

Sample Question Paper - 28
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours

Maximum Marks : 40

General Instructions :

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION - A

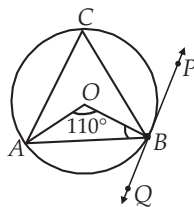
1. If mean of ten consecutive odd numbers is 120, then find the mean of first five odd numbers among them.

OR

If the mean of the following distribution is 5, then find the value of $-2p$.

x_i	2	4	6	10	$p + 5$
f_i	2	3	3	2	1

2. A wall 8 m long casts a shadow 5 m long. At the same time, a tower casts a shadow 50 m long, then find the height of tower.
3. A sphere of maximum volume is cut-out from a solid hemisphere of radius r . What is the ratio of the volume of the hemisphere to that of the cut-out sphere?
4. In figure, AB is a chord of circle, and PQ is a tangent at point B of the circle. If $\angle AOB = 110^\circ$, then find $\angle ABQ$.



OR

A chord of a circle is 12 cm in length and its distance from the centre is 8 cm. Find the length of the chord of the same circle which is at a distance of 6 cm from the centre.

5. In the following frequency distribution, if $f_2 : f_3 = 4 : 3$, and mean = 50, then find the missing frequencies f_1, f_2 and f_3 .

Class interval	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	Total
Frequency	17	f_1	f_2	f_3	19	120

6. A portion of a 45 m long tree is broken by tornado and the top struck up the ground making an angle of 30° with the ground level. Find the height of the point where the tree is broken.

SECTION - B

7. A sum of ₹ 1000 is invested at 8% simple interest per annum. Find the interest at the end of 30 years.
8. If the height of a tower and distance of the point of observation from its foot both are increased by 10%, then prove that the angle of elevation of its top remains unchanged.
9. Draw a circle of radius 5 cm. Take a point P on it. Without using the centre of the circle, draw a tangent to the circle at point P .

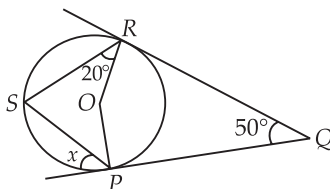
OR

Draw two concentric circles of radius 5 cm and 2 cm. Draw a pair of tangents to the smaller circle from a point on the larger circle.

10. For what value of n is the n^{th} term of the following two A.P.'s is same?
The two A.P.s are 1, 7, 13, 19, and 69, 68, 67,

SECTION - C

11. In the diagram, PQ and QR are tangents to the circle with centre O , at P and R respectively. Find the value of x .



12. A cylindrical bottle with radius 5 cm and height 14 cm has jam packed in it. The jam is spread over bread of length 11 cm and breadth 10 cm. How thick can the layer of jam be spread, if the entire jam is to be spread on 10 bread pieces?

OR

Two spheres of same metal weigh 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the new sphere.

Case Study - 1

13. Amit is preparing for his upcoming semester exam. For this, he has to practice the chapter of Quadratic Equations. So he started with factorization method. Let two linear factors of $ax^2 + bx + c$ be $(px + q)$ and $(rx + s)$.

$$\therefore ax^2 + bx + c = (px + q)(rx + s) = prx^2 + (ps + qr)x + qs.$$

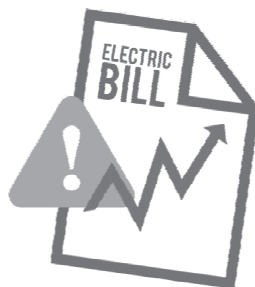
Based on the above information, answer the following questions.

- (i) Factorise $2x^2 + x - 300 = 0$ and find its roots.
- (ii) Factorise $100x^2 - 20x + 1 = 0$ and find its roots.

Case Study - 2

14. An inspector in an enforcement squad of electricity department visit to a locality of 100 families and record their monthly consumption of electricity, on the basis of family members, electronic items in the house and wastage of electricity, which is summarise in the following table.

Monthly Consumption (in kwh)	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Number of families	2	5	x	12	17	20	y	9	7	4



Based on the above information, answer the following questions.

- (i) Find the value of $x + y$.
- (ii) If the median of the above data is 525, then find the value of x .

