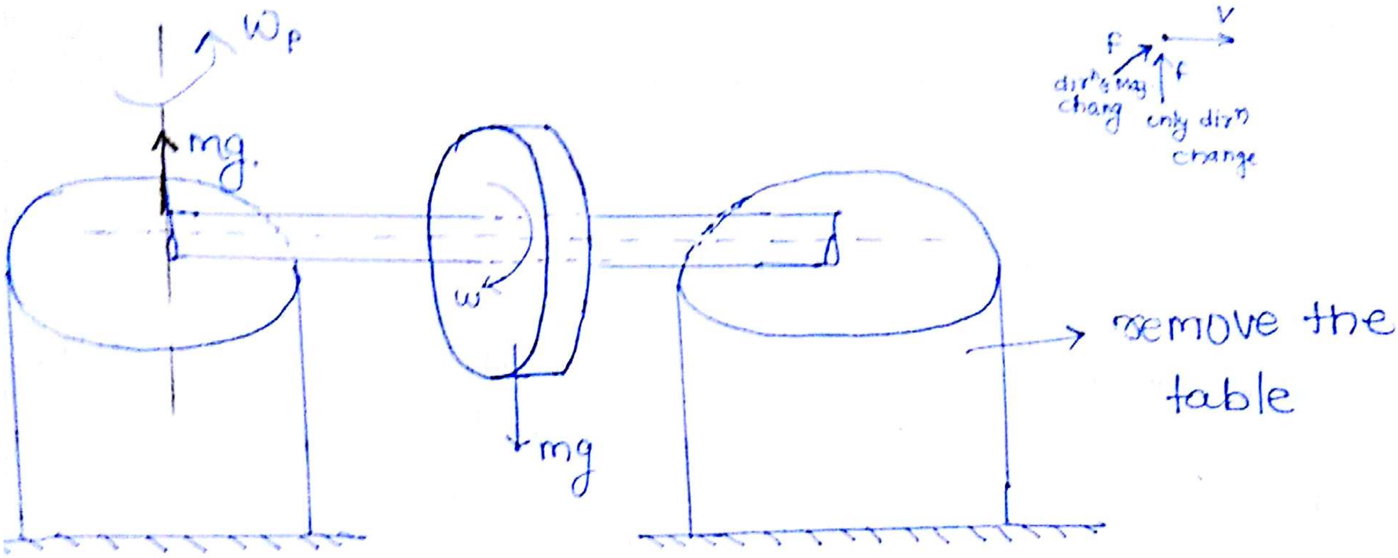
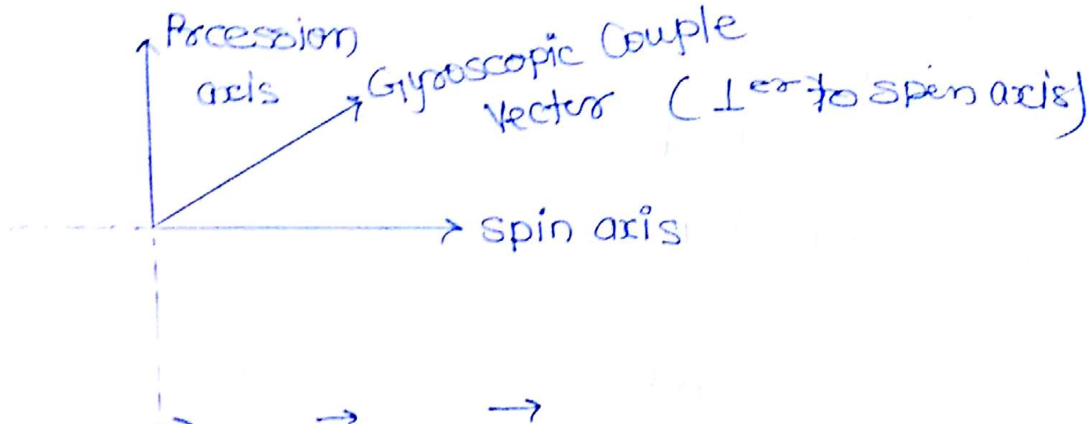
Gyroscope :





$$\vec{\alpha} = \vec{\alpha'} + \vec{\alpha''}$$

$\vec{\alpha'}$ — ‖el to $\vec{\omega}$

Cause change in mag. of ω (tangential accn)

$\frac{d\omega}{dt}$

$\vec{\alpha''}$ — ⊥r to $\vec{\omega}$

Cause change in dirn of ω

(Gyroscopic accn)

$\alpha'' = \omega \omega_p$

ω (tang accn) >> ω (Gyro)

