In alkaline medium, the reduction of permanganate anion involves a gain of ________ electrons. **[4]**

57. When Fe0.93O is heated in presence of oxygen, it converts to Fe2O3. The number of correct statement/s from the following is ________. **[4]**

A. The equivalent weight of $Fe_{0.93}O$ is $\dfrac{\text{Molecular weight}}{0.79}$

B. The number of moles of Fe^{2+} and Fe^{3+} in 1 mole of $Fe_{0.93}O$ is 0.79 and 0.14 respectively.

C. $Fe_{0.93}O$ is metal deficient with lattice comprising of cubic closed packed arrangement of O^{2-} ions.

D. The % composition of Fe^{2+} and Fe^{3+} in $Fe_{0.93}O$ is 85% and 15% respectively.

58. Total ________ electrons in an atom can have n = 3, l = 1. **[4]**

59. Total number of edges in T.B.P. (trigonal bipyramidal) polyhedron is: **[4]**

60. The internal energy change (in J) when 90 g of water undergoes complete evaporation at 100°C is ________. (Given: ΔH_{vap} for water at 373 K = 41 kJ/mol, R = 8.314 JK^{-1} mol^{-1}) **[4]**

MATHEMATICS (Section-A)

61. Let A = {2, 3, 4} and B = {8, 9,12}. Then the number of elements in the relation R = $\{((a_1, b_1), (a_2, b_2)) \in (A \times B, A \times B): a_1$ divides b_2 and a_2 divides $b_1\}$ is: **[4]**

a) 36 b) 18

c) 24 d) 12

62. If z_1, z_2, z_3 and z_4 are the complex roots of the equation $z^4 + 3z^2 + 1 = 0$, then the value of $\prod_{i=1}^{4}\left(4 + z_i^2\right)$ is: **[4]**

a) 9 b) 25

c) 12 d) 27

63. Number of triangles formed by joining 12 points, no three of which are collinear except 7 of them which are in a straight line, is **[4]**

a) 220 b) 792

c) 185 d) 892

64. If the coefficient of 4^{th} term in the expansion of $(a + b)^n$ is 56, then n is **[4]**

a) 8 b) 6

c) 12 d) 10

65. Suppose a_1, a_2,..., a_n,... be an arithmetic progression of natural numbers. If the ratio of the sum of the first five terms of the sum of first nine terms of the progression is 5 : 17 **[4]**

and $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to-

a) 510 b) 290

c) 380 d) 460

66. $$\lim_{x \to \infty} \frac{(x+1)^{10} + (x+2)^{10} + \dots + (x+100)^{10}}{x^{10} + 10^{10}}$$ is equal to **[4]**

a) 10 b) 1

c) 0 d) 100

67. Let f, g and h are differentiable function such that $g(x) = f(x) - x$ and $h(x) = f(x) - x^3$ are both strictly increasing functions, then the function $F(x) = f(x) - \frac{\sqrt{3}x^2}{2}$ is: **[4]**

a) strictly decreasing on $\left(-\infty, \frac{1}{\sqrt{3}}\right)$ and strictly increasing on $\left(\frac{1}{\sqrt{3}}, \infty\right)$

b) strictly increasing on $\left(-\infty, \frac{1}{\sqrt{3}}\right)$ and strictly decreasing on $\left(\frac{1}{\sqrt{3}}, \infty\right)$

c) strictly increasing $\forall x \in R$

d) strictly decreasing $\forall x \in R$

68. If $I_1 = \int_0^1 (1 - x^{50})^{100}\, dx$ and $I_2 = \int_0^1 (1 - x^{50})^{101}$ such that $I_2 = \alpha I_1$ then α equals to: **[4]**

a) $\frac{5049}{5050}$

b) $\frac{5051}{5050}$

c) $\frac{5050}{5051}$

d) $\frac{5050}{5049}$

69. The two adjacent sides of a parallelogram are given by $2x^2 + 2y^2 - 5xy = 0$ and $5x + 2y = 1$ is one of its diagonal. If area of parallelogram is S, then $\sqrt{S}$ is equal to : **[4]**

a) $\frac{2}{3}$

b) $\frac{1}{2}$

c) $\frac{1}{6}$

d) $\frac{3}{4}$

70. The equation of incircle of $\triangle OAB$, where AB is the intercept of the line $3x + 4y = 12$ between the coordinate axes and O is the origin, is **[4]**

a) $x^2 + y^2 - 6x - 8y - 24 = 0$

b) $x^2 + y^2 - 2x - 2y + 1 = 0$

c) $x^2 + y^2 - 8x - 6y - 24 = 0$

d) $x^2 + y^2 - 4x - 4y - 1 = 0$

71. The normal at point P to a parabola meets the curve again at Q. If O is the vertex, then **[4]**

a) $|OQ| < \sqrt{6}$ (length of the latus rectum)

b) $|OQ| \geq \sqrt{6}$ (length of the latus rectum)

c) $|OQ| < \sqrt{6}$ (length of the semi latus rectum)

d) $|OQ| \geq \sqrt{6}$ (length of the semi latus rectum)

72. The slope of normal at any point (x, y), x > 0, y > 0 on the curve y = y(x) is given by $\dfrac{x^2}{xy-x^2y^2, -1}$. If the curve passes through the point (1, 1), then e.y(e) is equal to **[4]**

a) $\dfrac{1-\tan(1)}{1+\tan(1)}$

b) tan (1)

c) 1

d) $\dfrac{1+\tan(1)}{1-\tan(1)}$

73. If the midpoints of the sides of a triangle are (0, $\frac{3}{2}$, 2), ($\frac{3}{2}$, 0, 2) and $\left(\frac{1}{2}, \frac{1}{2}, 3\right)$, then the centroid of the triangle is **[4]**

a) $\left(\frac{2}{3}, \frac{2}{3}, \frac{-7}{3}\right)$

b) $\left(\frac{1}{3}, \frac{1}{3}, \frac{5}{3}\right)$

c) $\left(\frac{2}{3}, \frac{1}{3}, 2\right)$

d) $\left(\frac{2}{3}, \frac{2}{3}, \frac{7}{3}\right)$

74. Let ABC be a triangle whose circumcentre is at P. If the position vectors of A, B, C and P are $\vec{a}, \vec{b}, \vec{c}$ and $\dfrac{\vec{a}+\vec{b}+\vec{c}}{4}$ respectively, then the position vector of the orthocentre of the triangle is **[4]**

a) $\left(\dfrac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

b) $\vec{a}+\vec{b}+\vec{c}$

c) $-\left(\dfrac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

d) $\vec{o}$

75. The S.D. of a variate x is σ. Then S.D. of the variate $\dfrac{ax+b}{c}$, where a, b, c are constants, is **[4]**

a) $\left(\dfrac{a^2}{c^2}\right)\sigma$

b) $\left|\dfrac{a}{c}\right|\sigma$

c) $\left|\dfrac{a^2}{c^2}\right|\sigma$

d) $\left(\dfrac{a}{c}\right)\sigma$

76. A person correctly recalls all but the last digit of a telephone number, calls from a public booth. He has money only to make two calls. He chooses the forgotten digit randomly. What is the probability that he calls the right person? **[4]**

a) $\dfrac{1}{10}$

b) $\dfrac{3}{10}$

c) $\dfrac{19}{100}$

d) $\dfrac{1}{5}$

77. If $\theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$, then the value of $\sqrt{4\cos^4\theta + \sin^2 2\theta} + 4\cot\theta\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$ is: [4]

a) $2\sin\theta$

b) $-2\cot\theta$

c) $2\cos\theta$

d) $2\cot\theta$

78. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is [4]

a) $\frac{2}{\sqrt{5}}$

b) $\frac{\sqrt{5}}{2}$

c) $\frac{3}{\sqrt{5}}$

d) $\frac{\sqrt{15}}{2}$

79. Let Z be the set of integers. If A = {x $\in$ Z: $2^{(x+2)\left(x^2-5x+6\right)} = 1$} and B = {x $\in$ Z: $-3 < 2x - 1 < 9$}, then the number of subsets of the set A $\times$ B, is: [4]

a) 2^{18}

b) 2^{10}

c) 2^{15}

d) 2^{12}

80. The matrix $\begin{bmatrix} 2 & \lambda & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is non-singular, if [4]

a) $\lambda \neq 3$ b) $\lambda \neq -2$

c) $\lambda \neq 2$ d) $\lambda \neq -3$

MATHEMATICS (Section-B)
Attempt any 5 questions

81. Find the absolute value of k ($k \in R$) for which $f(k) = \int_{-\pi}^{\pi} (x + 1 + k \sin x)^2 \, dx$ is minimum. **[4]**

