FUNCTIONS

13

INTRODUCTION

Quantities of various characters such as length, area, mass, temperature and volume either have constant values or they vary based on the values of other quantities. Such quantities are called constant and variable respectively.

Function is a concept of Mathematics that studies the dependence between variable quantities in the process of their change. For instance, with a change in the side of a square, the area of the square also varies. The question of how the change in the side of the square affects the area is answered by a mathematical relationship between the area of the square and the side of the square.

Let the variable *x* take on numerical values from the set *D*.

A function is a rule that attributes to every number *x* from *D* one definite number *y* where *y* belongs to the set of real numbers.

Here, x is called the independent variable and y is called the dependent variable.

The set D is referred to as the *domain of definition* of the function and the set of all values attained by the variable y is called the *range of the function*.

In other words, a variable y is said to be the value of function of a variable x in the domain of definition D if to each value of x belonging to this domain there corresponds a definite value of the variable y.

This is symbolised as $y = f(x)$, where f denotes the rule by which y varies with x.

BASIC METHODS OF REPRESENTING FUNCTIONS

Analytical representation

This is essentially representation through a formula.

This representation could be a uniform formula in the entire domain, for example, $y = 3x^2$

Or

By several formulae which are different for different parts of the domain.

Example: $y = 3x^2$ if $x < 0$

and $y = x^2$ if $x > 0$

In analytical representations, the domain of the function is generally understood as the set of values for which the equation makes sense.

For instance, if $y = x^2$ represents the area of a square then we get that the domain of the function is $x > 0$.

Problems based on the analytical representation of functions have been a favourite for the XLRI exam and have also become very common in the CAT over the past few years. Other exams are also moving towards asking questions based on this representation of functions.

Tabular Representation of Functions

For representing functions through a table, we simply write down a sequence of values of the independent variable x and then write down the corresponding values of the dependent variable y. Thus, we have tables of logarithms, trigonometric values and so forth, which are essentially tabular representations of functions.

The types of problems that appear based on tabular representation have been restricted to questions that give a table and then ask the student to trace the appropriate analytical representation or graph of the function based on the table.

Graphical Representation of Functions

This is a very important way to represent functions. The process is: on the coordinate xy-plane for every value of x from the domain D of the function, a point P(x, y) is constructed whose abscissa is x and whose ordinate y is obtained by putting the particular value of x in the formula representing the function.

For example, for plotting the function $y = x2$, we first decide on the values of x for which we need to plot the graph.

Thus, we can take $x = 0$ and get $y = 0$ (means the point (0, 0) is on the graph).

Then for $x = 1, y = 1$; for $x = 2, y = 4$; for $x = 3, y = 9$ and for $x = –1, y = 1$; for $x = –2, y = 4$, and so on.

EVEN AND ODD FUNCTIONS

Even Functions

Let a function $y = f(x)$ be given in a certain interval. The function is said to be even if for any value of x

$$\rightarrow f(x) = f(-x);$$

Properties of even functions:

(a) The sum, difference, product and quotient of an even function are also even functions.

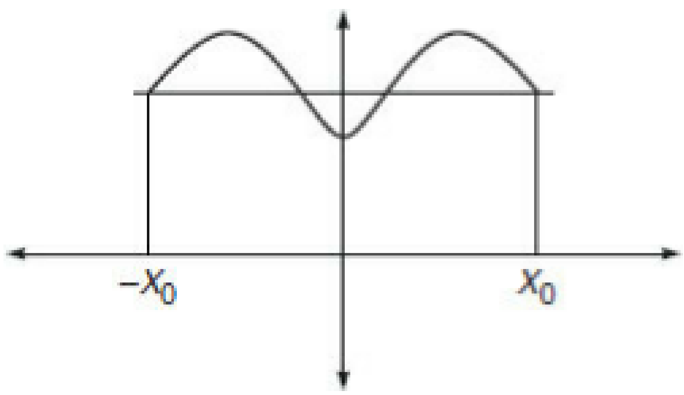
(b) The graph of an even function is symmetrical about the y-axis.

However, when y is the independent variable, it is symmetrical about the x-

axis. In other words, if $x = f(y)$ is an even function, then the graph of this function will be symmetrical about the x-axis. Example: $x = y^2$.

Examples of even functions: $y = x^2$, $y = x^4$, $y = -3x^8$, $y = x^2 + 3$, $y = x^{4/5}$, $y = |x|$ are all even functions.

The symmetry about the y-axis of an even function is illustrated below.



Odd Functions

Let a function $y = f(x)$ be given in a certain interval. The function is said to be odd, if for any value of x

$$f(x) = -f(-x)$$

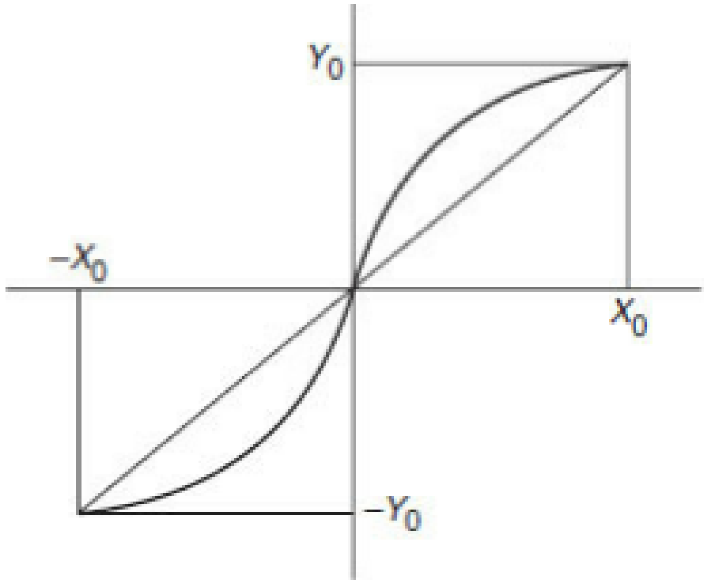
Properties of odd functions:

(a) The sum and difference of an odd function is an odd function.

(b) The product and quotient of an odd function is an even function.

(c) The graph of an odd function is symmetrical about the origin.

The symmetry about the origin of an odd function is illustrated below.



Examples of odd functions $y = x^3$, $y = x^5$, $y = x^3 + x$, $y = x/(x^2 + 1)$.

Not all functions need be even or odd. However, every function can be represented as the sum of an even function and an odd function.

Inverse of a function

Let there be a function $y = f(x)$, which is defined for the domain D and has a range R.

Then, by definition, for every value of the independent variable x in the domain D, there exists a certain value of the dependent variable y. In certain cases, the same value of the dependent variable y can be obtained for different values of x. For example, if $y = x^2$, then for $x = 2$ and $x = -2$ give the value of y as 4.

In such a case, the inverse function of the function $y = f(x)$ *does not exist*.

However, if a function $y = f(x)$ is such that for every value of y (from the range of the function R), there corresponds one and only one value of x from the domain D, then the inverse function of $y = f(x)$ exists and is given by $x = g(y)$. Here it can be noticed that x becomes the dependent variable and y becomes the independent variable. Hence, this function has a domain R and a range D.

Under the above situation, the graph of $y = f(x)$ and $x = g(y)$ are one and the same.

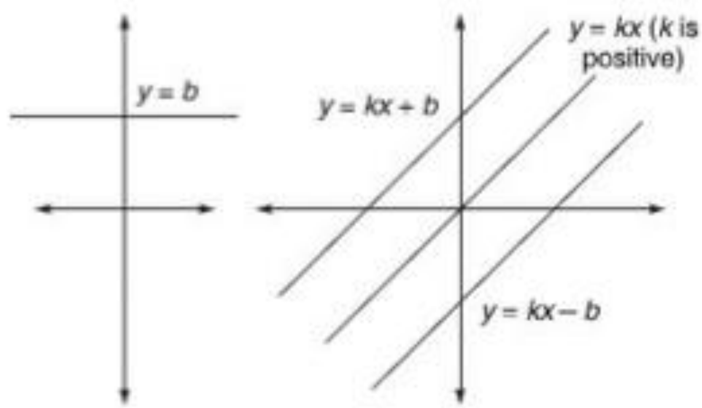
However, when denoting the inverse of the function, we normally denote the independent variable by y and, hence, the inverse function of $y = f(x)$ is denoted by $y = g(x)$ and not by $x = g(y)$.

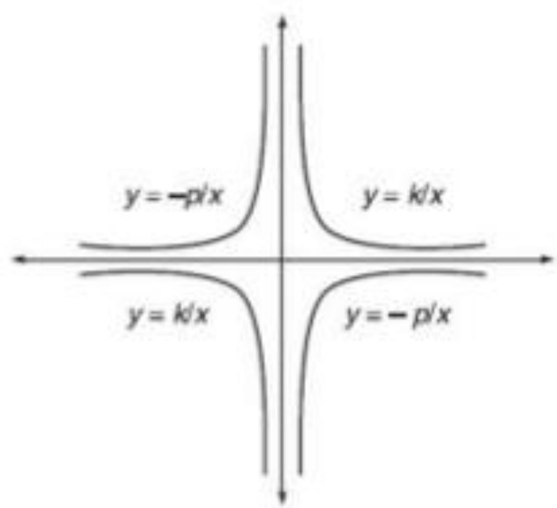
The graphs of two inverse functions, when this change is used are symmetrical about the line $y = x$ (which is the bisector of the first and the third quadrants).

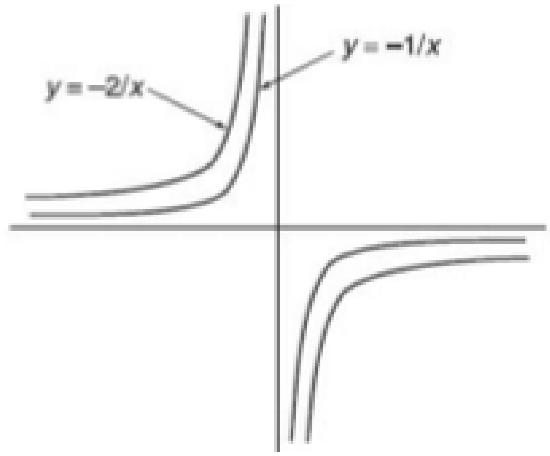
Graphs of Some Simple Functions The student is advised to familiarise himself/herself with the following figures.

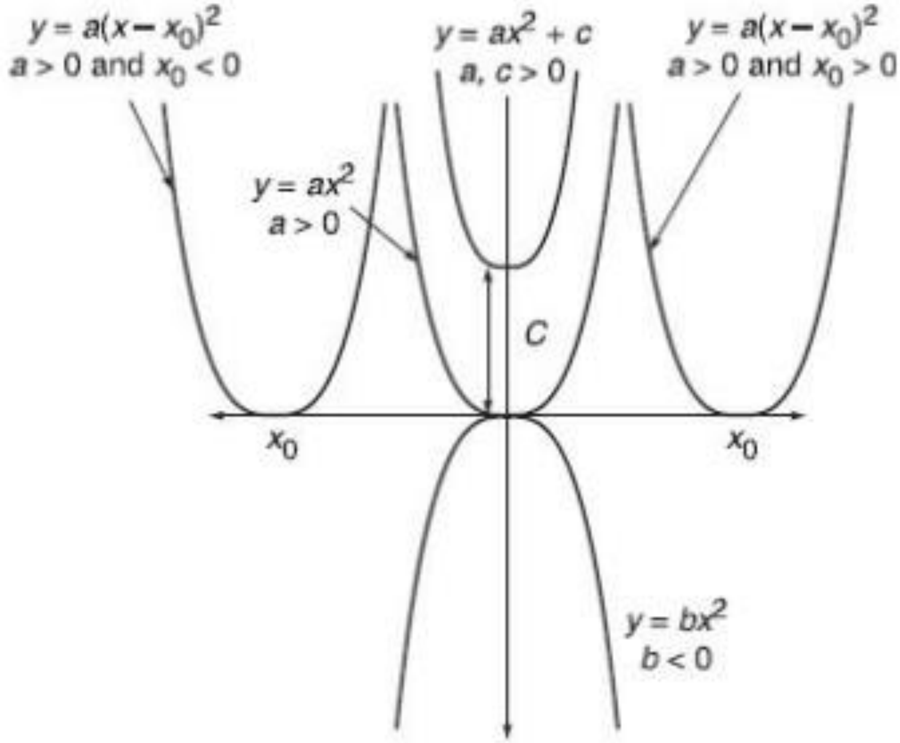
Graphs of $y = b$, $y = kx$, $y = kx + b$, $y = kx - b$

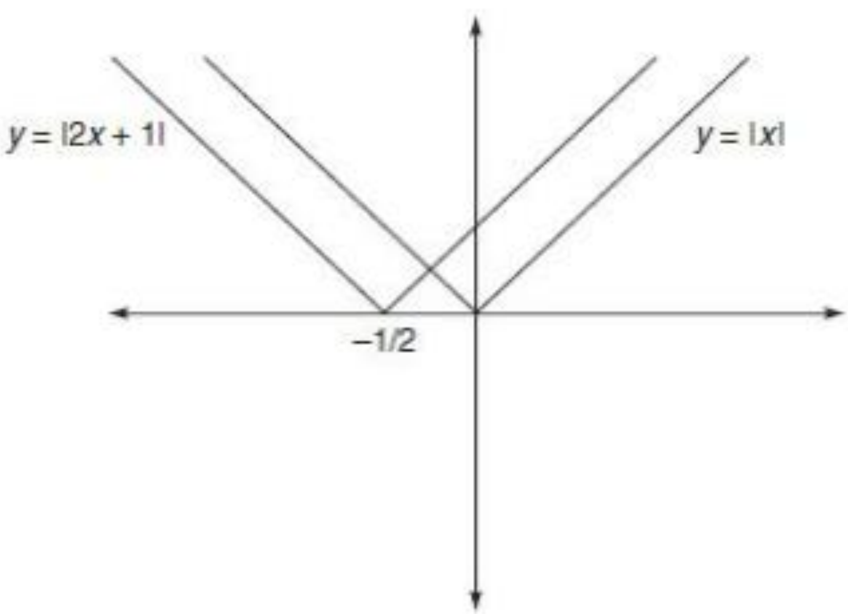
Note the shifting of the line when a positive number b is added and subtracted to the function's equation.






y = −1/x
y = −2/x


y = a(x − x0)^2
a > 0 and x0 < 0
y = ax^2 + c
a, c > 0
y = a(x − x0)^2
a > 0 and x0 > 0
y = ax^2
a > 0
C
x0
x0
y = bx^2
b < 0


y = |2x + 1|
y = |x|
−1/2

SHIFTING OF GRAPHS

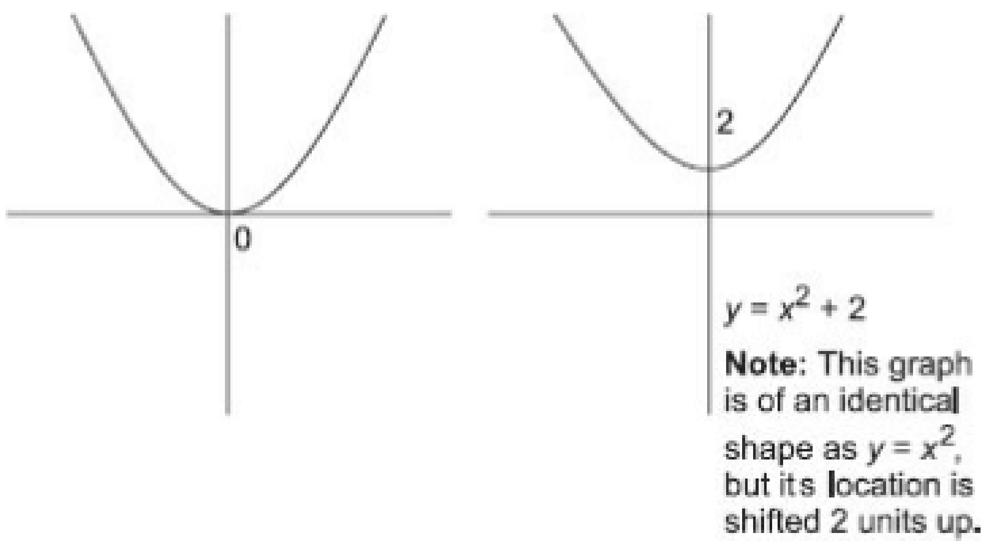
The ability to visualise how graphs shift when the basic analytical expression is changed is a very important skill. For instance, if you knew how to visualise the graph of $(x + 2)^2 - 5$, it will definitely add a lot of value to your ability to solve questions of functions and all related chapters of block V graphically.

In order to be able to do so, you first need to understand the following points clearly:

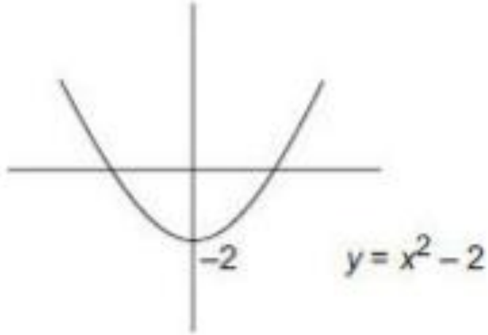
1. **Relationship between the graph of $y = f(x)$ and $y = f(x) + c$ (where c represents a positive constant.):** The shape of the graph of $y = f(x) + c$ will be the same as that of the $y = f(x)$ graph. The only difference would be in terms of the fact that $f(x) + c$ is shifted c units up on the $x - y$ plot.

   The following figure will make it clear for you:

   Example: Relationship between $y = x^2$ and $y = x^2 + 2$

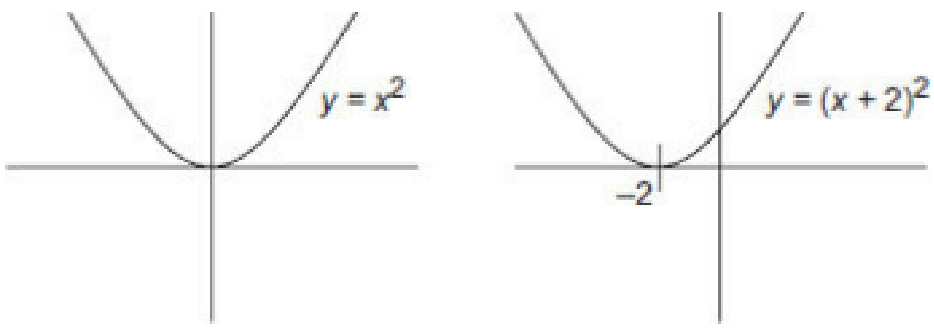


2. **Relationship between $y = f(x)$ and $y = f(x) - c$:** In this case, while the shape remains the same, the position of the graph gets shifted c units down.

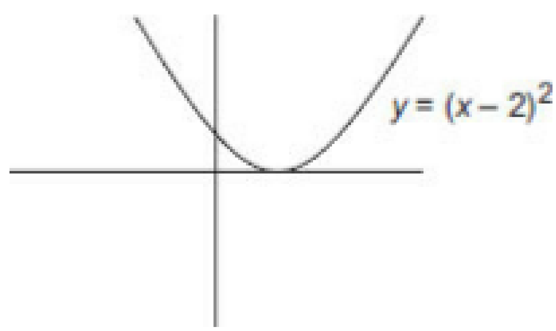


3. **Relationship between $y = f(x)$ and $y = f(x + c)$:** In this case, the graph will get shifted c units to the left. (Remember, c is a positive constant)

Example:



4. **Relationship between $y = f(x)$ and $y = f(x - c)$:** In this case, the graph will get shifted c units to the right on the $x - y$ plane.



COMBINING MOVEMENTS

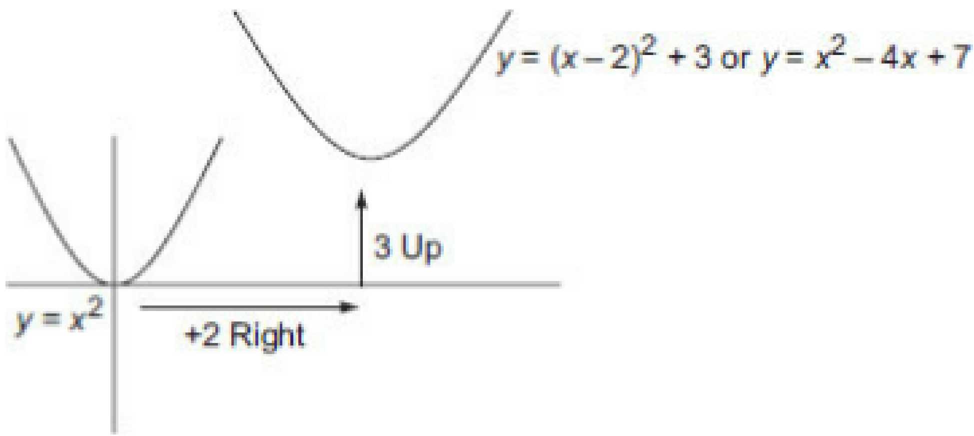
It is best understood through an example:

Visualising a graph for a function like $x_2 - 4x + 7$

First convert $x^2 - 4x + 7$ into $(x - 2)^2 + 3$

Note: In order to do this conversion, the key point of your thinking should be on the $-4x$. Your first focus has to be to put down a bracket $(x - a)^2$ which on expansion gives $-4x$ as the middle term. When you think this way, you will get $(x - 2)^2$. On expansion, $(x - 2)^2 = x^2 - 4x + 4$. But you wanted $x^2 - 4x + 7$. Hence, add $+3$ to $(x - 2)^2$. Hence the expression $x^2 - 4x + 7$ is equivalent to $(x - 2)^2 + 3$.

To visualise $(x - 2)^2 + 3$ shift the x^2 graph two units right [to account for $(x - 2)^2$] and 3 units up [to account for the +3] on the $x - y$ plot. This will give you the required plot.



Task for the student: The students are challenged and encouraged to think of how to add and multiply functions graphically.

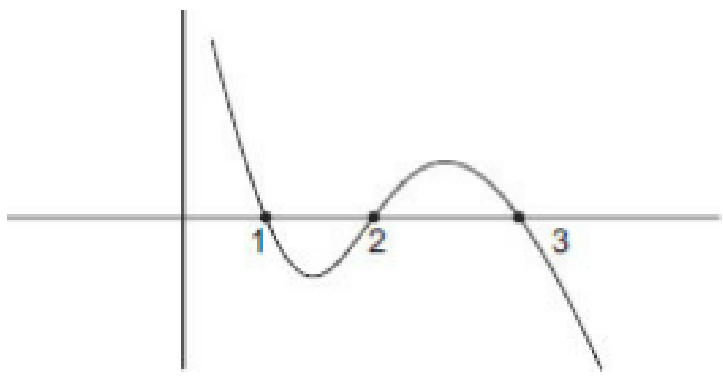
INEQUALITIES

Logical Graphical Process for Solving Inequalities

Your knowledge of the standard graphs of functions and how these shift can help you immensely while solving inequalities.

Thus, for instance if you are given an inequality question based on a quadratic will be *U* shaped. And the inequality would be satisfied between the roots of the quadratic equation $ax_2 + bx + c = 0$. [Remember, we have already seen and understood that the solution of an equation $f(x) = 0$ is seen at the points where the graph of $y = f(x)$ cuts the x axis.]

Similarly, for a cubic curve like the one shown below, you should realise that it is greater than 0 to the left of the point 1 shown in the figure. This is also true between points 2 and 3. At the same time, the function is less than zero between points 1 and 2 and to the right of point 3. (on the x-axis).

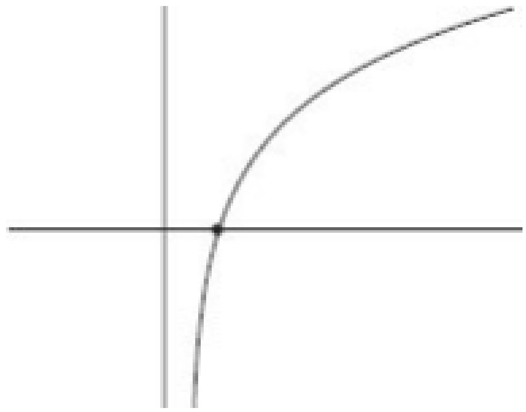


Another important point to note is that in the case of strict inequalities (i.e. inequalities with the '<' or '>' sign), the answer will also consist of strict inequalities only. On the other hand, in the case of slack inequalities (inequalities having £ or ≥ sign), the solution of the inequality will also have a slack inequality sign.

LOGARITHMS

Graphical View of the Logarithmic Function

The typical logarithmic function is shown in the graph below:



Note the following points about the logarithmic function, $y = \log x$.

1. It is only defined for positive values of x.
2. For values of x below 1, the logarithmic function is negative. At the same time for $x = 1$, the logarithmic function has a value of 0. (Irrespective of the base)
3. The value of $\log x$ becomes 2, when the value of x becomes equal to the square of the base.
4. As we go further right on the x-axis, the graph keeps increasing. However, this increase becomes more and more gradual and hence, the shape of the graph becomes increasingly flatter as we move further on the x-axis.

WORKED-OUT PROBLEMS

Problem 13.1 Find the domain of the definition of the function $y = 1/(x^2 - 2x)^{1/2}$

(a) $(-\infty, -2)$

(b) $(-\infty, +\infty)$ except $[0, 2]$

(c) $(2, +\infty)$

(d) $(-\infty, 0)$

Solution: For the function to be defined, the expression under the square root should be non-negative and the denominator should not be equal to zero.

So, $x^2 - 2x > 0$ and $(x^2 - 2x) \neq 0$

or, $x(x - 2) > 0$ or $(x^2 - 2x) \neq 0$

So, x will not lie in between 0 and 2 and $x \neq 0, x \neq 2$.

So, x will be $x(-\infty, +\infty)$ excluding the range $0 \leq x$ to ≤ 2.

In exam situations, to solve the above problem, you should check the options as below.

In fact, for solving all questions on functions, the student should explore the option-based approach.

Often you will find that going through the option-based approach will help you save a significant amount of time. The student should try to improve his/her selection of values through practice so that he/she is able to eliminate the maximum number of options on the basis of every check. The student should develop a knack for disproving three options so that the fourth automatically becomes the answer. It should also be noted that if an option cannot be disproved, it means that it is the correct option.

The meaning of what has been said will be clear from the following solution process.

For this question, if we check at $x = 3$, the function is defined. However, $x = 3$ is outside the ambit of options *a* and *d*. Hence, a and *d* are rejected on the basis of just one value check, and *b* or *c* has to be the answer.

Alternately, you can try to disprove each and every option one by one.

Problem 13.2 Which of the following is an even function?

(a) $|x^2| - 5x$

(b) $x^4 + x^5$

(c) $e^{2x} + e^{-2x}$

(d) $| x |^2/x$

Solution: Use options for solving.

If a function is even, it should satisfy the equation $f(x) = f(-x)$.

We now check the four options to see which of them represents an even function.

Checking option (a): $f(x) = | x^2 | - 5x$

Putting $(-x)$ in the place of x.

$$f(x) = | (-x)^2 | - 5(-x)$$

$$= | x^2 | + 5(x) \neq f(x)$$

Checking option (b) $f(x) = x^4 + x^5$

Putting $(-x)$ in the place of x,

$$f(-x) = (-x)^4 + (-x)^5 = x^4 - x^5 \neq f(x)$$

Checking option (c): $f(x) = e^{2x} + e^{-2x}$

Putting $(-x)$ in the place of x.

$$f(-x) = e^{-2x} + e^{-(-2x)} = e^{-2x} + e^{2x} = f(x)$$

So (c) is the answer.

You do not need to go further to check for (d). However, if you had checked, you would have been able to disprove it as follows:

Checking option (d): $f(x) = |x|^2/x$

Putting $f(-x)$ in the place of x,

$$f(-x) = | -x |^2/-x = | x |^2/-x \neq f(-x)$$

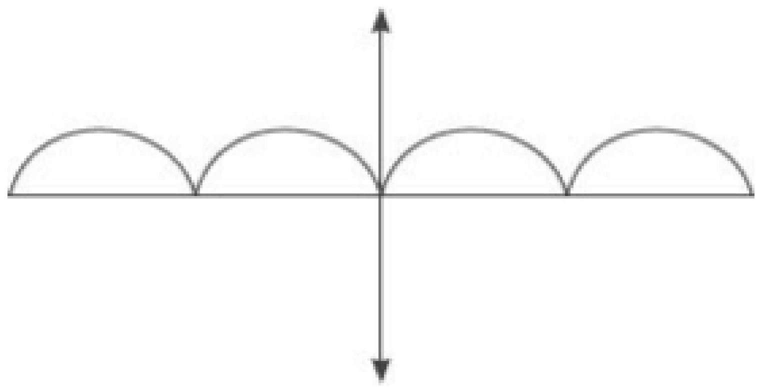
Directions for Problem 13.3 to 13.6:

Mark (a), if f $(-x) = f(x)$

Mark (b), if f $(-x) = -f(x)$

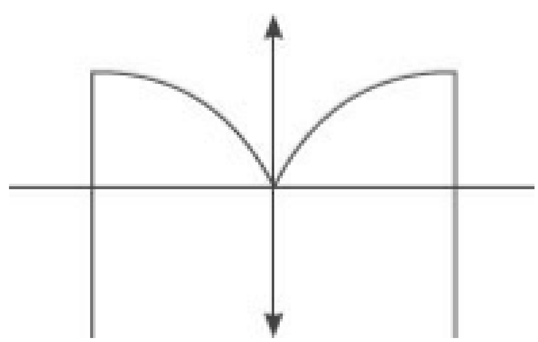
Mark (c), if neither (a) nor (b)

Problem 13.3



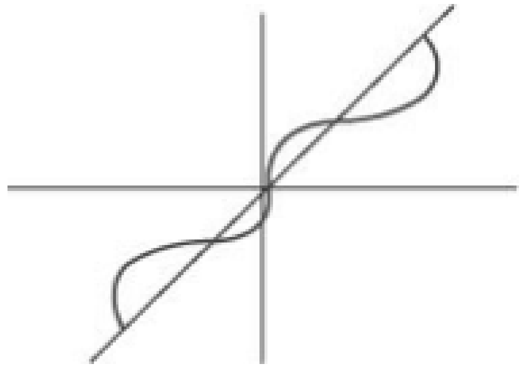
Solution: The graph is symmetrical about the y-axis. This is the definition of an even function. So (a) is the answer.

Problem 13.4



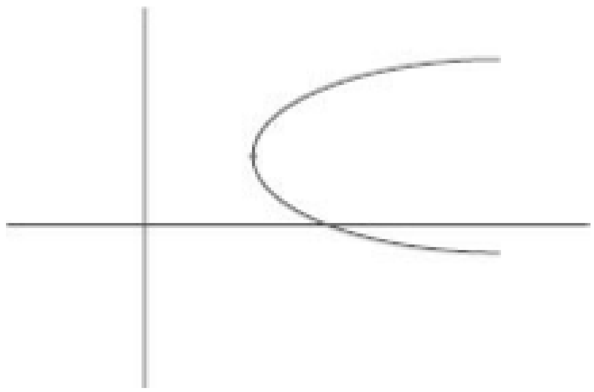
Solution: The graph is symmetrical about the y-axis. This is the definition of an even function. So (a) is the answer.

Problem 13.5



Solution: The graph is symmetrical about origin. This is the definition of an odd function. So (b) is the answer.

Problem 13.6



Solution: The graph is neither symmetrical about the y-axis nor about origin. So (c) is the answer.

Problem 13.7 Which of the following two functions are identical?

(a) $f(x) = x2/x$

(b) $g(x) = (\sqrt{x})^2$

(c) $h(x) = x$

(i) (a) and (b)

(ii) (b) and (c)

(iii) (a) and (c)

(iv) None of these

Solution: For two functions to be identical, their domains should be equal.

Checking the domains of $f(x)$, $g(x)$ and $h(x)$,

$f(x) = x^2/x$, x should not be equal to zero.

So, domain will be all real numbers except at $x = 0$.

$g(x) = (\sqrt{x})^2$, x should be non-negative.

So, domain will be all positive real numbers.

$h(x) = x$, x is defined everywhere.

So, we can see that none of them has the same domain.

Hence, (d) is the correct option.

Problem 13.8 If $f(x) = 1/x$, $g(x) = 1/(1 - x)$ and $h(x) = x^2$, then find *fogoh* (2).

