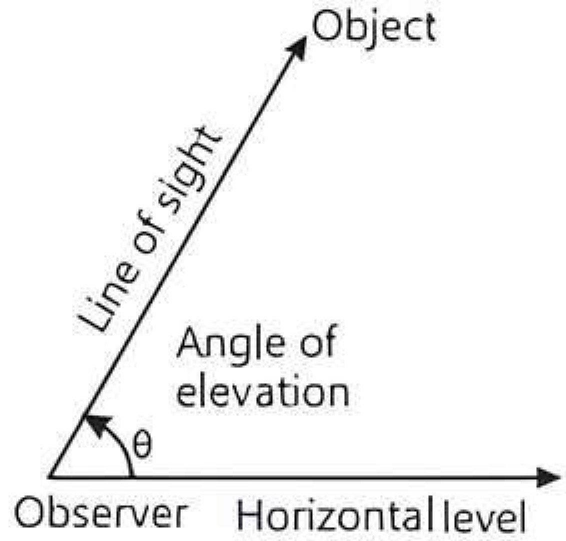
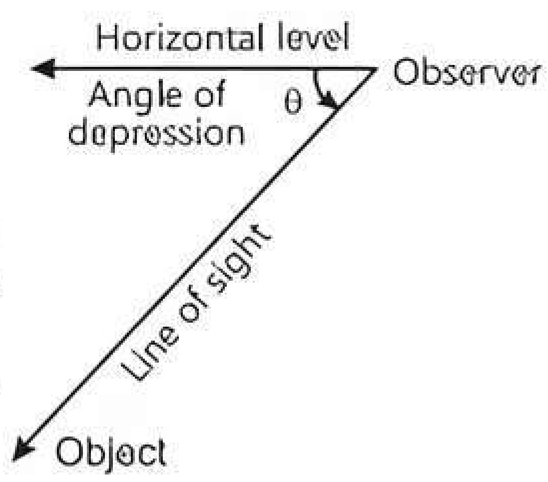
9 Some Applications of Trigonometry

Fastrack Revision

- **Line of Sight:** If an observer is viewing an object, the straight line joining the eye of the observer to that object is called line of sight.
- **Angle of Elevation:** The angle of elevation of an object viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level, *i.e.*, the case when we raise our head to look at the object.



- **Angle of Depression:** The angle of depression of an object viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level, *i.e.*, the case when we lower our head to look at the object.



- The height or length of an object or the distance between two distant objects can be determined through trigonometric ratios.

Knowledge BOOSTER

1. *The angles of elevation and depression are always acute angles.*
2. *If the angle of elevation of the tower (or Sun) decreases, the shadow of the tower (or Sun) increases.*
3. *If the observer moves towards (or moves away) the perpendicular line, the angle of elevation increases (or decreases).*
4. *If the height of tower is doubled and the distance between the observer and the foot of tower is also doubled, then the angle of elevation remains same.*

Practice Exercise

Multiple Choice Questions

Q 1. **If a pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then Sun's elevation is:**

[NCERT EXEMPLAR; CBSE 2023; CBSE SQP 2023-24]

a. 60° b. 45° c. 30° d. 90°

Q 2. **From a point P on a level ground, the angle of elevation of the top of tower is 30°. If the tower is 100 m high, the distance of point P from the foot of the tower is:**

a. 149 m b. 156 m c. 173 m d. 200 m

Q 3. **The angle of elevation of the top of the tower is 60° and the horizontal distance from the observer's eye to the foot of the tower is 100 m, then the height of the tower will be:**

a. $50\sqrt{3}$ m b. $\frac{100}{\sqrt{3}}$ m

c. $100\sqrt{3}$ m d. $60\sqrt{3}$ m

Q 4. **The ratio of the length of a rod and its shadow is $1:\sqrt{3}$, then the angle of elevation of the Sun is:**

a. 45° b. 30° c. 60° d. 90°

Q 5. **The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6 m away from the wall. The length of the ladder is:**

a. 3 m b. 6 m c. 8 m d. 9.2 m

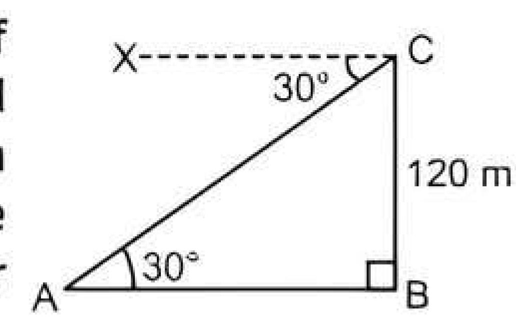
Q 6. **A boy standing on top of a tower of height 20 m observes that the angle of depression of a car on the road is 60°. The distance between the foot of the tower and the car must be: [Use $\sqrt{3} = 1.73$]**

a. 10.45 m b. 11.54 m c. 12.55 m d. 12.50 m

Q 7. **If the angle of depression of an object from a 50 m high tower is 30°, then the distance of the object from the tower is:**

a. $25\sqrt{3}$ m b. $\frac{50}{\sqrt{3}}$ m c. $50\sqrt{3}$ m d. 50 m

Q 8. **The angle of depression of a car parked on the road from the top of 120 m high tower is 30°. The distance of the car from the tower (in metres) is:**



a. $120\sqrt{3}$ m b. 120 m

c. $40\sqrt{3}$ m d. None of these

Q 9. A vertical straight tree of 15 m high, is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did the tree break? [Use $\sqrt{3} = 1.73$]

a. 6.9 m b. 9.6 m c. 5.9 m d. 7.9 m

Q 10. An observer 1.6 m tall is 20 m away from a tower. The angle of elevation from his eye to the top of the tower is 45°. The height of the tower is:

a. 21.6 m b. 2 m
c. 72 m d. None of these

Q 11. An observer from the top of a 100 m high lighthouse from the sea level observed that the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, then the distance between two ships is:

a. $100(\sqrt{3}+1)$ m b. $50(\sqrt{3}+1)$ m
c. $50(\sqrt{3}-1)$ m d. $100(\sqrt{3}-1)$ m

Q 12. When the Sun's altitude changes from 30° to 60°, the length of the shadow of a tower decreases by 70 m. What is the height of the tower?

a. 35 m b. 140 m c. $35\sqrt{3}$ m d. $2\sqrt{3}$ m

Q 13. From the top of a cliff 30 m high, the angle of elevation of the top of a tower from cliff top is found to be equal to the angle of depression of the foot of the tower. The height of the tower is:

a. 30 m b. 60 m c. 20 m d. 50 m

Assertion & Reason Type Questions

Directions (Q. Nos. 14-17): *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 14. Assertion (A): If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the Sun is 45°.

Reason (R): Trigonometric ratio, tangent is defined as

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

Q 15. Assertion (A): The angle of elevation of the top of a tower is 60°. If the height of the tower and its base is tripled then angle of elevation of its top will also be tripled.

Reason (R): In an equilateral triangle of side $3\sqrt{3}$ cm, the length of the altitude is 4.5 cm.

Q 16. Assertion (A): Suppose a bird was sitting on a tree. A person was sitting on a ground and saw the bird, which makes an angle such that $\tan\theta = \frac{12}{5}$. The distance from bird to the person is 13 units.

Reason (R): In a right-angled triangle,

$$(\text{Hypotenuse})^2 = (\text{Side})^2 + (\text{Base})^2.$$

Q 17. Assertion (A): The angle of elevation of the top of the tower is 30° and the horizontal distance from the observer's eye to the foot of the tower is 50 m, then the height of the tower will be $\frac{50}{3}\sqrt{3}$ m.

Reason (R): While using the concept of angle of elevation/depression, triangle should be a right angled triangle.

Fill in the Blanks Type Questions

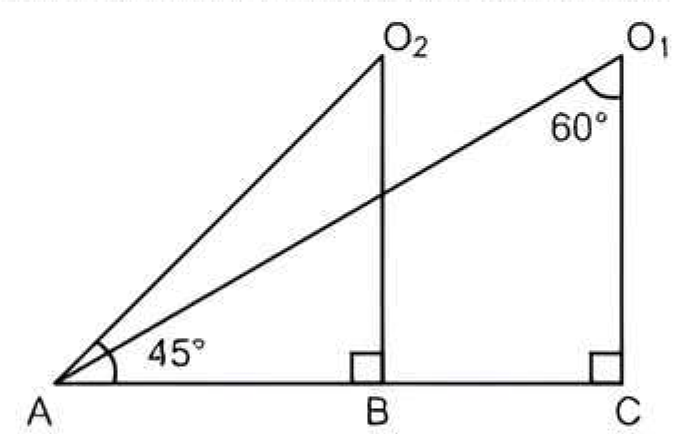
Q 18. The .... of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

Q 19. The angle of .... of an object viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level.

Q 20. If the length of the shadow of a tower is $\sqrt{3}$ times its height, the angle of elevation of the Sun is .... *[NCERT EXEMPLAR]*

Q 21. From a point 20 m away from the foot of a tower, the angle of elevation of the top of the tower is 30°, then the height of the tower is .... [Use $\sqrt{3} = 1.73$]

Q 22. In the given figure, the angles of depressions from the observing positions O_1 and O_2 respectively of the object A are .... and ....



True/False Type Questions

Q 23. The height of the tower if length of the shadow is 10 m and Sun's altitude is 45°, is 10 m.

Q 24. If length of shadow of tower is 20 m and angle of elevation is 60°, then the height of tower is $\frac{20}{\sqrt{3}}$ m.

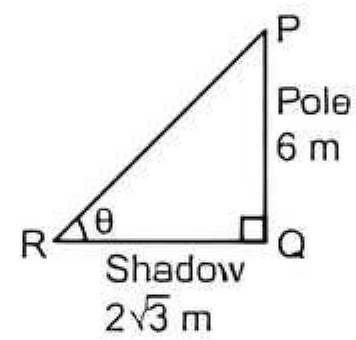
Q 25. A little boy is flying a kite. The string of kite makes an angle of 30° with the ground. If the height of the kite is 21m, then the length of the string is 35 m.

Q 26. The angle of elevation of the top of a tower is 30°. If the height of the tower is doubled then angle of elevation of its top will also be doubled. *[NCERT EXERCISE]*

Q 27. A person walking 20 m towards a chimney in a horizontal line through its base observer that its angle of elevation changes from 30° to 45°, then height of chimney is $\left(\frac{20}{\sqrt{3}-1}\right)$ m.

Solutions

1. (a) Let $PQ = 6$ m be the height of the pole and $RQ = 2\sqrt{3}$ m be its shadow.



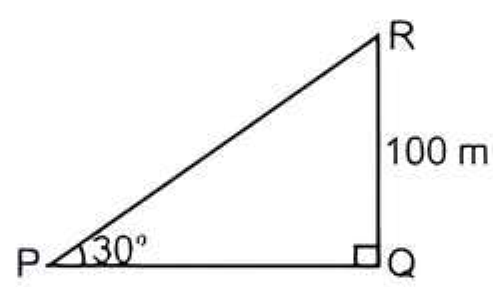
Let angle of elevation of the Sun be θ.
In right-angled ΔPQR,

$$\tan\theta = \frac{PQ}{RQ} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$= \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60^\circ$$

$\Rightarrow \quad \theta = 60^\circ$

2. (c) Let $QR = 100$ m be the height of the tower and point P makes an angle of elevation of the top of the tower *i.e.* $\angle QPR = 30^\circ$.



In right-angled ΔPQR,

$$\tan 30^\circ = \frac{RQ}{PQ} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

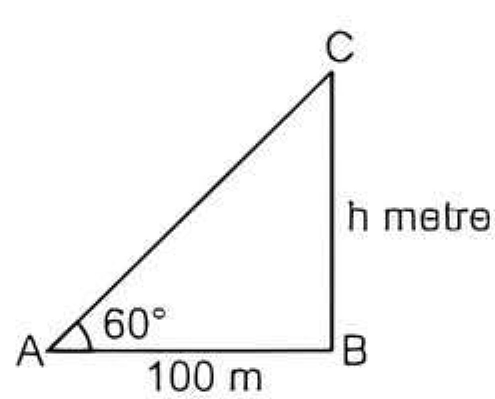
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{PQ}$$

$$\Rightarrow \quad PQ = 100\sqrt{3} \text{ m}$$

$$= 100 \times 1.73 \text{ m} = 173 \text{ m}$$

3. (c) Let $BC = h$ metre be the height of the tower and distance from observer to the foot of the tower be $AB = 100$ m.

In right-angled ΔABC,



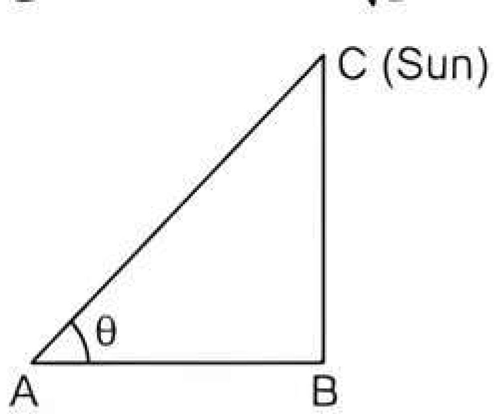
$$\tan 60^\circ = \frac{BC}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{100}$$

$$\Rightarrow \quad h = 100\sqrt{3} \text{ m}$$

4. (b) Let C be the position of the Sun.
Let BC and AB be the length of rod and length of the shadow.

Given, $\dfrac{\text{Length of rod}}{\text{Length of shadow}} = \dfrac{1}{\sqrt{3}} = \dfrac{BC}{AB}$ ...(1)



In right-angled ΔABC,

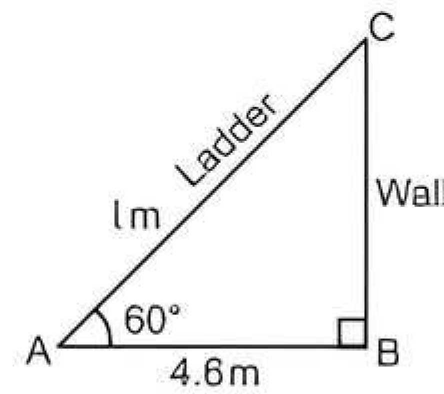
$$\tan\theta = \frac{BC}{AB}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \qquad (\because \text{from eq. (1)})$$

$$\Rightarrow \quad \tan\theta = \tan 30^\circ$$

$$\Rightarrow \quad \theta = 30^\circ$$

5. (d) Let BC be the height of the wall and $AC = l$ m be the length of the ladder leaning against a wall.



Ladder AC makes an angle of elevation of 60° to the wall *i.e.* $\angle CAB = 60^\circ$

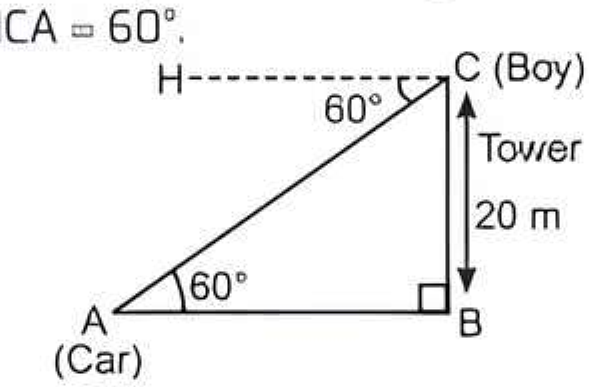
Let $AB = 4.6$ m be the foot of the ladder from the wall.
In right-angled ΔABC,

$$\cos 60^\circ = \frac{AB}{AC} \qquad \left(\because \cos\theta = \frac{B}{H}\right)$$

$$\Rightarrow \quad \frac{1}{2} = \frac{4.6}{l}$$

$$\Rightarrow \quad l = 9.2 \text{ m}$$

6. (b) Let $BC = 20$ m be the height of the tower. Let A be the position of car and C be the position of boy.
At point C, boy makes an angle of depression of 60° *i.e.*, $\angle HCA = 60^\circ$.



Here, $\angle BAC = \angle HCA = 60^\circ$ (alternate angles)
In right-angled ΔABC,

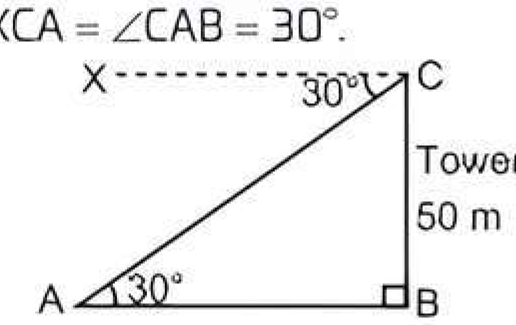
$$\tan 60^\circ = \frac{BC}{AB} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad \sqrt{3} = \frac{20}{AB}$$

$$\Rightarrow \quad AB = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{20}{3} \times 1.73$$

$$= 6.67 \times 1.73 = 11.54 \text{ m}$$

7. (c) Let A be the position of an object and $BC = 50$ m be the height of tower.
Then $\angle XCA = \angle CAB = 30^\circ$. (alternate angles)



In right-angled ΔABC,

$$\tan 30^\circ = \frac{BC}{AB} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{50}{AB}$$

$$\Rightarrow \quad AB = 50\sqrt{3} \text{ m}$$

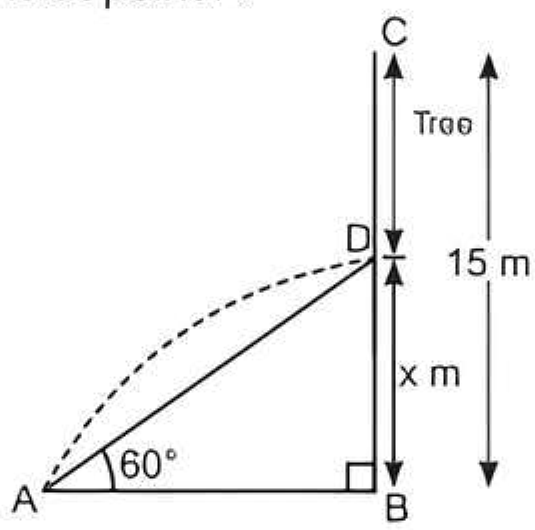
8. (a) In right-angled ΔABC,

$$\tan 30^\circ = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{120}{AB}$$

$$\Rightarrow \quad AB = 120\sqrt{3} \text{ m}$$

9. (a) Let $BC = 15$ m be the height of the tree. Let at point D, tree breaks and touches the top point C of tree to the ground at point A.