82. If α and β ($\alpha < \beta$) are the roots of the equation **[4]**

$$\lim_{t \to \infty} \cos^{-1}\left[\sin\left(\tan^{-1}\left(\frac{\sqrt{tx}}{\sqrt{tx^2 - 3tx + t - 1 - x}}\right)\right)\right] = \frac{\pi}{6}$$

then find the value of $\left(8^{\alpha} + 2^{\beta} - \alpha\beta\right)$.

83. Let $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ be vectors such that $\vec{\mathbf{u}} + \vec{\mathbf{v}} + \vec{\mathbf{w}} = \vec{\mathbf{0}}$. If $|\vec{u}| = 3$, $|\vec{v}| = 4$ and $|\vec{\mathbf{w}}| = 5$, then $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} + \vec{\mathbf{v}} \cdot \vec{\mathbf{w}} + \vec{\mathbf{w}} \cdot \vec{\mathbf{u}}$ is ________. **[4]**

84. If $\int_{0}^{1} [4x^3 - f(x)] \, f(x) \, dx = \frac{4}{7}$ then find the area fo region bounded by y = f(x), x-axis and coordinate x = 1 and x = 2. **[4]**

85. Let the co-ordinates of one vertex of ΔABC be A $(0, 2, \alpha)$ and the other two vertices lie on the line $\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. For $\alpha \in Z$ if the area of ΔABC is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to ________. **[4]**

86. Three balls are marked 1, 2 and 3. They are placed in a bowl and a ball is drawn, its number is recorded and the ball is returned to the bowl. The process is replaced two more times. If the sum of the three numbers is 6 then $P = \frac{a}{b}$ is the probability (express in lowest form) that the ball numbered 2 was drawn all the three times, then find the value of (a + b). **[4]**

87. Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in Z : |f(n)| \leq 800\}$. Then, the value of $\sum_{n \in S} f(n)$ is equal to ________. **[4]**

88. If m and n respectively are the numbers of positive and negative value of θ in the interval $[-\pi, \pi]$ that satisfy the equation $\cos 2\theta \cos\frac{\theta}{2} = \cos 3\theta \cos\frac{9\theta}{2}$, then mn is equal to ________. **[4]**

89. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in R$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to ________. **[4]**

90. If $a + \alpha = 1$, $b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$, $x \neq 0$, then the value of expression **[4]**

$$\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$$

is ________.

JEE MAIN 2024

Sample Paper - 1

Solution

PHYSICS (Section-A)

1.

(d) 4.40%

Explanation: Given, Length of simple pendulum, l = 25.0 cm

Time of 40 oscillation, T = 50s

Time period of pendulum,

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow T^2 = \frac{4\pi^2\ell}{g} \Rightarrow g = \frac{4\pi^2\ell}{T^2}$$

$$\Rightarrow \text{Fractional error in g} = \frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$$

$$\Rightarrow \frac{\Delta g}{g} = \left(\frac{0.1}{25.0}\right) + 2\left(\frac{1}{50}\right) = 0.044$$

$$\therefore \text{Percentage error in g} = \frac{\Delta g}{g} \times 100 = 4.4\%$$

2.

(d) $1.5\sqrt{3}$ m/s

Explanation: $|\langle\vec{v}\rangle| = \dfrac{\left|\vec{r}_f - \vec{r}_i\right|}{\Delta t}$

$$= \frac{2R\cos\left[\frac{\pi-\theta}{2}\right]}{\frac{2\pi R}{3v}} = \frac{2R\cos\left[\frac{180-120}{2}\right]}{2\pi R} = 3\cos 30^\circ$$

$= 1.5\sqrt{3}$ m/s

3.

(c) only iv

Explanation: As acceleration due to gravity acts against the motion upto the highest point, hence vertical component of the velocity first decreases. But during downward motion, acceleration due to gravity acts in the direction of motion, hence vertical component of velocity then starts increasing.

4.

(c) 1000 mile/hr

Explanation: $v = R_e\omega = R_e \times \frac{2\pi}{T}$

$= 4000 \text{ miles } \times \frac{2\pi}{24\text{hr}} = 1000\text{mile/hr}.$

5.

(b) x^2

Explanation: As we know that,

$\frac{dv}{dt} = kx$

We can write it like:

$\left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right) = kx$

dv(v) = kx dx

$\int_{v_1}^{v_2} v\, dv = k\int_0^x x\, dx$

$\frac{v_2^2}{2} - \frac{v_1^2}{2} = \frac{kx^2}{2}$

$\frac{mv_2^2}{2} - \frac{mv_1^2}{2} = \frac{mkx^2}{2}$

$(K_2 - K_1) = \frac{mkx^2}{2}$

Loss of kinetic energy is proportional to x^2

6.

(b) does not shift

Explanation: No external horizontal force is applied,

$\therefore\ a_{CM} = 0$

Since, $v_{CM} = 0$, hence $\Delta x_{CM} = 0$

7.

(b) 4.25 $cm^3\ s^{-1}$

Explanation: $V_A = 4\ cm^3\ s^{-1}$

$\therefore\ V_B = \frac{\pi P\left(\frac{r}{2}\right)^4}{8\eta l} = \frac{1}{16} \times\ V_A = \frac{1}{16} \times\ 4 = 0.25\ cm^3\ s^{-1}$

For parallel combination, rate of flow of volume,

$V = V_A + V_B = 4 + 0.25 = 4.25\ cm^3\ s^{-1}$

8.

(c) 11×10^{-5} cm shorter

Explanation: We know that; L - $L_0\{1 + \alpha\Delta\theta)$

$\therefore$ $10 = L_0(1 + 20\alpha)$ and $L' = L_0(1 + 19\alpha)$

$$\therefore \frac{L'}{10} = \frac{1 + 19\alpha}{1 + 20\alpha} = \frac{1 + 19\left(11 \times 10^{-6}\right)}{1 + 20\left(10 \times 10^{-6}\right)}$$

Solving, we get; L' = 9.99989

$\therefore$ L' is shorter by: (10 - 9.99989) = 0.00011 = 11×10^{-5} cm

9. **(a)** both Assertion & Reason are true and the reason is the correct explanation of the assertion

Explanation: both Assertion & Reason are true and the reason is the correct explanation of the assertion.

