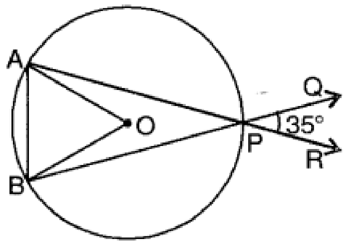
Chapter 4 Circles

Very Short Answer Type Questions

Q1. In the given figure, O is the centre of the circle with chords AP and BP being produced to R and Q respectively. If ∠QPR = 35°, find the measure of ∠AOB. [CBSE-14-17DIG1U]



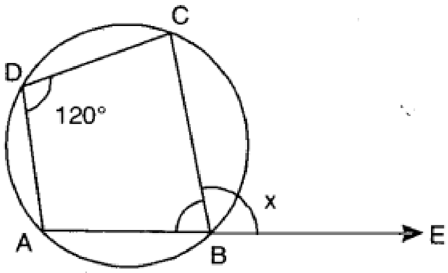
Answer.

$\angle APB = \angle RPQ = 35^\circ$ [vert. opp. ∠s]

Now, ∠AOB and ∠APB are angles subtended by an arc AB at centre and at the remaining part of the circle.

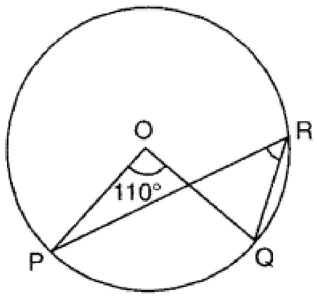
$\therefore \quad \angle AOB = 2\angle APB = 2 \times 35^\circ = 70^\circ$

Q2. In the given figure, what is the measure of angle x ?



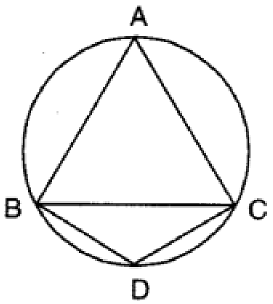
Answer. We know that exterior angle of a cyclic quadrilateral is equal to interior opposite angle.
.-. ∠CBE = ∠ADC
=> x = 120°

Q3. In the given figure, if O is the centre of circle and ∠POQ = 110°, then find ∠PRQ



Answer. We know that angle subtended by an arc at the centre is double the angle subtended by it at the remaining part of the circle.
∠PRQ =1/2 ∠POQ
= 1/2 x 110°
= 55°

Q4. In the given figure, ΔABC is an equilateral triangle and ABDC is a cyclic quadrilateral, then find the measure of ∠BDC.



Answer.

ΔABC is an equilateral triangle.

$$\therefore \quad \angle BAC = 60^\circ$$

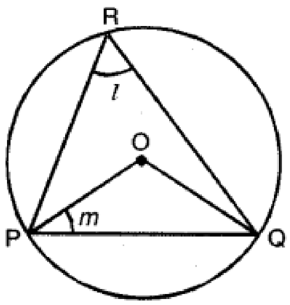
$$\text{Now,} \quad \angle BDC + \angle BAC = 180^\circ$$

$$\angle BDC + 60^\circ = 180^\circ$$

$$\angle BDC = 180^\circ - 60^\circ$$

$$\angle BDC = 120^\circ$$

Q5. In the given figure, O is the centre of the circle. PQ is a chord of the circle and R is any point on the circle. If ∠PRQ = l and ∠OPQ = m, then find l + m.



Answer.

$$\angle POQ = 2\angle PRQ$$

$$\Rightarrow \quad \angle POQ = 2l$$

$$\text{In } \Delta PQO,\ OP = OQ = r$$

$$\Rightarrow \quad \angle OQP = \angle OPQ = m$$

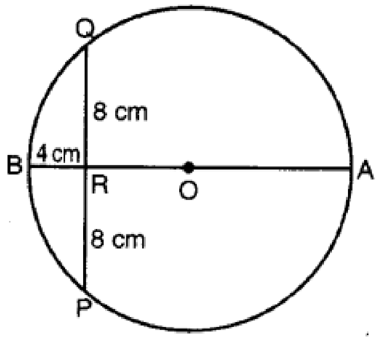
$$\text{Also, } \angle OPQ + \angle OQP + \angle POQ = 180^\circ$$

$$\Rightarrow \quad m + m + 2l = 180^\circ$$

$$\Rightarrow \quad 2(l + m) = 180^\circ$$

$$\Rightarrow \quad l + m = 90^\circ$$

Q6. The given figure shows a circle with centre O in which a diameter AB bisects the chord PQ at the point R. If PR = RQ = 8 cm and RB = 4 cm, then find the radius of the circle.



Answer.

Let r be the radius, then

$OQ = OB = r$ and $OR = (r - 4)$

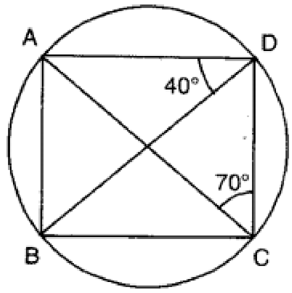
$\therefore \quad OQ^2 = QR^2 + RO^2$

$\Rightarrow \quad r^2 = 64 + (r - 4)^2$

$\Rightarrow \quad r^2 = 64 + r^2 + 16 - 8r$

$\Rightarrow \quad 8r = 80 \Rightarrow r = 10$ cm

Q7. In the given figure, ABCD is a cyclic quadrilateral such that ∠ADB= 40° and ∠DCA = 70°, then find the measure of ∠DAB .



Answer.

$\angle BCA = \angle ADB = 40°$ [angles in same segment of a circle are equal]

Now, $\angle BCD = 70° + 40° = 110°$

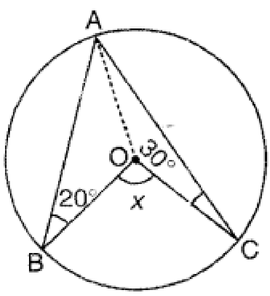
$\angle DAB + \angle BCD = 180°$

$\angle DAB + 110° = 180°$

$\angle DAB = 180° - 110°$

$\Rightarrow \quad \angle DAB = 70°$

Q8. In the figure, 'O' is the centre of the circle, ∠ABO= 20° and ∠ACO= 30°, where A, B, C are points on the circle. What is the value of x ?



Answer.

In ΔOAB, we have

$$OA = OB \quad \text{[radii of same circle]}$$

$$\Rightarrow \quad \angle OAB = \angle OBA = 20^\circ \quad [\angle\text{s opp. to equal sides}]$$

In ΔOAC, we have

$$OA = OC \quad \text{[radii of same circle]}$$

$$\Rightarrow \quad \angle OAC = \angle OCA = 30^\circ \quad [\angle\text{s opp. to equal sides}]$$

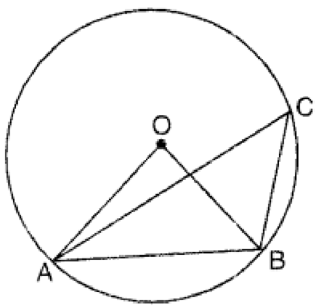
Now, $\angle BAC = \angle OAB + \angle OAC$

$$= 20^\circ + 30^\circ = 50^\circ$$

$$x = \angle BOC = 2\angle BAC$$

$$= 2 \times 50^\circ = 100^\circ$$

Q9. In the given figure, if O is the centre of circle. Chord AB is equal to radius of the circle, then find ∠ACB.



Answer.

