7. Rational Numbers

Exercise 7.1

1. Question

Identify the property in the following statements:

(i) 2 + (3 + 4) = (2 + 3) + 4;

(ii) 2·8 = 8·2;

(iii) 8· (6 + 5) = (8·6) + (8·5).

Answer

(i) The given statement shows the associative property for addition.

(ii) The given statement shows the commutative property for multiplication.

(iii) The given statement shows the distributive property for multiplication over addition

2. Question

Find the additive inverses of the following integers:

6, 9, 123, –76, –85, 1000.

Answer

Additive inverse property:

Case 1: If x is positive number, x+(-x)=0. So, (-x) is called as additive inverse of x.

Case 2: If x is negative number, -x+(-(-x))=0. So, (x) is called as additive inverse of x.

According to the above property,

The additive inverse of 6 is (-6).

The additive inverse of 9 is (-9).

The additive inverse of 123 is

(-123).

The additive inverse of -76 is 76.

The additive inverse of -85 is 85.

The additive inverse of 1000 is

(-1000).

3. Question

Find the integer m in the following:

(i) $m + 6 = 8$;

(ii) $m + 25 = 15$;

(iii) $m - 40 = -26$;

(iv) $m + 28 = -49$

Answer

(i) $M + 6 = 8$

$\Rightarrow m = 8 - 6$

$\Rightarrow m = 2$

(ii) $M + 25 = 15$

$\Rightarrow m = 15 - 25$

$\Rightarrow m = -10$

(iii) $M - 40 = -26$

$\Rightarrow m = 40 - 26$

$\Rightarrow m = 14$

(iv) $M + 28 = -49$

$\Rightarrow m = -28 - 49$

$\Rightarrow m = -77$

4. Question

Write the following in increasing order:

21, −8, −26, 85, 33, −333, −210, 0, 2011.

Answer

Increasing order of above numbers,

$-333<-210<-26<-8<0<21<33<85<2011$.

5. Question

Write the following in decreasing order:

85, 210, –58, 2011, –1024, 528, 364, –10000, 12.

Answer

Decreasing order of above numbers,

2011>528>364>210>85>12>-58>-1024>-10000.

Exercise 7.2

1. Question

Write down ten rational numbers which are equivalent to 5/7 and the denominator not exceeding 80.

Answer

The equivalent rational number can be calculated by multiplying and dividing with a positive number. In the question, it is given that the denominator should not be greater than 80.

First equivalent rational number of $\frac{5}{7}$ when multiplying and dividing with 2 is $\frac{5\times2}{7\times2} = \frac{10}{14}$

Second equivalent rational number of $\frac{5}{7}$ when multiplying and dividing with 3 is $\frac{5\times3}{7\times3} = \frac{15}{21}$

Third equivalent rational number of $\frac{5}{7}$ when multiplying and dividing with 4 is $\frac{5\times4}{7\times4} = \frac{20}{28}$

Similarly, other equivalent rational numbers of $\frac{5}{7}$ are $\frac{25}{35}, \frac{30}{42}, \frac{35}{49}, \frac{40}{56}, \frac{45}{63}, \frac{50}{70}, \frac{55}{77}$.

2. Question

Write down 15 rational numbers which are equivalent to $\frac{11}{5}$ and the numerator not exceeding 180.

Answer

The equivalent rational number can be calculated by multiplying and dividing with a positive number and it is given that numerator should not be greater than 180.

First equivalent rational number of $\frac{11}{5}$ when multiplying and dividing with 2 is $\frac{11\times2}{5\times2} = \frac{22}{10}$

Second equivalent rational number of $\frac{11}{5}$ when multiplying and dividing with 3 is $\frac{11\times3}{5\times3} = \frac{33}{15}$

Third equivalent rational number of $\frac{11}{5}$ when multiplying and dividing with 4 is $\frac{11\times4}{5\times4} = \frac{44}{20}$

Similarly, other equivalent rational numbers of $\frac{11}{5}$ are $\frac{55}{25}, \frac{66}{30}, \frac{77}{35}, \frac{88}{40}, \frac{99}{45}, \frac{110}{50}, \frac{121}{55}, \frac{132}{60}, \frac{143}{65}, \frac{154}{70}, \frac{165}{75}, \frac{176}{80}$.

3. Question

Write down ten positive rational numbers such that the sum of the numerator and the denominator of each is 11. Write them in decreasing order.

Answer

The positive rational numbers having the sum of numerator and denominator as 11 are: $\frac{1}{10}, \frac{10}{1}, \frac{2}{9}, \frac{9}{2}, \frac{3}{8}, \frac{8}{3}, \frac{4}{7}, \frac{7}{4}, \frac{5}{6}, \frac{6}{5}$.

According to the question, increasing order of above rational numbers is $\frac{10}{1}, \frac{9}{2}, \frac{8}{3}, \frac{7}{4}, \frac{6}{5}, \frac{5}{6}, \frac{4}{7}, \frac{3}{8}, \frac{2}{9}, \frac{1}{10}$.

4. Question

Write down ten positive rational numbers such that numerator – denominator for each of them is –2. Write them in increasing order.

Answer

The positive rational numbers having numerator-denominator as -2 with their increasing order are: $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}, \frac{6}{8}, \frac{7}{9}, \frac{8}{10}, \frac{9}{11}, \frac{10}{12}$.

5. Question

Is $\frac{3}{-2}$ a rational number? If so, how do you write it in a form conforming to the definition of a rational number (that is, the denominator as a positive integer)?

