4. Co-ordinate Geometry: Distance Formula

Let us Work 4

1. Question

Let us calculate the distances of the following points from origin:

(i) (7, –24)

(ii) (3, –4)

(iii) (a + b, a – b)

Answer

The distance of a point (x,y) from origin is given as $\sqrt{x^2+y^2}$

(i) Distance of a point (7,-24) from origin $=\sqrt{7^2+(-24)^2}$ units

$=\sqrt{49+576}=\sqrt{625}$ units

= 25 units

(ii) Distance of a point (3,-4) from origin $=\sqrt{3^2+(-4)^2}$

$=\sqrt{9+16}=\sqrt{25}$ units

= 5 units

(iii) Distance of a point (a+b, a-b) from origin $=\sqrt{(a+b)^2+(a-b)^2}$

$=\sqrt{a^2+2ab+b^2+a^2-2ab+b^2}$

$=\sqrt{2(a^2+b^2)}$

2. Question

Let us calculate the distances between the two points given below:

(i) (5, 7) and (8, 3)

(ii) (7, 0) and (2, –12)

(iii) $\left(-\frac{3}{2}, 0\right)$ and (0, –2)

(iv) (3, 6) and (−2, −6)

(v) (1, −3) and (8, 3)

(vi) (5, 7) and (8, 3)

Answer

We know that distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(i) Distance between the points (5, 7) and (8, 3) $= \sqrt{(8-5)^2 + (3-7)^2}$

$= \sqrt{(3)^2 + (-4)^2}$

$= \sqrt{9 + 16} = \sqrt{25}$

= 5 units

(ii) Distance between the points (7, 0) and (2, -12)
$= \sqrt{(2-7)^2 + (-12-0)^2}$

$= \sqrt{(-5)^2 + (-12)^2}$

$= \sqrt{25 + 144} = \sqrt{169}$

= 13 units

(iii) Distance between the points $\left(-\frac{3}{2}, 0\right)$ and (0, -2)

$= \sqrt{\left(0 - \frac{-3}{2}\right)^2 + (-2-0)^2}$

$= \sqrt{\left(\frac{3}{2}\right)^2 + (-2)^2}$

$= \sqrt{\frac{9}{4} + 4} = \sqrt{\frac{9+16}{4}}$

$= \sqrt{\frac{25}{4}}$

$= \frac{5}{2}$ units

(iv) Distance between the points (3, 6) and (−2, −6)
$= \sqrt{(-2-3)^2 + (-6-6)^2}$

$= \sqrt{(-5)^2 + (-12)^2}$

$= \sqrt{25 + 144} = \sqrt{169}$

= 13 units

(v) Distance between the points (1, -3) and (8, 3)
$= \sqrt{(8-1)^2 + (3-(-3))^2}$

$= \sqrt{(7)^2 + (6)^2}$

$= \sqrt{49 + 36} = \sqrt{85}$

= 9.21 units

(vi) Distance between the points (5, 7) and (8, 3) $= \sqrt{(8-5)^2 + (3-5)^2}$

$= \sqrt{(3)^2 + (-4)^2}$

$= \sqrt{9 + 16} = \sqrt{25}$

= 5 units

3. Question

Let us prove that the point (–2, –11) is equidistant from the two points (–3, 7) and (4, 6).

Answer

We know that distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Using the above distance formula,

Distance between (–2, –11) and (-3, 7) $= \sqrt{(-3-(-2))^2 + (7-(-11))^2}$

$= \sqrt{(-1)^2 + (18)^2}$

$= \sqrt{1 + 324} = \sqrt{325}$

$= 5\sqrt{13}$

Distance between (–2, –11) and (4, 6) $= \sqrt{(4-(-2))^2 + (6-(-11))^2}$

$= \sqrt{(6)^2 + (17)^2}$

$= \sqrt{36 + 289} = \sqrt{325}$

$= 5\sqrt{13}$

Hence proved that the point (−2, −11) is equidistant from the two points (−3, 7) and (4, 6).

4. Question

Let us show that the points (7, 9), (3, −7) and (−3, 3) are the vertices of a right angled triangle by calculation.

