Quadrilaterals

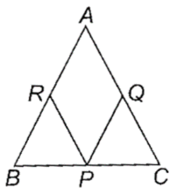
MATHEMATICAL REASONING

1. The bisectors of angles of a parallelogram forms
(a) Trapezium (b) Rectangle
(c) Rhombus (d) Kite

2. If a quadrilateral has two adjacent sides are equal and the opposite sides are unequal, then it is called a
(a) Parallelogram (b) Square
(c) Rectangle (d) Kite

3. If the angles of a quadrilateral are in the ratio $1:2:3:4$. Then, the measure of angles in descending order are
(a) $36°, 108°, 72°$ and $144°$
(b) $144°, 108°, 72°$ and $36°$
(c) $36°, 72°, 108°$ and $144°$
(d) None of these

4. In a $\Delta ABC, P, Q$ and R are the mid-points of sides BC, CA and AB respectively. If $AC = 21\ cm$, $B = 29 cm$ and $AB = 30 cm$. The perimeter of the quadrilateral $ARPQ$ is _____.



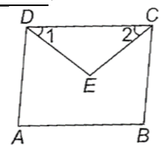
(a) 91 cm (b) 60 cm
(c) 51 cm (d) 70 cm

5. The diagonals AC and BD of a parallelogram $ABCD$ intersect each other at the point O. If $\angle DAC = 32°$ and $\angle AOB = 70°$, then, $\angle DBC$ is equal to ____.
(a) $38°$ (b) $86°$
(c) $24°$ (d) $32°$

6. If the angles of a quadrilateral are $x + x + 20°, x - 40°$ and $2x$. Then, the difference between greatest angle and the smallest angle is _____.
(a) $70°$ (b) $90°$
(c) $80°$ (d) None of these

7. Two adjacent angles of a parallelogram are in the ratio $2:3$. The angles are
(a) $90°, 180°$ (b) $36°, 144°$
(c) $72°, 108°$ (d) $52°, 104°$

8. In a quadrilateral $ABCD$, the line segments bisecting $\angle C$ and $\angle D$ meet at E. Then $\angle A + \angle B$ is equal to _____.



(a) $\angle CED$ (b) $\frac{1}{2}\angle CED$
(c) $2\angle CED$ (d) None of these

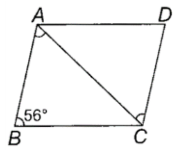
9. The diagonals of a rectangle $PQRS$ intersect at O. If $\angle ROQ = 60°$, then find $\angle OSP$.
(a) $70°$ (b) $50°$
(c) $60°$ (d) $80°$

10. If diagonals of a quadrilateral bisect each other at right angles, then it is a
(a) Parallelogram (b) Rectangle
(c) Rhombus (d) Trapezium

11. The measure of all the angles of a parallelogram, if an angle adjacent to the smallest angle is $24°$ less than twice the smallest angle, is
(a) $37°, 143°, 37°, 143°$ (b) $108°, 72°, 108°, 72°$
(c) $68°, 112°, 68°, 112°$ (d) None of these

12. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral $PQRS$, taken in order, is a rectangle if
(a) $PQRS$ is a rectangle
(b) $PQRS$ is a parallelogram
(c) Diagonals of $PQRS$ are equal
(d) Diagonals of $PQRS$ are at right angles

13. $ABCD$ is a rhombus with $\angle ABC = 56°$, then $\angle ACD$ is equal to



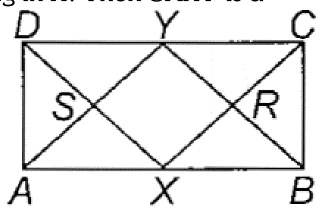
(a) $90°$ (b) $60°$
(c) $56°$ (d) $62°$

14. $ABCD$ is a parallelogram. If AB is produced to E such that ED bisects BC at O. Then which of the following is correct?
(a) $AB = OE$ (b) $AB = BE$
(c) $OE = OC$ (d) None of these