Join OA, OB

$\because \quad AB = OA = OB$

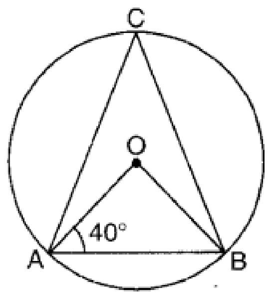
$\therefore$ ΔOAB is an equilateral triangle

$\therefore \quad \angle AOB = 60^\circ$

$$\angle ACB = \frac{1}{2}\angle AOB$$

$$= \frac{1}{2} \times 60^\circ = 30^\circ$$

Q10. In the given figure, if ∠OAB = 40°, then find the measure of ∠ACB. [NCERT Exemplar Problem]



Answer.

In ΔOAB,

$$OA = OB \quad \text{[radii of same circle]}$$
$$\angle OBA = \angle OAB = 40^\circ \quad [\angle\text{s opp. to equal sides in a } \Delta \text{ are equal}]$$
Now, $\angle OAB + \angle AOB + \angle OBA = 180^\circ$ [angle sum property of a Δ]

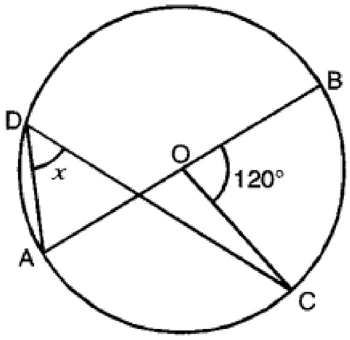
$\Rightarrow$ $40^\circ + \angle AOB + 40^\circ = 180^\circ$

$\Rightarrow$ $\angle AOB = 100^\circ$

Therefore, $\angle ACB = 50^\circ$

[$\because$ angle subtended by an arc at the centre is double the angle subtended by it at the remaining part of the circle]

Q11. In the given figure, O is the centre of the circle. If∠BOC= 120°, then find the value of x.



Answer.

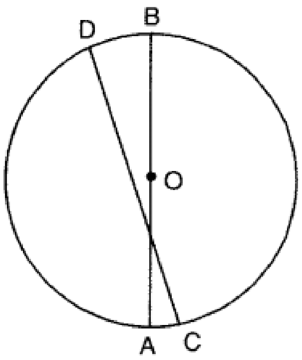
Here, $\angle AOC + \angle COB = 180^\circ$

$\angle AOC = 180^\circ - \angle COB$

$= 180^\circ - 120^\circ = 60^\circ$

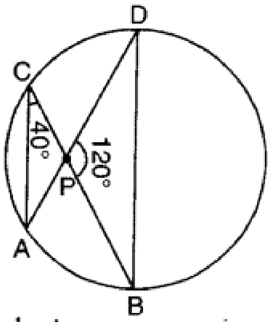
Now, $x = \frac{1}{2}\angle AOC = \frac{1}{2} \times 60^\circ = 30^\circ$

Q12. In the given figure, O is the centre of the circle, then compare the chords.



Answer. In chords AB and CD, AB is passing through the centre of the circle. AB is the diameter of circle. Thus, AB>CD [v diameter is the largest chord]

Q13. In the given figure, ∠ACP = 40° and ∠BPD = 120°, then find ∠CBD .



[angle in same segment]

Answer.

$\angle BDP = \angle ACP = 40°$ [angle in same segment]

Now, in ΔBPD, we have

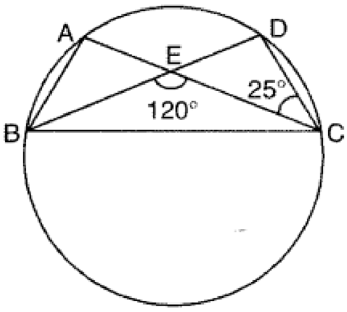
$\angle PBD + \angle BPD + \angle BDP = 180°$

$\Rightarrow \angle PBD + 120° + 40° = 180°$

$\Rightarrow \angle PBD = 180° - 160° = 20°$

or $\angle CBD = 20°$

Q14. In the given figure, if ∠SEC = 120°, ∠DCE = 25°, then find ∠BAC.



Answer.

$\angle BEC$ is exterior angle of ΔCDE.

$\therefore \angle CDE + \angle DCE = \angle BEC$

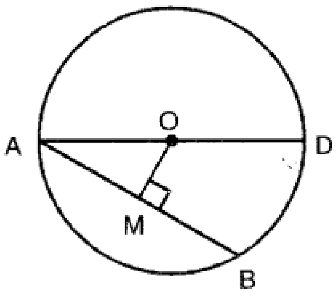
$\Rightarrow \angle CDE + 25° = 120°$

$\Rightarrow \angle CDE = 95°$

Now, $\angle BAC = \angle CDE$ [$\because$ angle in same segment are equal]

$\Rightarrow \angle BAC = 95°$

Q15. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, then find the distance of AB from the centre of the circle. [CBSE March 2012]



Answer.

$\because$ The perpendicular drawn from centre to the chord bisects it.

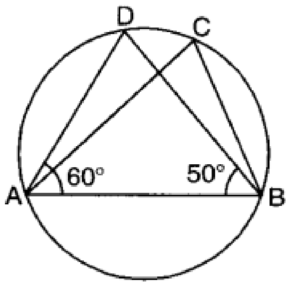
$$\therefore \quad AM = \frac{1}{2}AB = \frac{1}{2} \times 30 \text{ cm}$$
$$= 15 \text{ cm}$$

Also, $$OA = \frac{1}{2}AD$$
$$= \frac{1}{2} \times 34 \text{ cm}$$
$$= 17 \text{ cm}$$

In rt. ΔOAM, we have

$$OA^2 = OM^2 + AM^2$$
$$17^2 = OM^2 + 15^2$$
$$\Rightarrow \quad 289 = OM^2 + 225$$
$$\Rightarrow \quad OM^2 = 289 - 225$$
$$\Rightarrow \quad OM^2 = 64$$
$$\Rightarrow \quad OM = \sqrt{64} = 8 \text{cm}$$

Q16. In the given figure, if ∠DAB = 60°, ∠ABD= 50°, then find ∠ACB. [NCERT Exemplar Problem]



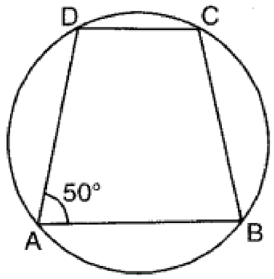
Answer.

In ΔABD, $\angle ABD + \angle ADB + \angle DAB = 180^\circ$ [angle sum property of Δ]

$$50^\circ + \angle ADB + 60^\circ = 180^\circ$$
$$\Rightarrow \quad \angle ADB = 70^\circ$$

Now, $\angle ACB = \angle ADB = 70^\circ$ [$\because$ angle in same segment]

Q17. In the given figure, AB || DC. If ∠A = 50°, then find the measure of ∠ABC. [NCERT Exemplar Problem]



Answer.

ABCD is a cyclic quadrilateral.

$\therefore \quad \angle A + \angle C = 180^\circ$ ...(i) [opposite $\angle$s of cyclic quad.]

Also, AB || DC

$\therefore \quad \angle B + \angle C = 180^\circ$ ...(ii)

From (i) and (ii), we have

$\angle B + \angle C = \angle A + \angle C$

$\Rightarrow \quad \angle B = \angle A = 50^\circ$

Short Answer Questions Type-I

Q18. Equal chords of a circle subtend equal angles at the centre. [CBSE March 2012]

Answer.

Given : In a circle C(O, r), chord AB = chord CD.

To Prove : $\angle AOB = \angle COD$.