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. Let ten consecutive odd numbers be

$$2x + 1, 2x + 3, \dots, 2x + 19.$$

$$\text{Hence, } (2x + 1) + (2x + 3) + \dots + (2x + 19) = 10 \times 120$$

$$\Rightarrow 20x + 100 = 1200 \Rightarrow x = \frac{1100}{20} = 55$$

\therefore Mean of first five odd numbers

$$= \frac{(2x + 1) + (2x + 3) + \dots + (2x + 9)}{5}$$

$$= \frac{10 \times 55 + 25}{5} = \frac{575}{5} = 115$$

OR

The frequency distribution table from the given data can be drawn as :

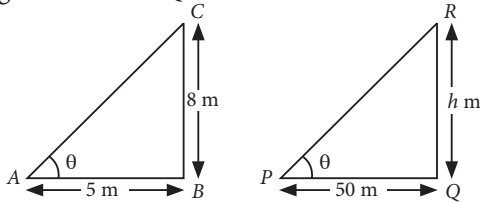
x_i	f_i	$f_i x_i$
2	2	4
4	3	12
6	3	18
10	2	20
$p + 5$	1	$p + 5$
Total	$\sum f_i = 11$	$\sum f_i x_i = p + 59$

$$\therefore \text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\Rightarrow 5 = \frac{p + 59}{11} \Rightarrow 55 = p + 59 \Rightarrow p = -4$$

$$\therefore -2p = -2 \times (-4) = 8$$

2. Let θ be the angle of elevation. Let h m be the height of tower RQ.



\therefore Angle of elevation in both cases is same.

\therefore In $\triangle ABC$ and $\triangle PQR$,

$$\tan \theta = \frac{8}{5} = \frac{h}{50} \Rightarrow h = \frac{8 \times 50}{5} = 80$$

Hence, the height of the tower is 80 m.

3. Here, radius of hemisphere = r

$$\therefore \text{Volume of hemisphere} = \frac{2}{3} \pi r^3$$

\therefore Diameter of sphere of maximum volume that is cut-out from it is r .

$$\Rightarrow \text{Radius of sphere} = \frac{r}{2}$$

$$\text{Volume of sphere} = \frac{4}{3} \pi \left(\frac{r}{2} \right)^3 = \frac{\pi}{6} r^3$$

$$\therefore \text{Required ratio} = \frac{\frac{2}{3} \pi r^3}{\frac{\pi}{6} r^3} = 4 : 1$$

4. In $\triangle OAB$, $OA = OB$ (radii of the circle)

$$\therefore \angle OAB = \angle OBA$$

$$\text{In } \triangle OAB, \angle AOB + \angle OAB + \angle OBA = 180^\circ$$

$$\Rightarrow 2\angle OBA = 180^\circ - 110^\circ \Rightarrow \angle OBA = 35^\circ$$

Since BQ is a tangent at B

$$\therefore \angle OBQ = 90^\circ \Rightarrow \angle OBA + \angle ABQ = 90^\circ$$

$$\Rightarrow 35^\circ + \angle ABQ = 90^\circ \Rightarrow \angle ABQ = 90^\circ - 35^\circ = 55^\circ$$

OR

$$AB = 12 \text{ cm}$$

$$BQ = \frac{1}{2} AB = \frac{1}{2} \times 12 = 6 \text{ cm}$$

$$OQ = 8 \text{ cm}$$

$$\text{In } \triangle OBQ, \angle Q = 90^\circ$$

$$\therefore OB^2 = OQ^2 + BQ^2 \text{ (By Pythagoras Theorem)}$$

$$\Rightarrow OB^2 = 8^2 + 6^2$$

$$\Rightarrow OB^2 = 64 + 36 = 100 \Rightarrow OB = 10 \text{ cm}$$

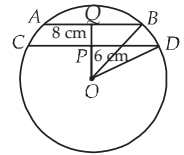
$$OB = OD \text{ [radii of the circle]}$$