TR!CK

The broken part CD of tree is equal to the slope line AD, i.e., CD = AD.

Let $BD = x$ m be the height of broken tree.
Then $CD = AD = 15 - x$.
Given, broken part of tree CD makes an angle of 60° with the ground *i.e.* $\angle DAB = 60^\circ$.
In right-angled ΔABD,

$$\sin 60^\circ = \frac{BD}{AD}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{x}{15-x}$$

$$\Rightarrow \quad 15\sqrt{3} - \sqrt{3}x = 2x$$

$$\Rightarrow \quad x(2+\sqrt{3}) = 15\sqrt{3}$$

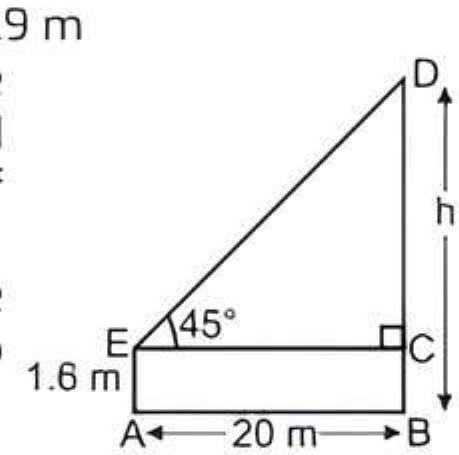
$$\Rightarrow \quad x = \frac{15\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{30\sqrt{3}-45}{(2)^2-(\sqrt{3})^2} = \frac{30\times 1.73 - 45}{4-3}$$

$$(\because (a+b)(a-b) = a^2 - b^2)$$

$$= 51.9 - 45 = 6.9 \text{ m}$$

10. (a) Let $AE = 1.6$ m be the height of an observer and $BD = h$ m be the height of the tower.
Let $EC = AB = 20$ m be the distance from observer to the tower.



In right-angled ΔECD,

$$\tan 45^\circ = \frac{CD}{EC}$$

$$(\because AB = EC = 20 \text{ m and } AE = BC = 1.6 \text{ m})$$

$$\Rightarrow \quad 1 = \frac{h-1.6}{20} \qquad \left(\begin{aligned}\because CD &= BD - BC \\ &= h - 1.6\end{aligned}\right)$$

$$\Rightarrow \quad h - 1.6 = 20$$

$$\Rightarrow \quad h = 21.6 \text{ m}$$

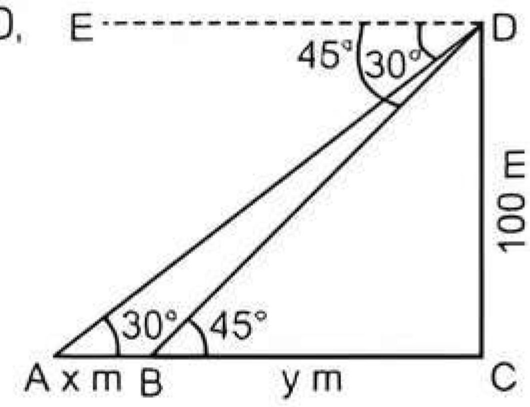
COMMON ERR(!)R

Sometimes students make an error of taking an angle from point A instead of taking at point E. So, continuous practice is required to make stronger concept.

11. (d) Let $CD = 100$ m be the height of the lighthouse. Let D be the position of observer, and A and B be the position of two ships. The angles of depression from point D to the ships A and B are

$$\angle EDA = 30^\circ \text{ and } \angle EDB = 45^\circ$$

Here, $\angle CAD = \angle EDA = 30^\circ$
and $\angle CBD = \angle EDB = 45^\circ$ (by alternate angles)
Let $AB = x$ m and $BC = y$ m.
Then in right-angled ΔBCD,



$$\tan 45^\circ = \frac{CD}{BC}$$

$$\Rightarrow \quad 1 = \frac{100}{y}$$

$$\Rightarrow \quad y = 100 \text{ m}$$

And in right-angled ΔACD,

$$\tan 30^\circ = \frac{CD}{AC}$$

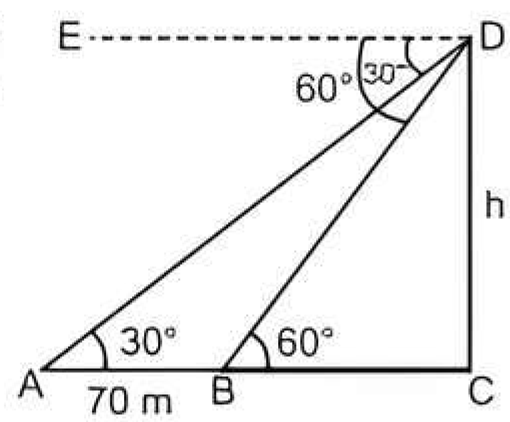
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{x+y} \qquad (\because AC = AB + BC = x + y)$$

$$\Rightarrow \quad x + y = 100\sqrt{3}$$

$$\Rightarrow \quad x + 100 = 100\sqrt{3} \qquad (\because y = 100 \text{ m})$$

$$\Rightarrow \quad x = 100(\sqrt{3} - 1) \text{ m}$$

12. (c) Let D be the position of the Sun and $DC = h$ m be the height of the tower.
Given that,



$$\angle EDA = 30^\circ$$

and $\angle EDB = 60^\circ$

Here, $\angle CAD = \angle EDA = 30^\circ$

and $\angle CBD = \angle EDB = 60^\circ$ (alternate angles)

Length of the shadow decrease, $AB = 70$ m.
In right-angled ΔACD,

$$\tan 30^\circ = \frac{CD}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{AB + BC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{70 + BC} \qquad \text{...(1)}$$

And in right-angled ΔBCD,

$$\tan 60^\circ = \frac{CD}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{BC} \quad \Rightarrow \quad BC = \frac{h}{\sqrt{3}}$$

From eq. (1), we get

$$\frac{1}{\sqrt{3}} = \frac{h}{70 + \frac{h}{\sqrt{3}}}$$

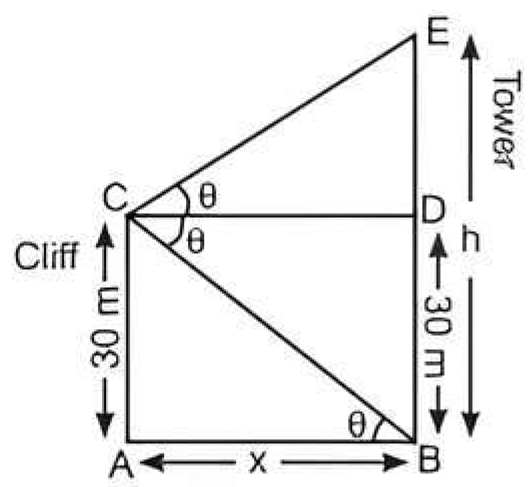
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h\sqrt{3}}{70\sqrt{3}+h}$$

$$\Rightarrow \quad 70\sqrt{3}+h = 3h$$

$$\Rightarrow \quad 70\sqrt{3} = 2h$$

$$\Rightarrow \quad h = 35\sqrt{3}\text{ m}$$

13. (b) Let C be the position of top of the cliff, then AC = 30 m. Let BE = h m be the height of the tower. Let θ be the angle of elevation from point C to the point E. Then



$\angle DCE = \theta$

Also given, angle of depression $\angle DCB = \theta$.

Here, $\angle ABC = \angle DCB = \theta$ (by alternate angle)

In right-angled ΔBAC,

$$\tan\theta = \frac{AC}{AB}$$

$$\Rightarrow \quad \tan\theta = \frac{30}{x} \Rightarrow x = \frac{30}{\tan\theta} \quad ...(1)$$

In ΔCDE, $\angle D = 90°$

Then $\tan\theta = \frac{ED}{CD}$ (Let AB = CD = x m)

$$\Rightarrow \quad \tan\theta = \frac{h-30}{x} \quad \left(\because ED = BE - BD = (h-30)\text{ m}\right)$$

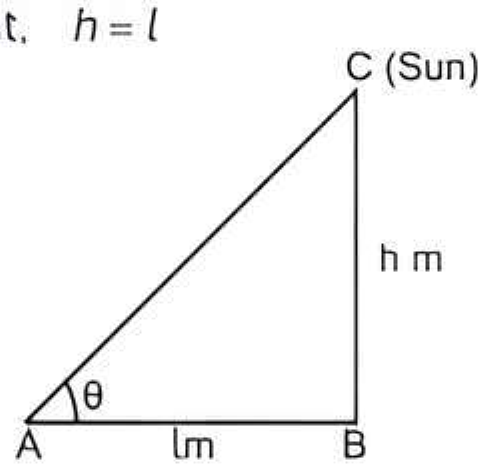
$$\Rightarrow \quad x = \frac{h-30}{\tan\theta} \quad ...(2)$$

From eqs. (1) and (2), we get

$$\frac{h-30}{\tan\theta} = \frac{30}{\tan\theta} \Rightarrow h = 60\text{ m}$$

14. (a) **Assertion (A):** Let BC = h m be the height of the pole and AB = l m be the length of the shadow. Let the Sun makes an angle θ from point A.

Given that, $h = l$



In right-angled triangle ABC,

$$\tan\theta = \frac{BC}{AB} \quad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad \tan\theta = \frac{h}{l} = \frac{l}{l} \quad [\because h = l \text{ (given)}]$$

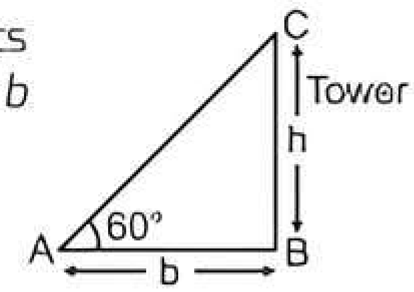
$$\Rightarrow \quad \tan\theta = 1 = \tan 45° \Rightarrow \theta = 45°$$

So, Assertion (A) is true.

Reason (R): It is also true that $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

15. (d) **Assertion (A):** Let BC = h units be the height of tower and AB = b units be the base of the tower.



Then $\tan 60° = \frac{BC}{AB}$

$$\Rightarrow \quad \tan 60° = \frac{h}{b} \quad ...(1)$$

If we tripled the height and base of tower *i.e.* BC = $3h$ and AB = $3b$, then angle will be

$$\tan\theta = \frac{BC}{AB} = \frac{3h}{3b} \Rightarrow \tan\theta = \frac{h}{b}$$

$$\Rightarrow \quad \tan\theta = \tan 60° \quad \text{[from eq. (1)]}$$

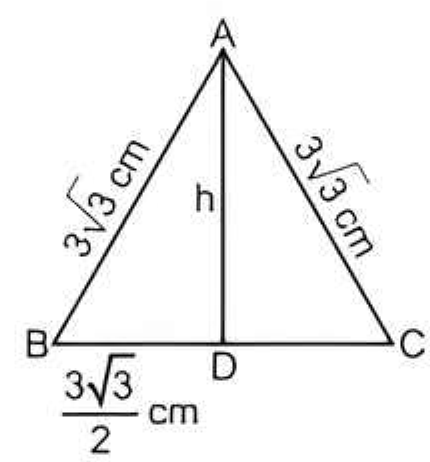
$$\Rightarrow \quad \theta = 60°,$$

which is not tripled the original angle.

So, Assertion (A) is false.

Reason (R): Let ABC be an equilateral triangle. Then

$$AB = BC = CA = 3\sqrt{3}\text{ cm}$$



Let h be the altitude of an equilateral triangle.

TIP

Altitude of an equilateral triangle divides the base into two equal parts.

$$\therefore \quad BD = DC = \frac{3\sqrt{3}}{2}\text{ cm}$$

In right-angled ΔADB, use Pythagoras theorem,

$$AD = \sqrt{(AB)^2 - (BD)^2} = \sqrt{(3\sqrt{3})^2 - \left(\frac{3\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{27 - \frac{27}{4}} = \sqrt{\frac{81}{4}} = \frac{9}{2} = 4.5\text{ cm}$$

So, Reason (R) is true.

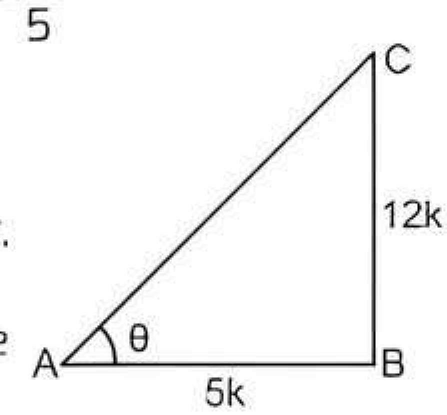
Hence, Assertion (A) is false but Reason (R) is true.

16. (a) **Assertion (A):** Given $\tan\theta = \frac{12}{5}$

$$\Rightarrow \quad \tan\theta = \frac{12}{5} = \frac{BC}{AB}$$

Let BC = $12k$ and AB = $5k$, where k is a constant.

In right-angled ΔABC, use Pythagoras theorem,



$$AC = \sqrt{(AB)^2 + (BC)^2}$$

$$= \sqrt{(5k)^2 + (12k)^2}$$

$$= \sqrt{25k^2 + 144k^2} = \sqrt{169k^2}$$

$$= 13k = 13 \text{ units} \quad (\text{Consider } k = 1)$$

So, Assertion (A) is true.

Reason (R): It is a true relation that

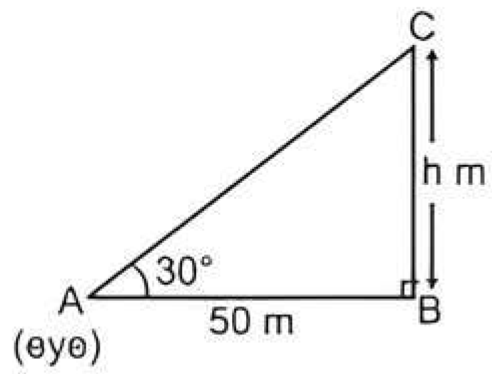
In a right-angled triangle,

$(\text{Hypotenuse})^2 = (\text{Side})^2 + (\text{Base})^2$

So, Reason (R) is true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

17. (a) **Assertion (A):** Let A be the position of observer eye and BC = h m be the height of the tower.



Let AB = 50 m be distance between observer's eye and foot of the tower.

In right-angled ΔABC,

$$\tan 30° = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{50}$$

$$\Rightarrow \quad h = \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{50}{3} \times \sqrt{3} \text{ m}$$

So, Assertion (A) is true.

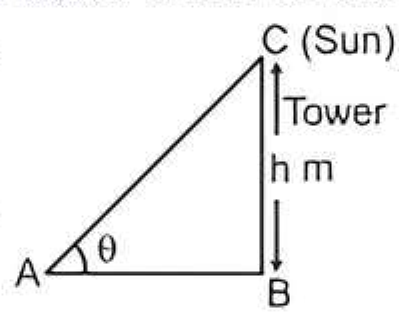
Reason (R): It is true to say that while solving the problem of angle of elevation/depression, triangle should be a right-angled triangle.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

18. line

19. depression

20. Let BC = h m be the height of the tower and C be the position of Sun. The length of the shadow will be $AB = \sqrt{3}h$ m.



Let the elevation of Sun from point A is ∠CAB = θ.

In right-angled ΔABC,

$$\tan\theta = \frac{BC}{AB} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

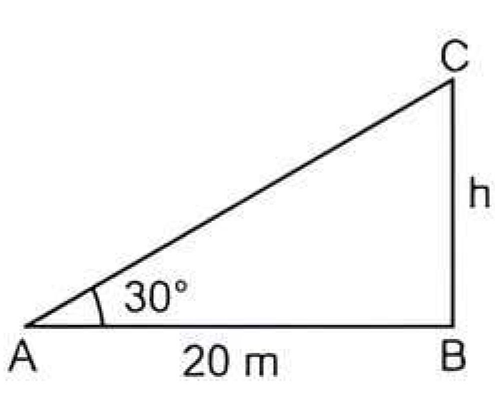
$$= \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$$

$$= \tan 30°$$

$$\Rightarrow \quad \theta = 30°$$

21. Let BC = h m be the height of the tower. Let A be the foot of the point such that AB = 20 m.

Also angle of elevation is ∠BAC = 30°.



In right-angled triangle,

$$\tan 30° = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{20}$$

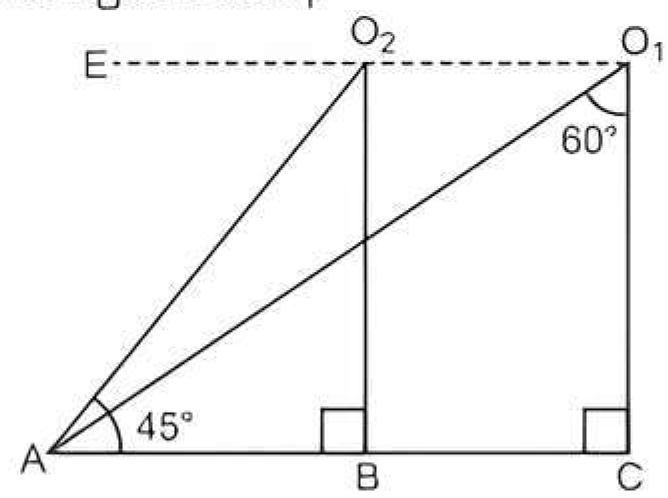
$$\Rightarrow \quad h = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{20\sqrt{3}}{3} \text{m}$$

$$= \frac{20}{3} \times 1.73 = \frac{34.6}{3} \text{m}$$

$$= 11.53 \text{ m}$$

Hence, height of the tower is 11.53 m.

22. In right-angled ΔACO_1,



$$\angle O_1AC + \angle ACO_1 + \angle CO_1A = 180°$$

(by angle sum property)

$$\Rightarrow \quad \angle O_1AC + 90° + 60° = 180°$$

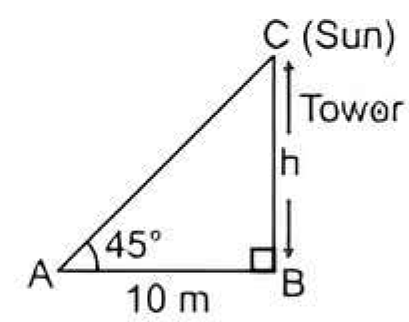
$$\Rightarrow \quad \angle O_1AC = 180° - 150° = 30°$$

Here, $\angle EO_1A = \angle O_1AC = 30°$ (alternate angles)

Also, $\angle EO_2A = \angle O_2AB = 45°$ (alternate angles)

Hence, angles of depressions from points O_1 and O_2 are respectively 30° and 45°.