(a) –1

(b) 1

(c) 1/2

(d) None of these

Solution: *fogoh* (2) is the same as $f(g(h(2)))$

To solve this, open the innermost bracket first. This means that, we first resolve the function $h(2)$. Since $h(2) = 4$, we will get

$f(g(h(2))) = f(g(4)) = f(-1/3) = -3$. Hence, the option (d) is the correct answer.

Directions for Problems 13.9 and 13.10: Read the instructions below and solve Problems 13.9 and 13.10.

$A * B = A^3 - B^3$

$A + B = A - B$

$A - B = A/B$

Problem 13.9 Find the value of $(3 * 4) - (8 + 12)$.

(a) 9

(b) 9.25

(c) –9.25

(d) None of these

Solution: Such problems should be solved by the BODMAS rule for sequencing of operations.

Solving, thus, we get: (3 * 4) – (8 + 12)

= –37 – (–4). (**Note:** The '–' sign between –37 and –4 is the operation defined above.)

= 37/4 = 9.25

Problem 13.10 Which of the following operation will give the sum of the reciprocals of x and y and unity?

(a) $(x + y) * (x - y)$

(b) $[(x * y) - x] - y$

(c) $(x + y) - (x - y)$

(d) None of these

For solving questions containing a function in the question as well as a function in the options (where values are absent), the safest process and most effective is to assume certain convenient values of the variables in the expression and checking for the correct option that gives us equality with the expression in the question. The advantage of this process of solution is that there is very little scope for making mistakes. Besides, if the expression is not simple and directly visible, this process takes far less time as compared to simplifying the expression from one form to another.

This process will be clear after perusing the following solution to the above problem.

Solution: The problem statement above defines the expression: $(1/x) + (1/y) + 1$ and asks us to find out which of the four options is equal to this expression. If we try to simplify, we can start from the problem expression and rewrite it to get the correct option. However, in the above case this will become extremely complicated since the symbols are indirect. Hence, if we have to solve through simplifying, we should start from the options one-by-one and try to get the problem expression. However, this is easier said than done and for this particular problem, going through this approach will take you at least two minutes plus.

Hence, consider the following approach:

Take the values of x and y as 1 each. Then,

$$(1/x) + (1/y) + 1 = 3$$

Put the value of x and y as 1 each in each of the four options that we have to consider.

Option (a) will give a value of $-1 \neq 3$. Hence, option (a) is incorrect.

Option (b) will give a value of 0: $0 \neq 3$. Hence, option (b) is incorrect.

Option (c) gives an answer of 0: $0 \neq 3$. Hence, option (c) is incorrect.

Now since options (a), (b) and (c) are incorrect and option (d) is the only possibility left, it has to be the answer.

LEVEL OF DIFFICULTY (I)

1. The points of intersection of the graphs of the functions $f(x) = 2x2 - 1$ and $g(x) = 1 - 3x$ are

   (a) $\{2, -1/2\}$

   (b) $\{-2, ½\}$

   (c) $\{1, 2\}$

(d) $\{-2, -1/2\}$

2. If $f(x) = 1/x$, $g(x) = 1/(1 - x)$ and $h(x) = x2$, then find *fogoh* (2).

(a) -1

(b) 1

(c) 1/2

(d) None of these

3. If $f(x) = \sqrt{x^3}$, then $f(3x)$ will be equal to

(a) $\sqrt{3x^3}$

(b) $3\sqrt{x^3}$

(c) $3\sqrt{(3x^3)}$

(d) $3\sqrt{x^5}$

4. Which of the following is not an odd function?

(a) $(x + 1)3$

(b) $x23$

(c) $x53$

(d) $x77$

Directions for Questions 5 to 7: Read the instructions below and solve.

$f(x) = f(x - 2) - f(x - 1)$, where x is a natural number

$f(1) = 0, f(2) = 1$

5. The value of $f(8)$ is

(a) 0

(b) 13

(c) –5

(d) –9

6. The value of $f(7) + f(4)$ is

(a) 11

(b) –6

(c) –12

(d) 12

7. What will be the value of $\sum_{n=1}^{9} f(n)$??

(a) –12

(b) –15

(c) –14

(d) –13

8. What will be the domain of the definition of the function $f(x) = {}^{8-x}C_{5-x}$ for positive values of x?

(a) {1, 2, 3}

(b) {1, 2, 3, 4, 5}

(c) {0, 1, 2, 3, 4, 5}

(d) {1, 2, 3, 4, 5, 6, 7, 8}

Directions for Questions 9 to 12

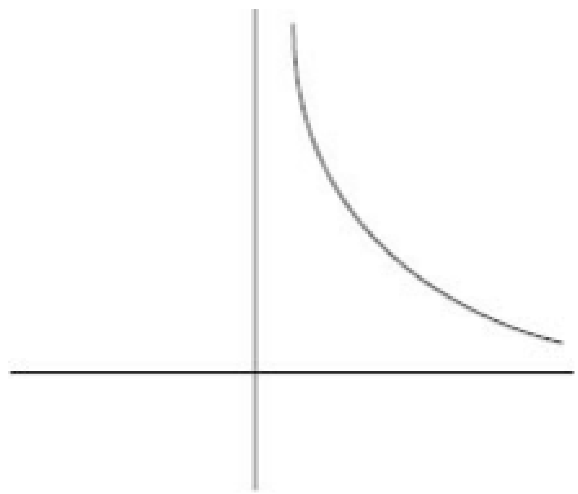
Mark a, if $f(-x) = f(x)$

Mark b, if $f(-x) = -f(x)$

Mark c, if neither a nor b is true

Mark d, if $f(x)$ does not exist at least one point of the domain

9.



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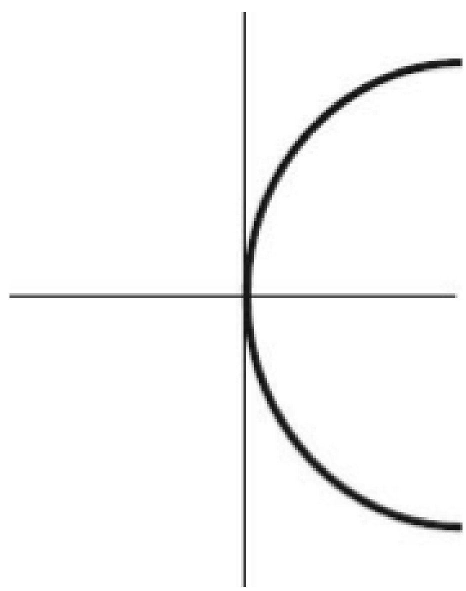
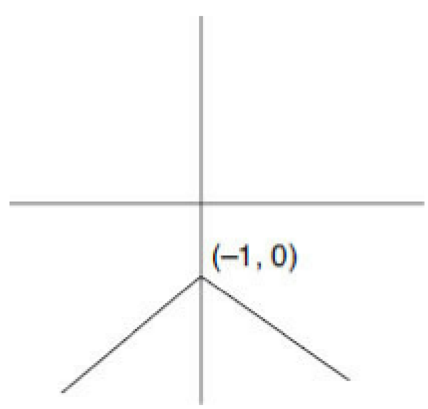


11.



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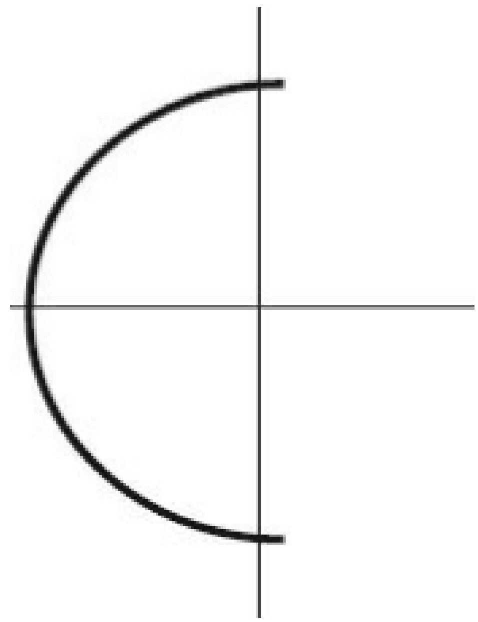


13. If $f(x) = 2x + 5$ and $h(x) = (x - 5)/2$, the value of $foh(x) = hof(x)$. What is the value of hofofohohof (x) × fohofoh(x)?

(a) $x - 5$

(b) $(2x + 5)/x + 5$

(c) x

(d) $x2$

14. For the above question, what is the value of $fo(foh)o(hof)(x)$?

(a) $2x + 5$

(b) x

(c) $x - 5$

(d) $x - 2$

15. Let $f(x) = |x - 4| + |x - 5| + |x - 6|$ and $A(x) = f(x + 1)$. Then

(a) $A(x)$ is an even function

(b) $A(x)$ is an odd function

(c) $A(x)$ is neither even or odd

(d) None of these

16. The inverse function of the function $f(x) = (e^x - e^{-x})/(e^x + e^{-x})$ is

(a) $y = \frac{1}{2} \log_e \frac{x+1}{1-x}$

(b) $y = \frac{1}{2} \log_e \frac{x+2}{2-x}$

(c) $y = \frac{1}{2} \log_e \frac{1-x}{1+x}$

(d) $y = \frac{1}{2} \log_a \frac{1-x}{1+x}$

17. The graph of the function $y = f(x)$ is symmetrical about the line $x - 3 = 0$, then

(a) $f(x + 3) = f(x - 3)$

(b) $f(3 + x) = f(3 - x)$

(c) $f(x) = f(-x)$

(d) $f(x) = 2f(x)$

18. If $f(x) = x^5 - 3x^4 + 2x^2 - 4x + a$, where '$a$' is an integer and $f(1)$ and $f(2)$ are of opposite signs, then which of the following is necessarily true?

(a) $5 < a < 16$

(b) $5 \leq a \leq 16$

(c) $5 \leq a \leq 15$

(d) None of these

19. Find the sum of the coefficients of the polynomial $(1 + x - 3x^2)^{2163}$.

(a) –2

(b) –1

(c) 0

(d) 1

20. If $f(x) = |x - 2|$, then

(a) $f(x^2) = [f(x)]^2$

(b) $f(|x|) = f(x)$

(c) $f(x + y) = f(x) + f(y)$

(d) None of these

21. If $f(a,b,c) = \min\{\min(a,b),\min(b,c),\min(c,a)\}$

$g(a,b,c) = \max\{\max(a,b),\max(a,c),\max(b,c)\}$

Find $f(a,b,c) - g(a,b,c)$, if a, b and c are in an AP with a common difference of 2?

(a) –4 or 4

(b) 4

(c) –4

(d) None of these

Directions for Questions 22 and 23: In these questions, the following functions are defined: Min (p, q) = least of p, q

Max (p, q) = larger of p, q

G(p, q) = GCD of p, q

L(p, q) = LCM of p, q

P(p, q) = product of p, q

22. What will be the maximum value of the expression $L(1, P(3,\max(3, p)))$ and for what value (or value range) of p would it occur?

(a) $-\infty$ when $p > 39$

(b) $3p$, when $p > 3$

(c) 0 when $p = 0$

(d) None of these

23. $P(9, \max(4, \min(L(12, 9), P(5, 3))))$. The value of above expression is

(a) 120

(b) 105

(c) 135

(d) 115

24. Let $f(x) = ax/(x + 1)$, x not equal to -1. Find the value of a for which $f(f(x)) = x$.

(a) 20.5

(b) -20.5

(c) 1

(d) -1

25. Find the domain of the definition of the function $y(x)$ given by the definition $2x + 2y = 2$.

(a) $0 < x < 1$

(b) $0 < x < 1$

(c) $x < 0$

(d) $x < 1$

26. If $f(x) = 2x^2 + 7x - 9$ and $g(x) = 2x + 3$, then find the value of $g(f(x))$ at $x = 3$.

(a) 63

(b) 46

(c) 57

(d) 49

27. Find the range of x, for which the function $y = (169 - x^2)^{1/2}$ is defined.

(a) $0 \le x \le 13$

(b) $(-\infty, -13] \cup [13, \infty)$

(c) $-13 \le x$

(d) $-13 \le x \le 13$

28. Which of the following is an even function?

(a) x^{12}

(b) $(x - 3)^2$

(c) $(x + 3)^4$

(d) More than one of the above

29. Find the inverse of the function $f(x) = 1/(x - 1)$.

(a) $(x - 1)$

(b) $(x + 1)$

(c) $1/(1 - x)$

(d) $(1 + x)/x$

30. Consider $f(x) = f(x - 1) + f(x - 2)$, x is a positive integer.

$f(1) = 2, f(2) = 3$

Now, find the value of $f(6)$.

(a) 21

(b) 13

(c) 19

(d) None of these

31. What will be the value of $f(7) - f(4)$ considering the function described in the previous question?

(a) 21

(b) 13

(c) 26

(d) None of these

Directions for Questions 32 to 36: Following functions are defined:

$x \$ y = x_2 + y_2$ $\quad$ $x \% y = (x - 1)(y - 1)$

$x \& y = (x + 1)(y + 1)$

32. Find the value of $(a \$ b)\%(a \& b)$, when it is given that $a = 2, b = 3$.

(a) 42

(b) 123

(c) 132

(d) None of these

33. What is the value of [(4\$3)%6]&[5\$(2%7)] ?

(a) 7502

(b) 7320

(c) 7140

(d) None of these

34. Which of the following operation will yield 3 as the result for p = 2 and q = 4?

(a) $p\$q$

(b) $p\%q$

(c) $p\&q$

(d) More than one of the above

35. For what values of a and b will $a\$b$ give 100 as the result?

(a) a = 101, b = 2

(b) a = 99, b = 0

(c) a = 8, b = 6

(d) More than one of the above

36. Which of the following will always be true for all real values of x and y?

(a) $x \$ y \geq 0$

(b) $x \% y \geq 0$

(c) $x \& y \geq 0$

(d) None of these

37. Find the maximum value of the function $1/(x^2 - 3x + 2)$.

(a) 11/4

(b) 1/4

(c) 0

(d) None of these

38. Find the minimum value of the function $f(x) = \log_2 (x^2 - 2x + 5)$.

(a) – 4

(b) 2

(c) 4

(d) – 2

Directions for Questions 39 to 43: Define the following functions:

(i) $a @ b = \frac{a+b}{2}$

(ii) $a \# b = a^2 - b^2$

(iii) $(a ! b) = \frac{a-b}{2}$

39. Find the value of {[(3@4)!(3#2)] @ [(4!3)@(2#3)]}.

(a) –0.75

(b) –1

(c) –1.5

(d) –2.25

40. Find the value of (4#3)@(2!3).

(a) 3.25

(b) 3.5

(c) 6.5

(d) 7

41. Which of the following has a value of 0.25 for a = 0 and b = 0.5?

(a) a @ b

(b) a # b

(c) Either a or b

(d) Cannot be determined

42. Which of the following expressions has a value of 4 for a = 5 and b = 3?

(a) $\frac{(a!b)}{(a\#b)}$

(b) $(a!b)(a@b)$

(c) $\frac{(a\#b)}{(a!b)(a@b)}$

(d) Both (b) and (c)

43. If we define $a\$b$ as $a^3 - b^3$, then for integers $a, b > 2$ and $a > b$ which of the following will always be true?

(a) $(a@b) > (a!b)$

(b) $(a@b) \geq (a!b)$

(c) $(a\#b) < (a\$b)$

(d) Both (a) and (c)

Directions for Questions 44 to 48: Define the following functions:

(a) $(a\ M\ b) = a - b$

(b) $(a\ D\ b) = a + b$

(c) $(a\ H\ b) = (ab)$

(d) $(a\ P\ b) = a/b$

44. Which of the following functions will represent $a2 - b2$?

(a) $(a\ M\ b)\ H\ (a\ D\ b)$

(b) $(a\ H\ b)\ M\ (a\ P\ b)$

(c) $(a\ D\ b)/(a\ M\ b)$

(d) None of these

45. Which of the following represents $a2$?

(a) $(a\ M\ b)\ H\ (a\ D\ b) + b2$

(b) $(a\ H\ b)\ M\ (a\ P\ b) + b2$

(c) $\frac{(a\ M\ b)}{(a\ P\ b)}$

(d) Both (a) and (c)

46. What is the value of $(3M4H2D4P8M2)$?

(a) 6.5

(b) 6

(c) –6.5

(d) None of these

47. Which of the four functions defined has the maximum value?

(a) ($a\ M\ b$)

(b) ($a\ D\ b$)

(c) ($a\ P\ b$)

(d) Cannot be determined

48. Which of the four functions defined has the minimum value?

(a) ($a\ M\ b$)

(b) ($a\ D\ b$)

(c) ($a\ H\ b$)

(d) Cannot be determined

49. If $0 < a < 1$ and $0 < b < 1$ and $a > b$, which of the four expressions will take the highest value?

(a) ($a\ M\ b$)

(b) ($a\ D\ b$)

(c) ($a\ P\ b$)

(d) Cannot be determined

50. If $0 < a < 1$ and $0 < b < 1$ and if $a < b$, which of the following expressions will have the highest value?

(a) ($a\ M\ b$)

(b) ($a\ D\ b$)

(c) $(a\ P\ b)$

(d) Cannot be determined

51. A function $F(n)$ is defined as $F(n-1) = \frac{1}{(2-F(n))}$ for all natural numbers 'n'.

If $F(1) = 3$, then

what is the value of $[F(1)] + [F(2)] + \ldots + [F(1000)]$?

(Here, $[x]$ is equal to the greatest integer less than or equal to 'x')

(a) 1001

(b) 1002

(c) 3003

(d) None of these

52. For the above question, find the value of the expression: $F(1) \times F(2) \times F(3) \times F(4) \times \ldots F(1000)$.

(a) 2001

(b) 1999

(c) 2004

(d) 1997

53. A function $f(x)$ is defined for all real values of x as $f(x) = ax2 + bx + c$. If $f(3) = f(-3) = 18$, $f(0) = 15$, then what is the value of $f(12)$?

(a) 63

(b) 159

(c) 102

(d) None of these

54. Two operations, for real numbers x and y, are defined as given below.

(i) $M(x \theta y) = (x + y)2$

(ii) $f(x \Psi y) = (x - y)2$

If $M(x2 \theta y2) = 361$ and $M(x2 \Psi y2) = 49$, then what is the value of the square root of $((x2\ y2) + 3)$?

(a) ±81

(b) ±9

(c) ±7

(d) ±11

55. The function $\Psi(m) = [m]$, where $[m]$ represents the greatest integer less than or equal to m. Two real numbers x and y are such that $\Psi(4x + 5) = 5y + 3$ and $\Psi(3y + 7) = x + 4$, then find the value of $x2 \times y2$.

(a) 1

(b) 2

(c) 4

(d) None of these

56. A certain function always obeys the rule: If $f(x.y) = f(x).f(y)$, where x and y are positive real numbers. A certain Mr. Mogambo found that the value of $f(128) = 4$, then find the value of the variable $M = f(0.5).f(1).f(2).f(4).f(8).f(16).f(32).f(64).f(128).f(256)$.

(a) 128

(b) 256

(c) 512

(d) 1024

57. x and y are non-negative integers such that $4x + 6y = 20$, and $x^2 \leq M/y^{2/3}$ for all values of x, y. What is the minimum value of M?

(a) $2^{2/3}$

(b) $2^{1/3}$

(c) $2^{8/3}$

(d) $4^{2/3}$

58. Let $\Psi(x) = \frac{x+3}{2}$ and $\theta(x) = 3x^2 + 2$. Find the value of $\theta\ (\Psi(-7))$.

(a) 12

(b) 14

(c) 50

(d) 42

59. If $F(a + b) = F(a). F(b) \div 2$, where $F(b) \neq 0$ and $F(a)\pi\ 0$, then what is the value of $F(12b)$?

(a) $(F(b))^{12}$

(b) $F(b))^{12} \div 2$

(c) $(F(b))^{12} \div 2^{12}$

(d) $(F(b))^{12} \div 2^{11}$

60. A function $a = \theta(b)$ is said to be reflexive if $b = \theta(a)$. Which of the following is/are reflexive functions?

(i) $\frac{3b+5}{4b-3}$

(ii) $\frac{3b+5}{5b-2}$

(iii) $\frac{2b+12}{12b-2}$

(a) All of these are reflexive

(b) Only (i) and (ii) are reflexive

(c) Only (i) and (iii) are reflexive

(d) None of these are reflexive

61. $f(x)=\frac{1}{x}, g(x)=|3x-2|$

Then $f(g(x))=?$

(a) $\frac{1}{|3x-2|}$

(b) $\left|\frac{1}{3x}-2\right|$

(c) $\frac{1}{|3x|}-2$

(d) None of these

Directions for Questions 62 to 66:

$f(x)=x^2+\frac{1}{x^2}$, $g(x)=|x|$, $h(x)=x^3-\frac{1}{x^3}$, $t(x),=x^2-\frac{1}{x^2}$, , then answer the following questions:

62. $f(g(x))$ is an:

(a) Even function

(b) Odd function

(c) Neither even nor odd

63. Which of the following options is true?

(a) $f(x) = g(x) + (g(x))^2$

(b) $f(x) = -f(g(x))$

(c) $f(g(x)) = g(f(x))$

(d) None of these

64. Out of $f(x)$, $g(x)$, $h(x)$, $t(x)$ how many are even functions?

65. Is $h(f(x))$ an even function or odd function? Type 1 if it is even; Type 2 if it is odd; Type 3 if it is neither even nor odd.

66. Is $h(t(x))$ an even function or an odd function or neither even nor odd? Type 1 if it is even; Type 2 if it is odd; Type 3 if it is neither even nor odd.

67. If $f(x) = \frac{x^2+1}{x-1}$, then- $f(f(f(2))) = 2$.

Directions for Questions 68 and 69:

$S(x, y) = x + y$

$P(x, y) = x \times y$

$D(x, y) = x/y$

$t(x, y) = |x - y|$

68. Find the value of $P(S(2,(D(3, 4)), 5)$.

69. $S(S(P(2,3), D(4,2)), t(1, 5))$ = ?

Directions for Questions 70 to 72: Define the following functions as:

$$xPy = \begin{cases} |x-y|, \text{if } x < y \\ xQy, \text{otherwise} \end{cases}$$

$$xQy = \begin{cases} \frac{x}{y}, \text{if } x > y \\ xRy, \text{otherwise} \end{cases}$$

$$xRy = \begin{cases} x \times y, if\ x \le y \\ xSy, otherwise \end{cases}$$

$$xSy = \begin{cases} \frac{1}{xy}, \text{if } x > y \\ xPy, \text{otherwise} \end{cases}$$

Here x & y are real numbers. Solve the following questions based on these definitions of the above functions.

70. Find the value of $[(5P6)Q(4Q2)]S(3S1)$.

71. Which of the following is true?

(a) $(4P2) \neq (2P4)$

(b) $(4Q2) = (2R4)$

(c) $(6Q3) = (2S(0.5))$

(d) None of these

72. If $(5P3)Q(4S2) = 20K1.5$. What is the correct operator to replace the 'K'?

Type 1 if your answer is 'P'; Type 2 if your answer is 'Q'; Type 3 if your answer is 'R'; Type 4 if your answer is 'S'

Directions for Questions 73 to 75:

$[x]$ is defined as the greatest integer less than equal to x.

$\{x\}$ is defined as the least integer greater than or equal to x

The functions f, g, h and i are defined as follows:

$f(a, b) = [a] + \{b\}$

$g(a,b) = [b] - \{a\}$
$h(a,b) = [a \div b]$
$i(a,b) = \{-a + b\}$

73. Find the value of $i(f(3,4)), g(3.5, 4.5))$.

(a) 2

(b) 1

(c) −1

(d) None of these

74. If $a^3 = 64, b^2 = 16$ and $8 + f(a,b) = -g(a,b)$ then $a - b$ =?

75. If $f(1.2,-2.3) + g(-1.2, 2.3) = i(a,-1.3)$,, then which of the following values can a take?

(a) −4.3

(b) −5.3

(c) 5.6

(d) −2.4

Directions of Questions 76 to 77:

Define the functions: $xPy = \frac{1}{1+\frac{y}{x}}, xQy = 1 + \frac{x}{y}$

76. Which of the following equals to $\frac{x}{y}$?

(a) (*xPy*) + (*xQy*)

(b) (*xPy*) − (*xQy*)

(c) (*xPy*) × (*xQy*)

(d) $(xPy) \div (xQy)$

77. If the functions $S(x, y) = (xPy)P(xQy)$ is defined, then find the value of $S(2, 3)$ (correct to two decimal points)?

Directions for Questions 78 to 80:

Define the functions:

$$aAb = |a - b|$$
$$aBb = [a \div b]$$
$$aCb = |a \times b|$$

$$\min(x, y) = \begin{cases} y, \text{when } x > y \\ x, \text{when } y > x \\ 0, \text{when } x = y \end{cases}$$

$$\max(x, y) = \begin{cases} x^2, \text{when } x \geq y \\ y^2, \text{when } y \geq x \end{cases}$$

Here $a, b, x, y \in R$..

78. Find the value of $(1 + \min(2A3, 1C2))B[\max(1A2, 1C1)]$.

79. The value of $\max(7A3, 16B2)$ will be equal to the value of which of the following options?

(a) $(32B16)C(\max(4,8))$

(b) $(32B2)C(\min(4,8))$

(c) $(16B2)C((\min(4,8))$

(d) None of these

80. $\max(3,4) \div \min(8, 4) = ?$

(a) $8A2$

(b) $28B7$

(c) 4C2

(d) None of these

Directions for Questions 81 to 85:

$f(a_1, a_2, a_3, \ldots, a_n)$ = minimum of $(a_1, a_2, \ldots a_n)$ and $g(a_1, a_2, a_3, \ldots a_n)$ = maximum of $(a_1, a_2, a_3 \ldots a_n)$

$h(x, y) = [x/y]$, where $[a]$ represents the greatest integer less than or equal to a.

$$t(a_1, a_2, a_3, \ldots, a_n) = a_1 \times a_2 \times a_3 \times \ldots \times a_n$$
$$i(a_1, a_2, a_3, \ldots, a_n) = a_1 + a_2 + a_3 + \ldots + a_n$$

81. If $f(1,3,5,7) + g(2,4,6,8) = h(aK, K)$

    Where a, K are both positive integers, then find the value of a.

82. The value of $f(t(1,2,3,4), i(1,2,3,4)) = ?$

83. The value of $h(f(5,6,7,8), i(1,2,3,4)) = ?$

84. If $P = f(2,3,4,6)$, $Q = g(1,2,3,4)$, $R = h(8,4)$, $S = t(1,2,3,4)$, $T = i(4,5,6)$ then which of the following options is true?

    (a) $P < R = Q < S < T$

    (b) $P = R < Q < S < T$

    (c) $P = R < Q < T < S$

    (d) $P = R < Q = T < S$

85. $f(f(1,2,3), g(2,3,4), f(0,1,2), g(-3,-2)) = ?$

    (a) −3

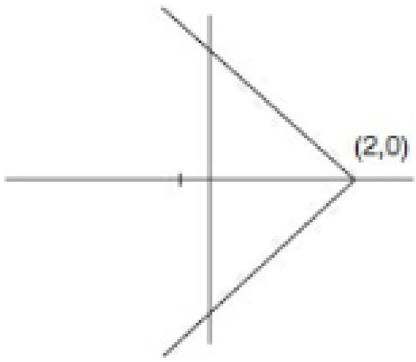
    (b) −2

(c) 1

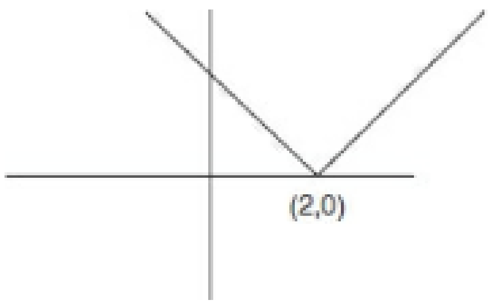
(d) 0

86. Which of the following curves correctly represents $y = |x - 2|$?

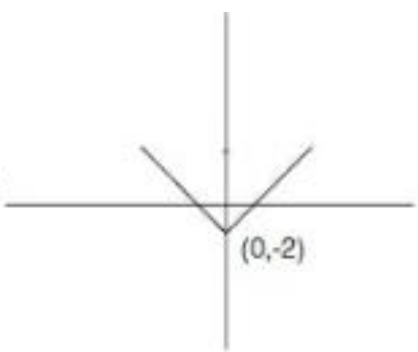
(a)



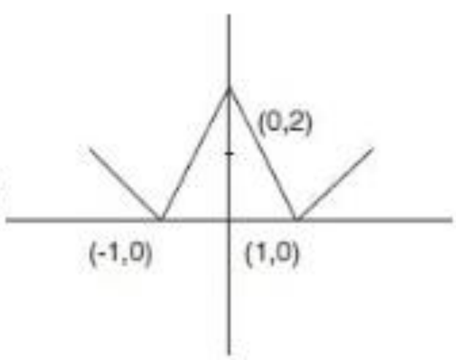
(b)



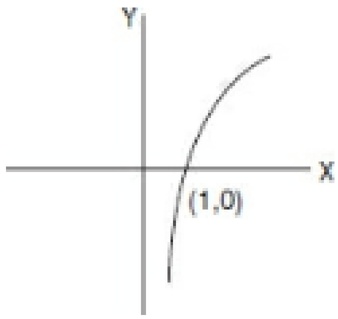
(c)

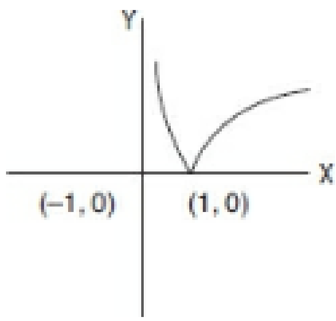


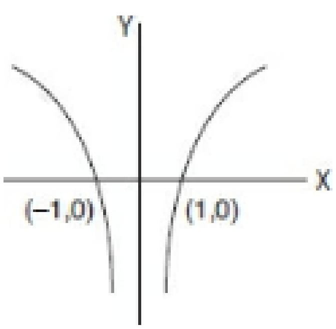
(d)



87. Which of the following represents the curve of $y = \log|x|$?

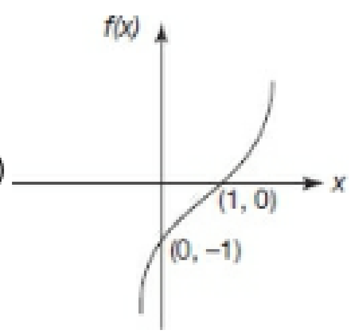
(a) 

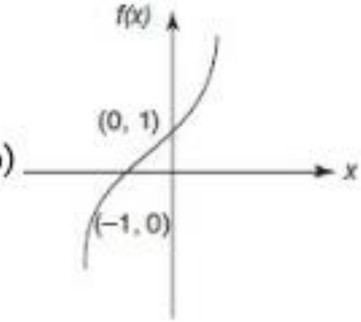
(b) 

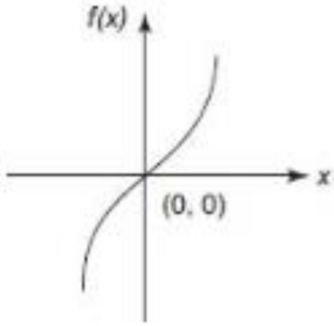
(c) 

(d) None of these

88. Which of the following options correctly represents the curve of $f(x) = (x - 1)^3$?

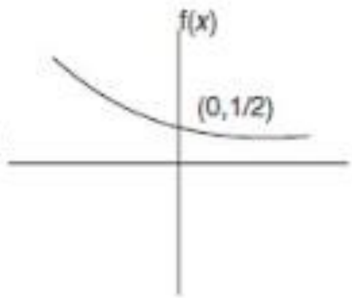
(a) 

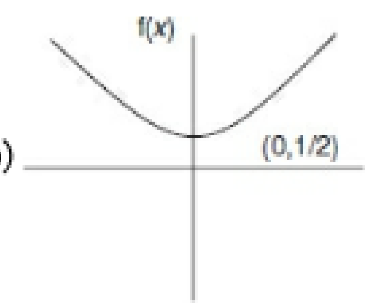
(b) 

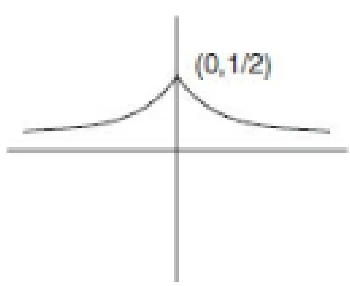
(c) 

(d) None of these

89. Which of the following options correctly represents the curve $f(x)=\frac{e^{|x|}}{2}$??

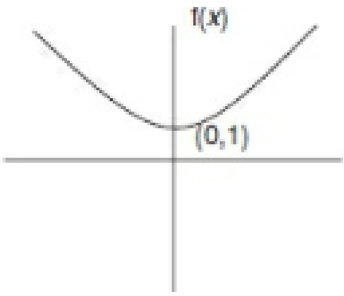
(a) 

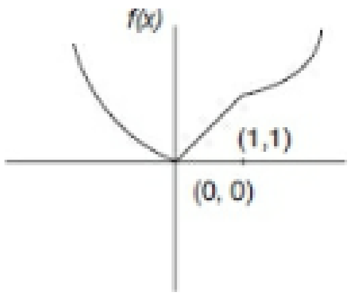
(b) 

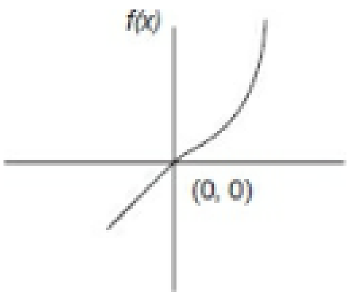
(c) 

(d) None of these

90. Which of the following options correctly represents the curve of $f(x) = \max(x, x^2)$?

(a) 

(b) 

(c) 

(d) None of these

91. Which of the following statements is true?

(a) If $f(x)$ and $g(x)$ are odd functions then their sum is an even function.