$$\text{In } \triangle OPD, \angle P = 90^\circ \therefore OD^2 = OP^2 + PD^2$$

$$\Rightarrow 10^2 = 6^2 + PD^2$$

$$\Rightarrow PD^2 = 100 - 36 = 64 \Rightarrow PD = 8 \text{ cm}$$

$$\therefore \text{Length of the chord } CD = 2PD = 2 \times 8 = 16 \text{ cm}$$



5. Let $f_2 = 4x$ and $f_3 = 3x$.

$$\text{Mean} = 50$$

Class interval	Frequency (f_i)	x_i	$f_i x_i$
0-20	17	10	170
20-40	f_1	30	$30f_1$
40-60	$4x$	50	$200x$
60-80	$3x$	70	$210x$
80-100	19	90	1710
Total	120		$\sum f_i x_i = 1880 + 30f_1 + 410x$

$$\text{Here, } 17 + f_1 + 4x + 3x + 19 = 120$$

$$\Rightarrow f_1 + 7x = 84$$

$$\Rightarrow f_1 = 84 - 7x$$

...(i)

$$\text{Also, } \bar{x} = \frac{\sum f_i x_i}{\sum f_i} \Rightarrow 50 = \frac{1880 + 30f_1 + 410x}{120}$$

$$\Rightarrow 6000 = 1880 + 30f_1 + 410x$$

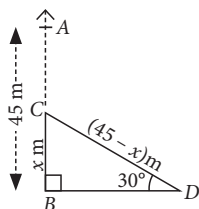
$$\Rightarrow 30f_1 + 410x = 4120 \Rightarrow 3f_1 + 41x = 412$$

$$\Rightarrow 3(84 - 7x) + 41x = 412$$

[Using (i)]

$\Rightarrow 252 - 21x + 41x = 412 \Rightarrow 20x = 160 \Rightarrow x = 8$
 When $x = 8$, (i) becomes $f_1 = 84 - 7 \times 8 = 84 - 56$
 $\Rightarrow f_1 = 28, f_2 = 4 \times 8 = 32, f_3 = 3 \times 8 = 24$
 So, $f_1 = 28, f_2 = 32, f_3 = 24$

6. Let AB is the tree which is broken at C , such that $BC = x$ m.



In $\triangle BCD$, $\frac{BC}{DC} = \sin 30^\circ$

$$\Rightarrow \frac{x}{45-x} = \frac{1}{2} \Rightarrow 2x = 45-x \Rightarrow x = 15$$

Hence, the height of the point where tree is broken is 15 m.

7. Here, $P = ₹ 1000$, $R = 8\%$ per annum

We know that

Simple interest (S.I.) = $\frac{PRT}{100}$, where T is the time in years

Putting $T = 1, 2, 3, \dots$ we get

$$\text{Interest at the end of first year} = \frac{1000 \times 8 \times 1}{100} = ₹ 80$$

$$\text{Interest at the end of two years} = \frac{1000 \times 8 \times 2}{100} = ₹ 160$$

$$\text{Interest at the end of three years} = \frac{1000 \times 8 \times 3}{100} = ₹ 240$$

And so on

Here, $a = 80, d = 80; n = 30$

$$\therefore \text{Interest at the end of 30 years} = 80 + (30-1)80$$

$$[\because a_n = a + (n-1)d]$$

$$= 80 + 2320 = ₹ 2400$$

8. Let AB be a tower of height h m and C be the point of observation at a distance of x m from the foot of the tower. Let θ be the angle of elevation of its top.

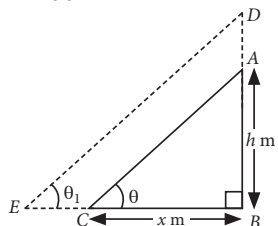
After 10% increase in both the height of the tower and the distance of the point of observation, let angle of elevation of its top becomes θ_1 .

Now, in $\triangle ABC$,

$$\tan \theta = \frac{AB}{BC} = \frac{h}{x} \quad \dots(i)$$

Height of tower after 10% increase = BD

$$= h + 10\% \text{ of } h = \frac{110}{100} h \text{ m}$$



Distance of the point of observation after 10% increase

$$= BE = x + 10\% x = \frac{110}{100} x \text{ m}$$

$$\text{In } \triangle DBE, \tan \theta_1 = \frac{BD}{BE} = \frac{\frac{110}{100} h}{\frac{110}{100} x} = \frac{h}{x} = \tan \theta \quad [\text{From (i)}]$$

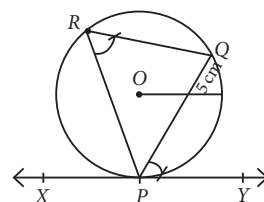
$$\Rightarrow \theta_1 = \theta$$

Hence proved.