Answer

Let A → (7, 9)

B → (3, -7)

C → (-3, 3)

be the vertices of a triangle.

We know that distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Using the above distance formula,

AB $= \sqrt{(3-7)^2 + (-7-9)^2}$

$= \sqrt{(-4)^2 + (-16)^2}$

$= \sqrt{16 + 256}$

$= \sqrt{272}$

BC $= \sqrt{(-3-3)^2 + (3-(-7))^2}$

$= \sqrt{(-6)^2 + (10)^2}$

$= \sqrt{36 + 100}$

$= \sqrt{136}$

CA $= \sqrt{(7-(-3))^2 + (9-3)^2}$

$= \sqrt{(10)^2 + (6)^2}$

$= \sqrt{100 + 36}$

$= \sqrt{136}$

Now, $(BC)^2 + (CA)^2 = \left(\sqrt{136}\right)^2 + \left(\sqrt{136}\right)^2 = 272$

and $(AB)^2 = \left(\sqrt{272}\right)^2 = 272$

We find that $(BC)^2 + (CA)^2 = (AB)^2$

Hence the points (7, 9), (3, −7) and (−3, 3) are the vertices of a right angled triangle.

5. Question

Let us prove that in both of the following cases, the three points are the vertices of an isosceles triangle:

(i) (1, 4), (4, 1) and (8, 8)

(ii) (−2, −2), (2, 2) and (4, −4)

Answer

(i) Let A → (1, 4)

B → (4, 1)

C → (8, 8)

be the vertices of a triangle.

We know that distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Using the above distance formula,

$AB = \sqrt{(4-1)^2 + (1-4)^2}$

$= \sqrt{(3)^2 + (-3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$BC = \sqrt{(8-4)^2 + (8-1)^2}$

$= \sqrt{(4)^2 + (7)^2}$

$= \sqrt{16+49}$

$= \sqrt{65}$

$CA = \sqrt{(1-8)^2 + (4-8)^2}$

$= \sqrt{(-7)^2 + (-4)^2}$

$= \sqrt{49 + 16}$

$= \sqrt{65}$

We find that BC = CA.

Since two sides of this triangle are equal, the points (7, 9), (3, −7) and (−3, 3) are the vertices of an isosceles triangle.

(ii) Let A → (-2, -2)

B → (2, 2)

C → (4, -4)

be the vertices of a triangle.

We know that distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Using the above distance formula,

AB $= \sqrt{(2 - (-2))^2 + (2 - (-2))^2}$

$= \sqrt{(4)^2 + (4)^2}$

$= \sqrt{16 + 16}$

$= \sqrt{32}$

BC $= \sqrt{(4 - 2)^2 + (-4 - 2)^2}$

$= \sqrt{(2)^2 + (-6)^2}$

$= \sqrt{4 + 36}$

$= 40$

CA $= \sqrt{(-2 - 4)^2 + (-2 - (-4))^2}$

$= \sqrt{(-6)^2 + (2)^2}$

$= \sqrt{36 + 4}$

$= \sqrt{40}$

We find that BC = CA.

Since two sides of this triangle are equal, the points (−2, −2), (2, 2) and (4, −4) are the vertices of an isosceles triangle.

6. Question

Let us prove that the three points A(3, 3), B(8, −2) and C(−2, −2) are the vertices of a right-angled isosceles triangle. Let us calculate the length of the hypotenuse of ΔABC.

Answer

Given A → (3, 3)

B → (8, -2)

C → (-2, -2)

are the vertices of a triangle.

We know that distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Using the above distance formula,

$AB = \sqrt{(8-3)^2 + (-2-3)^2}$

$= \sqrt{(5)^2 + (-5)^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50}$

$BC = \sqrt{(-2-8)^2 + (-2-(-2))^2}$

$= \sqrt{(-10)^2 + (0)^2}$

$= \sqrt{100}$

= 10

$CA = \sqrt{(3-(-2))^2 + (-2-3)^2}$

$= \sqrt{(5)^2 + (-5)^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50}$

Now, $(AB)^2 + (CA)^2 = \left(\sqrt{50}\right)^2 + \left(\sqrt{50}\right)^2 = 100$

and $(AB)^2 = (10)^2 = 100$

We find that $(AB)^2 + (CA)^2 = (BC)^2$ and AB = CA.