(b) If $f(x)$ and $g(x)$ are even functions then their sum is an odd function.

(c) If $f(x)$ and $g(x)$ are odd functions then their product is an even function

(d) None of these

92. If $f(x)$ is an even function, $g(x)$ is an odd function, then which of the following options is true?

(a) $f(g(x))$ is an odd function, $g(f(x))$ is an even function.

(b) $f(g(x))$, $g(f(x))$ are odd functions

(c) $f(g(x))$, $g(f(x))$ are even functions

(d) $g(f(x))$ is an odd function, $f(g(x))$ is an even function

Directions for Questions 93 and 94:

If $f(x) = x^3 - x^2 - 6x$ for all $x \in R$, then answer the following questions:

93. For how many values of x, is $f(x) = 0$?

94. If $f(x)$ is defined only for interval $(-2, 3)$, then $f(x)$ will attain its minima in the interval

(a) $(-2, 0)$

(b) $(-2, -1)$

(c) $(0, 3)$

(d) None of these

95. If for all $x \in R$, $f(x) \in R$, then which of the following options correctly represents $f(x)$?

(a) $f(x) = logx$

(b) $f(x) = 1/|x|$

(c) $f(x) = log(x^4+7)$

(d) $f(x) = 1/x$

LEVEL OF DIFFICULTY (II)

1. Find the coefficient of x^{203} in the expansion of the following expression. $(x - 2)(x(x + 1)(x + 2)(x + 3)(x + 4) \ldots (x + 202))$

2. Find the maximum value of $f(x) = (x + 5)(36 - 12x)$

3. Let $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x. If 'a' and 'b' are two real numbers such that $f(b - 2) = 3a + 4$ and $f(a - 4) = b - 12$. Find the product of a and b, assuming 'a' and 'b' are integers.

4. $f(x) = x^{[x]}$ and $g(x) = \frac{x^2}{9}$, where $[x]$ denotes greatest integer less than or equal to 'x' and $x \neq 0$. What is the minimum value of $f(g(x))$, if x cannot be 0?

5. If $f(x) = ax^2 + bx + c$ and $g(x) = px^2 + qx$ with $g(1) = f(1)$, $g(2) - f(2) = 1$, $g(3) - f(3) = 4$, then $g(4) - f(4)$ is

   (a) 0

   (b) 5

   (c) 6

   (d) None of these

Directions for Questions 6 to 8: A function $f(x)$ is defined for all real values of x and $5f(x) + 2f(1/x) = 4x + 2$. If s is the sum of all the values of x for which $f(x) = 70$, then find the value of s.

Answer the following questions based on the above information:

6. What is the value of $f(2)$?

7. Find the sum of all values of x for which $f(x) = \frac{18}{21}$.

8. Find the product of all possible values of x for which $f(x) = \frac{18}{21}$.

Directions for Questions 9 and 10: Let $f(x) = 8a2 - 8 + 8(x - 1) - x2$, where a is a constant. If the maximum of $f(x)$ is 72, then answer the following questions.

9. For what value of x, $f(x)$ is maximum?

10. What is the sum of all values of a?

Directions for Questions 11 to 13: A function $f(x)$ is defined for all natural numbers $x > 1$ as $f(x) = \left(1 - \frac{1}{x}\right) f(x - 1)$. Based on this information, answer the following questions:

11. Find the value of $f(1)$, if $f(3) = 10$.

12. Find the value of $f(1000)$. If $f(1) = 10$.

13. Find the value of $\frac{1}{f(1)} + \frac{1}{f(2)} + \frac{1}{f(3)} + \ldots + \frac{1}{f(9)}$. It is given that $f(1) = 12$.

Directions for Questions 14 and 15: Let $f(x) = \sqrt{f(x-1) f(x+1)}$ & $f(0) = 1, f(1) = ½$

14. What is the value of $f(3)$?

15. What is the value of $f(1) + f(2) + f(3) + f(4) + \ldots.. f(\infty)$?

16. $f(x) = \frac{(x+3)}{2x^2 + 18 + 12x}$. x is a non-negative integer. Find the maximum possible value of $f(x)$.

Directions for Questions 17 and 18:

A cubic polynomial $f(x)$ yields a remainder of 3 when divided by $(x - 2)$ and remainder of 2 when divided by $(x - 3)$. Answer the following questions based on the above information.

17. Find the value of $f(3)$.

18. Find the remainder when we divide $f(x)$ by $(x - 2)(x - 3)$.

(a) $x + 5$

(b) $5 - x$

(c) 5

(d) None of these

Directions for Questions 19 to 21:

Let x be a real number that always satisfies the equation: $\left[\frac{x}{7}\right] = \left[\frac{x}{9}\right]$, where [] is the greatest integer function.

19. Find the number of integral solutions of the above equation.

20. Find the sum of integral solutions of the above equation.

21. Find the maximum value of x.

Directions for Questions 22 and 23:

$$f(x) = \left(x^2 - 7x + 13\right)^{2x+5}$$

22. For how many values of x, is $f(x) = 1$?

23. Find the sum of values of x, for which $f(x) = 1$.

24. Let $f(x) = |x-3| + |x-4| + |x-5|$, $f(x + 2)$ is minimum at $x = k$. Find the value of k.

Directions for Questions 25 and 26:

$f(x) = 2 - 1/f(x - 1)$ for all natural number values of 'x'. If $f(2) = 3/2$, then answer the following questions:

25. Find the value of $f(500)$.

26. What is the value of $[f(100001)]$, where '[]' represents greatest integer function.?

27. If $[x]$ denotes the greatest integer less than or equal to x, then the solution set of the in-equation $\frac{[x]-2}{4-[x]}>0$, is

(a) (2, 3]

(b) [3, 4)

(c) [2, 3]

(d) [3, 4]

Directions for Questions 28 and 29:

$f(x)$ is an integer valued function, that satisfies the following conditions:

$f(xy) = f(x) f(y)$

If $f(x) > f(y)$, then $x > y$, where x and y are natural numbers. It is also known that $f(2) = 2$.

Answer the following questions based on the above information:

28. Find the value of $f(3)$.

29. Find the value of $f(2001)$.

30. If $n^2 f(n) + f(1-n) = 2n - n^4$, where n is a real number, then what is the value of $f(2) \times f(5)$?

31. If $f(t) = \sqrt{t}$, $g(t) = t/4$ and $h(t) = 4t - 8$, then the formula for $g(f(h(t)))$ will be

(a) $\frac{\sqrt{t-2}}{4}$

(b) $2\sqrt{t} - 8$

(c) $\frac{\sqrt{(4t-8)}}{4}$

(d) $\frac{\sqrt{(t-8)}}{4}$

32. In the above question, find the value of $h(g(f(t)))$.

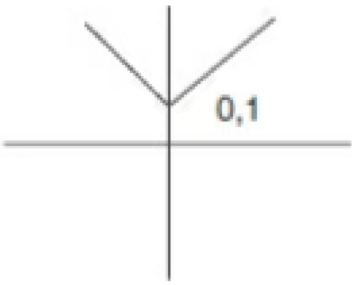
(a) $\sqrt{t} - 8$

(b) $2\sqrt{t-8}$

(c) $\dfrac{\sqrt{t}+8}{4}$

(d) None of these

Directions for Questions 33 to 36: If $f(x)$ is represented by the graph below.

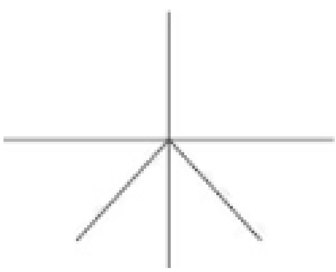


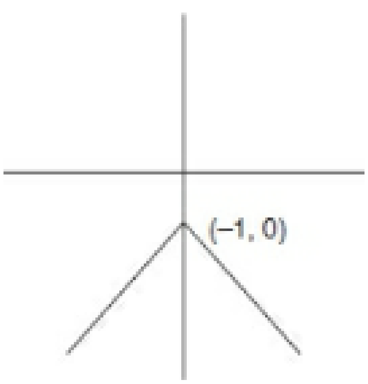
33. Which of the following will represent the function $-f(x)$?

34. Which of the following will represent the function $-f(x) + 1$?

35. Which of the following will represent the function $f(x) - 1$?

36. Which of the following will represent the function $f(x) + 1$?

(a) 

(b) 

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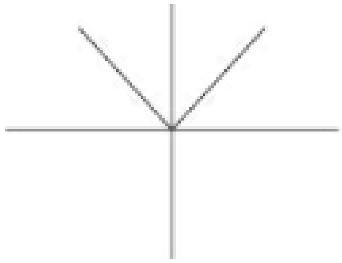
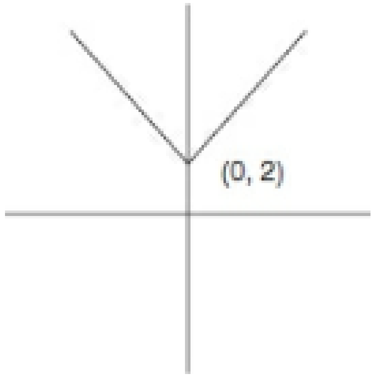


(d)



Directions for Questions 37 to 40: Define the following functions:

$f(x, y, z) = xy + yz + zx$

$g(x, y, z) = x2y + y2z + z2x$ and

$h(x, y, z) = 3\ xyz$

Find the value of the following expressions:

37. $h[f(2, 3, 1), g(3, 4, 2), h(1/3, 1/3, 3)]$

(a) 0

(b) 23760

(c) 2640

(d) None of these

38. $g[f(1, 0, 0), g(0, 1, 0), h(1, 1, 1)]$

(a) 0

(b) 9

(c) 12

(d) None of these

39. $f[f(1, 1, 1), g(1, 1, 1), h(1, 1, 1)]$

(a) 9

(b) 18

(c) 27

(d) None of these

40. $f(1, 2, 3) - g(1, 2, 3) + h(1, 2, 3)$

(a) –6

(b) 6

(c) 12

(d) 8

41. If $f(x) = 1/x$ and $g(x) = x$, then which of the following expressions would be true?

(a) $f(f(g(f(x)))) = f(g(g(f(f(x)))))$

(b) $f(g(g(f(f(x))))) = f(f(g(g(g(x)))))$

(c) $g(g(f(f(g(f(x)))))) = f(f(g(g(f(g(x))))))$

(d) $f(g(g(g(f(x))))) = g(g(f(f(f(x)))))$

42. If $f(x) = 1/g(x)$, then the minimum value of $f(x) + g(x)$, $f(x) > 0$ and $g(x) > 0$, will be

(a) 0

(b) 2

(c) Depends upon the value of $f(x)$ and $g(x)$

(d) None of these

Directions for Questions 43 to 45: If $R(a/b)$ = Remainder when a is divided by b;

$Q(a/b)$ = Quotient obtained when a is divided by b;

$SQ(a)$ = Smallest integer just bigger than the positive square root of a.

43. If $a = 12$, $b = 5$, then find the value of $SQ[R\{(a + b)/b\}]$.

(a) 0

(b) 1

(c) 2

(d) 3

44. If $a = 9$, $b = 7$, then the value of $Q[[Sq(ab)+b]/a]$ will be

(a) 0

(b) 1

(c) 2

(d) None of these

45. If $a = 18$, $b = 2$ and $c = 7$, then find the value of $Q[\{SQ(ab) + R(a/c)\}/b]$.

(a) 3

(b) 4

(c) 5

(d) 6

Directions for Questions 46 to 48: Read the following passage and try to answer questions based on them.

$[x]$ = Greatest integer less than or equal to x

46. If x is not an integer, what is the value of $([x] - \{x\})$?

(a) 0

(b) 1

(c) –1

(d) 2

47. If x is not an integer, then $(\{x\} + [x])$ is

(a) An even number

(b) An odd integer

(c) $> 3x$

(d) $< x$

48. What is the value of x if $5 < x < 6$ and $\{x\} + [x] = 2x$?

(a) 5.2

(b) 5.8

(c) 5.5

(d) 5.76

49. If $f(t) = t_2 + 2$ and $g(t) = (1/t) + 2$, then for $t = 2$, $f[g(t)] - g[f(t)] = ?$

(a) 1.2

(b) 2.6

(c) 4.34

(d) None of these

50. Given $f(t) = kt + 1$ and $g(t) = 3t + 2$. If $fog = gof$, find k.

(a) 2

(b) 3

(c) 5

(d) 4

51. Let $F(x)$ be a function such that $F(x)\,F(x + 1) = -F(x-1)\,F(x-2)\,F(x-3)\,F(x-4)$ for all $x \geq 0$. Given the values of If $F(83) = 81$ and $F(77) = 9$, then the value of $F(81)$ equals to

(a) 27

(b) 54

(c) 729

(d) Data insufficient

52. Let $f(x) = 121 - x^2$, $g(x) = |x-8| + |x+8|$ and $h(x) = \min\{f(x), g(x)\}$. What is the number of integer values of x for which $h(x)$ is equal to a positive integral value?

(a) 17

(b) 19

(c) 21

(d) 23

53. If the function $R(x) = \max(x^2 - 8, 3x, 8)$, then what is the max value of $R(x)$?

(a) 4

(b) $\frac{1+\sqrt{5}}{2}$

(c) ∞

(d) 0

54. If the function $R(x) = \min(x^2 - 8, 3x, 8)$, what is the maximum value of $R(x)$?

(a) 4

(b) 8

(c) ∞

(d) None of these

55. The minimum value of $ax^2 + bx + c$ is 7/8 at $x = 5/4$. Find the value of the expression at $x = 5$, if the value of the expression at $x = 1$ is 1.

(a) 75

(b) 29

(c) 121

(d) 129

56. Find the range of the function $f(x) = (x + 4)(5 - x)(x + 1)$.

(a) [–2, 3]

(b) (–∞, 20]

(c) (– ∞,+ ∞)

(d) [–20, ∞)

57. The function $f(x)$ is defined for positive integers and is defined as:

$f(x) = 6x - 3$, if x is a number in the form $2n$.

$= 6x + 4$, if x is a number in the form $2n + 1$.

What is the remainder when $f(1) + f(2) + f(3) + \ldots + f(1001)$ is divided by 2?

(a) 1

(b) 0

(c) –1

(d) None of these

58. p, q and r are three non-negative integers such that $p + q + r = 10$. The maximum value of $pq + qr + pr + pqr$ is

(a) ≥ 40 and < 50

(b) ≥ 50 and < 60

(c) ≥ 60 and < 70

(d) ≥ 70 and < 80

59. A function $\alpha(x)$ is defined for x as $3\alpha(x) + 2\alpha(2 - x) = (x + 3)^2$. What is the value of $[\alpha(-5)]$ where $[x]$ represents the greatest integer less than or equal to x?

(a) 37

(b) –38

(c) –37

(d) Cannot be determined

60. For a positive integer x, $f(x + 2) = 3 + f(x)$, when x is even and $f(x + 2) = x + f(x)$, when x is odd. If $f(1) = 6$ and $f(2) = 4$, then find $f(f(f(f(1)))) \times f(f(f(f(2))))$.

(a) 1375

(b) 1425

(c) 1275

(d) None of these

61. If $x > 0$, the minimum value of

$$\frac{\left(x+\frac{1}{x}\right)^6-\left(x^6+\frac{1}{x^6}\right)-2}{\left(x+\frac{1}{x}\right)^3+\left(x^3+\frac{1}{x^3}\right)}$$ is _____?

(a) 3

(b) 1

(c) 2

(d) 6

62. The domain of definition of the function

$$y=\frac{1}{\{\log_{10}(3-x)\}}+\sqrt{x+7}$$

(a) [−7, 3) − {2}

(b) [−7, 3] − {1}

(c) (−7, 3) − {0}

(d) (−7, 3)

63. If $[x]$ denotes the greatest integer $\le x$, then

$$\left[\frac{1}{3}\right]+\left[\frac{1}{3}+\frac{1}{99}\right]+\left[\frac{1}{3}+\frac{2}{99}\right]+\ldots\left[\frac{1}{3}+\frac{98}{99}\right]=?$$

(a) 98

(b) 33

(c) 67

(d) 66

64. If x and y are real numbers, then the minimum value of $x^2 + 4xy + 6y^2 - 4y + 4$ is

(a) – 4

(b) 0

(c) 2

(d) 4

65. What is the maximum possible value of $(21 \sin X + 72 \cos X)$?

(a) 21

(b) 57

(c) 63

(d) 75

66. The sum of the possible values of X in the equation $|x + 7| + |x - 8| = 16$ is

(a) 0

(b) 1

(c) 2

(d) 3

67. If $|3x+4| \le 5$ and a and b are the minimum and maximum values of x respectively, then $a + b$ = ?

68. For how many positive integer values of x, is $x^3 - 16x + x^2 + 20 \le 0$?

69. $3f(x)+2f\left(\frac{4x+5}{x-4}\right) = 7(x+3)$. where $x \in R$ and $x \neq 4$. What is the value of $f(11)$?

Directions for Questions 70 and 71: If $q = p \times [p]$ and 'q' is an integer such that $7 < q < 17$. Then answer the following questions:

70. Find the number of positive real values of 'p'.

71. Find the product of all possible values of p.

72. If $f(1) = -1, f(2x) = 4f(x) + 9, f(x + 2) = f(x) + 8(x + 1)$, find the value of $f(24) - f(7)$.

73. In the previous question, find the value of $f(1000)$

74. $f(x) = (x^2 + [x]^2 - 2x[x])^{1/2}$, where x is real and $[x]$ denotes the greatest integer less than or equal to x. Find the value of $f(10.08) - f(100.08)$.

75. Find the sum of coefficients of the polynomial $(x - 4)^7 (x - 3)^4 (x - 5)^2$.

(a) $2^8 3^7$

(b) $-2^6.3^8$

(c) $-2^8.3^7$

(d) $2^6.3^8$

76. A function $f(x)$ is defined as $f(x) = x - \frac{1}{9-3x} - 3$. If $x > 3$, then find the minimum possible value of $f(x)$.

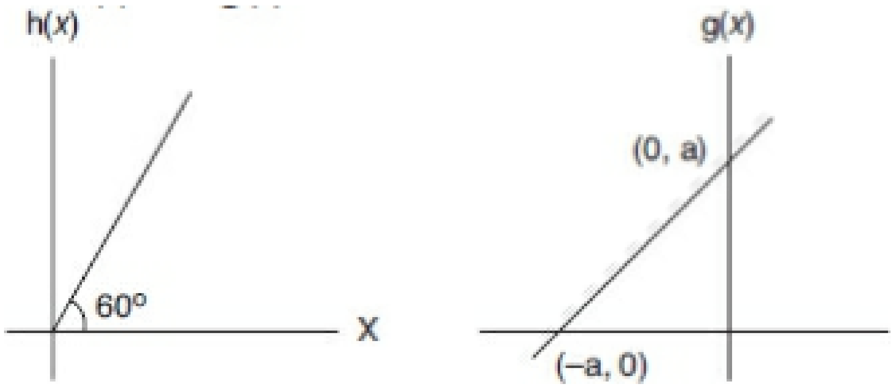
(a) $3-\frac{1}{\sqrt{3}}$

(b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}-\frac{1}{3}$

(d) $\sqrt{3}+\frac{1}{3}$

77. The graph of $h(x)$ and $g(x)$ are given below. Then which of the following defines the relation between $h(x)$ and $g(x)$?



(a) $\sqrt{3}g(x)-h(x)=a\sqrt{3}$

(b) $\sqrt{3}g(x)+h(x)=a\sqrt{3}$

(c) $g(x)+h(x)\sqrt{3}=\frac{a}{\sqrt{3}}$

(d) None of these

78. Consider a function 'f' is such that $\frac{f(xy)}{f(x+y)}=1$ for all real values of x, y. If $f(6) = 7$, then find the value of $f(-10) + f(10)$.

79. A function $f(x)$ is defined such that

$$f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$$

If $f(3) = 5$, then $f(81) = ?$

80. Find the sum of all the coefficients of the polynomial $(x - 4)^3 (x - 2)^{10} (x - 3)^3$.

81. $f(x) = \begin{cases} 3^x \text{ when } x \text{ is an odd number.} \\ 3^x + 4 \text{ when } x \text{ is an even number.} \end{cases}$

What is the value of

$\frac{1}{4}[f(1) + f(2) + f(3) + f(4) + \ldots + f(n)]$ if $n = 72$? if $n = 72$?

(a) $\frac{3}{8}(3^{72} - 1) + 36$

(b) $\frac{3}{8}(3^{72} + 1) + 36$

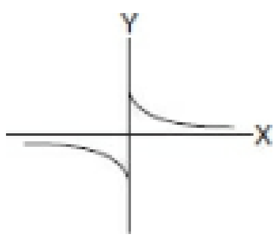
(c) $\frac{3}{8}(3^{72}) + 36$

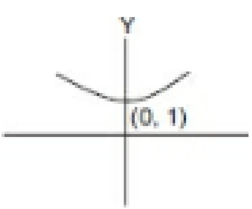
(d) None of these

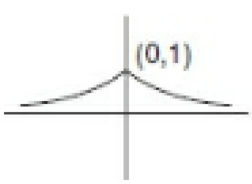
82. $f(x + y) = f(x \,.\, y)$, where x and y are real numbers and 'f' is a real function.

If $f(10) = 12$, then $[f(7)]^{143} - [f(11)]^{143} + f(5) = ?$

83. Which of the following represents the curve of $|e^{-|x|}|$?

(a) 

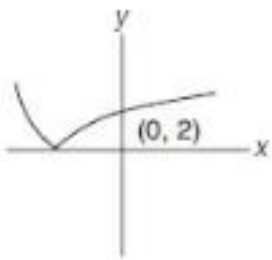
(b) 

(c) 

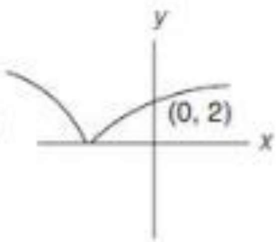
(d) None of these

84. Which of the following function correctly represents the curve of $|e^{-x} - 3|$?

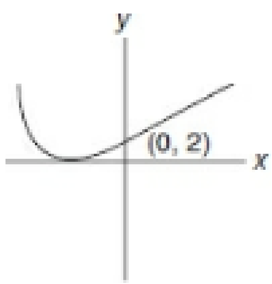
(a)



(b)



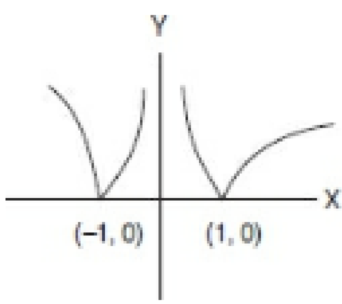
(c)



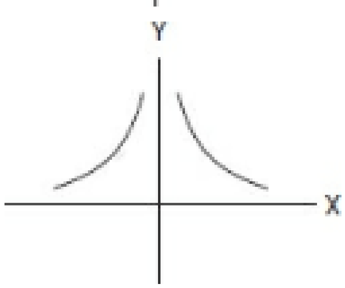
(d) None of these

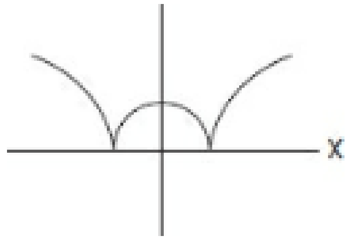
85. Which of the following represents the curve of $|\log|x - 3||$?

(a)



(b)



(c) 

(d) None of these.

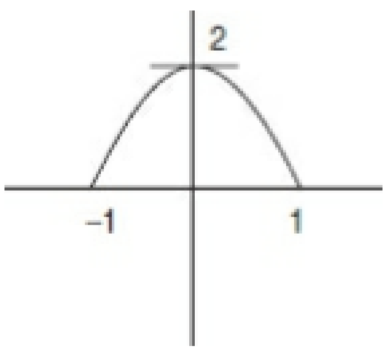
86. $f(x, y) = x^2 + y^2 - x - \frac{3y}{2} + 1$

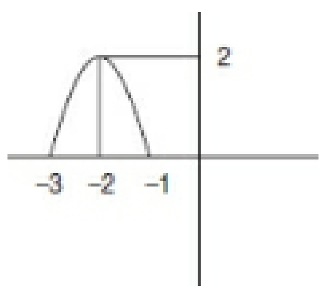
When $f(x, y)$ is minimum, then find the value of $x + y$.

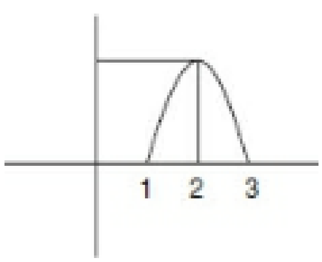
87. In the previous question, find the minimum value of $f(x, y)$.

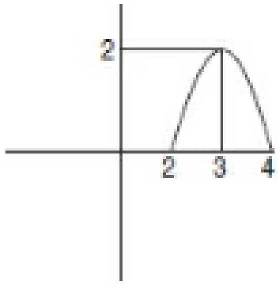
Directions for Questions 88 and 89:

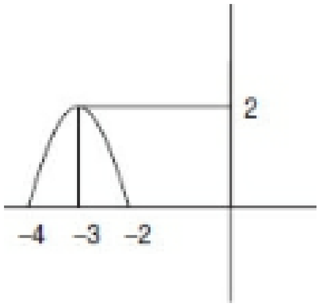
88. If the graph given below represents $f(x + 5)$, then which of the given options would represent the graph of $f(-2 - x)$?



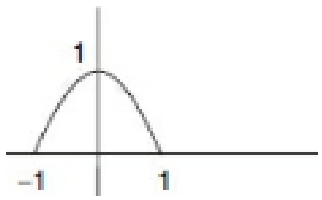
(a) 

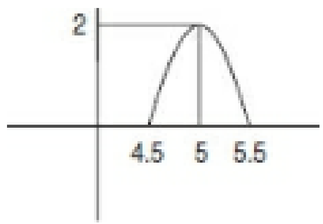
(b) 

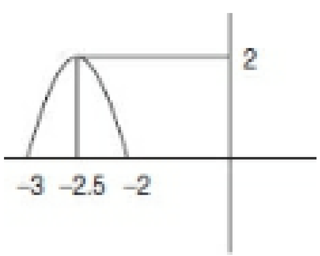
(c) 

(d) 

89. Which of the following options represents curve of $f(-2x)$?

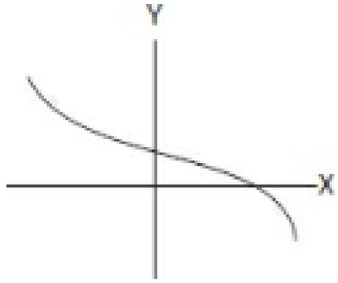
(a) 

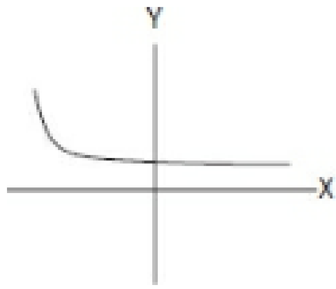
(b) 

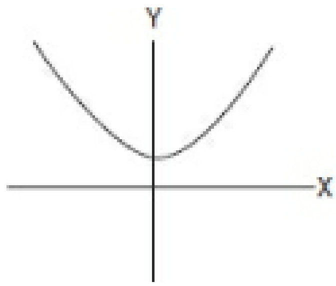
(c) 

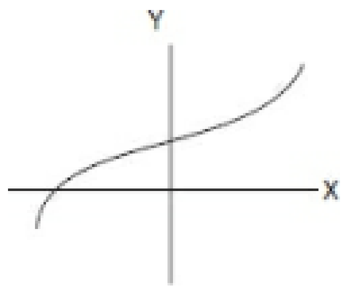
(d) None of these

90. If x, y are real numbers and function $g(x)$ satisfies $\frac{(g(x+y)+g(x-y))}{2} = g(x)g(y)$ and $g(0)$ is a positive real number, then which of the following options may represent graph of $g(x)$?