10. **(a)** $\sqrt{\frac{Ag}{V}}$

Explanation: The bulk modulus for gas is given by

$$B = -\frac{\Delta p}{\frac{\Delta V}{V}}$$

But the gas obeys Boyle's law, so pV = constant (isothermal process). In isothermal process, isothermal bulk modulus of gas is equal to the pressure of the gas at that instant of time

or B = p

$$\therefore p = -\frac{\Delta p}{\frac{\Delta V}{V}} \Rightarrow \Delta p = -\frac{p}{V}\Delta V$$

$$\Rightarrow \frac{F}{A} = -\frac{p}{V}Ax \Rightarrow F = -\frac{pA^2}{V}x$$

This equation is similar to

F = -kx

where, k = force constant of spring

So, $k = \frac{pA^2}{V}$

Hence, angular frequency

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{pA^2}{mV}} = \sqrt{\frac{(mg)A}{mV}} \ldots\left(\because p = \frac{mg}{A}\right)$$

$$\omega = \sqrt{\frac{Ag}{V}}$$

11.

(b) zero

Explanation: The electric potential at the equatorial point of a small dipole moment =

P (at r)
As we have learned,
if r > > 1

$$E_{axi} = \frac{1}{4\pi\epsilon_0}\frac{2P}{r^3}$$

$$V_{axi} = \frac{1}{4\pi\epsilon_0}\frac{P}{r^2}$$

Where the angle between E_{axi} and P will be 0.

12. **(a)** iii

Explanation: reduces to $\frac{r}{2}$

13.

(d) 100 A-m

Explanation: For vibration magnetometer,

$$T^2 = \frac{4\pi^2 I}{MH}$$

Given: $\frac{4\pi^2 I}{T^2} = 36 \times 10^{-4}$ ...(i)

For deflection magnetometer,

$$H = \frac{\mu_0}{4\pi} \times \frac{2M}{d^3}$$

Given: $\frac{4\pi d^3}{2\mu_0} = \frac{10^8}{36}$

From eqns. (i) and (ii)

$$M = \frac{4\pi^2 I}{T^2 H} = \frac{4\pi^2 I}{T^2\left(\frac{2\mu_0}{4\pi d^3}\right)M}$$

$$\text{or } M^2 = \frac{\left(4\pi^2 I/T^2\right)}{\left(2\mu_0/4\pi d^3\right)} = \frac{36 \times 10^{-4}}{\left(36/10^8\right)} = 10^4$$

$\therefore$ M = 100 A-m

14.

(b) 30 πV

Explanation: The current through the coil 1 is

$I_1 = I_0 \sin \omega t$

where I_0 is the peak value of current.

Magnetic flux linked with the coil 2 is

$\phi_2 = MI_1 = MI_0 \sin \omega t$

where M is the mutual inductance between the two coils.

The magnitude of induced emf in coil 2 is

$$|e_2| = \frac{d\phi_2}{dt} = \frac{d}{dt}\left(MI_0 \sin\omega t\right) = MI_0 \omega \cos\omega t$$

$\therefore$ Peak value of voltage induced in the coil 2 is

$$= MI_0\omega = 150 \times 10^{-3} \times 2 \times 2\pi \times 50 = 30\pi V$$

15.

(d) 0.5 ampere

Explanation: Wattless component of AC

$$= I_V \sin\theta = \frac{E_V}{Z}\sin\theta$$

$$= \frac{220}{\sqrt{R^2+\omega^2L^2}} \times \frac{\omega L}{\sqrt{R^2+\omega^2L^2}} = \frac{220 \times \omega L}{\left(R^2+\omega^2L^2\right)}$$

As $\omega L = 0.7 \times 2\pi \times 50$

Hence, wattless component of AC $= \dfrac{220 \times (0.7 \times 2\pi \times 50)}{\left(220^2 + 220^2\right)} = \dfrac{1}{2}$

= 0.5 amp.

16.

(b) 1.23×10^{-10} m

Explanation: 1.23×10^{-10} m

17.

(b) 1.856×10^{-6} amp

Explanation: Let n be the number of photons falling on the surface. Energy of n photons,

$$n\frac{hc}{\lambda} = P$$

$$\therefore \quad n = \frac{P\lambda}{hc} = \frac{10^{-3} \times 4.56 \times 10^{-7}}{6.62 \times 10^{-34} \times 3 \times 10^8}$$

$= 2.35 \times 10^{15}$

As quantum efficiency = 0.5% hence number of electrons liberated from surface,

$$n' = n \times \frac{0.5}{100} = 1.15 \times 10^{13}$$

$$\therefore \quad I = n'e = 1.15 \times 10^{13} \times 1.6 \times 10^{-19}$$

$= 1.84 \times 10^{-6}$ amp

18. **(a)** 79 eV

Explanation: Energy required to remove e^- from singly ionized helium atom =

$$\frac{(13.6)Z^2}{1^2} = 54.4 \text{ eV } (\because \ Z = 2)$$

Energy required to remove e^- from helium atom = x eV

According to question, 54.4 eV = 2.2x $\Rightarrow$ x = 24.73 eV

Therefore, energy required to ionize helium atom

= (54.4 + 24.73) eV = 79.12 eV

19. **(a)** $F_1 = F_2 = F_3$

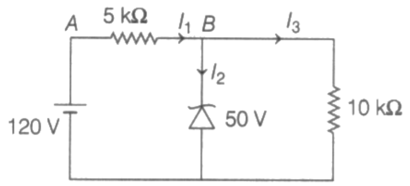
Explanation: The nuclear force is independent of the charge of the nucleons. Thus nuclear force is the same for any pair of two nucleons that are at the same distance apart.

$\therefore \ F_1 = F_2 = F_3$

20. **(a)** 9 mA

Explanation:

In the circuit, let the current in branches is as shown in figure below



By Kirchhoff s node law,

$I_1 = I_2 + I_3$ ...(i)

Now, when diode conducts, voltage difference between points A and B will be

V_{AB} = 120 - 50 = 70 V

So, current $I_1 = \frac{V_{AB}}{5k\Omega} = \frac{70}{5 \times 10^3}$

I_1 = 14 mA ...(ii)

Since, diode and 10 kΩ resistor are in parallel combination, so voltage across 10 kΩ resistor will be 50 V only.

$\Rightarrow \quad I_3 = \frac{50}{10k\Omega} = \frac{50}{10 \times 10^3}$

$\Rightarrow$ I_3 = 5 mA or current through diode, I_2 = 14 mA - 5 mA = 9 mA

PHYSICS (Section-B)

21. 20.0

Explanation:

in series current it will be i_1 will be $i_1 = \frac{20}{10+10n} = \frac{2}{1+n}$

Current in parallel will be $i_2 = \frac{20}{\frac{10}{n}+10} = \frac{2}{1+n}$

$$\frac{i_2}{i_1} = 20 \Rightarrow \frac{\left(\frac{2n}{1+n}\right)}{\left(\frac{2}{1+n}\right)} = 20 \Rightarrow \text{n} = 20$$

22. 36.0

Explanation:

When two spheres charges $q_1' = 2.1$ nC and $q_2' = -0.1$ nC are brought into contact and then separated by a distance r = 0.5 m then,

$$q_1' = q_2' = \frac{Q_1+Q_2}{2} = 1 \text{ nC}$$

Electrostatic force between the two charged sphere,

$$\text{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1' q_2'}{r^2} = 9 \times 10^9 \times \frac{10^{-9} \times 10^{-9}}{(0.5)^2} = 36 \times 10^{-9} \text{ N}$$

$\therefore$ x = 36.

23. 226

Explanation:

$e_0 = NAB\omega$

$= 30 \times 400 \times 10^{-4} \times 1 \times 3 \times 2\pi$

= 226 volt.

24. 0.5

Explanation:

By energy conservation,

Δ K.E. = Δ P.E.

