# Chapter 17

# Mensuration

## Exercise 17.1

**Take $\pi = \dfrac{22}{7}$ unless stated otherwise.**

**1. Find the total surface area of a solid cylinder of radius 5 cm and height 10 cm. Leave your answer in terms of $\pi$.**

**Solution:**

Given radius of the cylinder, r = 5 cm

Height of the cylinder, h = 10 cm

Total surface area = 2r (r +h)

= 2 × 5(5 + 10)

= 2 × 5 × 15

= $150 cm^2$

Hence the total surface area of the solid cylinder is $150 cm^2$.

**2. An electric geyser is cylindrical in shape, having a diameter of 35 cm abd height 1.2m. Neglecting the thickness of its walls, calculate**

**(i) its outer lateral surface area,**

**(ii) its capacity in litres.**

**Solution:**

Given diameter of the cylinder, d = 35 cm

radius, r = $\frac{d}{2}$ = $\frac{35}{2}$ = 17.5 cm

Height of the cylinder, h = 1.2m = 120 cm


(i) Outer lateral surface area = 2rh

$= 2 \times \left(\frac{22}{7}\right) \times 17.5 \times 120$

$= 13200 cm^2$


(ii) Capacity of the cylinder $= r^2 h$

$= \left(\frac{22}{7}\right) \times (17.5)^2 \times 120$

$= 115500 cm^3$

$= 115.5$ litres $[\because 1000 cm^3 = 1\ litre]$

Hence the outer lateral surface area and the capacity of the cylinder is $13200 cm^2$ and 115.5 litres respectively.

**3. A school provides milk to the students daily in cylindrical glasses of diameter 7 cm. if the glass is filled with milk upto a height of 12cm, find how, many litres of milk is needed to serve 1600 students.**

**Solution:**

Given diameter of the cylindrical glass, d = 7 cm

Radius, r = $\frac{d}{2}$ = $\frac{7}{2}$ = 3.5 cm

Height of the cylindrical glass, h = 12cm

Volume of the cylindrical glass, V = $r^2h$

= $\left(\frac{22}{7}\right) \times (3.5)^2 \times 12$
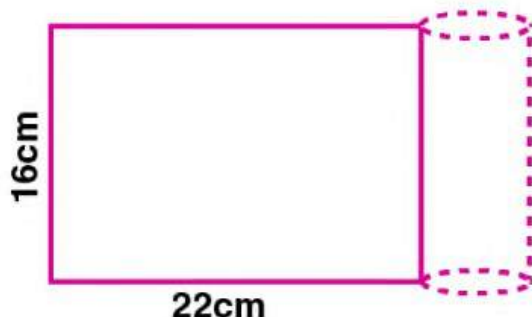
= 462 $cm^3$

Number of students = 1600

Milk needed for 1600 students = 1600 × 462

= 739200 $cm^3$

= 739.2 litres [∵ 1000$cm^3$ = 1 $litre$]

Hence the amount of milk needed to serve 1600 stuents is 739.2 litres.

**4. In the given figure, a rectangular tin foil of size 22 cm by 16 cm is wrapped around to form a cylinder of height 16 cm. Find the volume of the cylinder.**

**Solution:**

Given height of the cylinder, h = 16cm

When rectangular foil of length 22cm is folded to form cylinder, the base circumference of the cylinder is 22cm

2r = 22

$r = \dfrac{22}{2}$

= 3.5 cm

Volume of the cylinder, $V = r^2 h$

$= \left(\dfrac{22}{7}\right) \times (3.5)^2 \times 16$

$= 616\ cm^3$

Hence the volume of the cylinder is $616 cm^3$.


**5.  (i) How many cubic metres of soil must be dug out to make a well 20 metres deep and 2 metres in diameter ?**

**(ii) If the inner curved surface of the well in part (i) above is to be plastered at the rate of Rs 50 per $m^2$, find the cost of plastering.**

**Solution:**

(i) Given diameter of the well, d = 2m

Radius, $r = \dfrac{d}{2} = \dfrac{2}{2} = 1$m

Depth of the well, h = 20m

Volume of the well, $V = r^2 h$

$= \left(\dfrac{22}{7}\right) \times (1)^2 \times 20$

$= 62.85 \ m^3$

Hence the amount of soil dug out to make the well is $62.85 \ m^3$.

(ii) Curved surface area of the well $= 2rh$

$= 2 \times \left(\frac{22}{7}\right) \times 1 \times 20$

$= \frac{880}{7} \ m^2$

Cost of plastering the well per $m^2 = Rs. 50$

Total cost of plastering the well $= 50 \times \left(\frac{880}{7}\right)$

$=$ Rs. 6285.71

Hence the total cost of plastering is Rs. 6285.71.

**6. A road roller ( in the shape of a cylinder) has a diameter 0.7 m and its width is 1.2m. Find the least number of revolutions that the roller must make in order to level a playground of size 120m by 44m.**

**Solution:**

Given diameter of the road roller, d = 0.7 m

Radius, r $= \frac{d}{2} = \frac{0.7}{2} = 0.35m$

Width, h = 1.2 m

Curved surface area of the road roller $= 2rh$

$= 2 \times \left(\frac{22}{7}\right) \times 0.35 \times 1.2$

$= 2.64 \, m^2$

Area of the play ground $= l \times b$

$= 120 \times 44$

$= 5280 \, m^2$

Number of revolutions $= \dfrac{Area \ of \ play \ ground}{Curved \ surface \ area}$

$= \dfrac{5280}{2.64}$

$= 2000$

Hence the road roller must taken 2000 revolutions to level the ground.

**7. If the volume of a cylinder of height 7cm is $448\pi \, cm^3$, find its lateral surface area and total surface area.**

**Solution:**

Given Height of the cylinder, h $= 7$cm

Volume of the cylinder, V $= 448cm^3$

$r^2 h = 448$

$r^2 \times 7 = 448$

$r^2 = \dfrac{448}{7}$

$r^2 = 64$

$r = 8$

Lateral surface area $= 2rh$

$= 2 \times 8 \times 7$

$= 112 \; cm^2$

Total surface area $= 2r(r + h)$

$= 2 \times 8 \; \times (8 + 7)$

$= 2 \; \times 8 \; \times 15$

$= 240 \; cm^2$

Hence the lateral surface area and the total surface area of the cylinder are $112 \; cm^2$ and $240 \; cm^2$ .


**8. A wooden pole is 7m high and 20cm in diameter. Find its weight if the wood weighs 225 kg per $m^3$.**

**Solution:**

Given height of the pole, h = 7m

Diameter of the pole, d = 20 cm

radius, r $= \dfrac{d}{2} = \dfrac{20}{2} =$ 10cm = 0.1 m

Volume of the pole $= r^2 h$

$= \left(\dfrac{22}{7}\right) \times 0.1^2 \; \times 7$

$= 0.22 \; m^3$

Weight of wood per $m^3 = 225 kg$

weight of $0.22 m^3$ wood $= 225 \times 0.22 = 49.5 kg$

Hence the weight of the wood is 49.5kg.

**9.** The area of the curved surface of a cylinder is 4400 $cm^2$, and the circumference of its base is 110cm. Find

(i) The height of the cylinder.

(ii) The volume of the cylinder.

**Solution:**

Given curved surface area of a cylinder $= 4400 cm^2$

Circumference of its base $= 110$ cm

$2r = 110$

$r = \left(\frac{110}{2}\right) = \frac{110 \times 7}{2 \times 22} = 17.5$ cm

(i) Curved surface area of a cylinder, $2rh = 4400$

$2 \times \left(\frac{22}{7}\right) \times 17.5 \times h = 4400$

$h = \frac{(4400 \times 7)}{2 \times 22 \times 17.5}$

$= 40$ cm

Hence the height of the cylinder is 40 cm.

(ii) Volume of the cylinder, $V = r^2 h$

$= \left(\frac{22}{7}\right) \times 17.5^2 \times 40$

$= 38500 cm^3$

Hence the volume of the cylinder is $38500 cm^3$.

**10. A cylinder has a diameter of 20cm. The area of curved surface is $1000cm^2$. Find**

(i) The height of the cylinder correct to one decimal place.

(ii) The volume of the cylinder correct to one decimal place. ( Take $\pi = 3.14$)

**Solution:**

(i)  Given diameter of the cylinder, d = 20cm

(ii) Given diamter of the cylinder, d = 20 cm

Radius, r $= \dfrac{d}{2} = \dfrac{20}{2} = 10 \; cm$

Curved surface area $= 1000 \; cm^2$

$2rh = 1000$

$2 \times 3.14 \times 10 \times h = 1000$

$62.8h = 1000$

$h = \dfrac{1000}{62.8}$

$= 15.9$ cm

Hence the height of the cylinder is 15.9 cm.

**(ii) Volume of the cylinder, V $= r^2h$**

$= 3.14 \times 10^2 \times 15.9$

$= 4992.6 \; cm^3$

Hence the volume of the cylinder is $4992.6 \; cm^3$.

**11.  The barrel of a fountain pen, cylindrical in shape, is 7cm long and 5mm in diameter. A full barrel of ink in the pen will be used up when writing 310 words on an average. How many words would use up a bottle of ink containing one- fifth of a litre ?**

**Answer correct to the nearest. 100 words.**

**Solution:**

Height of the barrel of a pen, h = 7cm

Diameter, d = 5mm = 0.5 cm

Radius, r = $\frac{d}{2}$ = $\frac{0.5}{2}$ = 0.25 cm

Volume of the barrel of pen, V = $r^2 h$

= $\left(\frac{22}{7}\right) \times (0.25)^2 \times 7$

= 1.375 $cm^3$

Ink in the bottle, = One fifth of a litre

= $\frac{(1/5)}{1000}$ = 200ml

Number of words written using full barrel of ink = 310

Number of words written by using this ink

= $\frac{(200)}{1.375} \times 310$ = 45090.90 words

Round off to nearest hundred, we get 45100 words.

Hence the number of words written using the ink is 45100 words.

**12. Find the ratio between the total surface area of a cylinder to its curved surface area given that its height and radius are 7.5 cm and 3.5 cm.**

**Solution:**

Given radius of the cylinder, r = 3.5 cm

Height of the cylinder, h = 7.5 cm

Total surface area = $2r\,(r+h)$

Curved surface area = $2rh$

**Ratio of Total surface area to curved surface area** $= \dfrac{2r(r+h)}{2rh}$

$= \dfrac{(r+h)}{h}$

$= \dfrac{(3.5+7.5)}{7.5}$

$= \dfrac{11}{7.5}$

$= \dfrac{22}{15}$

Hence the required ratio is 22:15.

**13. The radius of the base of a right circular cylinder is halved and the height is doubled. What is the ratio of the volume of the new cylinder to that of the original cylinder ?**

**Solution:**

Let the radius of the base of a right circular cylinder be r and height be h.

Volume of the cylinder, $V_1 = r^2 h$

$$= \left(\frac{r}{2}\right)^2 \times 2h$$

$$= \frac{1}{2} r^2 h$$

So ratio of volume of new cylinder to the original cylinder, $\frac{V_2}{V_1} = \frac{\frac{1}{2} r^2 h}{r^2 h} = \frac{1}{2}$

Hence the required ratio is 1: 2.


**14. (i) The sum of the radius and the height of a cylinder is 37 cm and the total surface area of the cylinder is $1628 cm^2$. Find the height and the volume of the cylinder.**

**(ii)** The total surface area of a cylinder is $352\ cm^2$. If its height is 10 cm, then find the diameter of the base.

**Solution:**

(i)  Let r be the radius and h be the height of the cylinder.

Given the sum of radius and height of the cylinder, $r + h = 37cm$

Total surface area of the cylinder $= 1628\ cm^2$

$2r(r + h) = 1628$

$2 \times \left(\frac{22}{7}\right) \times r \times 37 = 1628$

$r = \frac{(1628 \times 7)}{(2 \times 22 \times 37)}$

$r = 7\ cm$

We have $r + h = 37$

$7 + h = 37$

$h = 37 - 7 = 30 \; cm$

Volume of the cylinder, $V = r^2 h$

$= \left(\frac{22}{7}\right) \times 7^2 \times 30$

$= \left(\frac{22}{7}\right) \times 49 \times 30$

$= 4620 \; cm^3$

Hence the height and volume of the cylinder is 30 cm and 4620 $cm^3$ respectively.

(ii) total surface area of the cylinder $= 353 cm^2$

Height , h = 10 cm

$2r(r + h) = 352$

$2 \times \left(\frac{22}{7}\right) \times r \times (r + 10) = 352$

$r^2 = 10r = \dfrac{(352 \times 7)}{2 \times 22}$

$r^2 + 10r = 56$

$r^2 + 10r - 56 = 0$

$(r + 14)(r - 4) = 0$

$r + 14 = 0 \; or \; r - 4 = 0$

$r = -14 \; or \; r = \; 4$

Radius cannot be nagative. So $r = 4$.

Diameter $= 2 \times r = 2 \times 4 = 8\ cm$.

**Hence the diamter of the base is 8 cm.**

**15. The ratio between the curved surface and the total surface of a cylinder is 1 : 2. Find the volume of the cylinder, given that its total surface area is 616 $cm^2$.**

**Solution:**

Given the ratio of curved surface area and the total surface area $= 1 : 2$

Total surface area $= 616\ cm^2$.

Curved surface area $= \frac{616}{2} = 308\ cm^2$

$2rh = 308$

$rh = \frac{308}{2}$

$rh = \frac{308 \times 7}{2 \times 22}$

$rh = 49$  ....(i)

Total surface area, $2rh + 2r^2 = 616$

$308 + 2r^2 = 616$

$2r^2 = 616 - 308$

$2r^2 = 308$

$r^2 = \frac{308}{2} = 154$

$r^2 = 154/$

$$= \frac{154 \times 7}{22}$$

$$= 49$$

Taking square root on both sides

$$r = 7$$

**Substitute r in (i)**

$$rh = 49$$

$$7 \times h = 49$$

$$h = \frac{49}{7} = 7 \text{ cm}$$

Volume of the cylinder, $V = r^2 h$

$$= \left(\frac{22}{7}\right) \times 7^2 \times 7$$

$$= 1078 \; cm^3$$

Hence the volume of the cylinder is $1078 cm^3$.

**16. Two cylindrical jars contain the same amount of milk. If their diameters are in the ratio 3 : 4, find the ratio of their heights.**

**Solution:**

Let $r_1$ and $r_2$ be the radius of the two cylinders and $h_1$ and $h_2$ be their heights.

Given ratio of the diameter = 3 : 4

Then the ratio of radius $r_1 : r_2 = 3 : 4$

**Given volume of both jars are same.**

$$r_1^2 h_1 : r_2^2 h_2$$

$$\frac{h_1}{h_2} = \frac{r_2^2}{r_1^2}$$

$$\frac{h_1}{h_2} = \frac{4^2}{3^2} = \frac{16}{9}$$

**Hence the ratio of the heights are 16 : 9.**

**17.** A rectangular sheet of tin foil of size 30 cm × 18 $cm$ can be rolled to form a cylinder in two ways along length and along breadth. Find the ratio of volumes of the two cylinders thus formed.

**Solution:**

Given size of the sheet = 30 $cm$ × 18 $cm$

If we roll it lengthwise, base circumference, $2r = 30$

$$2 \times \left(\frac{22}{7}\right) r = 30$$

$$r = 30 \times \frac{7}{2} \times 22 = \frac{210}{44} = \frac{105}{22} \text{cm}$$

Height, h = 18 $cm$

Volume of the cylinder, $V_1 = r^2 h$

$$= \left(\frac{22}{7}\right) \times \left(\frac{105}{22}\right)^2 \times 18$$

$$= 15 \times 105 \times \frac{9}{11}$$

If we roll it breadthwise, base circumference, $2r = 18$

$$2 \times \left(\frac{22}{7}\right) r = 18$$

$$r = 18 \times \frac{7}{2} \times 22$$

$$= \frac{126}{44}$$

$$= \frac{63}{22} \text{ cm}$$

Height, h = 30 cm

Volume of the cylinder, $V_2 = r^2 h$

$$= \left(\frac{22}{7}\right) \times \left(\frac{63}{22}\right)^2 \times 30$$

$$= 9 \times 63 \times \frac{15}{11}$$

$$\frac{V_1}{V_2} = \frac{\left(15 \times 105 \times \frac{9}{11}\right)}{\left(9 \times 63 \times \frac{15}{11}\right)}$$

$$= \left(15 \times 105 \times \frac{9}{11}\right) \times \left(\frac{11}{9} \times 63 \times 15\right)$$

$$= \frac{105}{63}$$

$$= \frac{15}{9}$$

$$= \frac{5}{3}$$

Ratio of the volumes of two cylinders is 5:3.


**18. A cylindrical tube open at both ends is made of metal. The internal diameter of the tube is 11.2 cm and its length is 21cm. The metal thickness is 0.4 cm. Calculate the volume of the metal.**

**Solution:**

Given internal diameter of the tube = 11.2 cm

Internal radius, r $= \frac{d}{2} = \frac{11.2}{2} = 5.6 \; cm$

Length of the tube, h = 21 cm

Thickness = 0.4 cm

Outer radius, R = 5.6 + 0.4 = 6 cm

Volume of the metal = $R^2h - ch$

$= h(R^2 - r^2)$

$= \left(\frac{22}{7}\right) \times 21 \times (6^2 - 5.6^2)$

$= 66 \times (6 + 5.6)(6 - 5.6)$
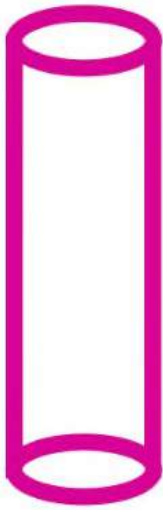
$= 66 \times 11.6 \times 0.4$

$= 306.24 \; cm^3$

Hence the volume of the metal is $306.24 cm^3$.


**19. The given figure shows a metal pipe 77 cm long. The inner diameter of a cross-secting is 4cm and the outer one is 4.4 cm. Find its**

(i) inner curved surface area

(ii) Outer curved surface area

(iii) Total surface area

**Solution:**

Given height of the metal pipe = 77 cm

Inner diameter = 4 cm

Inner radius, $r = \dfrac{d}{2} = \dfrac{4}{2} = 2\text{cm}$

Outer diameter = 4.4 cm

Outer radius, $R = \dfrac{d}{2} = \dfrac{4.4}{2} = 2.2$ cm

**(i) Inner curved surface area = $2rh$**

$= 2 \times \left(\dfrac{22}{7}\right) \times 2 \times 77$

$= 968\ cm^2$

Hence the inner surface area is 968 cm².

