Algorithms Test 3

Number of Questions: 25 **Section Marks: 30**

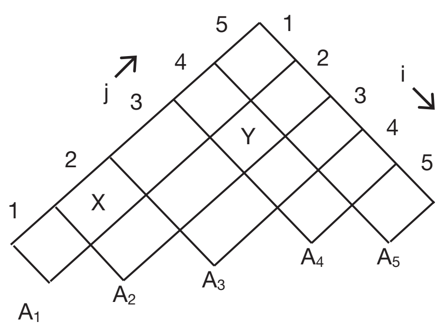
Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. Given the following input 564, 816, 726, 903, 1321, 1345, 1498 and the Hash function is $h(k) = k \bmod 3$. Which of the following statements are CORRECT?
   I. 564, 816, 726, 903 will hash to same Location.
   II. 1321, 1345, 1498 will hash to same Location.
   III. 816, 1345, 903 will hash to same Location.
   (A) I and III (B) I and II
   (C) II and III (D) I, II and III

2. Let M be an integer greater than 1. Which of the following represents the order of growth of the expression $\sum_{i=1}^{n} M^i$ as a function of 'n'?
   (A) $\theta(n^M)$ (B) $\theta(M^{2n+1})$
   (C) $\theta(M^{n \log n})$ (D) $\theta(M^n)$

Common Data For Questions 3 and 4:

To compute the Matrix product $M_1 M_2$, where M_1 has 'p' rows and 'q' columns and M_2 has 'q' rows and 'r' columns, takes time proportional to 'pqr', and result is a matrix of 'p' rows and 'r' columns. Consider the given 5 matrices, A_1, A_2, A_3, A_4 and A_5 and their dimensions are $P_0 \times P_1$, $P_1 \times P_2$, $P_2 \times P_3$, $P_3 \times P_4$, $P_4 \times P_5$, respectively. The values are $P_0 = 10$, $P_1 = 5$, $P_2 = 15$, $P_3 = 20$, $P_4 = 40$, $P_5 = 25$.



3. MATRIX-CHAIN-ORDER computes the rows from bottom to top and from left to right, The value of X (where $X = M[1, 2]$) is ________
   (A) 600 (B) 750
   (C) 800 (D) 900

4. The value of Y (where $Y = M[2, 4]$) is ________.
   (A) 1500 (B) 5500
   (C) 15000 (D) None of the above

5. Which of the following cannot be the time complexity of Quick sort algorithm, under any of the Average, Best, Worst cases?
   (A) $O(n \log n)$ (B) $O(n^2)$
   (C) $O(n^3)$ (D) $O(\log n)$

6. Consider the following:
   I. If $f(n) = O(g(n))$ and $g(n) = O(h(n))$ then $f(n) = \Omega(h(n))$
   II. The asymptotically tight upper bound for $T(n) = T(n-2) + 1$ is $O(\log n)$
   Which of the following is correct?
   (A) I–TRUE, II–TRUE (B) I–TRUE, II–False
   (C) I–False, II–TRUE (D) I–False, II–False

7. Let
   $$f(n) = 6n + 8n^2 + 200n^3,$$
   $$g(n) = n^2 \log n$$
   Which of the following is valid?
   (A) $f(n) = O(g(n))$ (B) $f(n) = \theta(g(n))$
   (C) $f(n) = \Omega(g(n))$ (D) $g(n) = \Omega(f(n))$

8. If $f(n) = n \log n$ then which of the following is FALSE?
   (A) $f(n) = O(n^2 \log n)$
   (B) $f(n) = \theta(n \log n)$
   (C) $f(n) = \Omega(n^2)$
   (D) $f(n) = \Omega(\log n * \log n)$

9. What is the average time complexity of the sequential search algorithm where the searched item is sequentially compared to each element in the list of 'n' elements?
   (A) $\frac{n}{2}$ (B) $\log n$
   (C) $\frac{n+1}{2}$ (D) $\frac{n-1}{2}$