15. D and E are the mid-points of the sides AB and AC, respectively of ΔABC. DE is produced to F. To prove that CF is equal and parallel to DA, we need an additional information which is

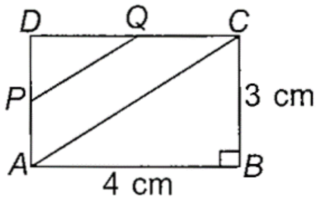
(a) $\angle DAE = \angle EFC$ (b) $AE = EF$
(c) $DE = EF$ (d) $\angle ADE = \angle ECF$

16. X, Y are the mid-points of opposite sides AB and DC of a parallelogram $ABCD$. AY and DX are joined intersecting in S; CX and BY are joined intersecting in R. Then $SXRY$ is a



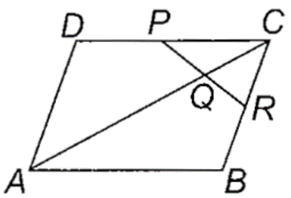
(a) Rectangle (b) Rhombus
(c) Parallelogram (d) Square

17. In the given figure, if $ABCD$ is a rectangle and P, Q are the mid-points of AD, DC respectively. Then, the ratio of lengths PQ and AC is equal to



(a) $1:1$ (b) $1:2$
(c) $2:1$ (d) $3:2$

18. In given figure, $ABCD$ is a parallelogram in which P is the midpoint of DC and Q is a point on AC such that $CQ = \frac{1}{4}AC$ and PQ produced meet BC at R, then

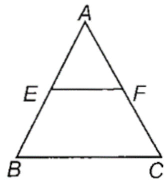


(a) $CR = \frac{1}{3}CB$ (b) $CR = RB$
(c) $CR = \frac{1}{2}RB$ (d) None of these

19. If APB and CQD are two parallel lines, then the bisectors of the angles APQ, BPQ, CQP and PQD form

(a) Kite (b) Rhombus
(c) Rectangle (d) Trapezium

20. In figure, E and F are the mid-points of sides AB and AC of a ΔABC. If $AB = 5\ cm$, $BC = 5\ cm$ and $AC = 6\ cm$, then EF is equal to



(a) 3 cm (b) 2.5 cm
(c) 4 cm (d) None of these

ACHIEVERS SECTION (HOTS)

21. Study the statements carefully.

Statement-1: If a sum of a pair of opposite angles of a quadrilateral is $180°$, the quadrilateral is cyclic.

Statement-2: A line drawn through mid-point of a side of a triangle, parallel to another side equal to third side.

Which of the following options holds?
(a) Both Statement-1 and Statement-2 are true.
(b) Statement-1 is true but Statement-2 is false.
(c) Statement-1 is false but Statement-2 is true.
(d) Both Statement-1 and Statement-2 are false.

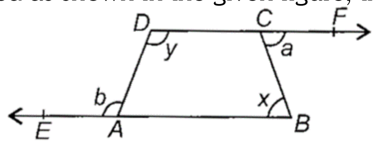
22. Read the statements carefully and state 'T' for true and 'F' for false.
(i) Diagonals of a parallelogram are perpendicular to each other.
(ii) All four angles of a quadrilateral can be obtuse angles.
(iii) If all sides of a quadrilateral are equal, then it is a rhombus.

	(i)	**(ii)**	**(iii)**
(a)	T	F	F
(b)	F	F	T
(c)	F	T	T
(d)	F	F	F

23. Fill in the blanks.
(a) If consecutive sides of a parallelogram are equal then it is necessarily a **P**.
(b) The figure formed by joining the mid-points of consecutive sides of a quadrilateral is **Q**.
(c) If the diagonals of a parallelogram are equal and perpendicular to each other, it is a **R**.