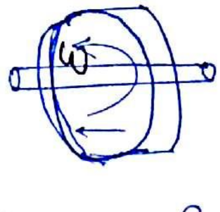
↓

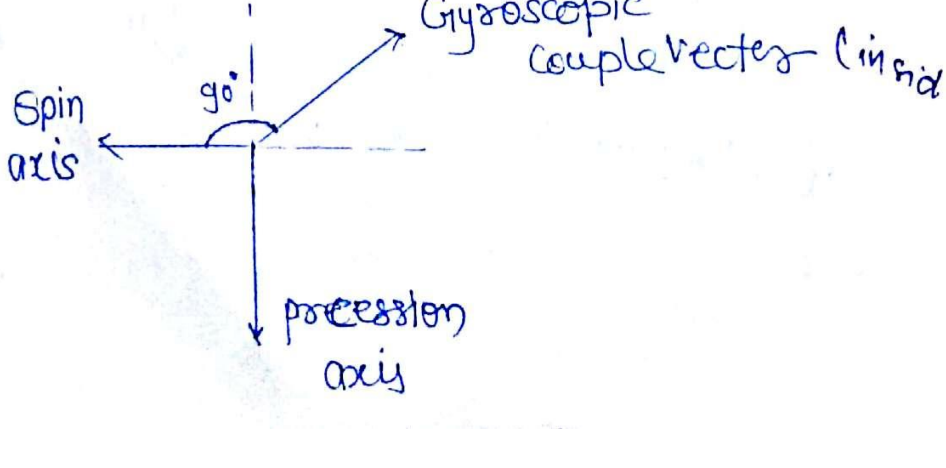
Const

A rotating rotor will not fall from left table when right hand table is suddenly remove as torque because couple of Mg is in the direction ⊥er to the dirn of angular velocity of spim So it will start processing accordingly to that torque and for the given case it will start processing about the axis ~~veti~~ vertically upward passing through both the table and rotor shaft (i.e. anticlock wise as viewed from top.)

Note:- Newton's second law is valid here also, shaft is not falling under gravity because net force on shaft is zero but net moment on shaft is non-zero. which is producing gyroscoping action.

★ ω dirn change

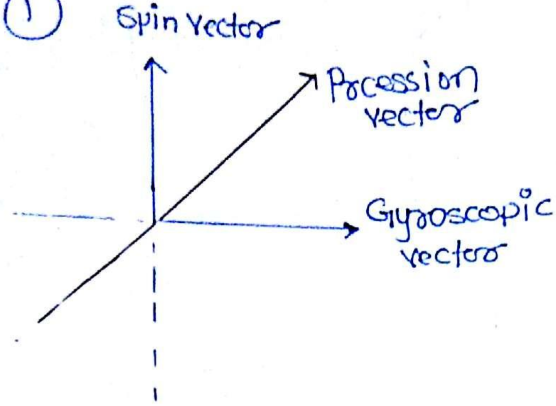




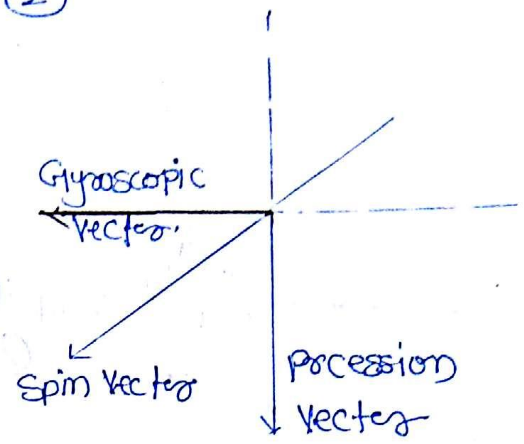
Direction of Gyroscope Couple Vector :-

Rotate the spin vector through 90° in the dir^n of procession about procession axis by right hand

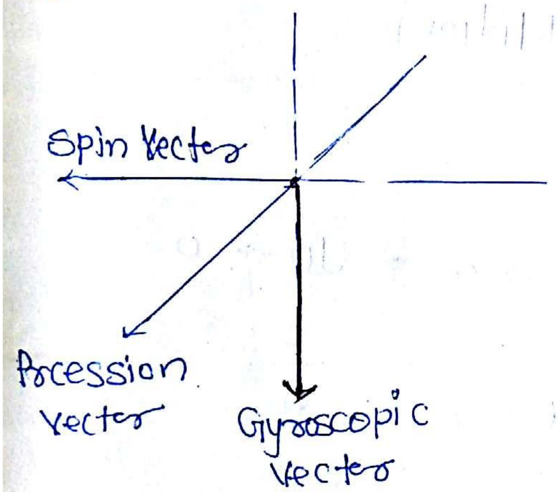
(1)



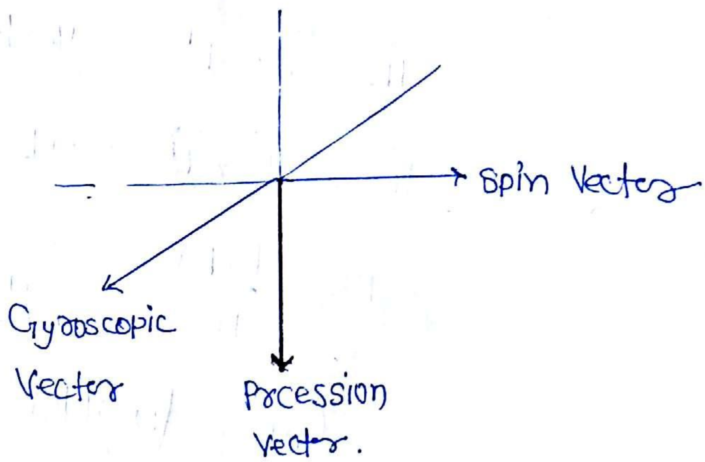
(2)