Hence proved that the three points A(3, 3), B(8, −2) and C(−2, −2) are the vertices of a right-angled isosceles triangle.

Length of hypotenuse BC = 10 units.

7. Question

Let us show by calculation that the points (2, 1), (0, 0), (−1, 2) and (1, 3) are the angular points of a square.

Answer

We have A → (2, 1)

B → (0, 0)

C → (-1, 2)

D → (1, 3)

We know that distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Using the above distance formula,

$AB = \sqrt{(0-2)^2 + (0-1)^2}$

$= \sqrt{(-2)^2 + (-1)^2}$

$= \sqrt{4+1}$

$= \sqrt{5}$

$BC = \sqrt{(-1-0)^2 + (2-0)^2}$

$= \sqrt{(-1)^2 + (2)^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$

$CD = \sqrt{(1-(-1))^2 + (3-2)^2}$

$= \sqrt{(2)^2 + 1^2}$

$= \sqrt{4+1}$

$= \sqrt{5}$

DA $= \sqrt{(2-1)^2 + (1-3)^2}$

$= \sqrt{(1)^2 + (-2)^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$

Now, consider the diagonals, AC and BD.

AC $= \sqrt{(-1-2)^2 + (2-1)^2}$

$= \sqrt{(-3)^2 + (1)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

BD $= \sqrt{(1-0)^2 + (3-0)^2}$

$= \sqrt{(1)^2 + (3)^2}$

$= \sqrt{1+9}$

$= \sqrt{10}$

We find that AB = BC = CD = DA with equal diagonals (AC = BD)

⇒ ABCD is a square.

8. Question

Let us calculate and see that for what value of y, the distance of the two points (2, y) and (10, –9) will be 10.

Answer

We know that distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Using the above distance formula,

Distance between (2, y) and (10, -9) $= \sqrt{(10-2)^2 + (-9-y)^2}$

$= \sqrt{(8)^2 + 81 - 18y + y^2}$

$= \sqrt{y^2 - 18y + 145}$

Given, distance of the two points (2, y) and (10, −9) = 10.

$\therefore \sqrt{y^2 - 18y + 145} = 10$

Squaring both sides, we get

$y^2 - 18y + 145 = 100$

$y^2 - 18y + 45 = 0$

On splitting the middle term,

$y^2 - 3y - 15y + 45 = 0$

$y(y - 3) - 15(y - 3) = 0$

$(y - 15)(y - 3) = 0$

So, y can take two values, 15 and 3.

9. Question

Let us find a point on x-axis which is equidistant from the two points (3, 5) and (1, 3).

Answer

Let (x, 0) be a point on the x-axis equidistant from the two points (3, 5) and (1, 3).

$\Rightarrow \sqrt{(x-3)^2 + (0-5)^2} = \sqrt{(x-1)^2 + (0-3)^2}$

$\Rightarrow \sqrt{x^2 - 6x + 9 + 25} = \sqrt{x^2 - 2x + 1 + 9}$

Squaring both sides

$x^2 - 6x + 34 = x^2 - 2x + 10$

$4x = 24$

$\therefore x = 6$

The required point is (6, 0).

10. Question

Let us write by calculation whether the three points O (0, 0), A(4, 3) and B(8, 6) are collinear.