When a particle of mass m is taken from the Earth's surface to a height h = nR, then the change in P.E. can be calculated as,

$$\Delta\text{U} = \text{mgR}\left(\frac{\alpha}{n+1}\right)$$

$$\frac{1}{2}\text{mv}^2 = \text{mgR}\left(\frac{\alpha}{\alpha+1}\right)$$

$$\therefore \text{v}^2 = \frac{2gR\alpha}{\alpha+1}$$

$$\therefore \text{v} = \sqrt{2gR}\left(\frac{\alpha}{\alpha+1}\right)^{\frac{1}{2}}$$

$$\therefore \text{n} = -\frac{1}{2} = -0.5$$

25. 2

Explanation:

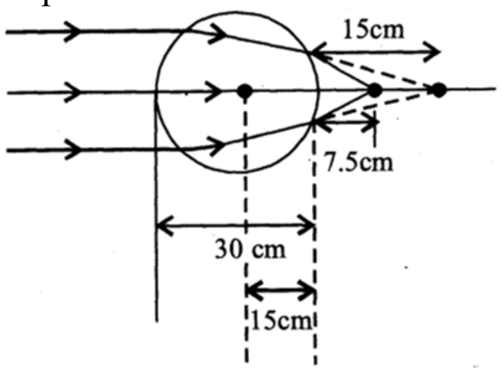
$T = 2\pi\sqrt{\frac{\mu}{K}}$

When μ = reduced mass = $\frac{6\times 3}{6+3}$ = 2 kg

$\therefore$ $T = 2\pi\sqrt{\frac{2}{2\pi^2}}$ = 2 sec.

26. 225.0

Explanation:



At surface 1

$\frac{1.5}{v_1} - \frac{1}{\infty} = \frac{1.5-1}{15}$

$\frac{1.5}{v_1} = \frac{1}{30}$ $\Rightarrow$ v = 45 cm

For surface 2

$\frac{1}{v} - \frac{1.5}{15} = \frac{1-1.5}{-15}$ $\Rightarrow$ $\frac{1}{v} - \frac{1}{10} = \frac{1}{30}$

$\frac{1}{v} = \frac{1}{30} + \frac{1}{10} = \frac{1+3}{30}$

$\therefore$ $v = \frac{30}{4}$ $\Rightarrow$ v = 7.5 cm, So required distance = (15 + 7.5) cm

= 22.5 cm = 225 mm.

27. 2

Explanation:

For a charge to be in equilibrium

qE = qvB or $E = vB = \frac{\omega LB}{2}$

$\therefore$ $E = \frac{2\times 1\times 2}{2}$ = 2 volt/m

28. 60.0

Explanation:

The resultant amplitude is given as,

$A_{\text{resultant}} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$\Rightarrow \sqrt{3}\ A = \sqrt{A^2 + A^2 + 2\ A^2 \cos\phi}$

$\Rightarrow 3\ A^2 = 2\ A^2 + 2\ A^2 \cos\phi \Rightarrow \cos\phi = \frac{1}{2}$

$\therefore \phi = 60^\circ$

$\therefore$ Phase difference = 60 degree

29. 1

Explanation:

$W = \pi\left(P_0 \times V_0\right) + 2V_0 \times 2P_0 = (\pi + 4)P_0V_0$

$\Delta U = nC_v\Delta T = 1 \times \frac{3R}{2}\left(T_f - T_i\right)$

$= \frac{3R}{2}\left[\frac{2P_0 \times 4V_0}{R} - \frac{2P_0 \times 2V_0}{R}\right] = 6P_0V_0$

$\therefore \Delta Q = W + \Delta U = (\pi + 10)P_0V_0$

or $K(\pi + 10)P_0V_0 = (\pi + 10)P_0V_0$ or K = 1

30. 2.0

Explanation:

$\text{Slope} = \frac{\text{Extension / Load}}{\text{Length of wire}} = \frac{\Delta l / w}{L}$

Young's modulus, $Y = \frac{mg/A}{\Delta\ell/L} = \frac{wL}{\Delta\ell A}$

$\therefore Y = \frac{1}{(\text{ slope })A} \Rightarrow Y = \frac{1}{2 \times 10^{-6}\left(0.25 \times 10^{-5}\right)}$

$\Rightarrow Y = 2 \times 10^{11}\ N/m^2$

CHEMISTRY (Section-A)

31.

(b) 5, 0, 0,$+\frac{1}{2}$

Explanation: The configuration of $_{37}Rb$ is :

$1s^2, 2s^2, 2p^6, 3p^6\ 3d^{10}, 4s^2\ 4p^6, 5s^1$

Thus valence electron is present in $5s^1$, i.e.,

$n = 5, l = 0, m = 0, s = +\frac{1}{2}$ or $-\frac{1}{2}$

32. **(a)** Ionisation potential

Explanation: Ionic radii = Increases

Atomic radii = Increases

I.E. = Decreases

33.

(d) C_2

Explanation: Molecular orbital configuration C_2 is:

$\sigma 1s^2, \sigma 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x{}^2 = \pi 2p_y{}^2$

34.

(c) ΔH (-ve), $T\Delta S$(+ve) and $\Delta H < T\Delta S$

Explanation: The feasibility of reaction is determined by free energy change (ΔG) value.

ΔG is given by the Gibbs Helmholtz equation:

$\Delta G = \Delta H - T\Delta S$

The reaction is feasible if ΔG value is negative. Therefore, if ΔH is negative and $T\Delta S$ is positive then the ΔG will always be negative.

35.

(b) $K_p = K_c(RT)^{\Delta n}$

Explanation: $K_p = K_c \times (RT)^{\Delta n}$

where Δn = mole of products - mole of reactants.

36.

(b) 1

Explanation: $2Cu^{2+} + 2e \rightarrow (Cu^+)_2$

$$\therefore E = \frac{M}{1}$$

37.

(d) It's formula is $PbO.2PbO_2$

Explanation: It's formula is $PbO.2PbO_2$

38.

(d) three

Explanation: A number of hyperconjugation structures = **No. of *α*-H atoms** adjacent to sp^2 hybridized C-atom. (Greater the number of hyperconjugation structures, greater will be the stability.)

Ethyl carbocation has 3 hyper conjugative structures as 3 α-H atoms are adjacent to sp^2 hybridized C-atom (i.e., C-atom carrying positive charge)

$$\begin{array}{l} \quad\quad\ {}^{1}H \\ \quad\quad\ |\ \alpha \quad\quad + \\ {}^{2}H - \ C\ \ |_{3}H - CH_2 \end{array}$$

39.

(d) 2-methyl pentane

Explanation:

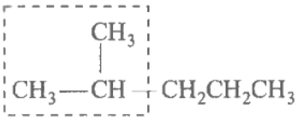
$$\begin{array}{l} \quad\quad\quad CH_3 \;\; CH_3 \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad CH_3 \\ \quad\quad\quad\;\; | \quad\quad\; | \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\; | \\ CH_3 - C - \; C \; - \; C\; H_2 - CH_3 \quad CH_3 - \; C \; - CH_2CH_2CH_3 \\ \quad\quad\quad\; | \quad\quad\; | \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\; | \\ \quad\quad\quad CH_3 \;\; CH_3 \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad CH_3 \end{array}$$

2,2,3,3 - tetramethyl pentane (no isopropyl group)

2,2 - dimethyl pentane (no isopropyl group)

$$\begin{array}{l} \quad\quad CH_3 \;\; CH_3 \\ \quad\quad\;\; | \quad\quad\; | \\ CH_3 - \; C \; - \; C\; H - CH_2CH_3 \\ \quad\quad | \\ \quad\quad CH_3 \end{array}$$

2,2,3 - trimethyl pentane
(no isopropyl group)



isopropyl group
2-methyl pentane

40.

(c) X is undergoing dissociation in water.