10. Consider the given data:
    — Let n be the number of elements
    — Number of levels used in sorting: $O(\log n)$
    — At each level $O(n)$ amount of work
    The given data is related to:
    (A) Heap sort (B) Merge sort
    (C) Selection sort (D) Bubble sort

11. Consider a graph, with all distinct Edge weights, which of the following is TRUE?
    (A) The shortest path is unique between every pair of vertices
    (B) The maximal matching is unique
    (C) The minimum spanning tree is unique
    (D) All the above

12. The Hash function $h(k) = k \bmod 7$ with Linear probing is used to insert the keys 37, 38, 72, 68, 98, 11, 74 into hash table indexed (0 – 6), The location of Key 74 is ________.
    (A) 1 (B) 2
    (C) 3 (D) 4

13. Given an undirected graph $G = (V, E)$ and a positive integer 'K', does 'G' have 'K' vertices that form

a complete sub graph and if it does, then what is the minimum value of 'K'?

(A) 2 (B) 3
(C) 4 (D) None of the above

14. In the Build-Heap algorithm, the Heapify routine is invoked 'n' times. This indicates that Build-Heap Complexity is ______.

(A) $O(\log n)$ (B) $O(n)$
(C) $O(n \log n)$ (D) $O(n^2)$

15. Give asymptotically tight Big-O bounds for the following recurrence relations. (By using Iterative substitution method)

$T(n) = T(n - 2) + 1$ if $n > 2$
$T(n) = 1$ if $n \leq 2$

(A) $O(n \log n)$ (B) $O(n)$
(C) $O(n^2)$ (D) $O(\log n)$

16. Consider the given code:

```
public static int f2(int n){
int x = 0;
for (int i= 0; i < n; i++)
for (int j = 0; j < i * i; j++)
x++;
return x;
}
```

What is the order of growth of the above code?

(A) $O(\log n)$ (B) $O(n^2)$
(C) $O(n^3)$ (D) $O(2^n)$

17. Consider the following:

I. $(3n)! = O(n!^3)$ II. $\log(n!) = \theta(n \log n)$

Which of the following is correct?

(A) I–False, II–False (B) I–False, II–TRUE
(C) I–TRUE, II–False (D) I–TRUE, II–TRUE

18. The Floyd-Warshall all pairs shortest path algorithm for finding the shortest distances between nodes in a graph is an example of:

(A) An iterative based divide - and - conquer
(B) A Dynamic programming formulation
(C) A greedy algorithm
(D) A recursive based divide - and - conquer

19. Which of the following is not having $O(n^2)$ complexity?

(A) $n + 2000n$ (B) $n^{1.999}$
(C) $10^6 n + 2^6 n$ (D) $\frac{n^3}{\sqrt{n}}$

20. Consider the given two strings str1 and str2.

Let str1 = < A B R A C A D A B R O A >
str2 = < Y A B B A D A B B A D O O B A >

Then the length of longest common subsequence would be ___________.

(A) 5 (B) 6
(C) 7 (D) 8

21. What is the computational complexity of the following piece of code:

```
for (i = n; i > 0; i/= 2){
for (int j = 1; j < n; j * = 2){
for (int k = 0; k < n; k + = 2){
      sum + = (i + j * k);
      }
   }
}
```

(A) $O(n^3)$
(B) $O(n^2 \log n)$
(C) $O(\log n * \log n * \log n)$
(D) $O(n * \log n * \log n)$

22. Consider the modified binary search algorithm so that it splits the input not into 2 sets of sizes equal sizes, but into three sets of sizes approximately one-third. What is the recurrence for this ternary search algorithm?

(A) $T(n) = T\left(\frac{n}{2}\right) + T(n-2) + C$

(B) $T(n) = T\left(\frac{n}{3}\right) + C$

(C) $T(n) = T\left(\frac{3n}{4}\right) + T\left(\frac{n}{4}\right) + C$

(D) $T(n) = T\left(\frac{n}{3}\right) + \log n$

23. Sort the following growth rate classes in increasing order of time complexity:

Exponential, quadratic, logarithmic, cubic, and factorial.