(a)


(b)


(c)


(d)


LEVEL OF DIFFICULTY (III)

1. Find the domain of the definition of the function $y = 1/(x - |x|)^{1/2}$.

   (a) $-\infty < x < \infty$

   (b) $-\infty < x < 0$

   (c) $0 < x < \infty$

   (d) nowhere

2. Find the domain of the definition of the function $y = (x-1)^{1/2} + 2(1-x)^{1/2} + (x^2 + 3)^{1/2}$.

(a) $x = 0$

(b) $[1, \infty)$

(c) $[-1, 1]$

(d) $x = 1$

3. Find the domain of the definition of the function $y = \log_{10} [(x-5)/(x^2 - 10x + 24)] - (x+4)^{1/2}$.

(a) $x > 6$

(b) $4 < x < 5$

(c) Both (*a*) and (*b*)

(d) None of these

4. Find the domain of the definition of the function $y = [(x-3)/(x+3)]^{1/2} + [(1-x)/(1+x)]^{1/2}$.

(a) $x > 3$

(b) $x < -3$

(c) $-3 \le x \le 3$

(d) nowhere

5. Find the domain of the definition of the function $y = (2x^2 + x + 1)^{-3/4}$.

(a) $x \ge 0$

(b) All x except $x = 0$

(c) $-3 \le x \le 3$

(d) Everywhere

6. Find the domain of the definition of the function $y = (x^2 - 2x - 3)^{1/2} - 1/(-2 + 3x - x^2)^{1/2}$.

(a) $x > 0$

(b) $-1 < x < 0$

(c) x^2

(d) None of these

7. Find the domain of the definition of the function $y = \log_{10} [1 - \log_{10}(x^2 - 5x + 16)]$.

(a) (2, 3]

(b) [2, 3)

(c) [2, 3]

(d) None of these

8. If $f(t) = (t - 1)/(t + 1)$, then $f(f(t))$ will be equal to

(a) $1/t$

(b) $-1/t$

(c) t

(d) $-t$

9. If $f(x) = e^x$ and $g(x) = \log_e x$ then value of fog will be

(a) x

(b) 0

(c) 1

(d) e

10. In the above question, find the value of gof.

(a) x

(b) 0

(c) 1

(d) e

11. The function $y = 1/x$ shifted 1 unit down and 1 unit right is given by

(a) $y - 1 = 1/(x + 1)$

(b) $y - 1 = 1/(x - 1)$

(c) $y + 1 = 1/(x - 1)$

(d) $y + 1 = 1/(x + 1)$

12. Which of the following functions is an even function?

(a) $f(t) = (a_t + a_{-t})/(a_t - a_{-t})$

(b) $f(t) = (a_t + 1)/(a_t - 1)$

(c) $f(t) = t.\ (a_t - 1)/(a_t + 1)$

(d) None of these

13. Which of the following functions is not an odd function?

(a) $f(t) = \log_2(t +$ $f(t) = \log_2(t + \sqrt{t^2} + 1)$ $+ 1)$

(b) $f(t) = (a_t + a_{-t})/(a_t - a_{-t})$

(c) $f(t) = (a^t + 1)/(a^t - 1)$

(d) All of these

14. Find *fof* if $f(t) = t/(1 + t^2)^{1/2}$.

(a) $1/(1+ 2t^2)^{1/2}$

(b) $t/(1 + 2t^2)^{1/2}$

(c) $(1 + 2t^2)$

(d) None of these

15. At what integral value of x will the function $\frac{(x^2 + 3x + 1)}{(x^2 - 3x + 1)}$ attain its maximum value?

(a) 3

(b) 4

(c) –3

(d) None of these

16. Inverse of $f(t) = (10^t - 10^{-t})/(10^t + 10^{-t})$ is

(a) $1/2 \log \{(1 - t)/(1 + t)\}$

(b) $0.5 \log \{(t - 1)/(t + 1)\}$

(c) $1/2 \log_{10} (2^t - 1)$

(d) None of these

17. If $f(x) = |x - 2|$, then which of the following is always true?

(a) $f(x) = (f(x))^2$

(b) $f(x) = f(-x)$

(c) $f(x) = x - 2$

(d) None of these

Directions for Questions 18 to 20: Read the instructions below and solve:

$f(x) = f(x-2) - f(x-1)$, x is a natural number

$f(1) = 0, f(2) = 1$

18. The value of $f(x)$ is negative for

(a) All $x > 2$

(b) All odd $x (x > 2)$

(c) For all even $x (x > 0)$

(d) $f(x)$ is always positive

19. The value of $f[f(6)]$ is

(a) 5

(b) – 1

(c) – 3

(d) – 2

20. The value of $f(6) - f(8)$ is

(a) $f(4) + f(5)$

(b) $f(7)$

(c) $-\{f(7) + f(5)\}$

(d) $-f(5)$

21. Which of the following is not an even function?

(a) $f(x) = e^{x} + e^{-x}$

(b) $f(x) = e^{x} - e^{-x}$

(c) $f(x) = e^{2x} + e^{-2x}$

(d) None of these

22. If $f(x)$ is a function satisfying $f(x).f(1/x) = f(x) + f(1/x)$ and $f(4) = 65$, what will be the value of $f(6)$?

(a) 37

(b) 217

(c) 64

(d) None of these

Directions for Questions 23 to 34:

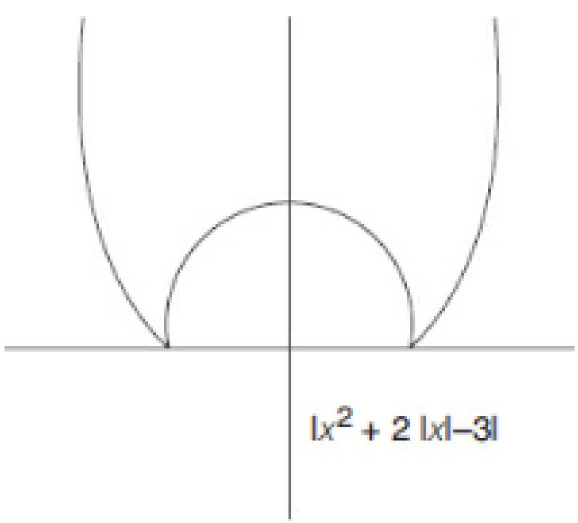
Mark (a), if $f(-x) = f(x)$,

Mark (b), if $f(-x) = -f(x)$

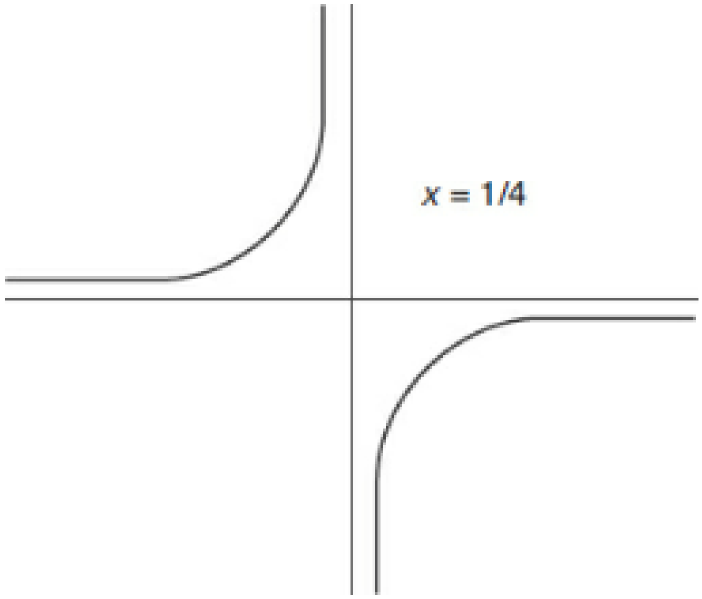
Mark (c), if neither (a) nor (b)is true

Mark (d), if $f(x)$ does not exist at least one point of the domain

23.



24.



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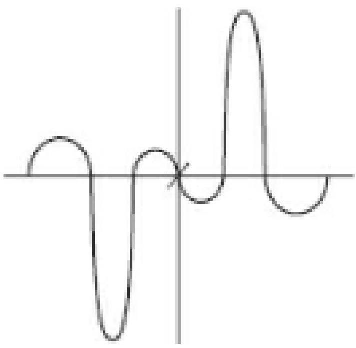

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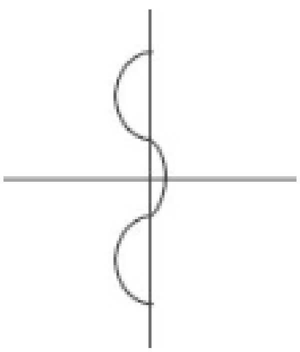
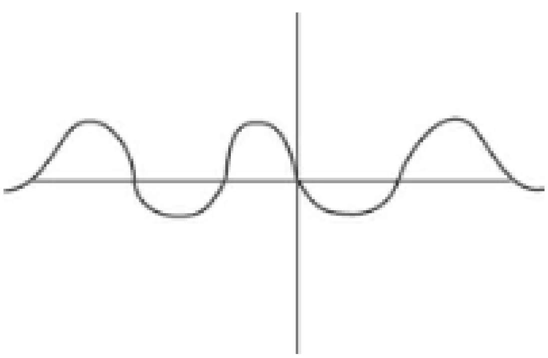
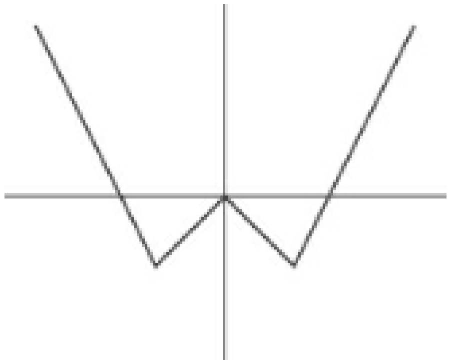


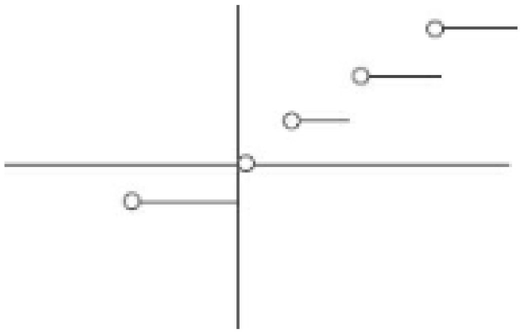
27.



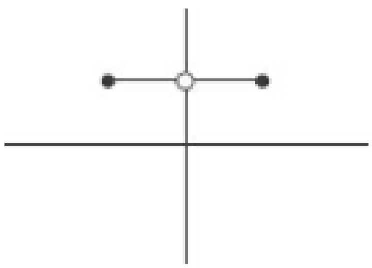
28.



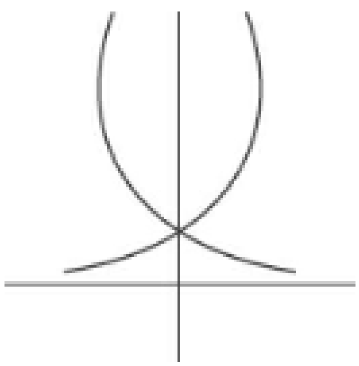
29.



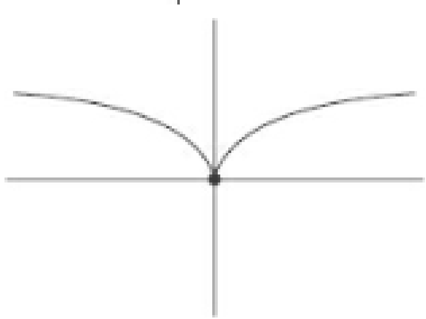
30.

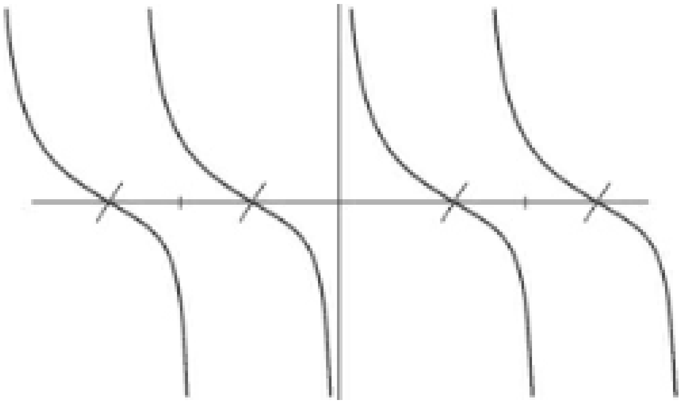


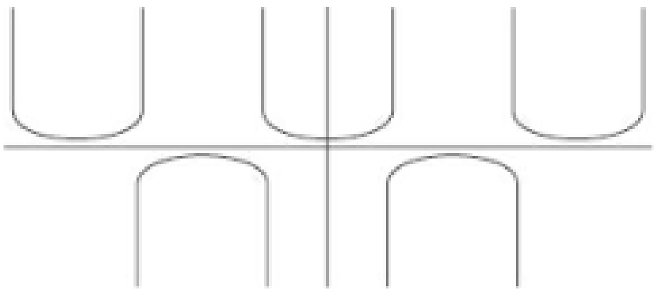
31.



32.



33. 

34. 

Directions for Questions 35 to 40: Define the functions:

$A(x, y, z)$ = Max (max (x, y), min (y, z) min (x, z))

$B(x, y, z)$ = Max (max (x, y), min (y, z) max (x, z))

$C(x, y, z)$ = Max (min (x, y), min (y, z) min (x, z))

$D(x, y, z)$ = Min (max (x, y), max (y, z) max (x, z))

Max (x, y, z) = Maximum of x, y and z

Min (x, y, z) = Minimum of x, y and z.

Assume that x, y and z are distinct integers.

35. For what condition will $A(x, y, z)$ be equal to Max (x, y, z)?

(a) When x is maximum

(b) When y is maximum

(c) When z is maximum

(d) Either (a) or (b)

36. For what condition will $B(x, y, z)$ be equal to Min (x, y, z)?

(a) When y is minimum

(b) When z is minimum

(c) Either (a) or (b)

(d) Never

37. For what condition will $A(x, y, z)$ not be equal to $B(x, y, z)$?

(a) $x > y > z$

(b) $y > z > x$

(c) $z > y > x$

(d) None of these

38. Under what condition will $C(x, y, z)$ be equal to $B(x, y, z)$?

(a) $x > y > z$
(b) $z > y > x$

(c) Both (a) and (b)

(d) Never

39. Which of the following will always be true?

(i) $A(x, y, z)$ will always be greater than Min (x, y, z)

(ii) $B(x, y, z)$ will always be lower than Max (x, y, z)

(III) $A(x, y, z)$ will never be greater than $B(x, y, z)$

(a) I only

(b) III only

(c) Both (a) and (b)

(d) All the three

40. The highest value amongst the following will be

(a) Max/Min

(b) A/B

(c) C/D

(d) Cannot be determined

Directions for Questions 41 to 49: Suppose x and y are real numbers. Let $f(x, y) = |x + y|$ $F(f(x, y)) = -f(x, y)$ and $G(f(x, y)) = -F(f(x, y))$

41. Which one of the following is true?

(a) $F(f(x, y)).G(f(x, y)) = -F(f(x, y)).\ G.(f(x, y))$

(b) $F(f(x, y)).G(f(x, y)) \leq -F(f(x, y)).\ G.(f(x, y))$

(c) $G(f(x, y)).f(x, y) = F(f(x, y)).\ (f(x, y)$

(d) $G(f(x, y)).F(f(x, y)) = f(x, y).\ f(x, y)$

42. Which of the following has $a2$ as the result?

(a) $F(f(a, -\text{a})).G(f(a, -a))$

(b) $-F(f(a, a).G(f(a, a))/4$

(c) $F(f(a, a)).G(f(a, a))/22$

(d) $f(a, a).f(a, a)$

43. Find the value of the expression.

$$\frac{G(f(3,2)) + F(f(-1,2))}{f(2,-3) + G(f(1,2)}\ldots$$

(a) 3/2

(b) 2/3

(c) 1

(d) 2

44. Which of the following is equal to

$\frac{G(f(32,13))+F(f(15,-5))}{f(2,3)+G(f(1.5,0.5))}$?

(a) $\frac{2G(f(1,2))+(f(-3,1))}{G(f(2,6)+F(f(-8,2))}$

(b) $\frac{3.G(f(3,4))+F(f(1,0))}{f(1,1)+G(f(2,0))}$

(c) $\frac{(f(3,4))+F(f(1,2))}{G(f(1,1))}$

(d) None of these

Now if $A(f(x, y) = f(x, y)$

$B(f(x, y) = -f(x, y)$

$C(f(x, y) = f(x, y)$

$D(f(x, y) = -f(x, y)$ and similarly

$Z\,(f(x, y) = -f(x, y)$

Now, solve the following:

45. Find the value of $A(f(0, 1)) + B(f(1, 2)) + C(f(2, 3)) + \ldots + Z(f(25, 26))$.

(a) –50

(b) –52

(c) –26

(d) None of these

46. Which of the following is true?

(i) $A(f(0, 1)) < B(f(1, 2)) < C(f(2, 3)\ldots$

(ii) $A(f(0, 1)).\, B(f(1, 2)) > B(f(1, 2)).C(f(2, 3)) > C(f(2, 3)).\, D(f(3, 4))$

(iii) $A(f(0, 0)) = B(f(0, 0)) = C(f(0, 0)) = \ldots = Z(f(0, 0))$

(a) Only (i) and (ii)

(b) Only (ii) and (iii)

(c) Only (ii)

(d) Only (i)

47. If max (x, y, z) = maximum of x, y and z

Min (x, y, z) = minimum of x, y and z

$f(x, y) = |x + y|$

$F(f(x, y)) = -f(x, y)$

$G(f(x, y) = -F(f(x, y))$

Then find the value of the following expression:

Min (max $[f(2, 3), F(f(3, 4)), G(f(4, 5))]$, min $[f(1, 2), F(f(-1, 2)), G(f(1, -2))]$, max $[f(-3\ -4),$

$f(-5\ -1), G(f(-4, -6))])$

(a) –1

(b) –7

(c) –6

(d) –10

48. Which of the following is the value of Max. $[f(a, b), F(f(b, c), G(f(c, d))]$ for all $a > b > c > d$?

(a) Anything but positive

(b) Anything but negative

(c) Negative or positive

(d) Any real value

49. If another function is defined as $P(x, y) = \frac{F(f(x, y))}{(x, y)}$, which of the following is second lowest in value?

(a) Value of $P(x, y)$ for $x = 2$ and $y = 1$

(b) Value of $P(x, y)$ for $x = 3$ and $y = 4$

(c) Value of $P(x, y)$ for $x = 3$ and $y = 5$

(d) Value of $P(x, y)$ for $x = 3$ and $y = 2$

50. If $f(s) = (b^s + b^{-s})/2$, where $b > 0$, find $f(s + t) + f(s - t)$.

(a) $f(s) - f(t)$

(b) $2\,f(s).f(t)$

(c) $4\,f(s).f(t)$

(d) $f(s) + f(t)$

Questions 51 to 60 are all actual questions from the XAT exam.

51. $A_0, A_1, A_2,\ldots$ is a sequence of numbers with $A_0 = 1$, $A_1 = 3$, and $A_t = (t + 1)A_{(t-1)} - tA_{(t-2)}$, where $t = 2,3,4\ldots$

Conclusion I. $A_8 = 77$

Conclusion II. $A_{10} = 121$

Conclusion III. $A_{12} = 145$

(a) Using the given statement, only Conclusion I can be derived.

(b) Only Conclusion II can be derived, using the given statement

(c) Only Conclusion III can be derived, using the given statement

(d) Conclusion, Using the given statement I, II and III can be derived.

(e) None of the three Conclusions, Using the given statement I, II and III can be derived.

52. A, B, C be real numbers satisfying $A < B < C, A + B + C = 6$ and $AB + BC + CA = 9$

Conclusion I. $1 < B < 3$

Conclusion II. $2 < A < 3$

Conclusion III. $0 < C < 1$

(a) Using the given statement, only Conclusion I can be derived.

(b) Using the given statement, only Conclusion II can be derived.

(c) Using the given statement, only Conclusion III can be derived.

(d) Using the given statement, Conclusion I, II and III can be derived.

(e) Using the given statement, none of the three Conclusions I, II and III can be derived.

53. If $F(x, n)$ be the number of ways of distributing 'x' toys to 'n' children so that each child receives at the most two toys, then $F(4, 3) =$ ___?

(a) 2

(b) 6

(c) 3

(d) 4

(e) 5

54. The figure below shows the graph of a function $f(x)$. How many solutions does the equation $f(f(x)) = 15$ have?

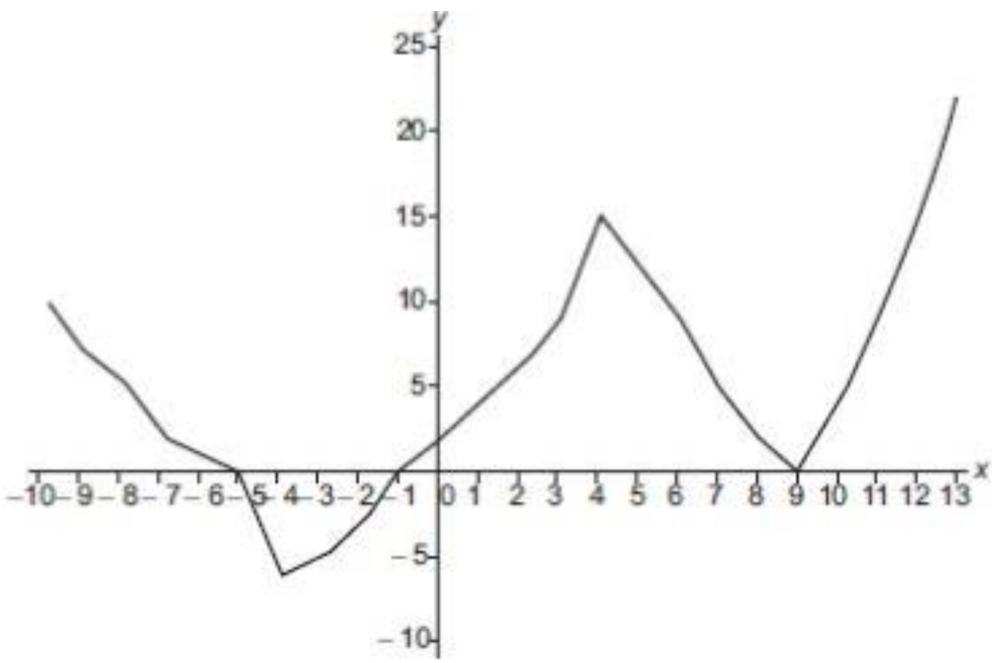
(a) 5

(b) 6

(c) 7

(d) 8

(e) Cannot be determined from the given graph



55. The following question is followed by two statements. Mark your answer as:

(a) If the question can be answered by the first statement alone but cannot be answered by the second statement alone;

(b) If the question can be answered by the second statement alone but cannot be answered by the first statement alone;

(c) If the question can be answered by both the statements together but cannot be answered by any one of the statements alone;

(d) If the question can be answered by the first statement alone as well as by the second statement alone;

(e) If the question cannot be answered even by using both the statements together.

A sequence of positive integers is defined as $A_{n+1} = A_n^2 + 1$ for each $n \geq 1$. What is the value of the Greatest Common Divisor of A_{900} and A_{1000}?

I. $A_0 = 1$

II. $A_1 = 2$

56. A manufacturer produces two types of products— A and B, which are subjected to two types of operations, viz., grinding and polishing. Each unit of product A takes two hours of grinding and three hours of polishing whereas product B takes three hours of grinding and two hours of polishing. The manufacturer has ten grinders and fifteen polishers. Each grinder operates for 12 hours/day and each polisher 10 hours/day. The profit margin per unit of A and B are ₹ 5/- and ₹ 7/- respectively. If the manufacturer utilises all his resources for producing these two types of items, what is the maximum profit that the manufacturer can earn?

(a) ₹ 280/-

(b) ₹ 294/-

(c) ₹ 515/-

(d) ₹ 550/-

(e) None of these

57. Consider a function $f(x) = x^4 + x^3 + x^2 + x + 1$, where x is a positive integer greater than 1. What will be the remainder if $f(x^5)$ is divided by $f(x)$?

(a) 1

(b) 4

(c) 5

(d) A monomial in x

(e) A polynomial in x

58. For all real numbers x, except $x = 0$ and $x = 1$, the function F is defined by

$$F\left(\frac{x}{x-1}\right) = \frac{1}{x}.$$

If $0 < \alpha < 90°$ then $F((\text{cosec}\ \alpha)^2) = ?$

(a) $(\sin \alpha)^2$

(b) $(\cos \alpha)^2$

(c) $(\tan \alpha)^2$

(d) $(\cot \alpha)^2$

(e) $(\sec \alpha)^2$

59. $F(x)$ is a fourth order polynomial with integer coefficients and with no common factor. The roots of $F(x)$ are $-2, -1, 1, 2$. If p is a prime number greater than 97, then the largest integer that divides $F(p)$ for all values of p is

(a) 72

(b) 120

(c) 240

(d) 360

(e) None of these

60. If $x = (9 + 4\sqrt{5})48 = [x] + f$, where $[x]$ is defined as integral part of x and f is a fraction, then $x(1 - f)$ equals

(a) 1

(b) Less than 1

(c) More than 1

(d) Between 1 and 2

(e) None of these

61. If $3f(x+2)+4f\left(\frac{1}{x+2}\right)=4x$, $x \neq -2$, then $f(4) = ?$

(a) 7

(b) −52 / 7

(c) 8

(d) None of these

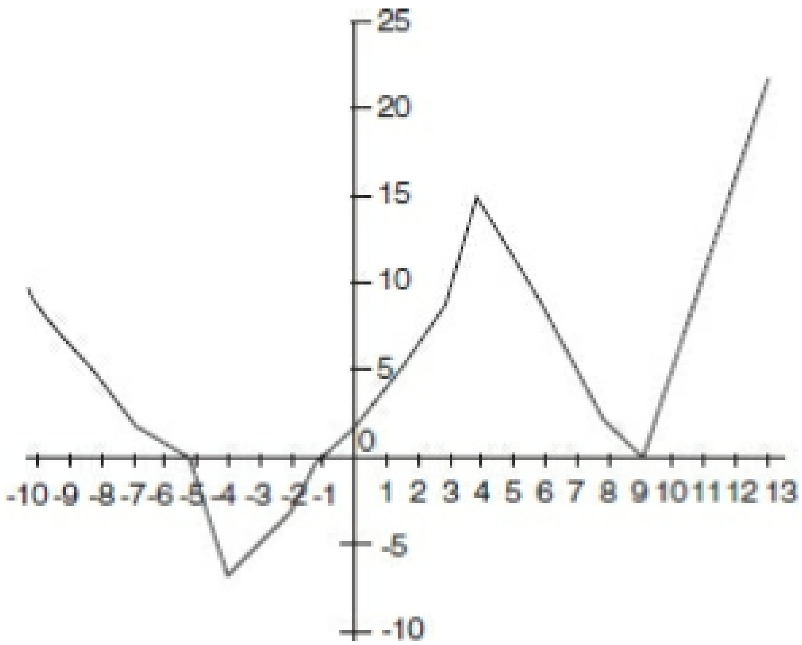
62. The figure below shows the graph of a function $f(x)$. How many solutions does the equation $f(f(x)) = 15$ have for the span of the graph shown?

(a) 5

(b) 6

(c) 7

(d) 8



63. If $f(x) = (x-6), g(x) = \frac{(x-9)(x-1)}{(x-7)(x-3)}$,

how many real values of x satisfy the equation $[f(x)]^{g(x)} = 1$?

64. A continuous function $f(x)$ is defined for all real values of x, such that $f(x) = 0$, only for two distinct real values of x. It is also known that $f(4) + f(6) = 0$, $f(5).f(7) > 0$, $f(4).f(8) < 0$, $f(1) > 0$ & $f(2) < 0$

Which of the following statement must be true?

(a) $f(1)f(2)f(4) < 0$

(b) $f(5)f(6)f(7) < 0$

(c) $f(1)f(3)f(4) > 0$

(d) None of these

65. $f(x) = 7[x] + 4\{x\}$

where $[x]$ = greatest integer less than or equals to x.

$\{x\} = x - [x]$

How many real values of x satisfy the equation?

(c) $f(1)f(3)f(4) > 0$

(d) None of these

65. $f(x) = 7[x] + 4\{x\}$

where $[x]$ = greatest integer less than or equals to x.

$\{x\} = x - [x]$

How many real values of x satisfy the equation?

$f(x) = 12 + x$

(a) 0

(b) 1

(c) 2

(d) None of these

66. How many non-negative integer solutions (x, y) are possible for the equation $x^2 - xy + y^2 = x + y$ such that $x \geq y$?

(a) 1

(b) 2

(c) 3

(d) 4

67. A function 'g' is defined for all natural numbers $n \geq 2$ as $\frac{g(n-1)}{g(n)} = \frac{n}{n-1}$.

If $g(1) = 2$ then what is the value of

$$\frac{\left[\frac{1}{g(1)}\times\frac{1}{g(2)}\times\frac{1}{g(3)}\times\ldots\frac{1}{g(8)}\right]}{\left[\frac{1}{g(1)}+\frac{1}{g(2)}+\frac{1}{g(3)}+\ldots\frac{1}{g(8)}\right]}??$$

(a) $8!/2^8$

(b) $8!/(2^8.18)$

(c) $8!/18.2^7$

(d) $8!/3^7.24$

68. $f(x)$ is a polynomial of degree 77 which when divided by $(x - 1)$, $(x - 2)$, $(x - 3)$, $(x - 4)$,…, $(x - 77)$, leaves 1, 2, 3, …, 77 respectively as the remainders. Find the value of $f(0) + f(78)$.

(a) 77

(b) 78

(c) –77

(d) 78!

69. A function $f(n)$ is defined as $f(n - 1)\,[2 - f(n)] = 1$ for all natural numbers 'n'. If $f(1) = 3$, then find the value of $f(21)$.

(a) 42/41

(b) 45/43

(c) 43/41

(d) 47/45

Directions for Questions 70 and 71: If $f(x) = 10[x] + 22\{x\}$, where $[x]$ denotes the largest integer less than or equal to x and $\{x\} = x - [x]$, (i.e. the fractional part of x), then answer the following questions.

70. How many solutions does the equation $f(x) = 250$ have?

    (a) 0

    (b) 1

    (c) 2

    (d) 3

71. What is the sum of all possible values of x?

72. If $f(x + 1) = f(x) - f(x-1)$ and $f(5) = 6$ and $f(17) = 2f(16)$, then $f(17) = ?$

    (a) 5

    (b) 6

    (c) 16

    (d) 18

73. If $h(x)$ is a positive valued function and

    $\frac{h(x)}{h(x-1)} = \frac{h(x-2)}{h(x+1)}$ for all $x \geq 0$

    If $h(56) = 16$ and $h(52) = 4$ then $h(54) = ?$

74. $f(x) = 1 - \frac{2}{(x+1)}$

    If $f^2(x) = f(f(x))$, $f^3(x) = f(f(f(x)))$, $f^4(x) = f(f(f(f(x))))$ and so on, then find $f^{802}(x)$ at $x = -1/2$.

Directions for Questions 75 and 76: If $\log_3(x + y) + \log_3(x - y) = 3$, where x and y are positive integers then answer the following questions:

75. If $y > 0$, then how many different pairs of (x, y) are possible?

76. The maximum value of $x + y$ = ____?

Directions for Questions 77 and 78:

$f(x) = |x + 2|$

$g(x) = x^2 - 7x + 10$

$h(x) = \min(f(x), g(x))$

77. For how many positive integer values of x, is $h(x) \leq 0$?

78. Find the sum of all integer values of x for which $h(x) < 0$.

79. If $[x]$ denotes the greatest integer less than or equal to x, if p and q are two distinct real numbers and $[2p - 3] = q + 7$, $[3q + 1] = p + 6$, then find the value of $p^2 \times q^2$.

80. If $f(a) = 3^a$ and $f(a + 1) = 3^{(a+1)} + 4$, where 'a' is an odd number, what is the value of

$\frac{1}{4}[f(1) + f(2) + f(3) + f(4) + \ldots + f(72)]$??

(a) $\frac{3}{8}(3^{72} - 1) + 36$

(b) $\frac{3}{8}(3^{72} - 1) - 36$

(c) $\frac{3}{8}(3^{72} + 1) + 36\frac{3}{8}(3^{72} + 1) + 36$

(d) None of these

Directions for Questions 81 and 82:

If $f(x) = \frac{x^2}{4}$ and $g(x) = 2x^{[3x]} + 2$, where $[x]$ is the greatest integer less than or equal to 'x', then answer the following questions:

81. Which of the following statement is true about $g(f(x))$?

(a) $g(f(x))$ is neither even nor odd

(b) $g(f(x))$ is maximum for $x = 11$

(c) $g(f(x))$ will have its minimum for a value of x that obeys $\frac{3x^2}{4} \leq 1$

(d) None of these

82. Which of the following is the value of $g(f(x))$ at $x = 2$?

(a) 66

(b) 34

(c) 18

(d) 64

83. The area bounded between $|x + y| = 2$ and $|x - y| = 2$ is

(a) 2

(b) 4

(c) 6

(d) 8

Directions for Questions 84 to 86:

If $f(x) = |x| + |x + 4| + |x + 8| + |x + 12| + \ldots + |x + 4n|$, where x is an integer and n is a positive integer.

84. If $n = 8$, what is the minimum value of $f(x)$?

85. If $n = 7$, then for how many values of x, $f(x)$ is minimum?

86. For $n = 9$, which of the following statements is true?

(a) $f(x)$ will be minimum for a total 5 values of x

(b) $f(-17) = f(-19)$

(c) Minimum value of $f(x)$ is 100

(d) All of these

87. Find the area enclosed by the graph $|x| + |y| = 3$.

88. Find the area enclosed by curve $|x - 2| + |y - 3| = 3$.

Directions for Questions 89 and 90:

$8\{x\} = x + 2[x]$

$\{x\}$ denotes the fractional part of x

$[x]$ denotes the greatest integer less than or equals to x

89. For how many positive values of x, is the given equation true?

90. Find the difference of the greatest and the least value of x for which the given equation is true? (Till two digits after the decimal point)

Directions for Questions 91 and 92:

If $f(x) = \dfrac{4^{x-1}}{4^{x-1}+1}$ and $g(x) = 2x$, then answer the following questions.