23. Let C be the position of Sun and BC = h m be the height of the tower. Let AB = 10 m be the length of the Sun.



In right-angled ΔABC,

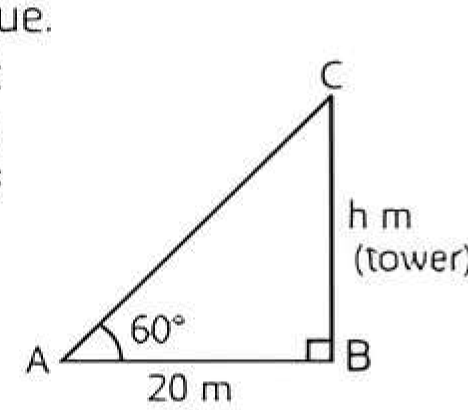
$$\tan 45° = \frac{BC}{AB} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad 1 = \frac{h}{10}$$

$$\Rightarrow \quad h = 10 \text{ m}$$

Hence, given statement is true.

24. Let BC = h metre be the height of the tower and AB = 20 m be the length of shadow of tower.



In right-angled ΔABC,

$$\tan 60° = \frac{BC}{AB}$$

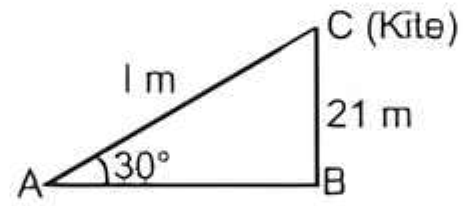
$$\Rightarrow \quad \sqrt{3} = \frac{h}{20}$$

$$\Rightarrow \quad h = 20\sqrt{3} \text{ m}$$

Hence, given statement is false.

25. Let C be the position of the kite and A be the position of the boy.

Let AC = l m be the length of the string and BC = 21m be the height of the kite. Then string of kite makes an angle $\angle CAB = 30^\circ$.



In right-angled ΔABC,

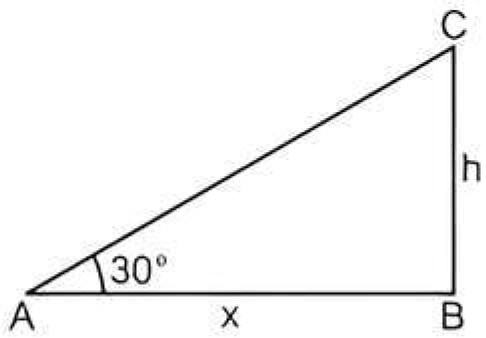
$$\sin 30^\circ = \frac{BC}{AC} = \frac{21}{l} \qquad \left(\because \sin\theta = \frac{P}{H}\right)$$

$$\Rightarrow \quad \frac{1}{2} = \frac{21}{l}$$

$$\Rightarrow \quad l = 21 \times 2 = 42 \text{ m}$$

Hence, given statement is false.

26. Let initially height of tower be h m and AB = x m.



In right angled ΔABC,

$$\tan 30^\circ = \frac{h}{x}$$

If the height of tower is doubled *i.e.*, BC = $2h$, then

$$\tan\theta = \frac{2h}{x}$$

Here we see that angle is not doubled when height is doubled.

Hence, given statement is false.

27. Let CD = h m be the height of the chimney and AB = 20 m be the distance covered from A to B.

Given, $\angle DAC = 30^\circ$

and $\angle DBC = 45^\circ$

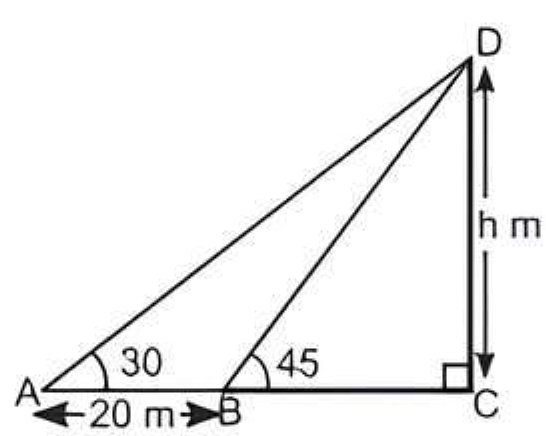
In right-angled ΔACD,

$$\tan 30^\circ = \frac{CD}{AC} = \frac{CD}{AB + BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{20 + BC} \qquad \ldots(1)$$

$$(\because AC = AB + BC = 20 + BC)$$

And right-angled ΔBCD,



$$\tan 45^\circ = \frac{CD}{BC}$$

$$\Rightarrow \quad 1 = \frac{h}{BC}$$

$$\Rightarrow \quad BC = h$$

From eq. (1), we get

$$\frac{1}{\sqrt{3}} = \frac{h}{20 + h}$$

$$\Rightarrow \quad 20 + h = \sqrt{3}h$$

$$\Rightarrow \quad h(\sqrt{3} - 1) = 20$$

$$\Rightarrow \quad h = \left(\frac{20}{\sqrt{3} - 1}\right) \text{ m.}$$

Hence, given statement is true.

Case Study Based Questions

Case Study 1

A group of students of class–X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name is Delhi Memorial, originally called All-India War Memorial, Monumental Sandstone Arch in New Delhi is dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 metres) in height.

Based on the above information, solve the following questions:

Q 1. What is the angle of elevation, if they are standing at a distance of $42\sqrt{3}$ m away from the monument?

a. 0° b. 30° c. 45° d. 60°

Q 2. They want to see the tower (monument) at an angle of 60°. So, they want to know the distance where they should stand and hence find the distance. [Use $\sqrt{3} = 1.732$]

a. 24.24 m b. 20.12 m c. 42 m d. 25.64 m

Q 3. If the altitude of the Sun is at 30°, then the height of the vertical tower that will cast a shadow of length 30 m is:

a. $10\sqrt{3}$ m b. $\frac{10}{\sqrt{3}}$ m

c. $\frac{20}{\sqrt{3}}$ m d. $20\sqrt{3}$ m

Q 4. The ratio of the length of a rod and its shadow is $24 : 8\sqrt{3}$. The angle of elevation of the Sun is:

a. 30° b. 60° c. 45° d. 90°

Q 5. The angle formed by the line of sight with the horizontal when the object viewed is above the horizontal level, is:

a. angle of elevation b. angle of depression

c. corresponding angle d. complete angle

Solutions

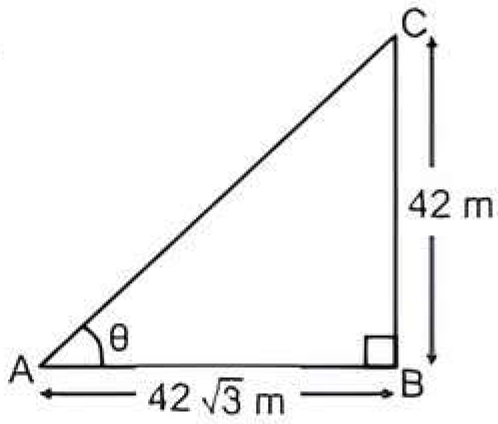
1. Let the angle of elevation be θ.

Given that,

Height of the monument (BC) = 42 m

and AB = $42\sqrt{3}$ m

Now, in right-angled ΔABC,



$$\tan\theta = \frac{BC}{AB} = \frac{42}{42\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} = \tan 30°$$

$$\therefore \quad \theta = 30°$$

So, option (b) is correct.

COMMON ERROR

Students take the value of $\frac{1}{\sqrt{3}} = \tan 60°$ in precocity.

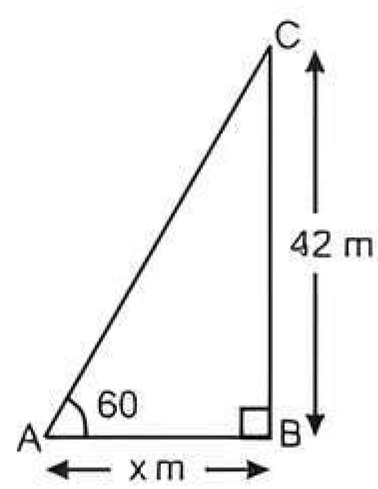
But it is wrong. The correct value of $\frac{1}{\sqrt{3}} = \tan 30°$.

2. Let the required distance be x m.
   Given, angle of elevation $(\theta) = 60°$
   and height of the monument $(BC) = 42$ m

TiP

Memorize the values of trigonometric angles properly and do practice more.

Now, in right-angled ΔABC,



$$\tan\theta = \frac{BC}{AB}$$

$$\Rightarrow \quad \tan 60° = \frac{42}{x}$$

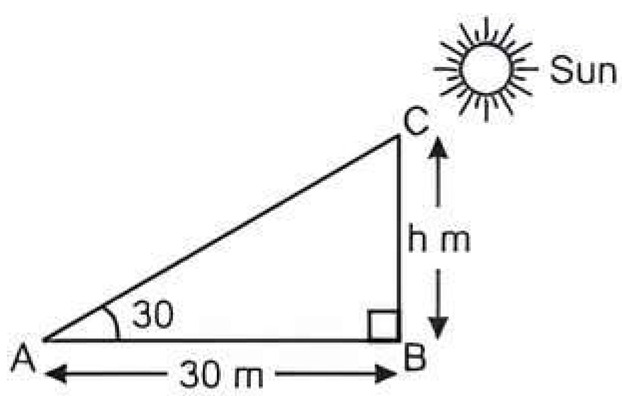
$$\Rightarrow \quad \sqrt{3} = \frac{42}{x}$$

$$\Rightarrow \quad x = \frac{42}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{42\sqrt{3}}{3} = 14\sqrt{3}$$

$$= 14 \times 1.732 = 24.24 \text{ m}$$

So, option (a) is correct.

3. Let the height of the vertical tower be h m.
   Given angle of elevation $(\theta) = 30°$ and length of the shadow $(AB) = 30$ m
   Now, In right-angled ΔABC,



$$\tan\theta = \frac{BC}{AB}$$

$$\Rightarrow \quad \tan 30° = \frac{h}{30}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{30}$$

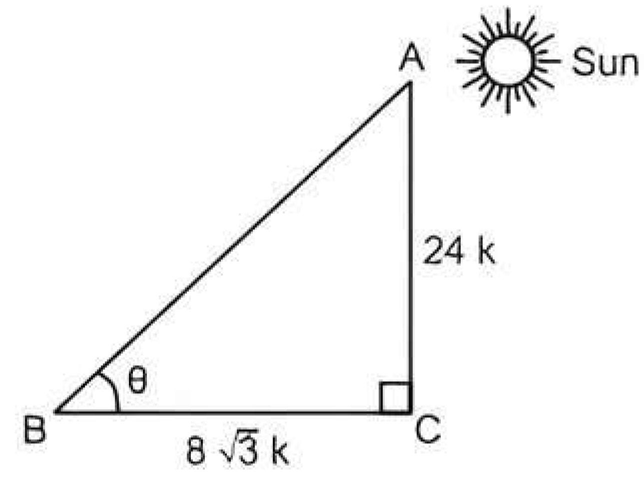
$$\Rightarrow \quad h = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3} \text{ m}$$

So, option (a) is correct.

COMMON ERROR

Some students confused the values of tan 30° and tan 60°. They take wrong value in haste.

4. Given, the ratio of the length of a rod and its shadow is $24 : 8\sqrt{3}$.
   Let $AC = 24k$
   and $BC = 8\sqrt{3}k$
   where k is a positive integer.
   Now, let angle of elevation of the Sun is θ.



In right-angled ΔACB:

$$\tan\theta = \frac{AC}{BC}$$

$$= \frac{24k}{8\sqrt{3}k} = \frac{3}{\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

$$\Rightarrow \quad \tan\theta = \tan 60°$$

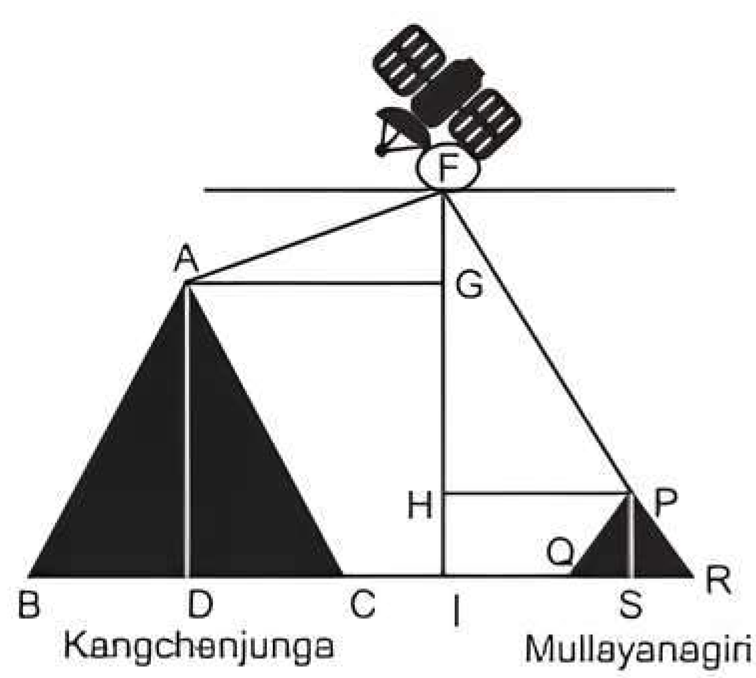
$$\therefore \quad \theta = 60°$$

So, option (b) is correct.

5. The angle formed by the line of sight with the horizontal, when the object viewed is above the horizontal level, is angle of elevation.
   So, option (a) is correct.

Case Study 2

A satellite flying at height h is watching the top of the two tallest mountains in Sikkim and Karnataka, which are in Kangchenjunga (height 8586 m) and Mullayanagiri (height 1930 m). The angles of depression from the satellite, to the top of Kangchenjunga and Mullayanagiri are 30° and 60° respectively. If the distance between middle of the bottom of both mountains is 2046 km, and the satellite is vertically above the mid-point of the distance between the two mountains.



Based on the above information, solve the following questions:

Q 1. The distance of the satellite from the top of Kangchenjunga is:

a. 1181.22 km
b. 577.52 km
c. 1937 km
d. 1025.36 km

Q 2. The distance of the satellite from the top of Mullayanagiri is:

a. 1139.4 km
b. 577.52 km
c. 2046 km
d. 1025.36 km

Q 3. The vertical distance of the satellite from the ground is:

a. 1139.4 km b. 599.2 km

c. 1937 km d. 1025.36 km

Q 4. What is the angle of elevation of the top of Kangchenjunga mountain, if a man is standing at a distance of 8586 m from Kangchenjunga?

a. 30° b. 45° c. 60° d. 0°

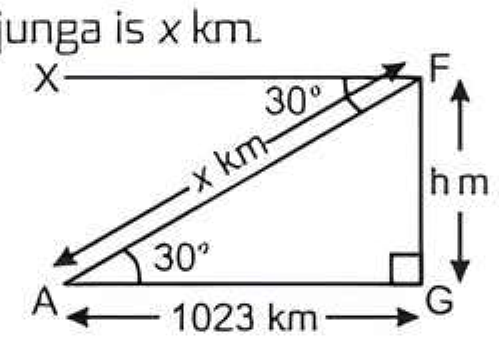
Q 5. If a stone very far away makes 45° to the top of Mullayanagiri mountain. So, find the distance of this stone from the mountain.

a. 1118.327 m b. 566.976 m

c. 1930 m d. 1025.36 m

Solutions

1. Let the distance of the satellite from the top of Kangchenjunga is x km.



Given that, height of the Kangchenjunga,

$$AD = 8586 \text{ m}$$

and the distance between middle of the bottom of both mountains (DS) = 2046 km.

But the satellite is vertically above the mid-point of the distance between the two mountains.

$$\therefore \quad DI = AG = \frac{DS}{2} = \frac{2046}{2} = 1023 \text{ km}$$

Now, in right-angled ΔAGF,

$$\cos\theta = \frac{AG}{AF} \quad \Rightarrow \quad \cos 30^\circ = \frac{1023}{x}$$

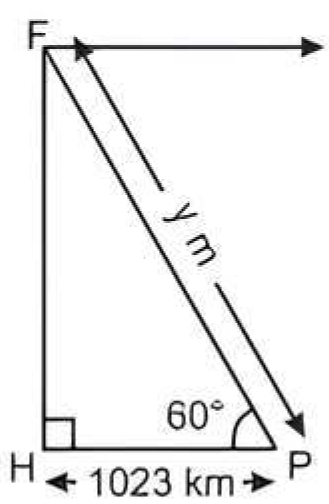
$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{1023}{x}$$

$$\Rightarrow \quad x = \frac{1023 \times 2}{\sqrt{3}} = \frac{2046 \times \sqrt{3}}{3} = 682 \times 1.732$$

$$\therefore \quad x = 1181.22 \text{ km}$$

So, option (a) is correct.

2. Let the distance of the satellite from the top of Mullayanagiri is y km.



Given that,

Height of the Mullayanagiri, PS = 1930 m and the distance between the middle of the bottom of both mountains (DS) = 2046 km.

But the satellite is vertically above the mid-point of the distance between the two mountains.

$$\therefore \quad SI = PH = \frac{DS}{2} = \frac{2046}{2} = 1023 \text{ km}$$

Now, in right-angled ΔPHF,

$$\cos\theta = \frac{PH}{PF}$$

$$\Rightarrow \quad \cos 60^\circ = \frac{1023}{y}$$

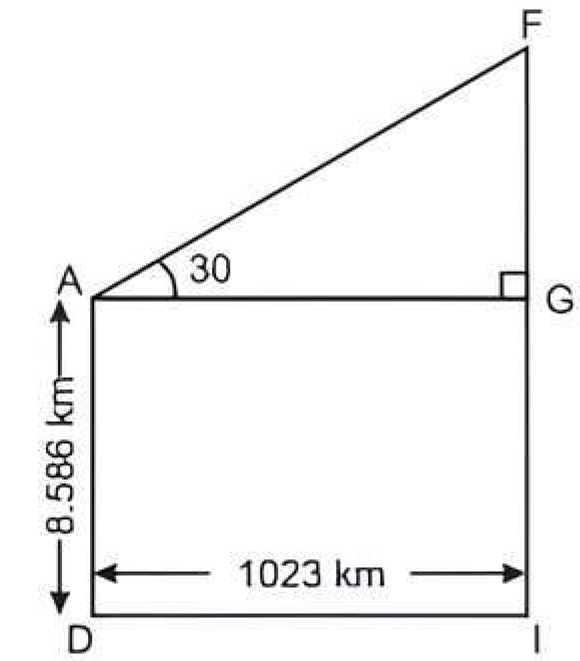
$$\Rightarrow \quad \frac{1}{2} = \frac{1023}{y}$$

$$\therefore \quad y = 2 \times 1023 = 2046 \text{ km}$$

So, option (c) is correct.