**(ii) Outer curved surface area = $2Rh$**

$= 2 \times \left(\frac{22}{7}\right) \times 2.2 \times 77$

$= 1064.8 cm^2$

Hence the outer curved surface area is $968 cm^2$.

(iii) Area of ring $= (R^2 - r^2)$

$= \left(\frac{22}{7}\right) \times (2.2^2 - 2^2)$

$= \left(\frac{22}{7}\right) \times (4.84 - 4)$

$= \left(\frac{22}{7}\right) \times 0.84$

$= 2.64$

**Total surface area** $=$ inner surface area $+$ Outer surface area $+$ area of two rings
$= 968 + 1064.8 + 2 \times 2.64$
$= 2038.08 \ cm^2$
Hence the total surface area of the metal pipe is $2038.08 \ cm^2$.

**20. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.**

**Solution:**

Given length of the pencil, h = 14 cm

Diameter of the pencil = 7mm

Radius, R = $\frac{7}{2}$ mm = $\frac{7}{20}$ cm

Diameter of the graphite = 1mm

Radius of graphite, r = $\frac{1}{2}mm$ = $\frac{1}{20}$ cm

Volume of graphite = $r^2h$

$= \left(\frac{22}{7}\right) \times \left(\frac{1}{20}\right)^2 \times 14$

$= \frac{11}{100}$

$= 0.11 \; cm^3$

Hence the volume of the graphite is $0.11 cm^3$.

Volume of the wood = $(R^2 - r^2)h$

$= \left(\frac{22}{7}\right) \times \left[\left(\frac{7}{20}\right)^2 - \left(\frac{1}{20}\right)^2\right] 14$

$= \left(\frac{22}{7}\right) \times \left[\left(\frac{49}{400}\right) - \left(\frac{1}{400}\right)\right] 14$

$= \left(\frac{22}{7}\right) \times \left(\frac{48}{400}\right) \times 14$

$= 11 \; \times \frac{12}{25}$

$= 5.28 \; cm^3$

Hence the volume of the wood is $5.28 \; cm^3$

**21. A soft drink is available in two packs**

**(i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and**

**(ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm.**
**Which containder has greater capacity and by how much ?**

**Solution :**

(i) Length of the can, $l = 5\ cm$

width, b = 4 cm

Height, h = 15 cm

Volume of the can = $lbh$

$= 5 \times 4\ \times 15$

$= 300\ cm^3$


(ii) Diameter of the cylinder,  d = 7 cm

Radius, $r = \dfrac{d}{2} = \dfrac{7}{2} = 3.5\ cm$

Height, h = 10 cm

Volume of the cylinder, $V = r^2 h$

$= \left(\dfrac{22}{7}\right) \times (3.5)^2 \times 10$

$= 385\ cm^3$

Difference of the volume $= 385 - 300 = 85 cm^3$

Hence the plastic cylinder has $85 cm^3$  more volume than the can.

**22. A cylindrical roller made of iron is 2 m long. Its inner diameter is 35 cm and ht thickness is 7cm all round. Find the weight of the roller in kg, if $1\ cm^3$ of iron weighs 8 g.**

**Solution:**

Length of the roller, h = 2m = 200 cm [1m = 100 cm]

Inner diameter = 35 cm

Inner radius, r = $\frac{35}{2}$ cm

Thickness = 7 cm

**Outer radius, R = $\left(\frac{35}{2}\right) + 7$**

$= \frac{(35+14)}{2}$

$= \frac{49}{2}$ cm

Volume of the iron in roller = $(R^2 - r^2)h$

$= \left(\frac{22}{7}\right)\left[\left(\frac{49}{2}\right)^2 - \left(\frac{35}{2}\right)^2\right]200$

$= \left(\frac{22}{7}\right)\left[49^2 - \frac{35^2}{4}\right]200$

$= \left(\frac{22}{7}\right) \times 50(49^2 - 35^2)$

$= \left(\frac{22}{7}\right) \times 50(2401 - 1225)$

$= \left(\frac{22}{7}\right) \times 50 \times 1176$

$= 184800\ cm^3$

Given 1 $cm^3$ of iron weighs 8 g.

Weight of the roller $= 184800 \times 8 = 1478400g$

= 1478.4 kg [1 kg = 1000 g]

Hence the weight of the roller is 1478.5 kg.

# Exercise 17.2

Take $\pi = \dfrac{22}{7}$ unless stated otherwise.

**1. Write whether the following statements are true or false. Justify your**
   **answer.**

(i) If the radius of a right circular cone is halved and its height is doubled, the volume will remain unchanged.

(ii) A cylinder and a right circular cone are having the same base radius and same height. The volume of the cylinder is three times the volume of the cone.

(iii) In a right circular cone, height, radius and slant height are always the sides of a right triangle.

**Solution:**

(i) Volume of cone $= \left(\dfrac{1}{3}\right) r^2 h$

If radius is halved and height is doubled, then volume $= \dfrac{1}{3} \left(\dfrac{r}{2}\right)^2 2h$

$= \left(\dfrac{1}{3}\right) \dfrac{r^2 h}{2}$

If the radius of a right circular cone is halved and its height is doubled, the volume will be halved.

So given statement is false.

(ii) Radius of cylinder = radius of cone

Height of cylinder = height of cone

Volume of cylinder = $r^2h$

= Volume of cone = $\left(\frac{1}{3}\right)r^2h$

The volume of the cylinder is three times the volume of the cone.

So given statement is true.


(iii) In a right circular cone,

$l^2 = h^2 + r^2$

In a right circular cone, height, radius and slant height are always the sides of a right triangle.

So, given statement is true.


**2. Find the curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 cm.**

**Solution:**

Given slant height of the cone, $l = 10\ cm$

Base radius, $r = 7\ cm$

Curved surface area of the cone = $rl$

= $\left(\frac{22}{7}\right) \times 7 \times 10 = 220 cm^2$

Hence the curved surface area of the cone is $220\ cm^2$.

**3. Diameter of the base of a cone is 10.5cm and slant height is 10 cm. Find its curved surface area.**

**Solution:**

Given diameter of the cone = 10.5 cm

Radius, r $= \frac{d}{2} = \frac{10.5}{2} = 5.25$ cm

Slant height of the cone, $l = 10 \ cm$

Curved surface area of the cone $= rl$

$= \left(\frac{22}{7}\right) \times 5.25 \times 10$

$= 165 \ cm^2$

Hence the curved surface area of the cone is $165 \ cm^2$.

**4. Curved surface area of a cone is $308cm^2$ and its slant height is 14 cm. Find,**
(i) radius of the base
(ii) Total surface area of the cone.
**Solution:**
(i) Given curved surface area of the cone = 308cm$^2$
Slant height of the cone, $l = 14 \ cm$
$rl = 308$
$\left(\frac{22}{7}\right) \times r \times 14 = 308$
$r = \dfrac{308 \times 7}{22 \times 14}$
$= 7$

Hence the radius of the cone is 7 cm.

**(ii)** Total surface area of the cone = Base area + curved surface area

$$= r^2 + rl$$

$$= \left(\frac{22}{7}\right) \times 7^2 + 308$$

$$= 154 + 308$$

$$= 462$$

Hence the total surface area of the cone is $462\text{cm}^2$.

## 5. Find the volume of the right circular cone with

(i) radius 6 cm and height 7 cm

(ii) radius 3.5 cm and height 12 cm.

**Solution:**

(i) Given radius, r = 6 cm

Height, h = 7 cm

Volume of the cone $= \left(\frac{1}{3}\right) r^2 h$

$$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 6^2 \times 7$$

$$= 22 \times 12$$

$$= 264\ cm^3$$

Hence the volume of the cone is $264\ cm^3$.

6. Find the capacity in litres of a conical vessel with

(i) radius 7 cm, slant height 25 cm

(ii) height 12 cm, slant height 13 cm

**Solution:**

Given radius, $r = 7\ cm$

Slant height, $l = 25\ cm$

We know that $l^2 = h^2 + r^2$

**Height of the conical vessel, h $= \sqrt{(l^2 - r^2)}$**

$= \sqrt{(25^2 - 7^2)}$

$= \sqrt{(625 - 49)}$

$= \sqrt{576}$

$= 24$ cm

Volume of the cone $= \left(\frac{1}{3}\right) r^2 h$

$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 7^2 \times 24$

$= 22 \times 7 \times 8$

$= 1232\ cm^3$

$= 1.232$ litres [1 litre $= 1000\ cm^3$]

Hence the volume of the cone is 1.232 litres.

(ii) Given height, h = 12 cm

Slant height, $l = 13$ cm

We know that $l^2 = h^2 + r^2$

Radius of the conical vessel, $r = \sqrt{(l^2 - h^2)}$

$= \sqrt{(13^2 - 12^2)}$

$= \sqrt{169 - 144}$

$= \sqrt{25}$

= 5 cm

Volume of the cone $= \left(\frac{1}{3}\right) r^2 h$

$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 5^2 \times 12$

$= \left(\frac{22}{7}\right) \times 25 \times 4$

$= \frac{2200}{7} cm^3$

$= \frac{2.2}{7}$ litres [ 1 litre = 1000 $cm^3$]

= 0.314 litres

Hence the volume of the cone is 0.314 litres.

**7. A conical pit of top diameter 3.5m is 12 m deep. What is its capacity in kiloliters ?**

**Solution:**

Given diameter, d = 3.5 m

So radius, $r = \frac{3.5}{2} = 1.75$

Depth, h = 12 m

**Volume of the cone $= \left(\frac{1}{3}\right) r^2 h$**

$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 1.75^2 \times 12$

$= \left(\frac{22}{7}\right) \times 1.75^2 \times 4$

$= 38.5 \ m^3$

$= 38.5$ kilolitres [1 kilolitre $= 1 m^3$]

Hence the volume of the conical pit is 38.5 kilolitres.


**8. If the volume of a right circular cone of height 9 cm is $48\pi \ cm^3$, find the diameter of its base.**

**Solution:**

Given height of a cone, h = 9 cm

Volume of the cone = 48

$\left(\frac{1}{3}\right) r^2 h = 48$

$\left(\frac{1}{3}\right) r^2 \times 9 = 48$

$3r^2 = 48$

$r^2 = \frac{48}{3} = 16$

$r = 4$

So diameter $= 2 \times radius$

$= 2 \times 4$

$= 8$ cm

$= 8$ cm

Hence the diameter of the cone is 8 cm.

**9. The height of a cone is 15cm. If its volume is 1570 $cm^2$, find the radius of the base. ( Use $\pi = 3.14$)**

**Solution:**

Given height of a cone, h = 15 cm

Volume of the cone = 1570 $cm^3$

$\left(\frac{1}{3}\right) r^2 h = 1570$

$\left(\frac{1}{3}\right) 3.14 \times r^2 \times 15 = 1570$

$5 \times 3.14 \times r^2 = 1570$

$r^2 = \frac{1570}{5} \times 3.14 = \frac{314}{3.14} = 100$

r = 10

**Hence the radius of the cone is 10 cm.**

**10. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost the white washing its curved surface area at the rate of Rs 210 per 100 $m^2$.**

**Solution:**

Given slant height of conical tomb, $l = 25m$

Base diameter , d = 14m

So, radius, r $= \dfrac{14}{2} = 7m$

Curved surface area $= rl$

$= \left(\dfrac{22}{7}\right) \times 7 \times 25 = 550m^2$

Hence the curved surface area of the cone is $550m^2$.

Rate of washing its curved surface area per $100m^2$ = Rs. 210

So total cost $= \left(\dfrac{550}{100} \times 210\right) = Rs.\,1155$

Hence the total cost of washing its curved surface area is Rs. 1155.


**11. A conical tent is 10m high and the radius of its base is 24m. Find :**

(i) slant height of the tent.

(ii) Cost of the canvas required to make the tent, if the cost of $1m^2$ canvas is Rs 70.

**Solution:**

(i)  Given height of the tent, h = 10m

Radius, r = 24m

We know that $l^2 = h^2 + r^2$

$l^2 = 10^2 + 24^2$

$l^2 = 100 + 576$

$l^2 = 676$

$l^2 = \sqrt{676}$

$l = 26$


(ii) Curved surface area $= rl$

$= \left(\frac{22}{7}\right) \times 24 \times 26 = \frac{13728}{7} m^2$

Cost of $1m^2$ $canvas = Rs.\,70$

Total cost $= \left(\frac{13728}{7}\right) \times 70$

= Rs. 137280

Hence the cost of the canvas required to make the tenet is Rs.137280.


**12. A Jocker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the cloth required to make 10 such caps.**

**Solution:**

**Given height of the cone, h = 24 cm**

**Radius, r = 7 cm**

**We know that**

$l^2 = h^2 + r^2$

$l^2 = 24^2 + 7^2$

$l^2 = 576 + 49$

$l^2 = 625$

$l = \sqrt{625}$

$l = 25$

Curved surface are $= rl$

$= \left(\frac{22}{7}\right) \times 7 \times 25$

$= 22 \times 25$

$= 550\ cm^2$

So the area of the cloth required to make  10 such caps

$= 10 \times 550 = 5500$

Hence the area of the cloth required to make 10 caps is 5500 $cm^2$.


**13. (a) The ratio of the base radii of two right circular cones of the same height is 3: 4. Find the ratio of their volumes.**

**(b) The ratio of the  heights of two right circular cones is 5 :2 and that of their base radii is 2 : 5. Find the ratio of their volumes.**

**(c) The height and the radius of the base of a right circular cone is half the corresponding height and radius of another bigger cone. Find :**

(i) The ratio of their volumes.

(ii) The ratio of their lateral surface areas.

**Solution:**

(a) Let $r_1$ and $r_2$ be the radius of the given cones and h be their height.

Ratio of radii, $r_1 : r_2 = 3 : 4$

Volume of cone, $V_1 = \left(\frac{1}{3}\right) r_1^2 h$

Volume of cone, $V_2 = \left(\frac{1}{3}\right) r_2^2 h$

$= \dfrac{r_1^2}{r_2^2}$

$= \dfrac{3^2}{4^2}$

$= \dfrac{9}{16}$

Hence the ratio of the volumes is 9 : 16.

(b) Let $h_1$ and $h_2$ be the heights of the given cones and $r_1$ and $r_2$ be their radii.

Ratio of heights, $h_1 : h_2 = 5 : 2$

Ratio of radii, $r_1 : r_2 = 2 : 5$

Volume of cone, $V_1 = \left(\frac{1}{3}\right) r_1^2 h_1$

Volume of cone, $V_2 = \left(\frac{1}{3}\right) r_2^2 h_2$

$\dfrac{V_1}{V_2} = \dfrac{\left(\frac{1}{3}\right) r_1^2 h_1}{\left(\frac{1}{3}\right) r_2^2 h_2}$

$= \dfrac{r_1^2 h_1}{r_2^2 h_2}$

$$= \frac{2^2 \times 5}{5^2 \times 2}$$

$$= \frac{4 \times 5}{25 \times 2}$$

$$= \frac{20}{50}$$

$$= \frac{2}{5}$$

Hence the ratio of the volumes is 2:5.

(c) Let r be the radius of bigger cone. Then the radius of smaller cone is $\frac{r}{2}$.

Let h be the height of bigger cone. Then the height of smaller cone is $\frac{h}{2}$.

(i) Volume of bigger cone, $V_1 = \left(\frac{1}{3}\right) r^2 h$

Volume of bigger cone, $V_2 = \left(\frac{1}{3}\right) \left(\frac{r}{2}\right)^2 \left(\frac{h}{2}\right) = \left(\frac{1}{3}\right) \frac{r^2 h}{8}$

Ratio of volume of smaller cone to bigger cone, $\frac{V_2}{V_1} = \frac{\frac{1}{3} r^2 \frac{h}{8}}{\frac{1}{3} r^2 h}$

$$= \left(\frac{1}{24}\right) r^2 h \times \left(\frac{3}{r^2 h}\right)$$

$$= \frac{1}{8}$$

Hence the ratio fo their volumes is 1 : 8.

(ii) Slant height of bigger cone $= \sqrt{h^2 + r^2}$

slant height of smaller cone $= \sqrt{\left(\frac{h}{2}\right)^2 + \left(\frac{r}{2}\right)^2}$

$$= \sqrt{\frac{h^2}{4} + \frac{r^2}{4}}$$

$$= \frac{1}{2}\sqrt{h^2 + r^2}$$

Curved surface area of bigger cone, $S_1 = rl$

$$= r\sqrt{h^2 + r^2}$$

Curved surface area of smaller cone, $S_2 = rl$

$$= x\left(\frac{r}{2}\right) \times \frac{1}{2}\sqrt{h^2 + r^2}$$

$$= \frac{1}{4}r\sqrt{h^2 + r^2}$$

Volume of smaller cone, $V_2 = \left(\frac{1}{3}\right)\left(\frac{r}{2}\right)^2\left(\frac{h}{2}\right)$

$$= \left(\frac{1}{3}\right)\frac{r^2 h}{8}$$

Ratio of volume of smaller cone to bigger cone,

$$\frac{V_2}{V_1} = \left(\frac{1}{3}\right)\frac{r^2 h}{8} \div \left(\frac{1}{3}\right)r^2 h$$

$$= \left(\frac{1}{24}\right)r^2 h \times \left(\frac{3}{r^2}h\right)$$

$$= \frac{1}{8}$$

Hence the ratio of their volumes is 1 : 8.

**(ii) slant height of bigger cone** $= \sqrt{h^2 + r^2}$

**slant height of smaller cone** $= \sqrt{\left(\left(\frac{h}{2}\right)^2 + \left(\frac{r}{2}\right)^2\right)}$

$= \sqrt{\left(\frac{h^2}{4} + \frac{r^2}{4}\right)}$

$= \frac{1}{2}\sqrt{(h^2 + r^2)}$

Curved surface area of bigger cone, $s_1 = rl$

$= r\sqrt{h^2 + r^2}$

Curved surface area of smaller cone, $s_2 = rl$

$= \left(\frac{r}{2}\right) \times \frac{1}{2}\sqrt{(h^2 + r^2)}$

$= \frac{1}{4}r\sqrt{(h^2 + r^2)}$

Ratio of curved surface area of smaller cone to bigger cone, $\frac{s_2}{s_1} = \frac{1}{4}$

$r\sqrt{(h^2 + r^2)} \div r\sqrt{(h^2 + r^2)}$

$= \frac{1}{4}r\sqrt{(h^2 + r^2)} \times \frac{1}{r(\sqrt{h^2 + r^2})}$

$= \frac{1}{4}$

Hence the ratio of curved surface area of smaller cone to bigger cone is 1 : 4.