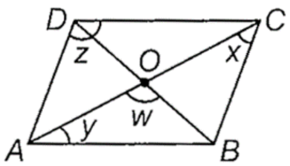
	P	**Q**	**R**
(a)	Square	Parallelogram	Rhombus
(b)	Kite	Rhombus	Square
(c)	Rhombus	Rectangle	Rectangle
(d)	Rhombus	Parallelogram	Square

24. If the sides BA and DC of quadrilateral $ABCD$ are produced as shown in the given figure, then



(a) $x+y=a+b$
(b) $x-y=a-b$
(c) $\frac{x-y}{2}=a-b$
(d) $2(x+y)=a+b$

25. By using a given figure of quadrilateral ABCD, match the following:



	Column-I	Column-II
(p)	If $ABCD$ is a parallelogram, then sum of the angles x, y and z is	(1) $25°$
(q)	If $ABCD$ is a rhombus, where $\angle D=130^\circ$, then the value of x is	(2) $180°$
(r)	If $ABCD$ is a rhombus, the value of w is	(3) $50°$
(s)	If $ABCD$ is a parallelogram, where $x+y=130^\circ$ then the value of B is	(4) $90°$

(a) (p) $\to$ (1), (q) $\to$ (2), (r) $\to$ (3), (s) $\to$ (4)
(b) (p) $\to$ (2), (q) $\to$ (1), (r) $\to$ (4), (s) $\to$ (3)
(c) (p) $\to$ (3), (q) $\to$ (1), (r) $\to$ (2), (s) $\to$ (4)
(d) (p) $\to$ (4), (q) $\to$ (3), (r) $\to$ (1), (s) $\to$ (2)

HINTS & EXPLANATIONS

1. (b):

2. (d) :

3. (b) : Let the angles of a quadrilateral be $x, 2x, 3x$ and $4x$.

$\therefore \quad x+2x+3x+4x=360^\circ$

($\because$ Sum of angles of a quadrilateral is 360°)

$\Rightarrow 10x=360^\circ \Rightarrow x=36^\circ$

$\therefore$ Angles are $36^\circ, 72^\circ, 108^\circ, 144^\circ$

And, angles in descending order are $144^\circ, 108^\circ, 72^\circ, 36^\circ$

4. (c) : Given, $AC=21\ cm$, $AB=30\ cm$

$\because$ Q is mid-point of AC

$\therefore\ AQ=\frac{AC}{2}=\frac{21}{2}cm$ ...(i)

$\because$ R is mid-point of AB

$AR=\frac{AB}{2}=\frac{30}{2}cm$ ...(ii)

In ΔBCA,

P is mid-point of BC, R is mid-point of BA then, by mid-point theorem,

$PR \parallel$ AC and $PR=\frac{1}{2}AC=\frac{21}{2}$ ...(iii)

Similarly, Q is mid-point of AC, P is mid-point of BC, then $QP \parallel$ AB and

$QP=\frac{1}{2}AB=\frac{30}{2}$ ....(iv)

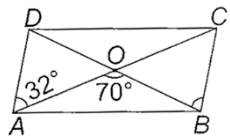
$\therefore$ Perimeter of quad. $ARPQ$

$=AR+RP+PQ+AQ$

$=\frac{30}{2}+\frac{21}{2}+\frac{30}{2}+\frac{21}{2}$ (from (i), (ii), (iii) and (iv))

$=2.\frac{30}{2}+2.\frac{21}{2}=30+21=51cm.$

5. (a) : Given, $\angle DAC=32^\circ$ As $DA \parallel$ BC and AC is transversal.



$\therefore\ \angle ACB=\angle DAC=32^\circ$

(alternate angles)

Also, $\angle AOB+\angle BOC=180^\circ$ (linear pair)

$\Rightarrow 70^\circ+\angle BOC=180^\circ$

$\Rightarrow \angle BOC=110^\circ$

In $\Delta BOC, \angle BOC+\angle OBC+\angle OCB=180^\circ$

(angle sum property)

$\Rightarrow 110^\circ+\angle OBC+32^\circ=180^\circ$

$\Rightarrow \angle OBC=180^\circ-(110^\circ+32^\circ)=38^\circ$

$\Rightarrow \angle DBC=38^\circ$

6. (d) : Since, sum of all angles of a quadrilateral is 360°.