3. In right-angled ΔAGF,

$$\tan 30^\circ = \frac{FG}{AG}$$



$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{FG}{1023}$$

$$FG = \frac{1023}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{1023 \times 1.732}{3} = 590.6 \text{ km}$$

and $GI = AD = 8586$ m

$= 8.586$ km ($\because$ 1 km = 1000 m)

$\therefore$ The distance of the satellite from the ground

$$(FI) = FG + GI$$

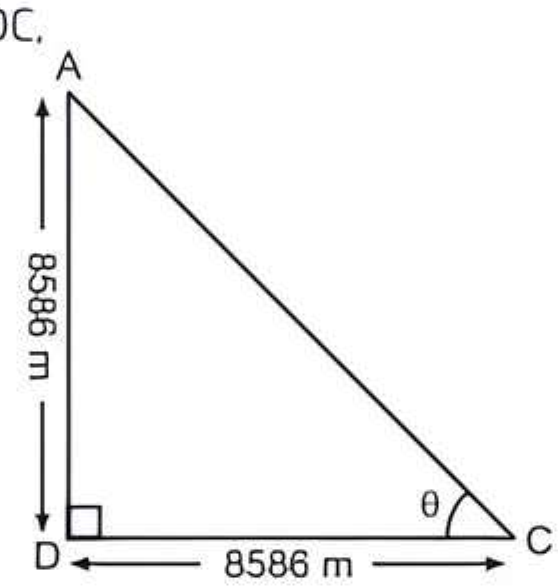
$$= 590.6 + 8.586 = 599.2 \text{ km}$$

So, option (b) is correct.

4. Let the angle of elevation be θ.

Given that, height of the Kangchenjunga (AD) = 8586 m and distance (CD) = 8586 m.

Now, in right-angled ΔADC,



$$\tan\theta = \frac{AD}{CD}$$

$$\Rightarrow \quad \tan\theta = \frac{8586}{8586} = 1$$

$$\therefore \quad \tan\theta = \tan 45^\circ$$

$$\Rightarrow \quad \theta = 45^\circ$$

So, option (b) is correct.

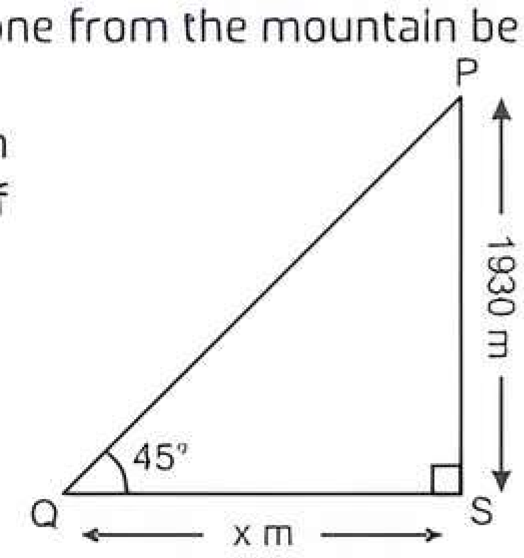
5. Let the distance of the stone from the mountain be x m.

Given, angle of elevation of a stone from the top of Mullayanagiri

$(\theta) = 45^\circ$

and height of the Mullayanagiri

(PS) = 1930 m



Now, in right-angled ΔQSP,

$$\tan 45^\circ = \frac{PS}{QS}$$

$$\Rightarrow \quad 1 = \frac{1930}{x}$$

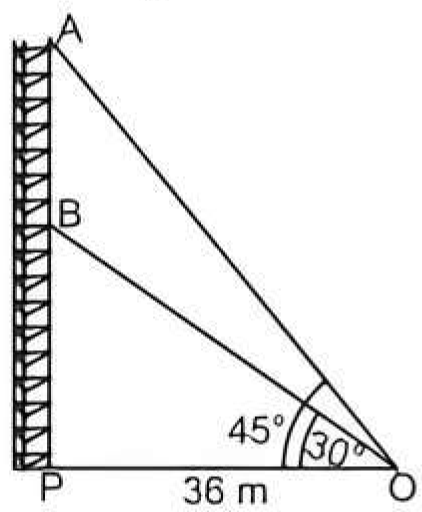
$$\therefore \quad x = 1930 \text{ m}$$

So, option (c) is correct.

Case Study 3

Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two Sections A and B. Tower is supported by wires from a point O.

Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of the Section B is 30° and the angle of elevation of the top of Section A is 45°.



Based on the above information, solve the following questions: [CBSE 2023]

Q 1. Find the length of the wire from the point O to the top of Section B.

Q 2. Find the distance AB.

Or

Find the area of ΔOPB.

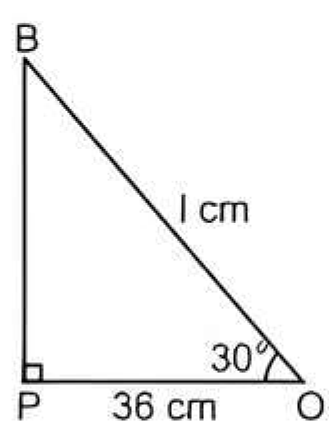
Q 3. Find the height of the Section A from the base of the tower.

Solutions

1. Let the length of the wire from the point O to the top of section B, *i.e.*, $OB = l$ m.

Given, $OP = 36$ cm and $\angle BOP = 30^\circ$

Now in right-angled ΔBPO,



$$\cos 30^\circ = \frac{OP}{OB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{36}{l}$$

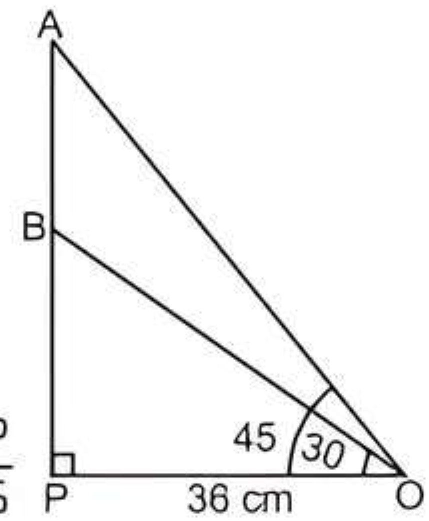
$$\Rightarrow l = \frac{72}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{72\sqrt{3}}{3} = 24\sqrt{3}$$

So, required length is $24\sqrt{3}$ cm.

2. Let $AB = x$ cm

Given, $\angle AOP = 45^\circ$ and $OP = 36$ cm

Now in right-angled ΔAPO,



$$\tan 45^\circ = \frac{AP}{OP}$$

$$\Rightarrow \quad 1 = \frac{AP}{36}$$

$$\Rightarrow \quad AP = 36 \text{ cm}$$

Again in right-angled ΔBPO,

$$\tan 30^\circ = \frac{BP}{OP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BP}{36}$$

$$\Rightarrow \quad BP = \frac{36}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{36\sqrt{3}}{3} = 12\sqrt{3} \text{ cm}$$

$$\therefore \quad AB = AP - BP$$

$$= 36 - 12\sqrt{3} = 12(3-\sqrt{3}) \text{ cm}$$

So, required distance AB is $12(3-\sqrt{3})$ cm.

Or

Since, ΔBPO is a right-angled triangle.

$$\therefore \text{ Area of } \Delta OPB = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times OP \times BP = \frac{1}{2} \times 36 \times 12\sqrt{3}$$

$$= 216\sqrt{3} \text{ cm}^2.$$

3. From part (2),

The height of the Section A from the base of the tower $= AP$

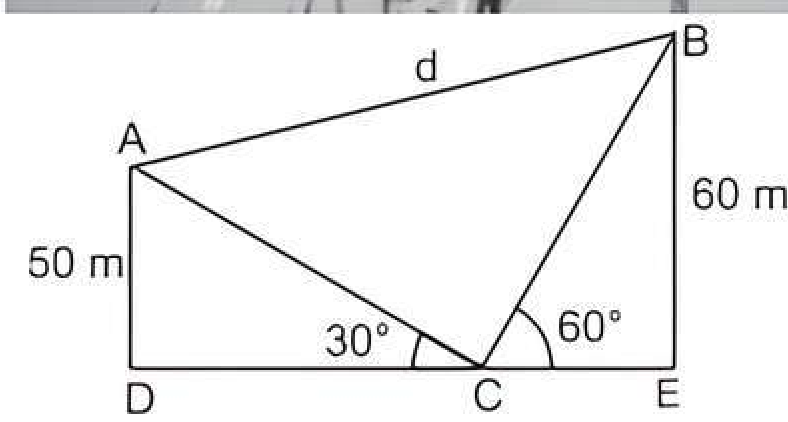
$= 36$ cm.

Case Study 4

Kite Festival

Kite festival is celebrated in many countries at different times of the year. In India, every year 14th January is celebrated as International Kite Day. On this day many people visit India and participate in the festival by flying various kinds of kites.

The picture given below, three kites flying together.



In figure, the angles of elevation of two kites (Points A and B) from the hands of a man (Point C) are found to be 30° and 60° respectively. Taking $AD = 50$ m and $BE = 60$ m.

Based on the above information, solve the following questions: [CBSE 2022 Term-II]

Q 1. Find the lengths of strings used (take them straight) for kites A and B as shown in the figure.

Q 2. Find the distance 'd' between these two kites.

Solutions

1. In right-angled ΔADC,

$$\sin 30^\circ = \frac{AD}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{50}{AC}$$

$$\Rightarrow \quad AC = 100\,m$$

and in right-angled ΔCEB,

$$\sin 60^\circ = \frac{BE}{BC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{60}{BC}$$

$$\Rightarrow \quad BC = \frac{60\times 2}{\sqrt{3}} = \frac{120}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{120}{3}\times\sqrt{3} = 40\sqrt{3}\,m$$

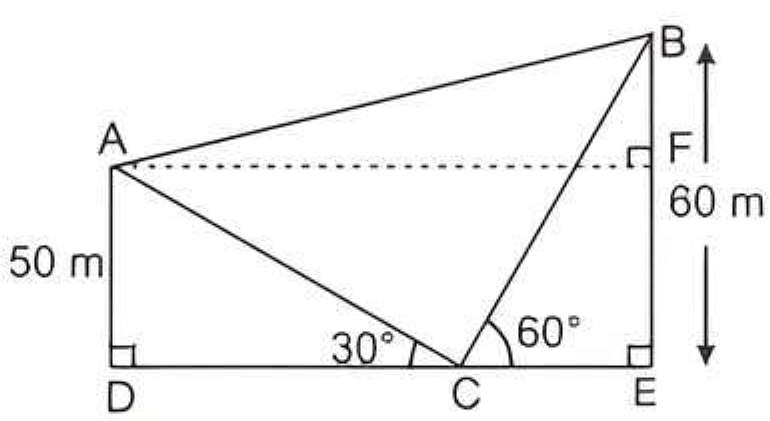
Hence, length of the strings from kites A and B are 100 m and $40\sqrt{3}$ m respectively.

2. In right-angled ΔADC,

$$\tan 30^\circ = \frac{AD}{DC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{50}{DC}$$

$$\Rightarrow \quad DC = 50\sqrt{3}\,m$$



and in right-angled ΔCEB,

$$\tan 60^\circ = \frac{BE}{CE}$$

$$\Rightarrow \quad \sqrt{3} = \frac{60}{CE}$$

$$\Rightarrow \quad CE = \frac{60}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{60\sqrt{3}}{3} = 20\sqrt{3}\,m.$$

From figure,

$$BF = BE - EF = BE - AD$$
$$= 60 - 50 = 10\,m$$

and

$$AF = DE = DC + CE$$
$$= 50\sqrt{3} + 20\sqrt{3}$$
$$= 70\sqrt{3}\,m$$

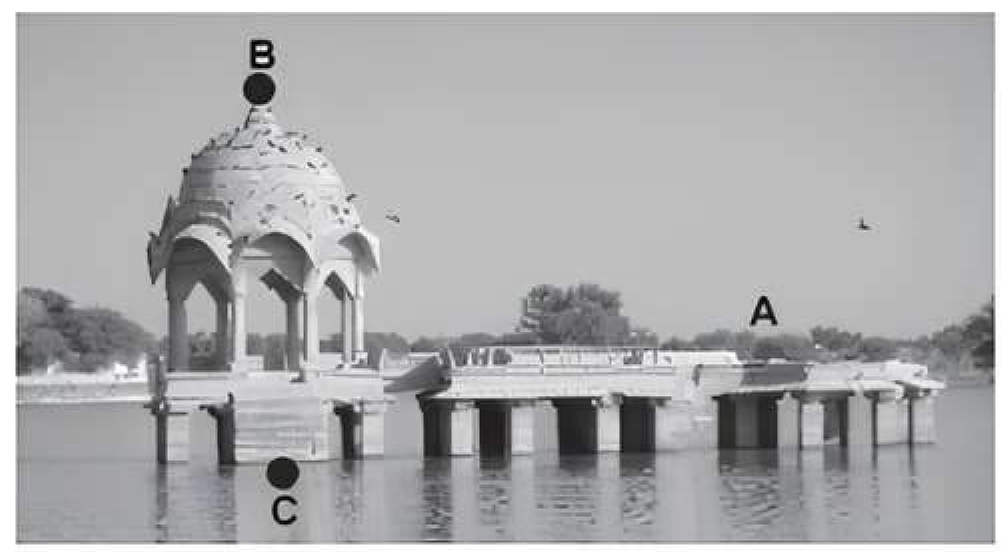
In right-angled ΔABF,

$$AB = \sqrt{(AF)^2 + (BF)^2}$$
$$= \sqrt{(70\sqrt{3})^2 + (10)^2}$$
$$= \sqrt{14700 + 100} = \sqrt{14800}$$
$$= 121.66\,m.$$

Hence, distance between two kites is 121.66 m.

Case Study 5

Gadisar Lake is located in the Jaisalmer district of Rajasthan. It was built by the King of Jaisalmer and rebuilt by Gadsi Singh in 14th century. The lake has many Chhatris. One of them is shown below:



Observe the picture. From a point A, h m above from water level, the angle of elevation of top of Chhatri (point B) is 45° and angle of depression of its reflection in water (point C) is 60°. If the height of Chhatri above water level is (approximately) 10 m, then.

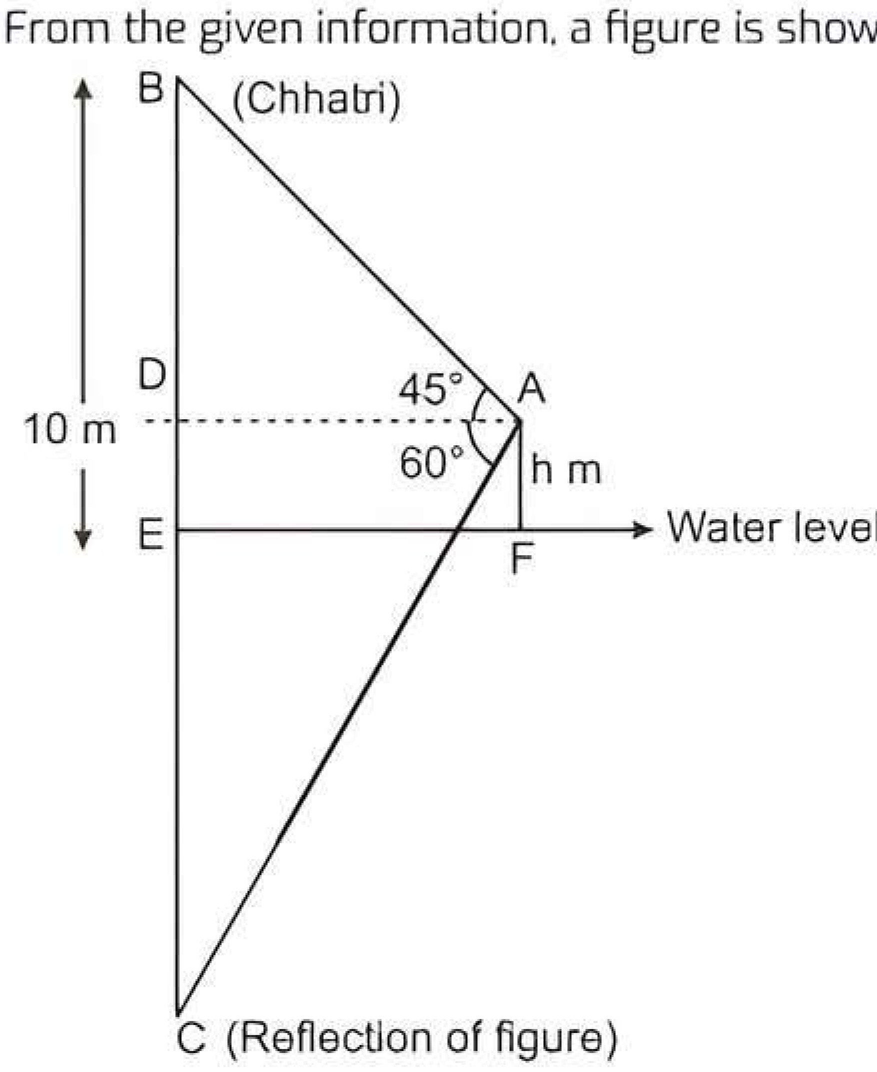
Based on the above information, solve the following questions: **[CBSE 2022 Term-II]**

Q 1. Draw a well-labelled figure based on the above information.

Q 2. Find the height (h) of the point A above water level. [Use $\sqrt{3} = 1.73$]

Solutions

1. From the given information, a figure is shown below.



2.