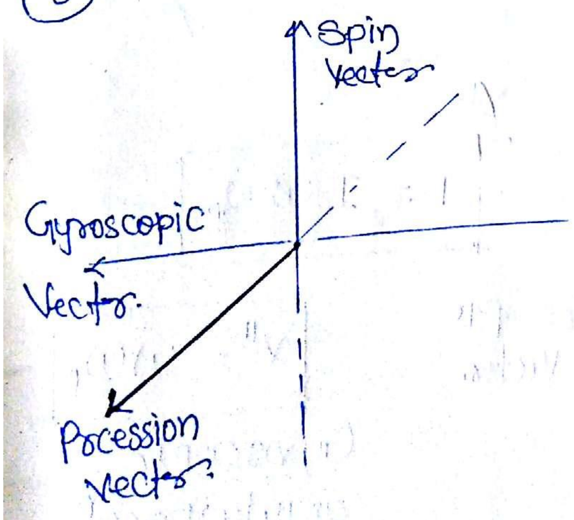
(3)



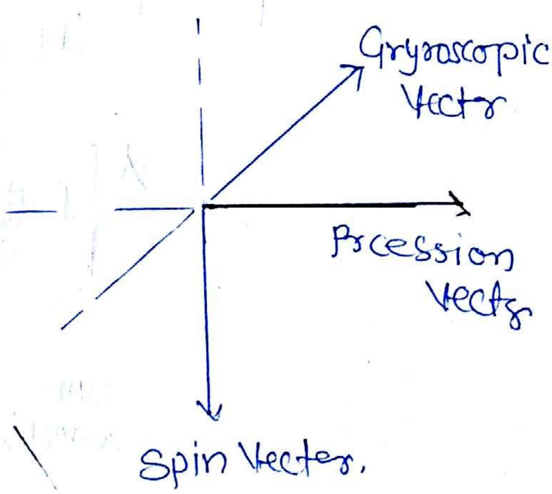
(4)



(5)



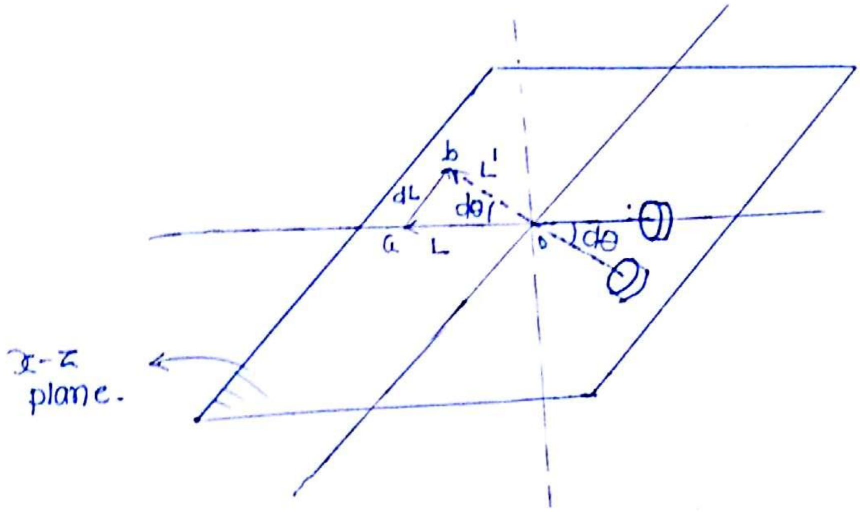
(6)



Magnitude of Gyroscopic Couple Vector :→

$$\vec{T} = \frac{d\vec{L}}{dt} \qquad L = I\omega$$

I → mass moment of Inertia about spin axis.



$\vec{dL} = \vec{L} - \vec{L'}$ (vector Addition)

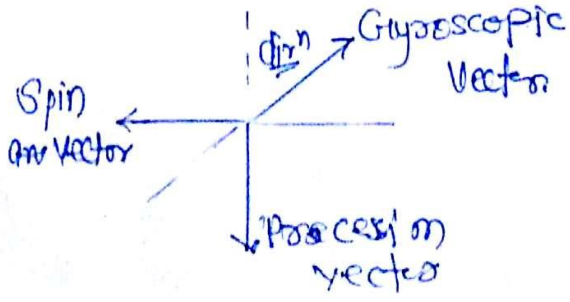
$\vec{dL} = \vec{L} - \vec{L'} = ab$

$T = \frac{(ab)}{dt}$ as $dt \to 0$ & $d\theta \to 0$

$(ab) = (oa)(d\theta) = L\,d\theta$

$$T = L\frac{d\theta}{dt} = I\omega\frac{d\theta}{dt}$$

$$\boxed{T = I\omega\frac{d\theta}{dt}} \qquad \boxed{T = I\omega\omega_p}$$



$$\boxed{\alpha'' = \omega\omega_p}$$

Gyroscopic angular accn

Question A flywheel of mass 10 kg has a radius of gyration 20 cm. It is given a spin 1000 rpm about its axis which is Hz. The flywheel is suspended at a point 15 cm from the plane of rotation of the flywheel as shown in fig. determine the motion of flywheel.

sol^n)

$I = mk^2$

$= (.20)^2 \times 10 = \frac{400}{10000} \times 10$

$I = 0.4 \ kgm^2$

$\omega = \frac{2\pi N}{60}$

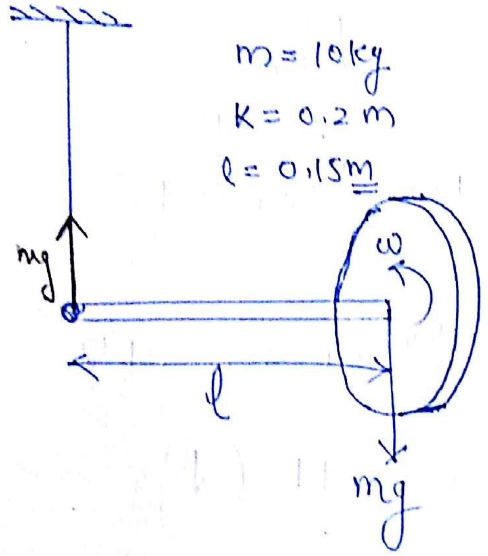
$\omega = \frac{2\pi \times 1000}{360}$

$\omega = \frac{100\pi}{3}$ rad/sec.