Answer

Rational numbers are those numbers which can be written in the form of $\frac{p}{q}$, where p and q are the integers and $q \neq 0$.

Now considering $\frac{3}{-2} = \frac{-3}{2}$, which is written in the form of $\frac{p}{q}$. Where $p = -3$ and $q = 2$. Here p and q both are integers and $q \neq 0$.

Hence, the number $\frac{-3}{2} = \frac{3}{-2}$ is a rational number.

6. Question

Earlier you have studied decimals 0.9, 0.8. Can you write these as rational numbers?

Answer

Yes the 0.9 and 0.8 can also be written in rational numbers.

0.9 can also be written as $\frac{9}{10}$, Which is a rational number.

0.8 can also be written as $\frac{8}{10} = \frac{4}{5}$, Which is a rational number.

Exercise 7.3

1. Question

Name the property indicated in the following:

(i) 315 + 115 = 430 (ii) $\frac{3}{4} \cdot \frac{9}{5} = \frac{27}{20}$

(iii) 5 + 0 = 0 + 5 = 5 (iv) $\frac{8}{9} \times 1 = \frac{8}{9}$

(v) $\frac{8}{17} + \frac{-8}{17} = 0$ (vi) $\frac{22}{23} \cdot \frac{23}{22} = 1$

Answer

(i) This is showing addition property.

(ii) This is showing multiplication property.

(iii) This is showing additive identity.

(iv) This is showing multiplicative identity.

(v) This is showing additive inverse.

(vi) This is showing multiplicative inverse.

2. Question

Check the commutative property of addition for the following pairs:

(i) $\frac{102}{201}, \frac{3}{4}$ (ii) $\frac{-8}{13}, \frac{23}{27}$

(iii) $\frac{-7}{9}, \frac{-18}{19}$

Answer

Commutative property of addition is $(a + b) = (b + a)$

(i) Let LHS(Left Hand Side) is
$(a + b) = \left(\frac{102}{201} + \frac{3}{4}\right) = \left(\frac{102\times4 + 3\times201}{804}\right) = \frac{1011}{804}$

Let RHS(Right Hand Side) is
$(b + a) = \left(\frac{3}{4} + \frac{102}{201}\right) = \left(\frac{3\times201 + 102\times4}{804}\right) = \frac{1011}{804}$

Hence, LHS=RHS i.e. $(a + b) = (b + a)$Which is showing commutative property of addition.

(ii) Let LHS(Left Hand Side) is
$(a + b) = \left(-\frac{8}{13} + \frac{23}{27}\right) = \left(\frac{-8\times27 + 23\times13}{351}\right) = \frac{83}{351}$

Let RHS(Right Hand Side) is
$(b + a) = \left(\frac{23}{27} + \left(\frac{-8}{13}\right)\right) = \left(\frac{23\times13 + (-8\times27)}{351}\right) = \frac{83}{351}$

Hence, LHS=RHS i.e. $(a + b) = (b + a)$Which is showing commutative property of addition.

(iii) Let LHS(Left Hand Side) is
$(a + b) = \left(\frac{-7}{9} + \left(\frac{-18}{19}\right)\right) = \left(\frac{-7\times19 + (-18\times9)}{171}\right) = \frac{-295}{171}$

Let RHS(Right Hand Side) is
$(b + a) = \left(\frac{-18}{19} + \left(\frac{-7}{9}\right)\right) = \left(\frac{-18\times9 + (-7\times19)}{171}\right) = \frac{-295}{171}$

Hence, LHS=RHS i.e. $(a + b) = (b + a)$Which is showing commutative property of addition.

3. Question

Check the commutative property of multiplication for the following pairs:

(i) $\frac{22}{45}, \frac{3}{4}$ (ii) $\frac{-7}{13}, \frac{25}{27}$

(iii) $\frac{-8}{9}, \frac{-17}{19}$

Answer

Commutative property of multiplication is $(a \times b) = (b \times a)$

(i) Let LHS(Left Hand Side) is$(a \times b) = \left(\frac{22}{45} \times \frac{3}{4}\right) = \left(\frac{66}{180}\right) = \frac{11}{30}$

Let RHS(Right Hand Side) is $(b \times a) = \left(\frac{3}{4} \times \frac{22}{45}\right) = \left(\frac{66}{180}\right) = \frac{11}{30}$

Hence, LHS=RHS i.e. $(a \times b) = (b \times a)$Which is showing commutative property of multiplication.

(ii) Let LHS(Left Hand Side) is$(a \times b) = \left(\frac{-7}{13} \times \frac{25}{27}\right) = \left(\frac{-175}{351}\right)$

Let RHS(Right Hand Side) is $(b \times a) = \left(\frac{25}{27} \times \frac{-7}{13}\right) = \left(\frac{-175}{351}\right)$

Hence, LHS=RHS i.e. $(a \times b) = (b \times a)$Which is showing commutative property of multiplication.

(iii) Let LHS(Left Hand Side) is$(a \times b) = \left(\frac{-8}{9} \times \frac{-17}{19}\right) = \frac{136}{171}$

Let RHS(Right Hand Side) is $(b \times a) = \left(\frac{-17}{19} \times \frac{-8}{9}\right) = \frac{136}{171}$

Hence, LHS=RHS i.e. $(a \times b) = (b \times a)$Which is showing commutative property of multiplication.