(A) Logarithmic, quadratic, cubic, exponential, factorial
(B) Logarithmic, quadratic, cubic, factorial, exponential
(C) Quadratic, cubic, logarithmic, exponential, factorial
(D) Quadratic, cubic, logarithmic, factorial, exponential

24. Determine if 'x' is present in an array of n-elements:

```
for i = 0 to n
if (a[i] = x) return TRUE
else
return false
```

What is the worst case, best case time complexities and space complexity respectively?

(A) $O(n)$, $O(1)$, $O(n)$
(B) $O(n)$, $O(1)$, $O(1)$
(C) $O(n)$, $O(\log n)$, $O(1)$
(D) $O(\log n)$, $O(\log n)$, $O(n)$

25. What is the space complexity of following sorting algorithms respectively?

I. Quick sort II. Merge sort
III. Selection sort IV. Insertion sort

(A) $O(n)$, $O(n)$, $O(1)$, $O(1)$
(B) $O(1)$, $O(n)$, $O(1)$, $O(n)$
(C) $O(1)$, $O(n)$, $O(1)$, $O(1)$
(D) $O(1)$, $O(n)$, $O(n)$, $O(1)$

Answer Keys

1. B	2. D	3. B	4. B	5. D	6. D	7. C	8. C	9. C	10. B
11. C	12. A	13. A	14. D	15. B	16. C	17. A	18. B	19. D	20. D
21. D	22. B	23. A	24. B	25. C					

Hints and Explanations

1. $h(k) = k \bmod 3$
564 mod 3 = 0
816 mod 3 = 0
726 mod 3 = 0
903 mod 3 = 0
1321 mod 3 = 1
1345 mod 3 = 1
1498 mod 3 = 1
564, 816, 726, 903 will hash to same location,
1321, 1345, 1498 will hash to same location.
Statement III is not TRUE. Choice (B)

2. Let M value be '3'

$$\sum_{i=1}^{n} 3^i = 3^1 + 3^2 + 3^3 ... + 3^n$$

The sequence is in Geometric Progression.
First term $= 3 = a$

$$r = \frac{3^2}{3} = 3$$

As $r > 1 \Rightarrow \text{sum} = a\frac{(r^n - 1)}{r-1}$

$$3 \cdot \frac{(3^n - 1)}{3-1} = \frac{3(3^n - 1)}{2}$$

$$= \frac{3.3^n - 3*1}{2} = \frac{3^{n+1} - 3^1}{2}$$

$= 3^{n+1} \Rightarrow 3^n$ (Ignore constants in Asymptotic Notations)
$\therefore$ $\theta(M^n)$ is the order of growth of given expression. Choice (D)

3. Computing the matrix product $A_{i....k}\, A_{k+1....j}$ takes $P_{i...l}\, P_k\, P_j$ scalar multiplications.
$M[i, j]$

$$= \begin{cases} 0 & \text{if } i = j \\ \min\limits_{i \le k < j} \{M[i,k] + M[k+1, j] + P_{i-1}P_kP_j, & \text{if } i < j \end{cases}$$

$X = M[1, 2]$
$i = 1, j = 2$
$\therefore$ $k = 1$ $(\because i \le k < j)$
$P_0 = 10, P_1 = 5, P_2 = 15, P_3 = 20,$
$P_4 = 40, P_5 = 25$
$M[1, 1] = 0$
$M[2, 2] = 0$
$M[3, 3] = 0$
$M[4, 4] = 0$
$M[5, 5] = 0$

$$M[1, 2] = \min_{i \le k < j} [M[i, k] + M[k+1, j] + P_{i-1}P_kP_j$$

$$K = 1 \Rightarrow M[1, 2] = M[1, 1] + M[2, 2] + P_0\, P_1\, P_2$$

$0 + 0 + 10 \times 5 \times 15$
$= 750$ Choice (B)

4. $M[2, 4] \rightarrow$ The possible values for 'K' are 2, 3.
(1) $\underset{k=2}{M[2,4]} = M[2, 2] + M[3, 4] + P_1\, P_2\, P_4$
(2) $\underset{k=3}{M[2,4]} = M[2, 3] + M[4, 4] + P_1\, P_3\, P_4$