**14. Find what length of canvas 2m in width is required to make a conical tent 20m in diameter and 42m in slant height allowing 10% for folds and the stitching. Also find the cost of the canvas at the rate of Rs 80 per metre.**

**Solution:**

Given diameter of the conical tent, d = 20 m

**radius,** $r = \dfrac{d}{2} = \dfrac{20}{2} = 10$ m

slant height, $l = 42m$

Curved surface area of the conical tent = rl

$= \left(\dfrac{22}{7}\right) \times 10 \times 42$

$= 22 \times 10 \times 6$

$= 1320 \; m^2$

So the area of canvas required is $1320 \; m^2$.

Since 10% of this area is used for folds and stitches, actual cloth needed
= 1320 + 10% of 1320

$= \; 1320 + 132$

$= 1452 \; m^2$

Width of the cloth = 2m

Length of the cloth $= \dfrac{Area}{width} = \dfrac{1452}{2} = 726$ m

Cost of canvas $= Rs. \, 80$ per metre.

Total cost $= \; 80 \times 726 =$ Rs, 58080

Hence the total cost of the canvas is Rs. 58080.

15. The perimeter of the base of a cone is 44cm and the slant height is 25 cm. Find the volume and the curved surface of the cone.

**Solution:**

Given perimeter of the base of a cone = 44 cm

$2r = 44$

$2 \times \frac{22}{7} \times r = 44$

$r = \frac{44 \times 7}{2 \times 22}$

$r = 7\ cm$

Slant height, $l = 25$

height, h $= \sqrt{(l^2 - r^2)}$

$h = \sqrt{25^2 - 7^2}$

$h = \sqrt{625 - 49}$

$h = \sqrt{576}$

$h = 24\ cm$

Volume of the cone, $V = \frac{1}{3}r^2h$

$V = \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 7^2 \times 24$

$V = \left(\frac{22}{7}\right) \times 49 \times 8$

$V = 22 \times 7 \times 8$

$V = 1232$

Hence the volume of the cone is $1232 cm^3$ Curved surface area of the cone $= rl$

$= \left(\frac{22}{7}\right) \times 7 \times 25$

$= 22 \times 25$

$= 550 \ cm^2$

Hence the curved surface area of the cone is $550 \ cm^2$.


**16. The volume of a right circular cone is $9856 cm^3$ and the area of its base is $616 \ cm^2$. Find**

(i) The slant height of the cone.

(ii) Total surface area of the cone.

**Solution:**
Given base area of the cone $= 616 \ cm^2$

$r^2 = 616$

$= \left(\frac{22}{7}\right) \times r^2 = 616$

$r^2 = 616 \times \frac{7}{22}$

$r^2 = 196$

$r = 14$

**Given volume of the cone $= 9856 \ cm^3$**

$\left(\frac{1}{3}\right) r^2 h = 9856$

$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 14^2 \times h = 9856$

$h = \frac{(9856 \times 3 \times 7)}{(22 \times 14^2)}$

$h = \frac{(9856 \times 3 \times 7)}{(22 \times 196)}$

$h = 48$

(i) Slant height, $l = \sqrt{(h^2 + r^2)}$

$l = \sqrt{48^2 + 14^2}$

$l = \sqrt{2304 + 196}$

$l = \sqrt{2500}$

$l = 50$

Hence the slant height of the cone is 50 cm.

(ii) Total surface area of the cone $= r(l + r)$

$= \left(\frac{22}{7}\right) \times 14 \times (50 + 14)$

$= 22 \times 2 \times 64$

$= 2816 \; cm^2$

Hence the total surface area of the cone is 2816 $cm^2$.

**17. A right triangle with sides 6 cm, 8 cm and 10 cm is revolved about the side 8 cm. Find the volume and the curved surface of the cone so formed. ( Take $\pi = 3.14$)**

**Solution:**
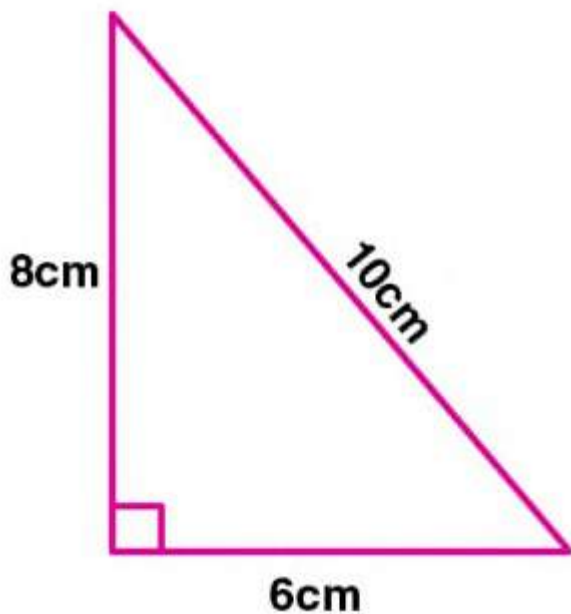


The triangle is rotated about the side 8 cm.
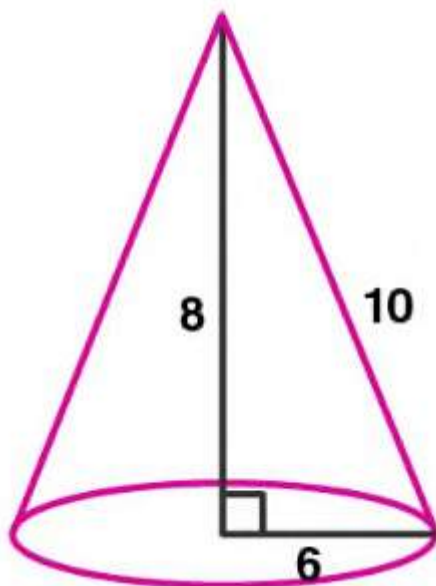
So the height of the resulting cone, $h = 8\ cm$

Radius, $r = 6\ cm$

Slant height, $l = 10\ cm$

Volume of the cone, $V = \left(\frac{1}{3}\right)r^2 h$

$V = \left(\frac{1}{3}\right) \times 3.14 \times 6^2 \times 8$

$V = \left(\frac{1}{3}\right) \times 3.14 \times 36 \times 8$

$V = 3.14 \times 12 \times 8$

$V = 301.44 cm^3$

Hence the volume of the cone is $301.44\ cm^3$.

Curved surface area of the cone $= rl$

$= 3.14 \times 6 \times 10$

$= 188.4$

Hence the curved surface area of the cone is $188.4\ cm^2$.


**18. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section cut ?**

**Solution:**

Given height of the cone, H = 30 cm

Let R be the radius of the given cone and r be radius of small cone.

Let h be the height of small cone.
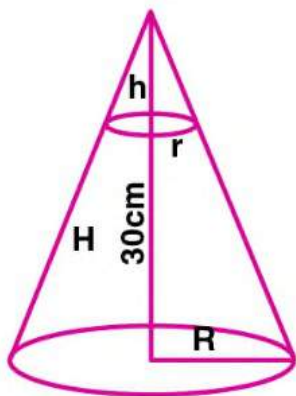
Volume of the given cone $= \left(\frac{1}{3}\right) R^2 H$

Volume of the small cone $= \frac{1}{27}$th of the volume of the given cone.

$$\left(\frac{1}{3}\right) r^2 h = \left(\frac{1}{27}\right) \times \left(\frac{1}{3}\right) R^2 H$$

Substitute H = 30

$$\left(\frac{1}{3}\right) r^2 h = \left(\frac{1}{27}\right) \times \left(\frac{1}{3}\right) R^2 \times 30$$

$$\frac{r^2 h}{R^2} = \frac{30}{27}$$

$$\frac{r^2 h}{R^2} = \frac{10}{9} \quad \ldots\ldots\ldots\text{(i)}$$

**From figure,** $\frac{r}{R} = \frac{h}{H}$

$$\frac{r}{R} = \frac{h}{30} \quad \ldots\ldots\text{(ii)}$$

**Substitute (ii) in (i)**

$$\left(\frac{h}{30}\right)^2 \times h = \frac{10}{9}$$

$$\frac{h^3}{900} = \frac{10}{9}$$

$$h^3 = 900 \times \frac{10}{9} = 1000$$

$Taking$ cube root on both sides.

$h = 10\ cm$

$H - h\ = 30 - 10 = 20$

The small cone is cut at a height of 20cm above the base.

**19. A semi- circular lamina of radius 35 cm is folded so that the two bounding radii are joined together to form a cone. Find**

**(i) The radius of the cone.**

**(ii) The ( lateral) surface area of the cone.)**

**Solution:**
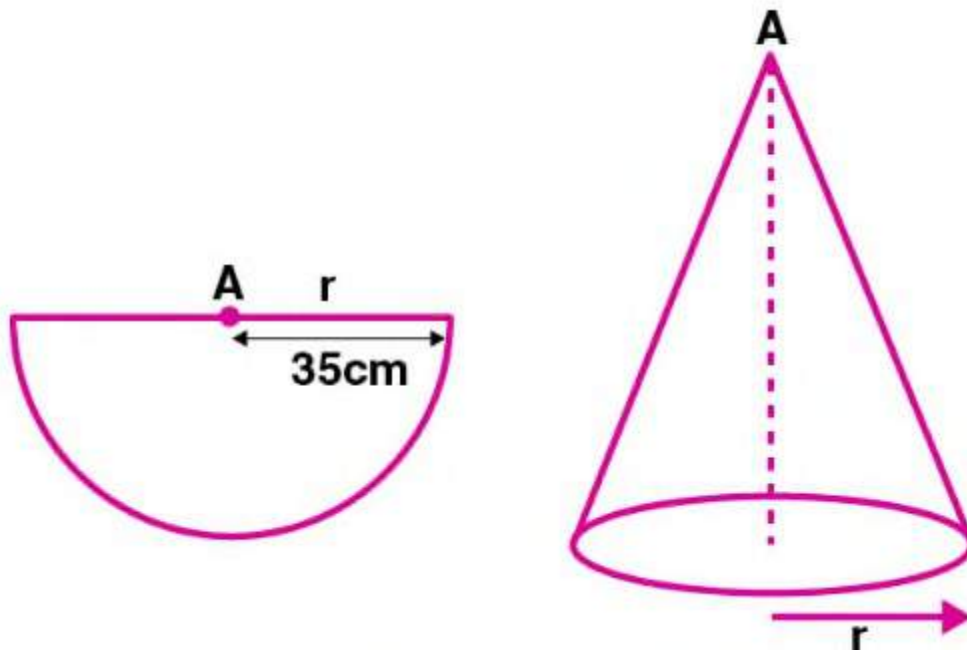
(i) Given radius of the sem circular lamina, $r = 35\ cm$

A cone is formed by folding it.

So the slant height of the cone, $l = 35\ cm$

Let $r_1$ be radius of cone.

Semicircular perimeter of lamina becomes the base of the cone.

$r = 2r_1$

$\boldsymbol{r = 2}\ r_1$

$35 = \boldsymbol{2}\ r_1$

$r_1 = \dfrac{35}{2}$

$= 17.5$ cm


Hence the  radius of the cone is 17.5 cm.


(ii) Curved surface area of the cone $= r_1 l$

$= \left(\dfrac{22}{7}\right) \times 17.5 \times 35$

$= \boldsymbol{22} \times 17.5\ \times 5$

$= 1925$ cm$^2$

Hence the lateral surface area of the cone is  1925 cm$^2$.

# Exercise 17.3

**1. Find the surface area of a sphere of radius :**

(i) 14 cm

(ii) 10.5 cm

**Solution:**

(i) Given radius of the spehere, $r = 14\ cm$

Surface area of the sphere $= 4r^2$

$= 4 \times \dfrac{22}{7} \times 14^2$

$= 4 \times 22 \times 14 \times 2$

$= 2464$

**Hence the surface area of the sphere is $2464 cm^2$.**


**(ii)** Given radius of the sphere, $r = 10.5\ cm$

Surface area of the sphere $= 4r^2$

$= 4 \times \left(\dfrac{22}{7}\right) \times 10.5^2$

$= 1386\ cm^2$

Hence the surface area of the sphere is $1386\ cm^2$.

**2. Find the volume of a sphere of radius:**

**(i) 0.63 m**

**(ii) 11.2 cm**

**Solution:**

(i) Given radius of the sphere, r = 0.63 m

Volume of the sphere, $V = \left(\frac{4}{3}\right)r^3$

$= \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 0.63^3$

$= 1.047 \ m^3$

$= 1.05 \ m^3$ (approx)

Hence the volume of the sphere is $1.05 m^3$ .


**(ii)** Given radius of the sphere, $r = 11.2 \ cm$

Volume of the sphere, $V = \left(\frac{4}{3}\right)r^3$

$= \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 11.2^3$

$= 5887.317 \ cm^3$

$= \mathbf{5887.32} \ cm^3$ (approx)

Hence the volume of the sphere is $5887.32 \ cm^3$.

**3. Find the surface area of a sphere of diameter :**

**(i) 21 cm**

**(ii) 3.5 cm**

**Solution:**

**(i)** Given diameter of the sphere, d = 21 cm

Radius, $r = \dfrac{d}{2} = \dfrac{21}{2} = 10.5$

Surface area of the sphere = $4r^2$

$= 4 \times \left(\dfrac{22}{7}\right) \times 10.5^2$

$= 1386 \; cm^2$

**Hence the surface area of the sphere is 1386 $cm^2$.**

**(ii) Given diamter of the sphere, $d = 3.5 \; cm$**

Radius, $r = \dfrac{d}{2} = \dfrac{3.5}{2} = 1.75$

Surface area of the sphere = $4r^2$

$= 4 \times \left(\dfrac{22}{7}\right) \times 1.75^2$

$= 38.5$

Hence the surface area of the sphere is 38.5 cm$^2$.

**4.  A shot-put is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g per $cm^3$, find the mass of the shot-put.**

**Solution:**

Given radius of the metallic sphere , r = 49 cm

Volume of the sphere, $V = \left(\frac{4}{3}\right) r^3$

$V = \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 4.9^3$

V = 493.005

$V = 493 \; cm^3$ ( approx)

Given density = 7.8g per $cm^3$

$\text{Density} = \frac{Mass}{Volume}$

Mass = Density  Volume

= b

= 7.8×493

= 3845.4 g

Hence the mass of the shot put is 3845.4 g.

**5. Find the diameter of a sphere whose surface area is 154 c$cm^2$**

**Solution :** Given surface area of the spherre $= 154 cm^2$

Surface area of the sphere $= 4r^2$

$= 4 \times \left(\frac{22}{7}\right) \times r^2 = 154$

$r^2 = \dfrac{154 \times 7}{(22 \times 4)}$

$r^2 = \dfrac{49}{4}$

$= r = \sqrt{\dfrac{49}{2}}$

Diameter $= 2 \times r = 2 \times \sqrt{49 \times 2} = \sqrt{49} = 7$

Hence the diameter of the sphere is 7 cm.

**6. Find:**

(i) The curved surface area.

(ii) The total surface area of a hemisphere of radius 21 cm.

**Solution:**

(i) Give radius of the hemisphere, $r = 21 \ cm$

Curved surface area of the hemisphere $= 2r^2$

$= 2 \times \left(\frac{22}{7}\right) \times 21^2$

$= 2 \times 22 \times 3 \times 21$

$= 2772 \ cm^2$

Hence the curved surface area of the hemisphere is $2772 \ cm^2$.

**(iii) Total surface area of the hemisphere = $3r^2$**

$= 3 \times \left(\frac{22}{7}\right) \times 21^2$

$= 3 \times 22 \times 3 \times 21$

$= 4158 \ cm^2$

Hence the total surface area of the hemisphere is $4158 \ cm^2$.

**7. A hemispherical brass bowl has inner – diameter 10.5cm. Find the cost of tin-plating it on the inside at the rate of Rs 16 per 100 $cm^2$ .**

**Solution :**

Given inner diameter of the brass bowl, d = 10.5 cm

Radius, r $= \frac{d}{2} = \frac{10.5}{2} = 5.25$ cm

Curved surface area of the bowl $= 2r^2$

$= 2 \times \left(\frac{22}{7}\right) \times 5.25^2$

$= 173.25 \ cm^2$

Rate of tine plating = Rs. 16 per 100 $cm^2$

So total cost $= 173.25 \times \frac{16}{100} = 27.72$

Hence the cost of tin plating the bowl on the inside is Rs. 27.72.

**8. The radius of a spherical balloon increases from 7 cm to 14 cm as air is jumped into it. Find the ratio of the surface areas of the balloon in two cases.**

**Solution:**

Given radius of the spherical balloon, r = 7 cm

Radius of the spherical balloon after air is pumped, R = 14 cm

Surface area of the sphere = $4r^2$

Ratio of surface areas of the balloons = $\dfrac{4r^2}{4R^2}$

$= \dfrac{r^2}{R^2}$

$= \dfrac{7^2}{14^2}$

$= \dfrac{1}{4}$

**Hence the ratio of the surface areas of the sphere is 1:4.**


**9. A sphere and a cube have the same surface area. Show that the ratio of the volume of the sphere to that of the cube is $\sqrt{6} : \sqrt{\pi}$**

**Solution:**

Let r be the radius of the sphere and a bee the side of the cube.

Surface area of sphere = $4r^2$

Surface area of cube = $6a^2$

Given sphere and cube has same surface area.

$4r^2 = 6a^2$

$\dfrac{r^2}{a^2} = \dfrac{6}{4}$

$$\therefore \frac{r}{a} = \frac{\sqrt{6}}{2\sqrt{\pi}}$$

Volume of the sphere, $V_1 = \left(\frac{4}{3}\right)r^3$

Volume of the cube, $V_2 = a^3$

$$\therefore \frac{V_1}{V_2} = \frac{4\pi r^3}{3a^3}$$

$$\therefore \frac{V_1}{V_2} = \frac{4\pi\sqrt{6}^3}{3\times(2\sqrt{\pi})^3}$$

$$\therefore \frac{V_1}{V_2} = \frac{4\pi\times\sqrt[6]{6}}{3\times8\pi\times\sqrt{\pi}}$$

$$\therefore \frac{V_1}{V_2} = \frac{\sqrt{6}}{\sqrt{\pi}}$$

**Hence Proved.**


**10. (a) If the ratio of the radii of two sphere is  3:7, find :**

(i) The ratio of their volumes.

(ii) The ratio of their surface areas.


(b) If the ratio of the volumes of the two sphere is 125 : 64, find the ratio of their surface areas.

