Sample Question Paper - 15
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

1. Solve for x (in terms of a and b) :

$\frac{a}{x-b}+\frac{b}{x-a}=2, x \neq a, b$

OR

Value of the roots of the quadratic equation, $x^2-x-6=0$ are ..... .

2. If the 1^{st} term of a series is 7 and 13^{th} term is 35. Find the sum of 13 terms of the sequence.

3. A circle is inscribed in a ΔABC touching AB, BC and AC at P, Q and R respectively. If $AB=10\,\text{cm}$ $AR=7\,\text{cm}$ and $CR=5\,\text{cm}$, then find the length of BC

4. A solid metallic of dimensions $9\text{m} \times 8\text{m} \times 2$ m is melted and recast into solid cubes of edge 2 m. Find the number of cubes so formed.

5. Write the relationship connecting three measures of central tendencies. Hence find the median of the give data if mode is 24.5 and mean is 29.75.

6. The following distribution shows the marks scored by 140 students in an examination. Calculate the mode of the distribution :

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	20	24	40	36	20

OR

Calculate the median from the following data :

Marks	0-10	10-20	20-30	30-40	40-50
Number of Students	5	15	30	8	2

Section B

7. Solve the following equation: $\frac{1}{x} - \frac{1}{x-2} = 3,\ x \neq 0, 2$

8. The 17^{th} term of an AP is 5 more than twice its 8^{th} term. If 11^{th} term of AP is 43, then find its n^{th} term.

9. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 18 minutes for the angle of depression to change from 30° to 60°, how soon after this will the car reach the tower?

10. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also to verify the measurement by actual calculation.

OR

raw a circle of radius 2 cm with centre O and take a point P outside the circle such that $OP = 6.5$ cm. From P , draw two tangents to the circle.

Section C

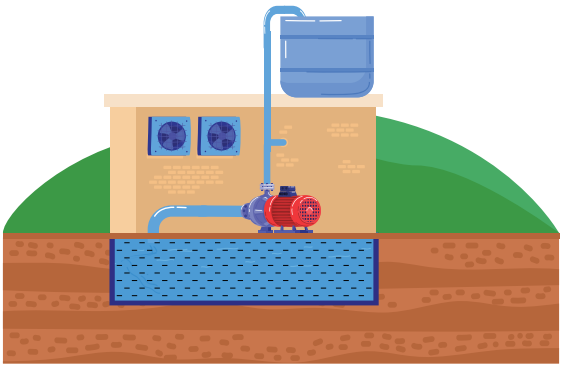
11. The angle of depression of two ships from an aeroplane flying at the height of 7500 m are 30° and 45°. If both the ships are in the same that one ship is exactly behind the other, find the distance between the ships.

12. Prove that tangent drawn at any point of a circle perpendicular to the radius through the point contact.

OR

Prove that tangent drawn at any point of a circle perpendicular to the radius through the point contact.

13. Underground water tank is popular in India. It is usually used for large water tank storage and can be built cheaply using cement-like materials. Underground water tanks are typically chosen by people who want to save space. The water in the underground tank is not affected by extreme weather conditions. The underground tanks maintain cool temperatures in both winter and summer. Electric pump is used to move water from the underground tank to overhead tank.



Ramesh has build recently his house and installed a underground tank and overhead tank. Dimensions of tanks are as follows :
Underground Tank : Base 2 m × 2 m and Height 1.1 m.
Overhead tank : Radius 50 cm and Height 175 cm

(i) What is the capacity of the underground tank ?
(ii) What is the ratio of the capacity of the underground tank to the capacity of the overhead tank?

14. An inspector in an enforcement squad of electricity department visit to a locality of 100 families and record their monthly consumption of electricity, on the basis of family members, electronic items in the house and wastage of electricity, which is summarise in the following table.

Monthly Consumption (in kwh)	Number of families
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Inspector calculated that median of the above data is 525 and after that he lost two data which is given as x and y in table.
Based on the above information, answer the following questions.
(i) What is the value of lost data x ?
(ii) What is the value of lost data y ?

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
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SECTION A

1. Solve for x (in terms of a and b) :

$\frac{a}{x-b}+\frac{b}{x-a}=2, x \neq a, b$

Ans :

We have $\frac{a(x-a)+b(x-b)}{(x-b)(x-a)}=2$

$a(x-a)+b(x-b) = 2\left[x^2-(a+b)x+ab\right]$

$ax-a^2+bx-b^2 = 2x^2-2(a+b)x+2ab$

$2x^2-3(a+b)x+(a+b)^2 = 0$

$2x^2-2(a+b)x-(a-b)x+(a+b)^2 = 0$

$\left[2x-(a+b)\right]\left[x-(a+b)\right] = 0$

Thus $x = a+b, \frac{a+b}{2}$

OR

Value of the roots of the quadratic equation, $x^2-x-6=0$ are ..... .