TiP

The image of point B with respect to water is of equal distance at point C. I.e., BE = EC

Given, $BE = EC = 10$ m

$\therefore$ $BD = BE - ED = 10 - h$

and $DC = DE + EC = h + 10$

In right-angled ΔADB,

$$\tan 45^\circ = \frac{BD}{AD}$$

$$\Rightarrow \quad 1 = \frac{10-h}{AD} \Rightarrow AD = 10 - h \quad ...(1)$$

In right-angled ΔADC,

$$\tan 60^\circ = \frac{CD}{AD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{10+h}{AD}$$

$$\Rightarrow \quad AD = \frac{10+h}{\sqrt{3}} \quad ...(2)$$

From eqs. (1) and (2), we get

$$10 - h = \frac{10+h}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}(10-h) = 10 + h$$

$$\Rightarrow \quad 10\sqrt{3} - \sqrt{3}h = 10 + h$$

$$\Rightarrow \quad 10(\sqrt{3}-1) = h(1+\sqrt{3})$$

$$\Rightarrow \quad h = \frac{10(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$$

$$= \frac{10(\sqrt{3}-1)^2}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{10(3+1-2\sqrt{3})}{3-1}$$

$$= \frac{10(4-2\sqrt{3})}{2}$$

$$= \frac{10 \times 2(2-\sqrt{3})}{2}$$

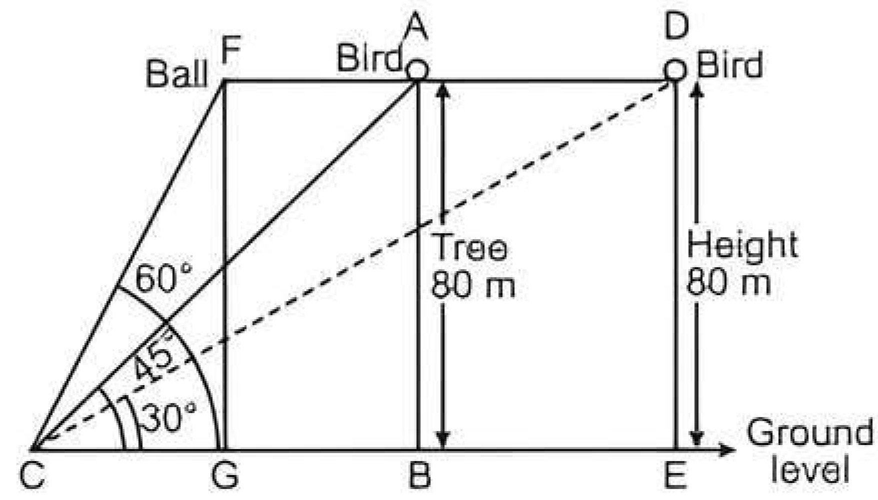
$$= 10\,(2 - 1.73)$$

$$= 10 \times 0.27 = 2.7 \text{ m}$$

Case Study 6

One evening, Kaushik was in a park. Children were playing cricket. Birds were singing on a nearby tree of height 80 m. He observed a bird on the tree at an angle of elevation of 45°.

When a sixer was hit, a ball flew through the tree frightening the bird to fly away. In 2 seconds, he observed the bird flying at the same height at an angle of elevation of 30° and the ball flying towards him at the same height at an angle of elevation of 60°.



Based on the above information, solve the following questions: **[CBSE SQP 2023-24]**

Q 1. At what distance from the foot of the tree was he observing the bird sitting on the tree?

Q 2. How far did the bird fly in the mentioned time?

Or

After hitting the tree, how far did the ball travel in the sky when Kaushik saw the ball?

Q 3. What is the speed of the bird (in m/min) if it had flown $20(\sqrt{3}+1)$ m?

Solutions

1. Given, height of tree $(AB) = 80$ m

and $\angle ACB = 45^\circ$

Now in right-angled ΔABC,

$$\tan 45^\circ = \frac{AB}{AC} \Rightarrow 1 = \frac{80}{BC}$$

$$\Rightarrow \quad BC = 80 \text{ m}$$

So, required distance is 80 m.

2. Given in 2 sec, the bird flying at the same height at an angle of elevation of 30°.

$\therefore \angle DCE = 30^\circ$ and $DE = 80$ m.

Now in right-angled ΔDEC,

$$\tan 30^\circ = \frac{DE}{CE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{80}{CE}$$

$$\Rightarrow \quad CE = 80\sqrt{3} \text{ m}$$

$$BE = CE - BC$$

$$= 80\sqrt{3} - 80 = 80(\sqrt{3}-1) \text{ m}$$

So, required distance the bird flew is $80(\sqrt{3}-1)$ m.

Or

After hitting the tree, the ball travel from A to F. Then angle of elevation of the ball from C is 60°.

i.e., $\angle FCG = 60^\circ$ and $FG = AB = 80$ m.

Now in right-angled ΔFGC.

$$\tan 60^\circ = \frac{FG}{CG} \Rightarrow \sqrt{3} = \frac{80}{CG}$$

$$\Rightarrow \quad CG = \frac{80}{\sqrt{3}} \text{ m}$$

$\therefore\ FA = BG = BC - CG = 80 - \frac{80}{\sqrt{3}} = 80\left(1 - \frac{1}{\sqrt{3}}\right) m$

So, required distance the ball travelled after hitting the tree is $80\left(1 - \frac{1}{\sqrt{3}}\right) m$.

3. Given, in 2 sec, the bird had flown $20(\sqrt{3}+1)$ m.

$\therefore$ Speed of the bird $= \frac{\text{Distance}}{\text{Time taken}}$

$$= \frac{20(\sqrt{3}+1)}{2} \text{ m/sec}$$

$$= \frac{20(\sqrt{3}+1)}{2} \times 60 \text{ m/min}$$

$$= 600(\sqrt{3}+1) \text{ m/min}$$

So, the required speed of the bird is

$600(\sqrt{3}+1)$ m/min.

Very Short Answer Type Questions

Q 1. The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. Find the angle of elevation of the Sun. *[CBSE 2023]*

Q 2. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3} : 1$. What is the angle of elevation of the Sun? *[CBSE 2017]*

Q 3. What is the angle of depression of the object at E from the observation point A, if AD = ED? *[CBSE 2017]*

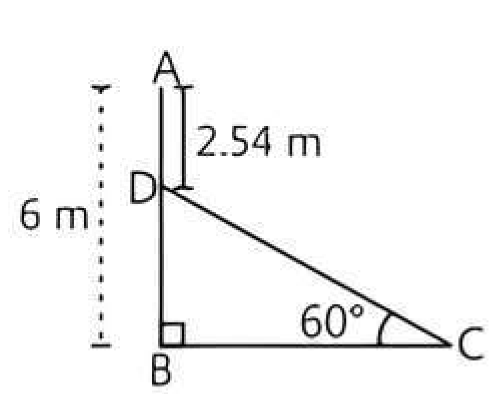
Q 4. The angle of elevation of the top of a tower from a point on the ground which is 30 m away from the foot of the tower, is 30°. Find the height of the tower. *[CBSE 2023]*

Q 5. A building casts a shadow of length $5\sqrt{3}$ m on the ground, when the Sun's elevation is 60°. Find the height of the building.

Q 6. A kite is flying, attached to a thread which is 140 m long. The thread makes an angle of 30° with the ground. Find the height of the kite from the ground, assuming that there is no slack in the thread. *[V. Imp.]*

Q 7. Find the length of the shadow on the ground of a pole of height 18 m when angle of elevation θ of the Sun is such that $\tan\theta = \frac{6}{7}$. *[CBSE 2023]*

Q 8. In figure, AB is a 6 m high pole and CD is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point D of pole. If AD = 2.54 m, find the length of the ladder. [Use $\sqrt{3} = 1.73$]. *[CBSE 2016]*



Short Answer Type-I Questions

Q 1. The length of a string between a kite and a point on the ground is 70 m. If the string makes an angle θ with the ground level such that $\tan\theta = \frac{4}{3}$, then the kite is at what height from the ground?

Q 2. The angle of depression of car parked on the road from the top of a 150 m high tower is 30°. Find the distance of the car from the tower.

Q 3. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree. *[NCERT EXERCISE; Imp.]*

Q 4. The top of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$. *[CBSE 2015]*

Short Answer Type-II Questions

Q 1. A boy 1.7 m tall is standing on a horizontal ground, 50 m away from a building. The angle of elevation of the top of the building from his eye is 60°. Calculate the height of the building.

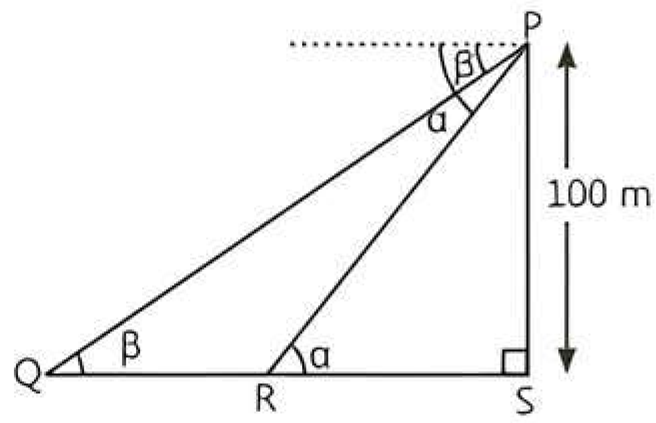
[Take $\sqrt{3} = 1.73$] *[CBSE SQP 2022 Term -II]*

Q 2. The shadow of a tower at a time is three times as long as its shadow when the angle of elevation of the Sun is 60°. Find the angle of elevation of the Sun at the time of the longer shadow. *[CBSE 2017]*

Q 3. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. Find the distance between the two towers and also the height of the other tower. *[CBSE 2023]*

Q 4. A person walking 45 m towards a tower in a horizontal line through its base observes that angle of elevation of the top of the tower changes from 45° to 60°. Find the height of the tower. [Use $\sqrt{3} = 1.732$] *[CBSE 2017]*

Q 5. As observed from the top of a 100 m high light-house from the sea level, the angles of depression of two ships are α and β. It is given that one ship is exactly behind the other on the same side of the light house. Based on the following figure, answer the following questions: *[Imp.]*



(i) In the given figure, if $\sin(3\beta - \alpha) = \frac{1}{\sqrt{2}}$ and $\cos(2\alpha - 3\beta) = 1$, $\alpha > \beta$, then find the values of α and β.

(ii) Find the distance between the two ships.

Q 6. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 3 m from the banks, then find the width of the river. *[CBSE 2022 Term-II]*

Q 7. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower and distance of the tower from the building. *[CBSE 2023]*

Q 8. Two vertical poles of different heights are standing 20m away from each other on the level ground. The angle of elevation of the top of the first pole from the foot of the second pole is 60° and angle of elevation of the top of the second pole from the foot of the first pole is 30°. Find the difference between the heights of two poles. [Take $\sqrt{3} = 1.73$] *[CBSE SQP 2022 Term-II]*

Q 9. A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 min. Find the speed of the boat (in m/min). *[CBSE 2019, 17]*

Q 10. The angle of elevation of a cloud from a point h m above a lake is α and the angle of depression of its reflection in the lake is β. Prove that the height of the cloud is $\frac{h(\tan\beta + \tan\alpha)}{(\tan\beta - \tan\alpha)}$ m. *[NCERT EXEMPLAR; CBSE 2017]*

Long Answer Type Questions

Q 1. From the top of a tower 100 m high, a man observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the two cars. [Use $\sqrt{3} = 1.73$] *[CBSE 2023]*

Q 2. From a point on the ground, the angles of elevation of the bottom and the top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower. *[NCERT EXERCISE; CBSE 2020]*

Q 3. Two ships are approaching a light house from opposite directions. The angles of depression of the two ships from the top of a lighthouse are 30° and 45°. If the distance between the two ships is 100 m, find the height of the lighthouse. [Use $\sqrt{3} = 1.732$]

Q 4. A straight highway leads to the foot of a tower. A man standing on the top of the 75 m high tower observes two cars at angles of depression of 30° and 60°, which are approaching the foot of the tower. If one car is exactly behind the other on the same side of the tower, find the distance between the two cars. [Use $\sqrt{3} = 1.73$] *[CBSE 2023]*

Q 5. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$] *[NCERT EXERCISE; CBSE 2018, 17]*

Q 6. There are two poles, one each on either bank of a river just opposite to each other. One pole is 60 m high. From the top of this pole, the angle of depression of the top and foot of the other pole are 30° and 60° respectively. Find the width of the river and height of the other pole. *[CBSE 2019]*

Q 7. A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of a 20 m high building, finds the elevation of the same bird to be 45°. The boy and the girl are on the opposite sides of the bird. Find the distance of the bird from the girl. (Given $\sqrt{2} = 1.414$) *[CBSE 2019]*

Q 8. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 sec, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of $3600\sqrt{3}$ m, find the speed of the plane (in km/h). *[CBSE 2019]*

Q 9. The angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water of lake is 60°. Find the height of the cloud from the surface of water. *[CBSE 2017]*

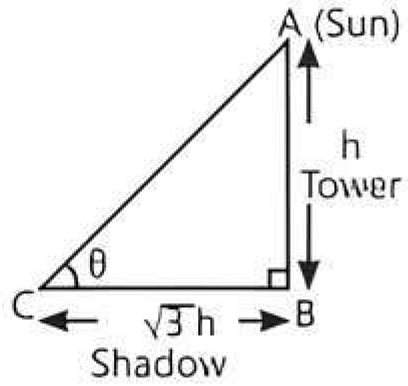
Solutions

Very Short Answer Type Questions

1. Let AB be the tower and BC be its shadow.

Let angle of elevation of the Sun be θ.

In right-angled Δ ABC,



$$\tan\theta = \frac{AB}{BC} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad \tan\theta = \frac{h}{h\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\Rightarrow \quad \theta = 30^\circ$$

Hence, the elevation of the Sun is 30°.

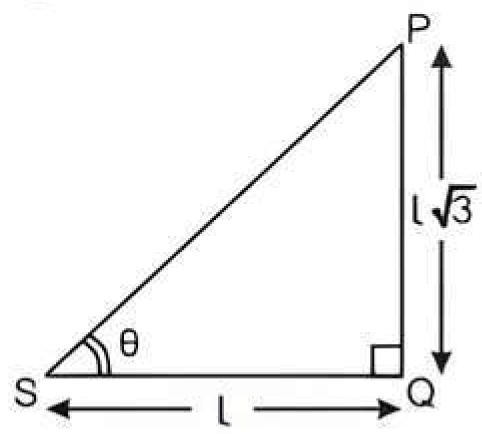
COMMON ERR(!)R

Students take the value of $\frac{1}{\sqrt{3}} = \tan 60^\circ$ in haste. But it is wrong. So, the correct value of $\frac{1}{\sqrt{3}} = \tan 30^\circ$.

2. Given that, the ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}:1$.

Let, tower height $(PQ) = l\sqrt{3}$

and its shadow length $(SQ) = l$



Let $\angle PSQ = \theta$ (angle of elevation of the Sun)

Now, in right-angled ΔPQS,

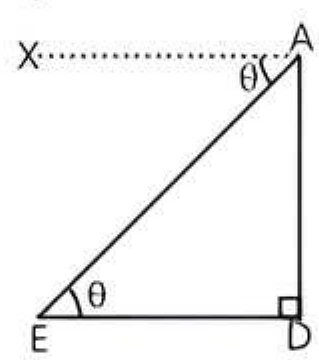
$$\tan\theta = \frac{PQ}{SQ} = \frac{l\sqrt{3}}{l} = \sqrt{3}$$

$$\Rightarrow \quad \tan\theta = \tan 60^\circ \quad \Rightarrow \quad \theta = 60^\circ$$

So, the angle of elevation of the Sun is 60°.

3. Let the angle of depression of the object at E from the observation point A is θ.

In right-angled ΔADE,



$$\tan\theta = \frac{AD}{ED} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$= \frac{AD}{AD} = 1 \qquad (\because AD = ED)$$

$$= \tan 45^\circ$$

$$\Rightarrow \quad \theta = 45^\circ$$

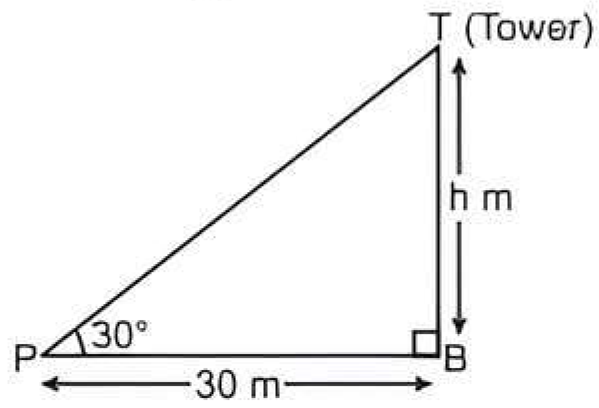
Hence, the angle of depression is 45°.

4. Given, the angle of elevation of the top (T) of a tower TB from a point (P) on the ground which is 30 m away from the foot (B) of the tower, is $\angle BPT = 30^\circ$.

i.e., BP = 30 m and let BT = h m

Now, in right-angled ΔTBP,

$$\tan 30^\circ = \frac{BT}{BP} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$



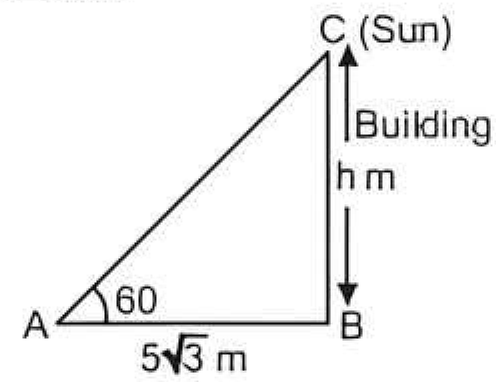
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{30}$$

$$\Rightarrow \quad h = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3}$$

$$\Rightarrow \quad h = 10\sqrt{3} \text{ m}$$

So, height of the tower is $10\sqrt{3}$ m.

5. Let BC = h m be the height of the building, AB = $5\sqrt{3}$ m and $\angle CAB = 60^\circ$.



In right-angled ΔABC,

$$\tan 60^\circ = \frac{BC}{AB} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

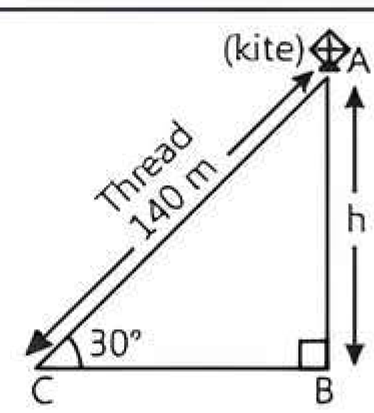
$$\Rightarrow \quad \sqrt{3} = \frac{h}{5\sqrt{3}} \quad \Rightarrow \quad h = 15 \text{ m}$$

Hence, height of the building is 15 m.