$m = 10 kg$

$K = 0.2 m$

$l = 0.15 m$



$\omega_p = \frac{d\theta}{dt}$

$T = I \omega \omega_p$

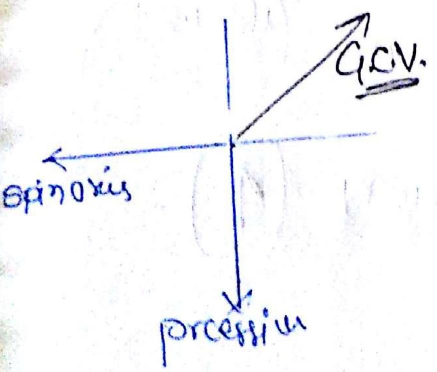
$T = 0.4 \times \frac{100\pi}{3} \times \omega_p$

T= F×r

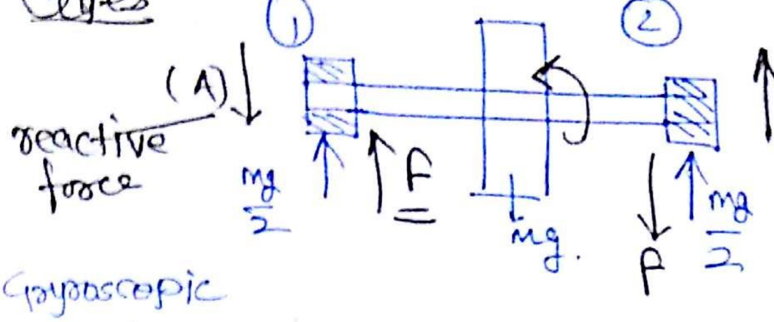
$mg \times l = 0.4 \times \frac{100\pi}{3} \times \omega_p$

$10 \times 9.81 \times 0.15 = 0.4 \times \frac{100\pi}{3} \times \omega_p$

~~$\omega_p = 0.527$~~ $\omega_p = 0.3513$ rad/s



Ques



d = 200 mm

M = 10 kg

l = 150 mm

N = 2000 rpm

N_p = 50 rpm

Gyroscopic Vector

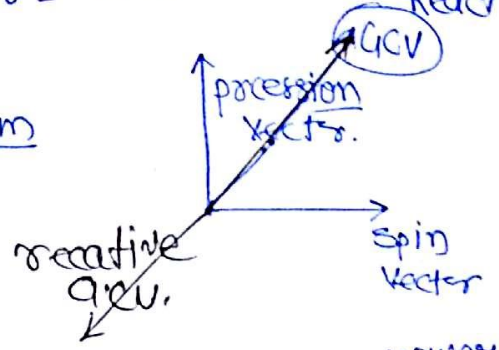
Couple $C = T = \left(\frac{mR^2}{2}\right) \omega \times \omega_p$

$I = \frac{mR^2}{2}$

$$C = \frac{10 \times (0.100)^2}{2} \times \left(\frac{2\pi}{60}\right)^2 \times 2000 \times 50$$

$C = T = 54.83$ Nm

$C = 54.83$ Nm



Reaction = 365.540 N

36.55 37.262

41.55 42.26

414.590

316.490

$F = \frac{C}{l}$

force due to Gyroscopic action

$F = \frac{54.83}{0.150} = 365.540$ upward / downward

$F_{net\,1} = \frac{mg}{2} + F = \frac{10 \times 9.81}{2} + 365.540$

$F_{net\,1} = 414.590$ N (↓)

$F_{net\,2} = F - \frac{mg}{2} = 365.540 - \left(\frac{10 \times 9.81}{2}\right)$

$F_{net\,2} = 316.490$ N (↑)