Proof : In ΔAOB and ΔCOD

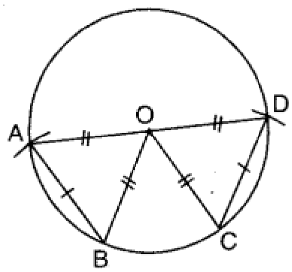
AO = CO [radii of same circle]

BO = DO [radii of same circle]

Chord AB = Chord CD [given]

$\Rightarrow \Delta AOB \cong \Delta COD$ [by SSS congruence axiom]

$\Rightarrow \angle AOB = \angle COD$ [c.p.c.t.]



Q19. If the angles subtended by the chords of a circle at the centre are equal, then chords are equal. [CBSE March 2012]

Answer.

Given : In a circle C(O, r), $\angle AOB = \angle COD$.

To Prove : Chord AB = Chord CD.

Proof : In ΔAOB and ΔCOD

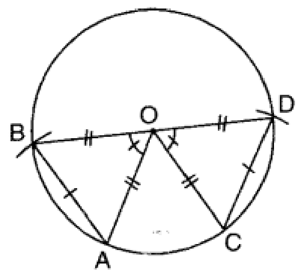
AO = CO [radii of same circle]

BO = DO [radii of same circle]

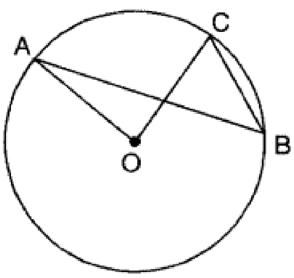
$\angle AOB = \angle COD$ [given]

$\Rightarrow \Delta AOB \cong \Delta COD$ [by SAS congruence axiom]

$\Rightarrow$ Chord AB = Chord CD [c.p.c.t.]



Q20. In the figure, O is the centre of the circle and ∠ABC= 45°. Show that OA⊥OC[/latex]. [CBSE March 2013]



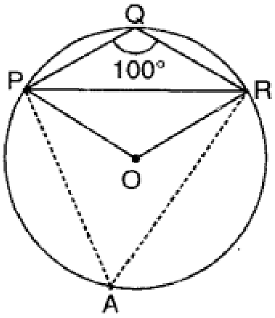
Answer.

Since angle subtended by an arc at the centre of the circle is double the angle subtended it at any point on the remaining part of the circle.

$\therefore \angle AOC = 2\angle ABC = 2 \times 45^\circ = 90^\circ$

Hence, $OA \perp OC$.

Q21. In the given figure, ∠PQR= 100°, where P,Q and R are points on a circle with centre O. Find ∠OPR. [CBSE March 2012]



Answer.

Take any point A on the circumcircle of the circle.

Join AP and AR.

$\because$ APQR is a cyclic quadrilateral.

$\therefore \quad \angle PAR + \angle PQR = 180^\circ$ [sum of opposite angles of a cyclic quad. is 180°]

$\angle PAR + 100^\circ = 180^\circ$

$\Rightarrow \quad \angle PAR = 80^\circ$

Since ∠POR and ∠PAR are the angles subtended by an arc PR at the centre of the circle and circumcircle of the circle.

$\therefore \quad \angle POR = 2\angle PAR = 2 \times 80^\circ = 160^\circ$

$\therefore$ In ΔPOR, we have $\quad OP = OR$ [radii of same circle]

$\angle OPR = \angle ORP$ [angles opposite to equal sides]

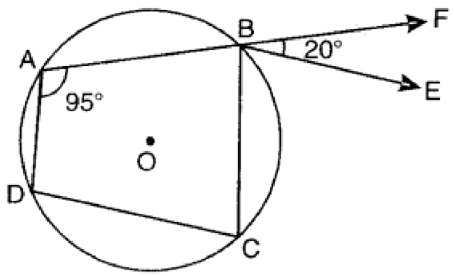
Now, $\angle POR + \angle OPR + \angle ORP = 180^\circ$

$\Rightarrow \quad 160^\circ + \angle OPR + \angle OPR = 180^\circ$

$\Rightarrow \quad 2\angle OPR = 20^\circ$

$\Rightarrow \quad \angle OPR = 10^\circ$

Q22. In figure, ABCD is a cyclic quadrilateral in which AB is extended till F and BE || DC. If ∠FBE= 20° and ∠DAB = 95°, then find∠ADC. [CBSE March 2012]



Answer.

Sum of opposite angles of a cyclic quadrilateral is 180°

$\therefore \quad \angle DAB + \angle BCD = 180°$

$\Rightarrow \quad 95° + \angle BCD = 180°$

$\Rightarrow \quad \angle BCD = 180° - 95° = 85°$

$\because \quad BE \parallel DC$

$\therefore \quad \angle CBE = \angle BCD = 85°$ [alternate interior angles]

$\therefore \quad \angle CBF = \angle CBE + \angle FBE$

$= 85° + 20° = 105°$

Now, $\angle ABC + \angle CBF = 180°$ [linear pair]

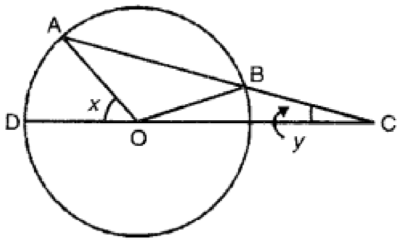
and $\angle ABC + \angle ADC = 180°$ [opposite angles of cyclic quad.]

Thus, $\angle ABC + \angle ADC = \angle ABC + \angle CBF$

$\Rightarrow \quad \angle ADC = \angle CBF$

$\Rightarrow \quad \angle ADC = 105°$ $[\because \angle CBF = 105°]$

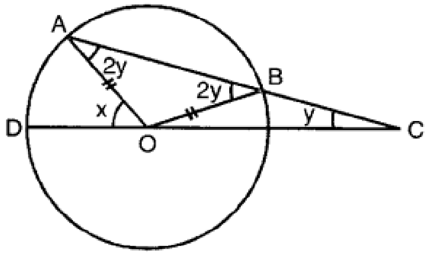
Q23. In the figure, chord AB of circle with centre O, is produced to C such that BC = OB. CO is joined and produced to meet the circle in D. If ∠ACD = y and ∠AOD= x, show that x = 3y. [CBSE March 2011]



Answer.

In ΔOBC, $OB = BC$

$\Rightarrow \quad \angle BOC = \angle BCO = y$ ...[angles opp. to equal sides are equal]



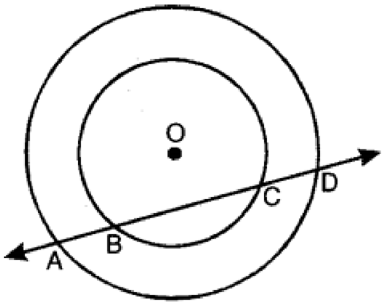
$\angle OBA$ is the exterior angle of ΔBOC

So, $\angle ABO = 2y$ ...[ext. angle is equal to the sum of int. opp. angles]

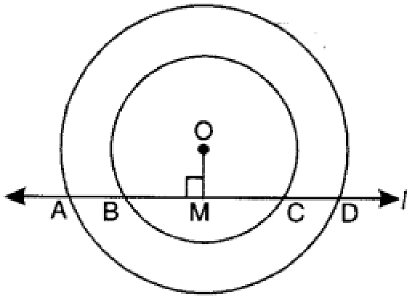
Similarly, $\angle AOD$ is the exterior angle of ΔAOC

$\therefore \quad x = 2y + y = 3y$

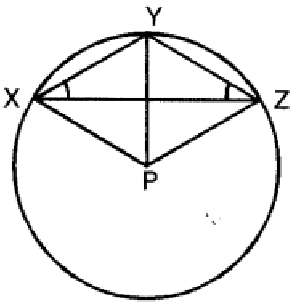
Q24. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see fig.). [CBSE March 2013]



Answer. Draw OM⊥ l
Since perpendicular from the centre of a circle to a chord of the circle bisects the chord.
BM = CM ...(i)
and AM = DM ...(ii)
Subtracting (i) from (ii), we have
AM – BM = DM – CM
AB = CD



Q25. In the given figure, P is the centre of the circle. Prove that : ∠ XPZ = 2(∠ XZY +∠ YXZ)



Answer.