6. Let AC be the length of thread and AB be the vertical height of the kite.

TiP

Students should do practice, use of trigonometrical ratios in the triangle.



Let AB = h m, AC = 140 m and $\angle ACB = 30^\circ$.

In right-angled Δ ABC,

$$\sin 30^\circ = \frac{AB}{AC} \qquad \left(\because \sin\theta = \frac{P}{H}\right)$$

$$\Rightarrow \quad \frac{1}{2} = \frac{h}{140}$$

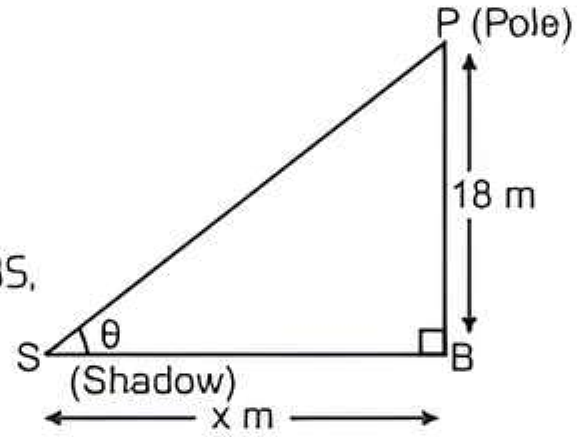
$$\Rightarrow \quad h = \frac{140}{2} = 70 \text{ m}$$

Hence, the height of the kite is 70 m.

7. Let the length of the shadow BS = x m on the ground of a pole BP of height 18 m.

Given, BP = 18 m

and $\tan\theta = \frac{6}{7}$



Now in right-angled ΔPBS,

$$\tan\theta = \frac{BP}{BS}$$

$$\Rightarrow \quad \frac{6}{7} = \frac{18}{x}$$

$$\Rightarrow \quad x = \frac{18 \times 7}{6} = 3 \times 7 = 21\,\text{m}$$

So, the required length of shadow is 21 m.

8. From figure, BD = AB – AD

$$= 6 - 2.54 = 3.46\text{ m}$$

In right-angled ΔDBC,

$$\sin 60° = \frac{BD}{CD} = \frac{3.46}{CD}$$

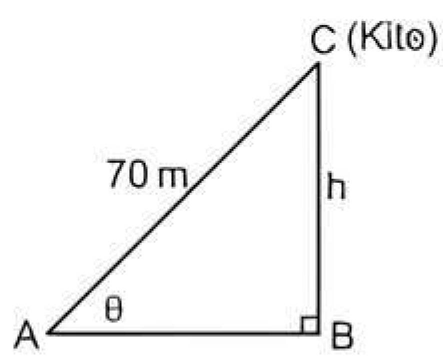
$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{3.46}{CD}$$

$$\Rightarrow \quad CD = \frac{3.46 \times 2}{\sqrt{3}} = \frac{3.46 \times 2}{1.73} = 4\,\text{m}$$

Hence, length of the ladder is 4 m.

Short Answer Type-I Questions

1. Let C be the position of kite and AC = 70 m be the length of the string. Let string makes an angle of θ from the ground. Let height of the kite from the ground be BC = h m.



Given $\tan\theta = \frac{4}{3}$

TR!CK

Use identities: $\text{cosec}\,\theta = \sqrt{1+\cot^2\theta}$ *and* $\tan\theta \cdot \cot\theta = 1$

$$\Rightarrow \quad \cot\theta = \frac{3}{4}$$

$$\because \quad \text{cosec}\,\theta = \sqrt{1+\cot^2\theta}$$

$$= \sqrt{1+\left(\frac{3}{4}\right)^2} = \sqrt{\frac{16+9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

Now in right-angled ΔABC,

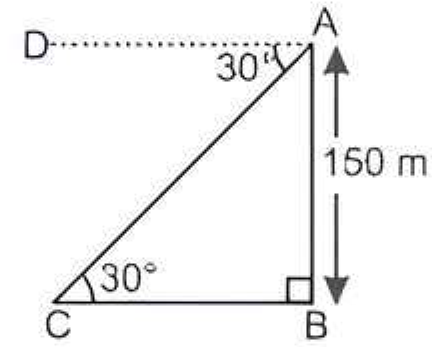
$$\text{cosec}\,\theta = \frac{AC}{BC}$$

$$\Rightarrow \quad \frac{5}{4} = \frac{70}{h}$$

$$\Rightarrow \quad h = 56\text{ m}$$

Hence, height of the kite is 56 m.

2. Let AB = 150 m be the height of the tower and angle of depression is ∠DAC = 30°.



Then, ∠ACB = ∠DAC = 30° (alternate angles)

In right-angled Δ ABC,

$$\tan 30° = \frac{AB}{BC} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{150}{BC}$$

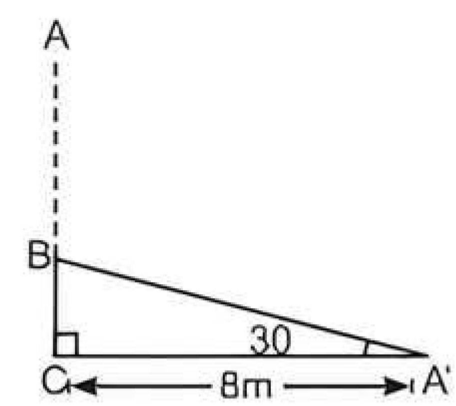
$$\Rightarrow \quad BC = 150\sqrt{3}\text{ m}$$

Hence, the distance of car from the tower is $150\sqrt{3}$ m.

COMMON ERR(!)R

Some candidates are unable to draw the diagram as per the given data and lose their marks.

3. Let AC was the original tree. Due to storm, it was broken into two parts. The broken part A'B is making an angle of 30° with the ground.



In right-angled Δ A′CB,

$$\tan 30° = \frac{BC}{A'C}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{BC}{8}$$

$$\Rightarrow \quad BC = \frac{8}{\sqrt{3}}\text{m}$$

and $\cos 30° = \frac{A'C}{A'B}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{8}{A'B}$$

$$\Rightarrow \quad A'B = \frac{16}{\sqrt{3}}\text{m}$$

∴ Height of the tree = AB + BC = A′B + BC

(∵ A′B = AB)

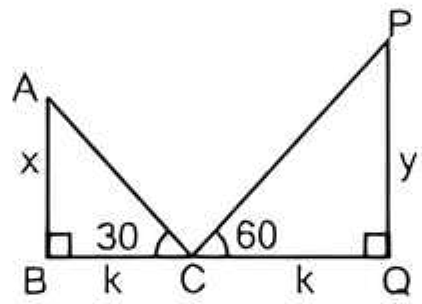
$$= \left[\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right]\text{m}$$

$$= \frac{24}{\sqrt{3}}\text{m} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\text{m}$$

$$= \frac{24\sqrt{3}}{3}\text{ m} = 8\sqrt{3}\text{ m}$$

Hence, the height of the tree is $8\sqrt{3}$m.

4. The base is same for both towers and their heights are given, *i.e.*, x and y respectively.



Let the base of towers be $BC = CQ = k$.

In right-angled ΔABC,

$$\tan 30^\circ = \frac{AB}{BC} = \frac{x}{k}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{k}$$

$$\Rightarrow \quad x = \frac{k}{\sqrt{3}} \quad ...(1)$$

In right-angled ΔPQC,

$$\tan 60^\circ = \frac{PQ}{CQ} = \frac{y}{k} \quad ...(2)$$

$$\Rightarrow \quad \sqrt{3} = \frac{y}{k}$$

$$\Rightarrow \quad y = k\sqrt{3}$$

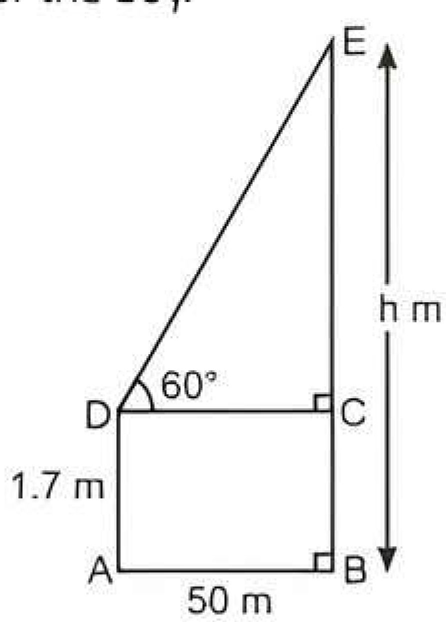
From eqs. (1) and (2), we get

$$\frac{x}{y} = \frac{k}{\sqrt{3}} \times \frac{1}{k\sqrt{3}} = \frac{1}{3}$$

$$\therefore \quad x : y = 1 : 3$$

Short Answer Type-II Questions

1. Let height of the tower be $BE = h$ m and $AD = 1.7$ m be the height of the boy.



Also, given the angle of elevation of the boy to the top of the tower is $\angle CDE = 60^\circ$.

Here, $\quad EC = EB - BC$

$\quad = h - 1.7 \quad (\because BC = AD = 1.7\text{ m})$

Also, $\quad DC = AB = 50$ m.

In right-angled triangle DCE,

$$\tan 60^\circ = \frac{CE}{DC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h-1.7}{50}$$

$$\Rightarrow \quad 50\sqrt{3} = h - 1.7$$

$$\Rightarrow \quad h = 50 \times 1.73 + 1.7$$

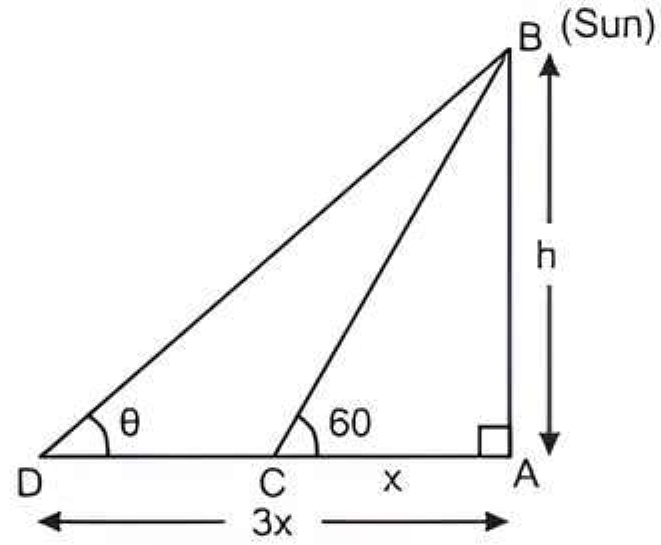
$$= 86.50 + 1.7$$

$$\Rightarrow \quad h = 88.20 \text{ m}$$

Hence, height of the building is 88.20 m.

2. Suppose B be the position of the Sun. Let the height of the tower be h m and the angle between the Sun and the ground at the time of longer shadow be θ.

AC and AD are the lengths of the shadow of the tower when the angle between the Sun and the ground are 60° and θ, respectively.



Let $AC = x$ unit, then

Given, $AD = 3\,AC \Rightarrow AD = 3x$

In right-angled ΔBAC,

$$\tan 60^\circ = \frac{AB}{AC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = x\sqrt{3} \quad ...(1)$$

In right-angled ΔBAD,

$$\tan\theta = \frac{AB}{AD} = \frac{h}{3x}$$

$$\Rightarrow \quad h = 3x \times \tan\theta \quad ...(2)$$

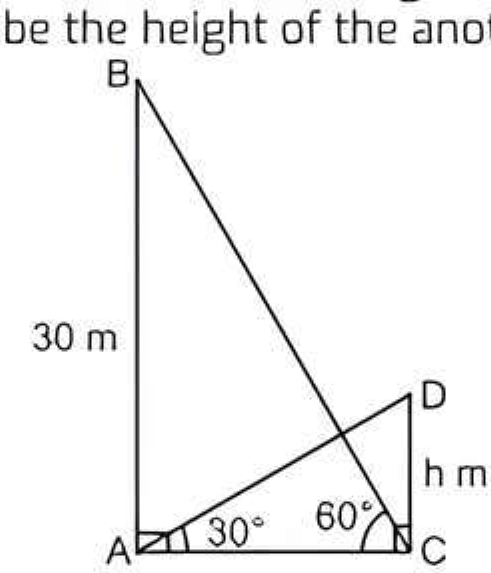
From eqs. (1) and (2), we get

$$x\sqrt{3} = 3x \cdot \tan\theta$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\Rightarrow \quad \theta = 30^\circ$$

3. Let $AB = 30$ m be the height of the tower and $CD = h$ m be the height of the another tower. Then



$\angle CAD = 30^\circ$ and $\angle ACB = 60^\circ$.

In right-angled ΔACB,

$$\tan 60^\circ = \frac{AB}{AC} \quad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad \sqrt{3} = \frac{30}{AC} \Rightarrow AC = \frac{30}{\sqrt{3}} \quad ...(1)$$

$$\Rightarrow \quad AC = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{3} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}\text{ m}$$

In right-angled ΔCAD,

$$\tan 30^\circ = \frac{CD}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{10\sqrt{3}} \quad [\because \text{from eq. (1)}]$$

$$\Rightarrow \quad h = \frac{10\sqrt{3}}{\sqrt{3}} = 10 \text{ m}$$

Hence, the distance between the two towers and height of the other tower are $10\sqrt{3}$ m and 10 m respectively.

4. Let AB = h m be the height of the tower.

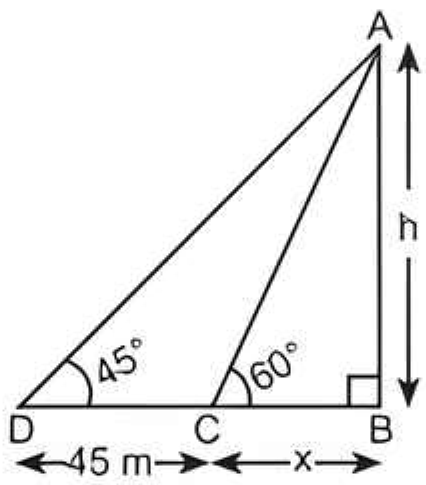
Let BC = x m and DC = 45 m

In right-angled ΔABC,

$$\tan 60^\circ = \frac{AB}{BC} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = \sqrt{3}\,x \qquad ...(1)$$



In right-angled ΔABD,

$$\tan 45^\circ = \frac{AB}{BD} = \frac{AB}{BC + CD}$$

$$\Rightarrow \quad 1 = \frac{h}{x + 45}$$

$$\Rightarrow \quad h = x + 45$$

$$\Rightarrow \quad h = \frac{h}{\sqrt{3}} + 45 \qquad \text{(using eq. (1))}$$

$$\Rightarrow \quad h - \frac{h}{\sqrt{3}} = 45$$

$$\Rightarrow \quad \frac{(\sqrt{3}-1)h}{\sqrt{3}} = 45$$

$$\Rightarrow \quad h = \frac{45\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

(rationalising the denominator)

$$= \frac{45(3+\sqrt{3})}{(\sqrt{3})^2 - 1} \qquad (\because (a-b)(a+b) = a^2 - b^2)$$

$$= \frac{45(3 + 1.732)}{3 - 1}$$

$$= \frac{45 \times 4.732}{2} = 106.47 \text{ m}$$

Hence, the height of the tower is 106.47 m.

5. (i) Given, $\sin(3\beta - \alpha) = \frac{1}{\sqrt{2}}$

$$\Rightarrow \quad \sin(3\beta - \alpha) = \sin 45^\circ$$

$$\Rightarrow \quad 3\beta - \alpha = 45^\circ \qquad ...(1)$$

Also, $\cos(2\alpha - 3\beta) = 1$

$$\Rightarrow \quad \cos(2\alpha - 3\beta) = \cos 0^\circ$$

$$\Rightarrow \quad 2\alpha - 3\beta = 0^\circ \qquad ...(2)$$

On adding eqs. (1) and (2), we get

$$\alpha = 45^\circ$$

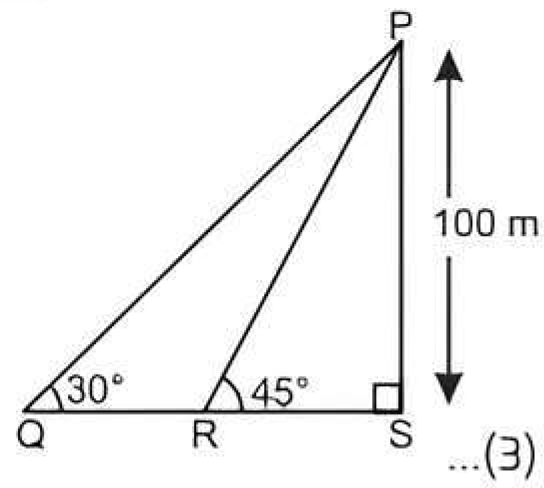
Put $\alpha = 45^\circ$ in eq. (1), we get

$$3\beta - 45^\circ = 45^\circ$$

$$3\beta = 90^\circ$$

$$\beta = 30^\circ$$

(ii) In right-angled ΔPSQ,



$$\tan 30^\circ = \frac{PS}{QS} = \frac{100}{QS}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{QS}$$

$$\Rightarrow \quad QS = 100\sqrt{3} \text{ m} \qquad ...(3)$$

In right-angled ΔPSR,

$$\tan 45^\circ = \frac{PS}{RS} = \frac{100}{RS}$$

$$\Rightarrow \quad 1 = \frac{100}{RS}$$

$$\Rightarrow \quad RS = 100 \text{ m} \qquad (\because \tan 45^\circ = 1) \; ...(4)$$

$$\therefore \quad QR = QS - RS$$

$$= 100\sqrt{3} - 100 \qquad \text{(from eq. (3))}$$

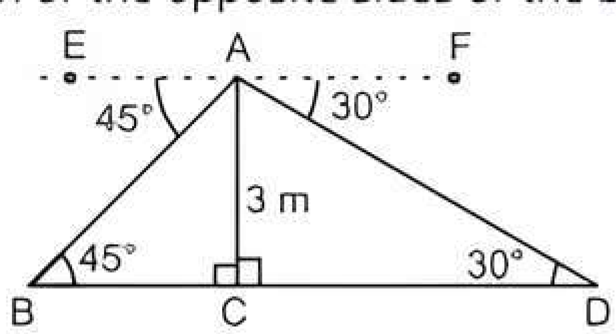
$$= 100(\sqrt{3} - 1) \text{ m}$$

Hence, distance between the two ships is $100(\sqrt{3}-1)$ m.