We need to get the values of $M[3, 4]$ and $M[2, 3]$
$\underset{k=3}{M[3,4]} = M[3, 3] + M[4, 4] + P_2\, P_3\, P_4$
$0 + 0 + 15 \times 20 \times 40 = 12000$
$\underset{k=2}{M[2,3]} = M[2, 2] + M[3, 3] + P_1\, P_2\, P_3$
$= 0 + 0 + 5 \times 15 \times 20$
$= 1500$
Choose min of (1) and (2)
$\underset{k=2}{M[2,4]} = M[2, 2] + M[3, 4] + P_1\, P_2\, P_4$
$0 + 12000 + 5 \times 15 \times 40$
$= 12000 + 3000 = 15000$
$\underset{k=3}{M[2,4]} = M[2, 3] + M[4, 4] + P_1\, P_3\, P_4$
$= 1500 + 0 + 5 \times 20 \times 40$
$= 1500 + 4000 = 5500$
$\therefore$ $M[2, 4] = \min\{15000, 5500\} = 5500$
Choice (B)

5. Average and Best case time complexities are $O(n \log n)$ and $O(n \log n)$
Worst case time complexity is $O(n^2)$
Since $(n \log n)$ and n^2 are less than n^3, $O(n^3)$ is also valid. Choice (D)

6. I. $f(n) = O(g(n))$ and $g(n) = O(h(n))$ then
$f(n) = \Omega(h(n))$
Lets assume
$f(n) = n$
$g(n) = n^2$
$h(n) = n^3$
$f(n) \le g(n)$
$\Rightarrow$ $n \le n^2$
$g(n) \le h(n) \Rightarrow n^2 \le n^3$
Then $f(n) \ge h(n)$
$\Rightarrow$ $n \ge n^3$ (false)

II. $T(n) = T(n-2) + 1$

Gives $O(n)$ time complexity (False) Choice (D)

7. $f(n) = 6n + 8n^2 + 200n^3 \Rightarrow n^3$

$g(n) = n^2 \log n$

Option (A):

$n^3 \le c * n^2 \log n$ (false)

Option (B):

$n^3 = c * n^2 \log n$ (false)

Option (C):

$n^3 \ge c * n^2 \log n$ (TRUE) Choice (C)

8. Option (A):

$n \log n \le c * n^2 \log n$ (TRUE)

Option (B):

$n \log n = c * n \log n$ (TRUE)

Option (C):

$n \log n \ge c * n^2$ (False) Choice (C)

9. In an average case, the probability that 'X' is in the K^{th} array slot is $1/n$, hence the average number of comparisons is

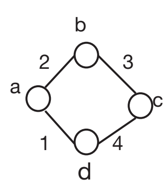
$$\sum_{i=1}^{n}\left(K \times \frac{1}{n}\right) = \frac{1}{n} \times \sum_{k=1}^{n} K$$

$$= \frac{1}{n} * \frac{n(n+1)}{2} = \frac{n+1}{2}$$ Choice (C)

10. In the worst, average, best cases, merge sort procedure will have $O(\log n)$ levels.

In each level $O(n)$ work is required. Choice (B)

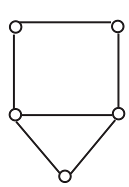
11. Option (A):



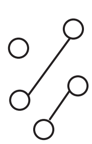
∴ There are 2 shortest paths From 'a' to 'c'

1. $a - b - c$
2. $a - d - c$

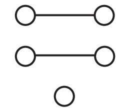
Option (B):



Maximal matching 1



Maximal matching 2



Option (C)

For any graph, there will be unique spanning tree.

Choice (C)

12. Hash function $h(k) = k \bmod 7$

Given elements are: 37, 38, 72, 68, 98, 11, 74

37 mod 7 = 2

38 mod 7 = 3

72 mod 7 = 2

68 mod 7 = 5

98 mod 7 = 0

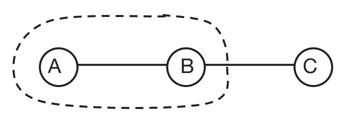
11 mod 7 = 4

74 mod 7 = 4

98	0
74	1
37	2
38	3
72	4
68	5
11	6

Choice (A)

13.