Arc XY subtends ∠XPY at the centre P and ∠XZY in the remaining part of the circle.

$\therefore \quad \angle XPY = 2(\angle XZY) \quad ...(i)$

Similarly, arc YZ subtends ∠YPZ at the centre P and ∠YXZ in the remaining part of the circle.

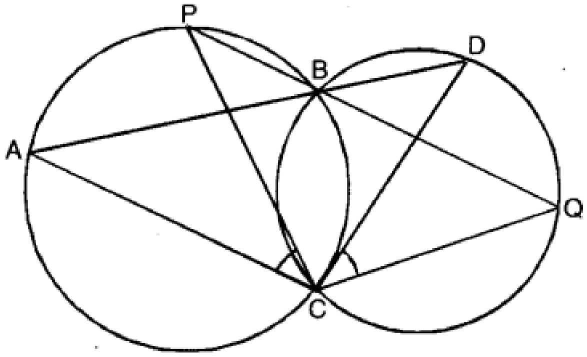
$\therefore \quad \angle YPZ = 2(\angle YXZ) \quad ...(ii)$

Adding (*i*) and (*ii*), we have

$$\angle XPY + \angle YPZ = 2(\angle XZY + \angle YXZ)$$

$$\angle XPZ = 2(\angle XZY + \angle YXZ)$$

Q26. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see fig.). Prove that ∠ ACP =∠ QCD.



Answer.

$\angle ACP = \angle ABP$ ...(*i*) [angles in the same segment]

$\angle QCD = \angle QBD$ ...(*ii*) [angles in the same segment]

Also, $\angle ABP = \angle QBD$ ...(*iii*) [vertically opposite angles]

$\therefore$ From (*i*), (*ii*) and (*iii*), we have

$\angle ACP = \angle QCD$

Q27. If the diagonals of a cyclic quadrilateral are diameters of the circle through the opposite vertices of the quadrilateral. Prove that the quadrilateral is a rectangle. [CBSE-14-GDQNI3W]

Answer.

Here, ABCD is a cyclic quadrilateral in which AC and BD are diameters.

Since AC is a diameter.

$\therefore \angle ABC = \angle ADC = 90°$

[$\because$ angle of a semicircle = 90°]

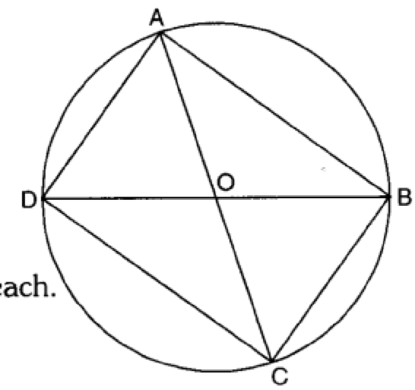
Also, BD is a diameter

$\therefore \angle BAD = \angle BCD = 90°$

[$\because$ angle of a semicircle = 90°]

Now, all the angles of a cyclic quadrilateral ABCD are 90° each.

Hence, ABCD is a rectangle.



Short Answer Questions Type-II

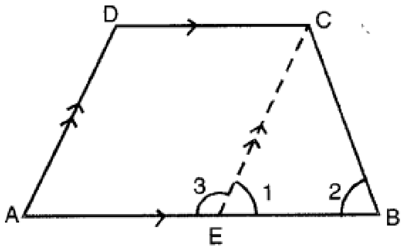
Q28. If the non-parallel sides of a trapezium are equal, prove that it is cyclic. [CBSE March 2013]

Answer.

Given : ABCD is a trapezium in which AD = BC.

To Prove : ABCD is a cyclic quadrilateral.

Const. : Draw CE || DA intersect AB in E.



Proof : Here, AB || DC [given]

or AE || DC

and CE || DA [by construction]

$\Rightarrow$ AECD is a parallelogram.

$\therefore$ AD = EC [opposite sides of a parallelogram]

$\angle D = \angle 3$ [opposite angles of a parallelogram]

Now, in ΔEBC CE = CB [$\because$ AD = BC (given) and AD = EC (proved)]

$\Rightarrow$ $\angle 1 = \angle 2$ [angles opposite to equal sides of a triangle]

$\angle 1 + \angle 3 = 180°$ [a linear pair]

$\angle 2 + \angle D = 180°$ [$\because$ $\angle 1 = \angle 2$ and $\angle 3 = \angle D$ (proved)]

or $\angle B + \angle D = 180°$ [$\because$ $\angle 2$ and $\angle B$ are same]

Hence, ABCD is a cyclic trapezium.

Q29. ABCD is a parallelogram. The circle through A, B and C intersects (produce if necessary) at E. Prove that AE = AD.

Answer.

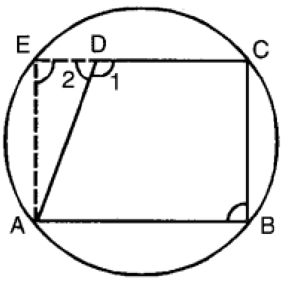
Given : ABCD is a parallelogram. Circle through A, B and C intersects CD produced in E.

To Prove : AE = AD

Proof : ABCE is a cyclic quadrilateral.

$\therefore$ $\angle B + \angle E = 180°$ ...(*i*)

ABCD is a parallelogram.



$\therefore$ $\angle B = \angle 1$ ...(*ii*)

Also, $\angle 1 + \angle 2 = 180°$ [a linear pair]

or $\angle B + \angle 2 = 180°$ ...(*iii*) [using (*ii*)]

Now, from (*i*) and (*iii*), we have

$\angle B + \angle E = \angle B + \angle 2$

$\Rightarrow$ $\angle E = \angle 2$

In ΔADE, we have

$\angle E = \angle 2$

$\Rightarrow$ AD = AE [sides opposite to equal angles of a Δ]

Q30. ABCD is a cyclic quadrilateral in which AB and CD when produced meet in E and EA = ED. Prove that : (i) AD\\BC (ii) EB = EC

Answer.

(*i*) In ΔAED, we have

$EA = ED$ [given]

$\Rightarrow \angle 1 = \angle 2$

[angles opposite to equal sides are equal]

Also, $\angle 3 = \angle 2$

[external angle of cyclic quad. = interior opposite angle]

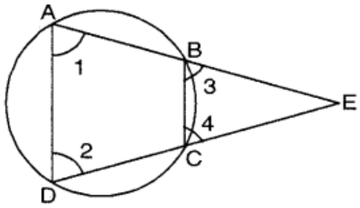
$\angle 1 = \angle 2$

and $\angle 3 = \angle 2$

$\Rightarrow \angle 1 = \angle 3$

These are correspondence angles.

$\therefore$ AD || BC



(*ii*) $\angle 1 = \angle 4$ [external angle of cyclic quad. = Interior opposite angle]

But $\angle 1 = \angle 3$ [proved above]

$\angle 1 = \angle 4$

and $\angle 1 = \angle 3$

$\Rightarrow \angle 3 = \angle 4$

In ΔBEC, we have $\angle 3 = \angle 4 \Rightarrow BE = CE$

[side opposite to equal angles are equal]

Q31. If two equal chords of a circle intersect within a circle, prove that the line segment joining the point of intersection to the centre makes equal angles with the chords. [CBSE-15-NS72LP7]

Answer.