COMMON ERR(!)R

Sometime students get confused with the values of trigonometric angles. They substitute wrong values which leads to the wrong result.

6. Let A be the point of the bridge and B and D be the position of the opposite sides of the bridge.



Also, given the angle of depressions are $\angle EAB = 45^\circ$ and $\angle FAD = 30^\circ$

Then,

$\angle ABC = \angle EAB = 45^\circ$ (by alternate angles)

and $\angle ADC = \angle FAD = 30^\circ$ (by alternate angles)

In right-angled ΔBCA,

$$\tan 45^\circ = \frac{AC}{BC}$$

$$\Rightarrow \quad 1 = \frac{AC}{BC} \Rightarrow BC = 3 \text{ m}$$

In right-angled ΔDCA,

$$\tan 30^\circ = \frac{AC}{CD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3}{CD} \quad \Rightarrow \quad CD = 3\sqrt{3} \text{ cm}$$

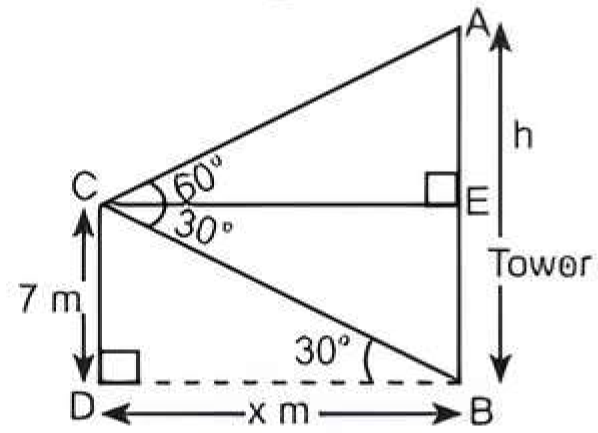
$$\therefore \quad BD = BC + CD$$

$$= 3 + 3\sqrt{3}$$

$$= 3(1+\sqrt{3}) \text{ m}$$

Hence, width of the river is $3(1+\sqrt{3})$ m.

7. Let AB = h m be the height of the tower and CD = 7 cm be the height of the building.



Here, $\angle ECB = \angle DBC = 30°$ (alternate angles)

Let $BD = x$ m, $CD = 7$ m

In right-angled ΔBDC,

$$\tan 30° = \frac{CD}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{7}{x} \quad \Rightarrow \quad x = 7\sqrt{3} \text{ m} \qquad ...(1)$$

In right-angled Δ AEC,

$$\tan 60° = \frac{AE}{CE} = \frac{AB - EB}{DB} = \frac{AB - CD}{DB}$$

$(\because CD = EB = 7 \text{ m}; DB = CE = x \text{ m})$

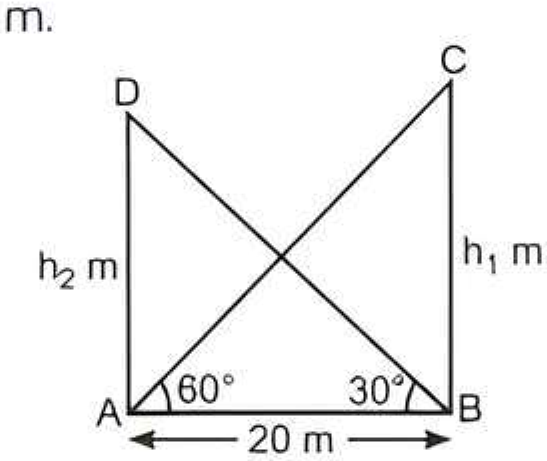
$$\Rightarrow \quad \sqrt{3} = \frac{h-7}{x} \quad \Rightarrow \quad \sqrt{3}x = h - 7$$

$$\Rightarrow \quad \sqrt{3} \times 7\sqrt{3} = h - 7 \qquad \text{(using eq. (1))}$$

$$\Rightarrow \quad h = 21 + 7 = 28 \text{ m}$$

Hence, the distance of the tower from the building is $7\sqrt{3}$ m and height of the tower is 30 m.

8. Let heights of two different poles be BC = h_1 m and AD = h_2 m.



Also given, $\angle BAC = 60°$

$\angle ABD = 30°$ and AB = 20 m

In right-angled Δ ABC,

$$\tan 60° = \frac{BC}{AB} \qquad \left(\because \tan\theta = \frac{P}{B}\right)$$

$$\Rightarrow \quad \sqrt{3} = \frac{h_1}{20}$$

$$\Rightarrow \quad h_1 = 20\sqrt{3} \text{ m}$$

In right-angled ΔBAD,

$$\tan 30° = \frac{AD}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h_2}{20}$$

$$\Rightarrow \quad h_2 = \frac{20}{\sqrt{3}} = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{20\sqrt{3}}{3} \text{ m}$$

$\therefore$ The difference between the height of two poles

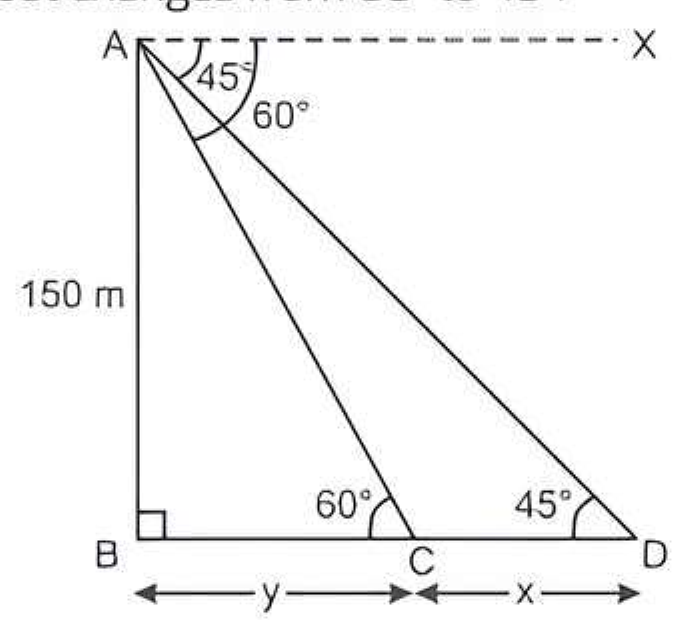
$$= h_1 - h_2 = 20\sqrt{3} - \frac{20}{3}\sqrt{3}$$

$$= \frac{60\sqrt{3} - 20\sqrt{3}}{3} = \frac{40\sqrt{3}}{3}$$

$$= \frac{40 \times 1.73}{3} = \frac{69.2}{3}$$

$$= 23.07 \text{ m}$$

9. In the figure, AB represents the 150 m high cliff. Initially, the boat is at point C and it moves to point D in 2 min and as it is given that the angle of depression of the boat changes from 60° to 45°.



So, $\angle DAX = 45°$ and $\angle CAX = 60°$

TiP

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

$\therefore \quad \angle DAX = \angle ADB = 45°$ and $\angle CAX = \angle ACB = 60°$

(alternate interior angles)

In right-angled ΔABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{150}{y} \quad (\because AB = 150 \text{ m and } BC = y \text{ m})$$

$$\Rightarrow \quad y = \frac{150}{\sqrt{3}} = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{150\sqrt{3}}{3} = 50\sqrt{3} \text{ m}$$

In right-angled ΔABD,

$$\tan 45° = \frac{AB}{BD} = \frac{AB}{BC + CD}$$

$$\Rightarrow \quad 1 = \frac{150}{y + x} \qquad (\because y = 50\sqrt{3} \text{ m})$$

$$\Rightarrow \quad x + y = 150$$

$$\Rightarrow \qquad x = 150 - 50\sqrt{3}\text{ m}$$

$\because$ The boat covers CD distance in 2 min.

$$\therefore \text{ The speed of boat} = \frac{\text{distance}}{\text{time}} = \frac{CD}{2}\text{ m/min}$$

$$= \frac{x}{(2)}\text{ m/min}$$

$$= \frac{1}{2}(150 - 50\sqrt{3})\text{ m/min}$$

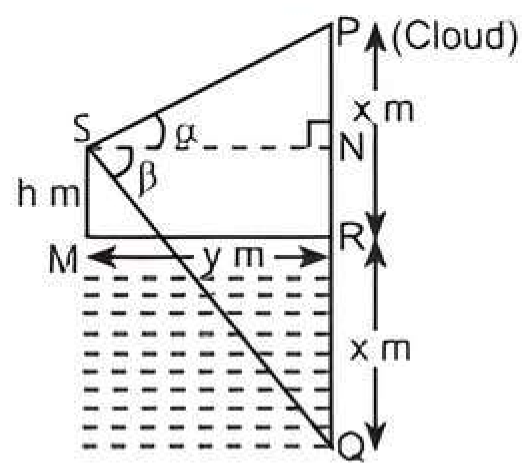
$$= 25(3 - \sqrt{3})\text{ m/min}$$

$$= 25\sqrt{3}(\sqrt{3} - 1)\text{ m/min}$$

10. Let P be the position of cloud, S be the point of observation and MR is the surface of lake.

TiP

The concept of angle of depression and angle of elevation must be understood deeply and clearly.



Let $\quad PR = RQ = x$ m

(height of cloud above the lake)

$SM = NR = h$ m

$\therefore \quad PN = PR - NR$

$= x - h$

$QN = QR + RN$

$= x + h$

and $\quad MR = SN = y$ m

In right-angled ΔPNS,

$$\tan\alpha = \frac{PN}{SN}$$

$$\Rightarrow \qquad \tan\alpha = \frac{x-h}{y} \qquad \text{...(1)}$$

In right-angled ΔSNQ,

$$\tan\beta = \frac{QN}{SN}$$

$$\Rightarrow \qquad \tan\beta = \frac{x+h}{y} \qquad \text{...(2)}$$

Dividing eq. (1) by eq. (2), we get

$$\frac{\tan\alpha}{\tan\beta} = \frac{\frac{x-h}{y}}{\frac{x+h}{y}}$$

$$\Rightarrow \qquad \frac{\tan\alpha}{\tan\beta} = \frac{x-h}{x+h}$$

$$\Rightarrow \qquad (x+h)\tan\alpha = (x-h)\tan\beta$$

$$\Rightarrow x\tan\alpha + h\tan\alpha = x\tan\beta - h\tan\beta$$

$$\Rightarrow h(\tan\alpha + \tan\beta) = x(\tan\beta - \tan\alpha)$$

$$\Rightarrow \qquad x = \frac{h(\tan\alpha + \tan\beta)}{\tan\beta - \tan\alpha}$$

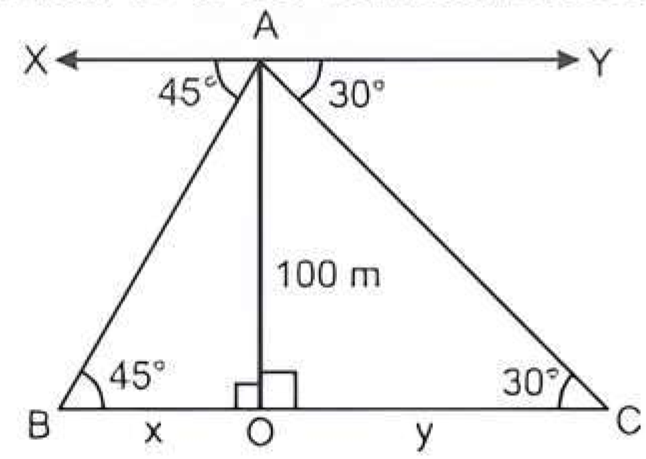
$\therefore$ The height of the cloud is

$$\frac{h(\tan\alpha + \tan\beta)}{(\tan\beta - \tan\alpha)}\text{ m.}$$

Hence proved.

Long Answer Type Questions

1. Let the top of the tower AO, a man at point A, observed the angle of depression 30° of car C and angle of depression 45° of car B on both sides of a tower.



TiP

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

$\therefore \qquad \angle XAB = \angle ABO = 45^\circ$

and $\qquad \angle YAC = \angle ACO = 30^\circ \qquad$ (alternate angles)

Given, $\qquad OA = 100$ m

TiP

The concept of angle of depression must be understood deeply and clearly.

In right-angled ΔAOB,

$$\tan 45^\circ = \frac{OA}{OB} = \frac{100}{x} \qquad (\text{let } OB = x\text{ m})$$

$$\Rightarrow \qquad 1 = \frac{100}{x} \Rightarrow x = 100\text{ m} \qquad \text{...(1)}$$

In right-angled ΔAOC,

$$\tan 30^\circ = \frac{OA}{OC} = \frac{100}{y} \qquad (\text{let } OC = y\text{ m})$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{100}{y}$$

$$\Rightarrow \qquad y = 100\sqrt{3}\text{ m} \qquad \text{...(2)}$$

Therefore, width of the river $= x + y$

$= 100 + 100\sqrt{3}$

$= 100 + 100 \times 1.73 = 100 + 173$

$= 273$m

Hence, distance between two cars is 273 m.

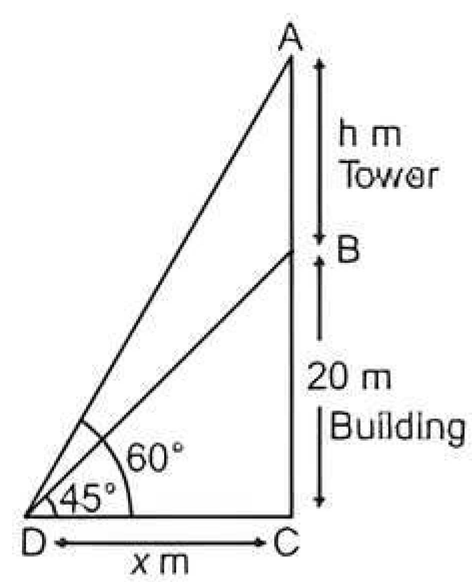
2. Let AB be the tower and BC be the building.

Let $DC = x$ m, $AB = h$ m and $BC = 20$ m.

In right-angled ΔBCD,

$$\tan 45^\circ = \frac{BC}{DC} \Rightarrow 1 = \frac{20}{x}$$

$$\Rightarrow \qquad x = 20\text{ m}$$



In right-angled $\triangle ACD$,

$$\tan 60^\circ = \frac{AC}{DC} = \frac{AB + BC}{DC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h+20}{x}$$

$$\Rightarrow \quad \sqrt{3}x = h + 20 \qquad \Rightarrow \sqrt{3} \times 20 = h + 20$$

$(\because x = 20 \text{ m})$

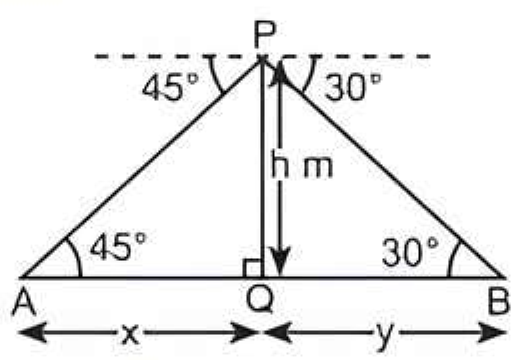
$$\Rightarrow \quad h = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1) \text{ m}$$

Hence, the height of tower is $20(\sqrt{3} - 1)$ m.

3. Let PQ be the lighthouse, and A and B are the position of two ships.

TiP

The concept of angle of depression must be understood deeply and clearly.



Let $PQ = h$ m, $AQ = x$ m and $QB = y$ m.

The distance between two ships (AB) $= x + y = 100$ m (Given)

In right-angled $\triangle PQA$,

$$\tan 45^\circ = \frac{PQ}{AQ} \Rightarrow 1 = \frac{h}{x} \Rightarrow x = h \qquad \text{...(1)}$$

In right-angled $\triangle PQB$,

$$\tan 30^\circ = \frac{PQ}{QB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{y}$$

$$\Rightarrow \quad y = \sqrt{3}\,h \qquad \text{...(2)}$$

Adding eqs. (1) and (2), we get

$$x + y = h + \sqrt{3}\,h$$

$$\Rightarrow \quad h + \sqrt{3}h = 100 \qquad (\because x + y = 100 \text{ m})$$

$$\Rightarrow \quad (\sqrt{3} + 1)h = 100$$

$$\Rightarrow \quad h = \frac{100}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

(rationalising the denominator)

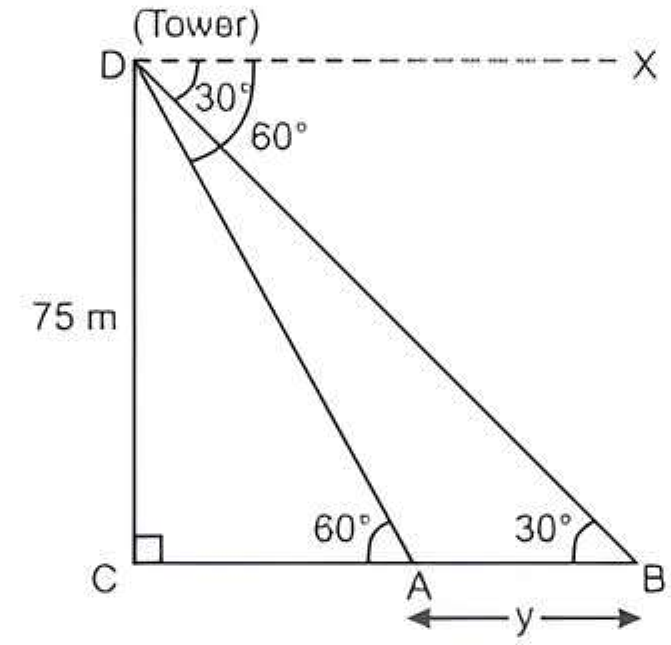
$$= \frac{100(\sqrt{3}-1)}{3-1} = \frac{100(1.732-1)}{2}$$

$(\because (a+b)(a-b) = a^2 - b^2)$

$$= 50 \times 0.732 = 36.6 \text{ m}$$

Hence, the height of the lighthouse is 36.6 m.

4. Given, the angles of depression of two cars A and B from the man standing on the top D of the tower CD (say) with height 75 m are 60° and 30° respectively.