Answer

We have O → (0, 0)

A → (4, 3)

B → (8, 6)

The distance of a point (x,y) from origin is given as $\sqrt{x^2 + y^2}$, so

OA $= \sqrt{4^2 + 3^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

= 5

and OB $= \sqrt{8^2 + 6^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100}$

= 10

We know that distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Using the above distance formula,

AB $= \sqrt{(8 - 4)^2 + (6 - 3)^2}$

$= \sqrt{(4)^2 + (3)^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

= 5

We find that,

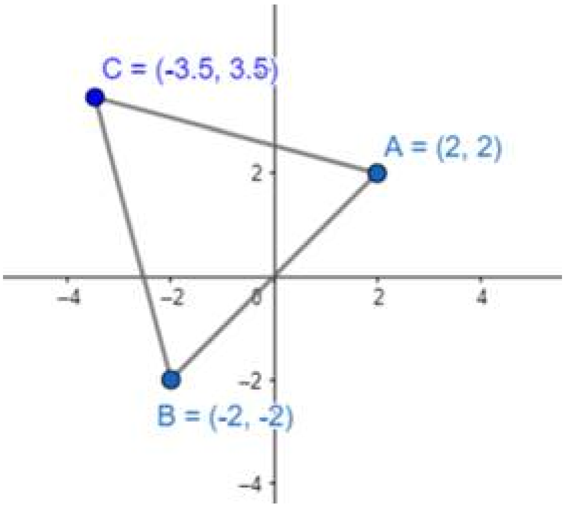
OA + AB = 5 + 5 = 10 = OB

⇒ The points A, B and C are collinear.

11. Question

Let us show that the three points (2, 2), (−2, −2) and $\left(-2\sqrt{3}, 2\sqrt{3}\right)$ are the vertices of an equilateral triangle.

Answer



Let us mark the points on a graph sheet and call them A, B, C and join the three points to form a triangle.

Using distance formula,

Let us calculate the distance between A-B, B-C and A-C.

The distance between two points, D = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AB = $\sqrt{[(2-(-2)]^2 + [2-(-2)]^2}$

$= \sqrt{(4)^2 + (4)^2} = \sqrt{32}$

BC = $\sqrt{[(-2)-(-2\sqrt{3})]^2 + [(-2)-(2\sqrt{3})]^2}$

$= \sqrt{\left[(-2)^2 + \left(-2\sqrt{3}\right)^2 - 2(-2)\left(-2\sqrt{3}\right)\right] + \left[(-2)^2 + \left(2\sqrt{3}\right)^2 - 2(-2)\left(2\sqrt{3}\right)\right]}$

$= \sqrt{4 + 12 - 8\sqrt{3} + 4 + 12 + 8\sqrt{3}} = \sqrt{32}$

CA = $\sqrt{[(-2\sqrt{3}) - (2)]^2 + [(2\sqrt{3}) - (2)]^2}$

$= \sqrt{12 + 4 + 8\sqrt{3} + 12 + 4 - 8\sqrt{3}} = \sqrt{32}$

AB=BC=CA

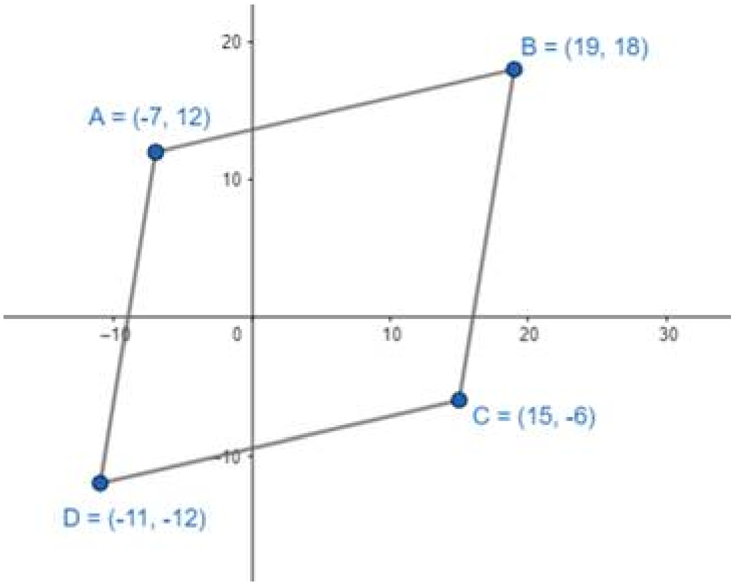
Hence, Triangle ABC is equilateral.

12. Question

Let us show that the points (−7, 12), (19, 18), (15, −6) and (−11, −12) form a parallelogram when they are joined orderly.