Ans :

$x^2-x-6 = 0$

$x^2-3x+2x-6 = 0$

$x(x-3)+2(x-3) = 0$

$(x-3)(x+2) = 0 \Rightarrow x = 3$ and $x = -2$

2. If the 1^{st} term of a series is 7 and 13^{th} term is 35. Find the sum of 13 terms of the sequence.

Ans :

Let the first term be a, common difference be d, n th term be a_n and sum of n term be S_n.

Here $a=7, a_{13}=35$

$a_n = a+(n-1)d$

$a_{13} = a+12d$

$35 = 7+12d \Rightarrow d = \frac{7}{3}$

Now $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$S_{13} = \frac{13}{2}\left[2\times 7+12\times\left(\frac{7}{3}\right)\right]$

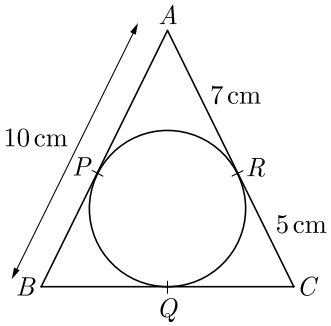
$= \frac{13}{2}[14+28] = \frac{13}{2}\times 42 = 273$

3. A circle is inscribed in a ΔABC touching AB, BC and AC at P, Q and R respectively. If $AB=10$ cm $AR=7$ cm and $CR=5$ cm, then find the length of BC

Ans :

As per given information we have drawn the figure below.

Here a circle is inscribed in a ΔABC touching AB, BC and AC at P, Q and R respectively.



Since, tangents drawn to a circle from an external point are equal,

$AP = AR = 7$ cm

$CQ = CR = 5$ cm

Now, $BP = (AB - AP) = 10 - 7 = 3$ cm

$BP = BQ = 3$ cm

$BC = BQ + QC = 3 + 5 = 8$ cm

4. A solid metallic of dimensions 9m $\times$ 8m $\times$ 2 m is melted and recast into solid cubes of edge 2 m. Find the number of cubes so formed.

Ans :

Volume of cuboid $= 9 \times 8 \times 2$ cm^3

Volume of cube $= 2^3$ cm^3

Let number of recast cubes be n.

Volume of n cubes = Volume of cuboid

$$n2^3 = 9 \times 8 \times 2$$

$$n \times 2 \times 2 \times 2 = 9 \times 8 \times 2$$

$$n = \frac{9 \times 8 \times 2}{2 \times 2 \times 2} = 18$$

Hence, number of cubes recast is 18.

5. Write the relationship connecting three measures of central tendencies. Hence find the median of the give data if mode is 24.5 and mean is 29.75.

Ans :

Mode, $M_o = 24.5$

and mean, $M = 29.75$

The relationship connecting measures of central tendencies is,

$$3M_d = M_o + 2M$$

Thus $3M_d = 24.5 + 2 \times 59.50$

$= 24.5 + 59.50 = 84.0$

Median $M_d = \frac{84}{3} = 28$

6. The following distribution shows the marks scored by 140 students in an examination. Calculate the mode of the distribution :

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	20	24	40	36	20

Ans :

Class 20-30 has the maximum frequency 40, therefore this is model class.

Here, $l = 20, f_1 = 40, f_0 = 24, f_2 = 36, h = 10$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$= 20 + \frac{(40 - 24)}{80 - 24 - 36} \times 10$$

$$= 20 + \frac{16 \times 10}{20} = 28$$

OR

Calculate the median from the following data :

Marks	0-10	10-20	20-30	30-40	40-50
Number of Students	5	15	30	8	2

Ans :

We prepare following cumulative frequency table to find median class.

Marks	No. of students	c.f.
0-10	5	5
10-20	15	20
20-30	30	50
30-40	8	58
40-50	2	60
	$N = 60$	

We have $N = 60$; $\frac{N}{2} = 30$

Cumulative frequency just greater than $\frac{N}{2}$ is 50 and the corresponding class is 20-30. Thus median class is 20-20.