91. $fog\left(\frac{1}{4}\right) + fog\left(\frac{3}{4}\right) = ?$

92. $fog\left(\frac{1}{2}\right) + fog\left(\frac{1}{4}\right) + fog\left(\frac{1}{8}\right) + f\left(\frac{1}{16}\right) + f\left(\frac{3}{4}\right) + fog\left(\frac{7}{8}\right) + fog\left(\frac{15}{16}\right) = ?$

Directions for Questions 93 to 96:

If for a positive integer x, $f(x + 2) = f(x) + 2(x + 1)$, when x is even and $f(x + 2) = f(x) + 1$, when x is odd. If $f(1) = 1$ and $f(2) = 5$, then answer the following questions.

93. $f(24) = ?$

94. $\left[\frac{f(14)}{f(11)}\right] = ?$, where [ ] denotes the greatest integer function

95. Which of the following statement is true?

(a) For even value of x value of $f(x)$ is also even

(b) For odd value of x, value of $f(x)$ is odd

(c) For even value of x, value of $f(x)$ is odd

(d) None of these

96. Value of $f(f(f(f(3)))) + f(f(f(2))) = ?$

Directions for Questions 97 and 98:

$F(x)$ is a 6th degree polynomial of x. It is given that $F(0) = 0$, $F(1) = 1$, $F(2) = 2$, $F(3) = 3$, $F(4) = 4$, $F(5) = 5$, $F(6) = 7 = ?$

97. Find the value of $F(8)$.

98. If x is a negative integer then find the minimum value of $F(x)$.

99. If $g(x + y) = g(x).g(y)$ and $g(1) = 5$, then find the value of $g(1) + g(2) + g(3) + g(4) + g(5)$.

100. In the previous question, if

$$\sum_{p=1}^{n} g(q+p) = \frac{1}{4}\left(5^{p+3} - 125\right)$$

where 'p' is a positive integer then find q.

ANSWER KEY

Level of Difficulty (I)

1. (b)

2. (d)
3. (c)
4. (a)
5. (b)
6. (b)
7. (a)
8. (c)
9. (c)
10. (d)
11. (a)
12. (d)
13. (d)
14. (a)
15. (c)
16. (a)
17. (b)
18. (c)
19. (b)
20. (d)
21. (c)
22. (b)
23. (c)
24. (d)
25. (d)
26. (a)
27. (d)
28. (a)

29. (d)
30. (a)
31. (c)
32. (c)
33. (a)
34. (b)
35. (c)
36. (a)
37. (d)
38. (b)
39. (c)
40. (a)
41. (a)
42. (d)
43. (d)
44. (a)
45. (a)
46. (c)
47. (d)
48. (d)
49. (d)
50. (d)
51. (b)
52. (a)
53. (a)
54. (b)
55. (d)

44. (a)

45. (a)

46. (c)

47. (d)

48. (d)

49. (d)

50. (d)

51. (b)

52. (a)

53. (a)

54. (b)

55. (d)

56. (d)

57. (c)

58. (b)

59. (d)

60. (c)

61. (a)

62. (a)

63. (c)

64. 3

65. 1

66. 1

67. 7.86

68. 13.75

69. 12

70. 1.5

71. (d)

72. 2

73. (d)

74. 8

75. (d)

76. (c)

77. 0.14

78. 2

79. (b)

80. (b)

81. 9

82. 10

83. 0

84. (c)

85. –2

86. (b)

87. (c)

88. (a)

89. (b)

90. (b)

91. (c)

92. (c)

93. 3

94. (c)

95. (c)

Level of Difficulty (II)

1. 20501
2. 192
3. 9
4. 1
5. (d)
6. 2
7. 3/5
8. –0.4
9. 4
10. 0
11. 30
12. 0.01
13. 3
14. 1/8
15. 1
16. 1/6
17. 2
18. (b)
19. 32
20. –16
21. 27
22. 3
23. 4.5
24. 2
25. 501/500
26. 1
27. (b)

28. 3

29. 2001

30. 72

31. (c)

32. (a)

33. (b)

34. (a)

35. (c)

36. (d)

37. (c)

38. (a)

39. (c)

40. (b)

41. (c)

42. (b)

43. (c)

44. (b)

45. (c)

46. (c)

47. (b)

48. (c)

49. (d)

50. (a)

51. (a)

52. (c)

53. (c)

54. (b)

55. (b)

56. (c)

57. (b)

58. (c)

59. (c)

60. (a)

61. (d)

62. (a)

63. (b)

64. (c)

65. (d)

66. (b)

67. −8/3

68. 1

69. 30.8

70. 4

71. 440/3

72. 1054

73. 1999997

74. 0

75. (c)

76. (b)

77. (a)

78. 14

79. 625

80. 216

81. (a)
82. 12
83. (c)
84. (a)
85. (d)
86. 1.25
87. 3/16
88. (d)
89. (c)
90. (c)

Level of Difficulty (III)

1. (d)
2. (d)
3. (c)
4. (d)
5. (d)
6. (d)
7. (d)
8. (b)
9. (a)
10. (a)
11. (c)
12. (c)
13. (a)
14. (b)
15. (a)
16. (b)
17. (d)

18. (b)
19. (c)
20. (b)
21. (b)
22. (b)
23. (a)
24. (b)
25. (b)
26. (d)
27. (b)
28. (a)
29. (c)
30. (a)
31. (d)
32. (a)
33. (d)
34. (a)
35. (d)
36. (d)
37. (c)
38. (d)
39. (c)
40. (d)
41. (b)
42. (b)
43. (c)
44. (b)
45. (c)

46. (b)

47. (a)

48. (b)

49. (b)

50. (b)

51. (e)

52. (a)

53. (b)

54. (e)

55. (d)

56. (b)

57. (c)

58. (b)

59. (d)

60. (a)

61. (b)

62. (c)

63. 3

64. (c)

65. (b)

66. (d)

67. (b)

68. (b)

69. (c)

70. (d)

71. 73.36

72. (b)

73. 8

74. 2

75. 2

76. 27

77. 4

78. 7

79. 784

80. (a)

81. (c)

82. (b)

83. (d)

84. 80

85. 5

86. (d)

87. 18

88. 18

89. 2

90. 2.86

91. 1

92. 3.5

93. 291

94. 16

95. (c)

96. 4

97. 36

98. 0

99. 3905

100. 2

Solutions and Shortcuts

Level of Difficulty (I)

1. $2x2 - 1 = 1 - 3x$

   $2x2 + 3x - 2 = 0 \Rightarrow (2x - 1)(x + 2) = 0 \Rightarrow x = -2, 1/2.$

2. $fogoh(2) = f(g(h(2)))$

   $h(2) = 4$

   $f(g(h(2))) = f(g(4)) = f(-1/3) = -3$. Hence, the Option (d) is correct answer.

3. $f(x) = \sqrt{x^3} \Rightarrow f(3x) = \sqrt{(3x)^3} = 3\sqrt{3x^3}$. Hence, Option (c) is correct.

4. $(x+1)^3$ is not odd as $f(x) \neq -f(-x)$. Hence, option (a) is correct.

Solutions for Questions 5 to 7:

$f(1) = 0, f(2) = 1$

$f(x) = f(x - 2) - f(x - 1)$

$f(3) = f(1) - f(2) = -1$

$f(4) = f(2) - f(3) = 2$

$f(5) = f(3) - f(4) = -3$

$f(6) = f(4) - f(5) = 5$

$f(7) = f(5) - f(6) = -8$

$f(8) = f(6) - f(7) = 13$

$f(9) = f(7) - f(8) = -21$

5. 13. Hence, Option (b) is correct.
6. $f(7) + f(4) = -8 + 2 = -6$. Hence, Option (b) is correct.
7. $0 + 1 - 1 + 2 - 3 + 5 - 8 + 13 - 21 = -12$. Hence, option (a) is correct.
8. For any ${}_nC_r$, n should be positive and $r \geq 0$.

   Thus, for positive x, $5 - x \geq 0$

   $\Rightarrow x = 0, 1, 2, 3, 4, 5$. Hence, Option (c) is correct.
9. (c)
10. (d)
11. (a)
12. (d)
13. $foh(x) = hof(x) = x$

    You will realise that if were to form a chain of these functions for even number of times, you would still end up getting x. For example, *Fohofo-h*(x) = *foho*$(x) = x$

    Since both the parts of the multiplication of the two functions, have the functions h and f repeated for even number of times; each of their values will be x and their product will be x^2.

    Hence, Option (d) is correct.
14. $fo(foh)o(hof)(x)$

$= fo(foh)(x)$

$= f(x) = 2x + 5$

Hence, option (a) is correct.

15. $A(x) = f(x + 1) \Rightarrow |x - 4 + 1| + |x - 5 + 1| + |x - 6 + 1|$

$A(x) \Rightarrow |x - 3| + |x - 4| + |x - 5|$

Obviously, this is neither odd or nor even alternatively, we know the graph of this function will neither be symmetrical to axis or origin.

Hence, Option (c) is correct.

16. Let $f(x) = y$

Then $y = \dfrac{e^x - e^{-x}}{e^x - e^{-x}} = \dfrac{e^{2x} - 1}{e^{2x} + 1}$

Now find the value x in terms of y.

$\Rightarrow \dfrac{y+1}{y-1} = - e2x$ [Applying componendo-dividendo]

$\Rightarrow \dfrac{y+1}{1-y} = e2x \Rightarrow x = ½ \log_e \dfrac{y+1}{1-y}$

Then put y in place of x and x in place of y.

Thus, we will get inverse function of $f(x)$ as: $y = ½ \log_e \dfrac{x+1}{1-x}$

$\Rightarrow$ Option (a) is correct.

17. If $f(x) = f(-x)$, i.e. $f(0 + x) = f(0 - x)$, the graph is symmetrical about $x = 0$.

Therefore, the graph is symmetrical about $x = 3$ if $f(3 + x) = f(3 - x)$

Hence, Option (b) is correct.

18. $f(1) = 15 - 3 \times (1)4 + 2 \times (1)2 - 4 \times (1) + a = a - 4$

$= a - 16$.

Once we have these expressions for $f(1)$ and $f(2)$, we need to use the information of $f(1)$ and $f(2)$ being of opposite signs. Applying a bit of common sense here, we can see that it is not possible for $f(2)$ to be positive, if $f(1)$ is negative, since $f(2) < f(1)$. Hence, the only situation where the condition of opposite signs will be true would be if $f(1)$ is positive and $f(2)$ is negative. $f(1)$ will be positive if $a > 5$, and $f(2)$ will be negative for values of a up to and including 15. Hence, the correct answer will be option (c).

19. Sum of coefficients can be found by putting $x = 1$.

$(1 + 1 - 3)2163 = -1$

Hence, Option (b) is correct.

20. Let $x = -2$ and check the options one by one:

(a) $f(-22) = |(-22) - 2| = 2$ and $f(x) = 4$

$[f(x)]2 = 16$. We do not get the equality of the two values. Hence, option (a) is not correct.

(b) $f(|-2|) = 0$ and $f(-2) = 4$. Hence, Option (b) is not correct.

(c) Let $x = -2$ and $y = 3$

$f(-2 + 3) = |1 - 2| = 1$ and $f(-2) = 4$ and $f(3) = 1$

Hence, Option (c) is not correct.

Thus, Option (d) is correct.

21. Since a, b, c are in AP with a common difference of 2, it means that the difference between the greatest value and the smallest value between a,

b and c would always be 4. The functions given to us $f(a,b,c)$ and $g(a,b,c)$ represent the minimum and the maximum value between a, b, and c, respectively. Hence $f(a,b,c) - g(a,b,c) = -4$. Hence, Option (c) is correct.

22. When $p < 3$, max $(p, 3) = 3$

    Expression = 9

    When $p > 3$, max $(p, 3) = p$

    Expression = $3p$ (as $p > 3$)

    Hence, Option (b) is correct.

23. $P(5, 3) = 15$

    $L(12, 9) = 36$

    Min (36, 15) = 15

    Max (4, 15) = 15

    $P(9, 15) = 135$

    Hence, Option (c) is correct.

24. $f(x) = \frac{ax}{x+1}$. $f(f(x) = f(f(\frac{ax}{x+1})) = \frac{a \times \frac{ax}{x+1}}{\frac{ax}{x+1}+1} = \frac{a^2x}{ax+x+1}$

    According to the question, we also have: $\frac{a^2x}{ax+x+1} = x$

    This, would occur at $x = -1$. The value of a, will also be -1 at $x = -1$. Hence, Option (d) is correct.

25. $2x + 2y = 2$

    $2y = 2 - 2x$

$y = \log_2(2 - 2^x)$. The logarithmic function, would exist only if $2 - 2^x$ is positive. Thus, we have:

$2 - 2^x > 0 \rightarrow 2 > 2^x \rightarrow x < 1$

Hence, Option (d) is correct.

26. $f(3) = 2 \times (3)^2 + 7 \times (3) - 9 = 30$

$g(30) = 2^{30} + 3 = 63$

Hence, option (a) is correct.

27. This function will be defined only when the base of the exponent (1/2) is non-negative.

Thus, for the domain,

$(169 - x^2) \geq 0$

Or $(13 - x)(13 + x) \geq 0$

This gives $-13 \leq x \leq 13$

Hence, (d) is the correct option.

28. Considering the definition of an even function, i.e. $f(x) = f(-x)$,

It is clear that only x^{12} satisfies this condition.

For option (b), $f(-x) = (-x - 3)^2$

which is not same as $f(x)$.

For option (c), $f(-x) = (-x + 3)^4$

which is again not equal to $f(x)$.

Hence, (a) is the correct option.

29. Putting $y = 1/(x - 1)$, we get: $x - 1 = 1/y \rightarrow x = 1 + 1/y$

Or $x = (1 + y)/y$.

Now we will replace y with x and x with y to get the inverse function as $y = (1 + x)/x$.

Hence, (d) is the correct option.

30. $f(1) = 2, f(2) = 3, f(3) = f(2) + f(1) = 5; f(4) = f(3) + f(2) = 8; f(5) = f(4) + f(3) = 13; f(6) = f(5) + f(4) = 21$

Hence, (a) is the correct option.

31. $f(7) = f(6) + f(5) = 34$. Hence, $f(7) - f(4) = 34 - 8 = 26$

Hence, (c) is the correct option.

32. $a\$b = 2\$3 = 4 + 9 = 13$

$a\&b = 2\&3 = 3 \times 4 = 12$

$13\%12 = (13 - 1)(12 - 1)$

$= 12 \times 11$

$= 132$

Hence, Option (c) is correct.

33. Solving the left hand side expression of sign '&' first,

$4\$3 = 4_2 + 3_2 = 25$

$25\%6 = (25 - 1)(6 - 1) = 120$.

Solving the right bracket of the expression (after the sign '&'),

$2\%7 = (2 - 1)(7 - 1) = 6$

$5\$6 = 5^2 + 6^2 = 61$

$120\&61 = (120 + 1)(61 + 1) = 7502$

Hence, (a) is the correct option.

34. Going by the options,

Gives $(2\$4) = 2^2 + 4^2 = 20$

Gives $(2\%4) = (2 - 1)(4 - 1) = 3$
Gives $(2\&4) = (2 + 1)(4 + 1) = 15$

Hence, (b) is the correct option.

35. As $a\$b = a^2 + b^2$

Clearly options (a) and

(b) both will yield results which will be far greater than 100.

Hence, (c) is the correct option.

36. As square of any number is always non negative, other expressions can give any value.

Hence, (a) is the correct option.

37. Since the denominator $x^2 - 3x + 2$ has real roots, the maximum value would be infinity.

38. The minimum value of the function would occur at the minimum value of $(x^2 - 2x + 5)$ as this quadratic function has imaginary roots.

For $y = x^2 - 2x + 5$

$dy/dx = 2x - 2 = 0 \Rightarrow x = 1$

$\Rightarrow x^2 - 2x + 5 = 4$

Thus, minimum value of the argument of the log is 4.

So, minimum value of the function is $\log_2 4 = 2$.

39. {[(3@4)! (3 #2)] @ [(4!3) @ (2 # 3)]}

{[(3.5) ! (5)] @ [(0.5) @ (–5)]}.

{[–0.75] @ [–2.25]} = –1.5

40. (7) @ (–0.5) = 3.25.

41. 0 @ 0.5 = 0.25. Thus, (a) is the correct option.

42. Option (b) gives us: (1) (4) = 4.

Option (c) gives us: $\frac{(16)}{(1)(4)}$

16/4 = 4

Hence, both (b) and (c) are correct.

43. (a) will always be true because $(a + b)/2$ would always be greater than $(a - b)/2$ for the given value range.

Further, $a^2 - b^2$ would always be less than $a^3 - b^3$. Thus, Option (d) is correct.

Solutions for Questions 44 to 48.

44. Option (a) = $(a - b)(a + b) = a^2 - b^2$

45. Option (a) = $(a^2 - b^2) + b^2 = a^2$.

46. 3 – 4 × 2 + 4/8 – 2 = 3 – 8 + 0.5 – 2 = – 6.5

(using BODMAS rule)

47. The maximum will depend on the values of a and b. thus, cannot be determined.

48. The minimum will depend on the values of a and b.

Thus, cannot be determined.

49. Any of $(a + b)$ or a/b could be greater and thus, we cannot determine this.

50. Again $(a + b)$ or a/b can both be greater than each other depending on the values, we take for a and b.

E.g. for $a = 0.9$ and $b = 0.91$, $a + b > a/b$

For $a = 0.1$ and $b = 0.11$, $a + b < a/b$

51. Given that $F(n-1) = \frac{1}{(2-F(n))}$, we can rewrite the expression as $F(n) = (2F(n-1)-1)/((F(n-1))$.

For $n = 2$: $F(2) = \frac{6-1}{3} \Rightarrow F(2) = \frac{5}{3}$.

The value of $F(3)$ will come out as $7/5$ and $F(4)$ comes out as $9/7$ and so on. What we realise is that for each value of n, after and including $n = 2$, the value of $F(n) = \frac{2n+1}{2n-1}$.

This means that the greatest integral value of $F(n)$ will always be 1 for $n = 2$ to $n = 1000$.

Thus, the value of the given expression will turn out to be:

$3 + 1 \times 999 = 1002$. Hence, Option (b) is the correct answer.

52. From the solution to the previous question, we already know how the value of the given functions at $n = 1, 2, 3$ and so on would behave.

Thus, we can try to see what happens when we write down the first few terms of the expression:

$F(1) \times F(2) \times F(3) \times F(4) \times \ldots. F(1000)$

$= 3 \times \frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \times \cdots \times \frac{2001}{1999} = 2001$

53. Since $f(0) = 15$, we get $c = 15$.

Next, we have $f(3) = f(-3) = 18$. Using this information, we get:

$9a + 3b + c = 9a - 3b + c \rightarrow 3b = -3b$

$\therefore 6b = 0 \rightarrow b = 0$.

Also, since

$f(3) = 9a + 3b + c = 18 \rightarrow$ we get: $9a + 15 = 18 \rightarrow a = 1/3$

The quadratic function becomes $f(x) = x^2/3 + 15$.

$f(12) = 144/3 + 15 = 63$.

54. What you need to understand about $M(x^2\, \theta\, y^2)$ is that it is the square of the sum of two squares. Since $M(x^2\, \theta\, y^2) = 361$, we get $(x^2 + y^2)^2 = 361$, which means that the sum of the sqaures of x and y, viz. $x^2 + y^2 = 19$. (**Note:** It cannot be –19 as we are talking about the sum of two squares, which cannot be negative under any circumstance).

Also, from $M(x^2\, \psi\, y^2) = 49$,we get $(x^2 - y^2)^2 = 49$, $\rightarrow (x^2 - y^2) = \pm 7$.

Based on these two values, we can solve for two distinct situations:

(a) When $x^2 + y^2 = 19$ and $x^2 - y^2 = 7$, we get $x^2 = 13$ and $y^2 = 6$

(b) When $x^2 + y^2 = 19$ and $x^2 - y^2 = -7$, we get $x^2 = 6$ and $y^2 = 13$

In both cases, we can see that the value of: $((x_2 y_2) + 3)$ will come out as $13 \times 6 + 3 = 81$ and the square root of its value will turn out to ± 9. Hence Option (b) is correct.

55. The first thing you need to understand while solving this question is that, since $[m]$ will always be integral, hence $\Psi(4x + 5)$ will also be integral. Since $\Psi(4x + 5) = 5y + 3$, naturally, the value of $5y + 3$ will also be integral. By a similar logic, the value of x will also be an integer considering the second equation: $\Psi(3y + 7) = x + 4$.

Using, this logic we know that $\Psi(4x + 5) = 4x + 5$ (because, whenever m is an integer the value of $[m] = m$).

This leads us to two linear equations as follows:

$4x + 5 = 5y + 3$ (1)

$3y + 7 = x + 4$ (2)

Solving simultaneously, we will get: $x = -3$ and $y = -2$.Thus, $x_2 \times y_2 = 9 \times 4 = 36$.

56. Since $f(128) = 4$, we can see that the product of $f(256).f(0.5) = f(256 \times 0.5) = f(128) = 4$.

Similarly, the products $f(1).f(128) = f(2).f(64)$

$= f(4).f(32) = f(8).f(16) = 4$.

Thus, $M = 4 \times 4 \times 4 \times 4 \times 4 = 1024$.

Hence, Option (d) is the correct answer.

57. The only values of x and y that satisfy the equation $4x + 6y = 20$ are $x = 2$ and $y = 2$ (since, x, y are non-negative integers). This gives us: $4 \le M/2^{2/3}$. M has to be greater than $2^{8/3}$ for this expression to be satisfied. Hence, Option (c) is correct.

58. $\theta(\Psi(-7)) = \theta(-2) = 14$. Hence, Option (b) is correct.

59. $F(2b) = F(b + b) = F(b).F(b) \div 2 = (F(b))^2 \div 2$.

Similarly, Ψ, $F(3b) = F(b + b + b) = F(b + b).F(b) \div 2 = \{F(b)^2 \div 2\}.\{F(b)\} \div 2 = (F(b))^3 \div 2^2$.

Similarly, Ψ, $F(4b) = (F(b))^4 \div 2^3$.

Hence, $F(12b) = (F(b))^{12} \div 2^{11}$. Option (d) is correct.

60. To test for a reflexive function as defined in the problem, use the following steps:

Step 1: To start with, assume a value of '*b*' and derive a value for 'a' using the given function.

Step 2: Then, insert the value, you got for '*a*' in the first step into the value of '*b*' and get a new value of '*a*'. This value of '*a*' should be equal to the first value of '*b*' that you used in the first step. If this occurs, the function would be reflexive. Else it is not reflexive.

Checking for the expression in (i), if we take $b = 1$, we get:

$a = 8/1 = 8$. Inserting, $b = 8$ in the function gives us $a = 29/29 = 1$. Hence, the function given in (i) is reflexive.

Similarly checking the other two functions, we get that the function in (ii) is not reflexive while the function in (iii) is reflexive.

Thus, Option (c) is the correct answer.

61. $f(g(x)) = f(|3x-2|) = \frac{1}{|3x-2|}$

Hence, option (a) is correct.

62. $f(g(x)) = f(|x|) = |x|^2 + \frac{1}{|x|^2}$. This function would take the same values when you try to use a positive value or a negative value of x. For instance, if you were to put x as 2, you would get the same answer as if you were to use x as −2. Hence, $f(g(x))$ is an even function.

63. For this question, you would have to go through each of the options checking them for their correctness in order to identify the correct answer. Thus,

For option (a): $g(x)+(g(x))^2 = |x|+|x|^2$

$\Rightarrow f(x) \neq g(x)+(g(x))^2$. Hence, option (a) is not correct.

For option (b): $f(x) = x^2 + \frac{1}{x^2}$. $f(g(x)) = |x|^2 + \frac{1}{|x|^2}$

$f(x) \neq -f(g(x))$. Hence, Option (b) is not correct.

For option (c): $g(f(x)) = \left|x^2 + \frac{1}{x^2}\right|$

$f(g(x)) = |x|^2 + \frac{1}{|x|^2}$ which is the same as $\left|x^2 + \frac{1}{x^2}\right|$.

Hence, $f(g(x)) = g(f(x))$

$\because$ Hence, Option (c) is correct.

64. $f(x) = f(-x)$

$g(x) = g(-x)$

$h(x) = -h(-x)$

$t(x) = t(-x)$

Therefore three functions are even.

65. $f(x) = f(-x)$

$\Rightarrow h(f(x)) = h(f(-x))$

$\Rightarrow$ Hence, $h(f(x))$ is an even function. So the correct answer is 1.

66. $t(x) = t(-x)$

Hence, $h(t(x)) = h(t(-x))$

$\Rightarrow h(t(x))$ is an even function. Thus, correct answer is 1.

67. $f(2) = \frac{2^2+1}{2-1} = 5$

$$f(f(2)) = f(5) = \frac{5^2+1}{5-1} = \frac{26}{4}$$

$$f(f(f(2))) = f\left(\frac{26}{4}\right) = \frac{\left(\frac{26}{4}\right)^2 + 1}{\frac{26}{4} - 1} = \frac{\frac{676+16}{16}}{\frac{22}{4}}$$

$$= \frac{692}{16} \times \frac{4}{22} = 7.86$$

68. $D(3,4) = \frac{3}{4} = 0.75$

$$S(2, D(3,4)) = S(2, 0.75) = 2.75$$

$$P(S(2, D(3,4)), 5) = P(2.75, 5) = 2.75 \times 5 = 13.75$$

69. $P(2, 3) = 2 \times 3 = 6$

$D(4, 2) = 4 \div 2 = 2$

$$S(P(2,3), D(4,2)) = S(6,2) = 8$$

$$t(1,5) = |1-5| = 4$$

$$S(8,4) = 8 + 4 = 12$$

Solutions for Questions 70 to 72:

70. $[(5\ P\ 6)Q(4\ Q\ 2)]S\ (3\ S\ 1)$

$= [(|5-6|)Q\ (4/2)]S\ (1/3)$

$= [1\ Q\ 2]S\left(\frac{1}{3}\right)$

$= [1\ Q\ 2]S\left(\frac{1}{3}\right)$

$= [1 \times 2)S\left(\frac{1}{3}\right)$

$= 2\ S\frac{1}{3}$

$= \left(\frac{1}{2\times\frac{1}{3}}\right) = \frac{3}{2} = 1.5$

71. For this Question, we would need to check each option and select the one that is true.

Checking option (a) we can see that:

$(4P2) = (4Q2) = 4/2 = 2, (2P4) = |2 - 4| = 2$

So, option (a), is incorrect.

Checking Option (b) we get:

$(4Q2) = 4/2 = 2, 2R4 = 2 \times 4 = 8$

Hence, option (b), is incorrect.

Checking Option (c) we get:

$(6Q3)= 6/3= 2$

$2\ S\ (0.5) = 1/(2 \times 0.5)= 1$

Hence, Option (c) is incorrect.

Hence, Option (d) is correct.

72. $(5P3)Q(4S2) = (5\,Q\,3)Q\left(\frac{1}{4.2}\right)$

$= \frac{5}{3}Q\frac{1}{8}$

$= \frac{40}{3}$

$20Q1.5 = 20 \div 1.5 = \frac{40}{3}$, therefore, the operator Q should replace 'K' in the equation.

Solutions for Questions 73 to 75:

73. $f(3,4) = [3] + \{4\} = 3 + 4 = 7$

$g(3.5, 4.5) = [4.5] - \{3.5\} = 4 - 4 = 0$

$i(f(3,4), g(3.5, 4.5)) = i(7,0) = -7$

Hence, Option (d) is correct.

74. $a^3 = 64 \Rightarrow a = 4$

$b^2 = 16 \Rightarrow b = 4 or -4$

When $a = 4$, $b = 4$

$f(4,4) = [4] + \{4\} = 4 + 4 = 8$

$g(4,4) = [4] - \{4\} = 4 - 4 = 0$

But these values do not satisfy the condition in the problem that $8 + f(a,b) = -g(a,b)$. Hence, we will try to use $a = 4$ and $b = -4$, to see whether that gives us the right set of values for the conditions to be matched.

When $a = 4, b = -4$

$$f(4,-4) = [4] + \{-4\} = 4 - 4 = 0$$

$$g(4,-4) = [-4] - \{4\} = -4 - 4 = -8$$

The given condition is satisfied here. Hence, $a = 4$ and $b = -4$. Therefore, $a - b = 4 - (-4) = 8$.

75. $f(1.2,-2.3) + g(-1.2,2.3)$

$= [1.2] + \{-2.3\} + [2.3] - \{-1.2\}$

$= -2 + 2 + 1$

$= 2$

$= i(a,-1.3) = \{-a-1.3\}$

For $a = -2.4 \Rightarrow i(-2.4,-1.3) = \{2.4-1.3\} = \{1.1\} = 2$

Hence, Option (d) is correct.

Solutions for Questions 76 and 77:

76. Given: $xPy = \frac{1}{1+\frac{y}{x}} = \frac{x}{x+y}$ and $xQy = 1 + \frac{x}{y} = \frac{x+y}{y}$. From this point you would need to read the options and check the one that gives you a value of $\frac{x}{y}$. It is easily evident here that:

$$(xPy) \times (xQy) = \frac{x}{x+y} \times \frac{x+y}{y} = \frac{x}{y}$$

Hence,Option (c) is correct.

77. $S(2,3) = (2P3)P(2Q3)$

$$= \left(\frac{2}{2+3}\right)P\left(\frac{2+3}{3}\right)$$

$$= \frac{2}{5}P\frac{5}{3} = \frac{\frac{2}{5}}{\frac{2}{5}+\frac{5}{3}} = 0.19$$

Solutions for Questions 78 to 80:

78. $(1 + \min(2A3, 1C2))B(\max(1A2), 1C1)$

$=[1 + \min(1, 2)]B \max(1, 1)$

$(1 + 1)B(1^2) = 2B1 = [2 \div 1] = 2$

79. $\max(7A3, 16B2) = \max(4, 8) = 8^2 = 64$

Now by checking the options we get only Option (b) that gives us the correct value.

$(32B2)C(\min(4, 8))$

$|[32 \div 2] \times 4| = |16 \times 4| = 64$

Hence, Option (b) is correct.

80. $\max(3, 4) \div \min(8, 4) = 4^2 \div 4 = 4$. Checking the options we see:

Option (a): $8A2 = |8 - 2| = 6$

Option (b): $28B7 = 28 \div 7 = 4$

Option (c): $4C2 = |4 \times 2| = 8$

Hence option (b) is correct.