Join OP, draw OL ⊥ AB and OM ⊥ CD, thus, L and M are the mid-points of AB and CD respectively. Also, equal chords are equidistant from the centre.

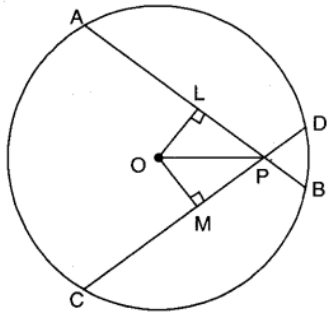
$\therefore OL = OM$

Now, in right-angled Δs OLP and OMP

$OL = OM$

$OP = OP$ [common]

$\angle OLP = \angle OMP$ [each = 90°]

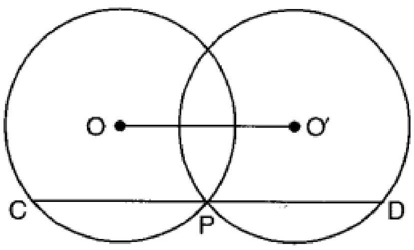


So, by RHS congruence axiom, we have

$\Delta OLP \cong \Delta OMP$

Hence, $\angle OPL = \angle OPM$ [c.p.c.t.]

Q32. Two circles whose centres are O and O' intersect at P. Through P, a line parallel to OO', intersecting the circles at C and D is drawn as shown in the figure. Prove the CD = 2OO'. [CBSE-15-6DWMW5A] [CBSE-14-ERFKZ8H]



Answer.

Draw OA ⊥ CD and O′B ⊥ CD

Now, OA ⊥ CD

⇒ OA ⊥ CP

⇒ $CA = AP = \frac{1}{2}CP$

⇒ $CP = 2AP$ ... (*i*)

Similarly, O′B ⊥ CD

⇒ O′B ⊥ PD

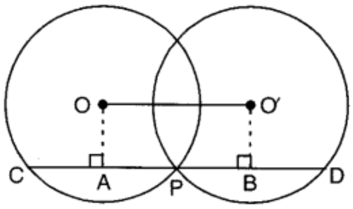
⇒ $PB = BD = \frac{1}{2}PD$

⇒ $PD = 2PB$ ... (*ii*)

Also, $CD = CP + PD$

$= 2AP + 2PB = 2(AP + PB) = 2AB$

$CD = 2OO'$ [∵ OABO′ is a rectangle]



Q33. If ABC is an equilateral triangle inscribed in a circle and P be any point on the minor arc BC which does not coincide with B or C, prove that PA is angle bisector of ∠BPC . [NCERT Exemplar Problem]

Answer.

Here, ΔABC is an equilateral triangle inscribed in a circle with centre O.

⇒ AB = AC = BC [∵ ΔABC is equilateral]

∠AOB = ∠AOC = ∠BOC

[equal chords subtend equal angles at centre)

⇒ ∠AOB = ∠AOC ...(*i*)

Now, ∠AOB and ∠APB are angles subtended by an arc AB at centre and at the remaining part of the circle by same arc.

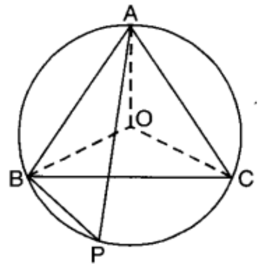
Therefore, $\angle APB = \frac{1}{2}\angle AOB$ ...(*ii*)

Similarly, $\angle APC = \frac{1}{2}\angle AOC$ ...(*iii*)

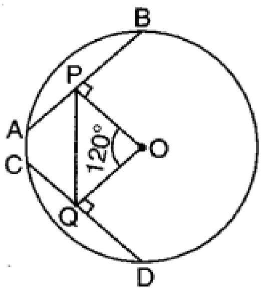
Using (*i*), (*ii*) and (*iii*), we have

∠APB = ∠APC

Hence, PA is angle bisector of ∠BPC.



Q34. In the given figure, AB and CD are two equal chords of a circle with centre O. OP and OQ are perpendiculars on chords AB and CD respectively. If ∠ POQ = 120°, find ∠APQ . [CBSE-14-ERFKZ8H]



Answer.

Since $AB = CD$

$\therefore$ $OP = OQ$ [$\because$ equal chords are equidistant from the centre]

$\therefore$ $\angle OPQ = \angle OQP$

[by using isosceles triangle property, angles opp. to equal sides of a Δ]

In ΔPOQ, by using angle sum property, we have

$\angle OPQ + \angle OQP + \angle POQ = 180^\circ$

$\Rightarrow$ $\angle OPQ + \angle OPQ + 120^\circ = 180^\circ$

$\Rightarrow$ $2\angle OPQ = 60^\circ$

$\Rightarrow$ $\angle OPQ = 30^\circ$

Now, $\angle APQ + \angle OPQ = 90^\circ$

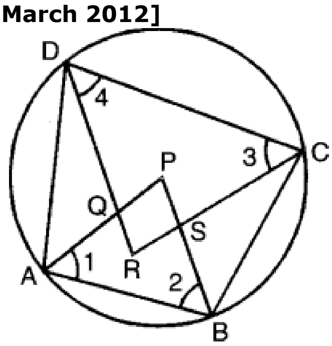
$\Rightarrow$ $\angle APQ + 30^\circ = 90^\circ$

$\Rightarrow$ $\angle APQ = 90^\circ - 30^\circ = 60^\circ$

Hence, $\angle APQ = 60^\circ$

LONG ANSWER TYPE QUESTIONS

Q35. Show that the quadrilateral formed by angle bisectors of a cyclic quadrilateral, is also cyclic. [CBSE March 2012]



Answer.

Given : A cyclic quadrilateral ABCD, in which AP, BP, CR and DR are the angle bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectively, such that a quadrilateral PQRS is formed.

To Prove : PQRS is a cyclic quadrilateral.

Proof : Since ABCD is a cyclic quadrilateral.

$$\therefore \quad \angle A + \angle C = 180^\circ \text{ and } \angle B + \angle D = 180^\circ \quad ...(i)$$

Also, AP, BP, CR and DR are the bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectively.

$$\therefore \quad \angle 1 = \frac{1}{2}\angle A,$$

$$\angle 2 = \frac{1}{2}\angle B,$$

$$\angle 3 = \frac{1}{2}\angle C \text{ and } \angle 4 = \frac{1}{2}\angle D$$

From (*i*), we have $\frac{1}{2}\angle A + \frac{1}{2}\angle C = \frac{1}{2}(\angle A + \angle C) = \frac{1}{2} \times 180^\circ = 90^\circ$

and $\quad \frac{1}{2}\angle B + \frac{1}{2}\angle D = \frac{1}{2}(\angle B + \angle D) = \frac{1}{2} \times 180^\circ = 90^\circ$

or $\quad \angle 1 + \angle 3 = 90^\circ$

and $\quad \angle 2 + \angle 4 = 90^\circ \quad ...(ii)$

Now, in ΔAPB, by angles sum property of a Δ

$$\angle 1 + \angle 2 + \angle P = 180^\circ \quad ...(iii)$$

Again, in ΔCRD, by angles sum property of a Δ

$$\angle 3 + \angle 4 + \angle R = 180^\circ \quad ...(iv)$$

Adding (*iii*) and (*iv*), we have

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle P + \angle R = 180^\circ + 180^\circ$$

$$90^\circ + 90^\circ + \angle P + \angle R = 360^\circ \quad \text{[using } (ii)\text{]}$$