Now $l = 20,\ f = 30,\ F = 20,\ h = 10$

Median, M_d

$$= l + \left(\frac{\frac{N}{2} - F}{f}\right) \times h$$

$$= 20 + \left(\frac{30 - 20}{30}\right) \times 10$$

$$= 20 + \frac{100}{30} = 20 + 3.33$$

Thus $Md = 23.33$

Section B

7. Solve the following equation: $\frac{1}{x} - \frac{1}{x-2} = 3,\ x \neq 0, 2$

Ans :

We have $\frac{1}{x} - \frac{1}{x-2} = 3 \qquad (x \neq 0, 2)$

$$\frac{x - 2 - x}{x(x-2)} = 3$$

$$\frac{-2}{x(x-2)} = 3$$

$$3x(x-2) = -2$$

$$3x^2 - 6x + 2 = 0$$

Comparing it by $ax^2 + bx + c$, we get $a = 3$, $b = -6$ and $c = 2$.

Now, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{6 \pm \sqrt{36 - 24}}{6} = \frac{6 \pm \sqrt{12}}{6}$$

$$= \frac{6 \pm 2\sqrt{3}}{6}$$

$$= \frac{3 + \sqrt{3}}{3}, \frac{3 - \sqrt{3}}{3}$$

8. The 17^{th} term of an AP is 5 more than twice its 8^{th} term. If 11^{th} term of AP is 43, then find its n^{th} term.

Ans :

Let a be the first term and d be the common difference.

n^{th} term of an AP,

$$a_n = a + (n-1)\,d$$

Since 17^{th} term of an AP is 5 more than twice of its 8^{th} term, thus

$$a + (17-1)\,d = 5 + 2[a + (8-1)\,d]$$

$$a + 16d = 5 + 2(a + 7d)$$

$$a + 16d = 5 + 2a + 14d$$

$$2d - a = 5 \quad \ldots(1)$$

Since 11^{th} term of AP is 43,

$$a + (11-1)\,d = 43$$

$$a + 10d = 43 \quad \ldots(2)$$

Solving equation (1) and (2), we have

$$a = 3 \text{ and } d = 4$$

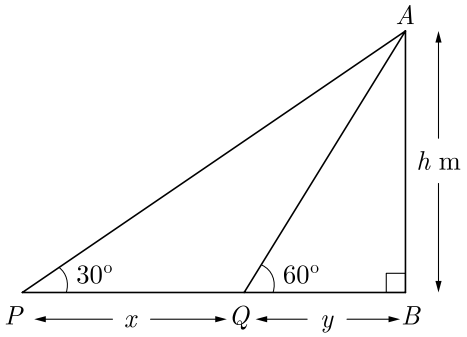
Hence, n^{th} term would be

$$a_n = 3 + (n-1)4 = 4n - 1$$

9. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 18 minutes for the angle of depression to change from 30° to 60°, how soon after this will the car reach the tower?

Ans :

Let h be the height of tower AB. Now as per given in question, we have drawn figure below.



In ΔABQ, $\tan 60^\circ = \frac{AB}{BQ}$

$$\sqrt{3} = \frac{h}{y}$$

$$y = \frac{h}{\sqrt{3}} = \frac{h\sqrt{3}}{3}$$

In ABP, $\tan 30^\circ = \frac{AB}{BP}$

$$\frac{1}{\sqrt{3}} = \frac{h}{x + y}$$

$$x + y = \sqrt{3}\,h$$

$$x = \sqrt{3}\,h - y$$

$$= \sqrt{3}\,h - \frac{\sqrt{3}\,h}{3}$$

$$= \frac{2\sqrt{3}\,h}{3}$$

Thus, speed of car $s = \frac{2\sqrt{3}\,h}{3 \times 18} = \frac{\sqrt{3}\,h}{27}$ m/min

Time for remaining distance,

$$t = \frac{\frac{h\sqrt{3}}{3}}{\frac{h\sqrt{3}}{27}} = 9 \text{ min}$$

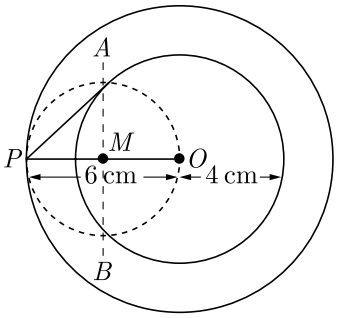
Hence, time taken by car is 9 min.

10. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also to verify the measurement by actual calculation.