4. Question

Check the distributive property for the following triples of rational numbers:

(i) $\frac{1}{8}, \frac{1}{9}, \frac{1}{10}$ (ii) $\frac{-4}{9}, \frac{6}{5}, \frac{11}{10}$

(iii) $\frac{3}{8}, 0, \frac{13}{7}$

Answer

theDistributive property of multiplication over addition is
$a \times (b + c) = a \times b + a \times c$.

(i) Let LHS(Left Hand Side) is$a \times (b + c) = \frac{1}{8} \times \left(\frac{1}{9} + \frac{1}{10}\right) = \frac{1}{8} \times \frac{19}{90} = \frac{19}{720}$

Let RHS (Right Hand Side) is

$a \times b + a \times c = \left(\frac{1}{8} \times \frac{1}{9}\right) + \left(\frac{1}{8} \times \frac{1}{10}\right) = \frac{1}{72} + \frac{1}{80} = \frac{19}{720}$

Hence, LHS=RHS i.e. a*(b+c)=a*b+a*c. Which is showing distributive property of multiplication over addition.

(ii) Let LHS(Left Hand Side) is

$a \times (b + c) = \frac{-4}{9} \times \left(\frac{6}{5} + \frac{11}{10}\right) = \frac{-4}{9} \times \frac{23}{10} = \frac{-92}{90} = \frac{-46}{45}$

Let RHS (Right Hand Side) is

$a \times b + a \times c = \frac{-4}{9} \times \frac{6}{5} + \frac{(-4)}{9} \times \frac{11}{10} = \frac{-24}{45} + \frac{(-44)}{90} = \frac{-8}{15} + \frac{(-22)}{45} = \frac{-24 + (-22)}{45} = \frac{-46}{45}$

Hence, LHS=RHS i.e. $a \times (b + c) = a \times b + a \times c$. Which is showing distributive property of multiplication over addition.

(iii) Let LHS(Left Hand Side) is$a \times (b + c) = \frac{3}{8}\left(0 + \frac{13}{7}\right) = \frac{3}{8} \times \frac{13}{7} = \frac{39}{56}$

Let RHS (Right Hand Side) is $a \times b + a \times c = \frac{3}{8} \times 0 + \frac{3}{8} \times \frac{13}{7} = \frac{39}{56}$

Hence, LHS=RHS i.e. $a \times (b + c) = a \times b + a \times c$. Which is showing distributive property of multiplication over addition.

5. Question

Find the additive inverse of each of the following numbers:

$\frac{8}{5}, \frac{6}{10}, \frac{-3}{8}, \frac{-16}{3}, \frac{-4}{1}.$

Answer

Case 1: If x is positive number, $x + (-x) = 0$. So, $(-x)$ is called as additive inverse of x.

Case 2: If x is negative number, $-x + (-(-x)) = 0$. So, (x) is called as additive inverse of x.

The additive inverse of $\frac{8}{5}$ is $(\frac{-8}{5})$

The additive inverse of $\frac{6}{10}$ is $(\frac{-6}{10})$

The additive inverse of $\frac{-3}{8}$ is $(\frac{3}{8})$

The additive inverse of $\frac{-16}{3}$ is $(\frac{16}{3})$

The additive inverse of $\frac{-4}{1}$ is $\left(\frac{4}{1}\right)$

6. Question

Find the multiplicative inverse of each of the following numbers:

$$2, \frac{6}{11}, \frac{-8}{15}, \frac{19}{18}, \frac{1}{1000}.$$

Answer

The multiplicative inverse of a digit is the reciprocal of given digit i.e.
$x \times x' = 1, x' = \frac{1}{x}$

The multiplicative inverse of 2 is $\frac{1}{2}$.

The multiplicative inverse of $\frac{6}{11}$ is $\frac{11}{6}$.

The multiplicative inverse of $\frac{-8}{15}$ is $\frac{-15}{8}$.

The multiplicative inverse $\frac{19}{18}$ is $\frac{18}{19}$.

The multiplicative inverse of $\frac{1}{1000}$ is $\frac{1000}{1}$.

Exercise 7.4

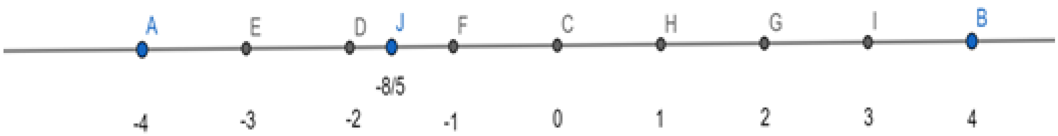
1. Question

Represent the following rational numbers on the number line:

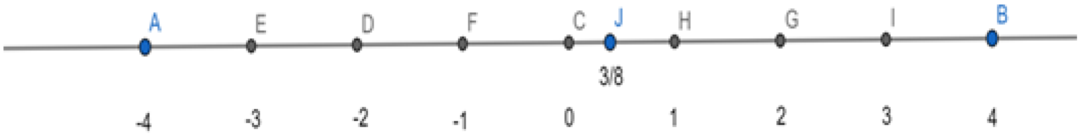
$$\frac{-8}{5}; \frac{3}{8}; \frac{2}{7}; \frac{12}{5}; \frac{45}{13}.$$

Answer

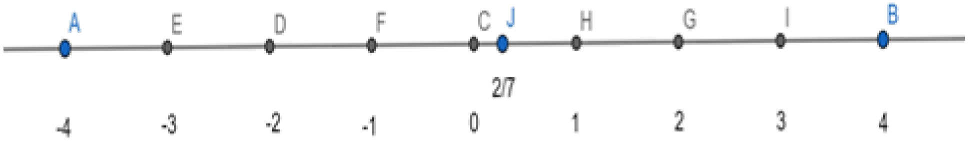
(i) $\frac{-8}{5}$ on a number line is shown below