Q17

$m = 3500\ kg$

$k = 0.45\ m$

$N = 3000\ rpm$ $\qquad \omega = \dfrac{2\pi \times 3000}{60} = 100\pi\ rad/s$

(i) $r = 100\ m$

$v = 36\ km/hr = \dfrac{36 \times 1000}{3600} = 10\ m/s.$

$V = r\omega \qquad \omega = \dfrac{10}{100} = 0.1\ rad/s.$

$C = (3500)(0.45)^2 \times 100\pi \times 0.1$

$C = 22.266\ kNm$

$\theta = \theta_0 \sin(\omega_n t + \phi)$

$\Omega = \theta_0 \omega_n \cos(\omega_n t + \phi)$

$(\Omega)_{max} = \theta_0 \omega_n$

$(\Omega)_{max} = 6 \times \dfrac{\pi}{180} \times \dfrac{2\pi}{40}$

$\omega_p =$

(ii) $t = 40\ sec.$ $\qquad t = \dfrac{\theta_0}{\omega}$

$\theta = 12^\circ = \dfrac{12 \times \pi}{180}$

$\omega_p = \dfrac{6 \times \pi / 180}{40} \times 2\pi$

$T = \dfrac{2\pi}{\omega} \Rightarrow$

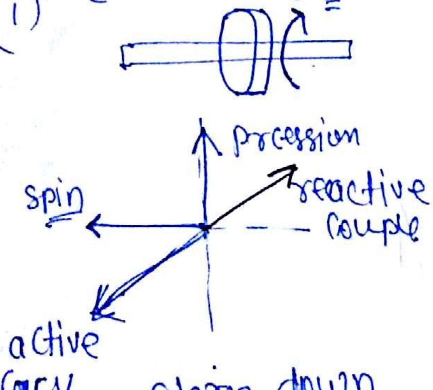
$\omega = 0.01047\ rad/sec.$

$C = 3500(0.45)^2 \times 100\pi \times 0.01047$

$C = 2.331\ kNm$ $\qquad$ 3.66261

effect

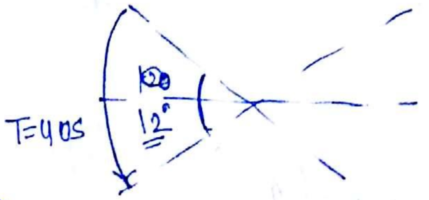
(i)

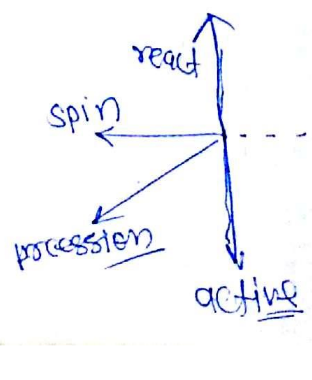


ccw → stern down

→ bow up

(ii)





Stern tow

$\theta = 6\sin(\omega_n t)$

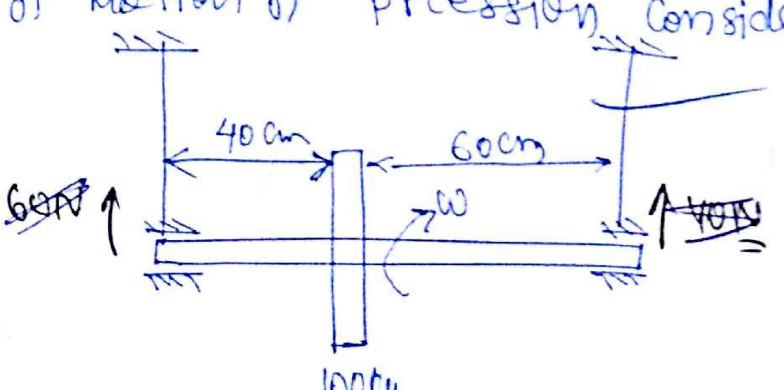
$\omega_n = \dfrac{2\pi}{T}$

$\omega_p = \theta_0 \omega_n$

Q. 16

Note! If both the bearings are on the same beam then there will be effect of reactive gyroscopic couple i.e. bearing (B) will try to lift up and bearing (A) will try to move down.

Ques A disc having mass of 100 kg with k = 40 cm is supported b/w two bearing on a shaft of negligible weight. The centre line of shaft, disc and the bearing lie in a Hz plane. The distance of disc from the bearings are as shown in fig. These bearing are supported by vertical thin chords when the disc is rotating at 98.1 rad/s in clockwise dirn looking from the rHS bearing the chord supporting the rHS bearing get broken discuss the motion of disc at the instant the chord is cut & determine the mag. of motion of precession. Consider only gyroscopic effect



is

Ans precession will be in Hz plane from top $\omega_p = 0.25$ rad/s

$T = 100 \times 0.40 = 100 \times 0.40^2 \times 98.1 \times \omega_p$

$m = 100\ kg,\ k = 40\ cm$

$\omega = 98.1$

$mg \times r = I \omega \omega_p$

$100 \times \frac{9.81}{100} \times 0.40 = 100 \times 0.40 \times \frac{0.40}{100} \times \frac{98.1}{10} \times \omega_p$