Solutions for Questions 81 to 85:

81. $f(1,3,5,7) + g(2,4,6,8) = 1 + 8 = 9$

$$h[aK, K] = \left[\frac{aK}{K}\right] = [a] = a, [a \in I]$$

$\Rightarrow a = 9$

82. $t(1,2,3,4) = 1 \times 2 \times 3 \times 4 = 24$

$i(1,2,3,4) = 1 + 2 + 3 + 4 = 10$

$f(t(1,2,3,4), i(1,2,3,4)) = f(24,10) = 10$

83. $f(5,6,7,8) = 5, i(1,2,3,4,) = 1 + 2 + 3 + 4 = 10$

$$h(5,10) = \left[\frac{5}{10}\right] = [0.5] = 0$$

84. $P = f(2,3,4,6) = 2$

$Q = g(1,2,3,4) = 4$

$$R = h(8,4) = \left[\frac{8}{4}\right] = 2$$

$S = t(1, 2, 3, 4) = 1 \times 2 \times 3 \times 4 = 24$

$T = i(4,5,6) = 4 + 5 + 6 = 15$

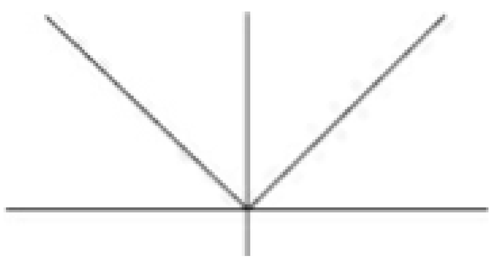
$\therefore P = R < Q < T < S.$

Hence, option (c) is correct

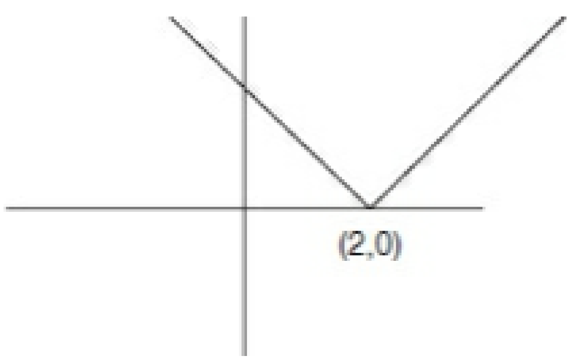
85. $f(1,2,3) = 1, g(2,3,4) = 4, f(0,1,2)$
$= 0, g(-3,-2) = -2$

$f(1,4,0,-2) = -2.$

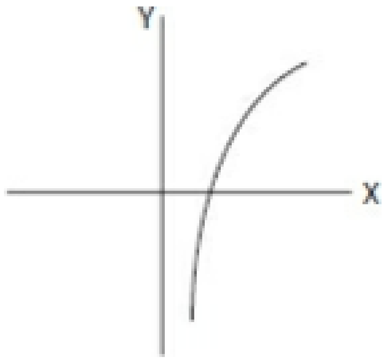
86. $|x|$



$|x - 2|$



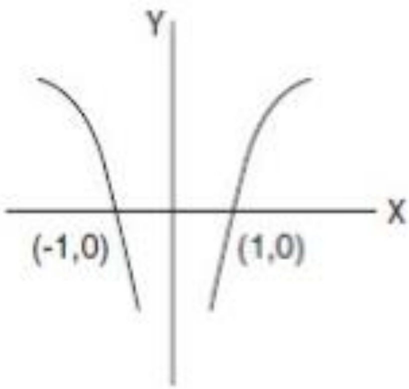
87. $\log x$



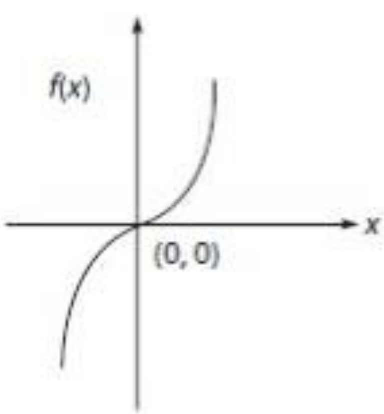
$f(x) \rightarrow f(|x|)$

Take mirror image about y-axis

$\log|x| \rightarrow$

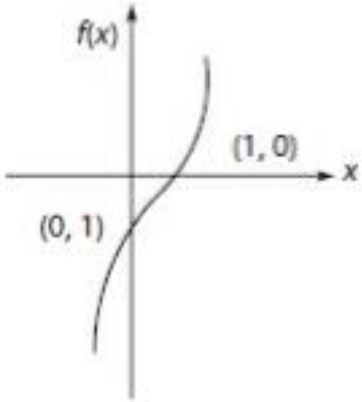


88. $x^3 \rightarrow$



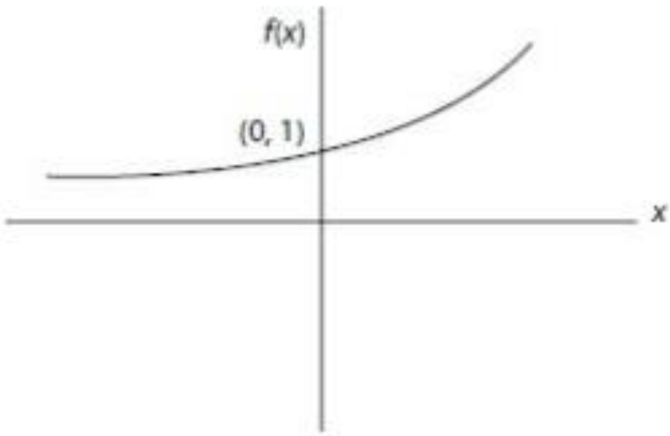
$(x - 1)^3 \rightarrow$

[Shift curve one unit right]

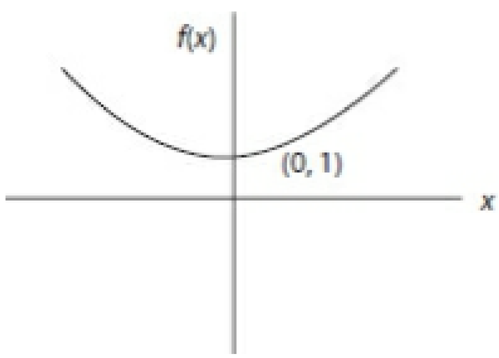


Hence, option (a) is correct.

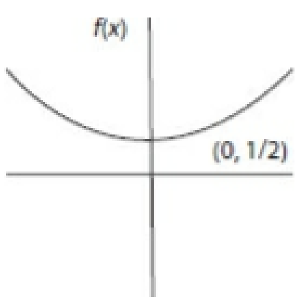
89. $e^x \rightarrow$



$e^{|x|} \rightarrow$



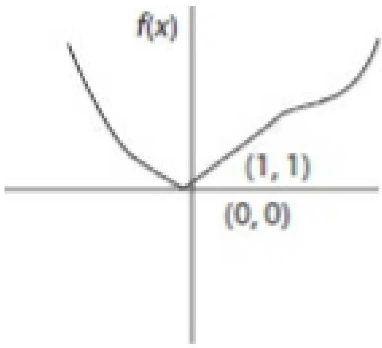
$\frac{e^{|x|}}{2} \rightarrow$



Hence, Option (b) is correct.

90. $\max(x,x^2) = \begin{cases} x^2, \text{for } -\infty < x \le 0 \\ x, \text{for } 0 \le x \le 1 \\ x^2, \text{for } x > 1 \end{cases}$

$\Rightarrow f(x) = \max(x, x2) \Rightarrow ?$



Hence, Option (b) is correct.

91. When $f(x)$ and $g(x)$ both are odd, then $S(x) = f(x) + g(x)$

$S(-x) = f(-x) + g(-x) = -[f(x) + g(x)]$, $S(x)$ is an odd function. This conclusion rejects option (a).

Their product $P(x) = f(x).g(x)$.

$P(-x) = f(-x).g(-x) = [-f(x)][-g(x)] = f(x)g(x)$. $P(x)$ is an even function. This is what is being said by the option (c). Hence, it is the correct answer.

If we check for Option (b) we can see that: when $f(x)$ and $g(x)$ both are even then $S(x) = f(x) + g(x)$
$S(-x) = f(-x) + g(-x) = [f(x) + g(x)]$, $S(x)$ is an even function.

Hence only Option (c) is true.

92. $f(g(-x)) = f(-g(x)) = f(g(x))$

$\therefore f(g(x))$ is an even function.

$g(f(x)) = g(f(-x)) = g(f(x))$

$\therefore g(f(x))$ is an even function.

Hence, Option (c) is true.

93. $f(x) = x^3 - x^2 - 6x$

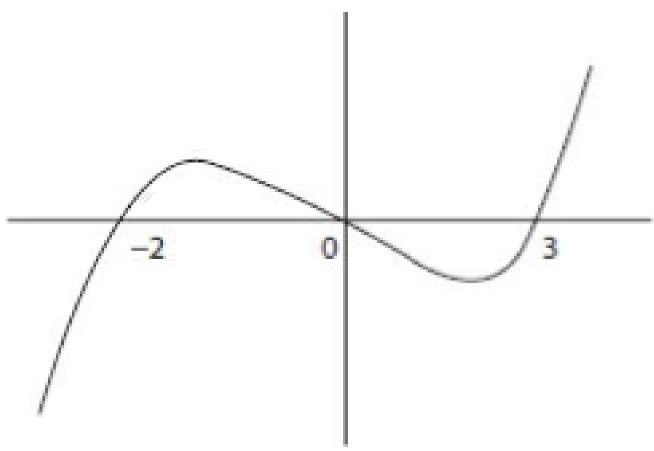
$= (x + 2)\,x\,(x - 3)$

$\Rightarrow f(x) = 0$

$\Rightarrow (x + 2)\,x\,(x - 3) = 0$

$X = 0, 3, -2$. There are three such values.

94. Curve of $f(x)$ will look like this:



For interval $(-2, 3)$, $f(x)$ will attain its minima in the interval $(0, 3)$.

95. Options (a), (b), (d) are undefined for $x = 0$.

As $x^4 + 7$ is always positive for $x \in R$, therefore, $\log(x^4 + 7) \in R$ for all $x \in R$. Hence, Option (c) is true. Each of the other options has at least one value where $f(x)$ does not remain real.

Level of Difficulty (II)

1. $(x-2)\left(x(x+1)(x+2)(x+3)(x+4)\ldots..(x+202)\right)$

$= (x-2)(x(x+1)(x+2)(x+3)(x+4).....(x+202))$

$= (x-2)(x^{203}+x^{202}(1+2+3+...+202)+..)$

Coefficient of x^{203} is $= -2+(1+2+3+...202) = -2+202\times\frac{203}{2} = -2+20503 = 20501$

2. $f(x) = (x + 3) \times (36 - 12x) = 12 \times (3 - x)(x + 5)$

The value of $f(x)$ is maximum when $3 - x = x + 5$ or $x = -1$.

Maximum value of $f(x) = 12 \times (3 + 1) \times (-1 + 5) = 192$.

3. '$3a + 4$' and '$b - 12$' must be integers. Also, it is given to us that a and b must be integers. Therefore, $b - 2$ and $a - 4$ must be integers.

$f(b - 2) = b - 2 = 3a + 4$ or $b - 3a = 6$

$f(a - 4) = b - 12$ or $a - 4 = b - 12$ or $a - b = -8$

On solving the above two equations, we get: $a = 1$, $b = 9$

Product of a and $b = 1 \times 9 = 9$.

4. $f(g(x)) = \left[\frac{x^2}{9}\right]^{\left[\frac{x^2}{9}\right]}$

The value of $f(g(x))$ will be minimum when value of x is as low as possible. Since, x cannot be 0; the values of the expression would get minimised if $0 < g(x) < 1$.

$f(g(x)) = 1$; this will be the minimum possible value of $f(g(x))$.

5. $g(x)-f(x)=(p-a)x^2+(q-b)x-c$ for all x

$\therefore g(1)=f(1), g(2)-f(2)=1 and g(3)-f(3)=4$

$g(1)-f(1)=(p-a)+(q-b)-c=0$ (1)

$g(2)-f(2)=4(p-a)+2(q-b)-c=1$ (2)

$$g(3)-f(3)=9(p-a)+3(q-b)-c=4 \quad (3)$$

By solving equations 1, 2, 3, we get $p-a=1$, $q-b=-2$ and $c=-1$

$$\therefore g(4)-f(4)=16(p-a)+4(q-b)-c$$
$$=16\times1+4\times-2+1=9$$

Solutions for Questions 6 to 8:

6. $5f(x)+2f\left(\frac{1}{x}\right)=4x+2$

   $5f\left(\frac{1}{x}\right)+2f(x)=\frac{4}{x}+2$ (Replacing x by $1/x$)

   On solving the above two equations for $f(x)$, we get:

   $f(x)=\frac{1}{21}\left[\frac{20x}{}-\frac{8}{x}+\frac{6}{}\right]$

   $f(2)=\frac{1}{21}\left(\frac{40}{}-\frac{8}{2}+\frac{6}{}\right)=\frac{84}{42}=2$

7. $f(x)=\frac{1}{21}\left[\frac{20x}{}-\frac{8}{x}+\frac{6}{}\right]=\frac{18}{21}\rightarrow 20x^2-12x-8=0$

   $5x^2-3x-2=0$

   Sum of all values of x for which $f(x)=\frac{18}{21}$ is $(-b/a)=3/5$

8. Product of roots= $c/a=-2/5=-0.4$

Solutions for Questions 9 and 10:

9. $f(x)=8a^2-8+8(x-1)-x^2=8a^2-(x^2-8x+16)=8a^2-(x-4)^2$

   Hence, $f(x)$ is maximum for $x=4$.

10. $8a^2=72$ or $a=3$ or -3

    Required Sum = $-3+3=0$

Solutions for Questions 11 to 13:

$f(x) = (1-\frac{1}{x})f(x-1)$

$f(x) = \frac{x-1}{x}f(x-1)$

$f(x-1) = \frac{x-2}{x-1}f(x-2)$

$f(x-2) = \frac{x-3}{x-2}f(x-3)$ and so on till $f(1)$

In the value of $f(x)$, if we replace the value of $f(x-1)$ with $\frac{x-2}{x-1}f(x-2)$

and then subsequently replace

$f(x-2)$ with $f(x-2) = \frac{x-3}{x-2}f(x-3)$, we will end with a chain that would

end with $f(1)$. After doing so, we will get the following equation:

$f(x) = \frac{x-1}{x}\times\frac{x-2}{x-1}\times\frac{x-3}{x-2}\times\ldots f(1)$

Thus, we get the expression: $f(x) = f(1)/x$.

11. According to the question, $f(3) = 10$.

    $f(3) = f(1)/3 = 10$. Thus, $f(1) = 30$.

12. $f(1000)=f(1)/1000=10/1000=0.01$

13. $\frac{1}{f(1)}+\frac{1}{f(2)}+\frac{1}{f(3)}+\ldots+\frac{1}{f(9)}=\frac{1}{f(1)}+\frac{2}{f(1)}+\frac{3}{f(1)}+\ldots+\frac{8}{f(1)}=\frac{36}{f(1)}=\frac{36}{12}=3$

Solutions for Questions 14 and 15:

14. $f(x)=\sqrt{f(x-1)f(x+1)}$. Squaring both the sides, we get: $f(x+1)=\frac{[f(x)]^2}{f(x-1)}$

    Since, we are given the values of $f(0)$ and $f(1)$, we get:

$$f(2)=\frac{[f(1)]^2}{f(0)}=\frac{1}{4}$$

$$f(3)=\frac{[f(2)]^2}{f(1)}=\frac{\frac{1}{16}}{\frac{1}{2}}=\frac{1}{8}$$

15. Similarly, $f(4)=\frac{1}{16}, f(5)=\frac{1}{32}$

$f(1) + f(2) + f(3) + \dots f(\infty) =$

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots=\frac{\frac{1}{2}}{1-\frac{1}{2}}=1$$

(Using the formula for the sum of an infinite geometric progression.)

16. $f(x) = \frac{x+3}{2\times(x^2+6x+9)}=\frac{x+3}{2(x+3)^2}=\frac{1}{2(x+3)}$

As x is non-negative integer so the maximum possible value of $f(x)$ would occur, when $x = 0$. We get the maximum possible value of $f(x)$ = $\frac{1}{2\times(0+3)}=\frac{1}{6}$.

Solutions for Questions 17 and 18:

Assume that the original polynomial is: $f(x) = Q[(x)(x-2)(x-3)] + px + q$, where p and q are constants.

(**Note:** We make this assumption, in order to solve the question through the process that follows. In this assumption.)

The remainder when a polynomial in x, is divided by $(x-a)$, is obtained by inserting the value of $x = a$, in the polynomial. Using this logic, we can see that:

For $x = 2, f(2) \to 2p + q = 3$

For $x = 3, f(3) \to 3p + q = 2$

On solving, we get: $p = -1$ and $q = 5$

Hence, the correct polynomial would be

$f(x) = Q[(x)(x-2)(x-3)] + (5 - x)$

17. $f(3) = 2$
18. When we divide $f(x)$ by $(x-2)(x-3)$, then the remainder is $5 - x$. Hence, Option (b) is correct.

Solutions for Questions 19 to 21:

19. $\left[\frac{x}{7}\right] = \left[\frac{x}{9}\right]$

For $0 \le x \le 6$, both LHS and RHS are equal to 0. (7 solutions)

For $9 \le x \le 13$, both LHS and RHS are equal to 1. (5 solutions)

For $18 \le x \le 20$ both LHS and RHS are equal to 2. (3 solutions)

For $x = 27$, both LHS and RHS are equal to 3. (1 solution)

After $x = 27$, there would be no value at which the LHS and RHS give the same answer.

Hence, the number of non-negative integral solutions is 7 + 5 + 3 + 1 = 16 solutions.

For $-7 \le x \le -1$, both LHS and RHS is equal to –1. (7 solutions)

For $-14 \le x \le -10$ both LHS and RHS is equal to –2. (5 solutions)

For $-21 \le x \le -19$ both LHS and RHS is equal to –3. (3 solutions)

For $x = -28$, both LHS and RHS is equal to –4. (1 solution)

Hence, the number of non-negative integral solutions is 7 + 5 + 3 + 1 = 16 solutions.

Total number of integral solutions = 16 + 16 = 32 solutions.

20. Required sum = 0 + 1 + 2 + 3 + …. + 6 + 9 + 10 + … + 13 + 18 + 19 + 20 + 27 – (1 + 2 + …. + 7 + 10 + … + 14 + 19 + 20 + 21 + 28) = (0 – 7 + 9 – 14 + 18 – 21 + 27 – 28) = –16

21. Maximum value of x = 27

Solutions for Questions 22 and 23:

22. $\left(x^2-7x+13\right)^{2x+5}=1$

The above equation is possible when $2x + 5 = 0$ or

When $2x + 5 = 0$, then $x = -5/2$.

When $x2 - 7x + 13 = 1$, then x = 3 or 4.

Hence, for three values of x, $f(x) = 1$.

23. Required sum = 3 + 4 – 5/2 = 4.5

24. $f(x + 2) = |x-1|+|x-2|+|x-3|$

It can be seen that this would reach its minimum value when $x = 2$. Since, the question tells us that $f(x + 2)$ is minimum at $x = k$, we get: $k = 2$.

Solutions for Questions 25 and 26:

25. $f(x) = 2 - 1/f(x - 1)$

For $x = 2$, $f(2) = 2 - 1/f(1)$ or $f(1) = 2$.

Similarly, $f(3) = 4/3$, $f(4) = 5/4$ and so on. We can observe that, $f(n) = (n + 1)/n$

$f(500) = 501/500$

26. $f(n) = (n + 1)/n = 1 + 1/n$

$[f(n)] = [1 + 1/n] = 1$. Hence, $[f(100001)] = 1$

27. $\frac{[x]-2}{4-[x]} > 0$ when $[x] < 4$ and $[x] > 2$

It is only possible when $[x] = 3 \Rightarrow x \in [3, 4)$. Hence, Option (b) is correct.

Solutions for Questions 28 and 29:

28. $f(xy) = f(x) \times f(y)$

$f(4) = f(2) \times f(2) = [f(2)]^2 = 2^2 = 4$

$f(2) < f(3) < f(4)$

$f(x)$ is an integer valued function. $f(3)$ must be an integer. Hence, $f(3) = 3$.

29. From the above analysis, we can say that $f(n) = n$.

Hence, $f(2001) = 2001$.

30. $n^2 f(n) + f(1-n) = 2n - n^4$

On putting $n = -1$, we get:

$f(-1) + f(2) = 2(-1) - (-1)^4 = -3$ (1)

On putting $n = 2$, we get:

$4f(2) + f(-1) = 2(2) - (2)^4 = -12$ (2)

On solving equation (1) and (2), we get $f(2) = -3$

On putting $n = 5$, we get:

$25f(5) + f(-4) = -615$ (3)

On putting $n = -4$, we get:

$16f(-4) + f(5) = -264$ (4)

On solving equation (3) and (4), we get: $f(5) = -24$

Hence, $f(2)\,f(5) = -3 \times -24 = 72$.

31. $g(f(h(t))) = g(f(4t - 8)) = g\left(\sqrt{4t-8}\right)$

$= \dfrac{\sqrt{4t-8}}{4}$

32. $h(g(f(t)) = h(g(\sqrt{t})) = h(\sqrt{t}/4)$

$= \sqrt{t} - 8$

33. $-f(x)$ will be the mirror image of the function, about the x-axis which is seen in option (b).

34. $-f(x) + 1$ will be mirror image about the x-axis and then shifted up by 1. Option (a) satisfies this.

35. $f(x) - 1$ will shift down by 1 unit. Thus Option (c) is correct.

36. $f(x) + 1$ will shift up by 1 unit. Thus, Option (d) is correct.

37. The given function will become $h[11, 80, 1] = 2640$.

38. The given function will become $g[0, 0, 3] = 0$.

39. The given function will become $f[3, 3, 3] = 27$.

40. $f(1, 2, 3) - g(1, 2, 3) + \text{h}(1, 2, 3) = 11 - 23 + 18 = 6$

41. The number of g's and f's should be equal on the LHS and RHS since both these functions are essentially inverse of each other.

Thus, Option (c) is correct.

42. The required minimum value will occur at $f(x) = g(x) = 1$.

43. $SQ[R[(a + b)/b]] = SQ[R[17/5]] \Rightarrow SQ[2] = 2$

44. $Q[[SQ(63) + 7]/9] = Q[[8 + 7]/9] = Q[15/9] = 1$

45. $Q[[SA(36) + R(18/7)]/2] = Q[(7 + 4)/2] = Q[11/2] = 5$

46. $[x] - \{x\} = -1$

47. $[x] + \{x\}$ will always be odd as the values are consecutive integers.

48. At $x = 5.5$, the given equation can be seen to be satisfied as: $6 + 5 = 2 \times 5.5 = 11$.

49. $f(g(t)) - g(f(t)) = f(2.5) - g(6) = 8.25 - 2.166 = 6.0833$

50. $fog = f(3t + 2) = k(3t + 2) + 1$

$gof = g(kt + 1) = 3(kt + 1) + 2$

$k(3t + 2) + 1 = 3(kt + 1) + 2$

$\Rightarrow 2k + 1 = 5$

$\Rightarrow k = 2$

51. When the value of $x = 81$ and 82 is substituted in the given expression, we get,

$F(81)F(82) = -F(80)F(79)F(78)F(77)$ (1)

$F(82)F(83) = -F(81)F(80)F(79)F(78)$ (2)

On dividing (1) by (2), we get

$\frac{F(81)}{F(83)} = \frac{F(77)}{F(81)}$ fi $F(81) \times F(81) = 81 \times 9$

$\Rightarrow F(81) = 27$

Hence, option (a) is the correct answer.

52. In order to understand this question, you first need to develop your thought-process about what the value of $h(x)$ is in various cases. A little bit of trial-and-error will show you that the value of $h(x)$ since it depends on the minimum of $f(x)$ and $g(x)$, would definitely be dependent on the value of $f(x)$; once x becomes greater than +11 or less than –11. Also, the value of $g(x)$ is fixed as an integer at 16, whenever x is between –8 to +8. Also, at $x = 9$, $x = 10$ and $x = -9$ and $x = -10$, the value of $h(x)$ would still be an integer.

With this thought, when you look at the expression of $f(x) = 121 - x^2$, you realise that the value of x can be –10, –9, –8, –7, …0, 1, 2, 3,….8, 9, 10, i.e. 21 values of x when $h(x) = g(x)$. When we use $x = 11$ or $x = -11$, the value of $f(x) = 0$ and is not a positive integral value.

Hence, the correct answer is option (c).

53. Since, $R(x)$ is the maximum amongst the three given functions, its value will always be equal to the highest amongst the three. It is easy to imagine that $x^2 - 8$ and $3x$ are increasing functions, therefore, the value of the function is continuously increasing as you increase the value of x. Similarly $x^2 - 8$ would be increasing continuously as you go farther and farther down on the negative side of the x-axis. Hence, the maximum value of $R(x)$ would be infinity. Thus, Option (c) is the correct answer.

54. In this case, the value of the function, is the minimum of the three values. If you visualise the graphs of the three functions (viz: $y = x^2 - 8$, $y = 3x$ and $y = 8$), you realise that the function $y = 3x$ (being a straight line) will keep going to negative infinity as you move to the left of zero on the negative side of the x-axis.

Hence, the minimum value of the function $R(x)$ after a certain point (when x is negative) would get dictated by the value of $3x$. This point will be the intersection of the line $y = 3x$ and the function $y = x^2 - 8$, when x is negative.

The two intersection points of the line ($3x$) and the quadratic curve ($x^2 - 8$) will be obtained by equating $3x = x^2 - 8$. Solving this equation tells us that the intersection points are:

$\frac{3-\sqrt{41}}{2}$ and $\frac{3+\sqrt{41}}{2}$.

$R(x)$ would depend on the following structures based on the value of x:

(i) When x is smaller than $\frac{3-\sqrt{41}}{2}$, the value of the function $R(x)$ will be given by the value of $3x$.

(ii) When x is between $\frac{3-\sqrt{41}}{2}$ and 4 is the value of the function $R(x)$ will be given by the value of $x^2 - 8$,since that would be the least amongst the three functions.

(iii) After $x = 4$, on the positive side of the x-axis, the value of the function will be defined by the third function, viz: $y = 8$.

A close look at these three ranges will give you that amongst these three ranges, the third range would yield the highest value of $R(x)$. Hence, the maximum possible value of $R(x) = 8$. Thus, Option (b) is correct.

55. The expression is $2x2 - 5x + 4$, and its value at $x = 5$ will be equal to $50 - 25 + 4 = 29$. Thus, Option (b) is correct.

56. At $x = 0$, the value of the function is 20 and this value rejects the first option. Taking some higher values of x, we realise that on the positive side, the value of the function will become negative when we take x greater than 5, since the value of $(5 - x)$ would be negative. Also, the value of $f(x)$ would start tending to $-\infty$, as we take bigger values of x.

Similarly, on the negative side, when we take the value of x lower than -4, $f(x)$ becomes positive and when we take it farther away from 0 on the negative side, the value of $f(x)$ would continue tending to $+\infty$. Hence, Option (c) is the correct answer.

57. The remainder when $6x + 4$ is divided by 2 would be 0 in every case (when x is odd).

Also, when x is even, we would get $6x - 3$ as an odd number. In every case, the remainder would be 1 (when it is divided by 2).

Between $f(2), f(4), f(6), \ldots f(1000)$, there are 500 instances when x is even. In each of these instances, the remainder would be 1 and hence, the remainder would be 0 (in total). Thus, Option (b) is correct.

58. The product of p, q and r will be maximum, if p, q and r are as symmetrical as possible. Therefore, the possible combination is (4, 3, 3).

Hence, maximum value of $pq + qr + pr + pqr = 4 \times 3 + 4 \times 3 + 3 \times 3 + 4 \times 3 \times 3 = 69$.

Hence, Option (c) is correct.

59. The equation given in the question is: $3\alpha(x) + 2\alpha(2-x) = (x + 3)^2$ (1)

Replacing x by $(2-x)$ in the above equation, weget

$3\alpha(2-x) + 2\alpha(x) = (5-x)^2$

Solving the above pairs of equation, we get

$5\alpha(x) = 3(x + 3)^2 - 2(5 - x)^2 = 3(x^2 + 6x + 9) - 2(25 - 10x + x^2) = 3x^2 + 18x + 27 - 50 + 20x - 2x^2 = x^2 + 38x - 23$

Thus, $\alpha(x) = (x^2 + 38x - 23)/5$

Thus, $\alpha(-5) = -188/5 = -37.6$. The value of $[-37.6] = -38$. Hence, Option (b) is the correct answer.

60. The first thing you do in this question is to create the chain of values of $f(x)$ for $x = 1, 2, 3$ and so on.

The chain of values will look something like this:

When x is odd			*When x is even*		
$f(1)$	Value is given	6	$f(2)$	Value is given	4
$f(3)$	$= 1 + f(1)$	7	$f(4)$	$= 3 + f(2)$	7
$f(5)$	$= 3 + f(3)$	10	$f(6)$	$= 3 + f(4)$	10
$f(7)$	$= 5 + f(5)$	15	$f(8)$	$= 3 + f(6)$	13
$f(9)$	$= 7 + f(7)$	22	$f(10)$	$= 3 + f(8)$	16
$f(11)$	$= 9 + f(9)$	31			

In order to evaluate the value of the embedded function represented by$(f(f(f(f(1)))))$, we can use the above values and think as follows:

$f(f(f(f(1)))) = f(f(f(6))) = f(f(10)) = f(16) = 25$

Also, $f(f(f(f(2)))) = f(f(f(4))) = f(f(7)) = f(15) = 55$

Hence, the product of the two values is $25 \times 55 = 1375$.

Thus, option (a) is correct.

61. For $x > 0$, $x + \frac{1}{x}$ has a minimum value of 2, when x is taken as 1. Why we would need to minimise $x + \frac{1}{x}$ is because it is raised to the power 6 in the numerator, so allowing $x^n + \frac{1}{x^n}$ to become greater than its minimum will increase the value of the expression. Also, the value of any expression of the form $x^n + \frac{1}{x^n}$ will also give us a value of 2. Hence, the value of the expression would be:

$$\frac{\left(x+\frac{1}{x}\right)^6 - \left(x^6 + \frac{1}{x^6}\right) - 2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3 + \frac{1}{x^3}\right)} = \frac{2^6 - 2 - 2}{2^3 + 2} = 6$$

Hence, (d) is the correct choice.

62. The function would be defined when the term $\frac{1}{\{\log_{10}(3-x)\}}$ is real, which will occur when $x < 3$. However, if $x = 2$, then the denominator of the term becomes 0, which should not be allowed. The other limit of the function gets defined by the constraint defined by the term $\sqrt{x+7}$. For $\sqrt{x+7}$ to be real, $x \geq -7$ is the requirement. Hence, the required domain is:

$= -7 \leq x < 3, x \neq 2$

i.e. $x \in [-7,3) - \{2\}$

Hence, option (a) is correct.

64. $x^2 + 4xy + 6y^2 - 4y + 4$

$= x^2 + 4y^2 + 4xy + 2y^2 - 4y + 2 + 2$

$= (x + 2y)^2 + 2(y^2 - 2y + 1) + 2$

The above expression is minimum for $y = 1, x = -2$.

So minimum value of the given expression

$= 0 + 0 + 2 = 2$.

Hence, Option (c) is correct.

65. Let $f(X) = 21 \sin X + 72 \cos X$

$\Rightarrow f'(X) = 21 \cos X - 72 \sin X$

If $f'(X) = 0$, $21 \cos X = 72 \sin X$

$\therefore \tan X = 21/72$ therefore, $\sin X = 21/75$, $\cos X = 72/75$ (Since, from the value of $\tan X$, we can think of a right-angled triangle with the legs as 21 and 72, respectively. This would give us the hypotenuse length of the triangle as 75 – using the Pythagoras theorem.)

Since $f''(x) = -21 \sin X - 72 \cos X < 0$, therefore, $f(X)$ has a maximum at $f'(X) = 0$. Thus, we can use the values of $\sin X = 21/75$ and $\cos X = 72/75$.

$\therefore$ Maximum value of

$$f(x) = \frac{21.21}{75} + \frac{72.72}{75} = \frac{75^2}{75} = 75$$

Thus, Option (d) is correct.

66. For $x < -7$

$|x + 7| + |x - 8| = -(x + 7) - (x - 8)$

$-(x + 7) - (x - 8) = 16$

$-2x + 1 = 16$

$x = -7.5$

For $-7 \le x \le 8$

$|x + 7| + |x - 8| = x + 7 - x + 8 = 15 \neq 16$

Therefore the given equation has no solution in this range.

For $x \ge 8$

$|x + 7| + |x - 8| = x + 7 + x - 8 = 2x - 1$

$2x - 1 = 16$

$\Rightarrow x = \frac{17}{2} = 8.5$

So the required sum = $-7.5 + 8.5 = 1$

Hence Option (b) is correct.

67. $|3x + 4| \le 5$

$-5 \le 3x + 4 \le 5$

$-3 \le x \le 1/3$

a = -3, $b = 1/3$

$a + b = -3 + \frac{1}{3}$

$= -\frac{8}{3}$

For any positive integer, the given expression can never be less than 0. Therefore, $x = 2$, is the only positive integer value of x for which the given inequality holds true. Alternately, you can also solve this question using trial-and-error, where you can start with $x = 1$ and then try to see the value of the expression at $x = 2$. At $x = 1$, the expression is positive, at x

= 2 it is 0, while at $x = 3$, it again becomes positive. Once, x crosses 3, the term x3 by itself will become so large that it would not be possible to pull the value of the expression into the non-positive territory because the magnitude of the negative term in the expression, viz. $16x$, would not be large enough to make the expression ≤ 0.

69. Putting $x = 7$ in the given equation, we get:

$3f(7) + 2f(11) = 70$ (1)

Similarly by putting $x = 11$ in the given equation, we get:

$3f(11) + 2f(7) = 98$ (2)

Solving equation (1) and (2) we get

$$f(11) = \frac{154}{5} = 30.8$$

70. $q = p \times [p]$

When you start to think about the values of q from 8 onwards to 16, the first solution is quite evident at $q = 9$ and $p = 3$. At $q = 10$, p can be taken to be 10/3 to give us the expression of $p \times [p]$ equal to 10. Similarly.

For $q = 11$, $p = 11/3$.