Explanation: $\Delta T_b \propto i \times$ molality

$\Delta T_b = K_b \times i \times$ molality

$\because$ molality is the same for two solutions, as well as K_b is the same because the solvent is the same

$\therefore \quad \Delta T_b \propto i$

Given $\left(\Delta T_b\right)_x > \left(\Delta T_b\right)_y$

$\therefore$ $(i)_x > (i)_y$

As per the choices given it is possible only when x dissociates. So, x is undergoing dissociation in water and y shows no change.

41.

(b) 6.023×10^{19}

Explanation: $Density = \frac{\text{Mass}}{\text{Volume}}$; $1 = \frac{g}{ml}$ or g = ml

0.0018ml = 0.0018gm

No. of moles $= \frac{\text{weight}}{\text{Molecular weight}} = \frac{0.0018}{18} = 1 \times 10^{-4}$

$\therefore$ No. of water molecules $= 6.023 \times 10^{23} \times 1 \times 10^{-4}$

$= 6.023 \times 10^{19}$

42.

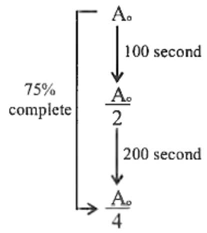
(d) 1.212 V

Explanation: 1.212 V

43.

(d) 1

Explanation:



The first-order reaction as the half-life is constant.

44.

(c) $CH_3Cl > CH_3F > CH_3Br$

Explanation: $CH_3Cl > CH_3F > CH_3Br$

Dipole moment in CH_3F is less than CH_3Cl as bond length is shorter in CH_3F and the other, it's all about electronegativity.

45. **(a)** $XeO_3 + 6HF \rightarrow XeF_6 + 3H_2O$

Explanation: XeF_6 has much tendency to hydrolyze. The reverse reaction is more spontaneous.

$XeF_6 + 3H_2O \rightarrow XeO_3 + 6HF$

46.

(b) does not contain any Cl^- ion

Explanation: does not contain any Cl^- ion

47. **(a)** 3

Explanation: $\underset{2,4-Dimethylpentane}{CH_3-\overset{\overset{CH_3}{|}}{C}H-CH_2-\overset{\overset{CH_3}{|}}{C}H-CH_3} + Cl_2 \xrightarrow{hv}$

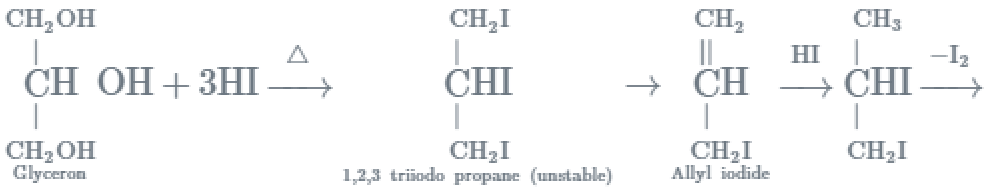
$$\underset{1-Chloro-2,4-dimethylpentane}{Cl-CH_2-\overset{\overset{CH_3}{|}}{C}H-CH_2-\overset{\overset{CH_3}{|}}{C}H-CH_3} + \underset{\text{2 - Chloro - 2,4 - dimethylpentane}}{CH_3-\underset{\underset{Cl}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH_2-\overset{\overset{CH_3}{|}}{C}H-CH_3} +$$

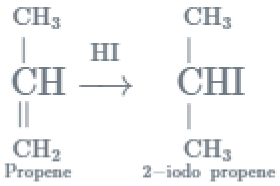
$$\underset{3-Chloro-2,4-dimethylpentane}{CH_3-\overset{\overset{CH_3}{|}}{C}H-\underset{\underset{Cl}{|}}{C}H-\overset{\overset{CH_3}{|}}{C}H-CH_3}$$

48.

(d) 2-iodopropane

Explanation:





49.

(c) 3-methylbut-2-enoic acid

Explanation: 3-methylbut-2-enoic acid

50. **(a)** methyl ammonium chloride

Explanation: Due to the presence of lone pair on nitrogen, methylamine acts as a Lewis base and reacts with HCl, H^+ ion from HCl forms an adduct (salt) methyl ammonium chloride, $CH_3NH_3{}^+Cl^-$.

CHEMISTRY (Section-B)

51. 2.12

Explanation:

Bohr radius (r_n) = $\in_0 n^2 h^2$

$$r_n = \frac{n^2h^2}{4\pi^2 me^2 kZ}$$

$$k = \frac{1}{4\pi\epsilon_0}$$

$$\therefore \quad r_n = \frac{n^2h^2\epsilon_0}{\pi me^2 Z} = n^2\frac{a_0}{Z}$$

where, m = mass of electron

e = charge of electron

h = Planck's constant

k = Coulomb constant

$$r_n = \frac{n^2 \times 0.53}{z} \overset{o}{A}$$

Radius of n^{th} Bohr orbit for H-atom

$= 0.53\ n^2\ \overset{o}{A}$ [Z = 1 for H-atom]

$\therefore$ Radius of 2^{nd} Bohr orbit for H-atom

$= 0.53 \times (2)^2 = 2.12\ \overset{o}{A}$

52. 0.983

Explanation:

$H_2S \rightleftharpoons H^+ + HS^-$

$$\therefore K_1 = \frac{[H^+][HS^-]}{[H_2S]}$$

Further, $HS^- \rightleftharpoons H^+ + S^{2-}$

$$\therefore K_2 = \frac{[H^+][S^{2-}]}{[HS^-]}$$

Dissociation constant of H_2S, $K = K_1 \times K_2$

i.e., $K = 1 \times 10^{-7} \times 1.3 \times 10^{-13} = 1.3 \times 10^{-20}$

Now we know that

$$K_{sp} = [M^{2+}][S^{2-}] \Rightarrow 6 \times 10^{-21} = 0.05 \times [S^{2-}]$$

$$[S^{2-}] = \frac{6 \times 10^{-21}}{0.05} = 1.2 \times 10^{-19}$$

Substituting the various values in the following relation

$$K = \frac{[H^+]^2[S^{2-}]}{[H_2S]}$$

$$1.3 \times 10^{-20} = \frac{[H^+]^2[1.2 \times 10^{-19}]}{0.1} = 1.04 \times 10^{-1}$$

$pH = -\log[H^+]$; $pH = -\log(1.04 \times 10^{-1})$

$= 1.0 - \log 1.04 = 1.0 - 0.017 = 0.983$

53. 24

Explanation:

24

54. 4

Explanation:

Non-metallic oxides produce acidic solution when dissolve in water.

55. 6.0

Explanation:

$[Mn(H_2O)_6]^{2+}$, $Mn^{2+} \Rightarrow 3d^5$

$\mu = \sqrt{5(5+2)} = 5.91$ BM

56. 3.0

Explanation:

In a faintly alkaline medium,

$$\overset{+7}{Mn}O_4^- + 3e^- + 2H_2O \rightarrow \overset{+4}{Mn}O_2 + 4OH^-$$

No. of electrons gained = 3

57. 4

Explanation:

A : $Fe_{0.93}O \rightarrow Fe_2O_3$

$$nf = \left(3 - \frac{200}{93}\right) \times 0.93$$

ng = 0.79

B : $2x + (0.93 - x) \times 3 = 2$

x = 0.79

$Fe^{2+} = 0.79, Fe^{3+} = 0.21$

$$D : \%Fe^{2+} = \frac{0.79}{0.93} \times 100 = 85\%; Fe^{3+} = 15\%$$

58. 6

Explanation:

Principal quantum number n = 3 - M shell

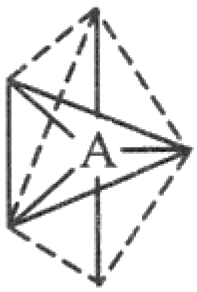
Azimuthal quantum number is l = 1-P subshell

P subshell has 3 orbitals. Each can hold two electrons.