**Solution:**

(i) Let the radii of two spheres be  $r_1$ and $r_2$.
Given ratio of their radii $= 3 : 7$

Volume of sphere $= \left(\frac{4}{3}\right) r^3$

Ratio of the volumes $= \dfrac{\left(\frac{4}{3}\right) r_1{}^3}{\left(\frac{4}{3}\right) r_2{}^3}$

$= \dfrac{r_1{}^3}{r_2{}^3}$

$= \dfrac{3^3}{7^3}$

$= \dfrac{27}{343}$

Hence the ratio of their volumes is

Hence the ratio of their volumes is 27 : 343.

(ii) Surface area of a sphere $= 4r^2$.

**Ratio of surface areas of the spheres** $= \dfrac{4r_1^2}{4r_2^2}$

$= \dfrac{r_1^2}{r_2^2}$

$= \dfrac{3^2}{7^2}$

$= \dfrac{9}{49}$

Hence the ratio of the surface areas is 9 : 49.

**(b) Given ratio of volume of two spheres** $= \dfrac{125}{64}$

$= \dfrac{\left(\frac{4}{3}\right) r_1^3}{\left(\frac{4}{3}\right) r_2^3}$

$$= \frac{125}{64}$$

Taking cube root on both sides

$$\frac{r_1}{r_2} = \frac{5}{4}$$

**Ratio of surface areas of the spheres** $= \frac{4r_1^2}{4r_2^2}$

$$= \frac{r_1^2}{r_2^2}$$

$$= \frac{5^2}{4^2}$$

$$= \frac{25}{16}$$

Hence the ratio of the surface areas is 25 : 16.

**11. A cube of side 4 cm contains a sphere touching its sides. Find the volume of the gap in between.**

**Solution:**

Given side of the cube, $a = 4cm$

Volume of the cube $= a^3$

$$= 4^3$$

$$= 4 \times 4 \times 4$$

$$= 64 \ cm^3$$

Diameter of the sphere $= 4$ cm

So radius of the sphere, r $= \frac{d}{2} = \frac{4}{2} = 2$ cm

Volume of the sphere $= \left(\frac{4}{3}\right) r^3$

$$= \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 2^3$$

$= 33.523$

$= 33.52 \; cm^3 \; (\text{approx})$

Volume of the gap in between $= 64 - 33.52$

$= 30.48$

$= 30.5 \; cm^3 \; (\text{approx})$

Hence the volume of the gap between the cube and sphere is $30.5 \; cm^3$ .


**12. Find the volume of a sphere whose surface area is 154 $cm^3$**

**Solution :**

Given surface area of the sphere $= 154 \; cm^2$

$4r^2 = 154$

$4 \times \left(\frac{22}{7}\right) \times r^2 = 154$

$r^2 = \frac{(154 \times 7)}{(4 \times 22)}$

$r^2 = \frac{49}{4}$

$r = \frac{7}{2}$

Volume of the sphere $= \left(\frac{4}{3}\right) r^3$

$$= \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times \left(\frac{7}{2}\right)^3$$

$= \frac{539}{3}$

$= 179.666$

$= 179.67 \ cm^3$ ( approx)

Hence the volume of the sphere is $179.67 \ cm^3$.

**13. If the volume of a sphere is $179\frac{2}{3}$. Find its radius and surface area.**

**Solution :**

Given volume of the sphere is $179\frac{2}{3}$.

$$\left(\frac{4}{3}\right)r^3 = \frac{539}{3}$$

$$r^3 = \frac{(539 \times 3 \times 7)}{(4 \times 22 \times 3)}$$

$$r^3 = 49 \times \frac{7}{8}$$

$$r^3 = \frac{7 \times 7 \times 7}{2 \times 2 \times 2}$$

**Taking cube root on both sides, we get**

$$r = \frac{7}{2} = 3.5 \ cm$$

Surface area of the sphere $= 4r^2$

$$= \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times \left(\frac{7}{2}\right)^2$$

$$= 22 \times 7$$

$$= 154 \ cm^2$$

Hence the radius and the surface area of the sphere is 3.5 cm and 154 $cm^2$ respectively.

**14. A hemispherical bowl has a radius 3.5cm. What would be the volume of water it would contain ?**

**Solution:**

Given radius of the hemispherical bowl, r = 3.5 cm = $\frac{7}{2}$ cm

Volume of the hemisphere = $\left(\frac{2}{3}\right) r^3$

$= \left(\frac{2}{3}\right) \times \left(\frac{22}{7}\right) \times \left(\frac{7}{2}\right)^3$

$= 11 \times \frac{49}{6}$

$= \frac{539}{6}$

$= 89 \frac{5}{6} \, cm^3$

Hence the volume of the hemispherical bowl is $89 \frac{5}{6} \, cm^3$.


**15. The water for a factory is stored in a hemispherical tank whose internal diameter is 14m. The tank contains 50 kilolitres of water. Water is pumped into the tank to fill to its capacity. Find the volume of water pumped into the tank.**

**Solution:**

Given internal diameter of the hemispherical tank, d = 14m

So radius, r = $\frac{14}{2}$ = 7m

Volume of the tank = $\left(\frac{2}{3}\right) r^3$

$= 718.667 \, m^3$

$= 718.67 \ m^3$ ( approx)

$= 718.67$ kilolitre

Quantity of water in tank $= 50$ kilolitre

Amount of water to be pumped into the tank

$= 718.67$ - $50$ $= 668.67$ kilolitre.

Hence the volume of water pumped into the tank is 668.67 kilolitre.


**16. The surface area of a solid sphere is 1256 $cm^2$. It is cut into two hemispheres. Find the total surface area and the volume of a hemisphere. Take $\pi = 3.14$.**

**Solution:**

Given surface area of the sphere $= 1256 \ cm^2$

$4r^2 = 1256$

$4 \times 3.14 \times r^2 = 1256$

$r^2 = \dfrac{1256}{3.14 \times 4}$

$r^2 = 100$

$r = 10 \ cm$

Total surface area of the hemisphere $= 3r^2$

$= 3 \times 3.14 \times 10^2$

$= 3 \times 3.14 \times 100$

$= 942 \ cm^2$

Hence the total surface area of the hemisphere is $942 \ cm^2$.

Volume of the hemisphere $= \left(\dfrac{2}{3}\right) r^3$

$$= \left(\frac{2}{3}\right) \times 3.14 \times 10^3$$

$$= \left(\frac{2}{3}\right) \times 3140$$

$$= \frac{6280}{3}$$

$$= 2093\frac{1}{3} \ cm^3$$

Hence the volume of the hemisphere is $2093\frac{1}{3} \ cm^3$.

**17. Write whether the following statements are true or false. Jutify your answer :**

**(i) The volume of a sphere is equal to two-third of the volume of a cylinder whose height and diameter are equal to the diameter of the sphere.**

**(ii) The volume of the largest right circular cone that can be fitted in a cube whose edge is 2r equals the volume of a hemisphere of radius r.**

**(iii) A cone, a hemisphere and a cylinder stand on equal bases and have the height. The ratio of their volumes is 1 : 2 : 3.**

**Solution:**

(i) Let the radius of sphere be r.

Then height of the cylider, h = $2r$.

Radius of cylinder = r

Volume of cyliner = $r^2 h$.
= $r^2 \times 2r$

= $2r^3$

Volume of spher $= \left(\frac{4}{3}\right) r^3$

$= \left(\frac{2}{3}\right) \times 2r^3$

$= \left(\frac{2}{3}\right) \times Volume\ of\ cylinder$

Hence the given statement is true.

(ii) Let the edge of the cube is 2r.

So radius of cone = r

Height of cone, h = 2r

Volume of cone $= \left(\frac{1}{3}\right) r^2 h.$

$= \left(\frac{1}{3}\right) r^2 \times 2r$

$= \left(\frac{2}{3}\right) r^3$

= Volume of a hemisphere of radius r

Hene the given statement is true.

(iii) Let r be radius of cone, hemisphere and cylinder.

So the height of the cone = r

Height of cylinder = r

Volume of cone $= \left(\frac{1}{3}\right) r^2 h$

$= \left(\frac{1}{3}\right) r^3$

Volume of hemisphere $= \left(\frac{2}{3}\right) r^3$

Volume of cylinder $= r^2h$

$= r^3$

Ratio of volume of cone, hemisphere and cylinder $= \left(\frac{1}{3}\right)r^3 : \left(\frac{2}{3}\right)r^3 : r^3$

$= \frac{1}{3} : \frac{2}{3} : 1$

$= 1 : 2 : 3$

Hence the given statement is true.

# Exercise 17.4

**Take $\pi = \dfrac{22}{7}$ unless stated otherwise.**

1. The adjoining figure shows a cuboidal block of wood through which a circular cylindrical hole of the biggest size is drilled. Find the volume of the wood left in the block.
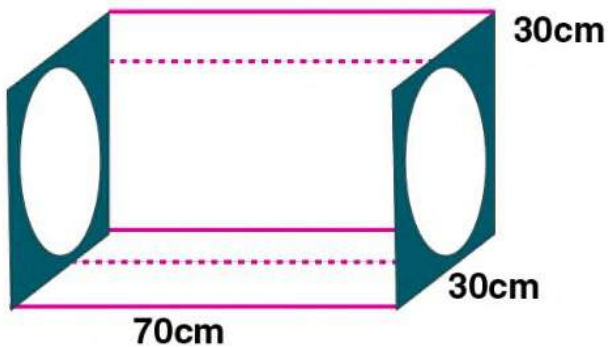


**Solution:**

Given diameter of the hold, d = 30 cm

Radius of the hole, r = $\dfrac{d}{2}$ = $\dfrac{30}{2}$ = 15 cm

Height of the cylindrical hole, h = 70 cm

Volume of the cuboidal block = $lbh$

= 70 × 30 × 30

= 63000 $cm^3$

Volume of cylindrical hole = $r^2h$

= $\left(\dfrac{22}{7}\right) \times 15^2 \times 70$

$= 22 \times 225 \times 10$

$= 49500 \ cm^3$

Volume of the wood left in the block $= 63000 - 49500 = 13500 \ cm^3$

Hence the volume of the wood left in the block is $13500 \ cm^3$.

**2. The given figure shows aa solid trophy made of shining glass. If one cubic centimetre of glass costs Rs. 0.75, Find the cost of the glass for making the trophy.**
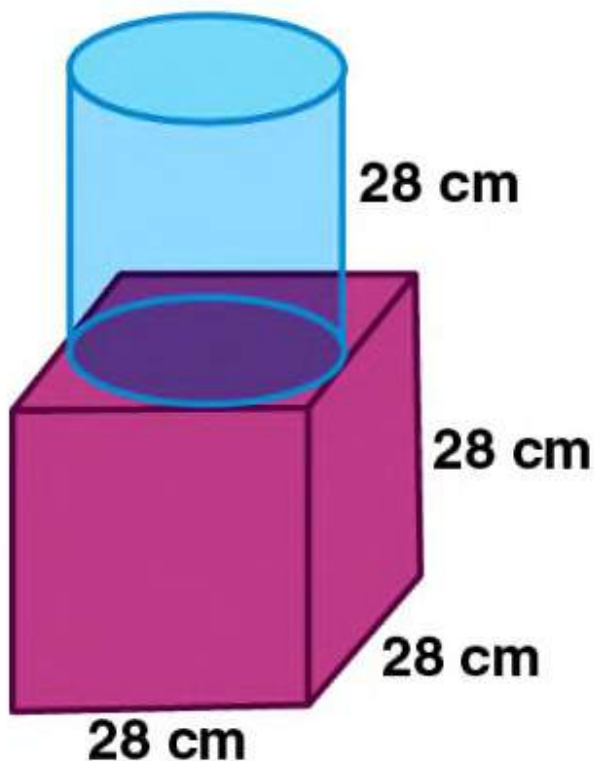


**Solution:**

Given side of the cube, a = 28cm

Radius of the cylinder, $r = \dfrac{28}{2} = 14$ cm

Height of the cylinder, h = 28cm

Volume of the cube = $a^3$

$= 28^3$

$= 28 \times 28 \times 28$

$= 21952 \ cm^3$

Volume of the cylinder = $r^2 h$

$= \left(\dfrac{22}{7}\right) \times 14^2 \times 28$

$= 22 \times 2 \ \times 14 \times 28$

$= 17248 \ cm^3$

Total volume of the solid = Volume of the cube + Volume of the cylinder

$= 21952 + 17248$

$= 39200 \ cm^3$

Cost of 1 $cm^3$ glass = Rs. 0.75

Total cost of glass = 39200 × 0.75

= Rs. 29400

Hence the cost of making the trophy is Rs. 29400.


**3. From a cube of edge 14cm, a cone of maximum size is carved out. Find the volume of the remaining material.**

**Solution:**
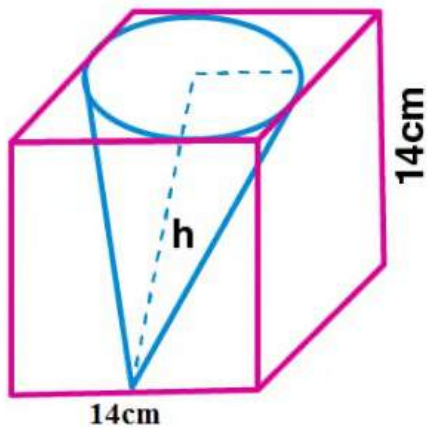
Given edge of the cube, a = 14 cm

Radius of the cone, r = $\frac{14}{2}$ = 7cm

Height of the cone, h = 14 cm

Volume of the cube = $a^3$

= $14^3$

= 14 × 14 × 14

= 2744 $cm^3$

Volume of the cone = $\left(\frac{1}{3}\right) r^2 h$

= $\left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 7^2 \times 14$

= 22 × 7 × $\frac{14}{3}$

= $\frac{2156}{3} cm^3$

Volume of the remaining meterial = Volume of the cube – Volume of the cone

$$= 2744 - \frac{2156}{3}$$

$$= \frac{(3 \times 2744 - 2156)}{3}$$

$$= \frac{(8232 - 2156)}{3}$$

$$= \frac{6076}{3}$$

$$= 2025\frac{1}{3} \; cm^3$$

Hence the volume of the remaining material is $2025\frac{1}{3} \; cm^3$.

4. A cone of maximum volume is curved out of a block of wood of size 20 cm $\times 10 \; cm \times 10 \; cm$. Find the volume of the remaining wood.

**Solution:**
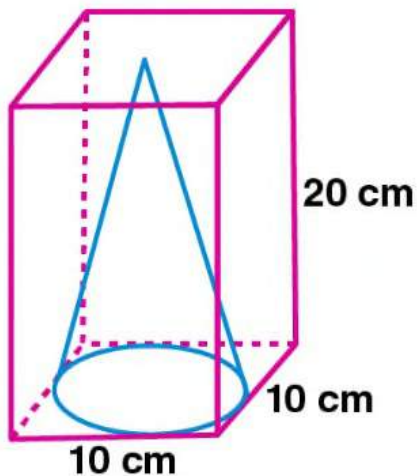


**Given**

dimensions of the block of wood = 20 cm $\times 10 \; cm \times 10 \; cm$

Volume of the block of wood = 20 $\times 10 \times 10 = 2000 cm^3$

[Volume $= lbh$]

Diameter of the cone, $d = 10cm$

Radius of the cone, $r = \frac{d}{2} = \frac{10}{2} = 5$cm

Height of the cone, $h = 20$ cm

Volume of the cone $= \left(\frac{1}{3}\right)r^2h$

$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 5^2 \times 20$

$= \frac{(22 \times 25 \times 20)}{3 \times 7}$

$= \frac{11000}{21} cm^3$

Volume of the remaining wood = Volume of block of wood – Volume of cone

$= 2000 - \frac{11000}{21}$

$= \frac{(21 \times 2000 - 11000)}{21}$

$= \frac{(42000 - 11000)}{21}$

$= \frac{31000}{21}$

$= 1476.19 \ cm^3$

Hence the volume of the remaining wood is $1476.19 \ cm^3$.


**5. 16 glass spheres each of radius 2cm are packed in a cuboidal box of internal dimensions 16 cm $\times$ 8 cm $\times$ 8 $cm$ and then the box is filled with water. Find the volume of the water filled in the box.**

**Solution :**

Given dimensions of the box = 16 cm × 8 cm × 8 cm

So volume of the box = $lbh$ = 16 × 8 × 8 = 1024$cm^3$

Radius of the glass sphere, $r = 2cm$

Volume of the sphere = $\left(\frac{4}{3}\right) r^3$

$= \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 2^3$

$= \frac{4 \times 22 \times 8}{(3 \times 7)}$

$= \frac{704}{21}$

Volume of 16 spheres = $\frac{16 \times 704}{21} = \frac{11264}{21}$ =536.38 $cm^3$

Volume of water filled in the box = Volume of the box – Volume of 16 spheres

= 1024 – 536.28

= 487.62 $cm^3$

Hence the volume of the water filled in the box is 487.62 $cm^3$.


**6. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depreesion is 0.5 cm and the depth is 1.4 cm. Find the volume of the wood in the entire stand, correct to 2 decimal places.**
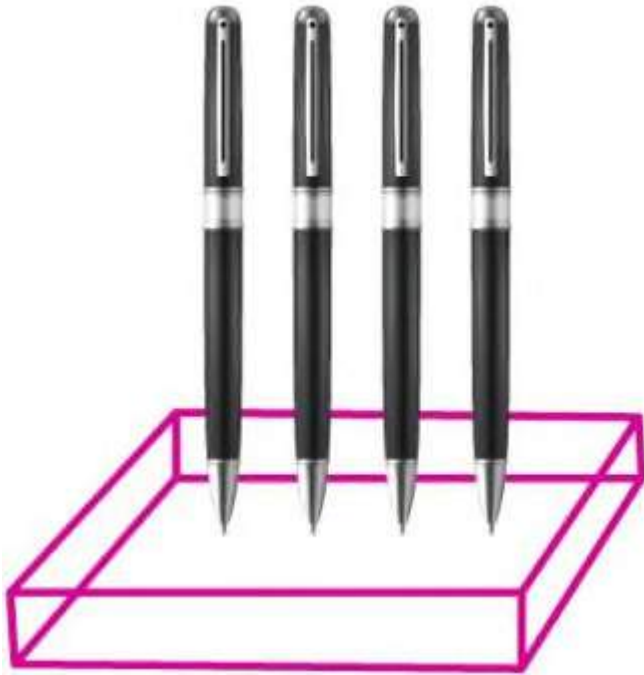
**Solution :**

Dimensions of the cuboid = 15 cm× 10 cm × 3.5 cm

Volume of the cuboid = $15 \times 10 \times 3.5 = 525 cm^3$

Radius of each depression, r = 0.5 cm

Depth, h = 1.4 cm

Volume of conical depression = $\left(\frac{1}{3}\right) r^2 h$

$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 0.5^2 \times 1.4$

$= 22 \times 0.25 \times \frac{1.4}{21}$

$= \frac{7.7}{21} cm^3$

Volume of 4 such conical depressions = $\frac{4 \times 7.7}{21}$

$= 1.467 \ cm^3$

Volume of wood in the stand = Volume of the cuboid – volume of 4 conical depressions

$= 525 - 1.467$

$= 523.533$

$= 523.53 \ cm^3$

Hence the volume of the wood in the stand is $523.53 \ cm^3$.