$\therefore\ x+x+20^\circ+x-40^\circ+2x=360^\circ$

$\Rightarrow 5x=360^\circ-20^\circ+40^\circ=380^\circ$

$\Rightarrow x=76^\circ$

$\therefore$ Angles are $76^\circ, 96^\circ, 36^\circ$ and 152°

$\therefore$ Required difference $=152^\circ-36^\circ=116^\circ$

7. (c) : Let the adjacent angles of a parallelogram be $2x$ and $3x$ and sum of adjacent angles of parallelogram is 180^o.

$\therefore\ 2x+3x=180^o$

$\Rightarrow\ 5x=180^o \Rightarrow x=36^o$

$\therefore$ Angles are 72^o and 108^o.

8. (c) : In quadrilateral $ABCD$, (angles sum property.)

$\angle A+\angle B+\angle C+\angle D=360^o$

$\Rightarrow\ \angle A+\angle B+2\angle 2+2\angle 1=360^o$

$(\because \angle 2=\frac{1}{2}\angle C \text{ and } \angle 1=\frac{1}{2}\angle D)$

$\Rightarrow\ \angle +\angle B=360^o-2(\angle 1+\angle 2)$ ...(i)

In ΔDEC,

$\angle 1+\angle 2+\angle CED=180^o$ (angle sum property)

$\Rightarrow\ \angle 1+\angle 2=180^o-\angle CED$ ...(ii)

From (i) and (ii), we get

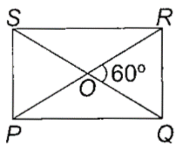
$\angle A+\angle B=360^o-2(180^o-\angle CED)$

$\angle A+\angle B=360^o-360^o+2\angle CED$

$\Rightarrow\ \angle A+\angle B=2\angle CED$

9. (c) : $\angle ROQ=\angle SOP=60^o$ ...(i)

[Vertically opposite angles]



$\therefore\ PR=SQ \Rightarrow PO=SO$

(Diagonals of a rectangle are equal and bisect each other)

$\Rightarrow\ \angle OPS=\angle OSP$

[$\because$ In a triangle, angles opposite to equal sides are equal]

In ΔPOS, by angle sum property

$\angle OSP+\angle OPS+\angle SOP=180^o$

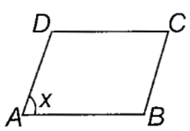
$\Rightarrow\ 2\angle OSP=180^o-60^o$ [Using (i) & (ii)]

$\Rightarrow\ \angle OSP=60^o$

10. (c) : In rhombus, diagonals bisect each other at right angles.

11. (c) : Let the smallest angle be $\angle A=x$, and other adjacent angle $\angle B=(2x-24)^o$

Now, sum of adjacent angles of parallelogram is 180^o.



$\therefore\ \angle A+\angle B=180^o$

$\Rightarrow\ x+2x-24^o=180^o$

$\Rightarrow\ 3x=204^o \Rightarrow x=68^o$

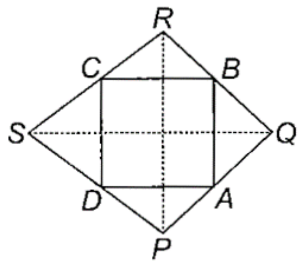
$\therefore\ A=x=68^o$

and $\angle B=(2x-24)^o=2\times 68^o-24^o=112^o$

Since, opposite angles of a parallelogram are equal.

So, $\angle A=\angle C=68^o, \angle B=\angle D=112^o$

12. (d) : Let A, B, C and D be the mid-points of PQ, QR, RS and SP respectively



Now, In $\Delta RSQ, C$ and B are the mid-points of RS and RQ respectively. So, by mid-point theorem.