Hence, the total 6 electrons can fit for n=3 and l=1

59. 9

Explanation:



Nine edges are present in PCI_5.

60. 189494

Explanation:

$\Delta H = \Delta U + \Delta n_g RT$

$$n = \frac{90}{18} = 5 \text{ mol}$$

$H_2O(l) \rightleftharpoons H_2O(g)$

$\Delta n = 1$

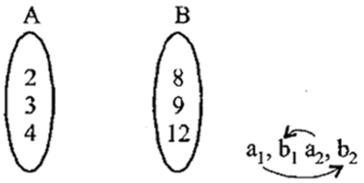
$41000 = \Delta U + 1 \times 8.314 \times 373$

$\Rightarrow \Delta U = 37898.875$ J

For 5 moles, $\Delta U = 37898.87 \times 5 = 189494$ J

MATHEMATICS (Section-A)

61. **(a)** 36

Explanation: 

a_1 divides b_2

Each elements has 2 choices $\Rightarrow$ $3 \times 2 = 6$

a_2 divides b_1

Each elements has 2 choices $\Rightarrow$ $3 \times 2 = 6$

Total = $6 \times 6 = 36$

62.

(b) 25

Explanation: 25

63.

(c) 185

Explanation: Any three points, not in a line, make a triangle. If all the 12 points were such that no three of them are in the same straight line, the number of triangles would have been ${}^{12}C_3$; but 7 points which are in a line make no triangle (instead of ${}^{7}C_3$)

$\Rightarrow$ The required number = ${}^{12}C_3 - {}^{7}C_3$

$= \dfrac{12!}{3!9!} - \dfrac{7!}{3!4!}$

= 220 - 35 = 185

64. **(a)** 8

Explanation: $T_4 = T_{3+1} = {}^{n}C_3\, a^{n-3}\, b^3$

According to the given condition,

${}^{n}C_3 = 56$

$\Rightarrow \dfrac{n!}{3!(n-3)!} = 56$

$\Rightarrow$ $n(n - 1)(n - 2) = 56 \times 6$

$\Rightarrow$ $n(n - 1)(n - 2) = 8 \times 7 \times 6$

$\Rightarrow$ $n = 8$

65.

(c) 380

Explanation: $\dfrac{S_5}{S_9} = \dfrac{5}{17} \Rightarrow \dfrac{\frac{5}{2}(2a+4d)}{\frac{9}{2}(2a+8d)} = \dfrac{5}{17}$

$34a + 68d = 18a + 72d \Rightarrow d = 4a$

$a_{15} = a + 14d = 57a$; Now, $10 < a_{15} < 120$ $\therefore$ $a_{15} = a + 14d$

$\Rightarrow$ $110 < 57a < 120$ $\Rightarrow$ $a = 2$ $\therefore$ $d = 8$

$S_{10} = \dfrac{10}{2}(2 \times 2 + 9 \times 8) = 380$

66.

(d) 100

Explanation: $\lim\limits_{x \to \infty} \dfrac{(x+1)^{10} + (x+2)^{10} + \ldots + (x+100)^{10}}{x^{10} + 10^{10}}$

$$= \lim_{x \to \infty} \frac{x^{10}\left[\left(1+\frac{1}{x}\right)^{10} + \left(1+\frac{2}{x}\right)^{10} + \ldots + \left(1+\frac{100}{x}\right)^{10}\right]}{x^{10}\left(1+\frac{10^{10}}{x^{10}}\right)}$$

= 100

67.

(c) strictly increasing $\forall$ x $\in$ R

Explanation: Assume f(x) = x^3 + x so that g(x) = x^3; h(x) = x

and F(x) = $x^3 - \dfrac{\sqrt{3}x^2}{2} + x$

$F'(x) = 3x^2 - \sqrt{3}x + 1$

D = 3 - 12 < 0

$F'(x) > 0$ $\forall$ x $\in$ R

$\Rightarrow$ F is increasing $\Rightarrow$ (A)

68.

(c) $\dfrac{5050}{5051}$

Explanation: $I_2 = \int_0^1 (1 - x^{50})^{100}\, dx = \int_0^1 (1 - x^{50})(1 - x)^{100}\, dx$

$I_2 = \int_0^1 (1 - x^{50})^{100}\, dx \int_0^1 xI\, x^{49}\left(1 - x^{50}\right)^{100} dx$

II

$I_2 = I_1 + 0 - \dfrac{I_2}{5050}$

$\Rightarrow \dfrac{5051}{5050} I_2 = I_1 \Rightarrow I_2 = \dfrac{5050}{5051} I_1$

$\Rightarrow \alpha = \dfrac{5050}{5051}$

69.

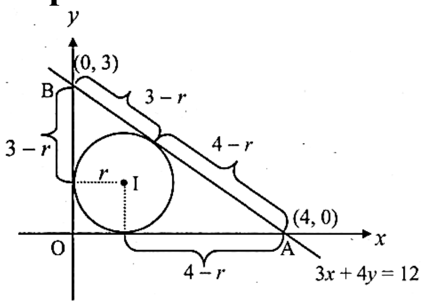
(c) $\dfrac{1}{6}$

Explanation: $\frac{1}{6}$

70.

(b) $x^2 + y^2 - 2x - 2y + 1 = 0$

Explanation:



$|AB| = 5 \Rightarrow (4 - r) + (3 - r) = 5$

$\Rightarrow r = 1$

$I = (1, 1)$

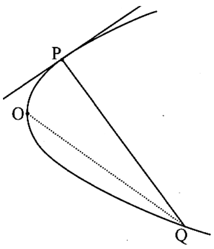
Equation of the incircle is

$(x - 1)^2 + (y - 1)^2 = 1$

$\Leftrightarrow x^2 + y^2 - 2x - 2y + 1 = 0$

71.

(b) $|OQ| \geq \sqrt{6}$ (length of the latus rectum)

Explanation: 

Let $P = (at_1^2, 2at_1)$. Then $Q = (at_2^2, 2at_2)$

where $t_2 = -t_1 - \frac{2}{t_1}$

$\Leftrightarrow t_1^2 + t_1t_2 + 2 = 0$ (A quadratic equation in t_1)

$\Delta \geq 0 \Leftrightarrow t_2^2 - 8 \geq 0$

$\Leftrightarrow t_2^2 \geq 8$ ...(i)

$|OQ|^2 = (at_2^2 - 0)^2 + (2at^2 - 0)^2$

$= a^2t_2^2\left(t_2^2 + 4\right)$ ...(ii)

(i) and (ii) $\Rightarrow |OQ|^2 \geq a^2 (8)(12) (=96a^2)$

$\Rightarrow |OQ| \geq 4\sqrt{6}a$

$\Leftrightarrow |OQ| \geq \sqrt{6}$ (length of the latus rectum)

72.

(d) $\frac{1 + \tan(1)}{1 - \tan(1)}$

Explanation: Given slope of normal

$$\frac{-dx}{dy} = \frac{x^2}{xy - x^2y^2 - 1}$$

x^2y^2 dx + dx - xy dx = x^2 dy; x^2y^2 dx + dx = x^2 dy + xy dx

x^2y^2 dx + dx = x(x dy + y dx)

x^2y^2 dx + dx = xd (xy); $\frac{dx}{x} = \frac{d(xy)}{1+x^2y^2}$

Take integral both sides,

ln kx = $\tan^{-1}$(xy) ...(i)

Satisfy y(1) = 1 for x = 1. ln k = $\frac{\pi}{2} \Rightarrow$ k = $e^{\frac{\pi}{4}}$

Put the value of link in eq. (i),

$\frac{\pi}{4}$ + ln x = $\tan^{-1}$(xy) $\Rightarrow$ xy = $\tan\left(\frac{\pi}{4} + \ln x\right)$

$\Rightarrow$ xy = $\left(\frac{1+\tan(\ln x)}{1-\tan(\ln x)}\right)$ ...(ii)

Satisfy x = e in (ii)

Therefore, ey(e) = $\frac{1+\tan 1}{1-\tan 1}$

73.