For $q = 16$, $p = 4$

So the required number of positive real values of $p = 4$.

71. Required product is = $3 \times \frac{10}{3} \times \frac{11}{3} \times 4 = \frac{440}{3}$

72. $f(3) = f(1) + 8(1 + 1)$

$= -1 + 16 = 15$

$f(5) = f(3) + 8(3 + 1)$

$= (15 + 32)$

$= 47$

$f(10) = 4.f(5) + 9$

$= 4 \times 47 + 9$

$= 197$

$f(20) = 4 \times 197 + 9$

$= 797$

$f(22) = f(20) + 8(20 + 1)$

$= 797 + 168$

$= 965$

$f(24) = 965 + 8(22 + 1) = 1149$

$f(7) = f(5) + 8(5 + 1) = 47 + 48 = 95$

Hence, $f(24) - f(7) = 1149 - 95 = 1054$

73. If we observe values of $f(x)$ for different values of x, then we can see that $f(x) = 2x^2 - 3$.

Hence, $f(1000) = 2(1000)^2 - 3$

$= 1999,997$

74. $f(x) = (x^2 + [x]^2 - 2x[x])^{1/2} = \left[(x-[x])^2\right]^{\frac{1}{2}} = x - [x]$

$f(x) = x - [x]$ represents the fractional part of x.

Hence, $f(10.08) = 0.08$

$f(100.08) = 0.08$

$f(10.08) - f(100.08) = 0.08 - 0.08 = 0$

75. Let $f(x) = (x - 4)^7 (x - 3)^4 (x - 5)^2$

$f(1) = (1 - 4)^7 (1 - 3)^4 (1 - 5)^2$

$= (-3)^7 (-2)^4 (-4)^2$

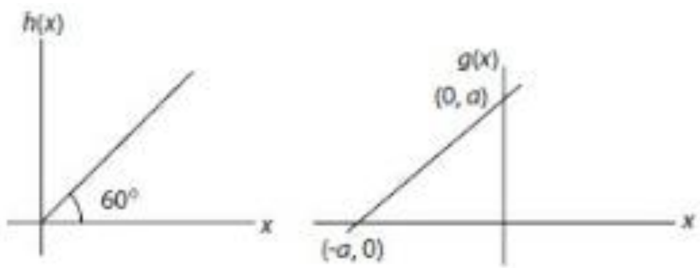
$= -2^8.3^7$

Hence, Option (c) is correct.

76. $f(x) = x - \frac{1}{3(3-x)} - 3 = (x-3) + \frac{1}{3(x-3)} \geq$

$\left[(x-3) \times \frac{1}{3(x-3)}\right]^{\frac{1}{2}}$. Hence, $(x-3) + \frac{1}{3(x-3)} \geq \frac{1}{\sqrt{3}}$

Hence, Option (b) is correct.

77. 

$h(x) = x \tan 60^\circ = x\sqrt{3}$

$x = \frac{h(x)}{\sqrt{3}}$

$\frac{x}{-a} + \frac{g(x)}{a} = 1$

$g(x) = \left[1 + \frac{x}{a}\right]a = a + x = a + \frac{h(x)}{\sqrt{3}}$

$\sqrt{3}g(x) = a\sqrt{3} + h(x)$

$\sqrt{3}g(x) - h(x) = a\sqrt{3}$

Hence, option (a) is correct.

Alternately, you can also solve this by looking at the values of the graphs. At $x = 0$, $h(x) = 0$ and $g(x) = a$. At $x = 1$, $h(x) = \sqrt{3}$(This can be visualised, since the triangle that is formed by the graph of $h(x)$ with the x-axis is a 30-60,90 triangle. Hence, if we take the side opposite the 30° angle as 1, the height (side opposite the 60° angle) would be $\sqrt{3}$. Also, the value of $g(x)$ would be $a + 1$ (since the gradient of the $g(x)$ slope is 45°). The first option satisfies both these pairs of values. Hence, it is the correct answer.

78. $\frac{f(xy)}{f(x+y)} = 1$ or $f(xy) = f(x + y)$

Put $x = 0$: $f(0.y) = f(0 + y) \Rightarrow f(y) = f(0)$

Put $y = 0$: $f(x.0) = f(x + 0) \Rightarrow f(x) = f(0)$

Therefore, function 'f' is a constant function. (This can also be interpreted since the function reads that the value of f when you put an argument equal to the product of x and y is the same as the value of f when you put the argument of the function as $x + y$).

$f(-10) = f(10) = f(6) = 7$

$f(-10) + f(10) = 7 + 7 = 14$

79. Putting $x = 9, y = 3$, in the above equation we get

$$f\left(\frac{9}{3}\right) = \frac{f(9)}{f(3)}$$

$$f(3) = \frac{f(9)}{f(3)}$$

$$f(9) = [f(3)]^2 = 5^2 = 25$$

Similarly, $x = 81, y = 9$

$$f\left(\frac{81}{9}\right) = \frac{f(81)}{f(9)}$$

$$f(9) = \frac{f(81)}{f(9)}$$

$f(81) = [f(9)]^2 = 25^2 = 625$

80. We can find the sum of all coefficients of a polynomial by putting each of the variables equals to 1:

Therefore, the required sum = $(1 - 4)^3 (1 - 2)^{10} (1 - 3)^3$

$= -3^3 \times 1 \times (-2)^3$

$= 27 \times 8 = 216$

81. $f(a) = 3^a$ (If a is an odd number)

$f(a + 1) = 3^{a+1} + 4 = 3.3^a + 4$

$$\frac{1}{4}[f(a) + f(a+1)] = \frac{3^a + 3.3^a + 4}{4}$$

$$= \frac{3^a.4 + 4}{4} = 3^a + 1$$

$$\Rightarrow \frac{1}{4}[f(1) + f(2)] + (f(3) + f(4)) + \dots + f(71) + f(72)]$$

$$= \frac{f(1) + f(2)}{4} + \frac{f(3) + f(4)}{4} + \dots + \frac{f(71) + f(72)}{4}$$

$= 3^1 + 1 + 3^3 + 1 + \dots + 3^{71} + 1$

$= (3^1 + 3^3 + \dots + 3^{71}) + 36$

$$= \frac{3\left(\left(3^2\right)^{36} - 1\right)}{3^2 - 1} + 36$$

(Using the formula for the sum of a geometric progression, since the series containing the powers of 3 is essentially a geometric progression).

$$= \frac{3}{8}\left(3^{72} - 1\right) + 36$$

82. Put $x = 0$, then $f(0 + y) = f(0) \rightarrow f(y) = p$

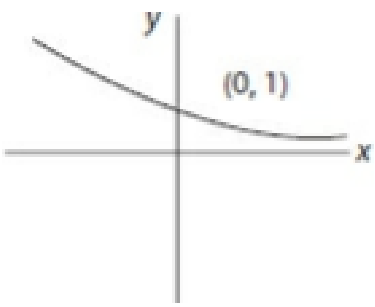
Put $y = 0$, then $f(x + 0) = f(0) \rightarrow f(x) = p$

Therefore, 'f' is a constant function.

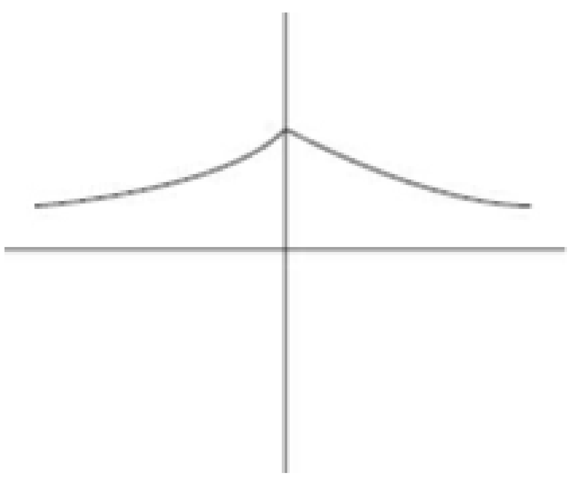
$f(7) = f(10) = f(5) = 12$

$[f(7)]^{143} - [f(11)]^{143} + f(5) = 12^{143} - 12^{143} + 12 = 12$

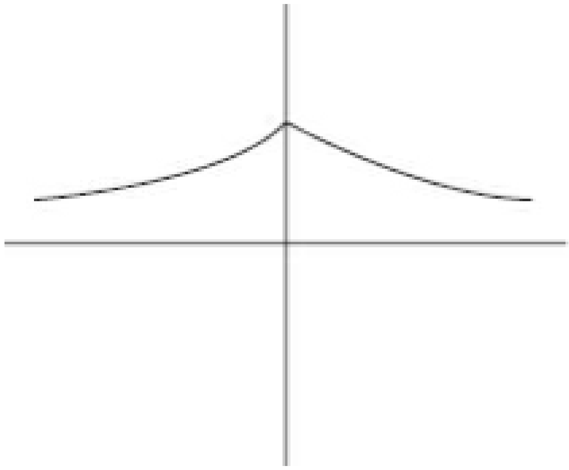
83. $e^{-x} \rightarrow$



$e^{-|x|} \rightarrow$

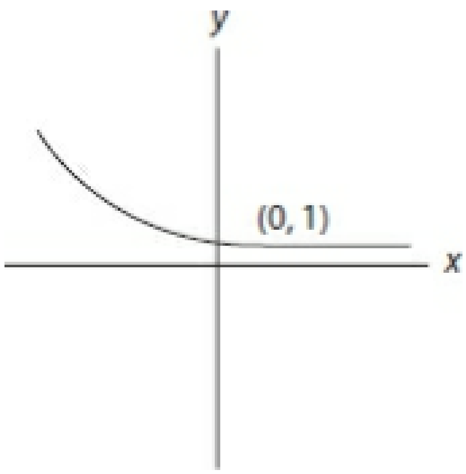


$|e^{-|x|}| \rightarrow$

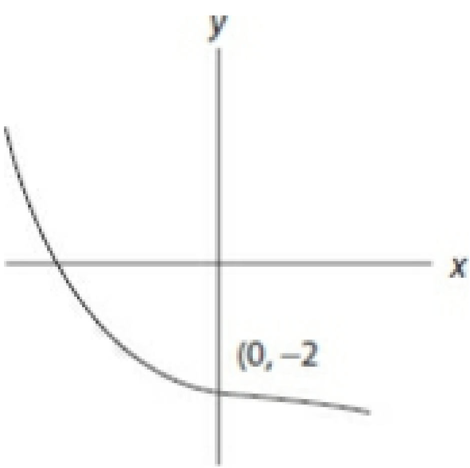


Hence, Option (c) is correct.

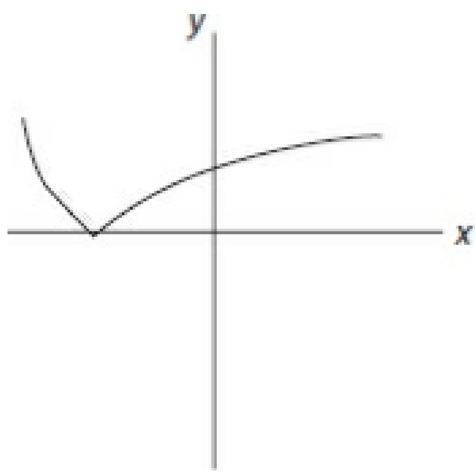
84. $e^{-x} \rightarrow$



$e^{-x} - 3 \rightarrow$

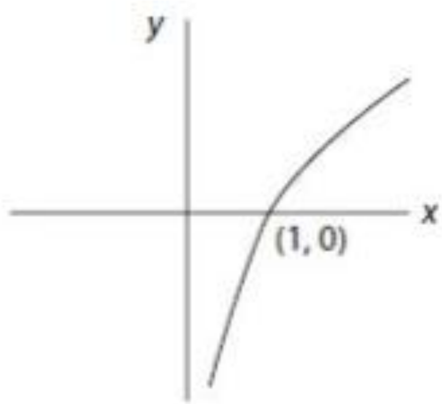


$|e^{-x} - 3| \rightarrow$

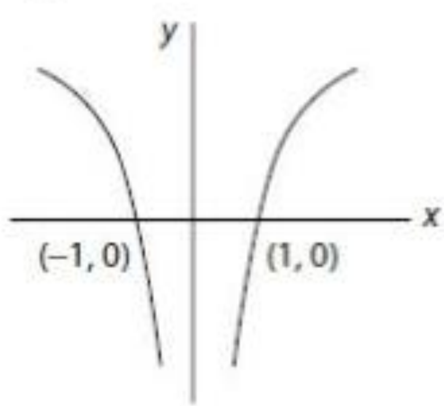


Hence, option (a) is correct.

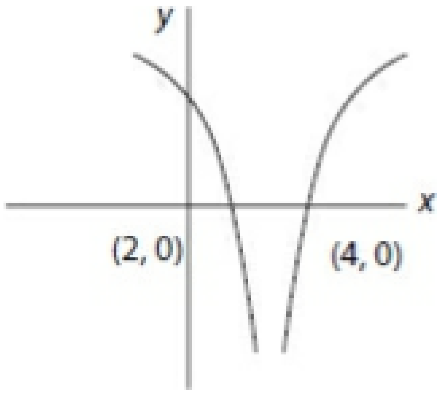
85. Log x →



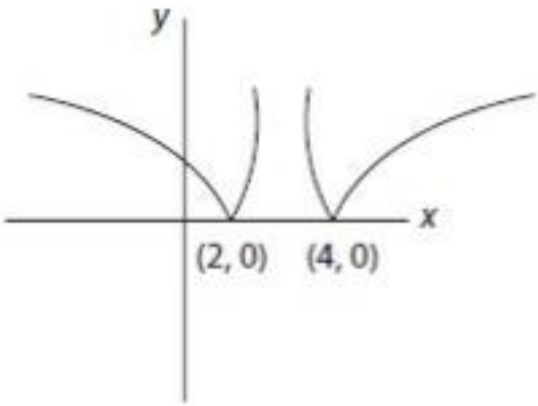
log|x| →



log|x−3| →



|log|x−3|| →



Hence, Option (d) is correct.

86. $f(x, y) = x^2 + y^2 - x - \frac{3y}{2} + 1$ can be split as:

$= x^2 - x + \frac{1}{4} + y^2 - \frac{3y}{2} + \frac{9}{16} + \frac{3}{16}$

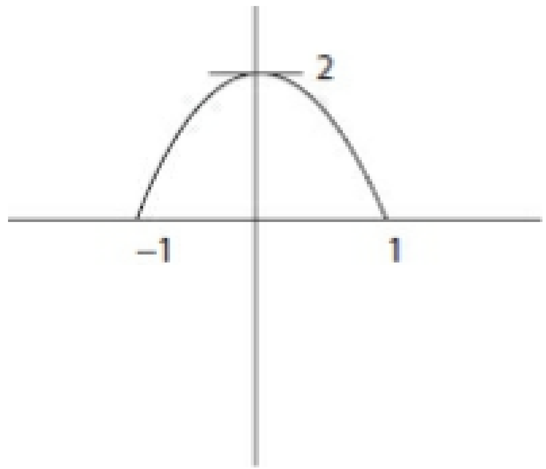
$= \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{3}{4}\right)^2 + \frac{3}{16}$

$f(x, y)$ will be minimum when $x = 1/2, y = 3/4$

Therefore, $x + y = \frac{1}{2} + \frac{3}{4} = \frac{5}{4} = 1.25$

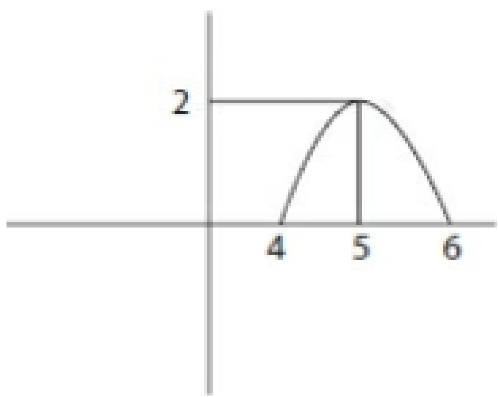
87. $f(x, y)$ min = 3/16

88. $f(x + 5)$

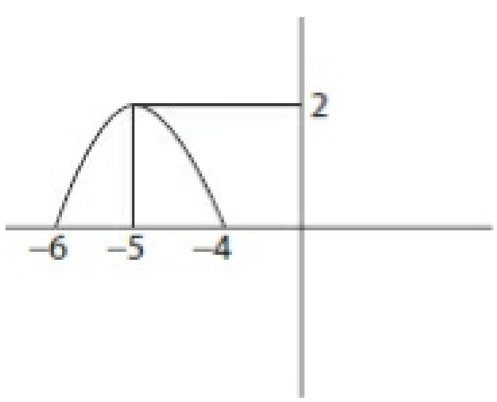


$f(x)$ can be obtained by shifting $f(x + 5)$ right by 5 units.

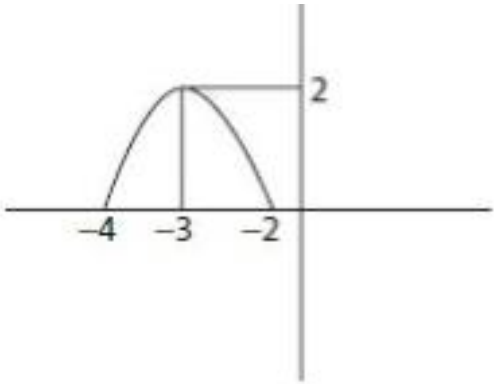
$f(x) \Rightarrow$



$f(-x)$can be obtained by reflecting the graph $f(x)$ about the y – axis

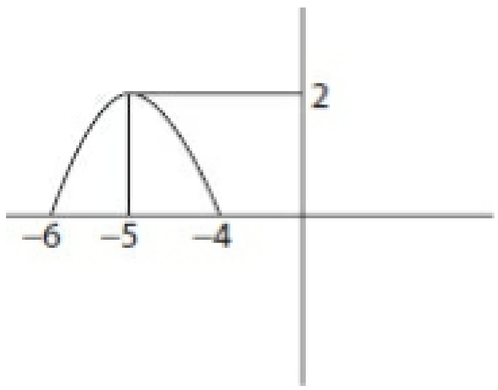


$f(-x - 2)$ can be obtained by shifting curve of $f(-x)$ to the right by 2 units

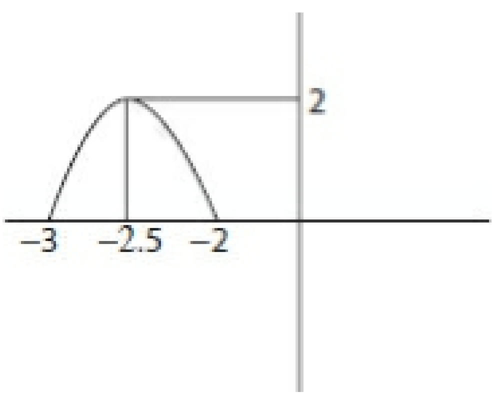


Hence, Option (d) is correct.

89. From our discussion of the previous question, we know that $f(-x)$ will look as below:



$f(-2x)$ would mean that the graph's value on the x-axis, will get halved at each of its points.



Hence, Option (c) is correct.

90. $\frac{g(x+y)+g(x-y)}{2} = g(x)g(y)$

$g(x + y) + g(x - y) = 2g(x)g(y)$ (1)

By replacing y with x and x with y, we get

$g(x + y) + g(y - x) = 2g(x)g(y)$ (2)

From equation (1) and equation (2),

$g(x + y) + g(x - y) = g(x + y) + g(y - x)$

$g(x - y) = g(y - x)$

By putting $y = 0$, we get $g(x) = g(-x)$

Therefore, $g(x)$ must be an even function: therefore only Option (c) satisfies because it also represents an even function.

Level of Difficulty (III)

1. $x - |x|$ is either negative for $x < 0$ or 0 for $x \geq 0$. Thus, Option (d) is correct.

2. The domain should simultaneously satisfy:

   $x - 1 \geq 0$, $(1 - x) \geq 0$ and $(x2 + 3) \geq 0$.

   Gives us: $x \geq 1$ and $x \leq 1$

   The only value that satisfies these two simultaneously is $x = 1$.

3. For the function to exist, the argument of the logarithmic function should be positive. Also, $(x + 4) \geq 0$ should be obeyed simultaneously.

   For $\frac{(x-5)}{(x^2-10x+24)}$ to be positive, both numerator and denominator should have the same sign. Considering all this, we get:

   $4 < x < 5$ and $x > 6$

   Hence, Option (c) is correct.

4. Both the brackets should be non-negative and neither $(x + 3)$ nor $(1 + x)$ should be 0.

For $(x - 3)/(x + 3)$ to be non negative, we have $x > 3$ or $x < -3$.

Also for $(1 - x)/(1 + x)$ to be non-negative, $-1 < x < 1$. Since there is no interference in the two ranges, Option (d) will be correct.

8. $f(f(t)) = f((t - 1)/(t + 1))$

   $$= \left[\left(\frac{t-1}{t+1}\right) - 1\right] \Big/ \left[\left(\frac{t-1}{t+1}\right) + 1\right] = \frac{t-1-t-1}{t-1+t+1}$$

   $= -2/2t = -1/t$

9. fog $= f(\log_e x) = e^{\log_e x} = x$

10. gof $= g(e^x) = \log_e e^x = x$

11. Looking at the options, one unit right means x is replaced by $(x - 1)$. Also, 1 unit down means -1 on the RHS.

    Thus, $(y + 1) = 1/(x - 1)$.

12. For option (c), we can see that $f(t) = f(-t)$. Hence, Option (c) is correct.

13. Option (b) is odd because:

    $$\frac{a^{-t} + a^{t}}{a^{t} - a^{-t}} = -1 \times \left(\frac{a^{-t} + a^{t}}{a^{-t} - a^{t}}\right)$$

    Similarly Option (c) is also representing an odd function. The function in option (a) is not odd.

14. $f(f(t)) = f[t/(1 + t^2)^{1/2}] = t/(1 + 2t^2)^{1/2}$

15. By trial-and-error, it is clear that at $x = 3$, the value of the function is 19. At other values of 'x', the value of the function is less than 19.

17. Take different values of x to check each option. All the options, i.e. (a), (b) and (c) can be ruled out. Hence, Option (d) is correct.

Solutions to Questions 18 to 20:

$f(1) = 0, f(2) = 1,$

$f(3) = f(1) - f(2) = -1$

$f(4) = f(2) - f(3) = 2$

$f(5) = f(3) - f(4) = -3$

$f(6) = f(4) - f(5) = 5$

$f(7) = f(5) - f(6) = -8$

$f(8) = f(6) - f(7) = 13$

18. It can be seen that $f(x)$ is positive wherever x is even and negative whenever x is odd, once x is greater than 2.

19. $f(f(6)) = f(5) = -3$

20. $f(6) - f(8) = 5 - 13 = -8 = f(7)$

21. Option (b) is not even since $e^x - e^{-x} \neq e^{-x} - e^x$.

22. We have $f(x) \cdot f(1/x) = f(x) + f(1/x)$

$\Rightarrow f(1/x)\,[f(x) - 1] = f(x)$

For $x = 4$, we have $f(1/4)\,[f(4) - 1] = f(4)$

$\Rightarrow f(1/4)\,[64] = 65$

$\Rightarrow f(1/4) = 65/64 = 1/64 + 1$

This means $f(x) = x^3 + 1$

For $f(6)$ we have $f(6) = 216 + 1 = 217$.

Solutions for Questions 23 to 34:

you essentially have to mark (a), if it is an even function, mark (b) if it is an odd function, mark (c) if the function is neither even nor odd.

Also, Option (d) would occur if the function does not exist at, at least one point of the domain. This means one of two things.

Either the function is returning two values for one value of x or the function has a break in between (as in questions 26, 31 and 33).

We see even functions in questions 23, 28, 30, 32 and 34 [Symmetry about the y axis]. We see odd functions in Questions 24, 25 and 27.

The figure in Question 29 is neither odd nor even.

Solutions for Questions 35 to 40:

In order to solve this set of questions first analyse each of the functions:

$A(x, y, z)$ = will always return the value of the highest between x and y.

$B(x, y, z)$ will return the value of the maximum amongst x, y and z.

$C(x, y, z)$ and $D(x, y, z)$ will return the second highest values in all cases while max (x, y, z) and min(x, y, z) would return the maximum and minimum values amongst x, y, and z respectively.

35. When either x or y is maximum
36. This would never happen.
37. When z is maximum, A and B would give different values. Thus, Option (c) is correct.
38. Never
39. I and III are always true
40. We cannot determine this because it would depend on whether the integers x, y, and z are positive or negative.

Solutions for Questions 41 to 44:

$f(x, y)$ is always positive or zero

$F(f(x, y)$ is always negative or zero

$G(f(x, y)$ is always positive or zero

41. $F \times G$ will always be negative while $-F \times G$ will always be positive except when they are both equal to zero.

    Hence, option (b), i.e. $F \times G \leq -F \times G$ is correct.

42. Option (b) can be seen to give us $4a2/4 = a2$.

43. (5 – 1)/(1+ 3) = 4/4 = 1

44. The given expression = (45 – 10)/(5 + 2) = 35/7 = 5.

    Option (b) = 20/4 = 5.

Solutions for Questions 45 to 49:

Do the following analysis:

$A\ (f(x, y))$ is positive

$B\ (f(x, y))$ is negative

$C\ (f(x, y))$ is positive

$D\ (f(x, y))$ is negative

$E\ (f(x, y))$ is positive and so on

45. 1 – 3 + 5 – 7 + 9 – 11 + ... – 51

    = (1 + 5 + 9 + 13 + ... 49) – (3 + 7 + 11 ... + 51)

    = –26

46. Verify each statement to see that (ii) and (iii) are true.

47. The given expression becomes:

Min (max [5, – 7, 9], min [3, – 1 , 1], max [7, 6, 10])

= Min [9, – 1, 10]

= – 1

48. The given expression becomes:

Max [$|a + b|$, – $|b + c|$, $|c + d|$]

This will never be negative.

49. The respective values are:

–3/2, –7/12, –8/15, and –5/6

Option (b) is second lowest.

50. Let $s = 1$, $t = 2$ and $b = 3$

Then, $f(s + t) + f(s - t)$

$= f(3) + f(-1) = (3^3 + 3^{-3})/2 + (3^{-1} + 3^1)/2$

= [(27 + (1/27)]/2 + [3 + (1/3)]/2

= 730/54 + 10/6

= 820/54 = 410/27

Option (b), i.e. 2 $f(s) \times f(t)$ gives the same value.

51. This question is based on the logic of a chain function. Given the relationship

$A_t = (t + 1)A_{(t-1)} - tA_{(t-2)}$

We can clearly see that the value of A_2 would depend on the values of A_0 and A_1. Putting $t = 2$ in the expression, we get:

$A_2 = 3A_1 - 2A_0 = 7$; $A_3 = 19$; $A_4 = 67$ and $A_5 = 307$.Clearly, $A6$ onwards will be larger than 307 and hence none of the three conclusions are true. Hence, option (e) is the correct answer.

52. In order to solve this question, we would need to check each of the value ranges given in the conclusions: Checking whether Conclusion I is possible

For $B = 2$, we get $A + C = 4$ (since $A + B + C = 6$).This transforms the second equation $AB + BC + CA = 9$ to:

$2(A + C) + CA = 9 \rightarrow CA = 1$.

Solving $CA = 1$ and $A + C = 4$ we get: $(4 - A)A = 1 \rightarrow A^2 - 4A + 1 = 0 \rightarrow A = 2 + 3^{1/2}$ and $C = 2 - 3^{1/2}$. Both these numbers are real and it satisfies $A < B < C$ and hence, Conclusion I is true.

Checking Conclusion II: If we chose $A = 2.5$, the condition is not satisfied since we get the other two variables as $(3.5 + 11.25^{1/2}) \div 2$ ª 3.4 and $(3.5 - 11.25^{1/2}) \div 2$ ª 0.1. In this case, A is no longer the least value and hence Conclusion II is rejected.

Checking Conclusion III we can see that $0 < C < 1$cannot be possible since C being the largest of the three values has to be greater than 3 (the largest amongst A, B, and C would be greater than the average of A, B, C).

Thus, option (a) is correct.

53. The number of ways of distributing n identical things to r people such that any person can get any number of things including 0 is always given by ${}^{n+r-1}C_{r-1}$. In the case of $F(4,3)$, the value of $n = 4$ and $r = 3$ and hence, the total number of ways without any constraints would be given by ${}^{4+3-1}C_{3-1} = {}^{6}C_{2} = 15$. However, out of these 15 ways of distributing the toys, we cannot count any way in which more than 2 toys are given to any one child. Hence, we need to reduce as follows:

The distribution of 4 toys as (3, 1 and 0) amongst three children A, B and C can be done in $3! = 6$ ways.

Also, the distribution of 4 toys as (4, 0 and 0) amongst three children A, B and C can be done in 3 ways.

Hence, the value of $F(4, 3) = 15 - 6 - 3 = 6$.

Thus, Option (b) is correct.

54. $f(f(x)) = 15$, when $f(x) = 4$ or $f(x) = 12$ in the given function. The graph given in the figure becomes equal to 4 at 4 points and it becomes equal to 12 at 3 points in the figure. This gives us 7 points in the given figure when $f(f(x) = 15$. However, the given function is continuous beyond the part of it which is shown between –10 and +13 in the figure. Hence, we do not know how many more solutions to $f(f(x) = 15$ will be there. Hence, option (e) is the correct answer.

55. The given function is a chain function where the value of A_{n+1} depends on the value A_n.

Thus for $n = 0$, $A_1 = A_0{}^2 + 1$.

For $n = 1$, $A_2 = A_{12} + 1$ and so on.

In such functions, if you know the value of the function at any one point, the value of the function can be calculated for any value till infinity.

Hence, statement I is sufficient by itself to find the value of the GCD of A_{900} and A_{1000}.

So also, the statement II is sufficient by itself to find the value of the GCD of A_{900} and A_{1000}.

Hence, Option (d) is correct.

56. This question can be solved by first putting up the information in the form of a table as follows:

	Product A	*Product B*	*Number of machines available*	*Number of hours/day per machine.*	*Total hours per day available for each activity*
Grinding	2 hr	3 hr	10	12	120
Polishing	3 hr	2 hr	15	10	150
Profit	₹ 5	₹ 7			

On the surface, the profit of Product B being higher, we can think about maximising the number of units of Product B. Grinding would be the constraint when we maximise Product B production and we can produce a maximum of 120 ÷ 3 = 40 units of Product B to get a profit of ₹ 280. The clue that this is not the correct answer comes from the fact that there is a lot of 'polishing' time left in this situation. In order to try to increase the profit, we can check that if we reduce production of Product *B* and try to increase the production of Product *A*, does the profit go up?

When we reduce the production of Product *B* by 2 units, the production of Product *A* goes up by 3 units and the profit goes up by +1 (–2 × 7 + 3 × 5 gives a net effect of +1). In this case, the grinding time remains the same (as there is a reduction of 2 units × 3 hours/unit = 6 hours in grinding time due to the reduction in Product *B*'s production, but there is also a simultaneous increase of 6 hours in the use of the grinders in producing 3 units of Product *A*). Given that a reduction in the production of Product *B*, with a simultaneous maximum possible increase in the production of Product *A*, results in an increase in the profit, we would like to do this as much as possible. To think about it from this point, this situation can be tabulated as under for better understanding:

	Product A Production (A)	*Product B Production (B)*	*Grinding Machine Usage = 3A + 2B*	*Polishing Machine Usage = 2A + 3B*	*Time Left on Grinding Machine*	*Time Left on Polishing Machine*	*Profit = 7A+5B*
Case 1	40	0	120	80	0	70	280
Case 2	38	3	120	85	0	65	281
Case 3	36	6	120	90	0	60	282
The limiting case would occur when we reduce the time left on the polishing machine to 0. That would happen in the following case:							
Optimal case	12	42	120	150	0	0	294

Hence, the answer would be 294.

57. The value of $f(x)$ as given is: $f(x) = x^4 + x^3 + x^2 + x + 1 = 1 + x + x^2 + x^3 + x^4 + x^5$. This can be visualised as a geometric progression with 5 terms with the first term 1 and common ratio x. The sum of the GP = $f(x) = \frac{x^5 - 1}{x - 1}$.