$\omega_p = \frac{1}{4} = 0.25\ rad/s.$

$\leftarrow$

Spin

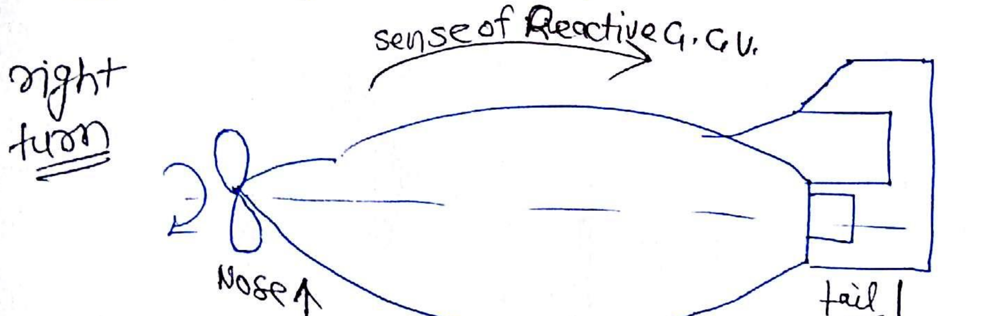
Effect on Aircraft

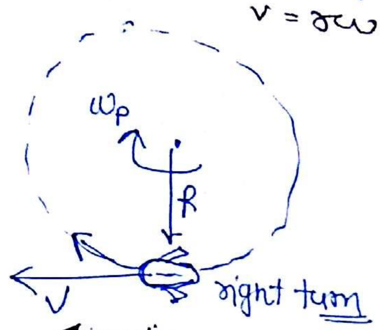
Ques The inertia of air screw with more than two blades and the rotating mass is all assumed to be spining about the same axis is 15 kg m^2 and the dirn of rotation is clockwise when ~~at~~ looked from ~~at of machine~~ front of the aircraft. The equivalent speed of rotation air screw & the rotating mass is 1600 rpm when the speed of fligh is 240 KM/hr if Aeroplane makes a right handed turn on a path of 170 m radius Find gyroscoping reaction of air screw of aeroplane and discuss it's effect

$v = \frac{r}{\omega}$

$\omega = \frac{r}{v}$

$v = r\omega$



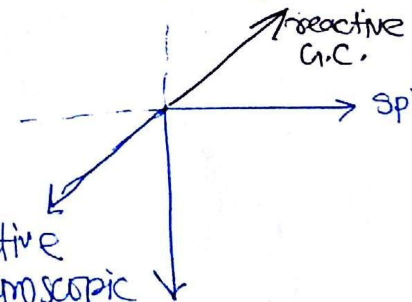


$I = 15$ kg-m^2

$N = 1600$ rpm

$V = 240$ km/hr

$r = 170$ m



Active G.C.V. $= I\omega\,\omega_p$

$v = r\omega_p$

$$C = 15 \times \frac{2\pi \times 1600}{60} \times \frac{240 \times 1000}{170 \times 3600}$$

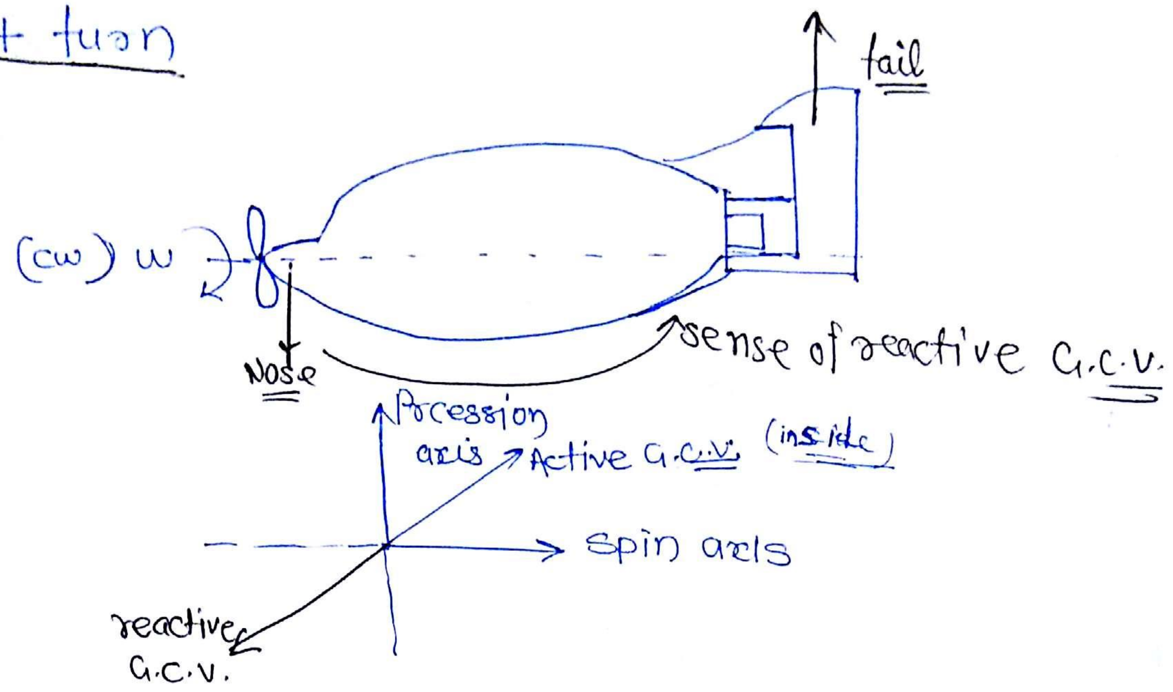
$$\omega_p = \frac{v}{r} = \frac{240 \times 1000}{170 \times 3600}$$