(d) $\left(\frac{2}{3}, \frac{2}{3}, \frac{7}{3}\right)$

Explanation: Let A(x_1, y_4, z_1), B(x_2, y_2, z_2) and C(x_3, y_3, z_3) be the vertices of the given triangle

Let P = (0, $\frac{3}{2}$, 2), Q = ($\frac{3}{2}$, 0, 2), R = $\left(\frac{1}{2}, \frac{1}{2}, 3\right)$ be the midpoints of AB, BC and CA respectively. Then

$\frac{x_1+x_2}{2} = 0, \frac{y_1+y_2}{2} = \frac{3}{2}, \frac{z_1+z_2}{2} = 2$

$\Rightarrow$ $x_1 + x_2 = 0$, $y_1 + y_2 = 3$, $z_1 + z_2 = 4$ ...(i)

$\frac{x_2+x_3}{2} = \frac{3}{2}, \frac{y_2+y_3}{2} = 0, \frac{z_2+z_3}{2} = 2$

$\Rightarrow$ $x_2 + x_3 = 3$, $y_2 + y_3 = 0$, $z_2 + z_3 = 4$ ...(ii)

and $\frac{x_1+x_3}{2} = \frac{1}{2}, \frac{y_1+y_3}{2} = \frac{1}{2}, \frac{z_1+z_3}{2} = 3$

$\Rightarrow$ $x_1 + x_3 = 1$, $y_1 + y_3 = 1$, $z_1 + z_3 = 6$ ...(iii)

Adding (i), (ii) and (iii), we get

$x_1 + x_2 + x_3 = 2, y_1 + y_2 + y_3 = 2, z_1 + z_2 + z_{.3} = 7$

$\Rightarrow$ Centroid $\equiv \left(\frac{2}{3}, \frac{2}{3}, \frac{7}{3}\right)$

74. **(a)** $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

Explanation: Let $\vec{r}$ be the position vector of the orthocentre of the triangle and $\vec{p}$ be the position vector of circumcentre.

Centroid $(\vec{g}) = \frac{2\vec{p}+\vec{r}}{3}$

$\Rightarrow \vec{r} = 3\vec{g} - 2\vec{p} = 3\left(\frac{\bar{a}+\vec{b}+\vec{c}}{3}\right) - 2\left(\frac{\vec{a}+\vec{b}+\vec{c}}{4}\right)$

$\Rightarrow \vec{r} = \frac{\vec{a}+\vec{b}+\vec{c}}{2}$

75.

(b) $\left|\frac{a}{c}\right|\sigma$

Explanation: Let y = $\frac{ax+b}{c}$ i.e, y = $\frac{a}{c}$x + $\frac{b}{c}$

i.e., y = Ax + B, where A = $\frac{a}{c}$, B = $\frac{b}{c}$

$\therefore \bar{y} = A\bar{x} + B$

$\therefore$ y - $\bar{y}$ = A(x - $\bar{x}$)

$\Rightarrow (y - \bar{y})^2 = A^2 (x - \bar{x})^2$

$\Rightarrow \sum (y - \bar{y})^2 = A^2 \sum (x - \bar{x})^2$

$\Rightarrow n \cdot \sigma_y^2 = A^2 \cdot n\sigma_x^2 \Rightarrow \sigma_y^2 = A^2\sigma_x^2$

$\Rightarrow \sigma_y = |A|\, \sigma_x$

$\Rightarrow \sigma_y = \left|\frac{a}{c}\right|\sigma_x$

Thus, new S.D. = $\left|\frac{a}{c}\right|\sigma$

76.

(d) $\frac{1}{5}$

Explanation: Case I: He chooses the correct digit in 1st attempt only

Case II: He chooses a wrong digit in 1st attempt and correct digit in the 2nd attempt

Required probability = $\frac{1}{10} + \frac{9}{10} \times \frac{1}{9}$ (He avoids picking up the wrong digit in 2nd attempt)

$= \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$

77.

(d) $2\cot\theta$

Explanation: $\sqrt{4\cos^4\theta + \sin^2 2\theta} + 4\cot\theta\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

$= \sqrt{4\cos^4\theta + 4\sin^2\theta\cos^2\theta} + 4\cot\theta\left[\frac{1 + \cos 2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{2}\right]$

$= \sqrt{4\cos^2\theta\left(\cos^2\theta + \sin^2\theta\right)} + 2\cot\theta[1 + \cos\left(\frac{\pi}{2} - \theta\right)]$

$= |2\cos\theta| + 2\cot\theta + 2\cos\theta \ldots\left[\because \theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)\right]$

$= 2\cot\theta$

78. **(a)** $\frac{2}{\sqrt{5}}$

Explanation: Given equation of hyperbola, is $\frac{x^2}{24} - \frac{y^2}{18} = 1$ ...(i)

Since, the equation of the normals of slope m to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, are given

by y = $mx \mp \frac{m\left(a^2 + b^2\right)}{\sqrt{a^2 - b^2m^2}}$

$\therefore$ Equation of normals of slope m, to the hyperbola (i), are

y = $mx \pm \frac{m(24 + 18)}{\sqrt{24 - m^2(18)}}$ ...(ii)

$\because$ Line y = mx $\pm$ $7\sqrt{3}$ is normal to hyperbola (i)

$\therefore$ On comparing with Eq. (ii) we get

$\pm\frac{m(42)}{\sqrt{24 - 18m^2}} = 7\sqrt{3}$

$$\Rightarrow \pm \frac{6m}{\sqrt{24-18m^2}} = \sqrt{3}$$

$$\Rightarrow \frac{36m^2}{24-18m^2} = 3 \text{ [squaring both sides]}$$

$\Rightarrow$ $12m^2 = 24 - 18m$

$\Rightarrow$ $30m^2 = 24$

$\Rightarrow$ $5m^2 = 4$ $\Rightarrow$ $m = \pm \frac{2}{\sqrt{5}}$

79.

(c) 2^{15}

Explanation: Let x $\in$ A, then

$\because$ $2^{(x+2)\left(x^2-5x+6\right)} = 1$

$\Rightarrow$ (x + 2)(x - 2)(x - 3) = 0

x = -2, 2, 3

A = {-2, 2, 3}

Then, n(A) = 3

Let x $\in$ B, then

-3 < 2x -1 < 9

-1 < x < 5 an d x $\in$ Z

$\therefore$ B = {0, 1, 2, 3, 4}

n(B) = 5

n(A $\times$ B) = 3 $\times$ 5 = 1 5

Hence, number of subsets of A $\times$ B = 2^{15}

80.