9. Steps of construction :

Step-I : Draw a circle with O as centre and radius 5 cm.

Step-II : Take a point P on circle and draw any chord PQ through the point P on the circle.



Step-III : Take a point R on the circle and join PR and QR .

Step-IV : Construct $\angle QPY = \angle PRQ$ on the opposite side of the point R with respect to the chord PQ .

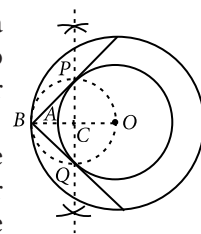
Step-V : Produce YP to X to get YPX , as the required tangent.

OR

Steps of construction :

Step-I : Take a point O and draw a circle of radius $OA = 2$ cm and also from the same point O draw another circle of radius $OB = 5$ cm.

Step-II : Bisect OB and taking the midpoint of OB as C , draw another circle of radius $OC = BC$, the circle intersect the circle having radius 2 cm at P and Q .



Step-III : Now, join BP and BQ to get the tangents from a point B on the circle of radius 5 cm.

10. Clearly, 1, 7, 13, 19, forms an A.P. with first term 1 and common difference 6. Therefore, its n^{th} term is given by

$$a_n = 1 + (n-1) \times 6 = 6n - 5$$

Also, 69, 68, 67, 66, forms an A.P. with first term 69 and common difference -1.

So, its n^{th} term is given by

$$a_n' = 69 + (n-1) \times (-1) = -n + 70$$

The two A.P.'s will have identical n^{th} terms, if $a_n = a_n'$

$$\Rightarrow 6n - 5 = -n + 70 \Rightarrow 7n = 75$$

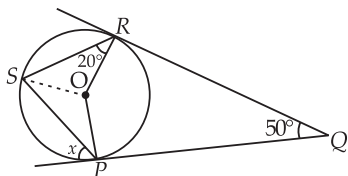
$$\Rightarrow n = 75/7, \text{ which is not a natural number.}$$

Hence, there is no value of n for which the two A.P.'s will have identical terms.

11. $\angle POR + \angle PQR = 180^\circ$

$$\therefore \angle POR = 180^\circ - 50^\circ = 130^\circ; \angle PSR = \frac{1}{2} \angle POR$$

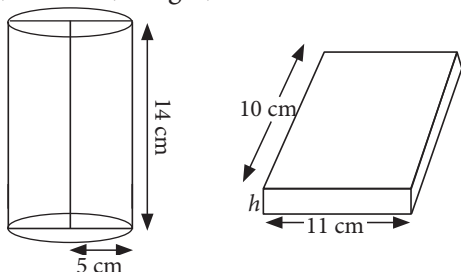
$$\therefore \angle PSR = \frac{1}{2} \times 130^\circ = 65^\circ$$



$$\begin{aligned} \Rightarrow \angle PSR &= \angle OSP + \angle OSR \\ \Rightarrow \angle PSR &= \angle OSP + 20^\circ & [\because \angle ORS = \angle OSR] \\ \Rightarrow 65^\circ &= \angle OSP + 20^\circ \\ \Rightarrow \angle OSP &= 45^\circ \\ \text{Now, } \angle OPQ &= 90^\circ & [\because PQ \text{ is a tangent}] \\ \Rightarrow x + \angle OPS &= 90^\circ \\ \Rightarrow x + \angle OSP &= 90^\circ & [\because \angle OPS = \angle OSP] \\ \Rightarrow x &= 90^\circ - 45^\circ = 45^\circ \end{aligned}$$

12. For cylindrical bottle :

Radius, $r = 5$ cm, Height, $h = 14$ cm



$$\begin{aligned} \therefore \text{Volume of the jam in cylindrical bottle} &= \pi r^2 h \\ &= \frac{22}{7} \times 5 \times 5 \times 14 = 1100 \text{ cm}^3 \end{aligned} \quad \dots(i)$$

Let the thickness of the layer of jam be h .