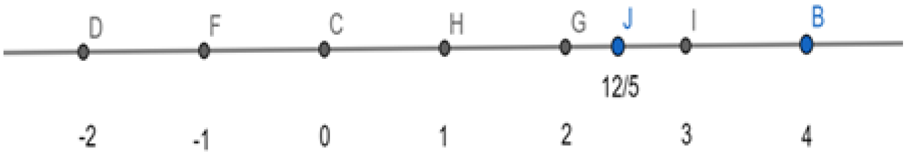
(ii) $\frac{3}{8}$ on the number, line is shown below



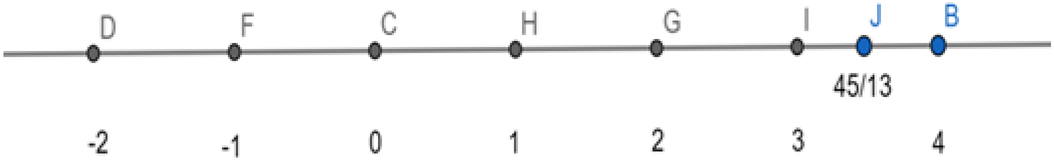
(iii) $\frac{2}{7}$ on the number, line is shown below



(iv) $\frac{12}{5}$ on the number, line is shown below



(v) $\frac{45}{13}$ on the number, line is shown below



2. Question

Write the following rational numbers in ascending order:

$$\frac{3}{4}, \frac{7}{12}, \frac{15}{11}, \frac{22}{19}, \frac{101}{100}, \frac{-4}{5}, \frac{-102}{81}, \frac{-13}{7}.$$

Answer

The ascending order of the above rational numbers is $\frac{-13}{7}, \frac{-102}{81}, \frac{-4}{5}, \frac{7}{12}, \frac{3}{4}, \frac{101}{100}, \frac{22}{19}, \frac{15}{11}$

3. Question

Write 5 rational number between $\frac{2}{5}$ and $\frac{3}{5}$, having the same denominators.

Answer

The rational numbers $\frac{2}{5}$ and $\frac{3}{5}$ can also be written as $\frac{2\times6}{5\times6} = \frac{12}{30}$ and $\frac{3\times6}{5\times6} = \frac{18}{30}$

So, the five rational number lies between $\frac{12}{30}$ and $\frac{18}{30}$ are $\frac{13}{30}, \frac{14}{30}, \frac{15}{30}, \frac{16}{30}, \frac{17}{30}$.

4. Question

How many positive rational numbers less than 1 are there such that the sum of the numerator and denominator does not exceed 10?

Answer

The positive rational numbers less than 1 whose sum of numerator and denominator does not exceed 10 are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{2}{3}, \frac{2}{5}, \frac{2}{7}, \frac{3}{4}, \frac{3}{5}, \frac{3}{7}, \frac{4}{5}$.

There are 15 rational numbers.

5. Question

Suppose $\frac{m}{n}$ and $\frac{p}{q}$ are two positive rational numbers. Where does $\frac{m+p}{n+q}$ lie, with respect to $\frac{m}{n}$ and $\frac{p}{q}$?

Answer

I didn't remember the concept used in this question.

There will be three cases for the question given above.

Case 1: when m/n >1 and p/q >1, this situation means that p > q and m > n

And this means that m + p > n + q and And this will mean $\frac{m+p}{n+q}$ > 1 and it will lie between m/p and n/q

Case 2: when m/n <1 and p/q <1, this situation means m < n and p < q

And thus m + p < n + q and this will mean $\frac{m+p}{n+q}$ < 1

And thus $\frac{m+p}{n+q}$ will lie less than m/p and n/q

Case 3: when any one is less than 1, then the case gets ambiguous.

Let m/n <1 and p/q >1

Now, m < n and p > q

Now we don't know the nature of $\frac{m+p}{n+q}$

And so this depends on the numbers taken.

6. Question

How many rational numbers are there strictly between 0 to 1 such that the denominator of the rational number is 80?

Answer

The rational numbers between 0 and 1 are in the form of $\frac{x}{80}$ having denominator as 80.

Value of x ranges from 1 to 79. The rational numbers are $\frac{1}{80}, \frac{2}{80}, \frac{3}{80} \ldots \ldots \ldots \ldots \ldots \ldots \frac{78}{80}, \frac{79}{80}.$

So there are 79 rational numbers lies between 0 and 1 having denominator as 80.

7. Question

How many rational numbers are there strictly between 0 to 1 with the property that the sum of the numerator and denominator is 70?

Answer

The rational number is in the form of $\frac{x}{y}$ and $x < y$,

According to the question, $x + y = 70$

So the rational numbers are $\frac{1}{69}, \frac{2}{68}, \frac{3}{67}, \frac{4}{66}, \frac{5}{65} \ldots \ldots \frac{30}{40}, \frac{31}{39}, \frac{32}{38}, \frac{33}{37}, \frac{34}{36}.$

There are 34 rational numbers following the conditions $x + y = 70$ and $x < y$.

Additional Problems 7

1. Question

Fill in the blanks:

(a) The number 0 is not in the set of ________ .

(b) The least number in the set of all whole numbers is _________.

(c) The least number in the set of all even natural numbers is ________.

(d) The successor of 8 in the set of all natural numbers is __________.