$$\angle P + \angle R = 360^\circ - 180^\circ = 180^\circ$$

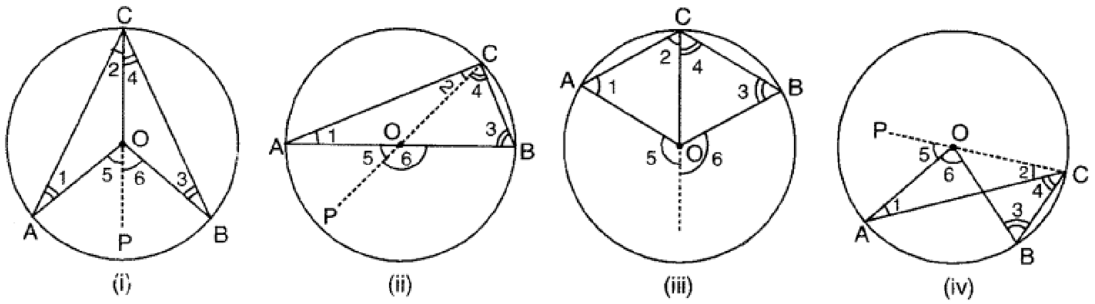
i.e., the sum of one pair of the opposite angles of quadrilateral PQRS is 180°.

Hence, the quadrilateral PQRS is a cyclic quadrilateral.

Q36. Prove that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle. [CBSE March 2012]

Answer.

Given : A circle with centre O. Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at any point C on the remaining part of the circle.



To Prove : $\angle AOB = 2\angle ACB$ in figure (*i*), (*ii*) and (*iv*) and reflex $\angle AOB = 2\angle ACB$ in figure (*iii*).

Const. : Join CO and produce it to P.

Proof : In ΔAOC

$\Rightarrow$ $OA = OC$ [radii of same circle]

$\Rightarrow$ $\angle 1 = \angle 2$ ...(*i*)

[angle opposite to equal sides of a Δ]

Ext. $\angle AOP = \angle 1 + \angle 2$

$\angle 5 = \angle 2 + \angle 2$ [using (*i*)]

$\angle 5 = 2\angle 2$ ...(*ii*)

Similarly, $\angle 6 = 2\angle 4$ ...(*iii*)

Adding (*ii*) and (*iii*), we have

$$\angle 5 + \angle 6 = 2\angle 2 + 2\angle 4$$

$$\angle 5 + \angle 6 = 2(\angle 2 + \angle 4)$$

$\Rightarrow$ $\angle AOB = 2\angle ACB$ [for figures (*i*), (*ii*)]

or reflex $\angle AOB = 2\angle ACB$ [for figure (*iii*)]

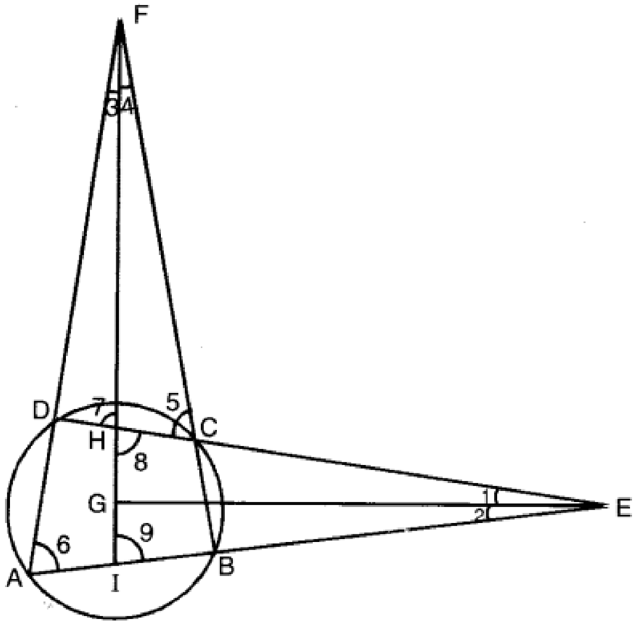
Subtracting (*ii*) from (*iii*) for figure (*iv*), we have

$$\angle 6 - \angle 5 = 2\angle 4 - 2\angle 2 = 2(\angle 4 - \angle 2)$$

$$\angle AOB = 2\angle ACB$$

Q37. Prove that the angle bisectors of the angle formed by producing opposite sides of a cyclic quadrilateral (provided they are not parallel) intersect at right angle.

Answer.



Given **:** In cyclic quadrilateral ABCD, opposite sides are produced to meet in E and F. Bisectors of ∠E and ∠F meet in G.

To Prove **:** ∠EGI = ∠EGH = 90°

Proof **:** EG and FI are the bisectors of ∠E and ∠F

∴ ∠1 = ∠2 and ∠3 = ∠4 ...(*i*)

In ΔFCH,

ext. ∠FHD = ∠7 = ∠4 + ∠5

Also, ∠7 = ∠8 [vertically opposite angles]

and ext. ∠FCD = ∠5 = ∠6

[ext. angle of a cyclic quadrilateral is equal to interior opposite angle]

⇒ ∠8 = ∠3 + ∠6 ...(*ii*) [using (*i*)]

Again, in ΔFAI, using ext. angle property

∠9 = ∠3 + ∠6 ...(*iii*)

From (*ii*) and (*iii*), we have

∠8 = ∠9

Now, in ΔEGI and ΔEGH

∠1 = ∠2

∠8 = ∠9

⇒ ∠EGI = ∠EGH

But ∠EGI + ∠EGH = 180° [a linear pair]

⇒ ∠EGI = ∠EGH = 90°

Q38. Bisectors of angles A, B and C of triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the ΔDEF are 90° -∠ A/2 ,90° – ∠ B/2 and 90°- ∠ C/2 respectively.

Answer.

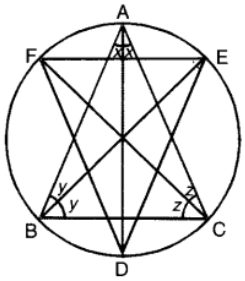
Let $\angle BAD = x$, $\angle ABE = y$

and $\angle ACF = z$, then

$\angle CAD = x$, $\angle CBE = y$

and $\angle BCF = z$

[AD, BE and CF is bisector of $\angle A$, $\angle B$ and $\angle C$]



In ΔABC, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow$ $2x + 2y + 2z = 180°$

or $x + y + z = 90°$ ...(*i*)

Now, $\angle ADE = \angle ABE$

and $\angle ADF = \angle ACF$ [angles in the same segment of a circle]

$\Rightarrow$ $\angle ADE = y$ and $\angle ADF = z$

$\Rightarrow$ $\angle ADE + \angle ADF = y + z$

or $\angle D = y + z$ ...(*ii*)

From (*i*) and (*ii*), we have

$$x + \angle D = 90°$$

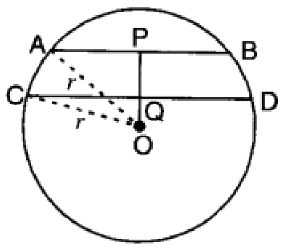
$\Rightarrow$ $\angle D = 90° - x$

or $\angle D = 90° - \frac{\angle A}{2}$ $\left[\because x = \frac{\angle A}{2}\right]$

Similarly, $\angle E = 90° - \frac{\angle B}{2}$

and $\angle F = 90° - \frac{\angle C}{2}$

Q39. In the given figure, O is the centre of a circle of radius r cm, OP and OQ are perpendiculars to AB and CD respectively and PQ = 1 cm. If AB II CD, AB = 6cm and CD = 8cm, determiner. [CBSE-15-6DWMW5A]



Answer. Since the perpendicular drawn from the centre of the circle to a chord bisects the chord. Therefore, P and Q are mid-points of AB and CD respectively.