It is a complete sub graph, with $K = 2$. Choice (A)

14. Heapify routine takes $O(\log n)$ time, if it is invoked 'n' times then Build Heap takes $n * \log n \Rightarrow O(n \log n)$.

Choice (D)

15. Given Recurrence Relation:

$T(n) = T(n-2) + 1$

Assume $n = 16$

$T(16) = T(14) + 1$

$T(14) = T(12) + 1$

$T(12) = T(10) + 1$

$T(10) = T(8) + 1$

$T(8) = T(6) + 1$

$T(6) = T(4) + 1$

$T(4) = T(2) + 1$

$T(2) = 1$

$T(4) = 2$

$T(6) = 3$

$T(8) = 4$

$T(10) = 5$

$T(12) = 6$

$T(14) = 7$

$T(16) = 8$

$\Rightarrow \frac{n}{2} \Rightarrow O\left(\frac{n}{2}\right) \Rightarrow O(n)$ Choice (B)

16. Each iteration of the inner loop is quadratic in the outer loop variable. The simplest way to do this is to realize we are just summing $\text{S}i^2$, which will just be $O(n^3)$, if we use the integration trick. Choice (C)

17. Let us assume some value for n

$n = 5$

I. $(3 \times 5)! = 15! =$ very large number

$(5!)^3 = 120^3 = 1728000$

$(3n)! < O(n!^3)$

I–false

II. $\log(n!) = \theta(n \log n)$

lets take $n = 8$

$\log(n!) = \log(8!) = \log(40320) = 15.299$

$n \log n = 24$

$\log(n!) < (n \log n)$

II–false Choice (A)

18. The Floyd-warshall all pairs shortest path algorithm in a graph is an example of a dynamic programming formulation.

Choice (B)

19. Option (A)

$n + 2000n \leq c * n^2$

$\Rightarrow \quad n \leq c * n^2$ (TRUE)

Option (B)

$n^{1.999} \leq c * n^2$ (TRUE)

Option (C)

$10^6 n + 2^6 n \leq c * n^2$

$n \leq c * n^2$ (TRUE)

Option (D)

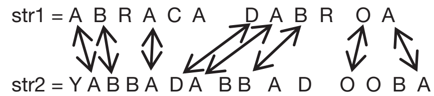
$\frac{n^3}{\sqrt{n}} \leq c * n^2$ (False)

$n^{2.5} \leq c * n^2$(False)

$\therefore$ for a very large value of 'n'

Choice (D)

20.



The longest common subsequence would be A B A D A B O A.

The length is '8'. Choice (D)

21. In the outer for loop, the variable 'i' keeps halving, so it goes a round ($\log n$) times. For each 'i' next loop goes round also ($\log n$) times, because of doubling the variable j. The inner most loop by 'k' goes round $\frac{n}{2}$ times.

Loops are nested, so the bounds may be multiplied to give that the algorithm is $O(n * \log n * \log n)$

Choice (D)

22. By analogy, we divide the elements instead of 2 subsets, we divide them into three subsets.

$\therefore \quad T(n) = T\left(\frac{n}{3}\right) + c$ Choice (B)

23. logarithmic $\rightarrow \log n$

cubic $\rightarrow n^3$

quadratic $\rightarrow n^2$

Exponential $\rightarrow 2^n$

Factorial $\rightarrow n!$

$\log n < n^2 < n^3 < 2^n < n!$

Logarithmic, quadratic, cubic, exponential, Factorial.

Choice (A)

24. The procedure given in problem performs linear search,

Worst case $\Rightarrow O(n)$

Best case $\Rightarrow O(1)$

We do not require extra space to search for an element in an array, so space complexity is $O(1)$. Choice (B)

25. Quick sort, selection sort, insertion sort are 'In-place' algorithms means they do not take extra space.

$\therefore O(1), O(1), O(1)$

Merge sort is Not In-Place algorithm.

It needs extra space

$\therefore O(n)$. Choice (C)