(e) The sum of two odd integers is ___________.

(f) The product of two odd integers is ___________.

Answer

(a) The number 0 is not in the set of **natural numbers.**

0 is added to set of natural number (1,2,3,4,.....) and it becomes set of whole numbers (0,1,2,3,4,....)

(b) The least number in the set of all the whole numbers is **0.**

0 is the smallest number which when added to the set of natural number make it a set of whole number.

(c) The least number in the set of all even natural numbers is **2.**

The set of even natural number starts from (2,4,6,....) so the least number in the set of all even natural numbers is number 2 .

(d) The successor of 8 in the set of all natural numbers is **9.**

The number succeeding number 8 is number 9 in the set of all natural numbers as the set of natural number consist of term (1,2,3,4,5,6,7,8,9,10,11,12,.....)

(e) The sum of two odd integers is **always an even number**

for e.g;- 3 + 5 = 8 , 5 + 7 = 12

Let m and n be odd integers. By definition of odd we have that m = 2a + 1 and

n = 2b + 1. Consider the sum m + n = (2a + 1) + (2b + 1) = 2ab + 2b + 2 = 2k, where

k = a + b + 1 is an integer. Therefore by definition of even we have shown that

m + n is even.

(f) The product of two odd integers is **odd.**

Let a and b be two odd integers.

By definition of odd we know that a = 2m + 1 and b = 2n + 1.

Now, the product ab = (2m + 1)(2n + 1)

= 4mn + 2m + 2n + 1

= 2(2mn + m + n) + 1 = 2k + 1, where k = (2mn + m + n) is an integer. Therefore by

Definition of odd number, the product of two odd integers is also odd.

2. Question

State whether the following statements are true or false:

(a) The set of all even natural numbers is a smallest element.

(b) Every non-empty subset of ℤ has the smallest element.

(c) Every integer can be identified with a rational number.

(d) For each rational number, one can find the next rational number.

(e) There is the largest rational number.

(f) Every integer is either even or odd.

(g) Between any two rational numbers, there is an integer.

Answer

(a) The set of all even natural numbers is a smallest element – False

The set of all even natural number consist of elements (2, 4, 6, 8,10,……) but the smallest element is 0 which is not included in the set of natural numbers and the smallest natural number is 1 which is also not included in set even numbers.

(b) Every non-empty subset of ℤ has the smallest element. – true

Every non empty subset of Z have the smallest element. It depends on the conditions attached to the non-empty subset such as the set of even natural numbers has 2 as its least element. The set of prime numbers bigger than 5 has 7 as its least element

(c) Every integer can be identified with a rational number- true

Every integer is a rational number as each integer n can be written in the form $\frac{n}{1}$.

For example 7 = 7/1 and thus 7 is a rational number.

(d) For each rational number, one can find the next rational number.- False

One cannot always find the next rational number for each rational number as there also lies irrational numbers in-between.

(e) There is the largest rational number.- False

The largest rational number cannot be determined as there are infinites numbers and each number can be expressed as a rational number.

(f) Every integer is either even or odd – true

We know n is even if n = 2k for some integer k and n is odd if n = 2k + 1 for some integer k.

Every integer can be expressed as 2k or 2k + 1 hence every integer is either odd or even.

(g) Between any two rational numbers, there is an integer. - false

Rational number can be any number 5/1, 2/1, which can be expressed in form of fraction and integers are the whole numbers which are not a fraction so there cannot lie a whole number between two rational number which are fractions.

3. Question

Simplify:

(i) 100(100 – 3) – (100 × 100 – 3);

(ii) (20–(2011–201) + (2011–(201–20))

Answer

(i) 100(100 – 3) – (100 × 100 – 3)

= 100 × 100 - 100 × 3 – 100 × 100 + 3

= - 300 + 3 (cancellation property)

= - 297

(ii) (20 – (2011 – 201) + (2011– (201 – 20))

= 20 - 2011 + 201 + 2011 – 201 + 20

= 40 (cancellation property)

4. Question

Suppose m is an integer such m ≠ −1 and m ≠ −2. Which is larger $\frac{m}{m+1}$ or $\frac{m+1}{m+2}$? State your reasons.

Answer

Given m is an integer and m ≠ -1 and -2

Let m = 1

Then $\frac{m}{m+1} = \frac{1}{2}$ and $\frac{m+1}{m+2} = \frac{2}{3}$

Comparing $\frac{1}{2}$ and $\frac{2}{3}$

⇒ $\frac{1}{6} < \frac{4}{6}$ (taking the LCM)

Hence $\frac{m+1}{m+2} > \frac{m}{m+1}$

If m < -2

Then let m = -3

$$\frac{m}{m + 1} = \frac{-3}{-3 + 1}$$

$$= \frac{-3}{-2}$$

$$= \frac{3}{2}$$

And $\frac{m+1}{m+2} = \frac{-3+1}{-3+2} = -\frac{2}{-1} = 2$

Here also $\frac{m+1}{m+2} > \frac{m}{m+1}$

If m > -1

Let m = 4

$$\frac{m}{m + 1} = \frac{4}{5}$$

And $\frac{m+1}{m+2} = \frac{5}{6}$

Comparing $\frac{4}{5}$ and $\frac{5}{6}$

Taking LCM

$\Rightarrow \frac{24}{30} < \frac{25}{30}$

For every integer this will hold true that $\frac{m+1}{m+2} > \frac{m}{m+1}$

5. Question

Define an operation * on the set of all rational numbers □ as follows:

r*s = r + s −(r × s),

for any two rational numbers r, s. Answer the following with justification:

(i) Is ℚ closed under the operation *?