**7. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter that the hemisphere can have ? Also, find the surface area of the solid.**

**Solution:**
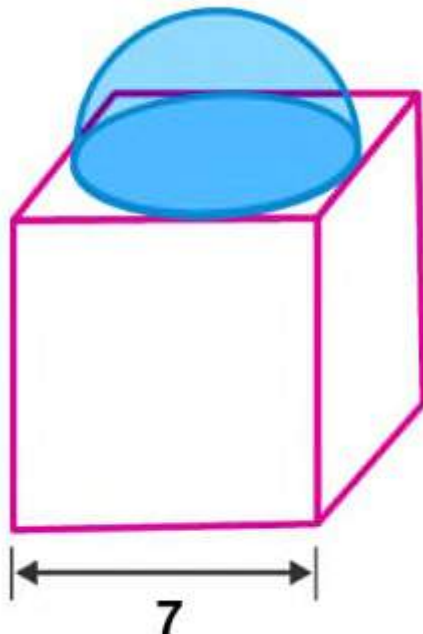
Given edge of the cube, $a = 7cm$

Diameter of the hemisphere, d = 7 cm

Radius, r $= \dfrac{d}{2} = \dfrac{7}{2} = 3.5$ cm

Surface area of the hemisphere $= 2r^2$

$= 2 \times \left(\dfrac{22}{7}\right) \times 3.5^2$

$= 44 \times \dfrac{12.25}{7}$

$= \dfrac{539}{7}$

$= 77 cm^2$

Surface area of the cube $= 6a^2$

$= 6 \times 7^2$

$= 6 \times 49$

$= 294 \ cm^2$

Surface area of base of hemisphere $= r^2$

$= \left(\dfrac{22}{7}\right) \times 3.5^2$

$= 22 \times \dfrac{12.25}{7}$

$= 38.5 \ cm^2$

hemisphere $-$ surface area of the base of hemisphere

$= 294 + 77 - 38.5$

$= 332.5 \ cm^2$

**8. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder (as shown in the given figure). If the height of the cylinder is 10 cm and its base is of radius 3.5cm, find the total surface area of the article.**
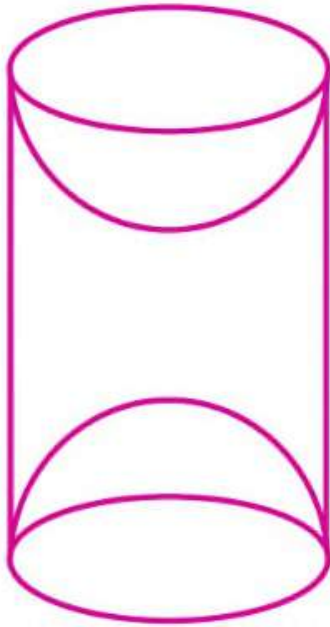


**Solution:**

Given height of the cylinder, h = 10cm

Radius of the cylinder, r = 3.5 cm

Radius of the hemisphere = 3.5 cm

Total surface area of the article = curved surface area of the cylinder + curved surface area of 2 hemispheres

$= 2rh + 2 \times 2r^2$

$= 2r(h + 2r)$

$= 2 \times \left(\frac{22}{7}\right) \times 3.5 \times (10 + 2 \times 3.5)$

$$= \left(\frac{154}{7}\right) \times (10 + 7)$$

$$= 22 \times 17$$

$$= 374 \ cm^2$$

Hence the total surface area of the article is $374 \ cm^2$.

**9. A toy is in the form of a cone a radius 3.5 cm mounted on a hemisphere of same radius. If the total height of the toy is 15.5 cm, find the total surface area of the toy.**

**Solution:**
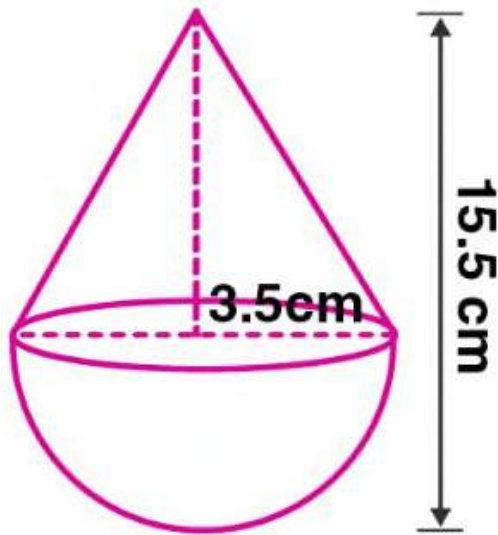


Given radius of the cone, r = 3.5 $cm$

Radius of hemisphere, r = 3.4 cm

Total height of the toy = 15.5 cm

Height of the cone = 15.5 -3.5 = 12 cm

Slant height of the cone, $l = \sqrt{h^2 + r^2}$

$$l = \sqrt{12^2 + 3.5^2}$$

$$l = \sqrt{144 + 12.25}$$

$$l = \sqrt{156.25}$$

$$l = 12.5 \ cm$$

**Total surface area of the toy = curved surface area of cone + curved surface area of the hemisphere**

$$= rl + 2r^2$$

$$= r(l + 2r)$$

$$= \left(\frac{22}{7}\right) \times 3.5 \times (12.5 + 2 \times 3.5)$$
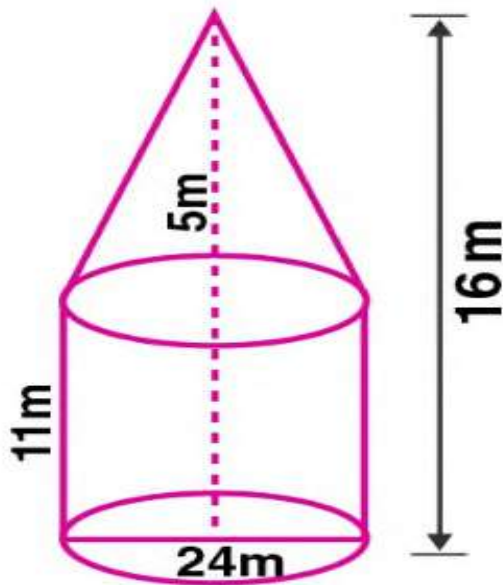
$$= \left(\frac{77}{7}\right) \times (\mathbf{12.5 + 7})$$

$$= 11 \times 9.5$$

$$= 214.5 \ cm^2$$

Hence the total surface area of the toy is $214.5 cm^2$

**10.** A circus tent is in the shape of a cylinder surmounted by a cone. The diameter of the cylindrical portion is 24m and its height is 11m. If the vertex of the cone is 16m above the ground, find the area of the canvas used to make the tent.

**Solution:**

Given diameter of the cylindrical part of tent, d = 24m

Radius, r $=\frac{d}{2}=\frac{24}{2}=$ 12m

Height of the cylindrical part, H = 11m

Since vertex of cone is 16m above the ground, height of cone,

h = 16 − 11

h = 5m

Slant height of the cone, $l = \sqrt{(h^2 + r^2)}$

$l = \sqrt{(5^2 + 12^2)}$

$l = \sqrt{25 + 144}$

$l = \sqrt{169}$

$l = 13m$

Radius of cone, r = 12m

Area of canvas used to make the tent = curved surface area of the cylindrical part + curved surface area of the cone.

Area of canvas used to make the tent = $2rH + rl$

$= r(2H + l)$

$= \left(\dfrac{22}{7}\right) \times 12 \times (2 \times 11 + 13)$

$= \left(\dfrac{264}{7}\right) \times (22 + 13)$

$= \left(\dfrac{264}{7}\right) \times 35$

$= 264 \times 5$

$= 1320 \ m^2$

Hence the area of canvas used to make the tent is $1320 \ m^2$.


**11.** An exhibition tent is in the form of a cylinder surmounted by a cone. The height of the tent above the ground is 85m and the height of the cylindrical part is 50m. If the diameter of the base is 168 m, find the quantity of canvas required to make the tent. Allow 20% extra for folds and stitching. Give your answer to the nearest $m^2$.

**Solution:**
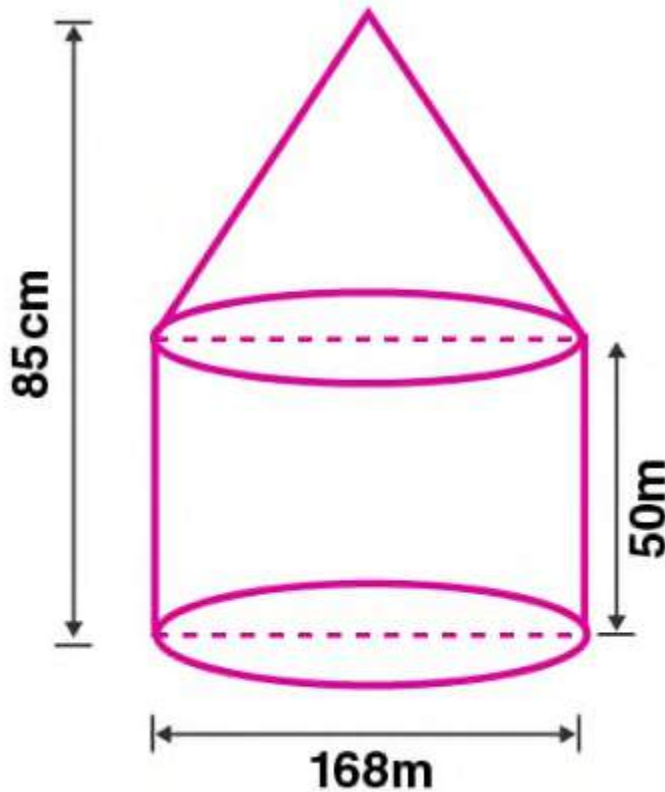
Given height of the tent above the ground = 85m

Height of the cylindrical part, H = 50 m

height of the cone, h = 85 – 50

h = 35 m

Diameter of the base, d = 168m

Radius of the base of cylindrical part, r = $\frac{d}{2} = \frac{168}{2}$ = 84m

Radius of cone, r = 84m

Slant height of the cone, $l = \sqrt{(h^2 + r^2)}$

$l = \sqrt{35^2 + 84^2}$

$l = \sqrt{1225 + 7056}$

$l = \sqrt{1225 + 7056}$

$l = \sqrt{8281}$

$l = 91m$

Surface area of tent = curved surface area of cylinder + curved surface area of cone

$= 2rH + rl$

$= r(2H + l)$

$= \left(\frac{22}{7}\right) \times 84 \times (2 \times 50 + 91)$

$= \left(\frac{22}{7}\right) \times 84 \times (100 + 91)$

$= \left(\frac{(22 \times 81)}{7}\right) \times 191$

$= \left(\frac{1848}{7}\right) \times 191$

$= 264 \times 191$

$= 50424 \ m^2$

Adding 20% extra for folds and stitches,

Area of canvas = 50424 + 20% of 50424

$= 50424 + \left(\frac{20}{100}\right) \times 50424$

$= 50424 + 0.2 \times 50424$

$= 50424 + 10084.8$

$= 60508.8 \ m^2$

$= 60509 \ m^2$

Hence the quantity of canvas required to make the tent is $60509 m^2$.

**12. From a solid cylinder of height 30cm and radius 7 cm, a conical cavity of height 24cm and of base radius 7 cm is drilled out. Find the volume and the total surface of the remaining solid.**

**Solution:**
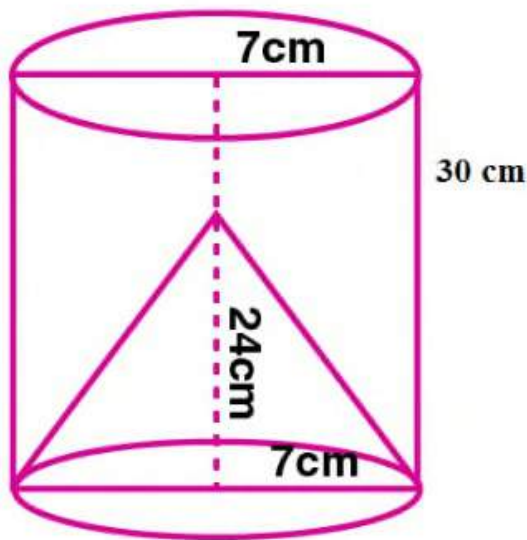


Given height of the cylinder, H = 30 cm

Radius of the cylinder, r = 7cm

Height of cone, h = 24 cm

Radius of cone, r = 7cm

Slant height of the cone, $l = \sqrt{h^2 + r^2}$

$l = \sqrt{24^2 + 7^2}$

$l = \sqrt{576 + 49}$

$l = \sqrt{625}$

$l = 25 \; cm$

Volume of the remaining solid = Volume of the cylinder − Volume of the cone

$$= r^2 H - \left(\frac{1}{3}\right) r^2 h$$

$$= r^2 \left(H - \frac{h}{3}\right)$$

$$= \left(\frac{22}{7}\right) \times 7^2 \times \left(30 - \frac{24}{3}\right)$$

$$= (22 \times 7) \times (30 - 8)$$

$$= (154) \times (22)$$

$$= 3388 \; cm^3$$

Volume of the remaining solid is $3388 \; cm^3$.

Total surface area of the remaining solid = Curved surface area of cylinder + Surface area of top of the cylinder + Curved surface area of the cone

Total surface area of the remaining solid $= 2rH + r^2 + rl$

$$= r(2H + r + l)$$

$$= \left(\frac{22}{7}\right) \times 7(2 \times 30 + 7 + 25)$$

$$= 22 \times (60 + 32)$$

$$= 22 \times 92$$

$$= 2024 \; cm^2$$

Hence the total surface area of the remaining solid is 2024 cm².

**13.** The adjoining figure shows a wooden toy rocket which is in the shape of a circular cone mounted on a circular cylinder. The total height of the rocket is 26cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3cm. If the conical portion is to be painted green and the cylindrical portion red, find the area of the rocket painted with each of these colours.

Also, find the volume of the wood in the rocket. Use $\pi$ = 3.14 and give answers correct to 2 decimal places.
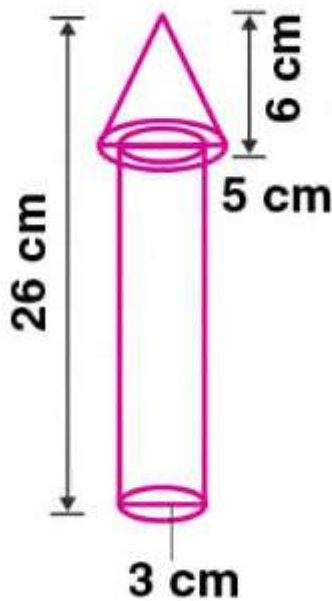


**Solution:**
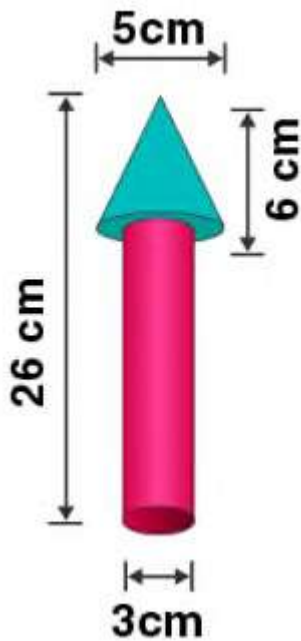
i) Given height of the rocket = 26 cm

Height of the cone, H = 6 cm

Height of the cylinder, h = 26 – 6= 20 cm

Diameter of the cone = 5cm

Radius of the cone, R = $\frac{5}{2}$ = 2.5 cm

Diameter of the cylinder = 3 cm

Radius of the cylinder, r = $\frac{3}{2}$ =1.5 cm

Slant height of cone, $l = \sqrt{h^2 + r^2}$

$l = \sqrt{6^2 + 2.5^2}$

$l = \sqrt{36 + 6.25}$

$l = \sqrt{42.25}$

$l$ = 6.5 cm

Curved surface area of the cone = R$l$

= 3.14×2.5×6.5

= 51.025 $cm^2$

Base area of cone = $R^2$

= 3.14 × $2.5^2$

= 3.14 × 6.25

= 19.625 $cm^2$

Curved surface area of the cylinder = 2$rh$

= 2 × 3.14 × 1.5 × 20

= 188.4 $cm^2$

Base area of cylinder = $r^2$

= 3.14 × $1.5^2$

= 3.14 × 2.25

= 7.065 $cm^2$

Total surface area of conical portion to be painted green = Curved surface area of the cone + Base area of cone-Base area of cylinder

= 51.025 + 19.625 − 7.065

= 63.585$cm^2$

= 63.59 $cm^2$

Hence the area of the rocket painted with green colour is 63.59$cm^2$.

Total surface area of the cylindrical portion to be painted red = Curved surface area of the cylinder + Base area of cylinder

= 188.4 + 7.065

=195.465 $cm^2$

=195.47 $cm^2$

Hence the area of the rocket painted with red colour is 195.47$cm^2$.


(ii) Volume of wood in the rocket = Volume of cone + Volume of cylinder

$= \left(\frac{1}{3}\right)R^2H + r^2h$

$= \left(\left(\frac{R^2H}{3}\right) + r^2h\right)$

$= 3.14 \times \left(2.5^2 \times \frac{6}{3}\right) + 1.5^2 \times 20$
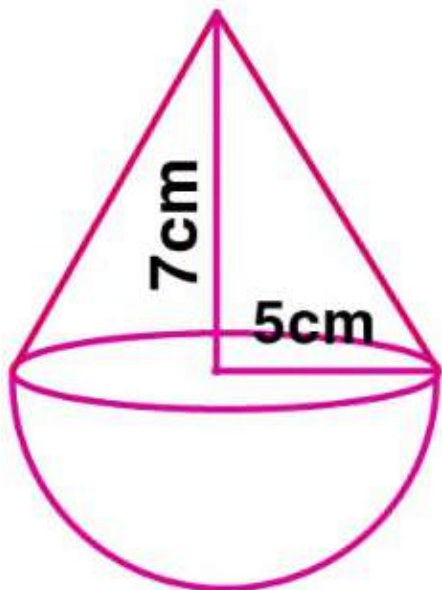
$= 3.14 \times (6.25 \times 2) + 2.25 \times 20$

$= 3.14 \times (12.5 + 45)$

$= 3.14 \times 57.5$

$= 180.55 \ cm^3$

Hence the volume of the wood in the rocket is 180.55 $cm^3$.