Answer

Let us mark the points on a graph sheet and call them A, B, C, D and join the four points in the same order to form a four-sided figure.



A = (- 7, 12)

B = (19, 18)

C= (15, - 6)

D= (- 11, - 12)

We know that slope of a line is given by $\frac{y_2 - y_1}{x_2 - x_1}$

Slopes of lines AB, BC, CD and DA are calculated as given below.

Slope (AB) $= \frac{18-12}{19+7} = 0.23$

Slope (BC) $= \frac{-6-18}{15-19} = 6$

Slope (CD) $= \frac{-12+6}{-11-15} = 0.23$

Slope (DA) $= \frac{12+12}{-7+11} = 6$

We see that slope of AB = Slope of CD

And

Slope of BC = Slope of DA

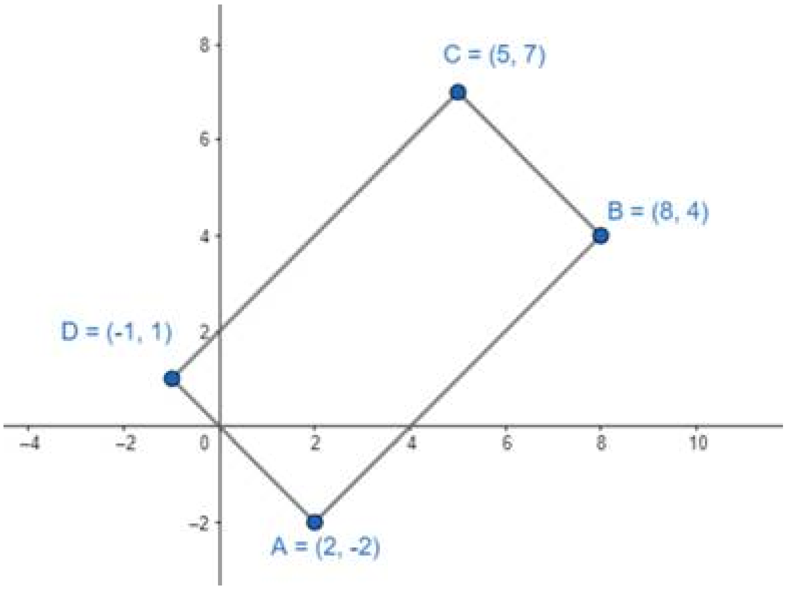
So AB || CD and BC || DA

$\Rightarrow$ABCD is a parallelogram.

13. Question

Let us show that the points (2, −2), (8, 4), (5, 7) and (−1, 1) are the vertices of a rectangle.

Answer



Let

A = (2, - 2)

B = (8, 4)

C = (5, 7)

D = (- 1, 1)

Distance between two points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$AB = \sqrt{\left[(8-2)^2 + \left(4-(-2)\right)^2\right]} = \sqrt{72}$$

$$BC = \sqrt{([(5-(8)]^2 + [7-(4)]^2)} = \sqrt{18}$$

$$CD = \sqrt{[(-1-(5)]^2 + [1-(7)]^2} = \sqrt{72}$$

$$DA = \sqrt{[(2-(-1)]^2 + [-2-(1)]^2} = \sqrt{18}$$

Let’s join A and C, and B and D.

$$AC = \sqrt{(5-2)^2 + \left(7-(-2)\right)^2} = \sqrt{90}$$

$$BD = \sqrt{[(-1-(8)]^2 + [1-(4)]^2} = \sqrt{90}$$

Since AB = CD, BC=AD and AC=BD, it is a rectangle.

(Opposite sides are equal; diagonals are equal)

14. Question

Let us show that the points (2, 5), (5, 9), (9, 12) and (6, 8) form a rhombus when they are joined orderly.