(ii) Is * an associative operation of ℚ ?

(iii) Is * a commutative operation of ℚ?

(iv) What is a * 1 for any a in ℚ?

(v) Find two integers a ≠ 0 and b ≠ 0 such that a * b = 0.

Answer

For the operation r*s = r + s −(r × s)**(i)** Let r = 1/2 and s = 1/4Now ,

$$\frac{1}{2} * \frac{1}{4} = \frac{1}{2} + \frac{1}{4} - \left(\frac{1}{2} \times \frac{1}{4}\right) = \frac{1}{2} + \frac{1}{4} - \frac{1}{8}$$

$= \frac{4+2-1}{8} = \frac{5}{8}$ Which is also a rational number.Hence Q is closed under the operation * .

(ii) Let three rational number be o,p,qFor the operation r*s = r + s −(r × s) (o*p)*q = [o + p -(o × p)]*q = [o + p - op]*q = [(o + p - op) + q - ((o + p - op) × q)] = [o + p - op + q - oq - pq +opq] ..... (1)Now,o*(p*q) = o* [p + q - (p × q)] = o* [p + q - pq] = [(o + (p + q - pq) - (o ×(p + q - pq))] = [o + p + q - pq - op - oq + opq] ..... (2)From 1 and 2(o*p)*q = o*(p*q)Hence * is an associative operation.

(iii) Let two rational number be p,q.For the operation r*s = r + s −(r × s)p*q = p + q - (p × q) = p + q - pqNow,q*p = q + p - (q × p) = q + p - qpHence * is a commutative operation.

(iv) For the operation r*s = r + s −(r × s)a * 1 = a + 1 - (a × 1) = a + 1 - a = 1For any a in Q a * 1 is 1.

(v) The value of two integers would be a = 2 and b = 2

For the operation r*s = r + s −(r × s)

a * b = 0 a + b - (a × b) = 0 a + b - ab = 0a + b (1 - a) = 0b (1- a) = -ab = - a/(1 - a)Hence the value is a , - a/(1 - a).

6. Question

Find the multiplicative inverses of the following rational numbers.

$$\frac{8}{13}, \frac{12}{17}, \frac{26}{23}, \frac{-13}{11}, \frac{-101}{100}.$$

Answer

The multiplicative inverse of number is given by the reciprocal of the integer, hence the multiplicative inverse of the given numbers are :-

$$\frac{8}{13} = \frac{13}{8}$$

$$\frac{12}{17} = \frac{17}{12}$$

$$\frac{26}{23} = \frac{23}{26}$$

$$-\frac{13}{11} = -\frac{11}{13}$$

$$-\frac{101}{100} = -\frac{100}{101}$$

7. Question

Write the following in increasing order:

$$\frac{10}{13}, \frac{20}{26}, \frac{5}{6}, \frac{40}{43}, \frac{25}{28}, \frac{10}{11}.$$

Answer

For arranging rational numbers in increasing or decreasing order, always make sure that the denominator is same.

For given numbers, make the denominator same by taking L.C.M of denominators.

Denominators are – 13, 26, 6, 43, 28, 11

Let us write down the prime factors of numbers

13 = 13 × 1

26 = 13 × 2

6 = 2 × 3

43 = 43 × 1

28 = 2 × 2 × 7

11 = 11 × 1

L.C.M of the given numbers will be = 13 × 2 × 3 × 2 × 11 × 43 × 7

= 516516

Now lets make the denominators equal by multiplying and dividing by terms.

$$\frac{10}{13} = \frac{10 \times 39732}{13 \times 39732} = \frac{397320}{516516}$$

And Similarly all our numbers will become as

$$\frac{397320}{516516}, \frac{397320}{516516}, \frac{430430}{516516}, \frac{480480}{516516}, \frac{461175}{516516}, \frac{469560}{516516}$$

Now arranging in increasing order we get,

$$\frac{397320}{516516} = \frac{397320}{516516} < \frac{430430}{516516} < \frac{461175}{516516} < \frac{469560}{516516} < \frac{480480}{516516}$$

Hence we have the order as

$10/13 = 20/26 < 5/6 < 25/28 < 10/11 < 40/43$

8. Question

Write the following in decreasing order:

$$\frac{21}{17}, \frac{31}{27}, \frac{13}{11}, \frac{41}{37}, \frac{51}{47}, \frac{9}{8}.$$

Answer

Now for arranging rational numbers in increasing or decreasing order make the denominator of numbers equal. But as we can see from the numbers that would make the whole process very long.

We can divide the numbers and can calculate the approximate values.

21/17 = 1.235

31/27 = 1.148

13/11 = 1.18

41/37 = 1.10

51/47 = 1.08

9/8 = 1.125

Now we can easily arrange the numbers as follows

21/17 > 13/11 > 31/27 > 9/8 > 41/37 > 51/47

9. Question

(a) What is the additive inverse of 0?

(b) What is the multiplicative inverse of 1?

(c) Which integers have multiplicative inverses?

Answer

(a) The additive inverse of 0 is 0 itself because 0 is neither positive nor negative number.

(b) The multiplicative inverse of 1 is 1 as 1 can be written as 1/1 which itself the reciprocal of the 1.

(c) (1, -1) is a pair of integers having multiplicative inverse.

10. Question

In the set of all rational numbers, give 2 examples each illustrating the following properties:

(i) associativity

(ii) commutativity

(iii) distributivity of multiplication over addition.

