Chapter - 2
Polynomial

Exercise 2.3

Divide the polynomial by the polynomial and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

(i)By long division method we have,

$$
\begin{array}{r|l}
 & x - 3 \\
\hline
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\
 & x^3 \qquad\quad - 2x \\
 & - \qquad\qquad + \\
\hline
 & -3x^2 + 7x - 3 \\
 & -3x^2 \qquad + 6 \\
\hline
 & 7x - 9
\end{array}
$$

Quotient = $x - 3$

Remainder = $7x - 9$

(ii) By long division method we have,

$$x^2 + x - 3$$

$$x^2 - x + 1 \sqrt{x^4 + ox^3 - 3x^2 + 4x + 5}$$

$$x^4 - \quad x^3 \quad + x^2$$

$$- \quad + \quad -$$

$$x^3 - 4x^2 + 4x + 5$$

$$x^3 - \quad x^2 + x$$

$$- \quad + \quad -$$

$$-3x^2 + 3x + 5$$

$$-3x^2 + 3x \; -3$$

$$+ \quad - \quad +$$

$$8$$

Quotient = $x^2 + x - 3$

Remainder = 8

(iii) By long division method we have,

$$-x^2 - 2$$

$$-x^2 + 2\sqrt{x^4 + 0x^2 - 5x + 6}$$

$$x^4 - 2x^2$$

$$- \quad +$$

$$2x^2 - 5x + 6$$

$$2x^2 \quad - 4$$

$$- \quad +$$

$$-5x \; +10$$

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

Q. 2 Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 -3, 2t^4 + 3t^3 - 2t^3 - 9t - 12$

(ii) $x^2 +3x +1, 3x^4 + 5x^3- 7x^2 + 2x + 2$

(iii) $x^3 - 3x +1, x^5- 4x^3 + x^2 + 3x + 1$

(i) $t^2-3 = t^2 + 0t - 3$

$$2t^2 + 3t + 4$$

$$t^2 + 0t - 3 \overline{\big) 2t^4 + 3t^3 - 2t^2 - 9t - 12}$$

$$2t^4 + 0t^3 - 6t^2$$

$$- \quad - \quad +$$

$$3t^3 + 4t^2 - 9t - 12$$

$$3t^3 + 0t^2 - 9t$$

$$- \quad - \quad +$$

$$4t^2 + 0t - 12$$

$$4t^2 + 0t - 12$$

$$- \quad - \quad +$$

$$0$$

Since the remainder is 0,

Hence, $t^2 - 3$ is a factor of $2t^4+3t^3-2t^2-9t-12$.

(ii)

$$3x^2 - 4x + 2$$

$$x^2+3x+1 \; \sqrt{3x^4 + 5x^3 - 7x^2 + 2x + 2}$$

$$3x^4 + 9x^3 + 3x^2$$

$$- \quad - \quad -$$

$$-4x^3 - 10x^2 + 2x + 2$$

$$-4x^3 - 12x^2 - 4x$$

$$+ \quad + \quad +$$

$$2x^2 + 6x + 2$$

$$2x^2 + 6x + 2$$

$$0$$

Since the remainder is 0,

Hence, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$.

(iii)

$$x^2 - 1$$

$$x^2 - 3x + 1 \; \sqrt{x^5 - 4x^3 + x^2 + 3x + 1}$$

$$x^5 - 3x^2 + x^2$$

$$- \quad + \quad -$$

$$-x^3 \qquad + 3x + 1$$

$$-x^3 \qquad + 3x - 1$$

$$+ \qquad - \quad +$$

$$2$$

Since the remainder $\neq 0$,

Hence, x^3-3x+1 is not a factor of $x^5-4x^3+ x^2+3x+1$.

Q. 3 Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

$p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right)$ is a factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$.

Therefore, we divide the given polynomial by $x^2 - \frac{5}{3}$.

$$
\begin{array}{r|l}
 & 3x^2 + 6x + 3 \\
x^2 + 0x - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 + 0x^3 - 5x^2 \\
 & - \quad - \quad + \\
\hline
 & 6x^3 + 3x^2 - 10x - 5 \\
 & 6x^3 + 0x^2 - 10x \\
 & - \quad - \quad + \\
\hline
 & 3x^2 + 0x - 5 \\
 & 3x^2 + 0x - 5 \\
 & - \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

We know, Dividend = (Divisor × quotient) + remainder

$3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$

$3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1)$

As $(a+b)^2 = a^2 + b^2 + 2ab$

So, $x^2 + 2x + 1 = (x+1)^2$

$3x^4 + 6x^3 - 2x^2 - 10x - 5 = 3\left(x^2 - \frac{5}{3}\right)(x + 1)^2$Therefore, its zero is given by $x + 1 = 0$.

$\Rightarrow x = -1, -1$

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$. and - 1 , - 1.

Q. 4 On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x) the quotient and remainder were $(x - 2)$ and $(-2x + 4)$, respectively. Find g(x).

Solution: Given,

Polynomial, $p(x) = x^3 - 3x^2 + x + 2$ (dividend)

Quotient = $(x - 2)$

Remainder = $(-2x + 4)$

To find: divisor = g(x)we know, Dividend = Divisor × Quotient + Remainder

$\Rightarrow x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

$\Rightarrow x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x - 2)$

$\Rightarrow x^3 - 3x^2 + 3x - 2 = g(x)(x - 2)$

g(x) is the quotient when we divide (x^3 - $3x^2$ + 3x - 2) by (x - 2)

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x - 2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & -x^2 + 3x - 2 \\ & -x^2 + 2x \\ & + \quad - \\ \hline & x - 2 \\ & x - 2 \\ & - \quad + \\ \hline \end{array}$$

∴ g(x) = ($x^2 - x + 1$)

Q. 5 Give examples of polynomials and which satisfy the division algorithm and

(i) deg p (x) = deg q (x)

(ii) deg q (x) = deg r (x)

(iii) deg r (x) = 0

Degree of a polynomial is the highest power of the variable in the polynomial. For example if f(x) = x^3 - $2x^2$ + 1, then the degree of this polynomial will be 3.

(i) By division Algorithm : p(x) = g(x) x q(x) + r(x)

It means when P(x) is divided by g(x) then quotient is q(x) and remainder is r(x)We need to start with p(x) = q(x)This means that the degree of polynomial p(x) and quotient q(x) is same. This can only happen if the degree of g(x) = 0 i.e p(x) is divided by a constant. Let p(x) = x^2 + 1 and g(x) = 2

$$\frac{p(x)}{g(x)} = \frac{x^2+1}{2}$$

The,

$p(x) = g(x) \times \left(\frac{x^2+1}{2}\right)$

Clearly, Degree of p(x) = Degree of q(x)

2. Checking for division algorithm,

p(x) = g(x) × q(x) + r(x)

= $6x^2$ +2x + 2 = 3 ($3x^2$ + x + 1)

= $6x^2$ +2x + 2

Thus, the division algorithm is satisfied.

(ii) Let us assume the division of x^3+ x by x^2,

Here,

p(x) = x^3+ x

g(x) = x^2

q(x) = x and r(x) = x

Clearly, the degree of q(x) and r(x) is the same i.e.,

Checking for division algorithm,

p(x) = g(x) × q(x) + r(x) x^3 + x

= (x^2) × x + x x^3 + x = x^3 + x

Thus, the division algorithm is satisfied.

(iii) Degree of the remainder will be 0 when the remainder comes to a constant.

Let us assume the division of x^3+ 1by x^2.

Here,

$p(x) = x^3 + 1$ $g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0. Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x) x^3 + 1$

$= (x^2) \times x + 1$ $x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.