(b) $\lambda \neq$ -2

Explanation: $\lambda \neq$ -2

MATHEMATICS (Section-B)

81. 2

Explanation:

$$f(x) = \int_{-\pi}^{\pi} (x^2 + 1 + k^2 \sin^2 x + 2x + 2k \sin x + 2\,kx \sin x)\, dx$$

$$= 2\int_0^{\pi} (x^2 + 1 + k^2 \sin^2 x + 2kx \sin x)\, dx = \frac{2\pi^3}{3} + 2\pi + 2k^2 \int_0^{\pi} \sin^2 x\, dx + 4k\underbrace{\int_0^{\pi} x \sin x\, dx}_{\pi}$$

$$= \frac{2\pi^3}{3} + 2\pi + 2k^2 \cdot 2 \cdot \frac{\pi}{4} + 4k\pi$$

$$f(k) = \frac{2\pi^3}{3} + \pi\,(k^2 + 4k + 2)$$

Minimum at $k = \frac{-b}{2a} = \frac{-4\pi}{2\pi} = -2$

Note: Minimum value $f(-2) = \frac{2\pi^3}{3} - 2\pi$

82. 9

Explanation:

$$\lim_{t\to\infty} \frac{\sqrt{tx}}{\sqrt{tx^2 - 3tx + t - 1 - x}} = \tan\left(\sin^{-1}\left(\cos\frac{\pi}{6}\right)\right)$$

$$\frac{\sqrt{x}}{\sqrt{x^2 - 3x + 1}} = \frac{\sqrt{3}}{1}$$

$\Rightarrow\ x = 3x^2 - 9x + 3$

$\Rightarrow\ 3x^2 - 10r + 3 = 0\ \Rightarrow\ (3x - 1)(r - 3) = 0\ \Rightarrow\ x = \frac{1}{x}, 3$

$\therefore\ \left(8^{\alpha} + 2^{\beta} - \alpha\beta\right) = 8^{\frac{1}{3}} + 2^3 - 1\ \Rightarrow\ 2 + 8 - 1 = 9$

83. -25

Explanation:

Since, $\vec{\mathbf{u}} + \vec{\mathbf{v}} + \vec{\mathbf{w}} = \vec{\mathbf{0}} \Rightarrow |\vec{\mathbf{u}} + \vec{\mathbf{v}} + \vec{\mathbf{w}}|^2 = 0$

$\Rightarrow |\vec{\mathbf{u}}|^2 + |\vec{\mathbf{v}}|^2 + |\vec{\mathbf{w}}|^2 + 2(\vec{\mathbf{u}} \cdot \vec{v} + \vec{\mathbf{v}} \cdot \vec{\mathbf{w}} + \vec{\mathbf{w}} \cdot \vec{\mathbf{u}}) = 0$

$\Rightarrow 9 + 16 + 25 + 2\,(\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} + \vec{\mathbf{v}} \cdot \vec{\mathbf{w}} + \vec{\mathbf{w}} \cdot \vec{\mathbf{u}}) = 0$

$\Rightarrow \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} + \vec{\mathbf{v}} \cdot \vec{\mathbf{w}} + \vec{\mathbf{w}} \cdot \vec{\mathbf{u}} = -25$

84. 8

Explanation:

$$\int_0^1 (4x^6 + (4x^3 - f(x))f(x) - 4x^6)dx = \frac{4}{7}$$

$$\int_0^1 (f(x) - 2x^3)^2\, dx = 0$$

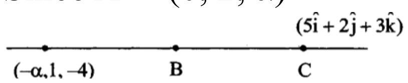
$\therefore\ f(x) = 2x^3$

$$\text{Area} = \int_0^2 2x^3 dx = 8$$

85. 9.0

Explanation:

Since A $\cdot$ (0, 2, α)



Now, $\left|\frac{1}{2}\cdot 2\sqrt{21}\cdot\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \alpha & 1 & \alpha+4\\ 5 & 2 & 3\end{vmatrix}\left|\frac{1}{\sqrt{25+4+9}}\right|\right| = 21\sqrt{21}$

$\Rightarrow\ \sqrt{(2\alpha+5)^2+(2\alpha+20)^2+(2\alpha-5)^2} = \sqrt{21}\sqrt{38}$

$\Rightarrow\ 12\alpha^2+80\alpha+450 = 798\ \Rightarrow\ 12\alpha^2+80\alpha - 348 = 0$

$\Rightarrow\ \alpha = 3\ \Rightarrow\ \alpha^2 = 9$

86. 8

Explanation:

8

87. 10620.0

Explanation:

$f(x) = 2x^2 - x - 1$

$\Rightarrow\ f(n) = 2n^2 - n - 1\ \because\ \text{if}(x)\ \leq\ 800$

$2n^2 - n - 1\ \leq\ 800$ or, $2n^2 - n - 801 < 0$

$n^2 - \frac{1}{2}n - \frac{801}{2} \leq 0;\ \left(n - \frac{1}{4}\right)^2 - \frac{801}{2} - \frac{1}{16} \leq 0$

$\left(n - \frac{1}{4}\right)^2 - \frac{6409}{16} \leq 0$

$\left(n - \frac{1}{4} - \frac{\sqrt{6409}}{4}\right)\left(n - \frac{1}{4} + \frac{\sqrt{6409}}{16}\right) \leq 0$

$\frac{1-\sqrt{6409}}{4} \leq n \leq \frac{1+\sqrt{6409}}{4}$

n = {-19, -18 - 17,.... 0, 1, 2,..., 20}

$\sum_{n\in S} f(x) = \sum\left(2x^2 - x - 1\right)$

$= 2[19^2 + 18^2 +\ldots+ 1^2 + 1^2 + 2^2 +\ldots+ 19^2 + 20^2]$

$= 4[1^2 + 2^2 +\ldots+ 19^2] + 2[202]-20-40$

$= \frac{4\times 19\times 20\times (2\times 19+1)}{6} + 2\times 400 - 60$

$= \frac{4\times 19\times 20\times 39}{6} + 800 - 60 - 9880 + 800 - 60 = 10620$

88. 25

Explanation:

Given $\cos 2\theta\cdot\cos\frac{\theta}{2} = \cos 3\theta\cdot\cos\frac{9\theta}{2}$

$\Rightarrow\ 2\cos 2\theta\cdot\cos\frac{\theta}{2} = 2\cos\frac{9\theta}{2}\cdot\cos 3\theta$

By using property 2cosAcosB = cos(A + B) + cos(A - B)

$$\Rightarrow \cos\frac{5\theta}{2} + \cos\frac{3\theta}{2} = \cos\frac{15\theta}{2} + \cos\frac{3\theta}{2}$$

$$\Rightarrow \cos\frac{15\theta}{2} = \cos\frac{5\theta}{2} \Rightarrow \frac{15\theta}{2} = 2k\pi \pm \frac{5\theta}{2}$$

$$5\theta = 2k\pi \text{ or } 10\theta = 2k\pi \Rightarrow \theta = \frac{2k\pi}{5}\ \theta = \frac{k\pi}{5}$$

$$\therefore \theta = \left\{-\pi, \frac{-4\pi}{5}, \frac{-3\pi}{5}, \frac{-2\pi}{5}, \frac{-\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi\right\}$$

m = 5, n = 5 $\Rightarrow$ m $\times$ n = 25

89. 10

Explanation:

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \left(x^2+1\right)^2 + x^2 & x\left(x^2+1\right) + x \\ x\left(x^2+1\right) + x & x^2+1 \end{bmatrix}$$

Given that $(x^2 + 1)^2 + x^2 = 109$

$x^4 + 3x^2 - 108 = 0$

$\Rightarrow (x^2 + 12)(x^2 - 9) = 0$

$\therefore x^2 = 9$

$a_{22} = x^2 + 1 = 9 + 1 = 10$

90. 2.0

Explanation:

Given that

$a + \alpha = 1$

$b + \beta = 2$

and, $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ ...(i)

replace x by $\frac{1}{x}$

$\Rightarrow af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x$ ...(ii)

Adding (i) and (ii),

$$(a + \alpha)\ f(x) + (a + \alpha) f\left(\frac{1}{x}\right) = x(b + \beta) + (b + \beta)\frac{1}{x}$$

$$\Rightarrow \frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{b+\beta}{a+\alpha} = \frac{2}{1} = 2$$