Answer

Let

A = (2, 5)

B = (5, 9)

C = (9, 12)

D = (6, 8)

Distance between two points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AB = $\sqrt{[(5-(2)]^2 + [9-(5)]^2} = 5$

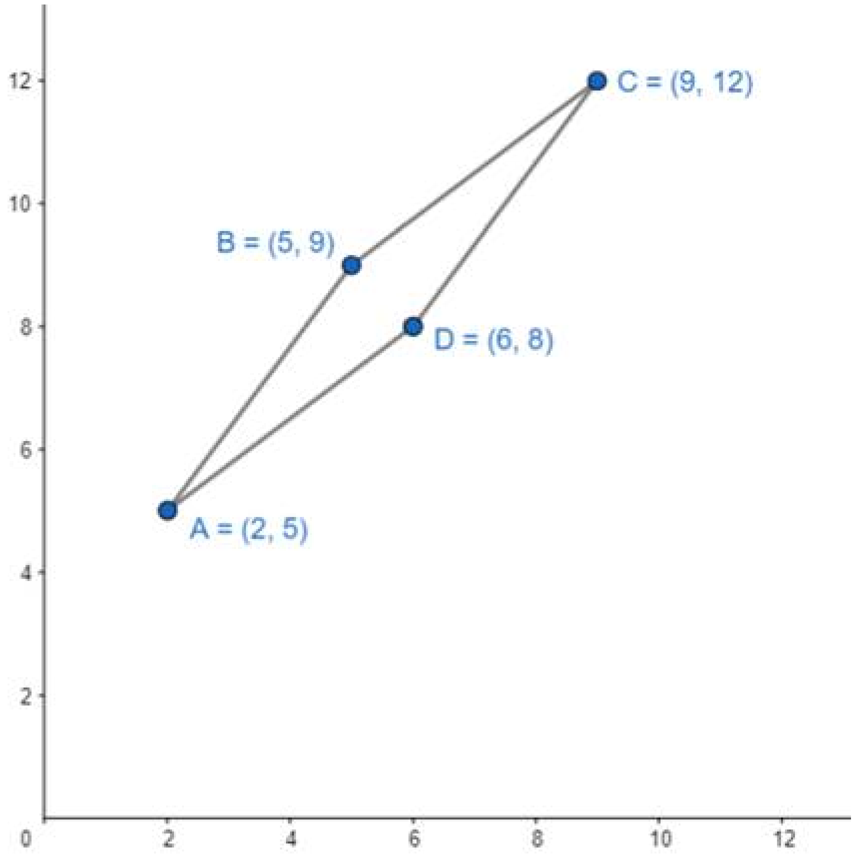
BC = $\sqrt{[(9-(5)]^2 + [12-(9)]^2} = 5$

CD = $\sqrt{[(6-(9)]^2 + [8-(12)]^2} = 5$

DA = $\sqrt{[(2-(6)]^2 + [5-(8)]^2} = 5$

Since all the sides are equal, the polygon qualifies as a rhombus.

The rhombus formed will look like:



15 A. Question

The distance between the two points (a + b, c – d) and (a – b, c + d) is

A. $2\sqrt{a^2 + c^2}$

B. $2\sqrt{b^2 + d^2}$

C. $\sqrt{a^2 + c^2}$

D. $\sqrt{b^2 + d^2}$

Answer

Distance between two points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{[(a - b) - (a + b)]^2 + [(c + d) - (c - d)]^2}$

= $\sqrt{4b^2 + 4d^2}$

= $2\sqrt{b^2 + d^2}$

15 B. Question

If the distance between the two points (x, –7) and (3, –3) is 5 units, then the values of x are

A. 0 or 3

B. 2 or 3

C. 5 or 1

D. −6 or 0

Answer

Distance between two points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$5 = \sqrt{(3 - x)^2 + (-3 + 7)^2}$

Squaring on both sides,

$25 = (3 - x)^2 + (-3 + 7)^2$

$25 = x^2 - 3x + 25$

Or

$x^2 - 3x = 0$

$= x(x - 3) = 0$

$x = 0$ or $x = 3$

15 C. Question

If the distance of the point (x, 4) from the origin is 5 units, then the values of x are

A. ± 4

B. ±

C. ± 3

D. none of these

Answer

Distance between two points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here the second point is the origin.