Ans :

Steps of Construction :

1. Draw two concentric circles with centre O and radii 4 cm and 6 cm.
2. Now take any point P on outer circle.
3. Join PO and bisect it and let the midpoint of PO is represented by M.
4. Taking M as centre and OM or MP as radius, draw a circle such that this circle intersects the circle (of radius 4 cm) at A and B.
5. Join AP. PA is the required tangent.
   By measurement, $PA = 4.5$ cm



Justification :

Join OA. As PO is diameter

$$\angle PAO = 90^\circ$$

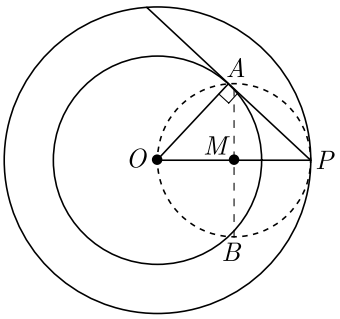
(Angle in a semi-circle)

$$PA \perp OA$$

OA is a radius of the inner circle.

Verification of length of PA. In right ΔPAO,

$$PO = 6\,\text{cm},\ OA = 4\,\text{cm}$$



$$PA = \sqrt{6^2 - 4^2} = \sqrt{36 - 16}$$
$$= \sqrt{20} = 4.47\,\text{cm}$$

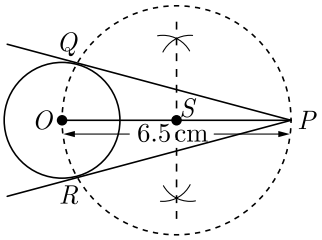
Hence, both lengths are approximately equal.

OR

raw a circle of radius 2 cm with centre O and take a point P outside the circle such that $OP = 6.5\,\text{cm}$. From P, draw two tangents to the circle.

Ans :

1. Draw a line segment OP of length 6.5 cm.
2. Draw a circle taking O as centre and radius 2 cm.
3. Taking OP as diameter draw another circle which intersects the first circle at Q and R.
4. Join P to Q and P to R. Hence PQ and PR are two tangents.

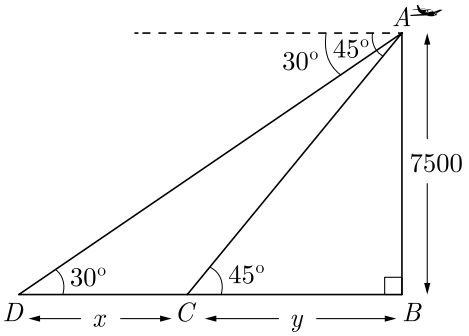


Section C

11. The angle of depression of two ships from an aeroplane flying at the height of 7500 m are 30° and 45°. If both the ships are in the same that one ship is exactly behind the other, find the distance between the ships.

Ans :

Let A, C and D be the position of aeroplane and two ship respectively. Aeroplane is flying at 7500 m height from point B. As per given in question we have drawn figure below.



In right ΔABC we have

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{7500}{y} = 1$$

$$y = 7500 \qquad ...(1)$$

In right ΔABD we have

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{7500}{x+y} = \frac{1}{\sqrt{3}}$$

$$x + y = 7500\sqrt{3} \qquad ...(2)$$

Substituting the value of y from (1) in (2) we have

$$x + 7500 = 7500\sqrt{3}$$
$$x = 7500\sqrt{3} - 7500$$
$$= 7500(\sqrt{3} - 1)$$

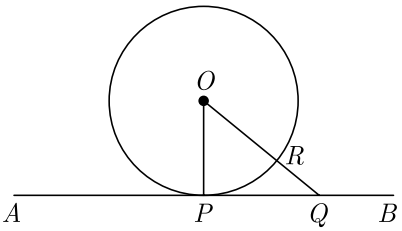
$$= 7500(1.73 - 1)$$
$$= 7500 \times 0.73 = 5475 \text{ m}$$

Hence, the distance between two ships is 5475 m.

12. Prove that tangent drawn at any point of a circle perpendicular to the radius through the point contact.

Ans :

Consider a circle with centre O with tangent AB at point of contact P as shown in figure below



Let Q be point on AB and we join OQ. Suppose it touch the circle at R.

We $\quad OP = OR \quad$ (Radius)

Clearly $\quad OQ > OR$

$\quad OQ > OP$

Same will be the case with all other points on circle. Hence OP is the smallest line that connect AB and smallest line is perpendicular.