TiP

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

$\therefore \quad \angle XDB = \angle CBD = 30^\circ$

and $\quad \angle XDA = \angle CAD = 60^\circ$ (alternate angles)

Let the distance between the cars, $AB = y$ m

In right-angled $\triangle ACD$,

$$\tan 60^\circ = \frac{CD}{CA} \Rightarrow \sqrt{3} = \frac{75}{CA}$$

$$\Rightarrow \quad CA = \frac{75}{\sqrt{3}} \qquad \text{...(1)}$$

In right-angled $\triangle BCD$,

$$\tan 30^\circ = \frac{CD}{BC} = \frac{CD}{CA + AB} = \frac{75}{CA + y}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{\frac{75}{\sqrt{3}} + y} \qquad \text{[from eq. (1)]}$$

$$\Rightarrow \quad y = 75\sqrt{3} - \frac{75}{\sqrt{3}} = \frac{75}{\sqrt{3}}(3-1) = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{150\sqrt{3}}{3} = 50\sqrt{3} = 50 \times 1.73 = 86.5$$

So, required distance between two cars is 86.5m.

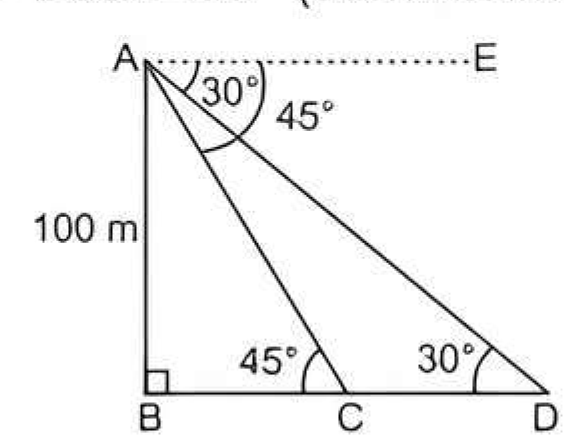
5. Let AB be the light house, C and D be the position of the ships.

TiP

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

$\therefore \quad \angle EAD = \angle ADB = 30^\circ$

and $\angle EAC = \angle ACB = 45^\circ$ (alternate interior angles)



In right-angled $\triangle ABC$,

$$\tan 45^\circ = \frac{AB}{BC} = \frac{100}{BC}$$

$$\Rightarrow \quad 1 = \frac{100}{BC} \quad \Rightarrow BC = 100 \text{ m}$$

In right-angled ΔABD,

$$\tan 30^\circ = \frac{AB}{BD} = \frac{100}{BD}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{BD} \Rightarrow BD = 100\sqrt{3}\,m$

$\therefore$ The distance between the two ships,

$$CD = BD - BC = 100\sqrt{3} - 100$$
$$= 100(\sqrt{3}-1) = 100(1.732-1)$$
$$= 100 \times 0.732 = 73.2\,m$$

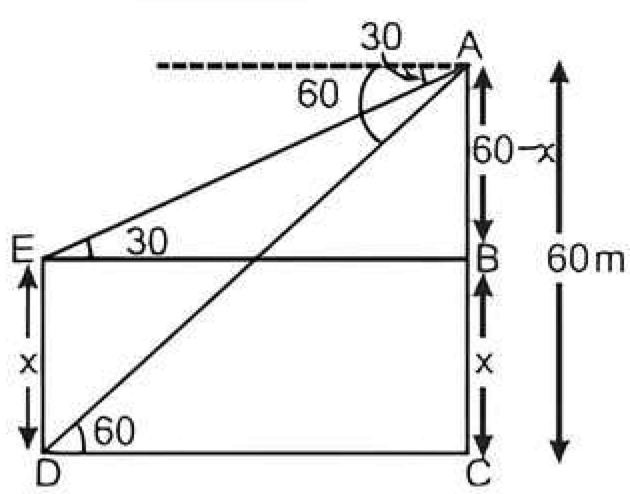
Hence, the distance between the two ships is 73.2 m.

6. In the figure, DE represents the small pole and AC represent 60 m high pole. The distance between two poles is DC.

Let the height of the small pole be x m.

TiP

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.



$\therefore \quad \angle XAE = \angle AEB = 30^\circ$

and $\angle XAD = \angle ADC = 60^\circ$ (alternate interior angles)

In right-angled ΔABE,

$$\tan 30^\circ = \frac{AB}{BE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{60-x}{BE}$$

$(\because DE = BC = x \therefore AB = AC - BC = AC - DE = 60 - x)$

$\Rightarrow \quad BE = \sqrt{3}(60-x) \quad ...(1)$

In right-angled ΔACD,

$$\tan 60^\circ = \frac{AC}{DC} = \frac{AC}{BE} \quad (\because BE = DC)$$

$\Rightarrow \quad \sqrt{3} = \frac{60}{\sqrt{3}(60-x)}$ [from eq. (1)]

$\Rightarrow \quad 60 = 3(60-x)$

$\Rightarrow \quad 20 = 60 - x$

$\Rightarrow \quad x = 40\,m$

$\therefore \quad DC = BE = \sqrt{3}(60-x)$ [$\because$ from eq. (1)]

$$= \sqrt{3}(60-40) = 20\sqrt{3}\,m$$

Hence, width of the river is $20\sqrt{3}$ m and height of the other pole is 40 m.

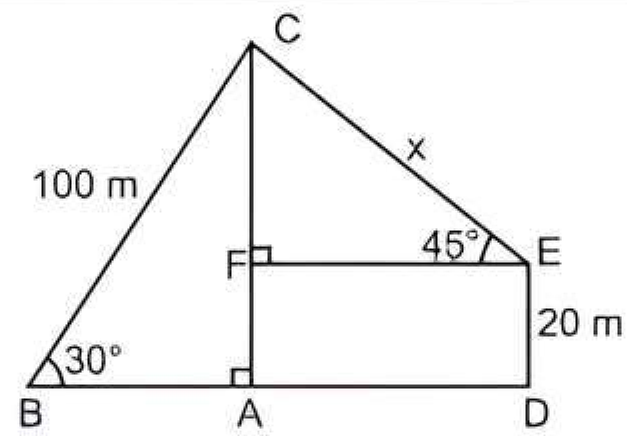
COMMON ERR(!)R

The concept of angle of depression and angle of elevation are not clear to many students. That's why they are not able to draw the diagram correctly.

7. Let C be the position of a bird flying. Let B be the position of the boy standing on the ground and E be the position of girl standing on the roof.

TiP

Students should practice mathematical calculations everyday to reduce calculation errors.



Given $BC = 100$ m, $DE = 20$ m, $\angle ABC = 30^\circ$

and $\angle CEF = 45^\circ$.

Let $CE = x$ m be the distance of bird from girl.

In right-angled ΔBAC,

$$\sin 30^\circ = \frac{AC}{BC} \Rightarrow \frac{1}{2} = \frac{AC}{100}$$

$\Rightarrow \quad AC = 50$ m

Now, $\quad CF = AC - AF$

$= 50 - 20 \quad (\because AF = ED = 20\text{ m})$

$= 30$ m

In right-angled ΔEFC,

$$\sin 45^\circ = \frac{CF}{EC} \Rightarrow \frac{1}{\sqrt{2}} = \frac{30}{x}$$

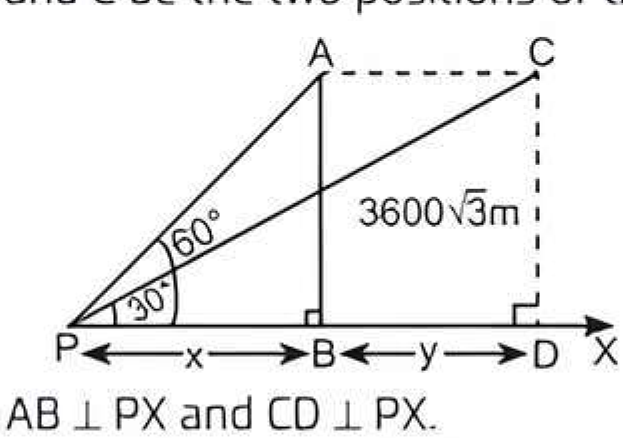
$\Rightarrow \quad = 30\sqrt{2} = 30 \times 1.414 = 42.42$ m

Hence, the distance of bird from girl is 42.4 m.

COMMON ERR(!)R

Some students express the answer in 3 significant or 4 significant figure which is not necessary until it is not asked in questions.

8. Let A and C be the two positions of the aeroplane.



Draw $AB \perp PX$ and $CD \perp PX$.

Let position of observer be P.

Here, $AB = CD = 3600\sqrt{3}$ m.

Let $\quad PB = x$ m and $BD = y$ m

In right-angled ΔABP,

$$\tan 60^\circ = \frac{AB}{PB} \Rightarrow \sqrt{3} = \frac{3600\sqrt{3}}{x}$$

$\Rightarrow \quad x = 3600$ m

In right-angled ΔCDP,

$$\tan 30^\circ = \frac{CD}{PD} = \frac{CD}{PB+BD}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{x+y}$

$\Rightarrow \quad x + y = 3600\sqrt{3} \times \sqrt{3}$

$\Rightarrow \quad 3600 + y = 10800 \qquad (\because x = 3600 \text{ m})$

$\Rightarrow \quad y = 10800 - 3600 = 7200 \text{ m}$

Since, distance (y) is covered by aeroplane in 30 sec. Therefore,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{7200}{30} = 240 \text{ m/s}$$

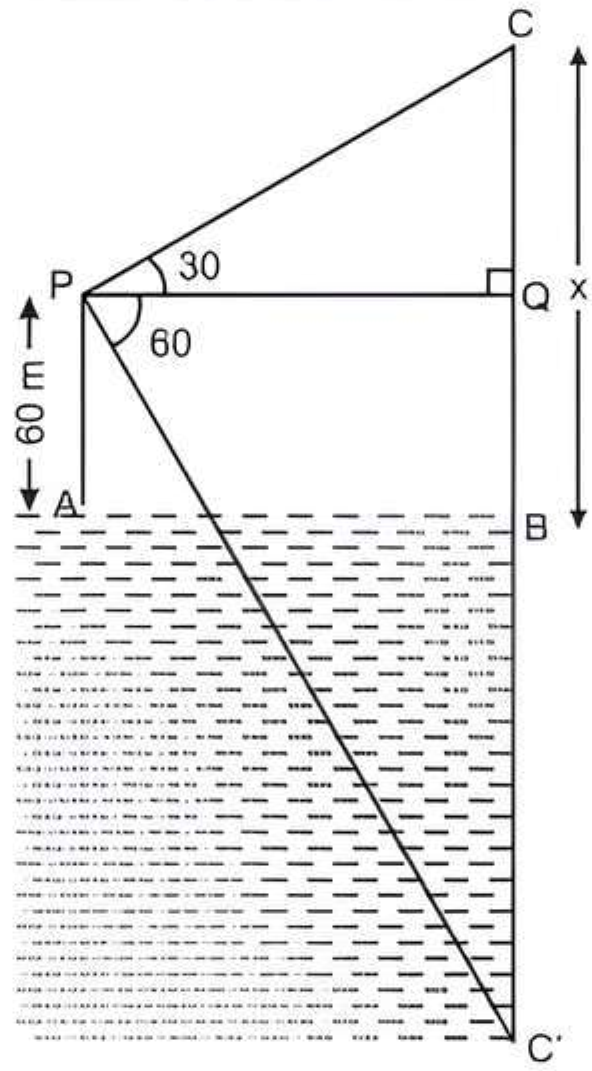
$$= \frac{18}{5} \times 240 = 864 \text{ km/h} \qquad \left(\because 1 \text{m/s} = \frac{18}{5} \text{km/h}\right)$$

Hence, the speed of aeroplane is 864 km/h.

9. Let two points A and B are on the surface of the water of a lake. A cloud C is at a height BC = x m from B. The angle of elevation of the cloud from P at 60 m vertically above the point A is $\angle CPQ = 30°$.

TiP

The concept of angle of depression and angle of elevation must be clear to the students.



Then in right-angled ΔCQP,

$$\tan 30° = \frac{CQ}{PQ} = \frac{BC - BQ}{PQ}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x - AP}{PQ} = \frac{x - 60}{PQ} \qquad (\because BQ = AP = 60 \text{ m})$$

$$\Rightarrow \quad PQ = \sqrt{3}(x - 60) \text{ m} \qquad ...(1)$$

From point B, the shadow of cloud C is C′ at depth x m from which $BC' = x$

Then, $\quad QC' = BQ + BC' = 60 + x$

$\because$ The angle of depression of the shadow at point P is 60°.

Then, in right-angled $\Delta C'QP$,

$$\tan 60° = \frac{QC'}{PQ} = \frac{QB + BC'}{PQ}$$

$$\Rightarrow \quad \sqrt{3} = \frac{PA + BC'}{PQ} = \frac{60 + x}{PQ} \qquad (\because QB = PA = 60 \text{ m})$$

From eq. (1), we get

$$\sqrt{3} = \frac{60 + x}{\sqrt{3}(x - 60)}$$

$$\Rightarrow \quad 3x - 180 = 60 + x \Rightarrow 2x = 240$$

$$\Rightarrow \quad x = 120 \text{ m}$$

Hence, the height of the cloud from the surface of water is 120 m.

COMMON ERROR

Most candidates are unable to draw the diagram as per the given data and lose their marks. So, adequate practice is required.

Chapter Test

Multiple Choice Questions

Q 1. The angle of depression of a car parked on the road from the top of 90 m high tower is 60°. The distance of the car from the tower (in metre) is:

a. 30 m
b. $30\sqrt{3}$ m
c. $90\sqrt{3}$ m
d. $60\sqrt{3}$ m

Q 2. A ladder 16 m long just reaches the top of a vertical wall. If the ladders makes an angle of 60° with the wall, then the height of the wall is :

a. 7 m
b. 8 m
c. 6 m
d. 5 m

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): If the length of shadow of a vertical pole is twice the height, then the angle of the sun is 60°.

Reason (R): Trigonometric ratio is used to determine the given statement as $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$.

Q 4. Assertion (A): If a ladder 10 cm long reaches a window 8 cm above the ground makes an angle $\sin\theta = \frac{4}{5}$ then the distance of the foot of the ladder from the base of the wall is 6 cm.

Reason (R): In an equilateral triangle of side $4\sqrt{3}$ cm the length of the altitude is 6 cm.

Fill in the Blanks

Q 5. If a person is standing on the ground and see the aeroplane flying in the sky, then this situation is based on angle of ....

Q 6. The angle of elevation of the top of the tower from the point on the ground which is 50 m away for the foot of the tower with height $50\sqrt{3}$ m is ....

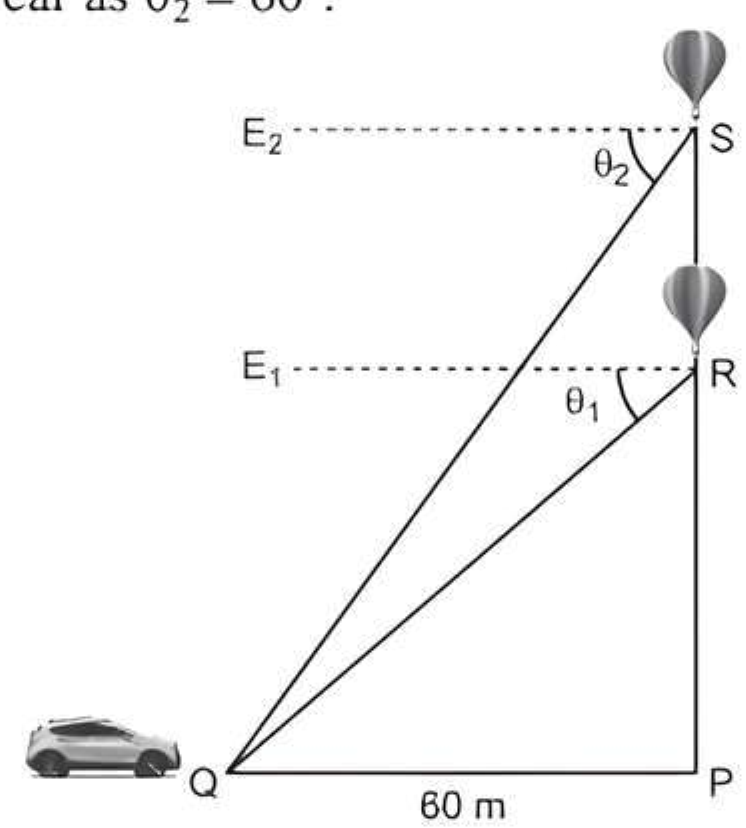
True/False

Q 7. If the angle of elevation of the tower decreases, the shadow of the tower increase.

Q 8. An observer 2.5 m tall is 20.5 m away from the tower 23 m high. The angle of elevation of the top of the tower from the eye of the observer is 60°.

Case Study Based Question

Q 9. A hot air balloon is rising vertically from a point P on the ground which is at a distance of 60 m from a car parked at a point Q on the ground. Sunita is riding the balloon, she observe that it took her 15 sec to reach a point R and makes an angle of depression θ_1 to point Q. She covers a distance equal to the horizontal distance of her starting point from the car parked at point Q. After certain time, Sunita observes that she reaches at point S and makes an angle of depression from the car as $\theta_2 = 60^\circ$.



Based on the above information, solve the following questions:

(i) Find the speed of the air balloon to reach at point R.

(ii) Find the distance of point R from car Q.

(iii) Find the distance of point S from foot of the ground.

Or

Find the distance of point S from point R.

Very Short Answer Type Questions

Q 10. A ramp for disabled people in a hospital have slope not more than 30°. If the height of the ramp be 2m, then find the length of the ramp. [Use $\sqrt{3} = 1.732$]

Q 11. If the Sun's angle of elevation is 60° and height of the pole is $9\sqrt{3}$ m, then find the length of shadow of pole.

Short Answer Type-I Questions

Q 12. If two towers of heights x m and y m subtends angles of 45° and 60° respectively at the centre of a line joining their feet, then find the ratio of $x : y$.

Q 13. The tops of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then find the length of the wire.

Short Answer Type-II Questions

Q 14. Two ships are there in the sea on either side of a light house in such a way that the ships and the base of the lighthouse are in the straight line. The angle of depression of two ships as observed from the top of the lighthouse are 60° and 45°. If the height of the lighthouse is 200 m, then find the distance between two ships. [Use $\sqrt{3} = 1.732$]

Q 15. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30s, the angle of elevation becomes 30°. If the aeroplane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane.

Long Answer Type Question

Q 16. From the top of a building 90 m high, the angles of depression of the top and the bottom of a tower are observed to be 30° and 60°. Find the height of the tower.