Answer

(i) In the set of all rational numbers associativity in addition holds true

e.g.; A) let $a = \frac{3}{2}$, $b = \frac{1}{2}$ and $c = \frac{5}{2}$

Now a + (b + c) = (a + b) + c

LHS $\frac{3}{2} + \left(\frac{1}{2} + \frac{5}{2}\right)$

$= \frac{3}{2} + \frac{6}{2}$

$= \frac{9}{2}$

RHS $\left(\frac{3}{2} + \frac{1}{2}\right) + \frac{5}{2}$

$= \frac{4}{2} + \frac{5}{2}$

$= \frac{9}{2}$

LHS = RHS hence associativity holds true over addition in rational numbers

(ii) let us check for subtraction a –(b - c) = (a- b) - c

RHS $\left(\frac{3}{2} - \frac{1}{2}\right) - \frac{5}{2}$

$= \frac{2}{2} - \frac{5}{2}$

$= -\frac{3}{2}$

LHS $\frac{3}{2} - \left(\frac{1}{2} - \frac{5}{2}\right) = \frac{3}{2} - \left(-\frac{4}{2}\right) = \frac{3}{2} + \frac{4}{2} = \frac{7}{2}$

Hence associativity doesn't hold true with subtraction in rational numbers

(ii) For two rational number addition and multiplication are commutative and subtraction and division are not commutative

e.g $\frac{2}{5} + \frac{4}{5} = \frac{6}{5}$

And $\frac{4}{5} + \frac{2}{5} = \frac{6}{5}$

$\frac{2}{5} \times \frac{7}{5} = \frac{17}{25}$ and $\frac{7}{5} \times \frac{2}{5} = \frac{14}{25}$

But $\frac{2}{5} - \frac{4}{5} = -\frac{2}{5}$ and $\frac{4}{5} - \frac{2}{5} = \frac{2}{5}$ are not same hence commutate property is not true with division and subtraction

(iii) E.g $\frac{3}{2}\left(\frac{3}{2} + \frac{5}{2}\right) = \frac{3}{2} \times \frac{3}{2} + \frac{3}{2} \times \frac{5}{2}$

LHS = $\frac{3}{2}\left(\frac{8}{2}\right) = \frac{24}{2}$

RHS = $\frac{9}{4} + \frac{15}{4} = \frac{24}{4}$

LHS = RHS

e.g $\frac{4}{5}\left(\frac{7}{4} + \frac{3}{4}\right) = \frac{4}{5} \times \frac{7}{4} + \frac{4}{5} \times \frac{3}{4}$

RHS

$= \frac{4}{5} \times \frac{7}{4} + \frac{4}{5} \times \frac{3}{4}$

$= \frac{28}{20} + \frac{12}{20}$

$= \frac{40}{20}$

LHS $\frac{4}{5}\left(\frac{7}{4} + \frac{3}{4}\right)$

$= \frac{4}{5}\left(\frac{10}{4}\right)$

$= \frac{40}{20}$

LHS = RHS

Hence the distributivity of multiplication over addition holds true for rational numbers

11. Question

Simplify the following using distributive property:

(i) $\frac{2}{5}\times\left(\frac{1}{9}+\frac{2}{5}\right)$

(ii) $\frac{5}{12}\times\left(\frac{25}{9}+\frac{32}{5}\right)$

(iii) $\frac{8}{9}\times\left(\frac{11}{2}+\frac{2}{9}\right)$

Answer

(i) $\frac{2}{5}\times\frac{1}{9}+\frac{2}{5}\times\frac{2}{5}$

$=\frac{2}{45}+\frac{4}{25}$

$=\frac{10+36}{225}$

$=\frac{46}{225}$

(ii) $\frac{5}{12}\times\frac{25}{9}+\frac{32}{5}\times\frac{5}{12}$

$=\frac{125}{108}+\frac{160}{60}$

$=\frac{125+288}{108}$

$=\left(\frac{413}{108}\right)$

(iii) $\frac{8}{9}\times\frac{11}{2}+\frac{2}{9}\times\frac{8}{9}$

$=\frac{88}{18}+\frac{16}{81}$

$=\frac{792+32}{162}$

$=\frac{824}{162}=\frac{412}{81}$

12. Question

Simplify the following:

(i) $\left(\frac{25}{9}+\frac{12}{3}\right)+\frac{3}{5}$

(ii) $\left(\frac{22}{7}+\frac{36}{5}\right)+\frac{6}{7}$

(iii) $\left(\frac{51}{2}+\frac{7}{6}\right)\div\frac{3}{5}$

(iv) $\left(\frac{16}{7}+\frac{21}{8}\right)\times\left(\frac{15}{3}-\frac{2}{9}\right)$

Answer

(i) $\frac{25+36}{9}+\frac{3}{5}$

$=\frac{61}{9}+\frac{3}{5}$

$=\frac{305+27}{45}$

$=\frac{332}{45}$

(ii) $\left(\frac{110+252}{35}\right)+\frac{6}{7}$

$=\left(\frac{262}{35}\right)+\frac{6}{7}$

$=\frac{262+30}{35}$

$=\frac{292}{30}$

(iii) $\frac{153+7}{6}\div\frac{3}{5}$

$=\frac{160}{6}\times\frac{5}{3}$

$=\frac{80}{3}\times\frac{5}{3}$

$= \frac{400}{9}$

(iv) $\left(\frac{16}{7}+\frac{21}{8}\right)\times\left(\frac{15}{3}-\frac{2}{9}\right)$

$= \left(\frac{128+147}{56}\right) \times \left(\frac{45-2}{9}\right)$

$= \frac{275}{56} \times \frac{43}{9}$

$= \frac{11825}{504}$

13. Question

Which is the property that is there in the set of all rationals but not in the set of all integers?