Consequently, $AP = BP = \frac{1}{2} AB = 3$ cm

and $CQ = QD = \frac{1}{2} CD = 4$ cm

In right-angled ΔOAP, we have

$$OA^2 = OP^2 + AP^2$$
$$r^2 = OP^2 + 3^2$$
$$r^2 = OP^2 + 9 \quad \dots (i)$$

In right-angled ΔOCQ, we have

$$OC^2 = OQ^2 + CQ^2$$
$$r^2 = OQ^2 + 4^2$$
$$r^2 = OQ^2 + 16 \quad \dots (ii)$$

From (*i*) and (*ii*), we have

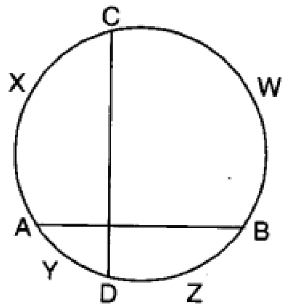
$$OP^2 + 9 = OQ^2 + 16$$
$$OP^2 - OQ^2 = 16 - 9$$
$$x^2 - (x - 1)^2 = 16 - 9 \quad \text{[where OP} = x \text{ and PQ} = 1 \text{ cm given]}$$
$$x^2 - x^2 - 1 + 2x = 7$$
$$2x = 7 + 1$$
$$x = 4$$

$\Rightarrow$ $OP = 4$ cm

From (*i*), we have

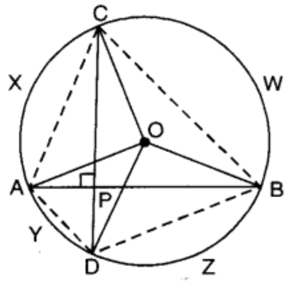
$$r^2 = (4)^2 + 9$$
$$r^2 = 16 + 9 = 25$$
$$r = 5 \text{ cm}$$

Q40. If two chords AB and CD of a circle AYDZBWCX intersect at right angles (see fig.), prove that arc CXA + arc DZB = arc AYD + arc BWC = semicircle. [NCERT Exemplar Problem]



Answer. Given two chords AB and CD of a circle intersect at right angle. Let P be the point of intersection of the chord and O be the centre of circle AYDZBWCX.

Join AC, AD, DB and BC and AO, BO, CO and DO.



Now, arc CXA subtends $\angle AOC$ and $\angle ABC$ at centre and at the remaining part of the circle.

Therefore, $\angle AOC = 2\angle ABC$ ...(i)

Similarly, arc DZB subtends $\angle BOD$ and $\angle DCB$ at the centre and at the remaining part of circle respectively

$\therefore$ $\angle BOD = 2\angle DCB$ ...(ii)

Adding (i) and (ii), we have

$$\angle AOC + \angle BOD = 2(\angle ABC + \angle DCB)$$
$$= 2 \times 90°$$
$[\because$ in rt. ΔCPB, $\angle P = 90° \Rightarrow \angle PCB + \angle PBC = 90°]$
$$= 180°$$

$\Rightarrow$ arc CXA + arc DZB = semicircle ...(iii)

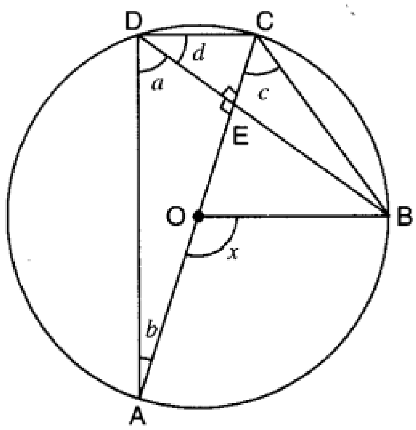
Similarly, $\angle AOD + \angle BOC = 2(\angle ACD + \angle CAB) = 2 \times 90° = 180°$

$\Rightarrow$ arc AYD + arc BWC = semicircle ...(iv)

From (iii) and (iv), we have

arc CXA + arc DZB = arc AYD + arc BWC = semicircle

Q41. In the given figure, AC is a diameter of the circle with centre O. Chord BD is perpendicular to AC. Write down the measures of angles a, b, c and d in terms of x. [CBSE-15-NS72LP7]



Answer.

Here, AC is a diameter of the circle.

$\therefore \quad \angle ADC = 90°$

$\Rightarrow \quad \angle a + \angle d = 90° \quad .... (i)$

In right-angled ΔAED, $\angle E = 90°$

$\therefore \quad \angle a + \angle b = 90° \quad ... (ii)$

From (*i*) and (*ii*), we obtain

$\angle b = \angle d \quad .... (iii)$

Also, $\quad \angle a = \angle c \quad ... (iv)$

[$\angle$s substended by the same segment are equal]

Now, $\angle AOB$ and $\angle ADB$ are angles subtended by an arc AB at the centre and at the remaining part of the circle.

$$\therefore \quad \angle ADB = \frac{1}{2} \angle AOB \Rightarrow \angle a = \frac{x}{2}$$

From (*iv*), we have $\angle a = \angle c = \frac{x}{2}$

Again, $\angle AOB + \angle BOC = 180°$

$\Rightarrow \quad \angle BOC = 180° - \angle AOB = 180° - x$

$\angle BOC$ and $\angle BDC$ are angles subtended by an arc BC at the centre and at the remaining part of the circle.

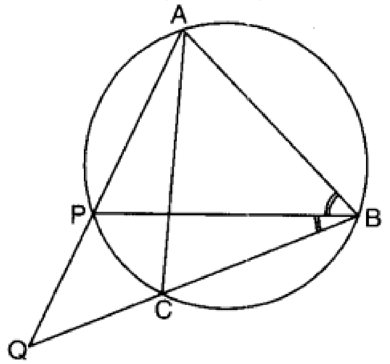
$$\therefore \quad \angle BDC = \frac{1}{2} \angle BOC$$

$$\Rightarrow \quad \angle d = \frac{1}{2} (180° - x) = 90° - \frac{x}{2}$$

From (*iii*), we have $\angle b = \angle d = 90° - \frac{x}{2}$

Hence, $\angle a = \frac{x}{2}$, $\angle b = 90° - \frac{x}{2}$, $\angle c = \frac{x}{2}$ and $\angle d = 90° - \frac{x}{2}$.

Q42. The bisector of $\angle B$ of an isosceles triangle ABC with AB = AC meets the circumcircle of ΔABC at P as shown in the given figure. If AP and BC produced meet at Q, prove that CQ = CA. [CBSE-14-GDQNI3W]



Answer.

Let $\angle ABP$ be x.

Since BP is angle bisector of $\angle B$

$\therefore \quad \angle ABP = \angle PBC = x$

$\Rightarrow \quad \angle ABC = 2x$

In ΔABC, $\quad AB = AC$ [given]

$\therefore \quad \angle ACB = \angle ABC$ [$\angle s$ opp. to equal sides of a Δ]

$\angle ACB = 2x$

$\angle PAC = \angle PBC = x$ [$\angle s$ in the same segment]

In ΔAQC, by using exterior angle property, we have

$\angle ACB = \angle AQC + \angle QAC$

$2x = \angle AQC + x$ [$\because \angle PAC$ and $\angle QAC$ are same angle]

$\Rightarrow \quad \angle AQC = 2x - x = x$

$\Rightarrow \quad \angle AQC = \angle QAC$

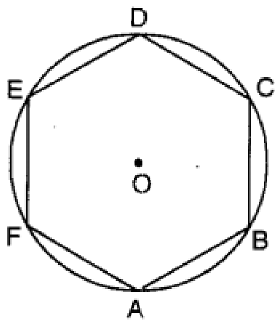
Now, in ΔQAC, we have

$\angle AQC = \angle QAC$

$\Rightarrow \quad CA = CQ$ [sides opp. to equal angles of a Δ]

Value Based Questions

Q1. A small cottage industry employing people from a nearby slum area prepares round table cloths having six equal designs in the six segment formed by equal chords AB, BC, CD, DE, EF and FA. If O is the centre of round table cloth (see figure). Find $\angle AOB$, $\angle AEB$ and $\angle AFB$.What value is depicted through this question ? [CBSE-14-17DJG1U]



Answer.