**14.** The adjoining figure shows a hemisphere of radius 5 cm surmounted by a right circular cone of base radius 5 cm. Find the volume of the solid if the height of the cone is 7cm. Give your answer correct to two places of decimal.



**Solution:**

Given radius of the hemisphere, r = 5 cm

Radius of cone, r = 5cm

Height of the cone, h = 7cm

Volume of the solid = Volume of the hemisphere + Volume of the cone

$$= \left(\frac{2}{3}\right)r^3 + \left(\frac{1}{3}\right)r^2h$$

$$= \left(\frac{1}{3}\right)r^2 + (2r + h)$$

$$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 5^2 (2 \times 5 + 7)$$

$$= \left(\frac{22}{21}\right) \times 25 \, (10 + 7)$$

$$= \left(\frac{22}{21}\right) \times 25 \times 17$$

$$= 445.238 \; cm^3$$

$$= 445.24 \; cm^3$$

Hence the volume of the solid is $445.24 \; cm^3$ .

**15. A buoy is made in the form of a hemisphere surmounted by a right cone whose circular base coincides with the plane surface of the hemisphere. The radius of the base of the cone is 3.5 metres and its volume is $\frac{2}{3}$ of the hemisphere. Calculate the height of the cone and the surface area of the buoy correct to 2 places of decimal.**

**Solution:**
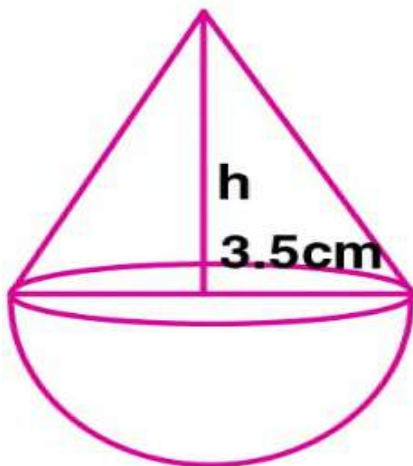


Given radius of the cone, r = 3.5 cm

Radius of hemisphere, $r = 3.5 \; cm = \frac{7}{2} \; cm$

Volume of hemisphere $= \left(\frac{2}{3}\right) r^3$

$$= \left(\frac{2}{3}\right) \times \left(\frac{22}{7}\right) \times \left(\frac{7}{2}\right)^3$$

$$= \left(\frac{2}{3}\right) \times \left(\frac{22}{7}\right) \times \left(\frac{7}{2}\right) \times \left(\frac{7}{2}\right) \times \left(\frac{7}{2}\right)$$

$$= \left(\frac{22}{3}\right) \times \left(\frac{7}{2}\right) \times \left(\frac{7}{2}\right)$$

$$= 11 \times \frac{49}{6}$$

$$= \frac{539}{6} m^3$$

Volume of cone $= \frac{2}{3}$ of volume of hemisphere

$$= \left(\frac{2}{3}\right) \times \frac{539}{6}$$

$$= \frac{539}{9} m^3$$

Volume of cone $= \left(\frac{1}{3}\right) r^2 h$

$$\left(\frac{1}{3}\right) r^2 h = \frac{539}{9}$$

$$\left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times \left(\frac{7}{2}\right)^2 \times h = \frac{539}{9}$$

$$h = 539 \times 3 \times \frac{2}{9} \times 11 \times 7$$

$$h = \frac{14}{3}$$

$$h = 4.667$$

$$h = 4.67 \ m$$

Hence the height of the cone is 4.67 m.

Slant height of cone, $l = \sqrt{(h^2 + r^2)}$

$$= \sqrt{\left(\left(\frac{14}{3}\right)^2 + \left(\frac{7}{2}\right)^2\right)}$$

$$= \sqrt{\left(\left(\frac{196}{9}\right)^2 + \left(\frac{49}{4}\right)^2\right)}$$

$$= \sqrt{\left(\frac{784}{36}\right) + \left(\frac{441}{36}\right)}$$

$$= \sqrt{\left(\frac{1225}{36}\right)}$$

$$= \frac{35}{6}m$$

Surface area of the buoy = Surface area of cone + Surface area of the hemisphere

$$= rl + 2r^2$$

$$= r(l + 2r)$$

$$= \left(\frac{22}{7}\right) \times \left(\frac{7}{2}\right) \times \left(\frac{35}{6} + 2 \times \left(\frac{7}{2}\right)\right)$$

$$= 11 \times \left(\frac{35}{6} + 7\right)$$

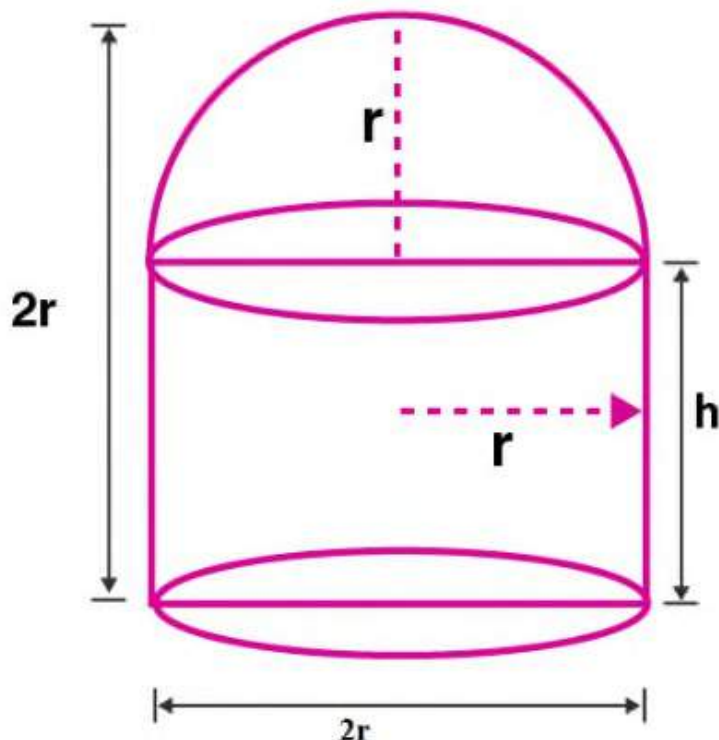$$= 11 \times (5.8333 + 7)$$

$$= 11 \times (12.8333)$$

$$= 141.166 \ m^2$$

$$= 141.17 \ m^2$$

Hence the Surface area of the buoy is $141.17m^2$.

**16. A circular hall (big room) has a hemispherical roof. The greatest height is equal to the inner diameter, find the area of the floor, given that the capacity of the hall is 48510 $m^3$.**

**Solution:**



Let the radius of the hemisphere be r.

Inner diameter = 2r

Given greatest height equal to inner diameter.

So total height of the hall = 2r

Height of the hemispherical part = r

Height of cylindrical area, h = $2r - r = r$

Capacity of the hall = Volume of cylindrical area + volume of hemispherical area

$$= r^2h + \left(\frac{2}{3}\right)r^3$$

$$= r^3 + \left(\frac{2}{3}\right)r^3 \; [\because h = r]$$

$$= r^3 + \left(1 + \frac{2}{3}\right)$$

$$= r^3 + \frac{(3+2)}{3}$$

$$= \left(\frac{5}{3}\right)r^3$$

Given capacity of hall $= 48510m^3$

$$= \left(\frac{5}{3}\right)r^3 = 48510$$

$$= \left(\frac{5}{3}\right) \times \left(\frac{22}{7}\right)r^3 = 48510$$

$$= r^3 = \frac{48510 \times 3 \times 7}{(22 \times 5)}$$

$$= r^3 = 9261$$

**Taking cube root on both sides,**

r = 21

Area of the floor $= r^2$

$$= \left(\frac{22}{7}\right) \times 21^2$$

$$= (\, 22 \times 21 \times 3)$$

$$= 1386 \; m^2$$

Hence the Area of the floor is $1386 \; m^2$.

**17.** A building is in the form of a cylinder surmounted by a hemisphere valted dome and contains $41\frac{19}{21}\,m^3$ of air. If the internal diameter of dome is equal to its total height above the floor, find the height of the building.

**Solution:**

Let the radius of the done be r.

Internal diameter = 2r

Given internal diameter is equal to total height.

Total height of the building = 2r

Height of the hemispherical area = r

So height of cylindrical area, h = 2r – r = r

**Volume of the building = Volume of cylindrical area + Volume of hemispherical area**

$$= r^2h + \left(\frac{2}{3}\right)r^3$$

$$= r^3 + \left(\frac{2}{3}\right)r^3 \quad [\because h = r]$$

$$= r^3\left(1 + \frac{2}{3}\right)$$

$$= r^3\left(\frac{3+2}{3}\right)$$

$$= \left(\frac{5}{3}\right)r^3$$

**Given Volume of the building** $= 41\frac{19}{21}\,m^3$

$$41\frac{19}{21} = \frac{880}{21}$$

$$= \left(\frac{5}{3}\right)r^3 = \frac{880}{21}$$

$$= \left(\frac{5}{3}\right) \times \left(\frac{22}{7}\right) \times r^3 = \frac{880}{21}$$

$$= r^3 = \frac{880 \times 3 \times 7}{5 \times 22 \times 21}$$

$$r^3 = \frac{880}{110}$$

$$r^3 = 8$$

Taking cube root

r = 2m

Height of the building $= 2r = 2 \times 2 = 4m$

Hence the height of the building is 4m.


**18. A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with same radius as that of the cylinder. The diameter and the height of the cylinder are 6 cm and 12 cm respectively. If the slant height of the conical portion is 5 cm, find the total surface area and the volume of the rocket. ( Use $\pi = 3.14$).**

**Solution:**
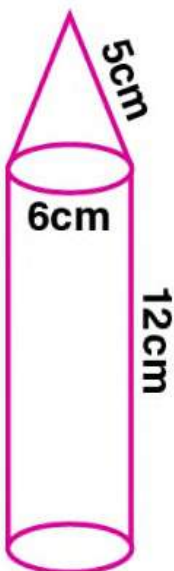
Given diameter of the cylinder = 6 cm

Radius of the cylinder, r $= \frac{6}{2} = 3cm$

Height of the cylinder, H = 12 cm

Slant height of the cone, $l = 5cm$

Radius of the cone, $r = 3cm$

**Height of the cone,** h $= \sqrt{(l^2 - r^2)}$

**h** $= \sqrt{(5^2 - 3^2)}$

**h =** $\sqrt{(25 - 9)}$

**h =** $\sqrt{16}$

**h = 4 cm**

Total surface area of the rocket = curved surface area of cylinder + base area of cylinder + curved surface area of cone

$= 2rH + r^2 + rl$

$= r(2H + r + l)$

$= 3.14 \times 3 \times (2 \times 12 + 3 + 5)$

$= 3.14 \times 3 \times (24 + 3 + 5)$

$= 3.14 \times 3 \times 32$

$= 301.44 \ cm^2$

Hence the total surface area of the rocket is $301.44 cm^2$.

**Volume of the rocket** = Volume of the cone + Volume of the cylinder

$= \left(\frac{1}{3}\right) r^2 h + r^2 H$

$$= r^2 \left( \left( \frac{h}{3} \right) + H \right)$$

$$= 3.14 \times 3^2 \times \left( \frac{4}{3} + 12 \right)$$

$$= 3.14 \times 9 \times \left( \frac{4+36}{3} \right)$$

$$= 3.14 \times 9 \times \left( \frac{40}{3} \right)$$

$$= 3.14 \times 3 \times 40$$

$$= 376.8 \; cm^3$$

Hence the volume of the rocket is $376.8 cm^3$

**19. The adjoining figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The heigh of the cylinder and the cone are each of 4 cm. Find the volume of the solid.**
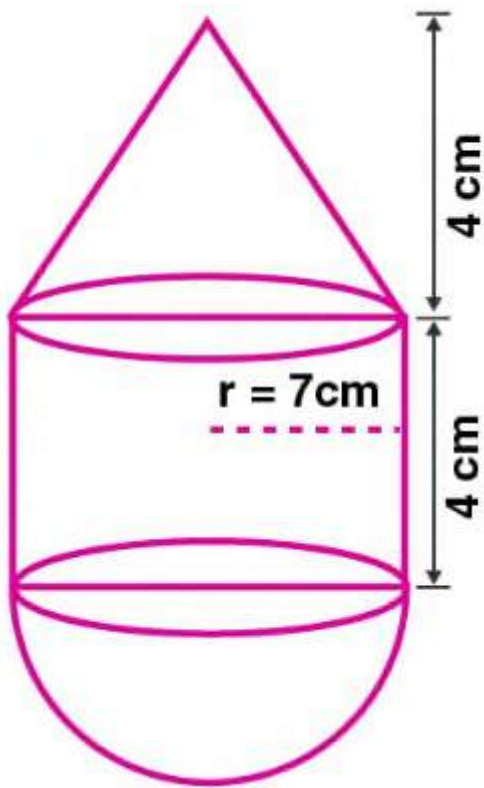
**Solution :**

Given common radius, r = 7cm

Height of the cone, h = 4cm

Height of the cylinder, H = 4 cm

Volume of the solid = Volume of the cone + Volume of the cylinder + Volume of the hemisphere

$$= \left(\frac{1}{3}\right)r^2h + r^2H + \left(\frac{2}{3}\right)r^3$$

Volume of the solid = Volume of the cone + Volume of the cylinder + Volume of the hemisphere

$$= \left(\frac{1}{3}\right) r^2 h + r^2 H + \left(\frac{2}{3}\right) r^3$$
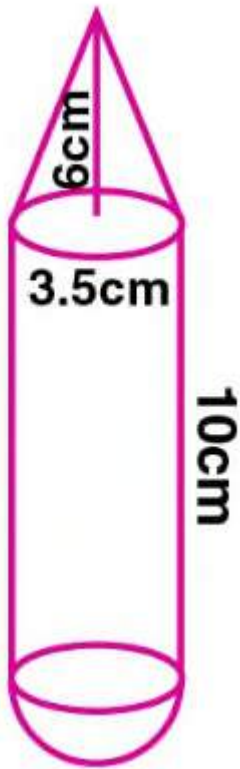
$$= r^2 \left(\left(\frac{h}{3}\right) + H + \left(\frac{2r}{3}\right)\right)$$

$$= \left(\frac{22}{7}\right) \times 7^2 \times \left(\left(\frac{4}{3}\right) + 4 + \frac{(2 \times 7)}{3}\right)$$

$$= \left(22 \times 7 \times \frac{(4+12+14)}{3}\right)$$

$$= 22 \times 7 \times \frac{30}{3}$$

$$= 1540 \; cm^3$$

Hence the volume of the solid is $1540 \; cm^3$.

20. A solid is in the form of a right circular cylinder with a hemisphere at one end and a cone at the other end. Their common diameter is 3.5 cm and the height of the cylindrical and conical portions are 10cm and 6 cm respectively. Find the volume of the solid. ( Take $\pi = 3.14$)

**Solution:**

Given height of the cylinder, H = 10 cm

Height of the cone, h = 6 cm

Common diameter = 3.5 cm

Common radius, r = $\frac{3.5}{2}$ = 1.75 cm

Volume of the solid = Volume of the cone + Volume of the cylinder + Volume of the hemisphere

$$= \left(\frac{1}{3}\right)r^2h + r^2H + \left(\frac{2}{3}\right)r^3$$

$$= r^2\left(\frac{h}{3}\right) + H + \left(\frac{2r}{3}\right)$$

$$= 3.14 \times 1.75^2 \left(\left(\frac{6}{3}\right) + 10 + \frac{(2\times1.75)}{3}\right)$$

$$= 3.14 \times 3.0625 \times (2 + 10 + 1.167)$$

$= 3.14 \times 3.0625 \times 13.167$

$= 9.61625 \times 13.167$

$= 126.617\ cm^3$

$= 126.62\ cm^3$

Hence the volume of the solid is $126.62\ cm^3$.

**21. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if the height of the conical part is 12 cm.**

**Solution:**
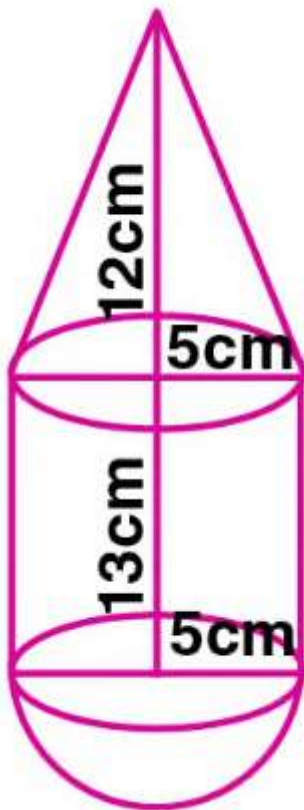
Given height of the cylinder, H = 13 cm

radius of the cylinder, r = 5cm

Radius of the hemisphere, r = 5cm

Height of the cone, h = 12 cm

Radius of the cone, r = 5 cm

Slant height of the cone, $l = \sqrt{(h^2 + r^2)}$

$= \sqrt{(12^2 + 5^2)}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

= 13 cm

Surface area of the toy = Curved surface area of cylinder + curved surface area of hemisphere + curved surface area of cone

$= 2rH + 2r^2 + rl$

$= r(2H + 2r + l)$

$= \left(\frac{22}{7}\right) \times 5(2 \times 13 + 2 \times 5 + 13)$

$= \left(\frac{110}{7}\right) \times (26 + 10 + 13)$
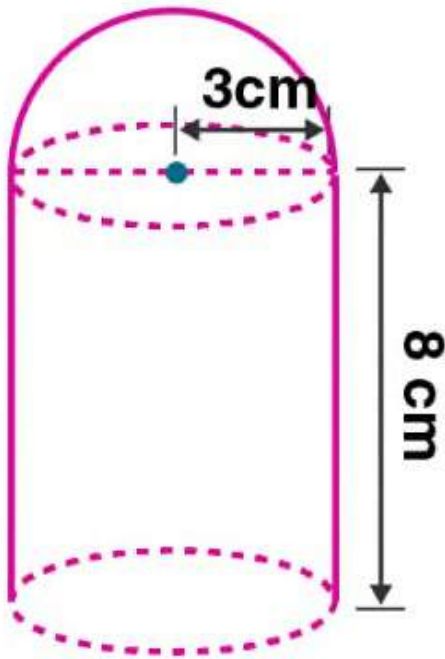
$= \left(\frac{110}{7}\right) \times 49$

$= 110 \times 7$

$= 770 \; cm^2$

Hence the surface area of the toy is $770 cm^2$.