Answer

Every non zero rational number is invertible, but only ± 1 are invertible integers.

14. Question

What is the value of

$$1+\frac{1}{1+\frac{1}{1+1}}$$

Answer

The value is

$= 1 + \frac{1}{\frac{2+1}{2}}$

$= 1 + \frac{2}{3}$

$= \frac{3 + 2}{3}$

$= \frac{5}{3}$

15. Question

Find the value of

$$\left(\frac{1}{3}-\frac{1}{4}\right)\div\left(\frac{1}{2}-\frac{1}{3}\right)$$

Answer

$$\left(\frac{1}{3}-\frac{1}{4}\right)\div\left(\frac{1}{2}-\frac{1}{3}\right)$$

$$= \left(\frac{4-3}{12}\right) \div \left(\frac{3-2}{6}\right)$$

$$= \frac{1}{12} \div \frac{1}{6}$$

$$= \frac{1}{12} \times 6$$

$$= \frac{1}{2}$$

16. Question

Find all rational numbers each of which is equal to its reciprocal.

Answer

The only rational number which is equal to its reciprocal is ± 1

17. Question

A bus shuttles between two neigbouring towns every two hours. It starts from 8 AM in the morning and the last trip is at 6 PM. On one day the driver observed that the first trip had 30 passengers and each subsequent trip had one passenger less than the previous trip. How many passengers travelled on that day?

Answer

The number of trips made by the driver on particular day are 5 (8-10, 10-12, 12-2, 2-4, 4-6)

Number of passenger keeps on decreasing with each trip so total passengers carried on that day

= 30 + 29 + 28 + 27 + 26

= 140

18. Question

How many rational numbers $\frac{p}{q}$ are there between 0 and 1 for which q < p?

Answer

There are no rational numbers between 0 and 1, of the form $\frac{p}{q}$ where $q < p$ as after the division the rational number obtained would always be greater than 1.

19. Question

Find all integers such that $\frac{3n+4}{n+2}$ is also an integer.

Answer

Let n = 0

Then $\frac{3n+4}{n+2} = \frac{4}{2} = 2$, which is an integer

Let n = 1

$\frac{3n + 4}{n + 2} = \frac{7}{3}$, is not integer

Let n = -1

$$\frac{3n + 4}{n + 2} = \frac{-3 + 4}{-1 + 2}$$

$$= \frac{1}{-1}$$

$= -1$ which is an integer

Let n = -3

$$\frac{3n + 4}{n + 2} = \frac{3 \times -3 + 4}{-3 + 2}$$

$$= \frac{-9 + 4}{-1}$$

$$= -\frac{5}{-1}$$

$= 5$, which is an integer

Let n = -4

$$\frac{3n + 4}{n + 2} = \frac{3 \times -4 + 4}{-4 + 2}$$

$$= \frac{-12 + 4}{-2}$$

$$= -\frac{8}{-2}$$

$= 4$, which is an integer

20. Question

By inserting parenthesis (that is brackets), you can get several values for 2 × 3 + 4 × 5. (For example ((2 × 3) + 5 is one way of inserting parenthesis.) How many such values are there?

Answer

The values are

(2 × 3) + 4 = 6 + 4 = 10

3 + (4 × 5) = 3 + 20 = 23

2 + (4 × 5) = 2 + 20 = 22

(2 × 3 + 4) × 5 = (6 + 4) × 5

= 10 × 5 = 50

21. Question

Suppose $\frac{p}{q}$ is a positive rational in its lowest form. Prove that $\frac{1}{q} + \frac{1}{p+q}$ is also in its lowest form.

Answer

Now, $\frac{1}{q} + \frac{1}{p+q}$

$= \frac{(p+q)+q}{q(p+q)}$

$= \frac{p+2q}{q(p+q)}$

Now note that a common factor of p + 2q and q is also a factor of (p + 2q)−2·q = p, therefore must be 1.

Also a common factor of p + 2q and p + q is a factor of both (p + 2q) − (p + q) = q and 2·(p + q) − (p + 2q) = p hence must be 1.

Hence it is in its lowest form.

22. Question

Show that for each natural number n, the fraction $\frac{14n+3}{21n+4}$ is in its lowest form.

Answer

Let n = 1

Then $\frac{14n+3}{21n+4} = \frac{14+3}{21+4} = \frac{17}{25}$ which is in its lowest form.

Let n = 3

$$\frac{14n + 3}{21n + 4}$$

$$= \frac{14 \times 3 + 3}{21 \times 3 + 4}$$

$= \frac{42+3}{63+4} = \frac{45}{67}$ which is again in its lowest form

Hence for every value of n natural number, $\frac{14n+3}{21n+4}$ is in its lowest form.

23. Question

Find all integers n for which the number (n + 3)(n – 1) is also an integer.

Answer

Let n = 0

= (0 + 3)(0-1)

= (3)(-1)

= -3 which is also an integer

Let n = -1

= (-1 + 3)(-1-1)

= (2)(-2)

= -4 which is also an integer

Let n = 1

= (1 + 3)(1-1)

$= (4)(0)$

$= 0$, which is also an integer

Therefore, for any integer value of n the equation will result an integer.