The value of $f(x^5) = x^{20} + x^{15} + x^{10} + x^5 + 1$ and this can be rewritten as:

$F(x^5) = (x^{20}-1) + (x^{15}-1) + (x^{10}-1) + (x^5-1) + 5$. When this expression is divided by $f(x) = \frac{x^5 - 1}{x - 1}$, we get each of the first four terms of the expression will be divisible by it, i.e. $(x^{20}-1)$ will be divisible by $f(x) = \frac{x^5 - 1}{x - 1}$, and would leave no remainder (because $x^{20}-1$ can be rewritten in the form $(x^5-1) \times (x^{15} + x^{10} + x^5 + 1)$ and when you divide this expression by $\left(\frac{x^5-1}{x-1}\right)$, we get the remainder as 0.)

A similar logic will also hold for the terms $(x^{15}-1)$, $(x^{10}-1)$ and (x^5-1). The only term that will leave a remainder would be 5 when it is divided by $\left(\frac{x^5-1}{x-1}\right)$

Also, for $x \geq 2$, we can see the value of $\left(\frac{x^5-1}{x-1}\right)$ will be more than 5. Hence, the remainder will always be 5 and Option (c) is the correct answer.

58. Start by putting $\frac{x}{x-1} = (\text{cosec } \alpha)^2$ in the given expression.

$$F\left(\frac{x}{x-1}\right) = \frac{1}{x}$$

Now for $0 < \alpha < 90°$

$$\frac{x}{x-1} = (\text{cosec } \alpha)^2 \Rightarrow x = \frac{1}{1-\sin^2 \alpha} \Rightarrow \frac{1}{x} = \cos^2 \alpha$$

Hence, Option (b) is correct.

59. Given that the roots of the equation $F(x) = 0$ are $-2, -1, 1$ and 2 respectively, and the $F(x)$ is a polynomial with the highest power of x as x^4, we can create the value of

$F(x) = (x + 2)(x + 1)(x - 1)(x - 2)$

Hence, $F(p) = (P + 2)(P + 1)(P - 1)(P - 2)$.

It is given to us that P is a prime number greater than 97. Hence, p would always be of the form $6n \pm 1$ where n is a natural number greater than or equal to 17.

Thus, we get two cases for $F(p)$.

$F(6n + 1) = (6n + 3)(6n + 2)(6n)(6n - 1)$

$= 3(2n + 1) \cdot 2(3n + 1)(6n)(6n - 1)$

$= (36)(2n + 1)(3n + 1)(n)(6n - 1)$ (1)

If you try to look for divisibility of this expression by numbers given in the options for various values of $n \geq 17$, we see that for $n = 17$ and 18, both 360 divides the value of $F(p)$. However at $n = 19$, none of the values in the four options divides $36 \times 39 \times 58 \times 19 \times 113$. In this case, however, at $n = 19, 6n + 1$ is not a prime number hence, this case is not to be considered. Whenever we put a value of n as a value greater than 17, such that $6n+1$ becomes a prime number, we also see that the value of $F(p)$ is divisible by 360. This divisibility by 360 happens since the expression $(2n + 1)(3n + 1)(n)(6n - 1)$ ... is always divisible by 10 in all such cases. A similar logic can be worked out when we take $p = 6n-1$. Hence, the Option (d) is the correct answer.

60. In order to solve this question, we start from the value of $x = (9 + 4\sqrt{5})^{48}$.

Let the value of $x(1 - f) = xy$. (We are assuming $(1 - f) = y$, which means that y is between 0 to 1).

The value of $x = (9 + 4\sqrt{5})^{48}$ can be rewritten as $[{}^{48}C_0\, 9^{48} + {}^{48}C_1\, 9^{47}(4\sqrt{5}) + {}^{48}C_2\, 9^{46}(4\sqrt{5})^2 + \ldots + {}^{48}C_{47}(9)(4\sqrt{5})^{47} + {}^{48}C_{48}\,(4\sqrt{5})^{48}]$ using the binary theorem.

In this value, it is going to be all the odd powers of the $(4\sqrt{5})$ which would account for the value of 'f' in the value of x. Thus, for instance it can be seen that the terms ${}^{48}C_0\, 9^{48}$, ${}^{48}C_2\, 9^{46}(4\sqrt{5})^2$, ...${}^{48}C_{48}\,(4\sqrt{5})^{48}$ would all be integers. It is only the terms: ${}^{48}C_1 9^{47}(4\sqrt{5})$, ${}^{48}C_3\, 9^{45}(4\sqrt{5})^3$,... ${}^{48}C_{47}\,(9)(4\sqrt{5})^{47}$ which would give us the value of 'f' in the value of x.

Hence, $x(1-f) = x\,[1 - {}^{48}C_1\, 9^{47}\,(4\sqrt{5}) - {}^{48}C_3\, 9^{45}(4\sqrt{5})^3 - \ldots - {}^{48}C_{47}\,(9)(4\sqrt{5})^{47}]$

In order to think further from this point, you would need the following thought. Let $y = (9-4\sqrt{5})^{48}$.

Also, $x + y = \{{}^{48}C_0\, 9^{48} + {}^{48}C_1\, 9^{47}(4\sqrt{5}) + {}^{48}C_2\, 9^{46}(4\sqrt{5})^2 + \ldots + {}^{48}C_{47}\,(9)(4\sqrt{5})^{47} + {}^{48}C_{48}\,(4\sqrt{5})^{48}\} + \{{}^{48}C_0\, 9^{48} - {}^{48}C_1\, 9^{47}(4\sqrt{5}) + {}^{48}C_2 9^{46}(4\sqrt{5})^2 + \ldots - {}^{48}C_{47}\,(9)(4\sqrt{5})^{47} + {}^{48}C_{48}\,(4\sqrt{5})^{48}\} = 2\{{}^{48}C_0\, 9^{48} + {}^{48}C_2\, 9^{46}\,(4\sqrt{5})^2 + \ldots {}^{48}C_{48}\,(4\sqrt{5})^{48}\}$ – the bracket in this expression has only retained the even terms which are integral. Hence, the value of $x+y$ is an integer.

Further, $x + y = [x] + f + y$ and hence, if $x+y$ is an integer, $[x] + f + y$ would also be an integer. This automatically means that $f+y$ must be an integer (as $[x]$ is an integer).

Now, the value of y is between 0 and 1 and hence, when we add the fractional part of x, i.e. 'f' to y, and we need to make it an integer, the only possible integer that $f + y$ can be equal to 1.

Thus, if $f + y = 1 \rightarrow y = (1 - f)$.

In order to find the value of $x(1 - f)$, we can find the value of x ¥ y.

Then, $x(1 - f) = x \times y = (9 + 4\sqrt{5})^{48} \times (9 - 4\sqrt{5})^{48} = (81 - 80)^{48} = 1$

$x(1 - f) = 1$

61. $3f(x + 2) + 4f\left(\frac{1}{x+2}\right) = 4x$

Let $x + 2 = t$

$3f(t) + 4f\left(\frac{1}{t}\right) = 4t - 8$ or $\frac{3}{4}f(t) + f\left(\frac{1}{t}\right)$

$= t - 2 \quad (1)$

Now replacing t with $\frac{1}{t}$ in the above equation, we get

$3f\left(\frac{1}{t}\right) + 4f(t) = \frac{4}{t} - 8$ or $f\left(\frac{1}{t}\right) + \frac{4}{3}f(t)$

$= \frac{4}{3t} - \frac{8}{3}$ (2)

From (1) and (2).

$f(t) = \frac{12}{7}\left\{\frac{4}{3t} - \frac{8}{3} - t + 2\right\}$

$f(4) = \frac{12}{7}\left\{\frac{1}{3} - \frac{8}{3} - 4 + 2\right\} = \frac{-52}{7}$

62. According to the graph, $f(4) = 15$ and $f(12) = 15$.

So $f(f(x)) = 15$ for $f(x) = 4, 12$.

According to the graph, $f(x) = 4$ has four solutions.

According to the graph, $f(x) = 12$ has three solutions.

Hence, the given equation has seven solutions.

63. $[f(x)]^{g(x)} = 1$

Now three cases are possible:
Case I: $f(x) = 1$ and $g(x)$ may be anything.

$x - 6 = 1$ or $x = 7$

But for $x = 7$, $g(x)$ is not defined.

Case II: $f(x) = -1$ and $g(x)$ is an even exponent

$x - 6 = -1$

$x = 5$

For $x = 5$

$$g(x)=\frac{(5-9)(5-1)}{(5-7)(5-3)}=\frac{-4\times4}{-2\times2}=4$$

So for $x = 5$, $g(x)$ is even, which satisfies the given equation.

Case III: $g(x) = 0$ and $f(x) \neq 0$

$\frac{(x-9)(x-1)}{(x-7)(x-3)}=0$ for $x = 1, 9$

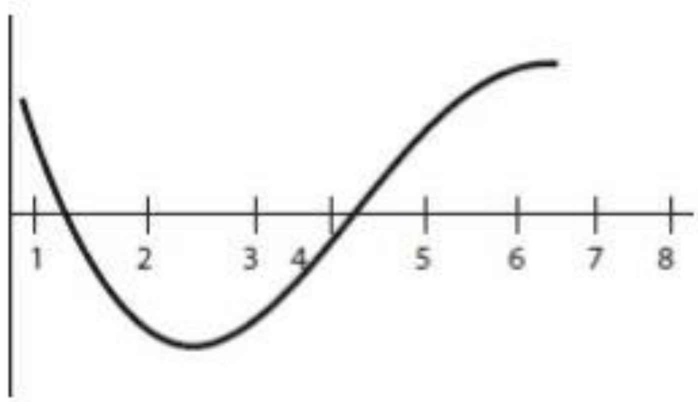
For $x = 1$ and 9 $f(x) \neq 0$. So both of these values of x satisfy the given equation.

So the given equation is satisfied for three values of x.

64. $f(4) + f(6) = 0$ implies that $f(4)$ and $f(6)$ are of opposite sign but same absolute value. Hence one root of the equation lies between 4 and 6.
$f(1) > 0$ and $f(2)<0$ implies that another root lies between 1 and 2.

$f(5).f(7) > 0$ implies that $f(5)$ and $f(7)$ are of same sign, so $f(4)$ and $f(5)$ must be of opposite sign. So the second root of $f(x) = 0$ must lie between $x = 4$ and $x = 5$.

So $f(x)$ would look like:



As $f(1) > 0$ & $f(2)$ & $f(4) < 0$

So $f(1)\,f(2)\,f(4) > 0$. Option (a) is incorrect.

As $f(5)$, $f(6)$ & $f(7)$ are greater than 0.

So $f(5)\,f(6)\,f(7) > 0$. So Option (b) is wrong.

As $f(1) > 0$ & $f(3)$ & $f(4) < 0$. So $f(1)\,f(3)\,f(4) > 0$

So Option (c) is true.

65. $f(x) = 12 + x$

$7[x] + 4\{x\} = 12 + x$

$3[x] + 4[[x] + \{x\}] = 12 + x$

$3[x] + 4x = 12 + x$

$3[x] + 3x = 12$

$[x] + x = 4$

Since 4 and $[x]$ are both integers, in the above equations x must also be an integer. This means that the value of $[x] = x$. So:

$2x = 4$

$x = 2$

Therefore, only one value of x satisfies the given equation.

66. $x^2 - xy + y^2 = x + y$

Multiplying both sides by 2, we get:

$2x^2 - 2xy + 2y^2 = 2x + 2y$

$x^2 - 2xy + y^2 + x^2 - 2x + 1 + y^2 - 2y + 1 = 2$

$(x - y)^2 + (x - 1)^2 + (y - 1)^2 = 2$

In the question, we are interested to find non-negative integer solutions, therefore, three cases are possible.

Case I: $x - y = 0, (x - 1)2 = 1, (y - 1)2 = 1$

Possible solutions (0, 0) and (2, 2)

Case II: $(x - y)2 = 1, (x - 1)2 = 1, (y - 1)2 = 0$

Possible solutions: (2, 1), (0, 1).

Case III: $(x - y)2 = 1, (y - 1)2 = 1, (x - 1)2 = 0$

Possible solutions: (1, 2) and (1, 0)

Possible solutions (x, y) such that $x \geq y$ are (0, 0), (2, 2), (1, 0), (2, 1). There are four such solutions.

67. $g(n) = \frac{n-1}{n} g(n-1)$

$$g(2) = \frac{1}{2} g(1)$$

$$g(3) = \frac{2}{3} g(2) = \frac{2}{3} \times \frac{1}{2} g(1) = \frac{1}{3} g(1).$$

Similarly:

$$g(4) = \frac{1}{4} g(1); g(5) = \frac{1}{5} g(1);$$

$$g(6) = \frac{1}{6} g(1); g(7) = \frac{1}{7} g(1); g(8) = \frac{1}{8} g(1)$$

Since $g(1) = 2$, the given expression will become:

$$\frac{\left[\frac{1}{2} \times \frac{2}{2} \times \frac{3}{2} \times \ldots \frac{8}{2}\right]}{\left[\frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \ldots \frac{8}{2}\right]}$$

Required answer is $\frac{8!}{2^8} \times \frac{1}{18}$.

68. Let $f(x) = a(x - 1)(x - 2)(x - 3)\ldots(x - 77) + x$

where 'a' is any constant.

Now putting $x = 78$ in the above equation, we get

$f(78) = a.77.76.75.74\ldots1 + 78 = a.77! + 78$

Similarly, $f(0) = a.(-1)(-2)(-3)\ldots(-77) + 0$

$f(0) = a(-1)7777! = -a.77!$

$f(78) + f(0) = a.77! + 78 - a.77! = 78$

69. $f(n - 1)(2 - f(n)) = 1$

$$2 - f(n) = \frac{1}{f(n-1)}$$

$$f(n) = 2 - \frac{1}{f(n-1)}$$

$$f(2) = 2 - \frac{1}{f(1)} = 2 - \frac{1}{3} = \frac{5}{3}$$

$$f(3) = 2 - \frac{1}{f(2)} = 2 - \frac{3}{5} = \frac{7}{5}$$

$$f(4) = 2 - \frac{1}{f(3)} = 2 - \frac{5}{7} = \frac{9}{7}$$

Observing this pattern, we can see that:

$$f(n) = \frac{2n+1}{2n-1}$$

$$f(21) = \frac{2\times21+1}{2\times21-1} = \frac{43}{41}$$

70. Since: $0 \le \{x\} < 1$

The expression: $10[x] + 22\{x\} = 250$ gives us the inequality: $228 < 10[x] \le 250$

$22.8 < [x] \le 25$

Possible values of $[x]$ = 23, 24, 25

For $[x] = 23$, $\{x\} = \frac{250-230}{22} = \frac{20}{22} = \frac{10}{11}$

For $[x] = 24\{x\} = \frac{250-240}{22} = \frac{10}{22} = \frac{5}{11}$

For $[x] = 25$, $\{x\} = 0$

So the possible values of x are $23\frac{10}{11}, 24\frac{5}{11}, 25$.

So there are three possible values of x.

71. $23\frac{10}{11} + 24\frac{5}{11} + 25 = 73\frac{4}{11} \approx 73.36$

72. $f(x+1) = f(x) - f(x-1)$

$f(x) = f(x+1) + f(x-1)$

$f(17) = f(18) + f(16)$

$2f(16) = f(18) + f(16)$

$f(16) = f(18)$

Let $f(16) = f(18) = x$

$f(17) = 2x$

$f(16) = f(15) + f(17) \rightarrow f(15) = -x;$

$f(15) = f(14) + f(16) \rightarrow f(14) = -2x;$

$f(14) = f(13) + f(15) \rightarrow f(13) = -x;$

$f(13) = f(12) + f(14) \rightarrow f(12) = x$

$f(12) = f(11) + f(13) \rightarrow f(11) = 2x$

$f(11) = f(10) + f(12) \rightarrow f(10) = x$

$f(10) = f(9) + (f(11) \rightarrow f(9) = -x$

If we observe the above pattern of values that we are getting, we can observe that $f(18) = f(12)$; $f(17) = f(11)$; $f(16) = f(10)$ and $f(15) = f(9)$. Here we can easily observe that values repeat for every six terms. So $f(5) = f(11) = f(17) = 6$

Thus, Option (b) is correct.

73. $\frac{h(x)}{h(x-1)} = \frac{h(x-2)}{h(x+1)}$

On putting $x = 54$, we get:

$\frac{h(54)}{h(53)} = \frac{h(52)}{h(55)}$ (1)

On putting $x = 55$, we get:

$\frac{h(55)}{h(54)} = \frac{h(53)}{h(56)}$ (2)

Equation (1) ÷ Equation (2)

$$\frac{[h(54)]^2}{h(53) \times h(55)} = \frac{h(52) \times h(56)}{h(55) \times h(53)}$$

$[h(54)]_2 = 4 \times 16$

$h(54) = 8$

74. $f(x) = 1 - \frac{2}{x+1} = \frac{x+1-2}{x+1} = \frac{x-1}{x+1}$

$$f^2(x) = f(f(x)) = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = -\frac{1}{x}$$

$$f^3(x) = f(f(f))) = -\frac{x+1}{x-1}$$

$$f^4(x) = f(f(f(f(x)))) = x$$

$$f^5(x) = f(x) = \frac{x-1}{x+1}$$

Here we can see that $f(x) = f^5(x)$, so the given function has a cyclicity of 4, therefore:

$f^n(x) = f^{n+4k}(x)$, where k is a whole number

$$f^{802} = f^{2+4\times 200}(x) = f^2(x) = -\frac{1}{x}$$

$$f^{802}(x)\, at\ x = -\frac{1}{2} = -\frac{1}{-\frac{1}{2}} = 2$$

75. $\log_3 (x + y) + \log_3 (x - y) = 3$

$\log_3 (x^2 - y^2) = 3$

$x^2 - y^2 = 3^3 = 27$

$(x - y)(x + y) = 27$

Here both $(x + y)$ and $(x - y)$ are positive integers (since they have to be used as the arguments of the logarithmic functions. Hence, $(x - y) > 0$ or $x > y$. From this point, we need to think of factor pairs of 27, in order to find out the values that are possible for x and y.

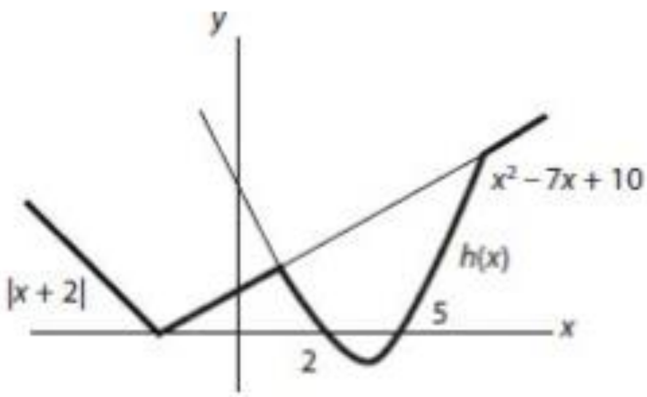
Case 1: $x + y = 9$, $x - y = 3$ or $x = 6$, $y = 3$

Case 2: when $x + y = 27$, $x - y = 1$ or $x = 14$, $y = 13$

Two pairs of (x, y) are possible.

76. The maximum value of $x + y = 14 + 13 = 27$.

77. In the following figure, the bold portion shows the graph of $h(x)$.



Therefore, $h(x) \leq 0$ for $x = 2, 3, 4, 5$. There are four such values.

78. We can observe from the graph that $h(x) < 0$ only for two integer values (3, 4) of x. So the required sum = 3 + 4 = 7.

79. $[2p - 3]$ is an integer. Hence, $q + 7$ is also an integer or q must be an integer.

Similarly p is also an integer (since $[3q + 1]$ is an integer, hence $p + 6$ should also be an integer.)

$\Rightarrow [2p - 3] = 2\text{p} - 3 = q + 7$

$2p - q = 10$ (1)

$\Rightarrow 3q + 1 = p + 6$

$3q - p = 5$ (2)

By solving equations (1) and (2), we get the values of p and q as:

$p = 7, q = 4$

The required answer is then given by 72 × 42 = 784.

80. $f(a) = 3a$ (If a is an odd number)

$f(a + 1) = 3(a + 1) + 4 = 3.3a + 4$

$$\frac{1}{4}[f(a) + f(a+1)] = \frac{3^a + 3.3^a + 4}{4} = \frac{3^a.4 + 4}{4} = 3^a + 1$$

$$\Rightarrow \frac{1}{4}[f(1)+f(2)+(f(3)+f(4))+\ldots+f(71)+f(72)]$$

$$=\frac{f(1)+f(2)}{4}+\frac{f(3)+f(4)}{4}+\ldots+\frac{f(71)+f(72)}{4}$$

$= 3_1 + 1 + 3_3 + 1 + \ldots + 3_{71} + 1$

$= (3_1 + 3_3 + \ldots + 3_{71}) + 36$

$$=\frac{3((3^2)^{36}-1)}{3^2-1}+36$$

$$=\frac{3}{8}(3^{72}-1)+36$$

81. $$g(f(x))=2.\frac{x^{2\left[\frac{3x^2}{4}\right]}}{4}+2$$

$$=\frac{x^{2\left[\frac{3x^2}{4}\right]}}{2}+2$$

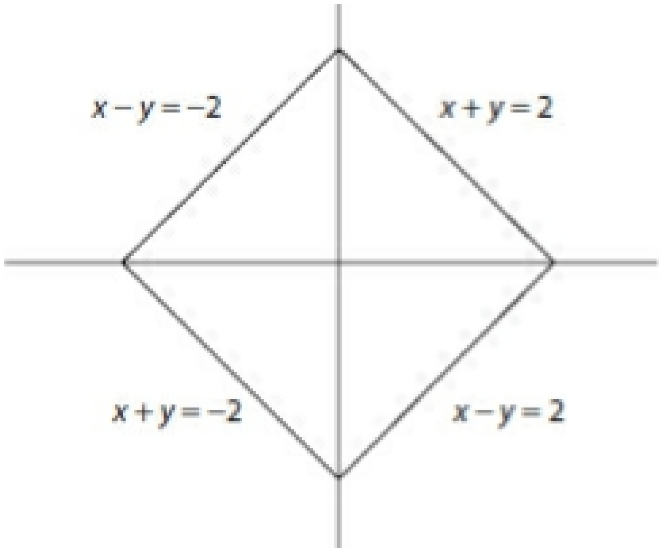
$g(f(x))$ is an even function, so option (a) is incorrect.

As we increase the value of x, value of $g(f(x))$ will get increased. Therefore, it will attain its maxima at ∞. So Option (b) is also incorrect.

$\frac{x^{2\left[\frac{3x^2}{4}\right]}}{2}+2$ will attain its minima when $0\le\frac{3x^2}{4}\le 1$. Since, the expression $\frac{3x^2}{4}$ is always going to be positive, hence, we can say that the only constraint we need to match for the minima of the function is $\frac{3x^2}{4}\le 1$. Therefore, option(c) is true.

82. $\frac{x^{2\left[\frac{3x^2}{4}\right]}}{2}+2 = 2_5 + 2 = 34$. Hence, Option (b) is correct.

83.



As shown in the above diagram, the region bounded by $|x + y| = 2$ and $|x - y| = 2$ is a square of side $\sqrt{2^2 + 2^2} = 2\sqrt{2}$.

Required area = $\left(2\sqrt{2}\right)^2 = 8$

84. For $n = 8$

$f(x) = |x| + |x + 4| + |x + 8| + \ldots + |x + 32|$

The minimum value of $f(x)$ will be when $x = -16$ when the middle term of this expression, viz. $|x + 16|$ becomes 0. (i.e. it is minimised)

We have: $f(-16) = |-16| + |-12| + |-8| + |-4| + 0 + |4| + |8| + |12| + |16| = 80$.

85. For $n = 7$, $f(x) = |x| + |x + 4| + |x + 8| + \ldots + |x + 28|$. In this case there would be two middle terms in the expression, viz. $|x + 12|$ and $|x + 16|$. The value of the expression would be minimised when the value of the sum of the middle terms is minimised.

We can see that $|x + 12| + |x + 16|$ gets minimised at $-16 \le x \le -12$. (Note that the values of the sum of the remaining six terms of the expression would remain constant whenever we take the values of x between −12 and −16.)

Thus, we have a total of five values at which the expression is minimised for $n = 7$.

86. For $n = 9, f(x) = |x| + |x + 4| + |x + 8| + \ldots + |x + 36|$. The middle terms of this expression are $|x + 16|$ and $|x + 20|$. Hence, this expression would attain its minimum value when

$-20 \leq x \leq -16$

Therefore $f(x)$ is minimum for a total of five values of x.

Minimum value of $f(x)$ can be seen at $x = -16 \rightarrow f(-16) = |-16| + |-16 + 4| + | + |-16 + 8| + |-16 + 12| + |-16 + 16| + |-16 + 20| + |-16 + 24| + |-16 + 28| + |-16 + 32| + |-16 + 36|$

$= 16 + 12 + 8 + 4 + 0 + 4 + 8 + 12 + 16 + 20$

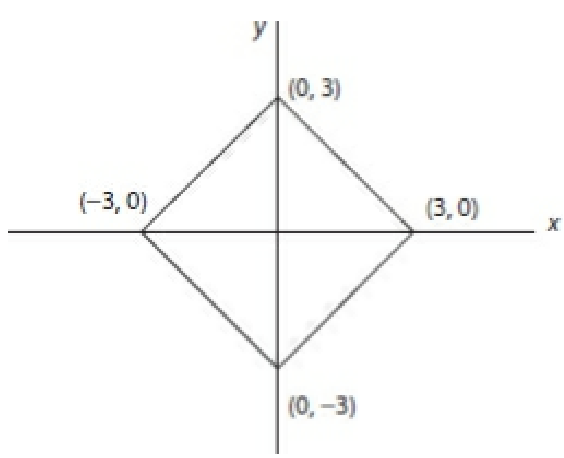
$= 100$

For $n = 9$, $f(x)$ will be minimum for $x = -16$ to -20

$\therefore f(-17) = f(-19) = 100$ minimum value of $f(x)$

Hence, Option (d) is correct.

87. The curve of $|x| + |y| = 3$ is shown below.



The curve is a square of side of length $3\sqrt{2}$ units.

Therefore, required area = $(3\sqrt{2})^2$ = 18 square units.

88. In the previous question, we found the area of curve $|x| + |y| = 3$.

$|x - a| + |y - b| = 3$ also has the same graph with the same shape and size only its center shifted to a new point (a, b). (Previous centre was (0, 0)).

Hence, area enclosed by curve $|x - 2| + |y - 3| = 3$ is same as area enclosed by curve $|x| + |y| = 3$ = 18 square units.

89. $8\{x\} = x + 2[x] \rightarrow 8\{x\} = [x] + \{x\} + 2[x] \rightarrow 7\{x\} = 3[x] \rightarrow [x] = \frac{7}{3}\{x\}$. This gives us the relationship between $[x]$ and $\{x\}$ and can also be expressed as $\{x\} = \frac{3}{7}[x]$.

Further, since $\{x\}$ is a fraction between 0 and 1, we get: $0 \le \frac{3}{7}[x] < 1 \rightarrow$
$0 \le 3[x] < 7 \rightarrow 0 \le [x] < \frac{7}{3}$

Thus, $[x]$ = 0, 1, 2 (we obtained three possible values between the limits).

Then using the relationship between $\{x\}$ and $[x]$, we get the possible values of $\{x\} = 0, \frac{3}{7}, \frac{6}{7}$ when $[x]$ is 0,1 and 2 respectively.
Since $x = [x] + \{x\}$ we get $x = 0, \frac{10}{7}, \frac{20}{7}$.

Therefore, there are two positive values of x for which the given equation is true.

90. Difference between the greatest and least value of $x = \frac{20}{7} - 0 = \frac{20}{7}$ = 2.85

91. $f(x) = \frac{4^{x-1}}{4^{x-1}+1} = \frac{4^x}{4^x+4}$

$$fog(x)=\frac{4^{2x}}{4^{2x}+4}$$

$$fog(1-x)=\frac{4^{2(1-x)}}{4^{2(1-x)}+4}$$

$$=\frac{4^2.4^{-2x}}{4^2.4^{-2x}+4}$$

$$=\frac{4^2}{4^2+4.4^{2x}}$$

$$=\frac{4}{4+4^{2x}}$$

$$fog(x)+fog(1-x)=\frac{4^{2x}}{4^{2x}+4}+\frac{4}{4+4^{2x}}$$

$$=\frac{4^{2x}+4}{4^{2x}+4}=1$$

put $x=\frac{1}{4}$

we get $fog\left(\frac{1}{4}\right)+fog\left(1-\frac{1}{4}\right)=fog\left(\frac{1}{4}\right)+fog\left(\frac{3}{4}\right)=1$

$$\Rightarrow fog\left(\frac{1}{4}\right)+fog\left(\frac{3}{4}\right)=1$$

92. Put $x = \frac{1}{2}$

$$fog\left(\frac{1}{2}\right)+fog\left(1-\frac{1}{2}\right)=1$$

$$2fog\left(\frac{1}{2}\right)=1\Rightarrow fog\left(\frac{1}{2}\right)=\frac{1}{2}$$

$$fog\left(\frac{1}{2}\right)+fog\left(\frac{1}{4}\right)+fog\left(\frac{3}{4}\right)+fog\left(\frac{1}{8}\right)$$

$$+fog\left(\frac{7}{8}\right)+fog\left(\frac{1}{16}\right)+fog\left(\frac{15}{16}\right)$$

$= \frac{1}{2} + 1 + 1 + 1 = 3\frac{1}{2} = 3.5$

93. $f(x + 2) = f(x) + 2(x + 1)$ when x is even.

$f(2) = 5$

$f(4) = f(2) + 2(2 + 1) = 5 + 6 = 11$

$f(6) = f(4) + 2(4 + 1) = 11 + 10 = 21$

Therefore, for even values of x, $\frac{x^2}{2} + 3$

$f(1) = 1$

$f(3) = 1 + 1 = 2$

$f(5) = 2 + 1 = 3$

⇒ For odd value of x, $\frac{x+1}{2}$

$$f(24) = \frac{(24)^2}{2} + 3$$

$= 291$

94. $f(14) = \frac{(14)^2}{2} + 3 = 101$

$$f(11) = \frac{11+1}{2} = 6$$

$$\left[\frac{f(14)}{f(11)}\right] = \left[\frac{101}{6}\right] = [16.83] = 16$$

95. From the solution of question 93, it is clear that only Option (c) is correct.

96. $f(f(f(f(3)))) + f(f(f(2)))$

$= f(f(f(2))) + f(f(5))$

$= f(f(5)) + f(3)$

$= f(3) + 2$

$= 2 + 2$

$= 4$

97. From the given information we can assume $F(x)$ as a sum of $P(x)$ and x, where

$P(x) = kx(x - 1)(x - 2)(x - 3)(x - 4)(x - 5)$, k is a constant.

$F(x) = kx(x - 1)(x - 2)(x - 3)(x - 4)(x - 5) + x$

$F(6) = k \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 6$

It is given $F(6) = 7$

$\therefore k \times 6! + 6 = 7$

$k \times 6! = 1$

Hence, $\frac{1}{6!}$.

Thus, $F(x) = \frac{x(x-1)(x-2)(x-3)(x-4)(x-5)}{6!} + x$.

$$F(8) = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3}{6!} + 8$$

$= 28 + 8 = 36$

98. By putting negative values of x, we can see that $F(x)$ is a decreasing function for negative integer values of x, therefore, $F(x)$ will be minimum for $x = -1$

Minimum value of $f(x) = \frac{-1 \times -2 \times -3 \times -4 \times -5 \times -6}{6!} - 1$

$\Rightarrow 1 - 1 = 0$

99. $g(x + y) = g(x)\,g(y)$

$g(1 + 1) = g(1) . g(1) = g(1)^2 = 5^2 = 25$

$g(2) = 5^2$

Similarly, $g(3) = 5^3, g(4) = 5^4, g(5) = 5^5$

$g(1) + g(2) + g(3) + g(4) + g(5) = 5 + 25 + 125 + 625 + 3125 = 3905$

100. $g(x) = 5^x$

If we put $n = 1$ in the given summation then

$$g(q+1) = \frac{1}{4}(5^4 - 125) = \frac{500}{4} = 125$$

$5^{q+1} = 125 \Rightarrow q + 1 = 3$ or $q = 2$