$CB\,||$ SQ ...(i)

Similarly, In ΔPSQ,

$DA\,||$ SQ ...(ii)

In ΔSPR,

$CD\,||$ RP ...(iii)

Also, in ΔQRP

$AB\,||$ RP ...(iv)

From (i) and (ii),

$CB\,||$ DA ...(v)

From (iii)and (iv),

$CD\,||$ AB

Hence, from (v) and (vi), $ABCD$ is a parallelogram.

Now, if diagonals bisect SQ and PR are at 90^o

Then,

$CB\perp CD, CB\perp AB, AB\perp DA$ and $AD\perp CD$.

So, $ABCD$ is a rectangle.

13. (d) : As diagonals of rhombus bisect the angles.

$\therefore\ \angle BAC=\angle CAD$

Also, In rhombus $ABCD$.

$\angle A+\angle B=180^o$ (Sum of adjacent angles)

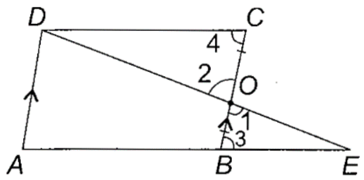
$\Rightarrow\ \angle A+56^o=180^o \Rightarrow \angle A=124^o$

$\therefore\ \angle BAC=\angle CAD=\frac{\angle A}{2}=62^o$

Now $\angle ACD=\angle BAC$ (Alternate angles)

$\Rightarrow\ \angle ACD=62^o$

14. (b) :



In the figure, $ABCD$ is a parallelogram where AB is produced to E such that

$OC = OB$

In ΔOBE and ΔOCD,

$\angle 1 = \angle 2$ (Vertically opposite angles)

$\angle 3 = \angle 4$ (Alternate angles)

$OB = OC$ (given)

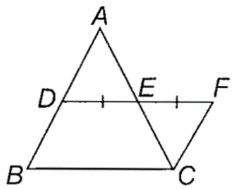
$\therefore\ \Delta OBE \cong \Delta OCD$ (By ASA congruency)

$\Rightarrow\ BE = CD$ (By CPCT)

Also, $AB = CD$ ($\because$ ABCD is parallelogram)

$\therefore\ AB = BE$

15. (c) : We have produced DE to F such that $DE=EF$ ...(i)



In ΔADE and ΔCFE,

$AE = CE$ [$\therefore E$ is the mid-point of AC]

$\angle AED = \angle CEF$ [Vertically opposite angles]

$DF = FE$ [By (i)]

$\therefore\ \Delta ADE \cong \Delta CFE$ [By SAS congruency]

$\therefore\ AD = CF$ and $\angle ADE = \angle CFE$ [By CPCT]

This show that alternate interior angles are equal.

Thus, $AD \parallel CF$

Therefore, the additional information which we need is $DE = EF$

16. (c) : In quadrilateral $AXCY$,

$AX \parallel CY$ ($\because AB \parallel CD$) ...(i)

$AX = \frac{1}{2}AB$ and $CY = \frac{1}{2}CD$

($\therefore X$ and Y are midpoint of AB and CD)

Also, $AB = CD$

(Opposite sides of parallelogram)

So, $AX = CY$

$\Rightarrow\ AXCY$ is a parallelogram (from (i) and (ii))

Similarly, quadrilateral $DXBY$ is a parallelogram.

In quadrilateral $SXRY$, $SX \parallel YR$

($\because\ SX$ is a part of DX and YR is a part of YB)

Similarly, $SY \parallel XR$

So, $SXRY$ is a parallelogram.

17.

(b) : In $\Delta ACD, P$ and Q are mid-points of AD and DC.

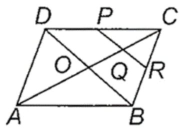
By mid-point theorem,

$PQ \parallel AC$ and $PQ = \frac{1}{2}AC$

$\therefore\ \frac{PQ}{AC} = \frac{1}{2}$

Now, $PQ : AC = 1 : 2$

18. (b) : Join B and D. Suppose AC and BD intersect at O. Then. $OC = \frac{1}{2}AC$



Now, $CQ = \frac{1}{4}AC$ [Given]

$\Rightarrow CQ = \frac{1}{2}OC$

In $\Delta COD, P$ and Q are the mid-points of DC and OC respectively.