$$C = 985.59 \text{ Nm}$$

$\omega_p = 0.39215$ rad/s

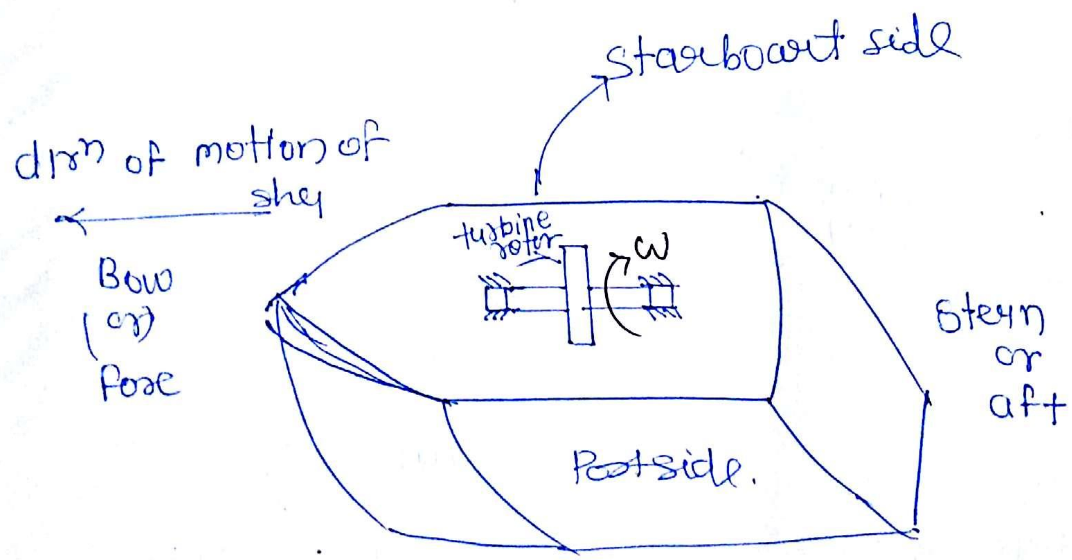
effect:- if ~~air~~ air craft is taking right turn nose will be lifted and tail will go down

Left turn



Tail will go up and nose will go down.

Gyroscopic effect on ship:-



① Bow/Fore:- it is the front end of ship

② stern/aft :- it is the rare end of ship

③ Port is LHS of ship when looking from the rare end.

④ starboard is RHS of ship when looking from the rare end.

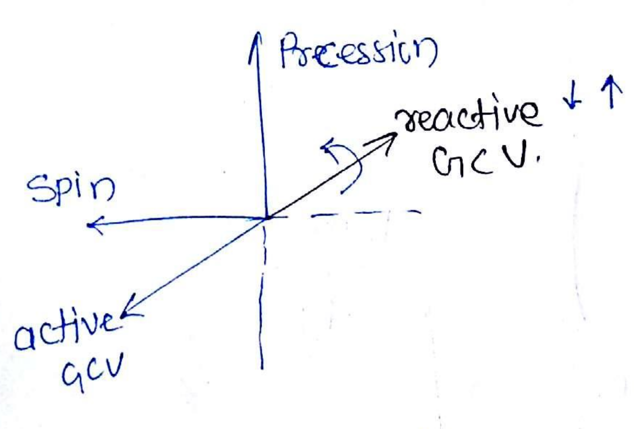
⑤ steering is taking turn to right/left when moving forward

⑥ Pitching is up & down motion of ship about transverse axis

⑦ Rolling is angular motion of the ship about longitudnal axis

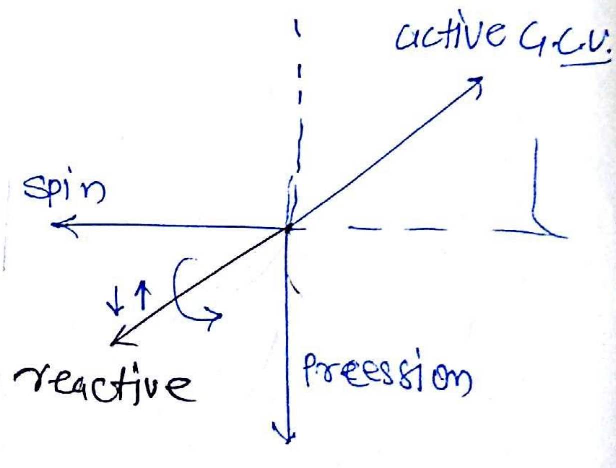
- steering:-

ⓐ left turn



→ stern will go down
→ Bow will go up

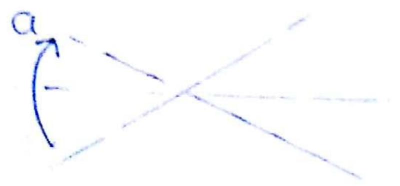
(b) Righ turn

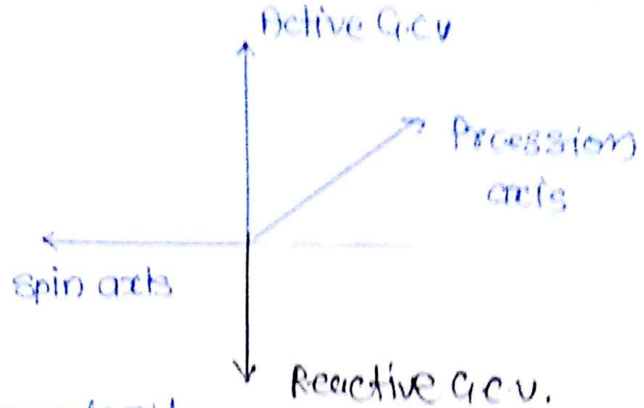


→ stern will go up.
→ Bow will go down

Pitching

a) Bow is moving up.