Thus $\quad OP \perp AB$

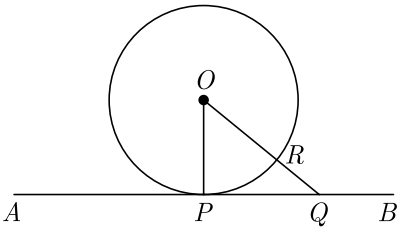
or, $\quad OP \perp PQ \quad$ Hence Proved

OR

Prove that tangent drawn at any point of a circle perpendicular to the radius through the point contact.

Ans :

Consider a circle with centre O with tangent AB at point of contact P as shown in figure below



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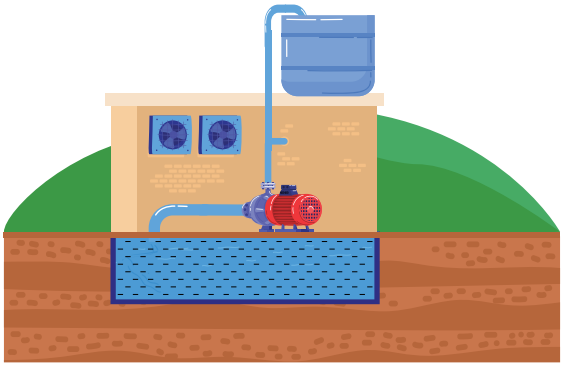
13. Underground water tank is popular in India. It is usually used for large water tank storage and can be built cheaply using cement-like materials. Underground water tanks are typically chosen by people who want to save space. The water in the underground tank is not affected by extreme weather conditions. The underground tanks maintain cool temperatures in both winter and summer. Electric pump is used to move water from the underground tank to overhead tank.



Ramesh has build recently his house and installed a underground tank and overhead tank. Dimensions of tanks are as follows :

Underground Tank : Base 2 m × 2 m and Height 1.1 m.

Overhead tank : Radius 50 cm and Height 175 cm

(i) What is the capacity of the underground tank ?

(ii) What is the ratio of the capacity of the underground tank to the capacity of the overhead tank?

Ans :

(i) Volume of underground tank,

$$lbh = 2 \times 2 \times 1.1 = 4.4 \text{ m}^3$$

Since 1 m^3 is equal to 1000 litres,

$$4.4 \text{ m}^3 = 4.4 \times 1000 = 4400 \text{ litres}$$

(ii) Radius of overhead is 50 cm i.e. $\frac{1}{2}$ meter and height is 175 cm i.e $1.75 = \frac{7}{4}$ metre.

Thus volume of overhead tank,

$$\pi r^2 h_{cy} = \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{7}{4} = \frac{11}{8} \text{ m}^3$$

$$\frac{\text{Capacity of sump}}{\text{Capacity of Overhead tank}} = \frac{lbh}{\pi r^2 h_{cy}} = \frac{4.4}{\frac{11}{8}} = 3.2$$

14. An inspector in an enforcement squad of electricity department visit to a locality of 100 families and record their monthly consumption of electricity, on the basis of family members, electronic items in the house and wastage of electricity, which is summarise in the following table.

Monthly Consumption (in kwh)	Number of families
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Inspector calculated that median of the above data is 525 and after that he lost two data which is given as x and y in table.
Based on the above information, answer the following questions.
(i) What is the value of lost data x ?
(ii) What is the value of lost data y ?

Ans :

We prepare following cumulative frequency table

Monthly Consumption (in kwh)	Number of families	Cumulative Frequency
0-100	2	2
100-200	5	7
200-300	x	$7+x$
300-400	12	$19+x$
400-500	17	$36+x$
500-600	20	$56+x$
600-700	y	$56+x+y$
700-800	9	$65+x+y$
800-900	7	$72+x+y$
900-1000	4	$76+x+y$
Total	$76+x+y$	

Since total frequency is 100,

$$76+x+y = 100$$
$$x+y = 100-76 = 24$$

Here median is 525, thus median class is 500-600.
Also $\frac{N}{2} = \frac{100}{2} = 50$.

Now, $l = 500$, $\frac{N}{2} = 50$, $F = 36+x$, $f = 20$ and $h = 100$.

$$\text{Median, } M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$525 = 500 + \left(\frac{50-36-x}{20}\right) \times 100$$

$$25 = (14-x) \times 5$$
$$25 = 70 - 5x$$
$$x = \frac{70-25}{5} = 9$$

Now $y = 24 - 9 = 15$

(i) Thus $x = 9$
(ii) $y = 15$