Since six equal designs in the six segment formed by equal chords AB, BC, CD, DE, EF and FA.

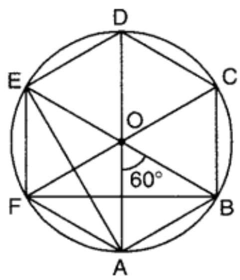
Therefore, we have six equilateral triangles as shown in the figure.

Since ΔAOB, ΔBOC, ΔCOD, ΔDOE, ΔEOF and ΔFOA are equilateral.

∴ Each angle is equal to 60°.

$\angle AOB = 60°$

∠AOB, ∠AEB and ∠AFB are angles subtended by an arc AB at the centre and at the remaining part of the circle.



$$\therefore \quad \angle AEB = \angle AFB = \frac{1}{2} \angle AOB$$

$$= \frac{1}{2} \times 60° = 30°$$

Thus, $\angle AEB = \angle AFB = 30°$

Value depicted : By employing people from a slum area to prepare round table cloths realize their social responsibility to work for helping the ones in need.

Q2. Three students Priyanka, Sania and David are protesting against killing innocent animals for commercial purposes in a circular park of radius 20 m. They are standing at equal distances on its boundary by holding banners in their hands.

(i) Find the distance between each of them.

(ii) Which mathematical concept is used in it ?

(iii) How does an act like this reflects their attitude towards society ?

Answer.

(*i*) Let us assume that A, B and C be the positions of Priyanka, Sania and David respectively on the boundary of circular park with centre O.

Draw AD ⊥ BC.

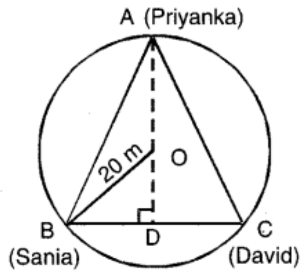
Since the centre of the circle coincides with the centroid of the equilateral ΔABC.

$$\therefore \text{Radius of circumscribed circle} = \frac{2}{3} AD$$

$$\Rightarrow \quad 20 = \frac{2}{3} AD$$

$$\Rightarrow \quad AD = 20 \times \frac{3}{2}$$

$$\Rightarrow \quad AD = 30 \text{ m}$$



Now, AD ⊥ BC, and let AB = BC = CA = x m

$$\Rightarrow \quad BD = CD = \frac{1}{2} BC = \frac{x}{2}$$

In rt. ΔBDA, ∠D = 90°

∴ By Pythagoras Theorem, we have

$$AB^2 = BD^2 + AD^2$$

$$x^2 = \left(\frac{x}{2}\right)^2 + (30)^2$$

$$x^2 - \frac{x^2}{4} = 900$$

$$\frac{3}{4}x^2 = 900$$

$$x^2 = 900 \times \frac{4}{3}$$

$$x^2 = 1200$$

$$x = \sqrt{1200} = 20\sqrt{3}$$

Hence, distance between each of them is $20\sqrt{3}$ m.

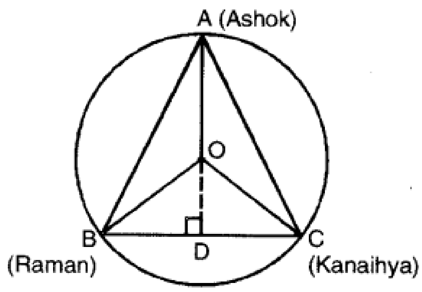
(*ii*) Properties of circle, equilateral triangle and Pythagoras theorem.

(*iii*) Live and let live !

Q3. A circular park of radius 10 m is situated in a colony. Three students Ashok, Raman and Kanaihya are standing at equal distances on its circumference each having a toy telephone in his hands to talk each other about Honesty, Peace and Discipline.
(i) Find the length of the string of each phone.
(ii) Write the role of discipline in students' life.

Answer. Let us assume, A, B and C be the position of three students Ashok, Raman and Kanaihya respectively on the circumference of the circular park with centre O and radius 10 m. Since the centre of circle coincides with the centroid of the equilateral ΔABC.

∴ Radius of circumscribed circle $= \frac{2}{3}$ AD



$\Rightarrow \quad 10 = \frac{2}{3}AD$

$\Rightarrow \quad AD = 15$ m

Now, AD ⊥ BC and let AB = BC = CA = x

$\Rightarrow \quad BD = CD = \frac{1}{2}BC = \frac{x}{2}$

In rt. ∠ed ΔBDA, ∠D = 90°

$$AB^2 = BD^2 + AD^2$$

$$x^2 = \frac{x^2}{4} + 225$$

$$x^2 - \frac{x^2}{4} = 225$$

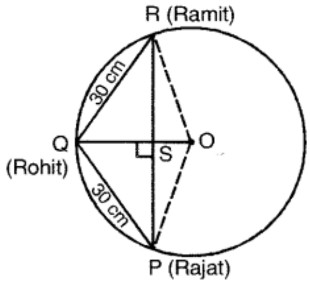
$\Rightarrow \quad x^2 = 225 \times \frac{4}{3} = 300$

$\Rightarrow \quad x = 10\sqrt{3}$ m

Thus, the length of each string is $10\sqrt{3}$ m.
In students' life, discipline is necessary. It motivates as well as it nurture the students to make him as a responsible citizen.

Q4. Three scouts Rajat, Rohit and Ramit in the cultural show holded three stringed balloons with a message 'Stop Child Labour'. Keeping themselves on the boundary of a circle of radius 25 cm, each scout holded the string tightly. Find the distance between Rajat and Ramit, when distance between Rajat and Rohit and Rohit and Ramit is 30 cm. What message was given by scouts and why ?

Answer.



Let P, Q and R be the position of Rajat, Rohit and Ramit respectively.
Let O be the centre of circle. Points P and R are equidistant from Q.
Now, OQ is the perpendicular bisector of PR.
Let OQ bisect PR at S.

In rt. ΔPSQ, $\angle S = 90^\circ$

$$PS^2 = PQ^2 - SQ^2$$

$$\Rightarrow \quad PS^2 = 900 - SQ^2 \quad \ldots(i)$$

Also, in rt. ΔOSP, we have

$$PS^2 = OP^2 - OS^2$$

$$\Rightarrow \quad = 625 - (OQ - SQ)^2$$

$$\Rightarrow \quad 900 - SQ^2 = 625 - (25 - SQ)^2$$

$$= 625 - 625 - SQ^2 + 50\,SQ$$

$$\Rightarrow \quad 50\,SQ = 900 \Rightarrow SQ = 18 \text{ cm} \quad \ldots(ii)$$

From (*i*) and (*ii*), we have

$$PS^2 = 900 - (18)^2 = 576$$

$$\Rightarrow \quad PS = 24 \text{ cm}$$

$$\Rightarrow \quad PR = 2 \times 24 = 48 \text{ cm}$$

Thus, distance between Rajat and Ramit is 48 cm.
Child labour is curse to society. Scouts appealed everyone to stop child labour.