$$\begin{aligned} \text{Area of one bread piece} &= \text{length} \times \text{breadth} \\ &= 11 \times 10 = 110 \text{ cm}^2 \end{aligned}$$

$$\therefore \text{Area of 10 bread pieces} = 110 \times 10 = 1100 \text{ cm}^2$$

$$\therefore \text{Volume of jam spread over all bread pieces} = 1100 h \text{ cm}^3 \quad \dots(ii)$$

$$\therefore \text{Volume of jam spread over all bread pieces} = \text{Volume of jam in cylindrical bottle}$$

$$\therefore 1100h = 1100 \quad [\text{Using (i) and (ii)}]$$

$$\Rightarrow h = 1 \text{ cm}$$

OR

Given, radius of smaller sphere (r) = 3 cm

\therefore Volume of smaller sphere

$$= \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (3)^3 = \frac{4}{3} \pi (27) = 36\pi \text{ cm}^3$$

$$\therefore \text{Volume of smaller sphere} \times \text{Density} = \text{Mass}$$

$$\therefore 36\pi \times \text{Density of metal} = 1$$

$$\Rightarrow \text{Density of metal} = 1/36\pi$$

Let radius of bigger sphere be R cm.

$$\therefore \text{Volume of bigger sphere} \times \text{density} = \text{Mass}$$

$$\Rightarrow \frac{4}{3} \pi (R)^3 \times \frac{1}{36\pi} = 7$$

$$\Rightarrow R^3 = \frac{7 \times 36 \times 3}{4} = 7 \times 9 \times 3 = 189 \quad \dots(i)$$

$$\therefore \text{Volume of new sphere} = \text{Volume of smaller sphere} + \text{Volume of bigger sphere}$$

$$\Rightarrow \frac{4}{3} \pi (R')^3 = \frac{4}{3} \pi r^3 + \frac{4}{3} \pi R^3$$

[Here R' is the radius of new sphere]

$$\Rightarrow \frac{4}{3} \pi (R')^3 = \frac{4}{3} \pi (3)^3 + \frac{4}{3} \pi (189)$$

$$\Rightarrow (R')^3 = 27 + 189 \Rightarrow (R')^3 = 216 \Rightarrow R' = 6 \text{ cm}$$

$$\therefore \text{Diameter of new sphere} = 2 \times 6 = 12 \text{ cm}$$

13. (i) We have, $2x^2 + x - 300 = 0$

$$\Rightarrow 2x^2 - 24x + 25x - 300 = 0$$

$$\Rightarrow 2x(x - 12) + 25(x - 12) = 0$$

$$\Rightarrow (x - 12)(2x + 25) = 0$$

$$\Rightarrow x = 12 \text{ or } x = \frac{-25}{2}$$

(ii) We have, $100x^2 - 20x + 1 = 0$

$$\Rightarrow 100x^2 - 10x - 10x + 1 = 0$$

$$\Rightarrow 10x(10x - 1) - 1(10x - 1) = 0$$

$$\Rightarrow (10x - 1)(10x - 1) = 0$$

$$\Rightarrow 10x - 1 = 0 \text{ or } 10x - 1 = 0$$

$$\Rightarrow x = \frac{1}{10} \text{ or } \frac{1}{10}$$

14. We have the following table :

Class interval	Frequency	Cumulative frequency
0-100	2	2
100-200	5	7
200-300	x	$7 + x$
300-400	12	$19 + x$
400-500	17	$36 + x$
500-600	20	$56 + x$
600-700	y	$56 + x + y$
700-800	9	$65 + x + y$
800-900	7	$72 + x + y$
900-1000	4	$76 + x + y$
Total	$76 + x + y$	

(i) Here, it is given that total frequency = 100

$$\therefore 76 + x + y = 100 \Rightarrow x + y = 24$$

$$(ii) \text{ Here, } \frac{N}{2} = \frac{100}{2} = 50$$

Also, median = 525

\therefore Median class is 500-600.

$$\text{Now, median} = l + \left(\frac{N/2 - c.f.}{f} \right) \times h$$

$$\Rightarrow 525 = 500 + \left(\frac{50 - (36 + x)}{20} \right) \times 100$$

$$\Rightarrow 5 = 50 - 36 - x \Rightarrow x = 9$$