$5 = \sqrt{(0 - x)^2 + (0 - 4)^2}$

Squaring on both sides,

$25 = x^2 + 16$

Or

x= + 3 or - 3

15 D. Question

The triangle formed by the points (3, 0), (−3, 0) and (0, 3) is

A. Equilateral

B. Isosceles

C. Scalene

D. Isosceles right-angled

Answer

Here we need to calculate each side of the triangle.

Considering, A=(3,0), B=(0,3) and C=(- 3,0)

We have,

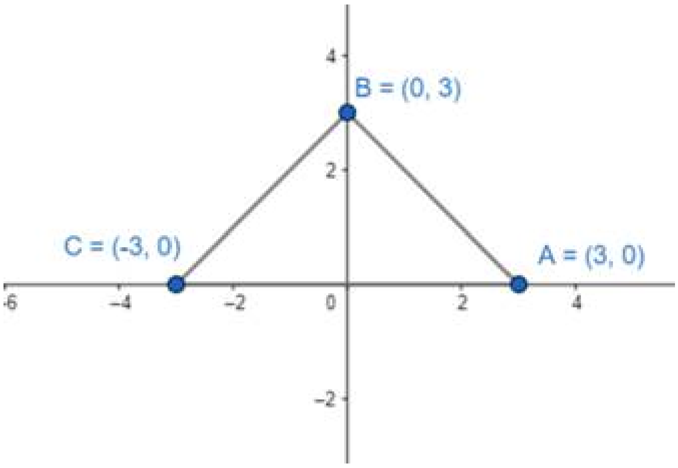
$$AB = \sqrt{(0-3)^2 + (3-0)^2} = \sqrt{18}$$

$$BC = \sqrt{(-3-0)^2 + (0-3)^2} = \sqrt{18}$$

$$AC = \sqrt{(-3-3)^2 + (0-0)^2} = \sqrt{36}$$

Since the two sides AB and BC are equal, so the triangle is isosceles.

Refer figure.



15 E. Question

The co - ordinates of the center of the circle are (0, 0) and the co - ordinates of a point on the circumference are (3, 4), the length of the circle is

A. 5 units

B. 4 units

C. 3 units

D. None of these

Answer

The length of the circle or radius is simply the distance between the two points.

The distance between two points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here the first point is the origin.

R = $\sqrt{(3-0)^2 + (4-0)^2}$

= 5 units

16. Question

Short answer type questions:

(i) Let us write the value of y if the distance of the point (−4, y) from the origin is 5 units.

(ii) Let us write the co - ordinates of a point on y - axis which is equidistant from two points (2, 3) and (−1, 2).

(iii) Let us write the coordinates of two points on the x-axis and y-axis for which an isosceles right-angled triangle is formed with the x-axis, y-axis and the straight line joining the two points.

(iv) Let us write the co - ordinates of two points on opposite sides of x - axis which are equidistant from x - axis.

(v) Let us write the co - ordinates of two points on opposite sides of y - axis which are equidistant from y - axis.

Answer

(i) Distance between two points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here the second point is origin.

5 = $\sqrt{(0-(-4))^2 + (y-0)^2}$

Squaring on both sides,

25 = 16 + y^2

Or y = + 3 or - 3.

(ii) A point on y-axis will have x coordinate 0.

So, let the point on y-axis be (0, y) .

If this point is equidistant from given two points,

$\sqrt{(0-2)^2 + (y-3)^2} = \sqrt{(0+1)^2 + (y-2)^2}$

Squaring on both sides,

$(y - 3)^2 + 4 = (y - 2)^2 + 1$

Or $y^2 - 3y + 9 + 4 = y^2 - 2y + 4 + 1$

$13 - 5 = y$

$y = 8$

So the coordinate on the y-axis is (0, 8)

(iii) Let the point on x-axis be A (x, 0)

Let the point on y-axis be B (0, y)

To satisfy the conditions given in the problem,

The distance of A from origin should be the same as the distance of B from origin.

The distance between two points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(0-x)^2 + (0-0)^2} = \sqrt{(0-0)^2 + (0-y)^2}$

Or x= + y or x = - y

The coordinates should be (0, x) and (x, 0) where x is any real number.

(iv) (x, 0) and (- x, 0)

(v) (0, y) and (0, - y)