**22. The adjoining figure shows a model of a solid consisting of a cylinder surmounted by a hemisphere at one end. If the model is drawn to a scale of 1:200, find**

(i) the total surface area of the solid in $\pi\ m^2$.

(ii) the volume of the solid in $\pi$ litres.



**Solution:**

Given height of the cylinder, h = 8cm

Radius of the cylinder, r = 3cm

Radius of hemisphere, r = 3 cm

Scale = 1 : 200

Hence actual radius, r = 200 × 3 = 600

Actual height, h = 200 × 8 = 1600

**(i) Total surface area of the solid = Base area of the cylinder + Curved surface area of the cylinder + curved surface area of the hemisphere**

$= r^2 + 2rh + 2r^2$
$= r(r + 2h + 2r)$
$= 600 \,(600 + 2 \times 1600 + 2 \times 600)$
$= 600 \,(600 + 3200 + 1200)$
$= 600 \times (5000)$
$= 3000000 \; cm^2$
$= 300 \; m^2$

Hence the total surface area of the solid is $3000 m^2$.

**(ii)** Volume of the solid = Volume of the cylinder + Volume of the hemisphere

$= r^2 h + \left(\dfrac{2}{3}\right) r^3$

$= r^2 \left(h + \left(\dfrac{2}{3}\right) r\right)$

$= 600^2 \left(1600 + \left(\dfrac{2}{3}\right) \times 600\right)$

$= 360000 \,(1600 + 400)$

$= 360000 \times 2000$

$= 720000000 \; cm^3$

$= 720 \; m^3$

$= 720000$ litres $[1 \; m^3 = 1000 \; litres]$

Hence the volume of the solid is 720000 litres.

**1. The diameter of a metallic sphere is 6cm. The sphere is melted and drawn into a wire of uniform cross-section. If the length of the wire is 36m, find its radius.**

**Solution:**

Given diameter of the metallic sphere = 6cm

Radius of the sphere, r $= \dfrac{6}{2} = 3$cm

Volume of the sphere, V $= \left(\dfrac{4}{3}\right) r^3$

$= \left(\dfrac{4}{3}\right) \times 3^3$

$= 4 \times 3 \times 3$

$= 4 \times 9$

$= 36 \ cm^3$

Length of the wire, h = 36m = 3600 cm
Since the sphere is melted and drawn into a wire, volume remains the same.
Volume of the wire, $r^2 h = 36$
$r^2 \times 3600 = 36$
$r^2 = \dfrac{1}{100}$
$r = \dfrac{1}{10}$
= 0.1 cm
= 1mm
Hence the radius of the wire is 1mm.

**2. The radius of a sphere is 9 cm. It is melted and drawn into a wire of diameter 2mm. Find the length of the wire in metres.**

**Solution :**

Radius of the sphere, r = 9 cm

Volume of the sphere, V = $\left(\frac{4}{3}\right) r^3$

= $\left(\frac{4}{3}\right) \times 9^3$

= 12 × 81

= 972 $cm^3$

Diameter of the wire = 2mm

So radius of the wire = $\frac{2}{2}$ = 1mm = 0.1 cm

Since the sphere is melted and drawn into a wire, volume remains the same.

Volume of the wire, $r^2 h = 972$

$0.1^2 \times h = 972$

$h = \frac{972}{0.1^2}$

$h = 97200$ cm

$h = 972m$

Hence the length of the wire is 972 m.

**3. A solid metallic hemisphere of radius 8 cm is melted and recasted into right circular cone of base radius 6cm. Determine the height of the cone.**

**Solution:**

Given radius of the hemisphere, r = 8 cm

Volume of the hemisphere, $V = \left(\frac{2}{3}\right) r^3$

$= \left(\frac{2}{3}\right) \times 8^3$

$= \left(\frac{1024}{3}\right) cm^3$

Radius of cone, R = 6cm

Since hemisphere is melted and recasted into a cone, the volume remains the same.

Volume of the cone, $\left(\frac{1}{3}\right) R^2 h = \left(\frac{1024}{3}\right)$

$= \left(\frac{1}{3}\right) \times 6^2 \times h = \left(\frac{1024}{3}\right)$

$= 36h = 1024$

$h = \frac{1024}{36}$

$= 28.44$ cm

Hence the height of the cone is 28.44 cm.

**4. A rectangular water tank of base 11m $\times$ 6m contains water upto a height of 5m. If the water in the tank is transferred to a cylindrical tank of radius 3.5m, find the height of the water level in the tank.**

**Solution:**

Given dimensions of the rectangular water tank = 11m $\times$ 6m

Height of water in tank = 5m

Volume of water in tank = $11 \times 6 \times 5 = 330m^3$

Radius of the cylindrical tank, r = 3.5m

Volume of cylindrical tank = $r^2h$

$r^2h = 330$

$\left(\frac{22}{7}\right) \times 3.5^2 \times h = 330$

$= \left(\frac{22}{7}\right) \times 12.25 \times h = 330$

$= h = 330 \times \frac{7}{22} \times 12.25$

= 8.57 m

Hence the height of the water level in the tank is 8.57m.

**5. The rain water from a roof of dimensions 22m × 20m drains into a cylindrical vessel having diameter of base 2m and height; 3.5m. If the rain water collected from the roof just fill the cylindrical vessel, then find the rainfall in cm.**

**Solution:**

Given dimensions of the cylindrical vessel = 22m × 20 $m$

Let the rainfall be $x$ cm.

Volume of water = $(22 \times 20 \times x)m^3$

Diameter of the cylindrical base = 2m

So radius of the cylindrical base = $\frac{2}{2}$ =1m

Height of the cylindrical base, h = 3.5 m

Volume of cylindrical vessel = $r^2h$

$= \left(\frac{22}{7}\right) \times 1^2 \times 3.5$

$= 11m^3$

Since the rain water collected from the roof just fill the cylindrical vessel, the volumes are equal.

$22 \times 20 \times x = 11$

$x = \frac{11}{22} \times 20$

$= \frac{1}{40}m$

$= \left(\frac{1}{40}\right) \times 100 \; cm$

$= 2.5$ cm

Hence the rainfall is 2.5 cm.

6. The volume of a cone is the same as that of the cylinder whose height is 9cm and diameter 40 cm. Find the radius of the base of the cone if its height is 108 cm.

**Solution:**

Given height of the cylinder, h = 9 cm

Diameter of the cylinder = 40 cm

Radius of the cylinder, r = $\frac{40}{2}$ = 20 cm

Volume of the cylinder = $r^2h$

= $20^2 \times 9$

= $400 \times 9$

= 3600 $cm^3$

Height of the cone, H = 108 cm

Volume of cone = $\left(\frac{1}{3}\right)r^2h$

= $\left(\frac{1}{3}\right)r^2 \times 108$

= $36r^2$

Since volume of cone is equal to the volume of the cylinder, we get

$36r^2 = 3600$

$r^2 = \frac{3600}{36}$

$r^2 = 100$

Taking square root on both sides,

r = 10 cm

Hence the radius of the cone is 10 cm.

**7. Eight metallic spheres, each of radius 2 cm, are melted and cast into a single sphere. Calculate the radius of the new ( single ) sphere.**

**Solution:**

Given radius of each sphere, r = 2cm

Volume of a sphere $= \left(\frac{4}{3}\right) r^3$

$= \left(\frac{4}{3}\right) \times 2^3$

$= \left(\frac{4}{3}\right) \times 8$

$= \left(\frac{32}{3}\right) cm^3$

Volume of 8 spheres $= 8 \times \left(\frac{32}{3}\right)$

$= \left(\frac{256}{3}\right) cm^3$

Let R be radius of new sphere.

Volume of the new sphere $= \left(\frac{4}{3}\right) R^3$

Since 8 spheres are melted and casted into a single sphere, volume remains same.

$\left(\frac{4}{3}\right) R^3 = \left(\frac{256}{3}\right)$

$4R^3 = 256$

$R^3 = \frac{256}{4} = 64$

Taking cuve root

R = 4 cm

Hence the radius of the new sphere is 4 cm.

**8. A metallic disc, in the shape of a right circular cylinder, is of height 2.5 mm and base radius 12 cm. Metallic disc is melted and made into a sphere. Calculate the radius of the sphere.**

**Solution:**

Given height of the cylinder, h = 2.5 mm = 0.25cm

Radius of the cylinder, r = 12 cm

Volume of the cylinder = $r^2h$

$= 12^2 \times 0.25$

$= 144 \times 0.25$

$= 36 \; cm^3$

Let R be the radius of the sphere.

Volume of sphere = $\left(\frac{4}{3}\right)R^3$

Since metallic disc is melted and made into a sphere, their volumes remains same.

$\left(\frac{4}{3}\right)R^3 = 36$

$R^3 = 36 \times \frac{3}{4}$

$R^3 = 27$

Taking cube root

R = 3 cm

Hence the radius of the sphere is 3 cm.

**9. Two spheres of the same metal weigh 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the big sphere.**

**Solution;**

For same meterial, density will be same.

Density = mass/ Volume

Mass of the smaller sphere, $m_1 = 1kg$

Mass of the bigger sphee, $m_2 = 7kg$

The spheres are melted to form a new sphere.

So the mass of new sphere, $m_3 = 1 + 7 = 8\ kg$

Density of smaller sphere = density of new sphere

Let $V_1$ be volume of smaller sphere and $V_3$ be volume of bigger sphere.

$$\frac{m_1}{V_1} = \frac{m_3}{V_3}$$

$$\frac{1}{V_1} = \frac{8}{V_3}$$

$$\frac{V_1}{V_3} = \frac{1}{8} \ \dots\dots.(i)$$

Given radius of the smaller sphere, r = 3 cm

$$V_1 = \left(\frac{4}{3}\right)r^3$$

$$V_1 = \left(\frac{4}{3}\right) \times 3^3$$

$$V_1 = 36$$

Let R be radius of new sphere.

$$V_3 = \left(\frac{4}{3}\right)R^3$$

$$\frac{V_1}{V_3} = \frac{36}{\left(\frac{4}{3}\right)R^3}$$

$$\frac{V_1}{V_3} = \frac{27}{R^3} \quad \dots\dots(ii)$$

From (i) and (ii)

$$\frac{1}{8} = \frac{27}{R^3}$$

$$R^3 = 27 \times 8 = 216$$

Taking cube root on both sides,

R = 6 cm

So diameter of the new sphere = 2R = $2 \times 6 = 12\ cm$

Hence diameter of the new sphere is 12 cm.

**10. A hollow copper pipe of inner diameter 6cm and outer diameter 10 cm is melted and changed into a solid circular cylinder of the same height as that of the pipe. Find the diameter of the solid cylinder.**

**Solution:**

Given inner diameter of the pipe = 6cm

So inner radius, r = $\frac{6}{2}$ = 3cm

Outer diameter = 10 cm

Outer radius, R = $\frac{10}{2}$ = 5cm

Let h be the height of the pipe.

Volume of pipe = $(R^2 - r^2)h$

$= (5^2 - 3^2) \times h$

$= h(25 - 9)$

$= 16h \ cm^3$

Let r be the radius of solid cylinder.

Volume of solid cylinder $= r^2h$

Since pipe is melted and changed into a cylinder, their volumes remains same.

$r^2h = 16h$

$r^2 = 16$

$r = 4 \ cm$

Diameter $= 2r = 2 \times 4 = 8cm$

Hence the diameter of the cylinder is 8 cm.


**11. A solid sphere of radius 6cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 4 cm and height is 72 cm, find the uniform thickness of the cylinder.**

**Solution:**

Given radius of the sphere, r = 6cm

Volume of the sphere $= \left(\dfrac{4}{3}\right) r^3$

$= \left(\dfrac{4}{3}\right) \times 6^3$

$= 288 \ cm^3$

Let r be the internal radius of the hollow cylinder.

External radius of the hollow cylinder, R = 4 cm

Height of hollow cylinder, h = 72 cm

Volume of hollow cylinder $= (R^2 - r^2)h$

Since sphere is melted and changed into a hollow cylinder, their volumes remains same.

$(R^2 - r^2)h = 288$

$(4^2 - r^2)72 = 288$

$(4^2 - r^2) = \frac{288}{72}$

$(4^2 - r^2) = 4$

$16 - r^2 = 4$

$r^2 = 16 - 4$

$r^2 = 12$

$r = \sqrt[2]{3}$ cm

So thickness = R − r = 4 - $\sqrt[2]{3}$

= 4 − 3.464

= 0.536 cm

= 0.54 cm ( approx)

Hence the thickness of the cylinder is 0.54 cm.


12. A hollow metallic cylindrical tube has an internal radius of 3 cm and height 21 cm. The thickness of the metal of the tube is $\frac{1}{2}$ cm. The tube is melted and cast into a right circular cone of height 7cm. Find the radius of the cone correct to one decimal place.

**Solution:**

Given internal radius of the tube, r = 3 cm

External radius of tube = 3 + 0.5 = 3.5 cm

Height of the tube, h = 21 cm

Volume of the tube = $(R^2 - r^2)h$

$= (3.5^2 - 3^2) \times 21$

$= (12.25 - 9) \times 21$

$= (3.25) \times 21$

$= 68.25 \ cm^3$

Height of the cone, h = 7 cm

Let r be radius of cone.

Volume of cone = $\left(\frac{1}{3}\right) r^2 h$

$= \left(\frac{1}{3}\right) r^2 \times 7$

$= \left(\frac{7}{3}\right) r^2$

Since tube is melted and changed into a cone, their volumes remain same.

$= \left(\frac{7}{3}\right) r^2 = 68.25$

$= r^2 = 68.25 \times \frac{3}{7} = 29.25$

Taking square root on both sides

r = 5.4 cm

Hence the radius of the cone is 5.4 cm.

**13. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively, is melted into a cone of base diameter 8 cm. Find the height of the cone.**

**Solution:**

Given internal diameter of hollow sphere = 4 cm

Internal radius, r $= \dfrac{4}{2} = 2$ cm

External diameter = 8 cm

External radius, R $= \dfrac{8}{2} = 4$ cm

Volume of the hollow sphere, V $= \left(\dfrac{4}{3}\right)(R^3 - r^3)$

V $= \left(\dfrac{4}{3}\right)(4^3 - 2^3)$

V $= \left(\dfrac{4}{3}\right)(64^3 - 8^3)$

V $= \left(\dfrac{4}{3}\right) \times (56)$

Base diameter of the cone = 8 cm

Radius, r $= \dfrac{8}{2} = 4$ cm

Volume of cone $= \left(\dfrac{1}{3}\right)r^2 h$

$= \left(\dfrac{1}{3}\right) \times 4^2 \times h$

$= \left(\dfrac{16}{3}\right) \times h$

Since sphere is melted and changed into a cone, their volumes remain same.

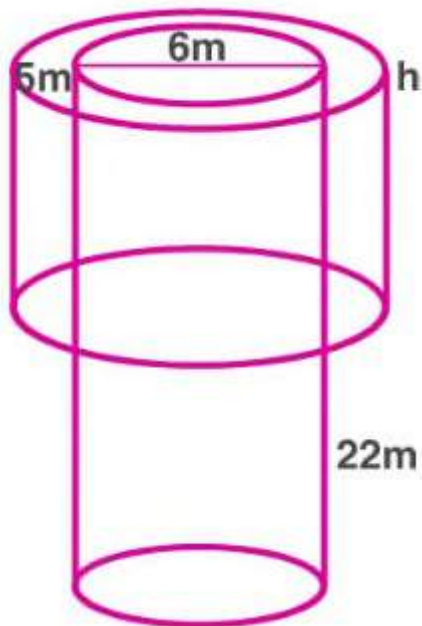$$\left(\frac{4}{3}\right) \times 56 = \left(\frac{16}{3}\right) \times h$$

$$h = 4 \times \frac{56}{16}$$

$$h = 14 \; cm$$

Hence the height of the cone is 14 cm.

**14. A well with inner diameter 6 m is dug 22 m deep. Soil taken out of it has been spread evenly all round it to a width of a 5m to form an embankment. Find the height of the embankment.**

**Solution:**



Given inner diameter of the well = 6m

Radius of the well, r = $\frac{6}{2}$ = 3m

Depth of the well, H = 22m

Volume of the soil dug out of well $= r^2 H$

$= 3^2 \times 22$

$= 198 \ m^3$

Width of the embankment $= 5m$

Inner radius of embankment, $r = 3m$

Outer radius of embankment, $R = 3 + 5 = 8m$

Let h be height of the soil embankment.

Volume of the soil embankment $= (R^2 - r^2)h$

$= (8^2 - 3^2)h$

$= (64 - 9)h$

$= 55h$

Volume of the soil dug out = Volume of the soil embankment

$198 = 55h$

$h = \frac{198}{55}$

$h = 3.6 \ m$

Hence the height of the soil embankment is 3.6 m.


**15. A cylindrical can of internal diameter 21 cm contains water. A solid sphere whose diameter is 10.5 cm is lowered into the cylindrical can. The sphere is completely immersed in water. Calculate the rise in water level, assuming that no water overflows.**

**Solution:**
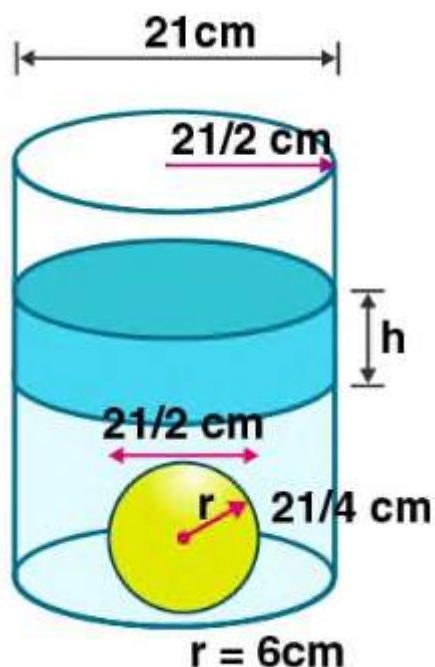


Given internal diameter of cylindrical can = 21 cm

Radius of the cylindrical can, R = $\frac{21}{2}$ cm

Diameter of spher = 10.5 cm

Radius of the sphere, r = $\frac{10.5}{2} = \frac{21}{4}$ cm

Let the rise in water level be h.