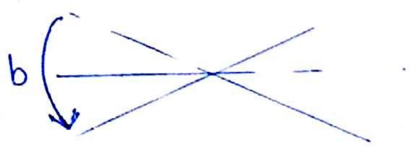


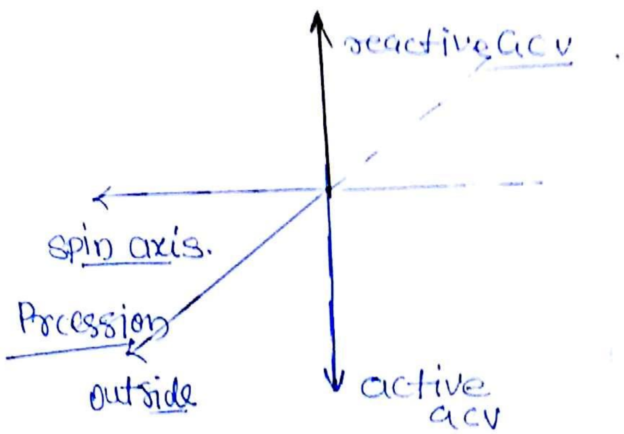
→ stern will move towards port side

→ Bow will move towards starboard side

(ship will rotate in CW sense as viewed from top)

b) Bow is moving down





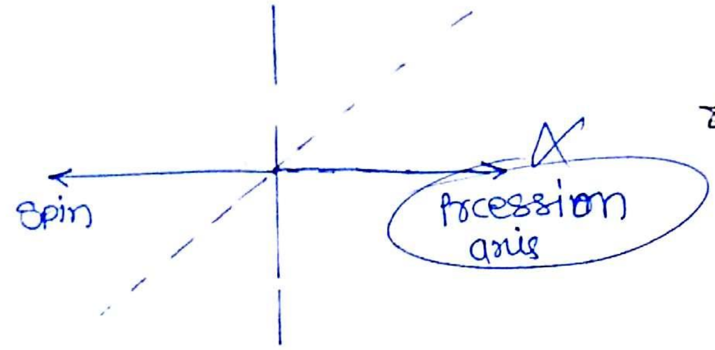
→ stern will move towards starboard side

→ bow will move towards port side

(ship will rotate in ACW sence as viewed from TOP)

Rolling

(a) Port side going down



* There is no reactive gyroscoping effect on ship during rolling

As in rolling axis of rotation of ship is parallel to the axis spin of rotor so there will not be any precession in turbine rotor. So there ~~will~~ is no gyroscopic effect on the ship

Q.17 before 4 page

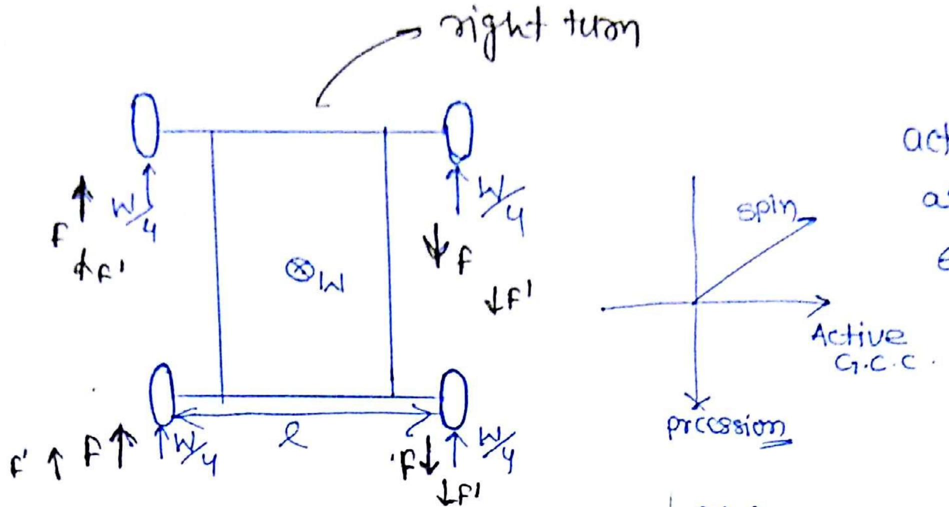
SHM $\theta = \theta_0 \sin(\omega_n t + \phi)$

Angular Velocity $\Omega = \theta_0 \omega_n \cos(\omega_n t + \phi)$

ω_n - Natural freq.

$$(\Omega)_{max} = \theta_0 \omega_n = \theta_0 \times \pi/180 \times \frac{2\pi}{T}$$

Gyroscope effect on 4- wheel Automobile:—



active & reactive are on same dir^n

Gyroscope effect $T = 4 I \omega \omega_p$

Speed of vehical $= V = r\omega = R\omega_p$

r = radius of wheel

$C = 4 I \omega \omega_p$

$C = 2(F \times l)$

$\omega = \frac{V}{r}$

$\omega_p = \frac{V}{R}$

★ $F = \frac{C}{2l}$ (↓) — inner wheel

(↑) → outer wheels → Always

Engine

$C' = I_E \omega_E \omega_p$

$C' = 2(F' \times l)$

$F' = \frac{C'}{2l}$ (↑) inner (↓) outer

Q.10

$C = 4 I \omega \omega_p = 4 \times 0.70 \times \left(100 \times \frac{5}{18}\right)^2 \times \frac{1}{0.30} \times \frac{1}{100}$

$C = 72.016\,Nm$

★ wheather wheel is taking left/right turn force on inner wheel will be downward and on outer wheels will be upward.