$\therefore\ PQ \mid DO$ [By mid-point theorem]

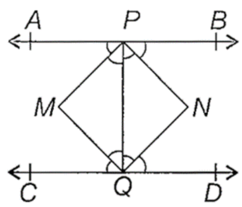
Also, in $\Delta COB, Q$ is the mid-point of OC and $QR \parallel OB$

$\therefore\ R$ is the mid-point of BC.

[By converse of mid-point theorem]

$\Rightarrow\ CR = RB$

19. (c) : Given, APB and CQD are two parallel lines. Let the bisectors of angles APQ and CQP meet at a point M and bisectors of angles BPQ and PQD meet at a point N.



Since, $APB \parallel CQD$

$\therefore\ \angle APQ = \angle PQD$

[Alternate interior angles]

$\Rightarrow \frac{1}{2}\angle APQ = \frac{1}{2}\angle PQD$

$\Rightarrow \angle MPQ = \angle NQP$

This shows that alternate interior angles are equal.

$\therefore\ PM \parallel QN$

Similarly, $\angle NPQ = \angle MQP$, which shows that alternate interior angles are equal.

$\therefore\ PN \parallel QM$

So, quadrilateral *PMQN* is a parallelogram.

Also, $\angle CQP + \angle DQP = 180^o$ [Linear pair]

$\Rightarrow 2\angle MQP + 2\angle NQP = 180^o$

$\Rightarrow 2(\angle MQP + \angle NQP) = 180^o$

$\Rightarrow \angle MQN = 90^o$

Thus, *PMQN* is a rectangle.

20. (b) : In ΔABC,

Given, *E* is mid-point of AB and F is mid-point of AC.

Then, by mid-point theorem

$BC \,||\, \text{EF and } EF = \frac{1}{2}BC$

$\therefore \quad EF = \frac{1}{2}(5) = 2.5\,cm$

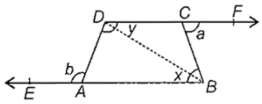
21. (b) :

22. (b) :

23. (d) :

24. (a) : Join BD. In ΔABD, we have

$\angle ABD + \angle ADB = b$ ...(i)



In ΔCBD, we have $\angle CBD + \angle CDB = a$ ...(ii)

Adding (i) and (ii), we get

$(\angle ABD + \angle CBD) + (\angle ADB + \angle CDB) = a + b$

$\Rightarrow x + y = a + b$

25. (c) : (P) In ΔABC, by angle sum property

$x + y + \angle ABC = 180^o$

$\Rightarrow \angle ABC = 180^o - (x + y)$ ...(i)

$\therefore \angle ABC = \angle ADC$

[$\because$ Opposite angles of a parallelogram are equal]

$\therefore \quad z = 180^o - (x + y)$ [using (i)]

$\Rightarrow x + y + z = 180^o$

(Q) $\angle C = 2x$

[Since, diagonals bisects the angles in rhombus]

Now, we have

$\angle D + \angle C = 180^o$ [Co-interior angles]

$130^o + 2x = 180^o$ $(\angle D = 130^o)$

$\Rightarrow 2x = 180^o - 130^o = 50^o \Rightarrow x = 25^o$

(R) Since, in a rhombus, diagonals bisect each other.

$\therefore \quad w = 90^o$

(S) Since in a parallelogram opposite angles are equal.

$\therefore \quad \angle B = \angle D = z$ ...(i)

In ΔABC, by angle sum property,

$\angle B + x + y = 180^o$

$\angle B = 180^o - (x + y)$

$\therefore \; z = 180^o - (x + y)$ (using (i))

$= 180^o - 130^o = 50^o$