Rise in volume of water = Volume of sphere immersed

$$R^2 h = \left(\frac{4}{3}\right) r^3$$

$$= \left(\frac{21}{2}\right)^2 h = \left(\frac{4}{3}\right) \times \left(\frac{21}{4}\right)^3$$

$$= \left(\frac{21}{2}\right) \times \left(\frac{21}{2}\right) \times h = \left(\frac{4}{3}\right) \times \left(\frac{21}{4}\right) \times \left(\frac{21}{4}\right) \times \left(\frac{21}{4}\right)$$
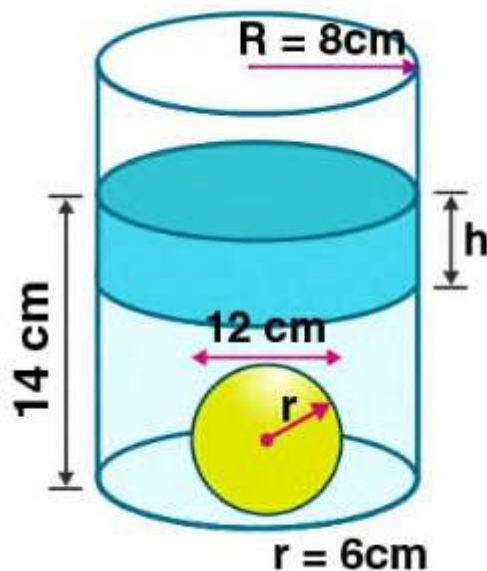
$h = \dfrac{21}{12}$

$h = \dfrac{7}{4}$

$h = 1.75\ cm$

Hence the rise in water level is 1.75 cm.

**16. There is water to a height of 14 cm in a cylindrical glass jar of radius 8 cm. Indise the water there is a sphere of diameter 12 cm completely immersed . By what height will the water go down when the sphere is removed ?**

**Solution:**



Given radius of the glass jar, R = 8 cm

Diameter of the sphere = 12 cm

Radius of the sphere, r $= \dfrac{12}{2} = 6$ cm

When the sphere is removed from the jar, volume of water decreases.

Let h be the height by which water level will decrease.

Volume of water decreased = Volume of the sphere

$$R^2 h = \left(\frac{4}{3}\right) r^3$$

$$8^2 h = \left(\frac{4}{3}\right) 6^3$$

$$h = \left(\frac{4}{3}\right) \times \frac{6 \times 6 \times 6}{(8 \times 8)}$$

$$= \frac{18}{4} = \frac{9}{2} = 4.5 \text{ cm}$$

Hence the height by which water level decreased is 4.5 cm.

**17. A vessel in the form of an inverted cone is filled with water to the brim. Its height is 20 cm and diameter is 16.8 cm. Two equal solid cones are dropped in it so that they are fully submerged. As a result, one-third of the water in the original cone overflows. What is the volume of each of the solid cone submerged ?**

**Solution:**
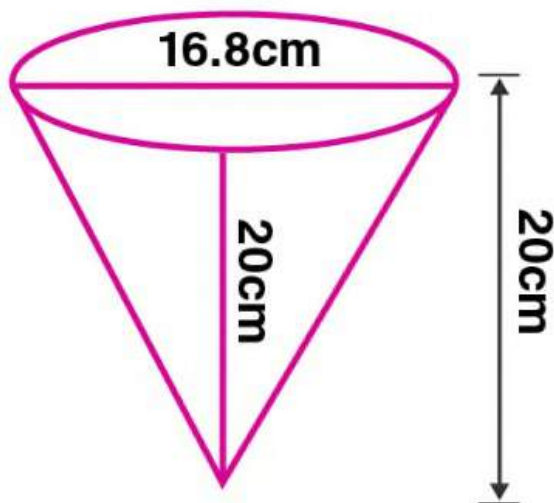
Given height of the cone, h = 20 cm

Diameter of the cone = 16.8 cm

Radius of the cone, r = $\frac{16.8}{2}$ = 8.4 cm

Volume of water in the vessel = $\left(\frac{1}{3}\right)r^2 h$

= $\left(\frac{1}{3}\right) \times 8.4^2 \times 20$

= $\left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 8.4 \times 8.4 \times 20$

= 1478.4 $cm^3$

One third of volume of water in the vessle = $\left(\frac{1}{3}\right) \times 1478.4$

= 492.8 $cm^3$

One third of volume of water in the vessel = Volume of water over flown

Volume of water over flown = Volume of two equal solid cones dropped into the vessel.

Volume of two equal solid cones dropped into the vessel = 492. 8 $cm^3$

Volume of one solid cone dropped into the vessel = $\frac{492.8}{2}$ = 246.4 $cm^3$

Hence the volume of each of the solid cone submerged is 246.4 $cm^3$.

18. A solid metallic circular cylinder of radius 14 cm and height 12 cm is melted and recast into small cubes of edge 2 cm. How many such cubes can be made from the solid cylinder ?

**Solution:**

Radius of the solid circular cylinder, r = 14 cm

Height, h = 12 cm

Volume of the cylinder $= r^2 h$

$= 14^2 \times 12$

$= 196 \times 12$

$= 2352$

$= 2352 \times \dfrac{22}{7}$

$= 7392 \; cm^3$

Edge of the cube, a = 2 cm

Volume of cube $= a^3$

$= 2^3 = 8 cm^3$

Number of cubes made from solid cylinder $= \dfrac{7392}{8} = 924.$

Hence the number of cubes made from solid cylinder is 924.

**19.** How many shots each having diameter 3 cm can be made from a cuboidal lead solid of dimensions 9cm × 11 cm × 12 $cm$ ?

**Solution :**

Given dimensions of the cuboidal solid = 9 cm × 11 $cm$ × 12 $cm$

Volume of the cuboidal solid = 9 × 11 × 12 = 1188$cm^3$

Diameter of shot = 3 cm

So radius of shot, r = $\frac{3}{2}$ = 1.5 cm

Volume of shot = $\left(\frac{4}{3}\right) r^3$

$= \left(\frac{4}{3}\right) \times 1.5^3$

$= \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 1.5^3$

$= \frac{297}{21} cm^3$

Number of shots made from cuboidal lead of solid = 1188 ÷ $\left(\frac{297}{21}\right)$

$= 1188 \times \frac{21}{297}$

= 84

Hence the number of shots made from cuboidal lead of solid is 84.

**20. How many spherical lead shots of diameter 4 cm can be made out of a solid cube of lead whose edge measures 44 cm ?**

**Solution:**

Edge of the cube, a = 44 cm

Volume of cube = $a^3$

= $44^3$

= 85184 $cm^3$

Diameter of shot = 4 cm

So radius of shot, r = $\frac{4}{2}$ = 2 cm

Volume of a shot = $\left(\frac{4}{3}\right) r^3$

= $\left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 2^3$

= $\frac{704}{21}$ $cm^3$

Number of lead shots made from cube = 85184 ÷ $\left(\frac{704}{21}\right)$

= 85184 × $\frac{21}{704}$

= 2541

Hence the number of shots made from cube is 2541.

**21. Find the number of metallic circular discs with 1.5 cm base diameter and height 0.2cm to be melted to form a circular cylinder of height 10 cm and diameter 4.5 cm.**

**Solution:**

Given height of the circular cylinder, h = 10cm

Diameter of circular cylinder= 4.5 cm

So radius, $r = \frac{4.5}{2} = 2.25$ cm

Volume of circular cylinder $= r^2h$

$= 2.25^2 \times 10 = 50.625 \; cm^3$

Base diameter of circular disc = 1.5 cm

So radius, $r = \frac{1.5}{2} = 0.75$ cm

Height of the circular disc, h = 0.2 cm

Volume of circular dics $= r^2h$

$= 0.75^2 \times 0.2$

$= 0.1125 \; cm^3$

Number of circular discs made from cylinder $= \frac{50.625}{0.1125} = 450$

Hence the number of circular discs made from cylinder is 450.

**22. A solid metal cylinder of radius 14 cm and height 21 cm is melted down and recast into spheres of radius 3.5 cm. Calculate the number of spheres that can be made.**

**Solution:**

Given radius of the metal cylinder, r = 14 cm

Height of the metal cylinder, h = 21 cm

Radius of the sphere, R = 3.5 cm

Volume of the metal cylinder = $r^2h$

$= \left(\frac{22}{7}\right) \times 14^2 \times 21$

$= 22 \times 2 \times 14 \times 21$

$= 12936 \ cm^3$

Volume of sphere $= \left(\frac{4}{3}\right)R^3$

$= \left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 3.5^3$

$= 11 \times \frac{49}{3}$

$= \frac{539}{3} \ cm^3$

Number of spheres that can be made $= \dfrac{\text{Volume of the metal cylinder}}{Volume\ of\ sphere}$

$= 12396 \div \frac{539}{3}$

$= 12936 \times \frac{3}{539}$

$= 72$

Hence the number of spheres that can be made is 72.

**23. A metallic sphere of radius 10.5 cm is melted and then recast into small cones, each of radius 3.5 cm and height 3 cm. Find the number of cones thus obtained.**

**Solution:**

Given radius of the metallic sphere, R = 10.5 cm

Volume of the sphere = $\left(\frac{4}{3}\right) R^3$

$= \left(\frac{4}{3}\right) \times 10.5^3$

$= 1543.5 \; cm^3$

Radius of cone, r = 3.5 cm

Height of the cone,  h = 3cm

Volume of the cone = $\left(\frac{1}{3}\right) r^2 h$

$= \left(\frac{1}{3}\right) \times 3.5^2 \times 3$

$= 12.25 \; cm^3$

Number of cones made from sphere $= \dfrac{Volume\ of\ sphere}{Volume\ of\ cone}$

$= \dfrac{1543.5}{12.25}$

$= 126$

Hence the number of cones that can be made is 126.

**24. A certain number of metallic cones each of radius 2cm and height 3 cm are melted and recast in a solid sphere of radius 6 cm. Find the number of cones.**

**Solution:**

Given radius of metallic cones, r = 2 cm

Height of cone, h = 3 cm

Volume of cone = $\left(\frac{1}{3}\right) r^2 h$

$= \left(\frac{1}{3}\right) \times 2^2 \times 3$

$= 4 \ cm^3$

Radius of the solid sphere, R = 6 cm

Volume of the solid sphere = $\left(\frac{4}{3}\right) R^3$

$= \left(\frac{4}{3}\right) 6^3$

$= 4 \times 2 \times 6 \times 6$

$= 288 \ cm^3$

Number of cones made from sphere $= \dfrac{Volume\ of\ solid\ sphere}{Volume\ of\ the\ cone}$

$= \dfrac{288}{4}$

$= 72$

Hence the number of cones that can be made is 72.

**25. A vessel is in the form of an inverted cone. Its height is 11 cm and the radius of its top, which is open, is 2.5 cm. It is filled with water upto the rim. When some lead shots, each of which is a sphere of radius 0.25cm, are dropped into the vessel, $\frac{2}{5}$ of the water flows out. Find the number of lead shots dropped into the vessel.**

**Solution:**

Given height of the cone, h = 11 cm

Radius of the cone, r = 2.5 cm

Volume of the cone $= \left(\frac{1}{3}\right) r^2 h$

$= \left(\frac{1}{3}\right) \times 2.5^2 \times 11$

$= \left(\frac{11}{3}\right) \times 6.25 \; cm^3$

When lead shots are dropped into vessel, $\left(\frac{2}{5}\right)$ of water flows out.

Volume of water flown out $= \left(\frac{2}{5}\right) \times \left(\frac{11}{3}\right) \times 6.25$

$= \left(\frac{22}{15}\right) \times 6.25$

$= \left(\frac{137.15}{15}\right) cm^3$

Radius of sphere, R = 0.25 cm $= \frac{1}{4}$ cm

Volume of sphere $= \left(\frac{4}{3}\right) R^3$

$= \left(\frac{4}{3}\right) \times \left(\frac{1}{4}\right)^3$

$= 48 \; cm^3$

Number of lead shots dropped $= \dfrac{Volume \; of \; water \; flown \; out}{Volume \; of \; sphere}$

$$= \frac{137.5}{15} \div \frac{1}{48}$$

$$= \frac{137.5}{15} \times 48$$

$$= \frac{137.5 \times 48}{15}$$

$$= 440$$

Hence the number of lead shots dropped is 440.

**26. The suurface area of a solid metallic sphere is 616 $cm^2$. It is melted and reacast into smaller spheres of diameter 3.5 cm. How many such spheres can be obtained ?**

**Solution:**

Given surface area of the sphere $= 616 \ cm^2$

$4R^2 = 616$

$4 \times \left(\frac{22}{7}\right) R^2 = 616$

$R^2 = 616 \times \frac{7}{4} \times 22$

$R^2 = 49$

$R = 7$

Volume of the solid metallic sphere $= \left(\frac{4}{3}\right) R^3$

$= \left(\frac{4}{3}\right) \times 7^3$

$= \left(\frac{1372}{3}\right) cm^3$

Diameter of smaller sphere = 3.5 cm

So radius, $r = \frac{3.5}{2} = \frac{7}{4}$ cm

Volume of the smaller sphere $= \left(\frac{4}{3}\right) r^3$

$= \left(\frac{4}{3}\right) \times \left(\frac{7}{4}\right)^3$

$= \left(\frac{343}{48}\right) cm^3$

Number of sphere made $= \dfrac{\text{Volume of the solid metallic  sphere}}{\textit{Volume of the smaller sphere}}$

$= \left(\frac{1372}{3}\right) \div \left(\frac{343}{48}\right)$

$= \dfrac{1372 \times 48}{3 \times 343}$

$= 64$

Hence the number of spheres made is 64.


27. The surface area of a solid metallic sphere is 1256 $cm^2$. It is melted and recast into solid right circular cones of radius 2.5 cm and height 8cm. Calculate (i) the radius of the solid sphere.

(ii) The number of cones recast. ( Use $\pi = 3.14$).

**Solution:**

(i) Given surface area of the  solid metallic sphere= 1256 $cm^2$.

$4R^2 = 1256$

$4 \times 3.14 \times R^2 = 1256$

$R^2 = \dfrac{1256}{4 \times 3.14}$

$R^2 = 100$

$R = 10$

Hence the radius of solid sphere is 10 cm.

(ii) Volume of the solid sphere $= \left(\frac{4}{3}\right) R^3$

$= \left(\frac{4}{3}\right) \times 10^3$

$= \left(\frac{4000}{3}\right) cm^3$

$= \frac{12560}{3} cm^3$

Radius of the cone, r = 2.5 cm

Height of the cone, h = 8 cm

Volume of the cone $= \left(\frac{1}{3}\right) r^2 h$

$= \left(\frac{1}{3}\right) \times 3.14 \times 2.5^2 \times 8$

$= \frac{157}{3} cm^3$

**Nmber of cones made** $= \frac{Volume\ of\ the\ solid\ sphere}{Volume\ of\ the\ cone}$

$= \left(\frac{12560}{3}\right) \div \left(\frac{3}{157}\right)$

$= \frac{12560}{157}$

$= 80$

Hence the number of cones made is 80.

**28. Water is flowing at the rate of 15km/h through a pipe of diameter 14 cm into a cuboid pond which is 50m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm ?**

**Solution:**

Given speed of water flow = 15km/h

Diameter of pipe = 14 cm

So radius of pipe, r = $\frac{14}{2}$ = 7cm = 0.07 m

Dimensions of cuboidal pond = 50m × 44$m$

Height of water in pond = 21cm = 0.21m

So volume of water in pond = 50 × 44 × 0.21

= 462 $m^3$

Volume of water in pipe = $r^2h$

= $0.07^2$ × $h$

= 0.0049h

Volume of water in pond = Volume of water in pipe

462 = 0.0049h

h = $\frac{462}{0.0049}$

= $\frac{462 \times 7}{0.0049 \times 22}$

= 30000 m

= 30 km  [ 1 km = 1000m]

Speed = $\frac{distance}{time}$

$$\text{Time} = \frac{distance}{time} = \frac{30}{15} = 2 \text{ hr}$$

Hence the time takne is 2 hours.

**29.** A cylindrical can whose base is horizontal and of radius 3.5 cm contains sufficient water so that when a sphere is placed in the can, the water just covers thesphere. Given that the sphere just fits into the can, calculate :

(i) the total surface area of the can in contact withwater when the sphere is in it.

(ii) The depth of the water in the can before the sphere was put into the can. Given your answer as proper fractions.

**Solution:**
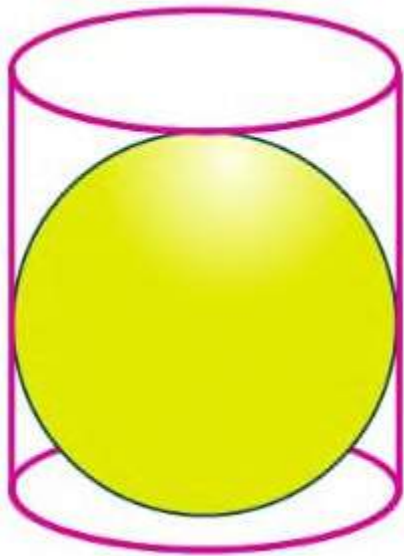


(i) Given radius of the cylinder, r = 3.5 cm

Diameter of the sphere = height of the cylinder

$= 3.5 \times 2$

= 7 cm

So radius of sphere, r = $\frac{7}{2}$ = 3.5 cm

Height of cylinder, h = 7 cm

Total surface area of can in contact with water = curved surface area of cylinder + base area of cylinder.

$= 2rh + r^2$

$= r(2h + r)$

$= \frac{22}{7} \times 3.5 \times (2 \times 7 + 3.5)$

$= \frac{22}{7} \times 3.5 \times (14 + 3.5)$

$= 11 \times 17.5$

$= 192.5 \; cm^2$

Hence the surface area of can in contact with water is 192.5 $cm^2$.


(ii) Let the depth of the water before thee sphere was put be d.

Volume of cylindrical can = Volume of sphere + volume of water

$r^2h = \left(\frac{4}{3}\right)r^3 + r^2d$

$r^2h = r^2\left\{\left(\frac{4}{3}\right)r + d\right\}$

$h = \left(\frac{4}{3}\right)r + d$

$d = h - \left(\frac{4}{3}\right)r$

$d = 7 - \left(\frac{4}{3}\right) \times 3.5$

$$d = \frac{(21-14)}{3}$$

$$d = \frac{7}{3}$$

Hence the depth of the water before the sphere was put into the can is $\frac{7}{3}$ cm.